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Mortgage Choice: What's the Point?

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Mortgage Choice: What's the Point?

ABSTRACT

This paper develops a general equilibrium model of mortgage lending, combining self-selection theory with option pricing. We construct a separating equilibrium, in which lenders offer a menu of prepayable, fixed rate mortgage contracts, differing in their tradeoff between coupon rate and points (prepaid interest). Borrowers select the optimal contract from the menu, revealing their mobility via their choice of loan, and lenders make zero profit on each loan taken out. Such a separating equilibrium can only exist if borrowers face frictions, such as refinancing costs. Our model provides an explanation for the large menus of mortgages typically encountered by potential borrowers, as well as for the prepayment options that are embedded in mortgage contracts, despite the significant deadweight costs associated with refinancing. We also show that the recent proliferation of loans with many different horizons represents an alternative means of persuading borrowers to self-select, with lower deadweight costs. Finally, our model suggests that the menu of contracts available at the time of origination should be an important predictor of future prepayment. Most commonly used prepayment models, which do not take this into account, are therefore misspecified, leading to errors in pricing and hedging mortgages and mortgage-backed securities.

1 Introduction

The mortgage market is of crucial importance to the U.S. economy. The family home represents by far the largest single investment for millions of Americans. Approximately 75% of new residential mortgages are securitized,[1] and there are roughly $2.4 trillion of outstanding mortgage-backed securities of all types, with billions of dollars in daily trading.[2] Recent multimillion dollar losses by firms engaged in mortgage related trading and hedging, such as the collapse of Askin Capital Management in April 1994, underline the need for a fuller understanding of pricing in this market.

Mortgages can be thought of as being roughly equivalent to a coupon bond (the scheduled stream of monthly payments) minus a call option on that bond (mortgage holders' right to prepay their loans early). Valuing and hedging mortgages and mortgage-backed securities requires modeling both interest rates and the prepayment behavior of mortgage holders.[3] Current valuation models consider only the behavior of the borrower *after* the loan has been taken out – the initial choice of the loan, and the objectives of lenders, are typically ignored. However, these are far from trivial issues. Lenders typically present potential borrowers with a huge array of loans from which to choose. Not only are there different types of loan (e.g. fixed vs. adjustable rate), but even within a single type there are loans with many different combinations of interest rate and points (prepaid interest). As an illustration, Table 1 shows a sample of the loans available one day in October 1993 from one of the largest U.S. mortgage lenders.

This paper develops a general equilibrium model of mortgage lending, simultaneously considering the objectives of *both* mortgage lenders and borrowers, that suggests explanations

[1]The main issuers of mortgage-backed securities (MBS) are the Government National Mortgage Association (GNMA), Federal Home Loan Mortgage Corporation (FHLMC), and Federal National Mortgage Association (FNMA). A mortgage issuer either sells loans to an agency or swaps them for mortgage-backed securities. The loans are packaged into pools that conform to agency requirements for quality, size, mortgage type and property type, and securities are issued backed by these pools. Security holders receive the payments from the underlying mortgages, less a servicing spread.

[2]Source: Inside Mortgage Securities, January 27 1995.

[3]Default behavior may also be modeled. See, for example, Kau *et al.* (1992).

for several features of existing mortgage markets and contracts. Among the questions we address are:

1. Why do lenders offer large menus of contracts, differing only in their tradeoff between interest rate and points?

2. Why do mortgages contain prepayment options, despite the significant cost and inconvenience associated with refinancing?

3. Why have lenders recently started offering loans with many different maturities?

4. What does the initial choice of contract tell us about a borrower's likely future refinancing behavior?

In our model, multiple classes of borrower, differing only in how long they expect to remain in their current home, select loans from a menu of fixed rate, prepayable contracts offered by lenders. Lenders in turn make zero expected profit on each loan that is taken out. Borrowers prepay their loans either in order to move, or because interest rates have fallen and it is optimal to refinance. We show that offering a menu of loans with differing combinations of interest rate and points can provide lenders with a mechanism for learning private information about potential borrowers' mobility. In equilibrium, the sooner a borrower expects to move, the higher the periodic interest rate, and the lower the points, on the loan taken out.[4] This separation only works if borrowers face some form of friction. In our model, the friction is a transaction cost payable by the borrower on refinancing. In the presence of this friction, the points/coupon choice serves as a self-selection mechanism. This justifies the presence of the prepayment options embedded in mortgage contracts, despite the significant deadweight costs associated with these options.

[4]This agrees with the informal rule for mortgage choice advocated by most mortgage lenders. However, while the intuition seems obvious, it may break down when we try to impose an equilibrium condition [see Section 3].

In addition to being able to induce self-selection via a tradeoff between points and coupon rate, we show that lenders can also induce borrowers to self-select according to mobility by offering them a menu of loans with different maturities. If it is costly to refinance, long term borrowers prefer not to take out a succession of short term loans. We find that separation using loan maturity results in lower deadweight costs than separation using points. This provides an explanation for lenders' recent shift towards offering loan contracts with many different maturities.

Finally, our results suggest that the number of points paid on taking out a loan, and even more generally, the entire selection of loans available at the time the loan was made, should be helpful in predicting the future prepayment of a mortgage borrower. Prepayment models that fail to take this into account may be misspecified, leading to the possibility of significant errors in pricing and hedging mortgages and mortgage-backed securities.

The paper is organized as follows. Section 2 describes the theory and practice of mortgage valuation, emphasizing the importance of modeling the prepayment behavior of mortgage holders, and summarizing previous attempts to explain the existence of points. Section 3 lays out the model, describing lender and borrower objectives. Section 4 describes the detailed implementation of the model, and constructs a separating equilibrium in which lenders offer loans differing only in their combination of points and coupon rate. Section 5 extends the model by allowing lenders to offer loans that differ not only in points vs. coupon. but also in maturity. Section 6 summarizes our results.

2 Valuing Mortgages and Mortgage-Backed Securities

Mortgage-backed securities (MBS) are claims on the cash flows from mortgages which have been pooled together and packaged as a financial asset. Investors receive all payments (principal plus interest) made by mortgage holders in a particular pool, less a servicing fee. For most mortgage-backed securities, the payments are guaranteed by government or private agencies. In the case of a household default, the agency pays the remaining principal of that mortgage in the pool. Default (from the investor's perspective) is equivalent to prepayment.

The pricing of mortgages and mortgage-backed securities is essentially an issue of estimating the magnitude and timing of their cash flows under different economic scenarios. These cash flows are in turn determined by borrowers' prepayment behavior. Borrowers prepay their loans when they move. In addition, they have an option to prepay their existing mortgage and refinance their property should interest rates fall. Thus, a mortgage or MBS is roughly equivalent to a default-free coupon-bearing bond (with monthly payments equal to the scheduled interest plus principal on the loan) plus a short position in a call option on that bond (with an exercise price of par). Mortgage holders become more likely to exercise their prepayment options as interest rates fall further and further below the coupon rate on their current loan.

Pricing and hedging MBS therefore involves modeling (i) the dynamics of the term structure of interest rates, and (ii) the prepayment behavior of mortgage holders. In one of the earliest academic studies in this area, Dunn and McConnell (1981a,b) apply an option pricing approach to the valuation of MBS, assuming interest rates to be described by the Cox, Ingersoll and Ross (1985) model. Their approach determines the optimal prepayment strategy as part of the MBS valuation process. The Dunn and McConnell model, however, has two problems. First, it implies that the price of an MBS can never exceed par.[5] Second, it implies that all mortgage holders refinance simultaneously, as soon as interest rates fall below a critical level.[6] To correct the first problem, Timmis (1985), Dunn and Spatt (1986) and Johnston and Van Drunen (1988) add transaction costs which must be paid by borrowers should they decide to refinance their loans early. To relax the second restriction, Stanton (1995) extends these models by making transaction costs heterogeneous across mortgage holders, and allowing borrowers to make prepayment decisions only at discrete intervals. This leads to prepayment behavior which exhibits most of the features found in real life,

[5] MBS prices are often well above par.

[6] In reality, after interest rates fall, prepayment occurs over a long period of time.

such as *burnout*.[7] An alternative approach, used by Schwartz and Torous (1989) and many Wall Street firms, is to value MBS using Monte Carlo simulation. Expected prepayment is written as some function of the entire history of interest rates (and often other, endogenous, variables such as cumulative prepayment to date), then large numbers of interest rate paths are simulated under a risk-adjusted interest rate process.[8] The sum of discounted cash flows along each simulated path is calculated, and the average of these is used as an estimate of the asset's value. While, in principle, any of these models could include variables summarizing the economic environment at the time the mortgages were issued, this is typically *not* done in practice.

2.1 Points

Several explanations have been advanced for the existence of points. They are sometimes regarded as compensation for the costs associated with originating a mortgage. Dunn and McConnell (1981b) suggest that points serve to pay for the prepayment option embedded in fixed rate mortgages. However, in both cases, why should the payment be made in the form of points, rather than via higher interest rates? As an alternative explanation, Kau and Keenan (1987) suggest tax reasons for the existence of points. They can also be regarded as a means of imposing a prepayment penalty [see, for example, Leroy (1994)]. However, none of these stories explains why we should see such a large number of different combinations of rates and points.

A different explanation for the existence of points was given by Dunn and Spatt (1988). They suggest that borrowers who plan to prepay their loans soon ought to take out loans with a high periodic interest rate and low points, whereas those who plan not to prepay (except

[7]For substantially prepaid pools, there is a tendency for low future prepayments. The intuition is that if mortgage holders prepay at different speeds, the more of a pool that has prepaid, the fewer fast prepayers are left, and so the slower the prepayment speed of the remaining pool.

[8]This is not the true interest rate process, but a process with an appropriately modified drift. It can be shown that the value of any interest rate dependent asset equals the sum of the expected discounted cash flows from the asset under this modified process [see, for example, Ingersoll (1987) for details].

possibly for interest rate related reasons) should take out loans with higher points and a lower periodic interest rate. While this intuition is attractive, previous attempts to construct a formal model which yields this sort of separation have met with limited success. Chari and Jagannathan (1989) develop a model in which two borrowers with different expected times to their next move choose different loans. Unfortunately, their model produces the counterintuitive result that borrowers who expect to move soon choose a loan with high points and a low interest rate, while borrowers who expect to wait longer before they move choose a loan with low points and a high coupon rate. Leroy (1994) develops a model in which two classes of borrower self-select into different loans, with the longer term borrowers selecting loans with higher points and a lower coupon rate. However, this model is restricted to only two classes of borrower. Brueckner (1994) also develops a simple model which exhibits separation by mobility, but his model relies on differences in preferences which essentially amount to the borrowers facing different discount rates, and hence preferring different time patterns of cash flows. Yang (1992) constructs a loan schedule which induces self-selection by multiple classes of borrower, but he allows the (non-competitive) lender to make arbitrarily large profits.

In the next section, we describe a model which overcomes these limitations. It predicts that borrowers with higher mobility will take out loans with lower points and higher coupon rate than borrowers with lower mobility. It can handle multiple classes of borrower. It is an equilibrium model, in which competitive lenders make zero expected profit on each type of loan offered and have no incentive to deviate from the equilibrium set of loans. In addition, it allows for a realistic specification for the dynamics of the term structure of interest rates, thus yielding realistic quantitative results as well as qualitative intuition.

3 The Model

3.1 Borrowers and Prepayment

We assume that all borrowers are outwardly identical, and differ only in their mobility, measured by time τ. Before time τ, the borrower knows with certainty that he or she will not move from the current house. After time τ, the borrower may move. We assume that moving is a Poisson process, governed by the parameter λ.[9] The expected time until the mortgage holder moves is thus $\tau + 1/\lambda$. The higher the value of λ, the more likely the borrower is to move within a short time after date τ. In the limit, as $\lambda \to \infty$, the borrower knows with certainty that moving will occur exactly at time τ. On moving, the borrower must repay the outstanding balance on the mortgage. In addition to moving, borrowers may also decide to refinance their mortgages, if interest rates have fallen sufficiently since the original loan was taken out. Prior to time τ, any prepayment will be associated with refinancing. Subsequent to time τ, prepayment may accompany either refinancing or moving.

To determine when borrowers find it optimal to refinance, it is convenient to split their liabilities into two parts. They owe the scheduled stream of payments on their loans, and also own a call option giving them the right at any time to receive an amount equal to each of the remaining mortgage payments, in exchange for payment of the remaining principal on the loan. We assume that, in addition to paying back the remaining principal on their loan, borrowers also face a proportional transaction cost X on refinancing. This represents the direct monetary costs of refinancing (appraisal fees, title search etc.) as well as non-monetary costs (representing, for example, the inconvenience and time involved in the refinancing process). Thus, if the remaining principal amount outstanding at time t is F_t, then on refinancing the borrower has to pay out

$$F_t(1 + X).$$

[9] In other words, after time τ, the probability of moving in the next instant of length δt is approximately $\lambda\,\delta t$.

Lenders are paid only F_t on refinancing, since they do not receive the transaction costs paid by borrowers.

It is common in the self-selection literature [see, for example, Rothschild and Stiglitz (1976)] to have different preferences for different agents in the economy. This approach was followed in a mortgage context by Brueckner (1994), who assumed that borrowers were risk-averse, and lenders risk-neutral. However, this ignores borrowers' ability to undertake additional financial transactions to improve their utility. As long as borrowers have unlimited access to capital markets, they rank mortgages by *market value*, regardless of their individual utility functions, since this puts them on the highest possible budget line.[10] We therefore assume borrowers act to minimize the market value of their mortgage liability in two ways.[11] First, they select the best loan from the set of available contracts. Second, having taken out a particular loan, they follow the optimal prepayment strategy for that loan, subject to their transaction costs. We assume that each borrower may refinance at most once before time τ. If a borrower decides to prepay early, we assume that he or she pays off the loan using cash.[12]

[10] In reality, borrowers do face some restrictions on their access to capital markets compared with lenders. However, they are certainly closer to having perfect access than none at all, as assumed by Brueckner (1994). Moreover, while differences in risk aversion help to explain many economic phenomena, including possibly the choice between different mortgage types (such as fixed rate versus adjustable), their impact should be negligible in this case, where the differences between the risks involved with two 30 year fixed rate loans are miniscule.

[11] We assume the amount borrowed is \$1. The homogeneity of the model means that the actual amount borrowed is irrelevant.

[12] This is the approach followed by most existing models of rational mortgage prepayment, such as Dunn and McConnell (1981a,b) and Stanton (1995), and in the mortgage choice models of Yang (1992), Kazarian (1993) and Leroy (1994). A major advantage of this approach is that it allows us to use option pricing theory to simplify the analysis, and to make specific predictions about borrowers' and lenders' optimal behavior, and the market values of different loans. Some authors, including Dunn and Spatt (1986), have suggested that a more realistic model would be to assume that borrowers refinance into another mortgage, which they may in turn wish to refinance (incurring additional costs) at some later date. It is possible to extend our results to such a model.

3.2 Lenders and Adverse Selection

Lenders operate in a competitive market, with costless entry and exit, so that, in equilibrium, they make zero expected profit on each loan offered.[13] Lenders may know the distribution of borrowers' investment horizons, but cannot observe the horizon for any individual borrower. This leads to a potential adverse selection problem for lenders, since the borrower's horizon has a significant impact on the value of the cash flows received by the lender from any given mortgage. First, if the term structure is not flat, then the value of a non-callable bond with a given coupon rate will depend on its maturity. If the term structure is upward sloping, the value will tend to decrease in maturity. If it is downward sloping, the value will tend to increase with maturity. Second, the value of the prepayment option embedded in a mortgage increases with maturity. Thus lenders have a strong incentive to discover borrowers' investment horizons.

In an attempt to discover borrowers' horizons. lenders may offer a menu of different contracts. in the hope that different borrowers will choose different contracts from the menu. To focus on the role of the points/coupon tradeoff, we assume initially that lenders may offer only prepayable. 30 year fixed rate loans, differing only in the initial discount points charged. and the coupon rate on the loan. Later, we shall extend their strategy space by allowing them in addition to vary the maturity of the loans.[14] Write the market value of the liability incurred by a borrower with terminal horizon t when taking out a loan with points P and coupon rate C as

$$V^B(P, C, t) = P + M^B(C, t) - 1. \tag{1}$$

[13]Zero profit on average is not sufficient, since entry would occur only in the profitable markets.

[14]This is still a fairly simple strategy space, and we do not attempt to prove optimality. In a much simpler environment, Chari and Jagannathan (1989) show that the optimal set of contracts to offer differs in the tradeoff between coupon and points. In the richer environment considered here, we are giving up the ability to determine the optimal strategy for lenders. In return, however, by using a more realistic process for interest rate movements, we are able to derive specific values for mortgage contracts, rather than merely making qualitative predictions. Moreover, if there are significant costs associated with writing or enforcing complex contracts (in the mortgage market, lenders frequently get sued for miscalculating payments due on adjustable rate mortgages [see Kazarian (1993)]), a simple strategy may indeed be optimal.

Here M^B is the value assigned by the borrower to the stream of payments (excluding points) made on the mortgage, taking into account the embedded prepayment option. Note that M^B (and hence V^B) will also depend, in general, on the current level of interest rates, on the process governing movements in interest rates, and on time. Dependence on these variables is suppressed for clarity. Write the value to a lender of the same loan as

$$V^L(P, C, t) = P + M^L(C, t) - 1. \tag{2}$$

Note that the valuation functions V^L and V^B differ because of the transaction costs which are paid by the borrower on refinancing, but are not received by the lender.[15]

3.3 Equilibrium

We define an equilibrium to be a set of contracts offered by lenders to borrowers, which satisfies the conditions:

1. (Zero profit) For each loan offered, the expected profit over all mortgage holders who take the loan is zero.

2. (Incentive compatibility) If there is more than one loan offered by lenders, each borrower chooses the best loan offered (for that borrower).

3. (Reaction to additional loans) For any additional contract which, offered in addition to the original set, yields positive profits to the lender, there exists another contract which, if offered by another lender, yields a profit to the second lender, and a loss to the first. Moreover, no further addition to the set of contracts results in a loss to the second lender.

This corresponds to Riley's (1979) definition of a *reactive equilibrium.*

[15]See Dunn and Spatt (1986).

We present here some background results on what an equilibrium mortgage offering must look like. These are mainly applications of standard screening results, and we therefore give only intuitive proofs. The key result is Proposition 3, which states that in the absence of transaction costs, it is *not* possible to construct a separating equilibrium. This is in stark contrast to previous models, such as Rothschild and Stiglitz (1976), and is driven by our assumption that both borrowers and lenders have the same objective function, the market value of their loans.

First consider Figure 1, which is the basis for much of what follows. This figure shows all possible combinations of coupon rate and initial discount points that may be offered. The lower the coupon rate, and the lower the points, the better off are mortgage borrowers. On the figure, moving downward and to the left corresponds to paying both lower points and a lower coupon rate, making a borrower better off. The solid line represents lenders' zero profit line for a particular borrower with horizon t, i.e. the set of (P, C) pairs that satisfy the equation

$$V^L(P, C, t) = 0.$$

The dashed lines are borrower indifference curves, i.e. each line shows the set of (P, C) pairs that satisfy the equation

$$V^B(P, C, t) = K,$$

for some constant K. Note that

1. Zero-profit lines and borrower indifference curves are convex. The intuition here is that, since a borrower is more likely to refinance a high coupon loan, it has a shorter expected life than a low coupon loan taken out by the same borrower. As the coupon rate drops, the increase in points needed to compensate the lender for a given drop in the coupon rate decreases, since the lower coupon payment will be made, on average, over a shorter period.

2. Borrower indifference curves are less steep than the lenders' zero-profit lines. This is because of the refinancing costs, paid by borrowers, but not received by lenders. As the coupon rate on the loan increases, the likelihood of future interest rate driven refinancing increases, thus increasing the present value of future refinancing costs paid by borrowers, and increasing the difference between the value of the loan to the borrower and the value to the lender. This means that to be indifferent between a low coupon loan and a high coupon loan, the borrower will insist on lower points for the high coupon loan than the lender.

Proposition 1 *In the absence of asymmetric information, with competition between lenders, all borrowers will choose loans with the highest possible points (lowest possible coupon rate).*

The intuition here is that taking out a loan with the lowest possible coupon rate minimizes the expected refinancing costs paid by borrowers. Since there is no asymmetric information (so, in particular, lenders know borrowers' mobility precisely), loans can be made contingent on borrower mobility. We give the argument for a single type borrower, but exactly the same argument applies to each borrower type.

Consider Figure 1. In equilibrium, lenders must offer some contract on the zero profit line. Suppose contract Z^0 were offered. Another lender could offer contract Z^*, which would make positive profits to the lender, and would be preferred by the borrower to contract Z^0. Thus offering contract Z^0 cannot be an equilibrium. The same argument holds for any contract on the zero-profit line except contract $\overline{Z}$, the contract with the highest possible poinys (lowest possible coupon rate).

Proposition 2 *In the presence of asymmetric information, no pooling equilibrium (where all borrowers choose the same contract) can exist.*

Consider Figure 2. Contract Z_P is the contract which would prevail in such an equilibrium, with lenders making a profit on type 1 borrowers, and an offsetting loss on type

2 borrowers. However, consider contract Z^*. This contract is preferred to Z_P by type 1 borrowers, but is less attractive than Z_P to type 2 borrowers. As a result, if a new lender were to enter the market offering this loan, only type 1 borrowers would take it, the lender would make positive profits on the loan, and the old lender(s) would make a loss on the old loan. This argument extends immediately to multiple classes of borrower. For any suggested equilibrium in which two or more borrower types pool, there is a contract arbitrarily close to the "pooling" contract, which attracts only the most profitable borrowers.[16]

Proposition 3 *With no refinancing costs, no zero-profit separating equilibrium can exist.*

With no transaction costs, lender isoprofit lines and borrower indifference curves in Figure 1 coincide, since borrowers and lenders assign the same value to any set of interest rate contingent cash flows. Suppose a separating equilibrium exists. Consider the loan chosen by a borrower with a horizon shorter than the maximum horizon. In equilibrium, this short horizon borrower must choose a loan with 0 NPV. However, any loan with a 0 NPV to the short horizon borrower has a positive NPV to the long horizon borrower,[17] so this loan would be better for the long borrower than the one he or she is supposed to choose. Thus the equilibrium cannot exist.

Proposition 4 *If a separating mortgage schedule exists, borrowers with the longest investment horizon will choose their first best loan, even in the presence of asymmetric information.*

Borrowers reveal that they are not the longest horizon borrowers by choosing suboptimal loans. The longest horizon borrower has no incentive to reveal his or her type, since in the absence of any information the worst that the lender can believe is that this borrower is indeed the longest horizon borrower. In any suggested equilibrium in which the longest horizon borrowers do *not* take out their first best loan, it will be possible for another lender to offer a profit making contract that is preferred by by those borrowers (see the discussion of Proposition 1).

[16]Wilson (1977) discusses modifications of our assumptions under which such pooling equilibria *can* exist.
[17]The long horizon borrower can always pretend to be a short horizon borrower.

3.4 Construction of a Separating Schedule

We can now construct a separating mortgage schedule. Define a mortgage schedule as a set of contracts $\{Z_\tau : \tau \in \mathcal{T}\}$, where $\mathcal{T} \subseteq I\!R^+$. Write the value (NPV) of contract Z_τ to a borrower with terminal horizon t as $V^{B,t}(Z_\tau)$. Write the value to a lender, assuming the loan is taken out by a borrower with terminal horizon t, as $V^{L,t}(Z_\tau)$. A mortgage schedule is called *fully separating* if

1. For every mortgage holder's horizon t, $t \in \mathcal{T}$.

2. For every horizon t, $V^{L,t}(Z_t) = 0$ (*Zero Profit*).

3. For every horizon t, $V^{B,t}(Z_t) \leq V^{B,t}(Z_{t'})$ for all $t' \in \mathcal{T}$ (*Incentive Compatibility*).

Start by assuming that there are just two classes of borrower, with horizons $t_1 < t_2$. Assume the values t_1 and t_2, and the proportions of each type of borrower in the community, are known to lenders, but that they cannot identify the horizon corresponding to any given borrower. By Proposition 4, the long horizon borrower (borrower 2) will choose a loan with the lowest possible coupon rate in any separating equilibrium. The appropriate number of points on the loan is determined from the lender's zero profit condition as the solution to the equation (in P),

$$V^L(P, 0, t) = 0. \tag{3}$$

The loan chosen by borrower 1 must satisfy both the lender's zero profit condition and the borrower's incentive compatibility condition. Assuming that the mortgage schedule we are looking for is that in which each borrower bears the least cost required to persuade a lender of his or her type, this implies that borrower 1's loan will be at the intersection of the lender's zero profit line for borrowers with horizon t_1 and the indifference curve of borrower 2 that passes through loan Z_2. This is depicted in Figure 3.

Extending this construction to more than two classes of borrower is straightforward. For example, Figure 4 shows the construction for three borrowers, with horizons t_1, t_2 and t_3.

The construction for borrowers 2 and 3 is exactly as above. The loan for borrower 1 lies at the intersection of the lender's zero profit line for borrowers with horizon t_1, and the indifference curve of borrower 2 that passes through loan Z_2. This process can be extended in principle to an arbitrary number of borrower types.

3.5 The Reaction Condition

Equilibrium condition 3 was not used in constructing this loan schedule. However, in the absence of this condition, the schedule we have constructed may *not* represent an equilibrium. Figure 5 shows a two borrower separating schedule, as described above, in which borrowers 1 and 2 are supposed to select loans Z_1 and Z_2 respectively. However, now consider loan Z^*. Type 1 borrowers prefer Z^* to Z_1, and type 2 borrowers prefer loan Z^* to Z_2. A lender will make a profit if this loan is taken out by a type 1 borrower, and a loss if it is taken out by a type 2 borrower. Whether the lender makes a profit on average depends on the relative proportions of type 1 and type 2 borrowers in the economy, or equivalently on where loan Z^* lies relative to the pooling contract Z_P (the contract that would make the lender a zero profit on average, if all borrowers took out that loan). As in Rothschild and Stiglitz (1976), it may thus be possible to "break" the equilibrium by offering a single contract that is preferred by both borrower types. However, by the same argument used to prove Proposition 2, any such pooling contract can be made unprofitable by the creation of another contract which attracts only the best borrowers. This loan cannot itself be made unprofitable, since it appeals only to a single class of borrower. As a result, condition 3 precludes the offering of any contract such as Z^*, and our separating schedule is indeed an equilibrium.

4 Implementation

4.1 Interest Rates

To implement the model we need to make assumptions about movements in interest rates. We assume interest rate movements are described by the Cox, Ingersoll and Ross (1985) one-factor model.[18] In this model, the instantaneous risk-free interest rate r_t satisfies the stochastic differential equation

$$dr_t = \kappa(\mu - r_t)\,dt + \sigma\sqrt{r_t}\,dz_t. \tag{4}$$

This equation says that, on average, the interest rate r converges toward the value μ. The parameter κ governs the rate of this convergence. The volatility of interest rates is $\sigma\sqrt{r_t}$. One further parameter, q, which measures the market price of interest rate risk, is needed to price interest rate dependent assets. The parameter values used in this paper are those estimated by Pearson and Sun (1989):

$$\begin{aligned}
\kappa &= 0.29368, \\
\mu &= 0.07935, \\
\sigma &= 0.11425, \\
q &= -0.12165.
\end{aligned}$$

The long run mean interest rate is 7.9%. Ignoring volatility, the time required for the interest rate to drift half way from its current level to the long run mean is $\ln(1/2)/(-\kappa) \approx 2.4$ years.

Given this model for movements in r_t, we can now calculate the value of the mortgage using the fact that $V(r_t, t)$, the value of any interest rate contingent claim paying coupons

[18] This is one of the most commonly used interest rate models. It has been applied to the valuation of mortgages by, for example, Dunn and McConnell (1981a,b) and Stanton (1995).

or dividends at rate $C(r_t, t)$, satisfies the partial differential equation[19]

$$\frac{1}{2}\sigma^2 r V_{rr} + [\kappa\mu - (\kappa + q)r] V_r + V_t - rV + C = 0. \tag{5}$$

Solving this equation, subject to a payout rate $C(r_t, t)$ and boundary conditions appropriate to the asset being valued,[20] yields the asset value $V(r_t, t)$.

4.2 Valuation and Optimal Prepayment Strategy

Natural boundaries for the interest rate, r, are 0 and ∞. Rather than solving Equation (5) directly, we use the transformation

$$y = \frac{1}{1 + \gamma r}, \tag{6}$$

for some constant $\gamma > 0$,[21] to map the infinite range $[0, \infty)$ for r onto the finite range $[0, 1]$ for y. The inverse transformation is

$$r = \frac{1 - y}{\gamma y}. \tag{7}$$

Equation (6) says that $y = 0$ corresponds to "$r = \infty$" and $y = 1$ to $r = 0$. Next, rewrite Equation (5) using the substitutions

$$U(y, t) \equiv V(r(y), t), \tag{8}$$

$$V_r = U_y \frac{dy}{dr}, \tag{9}$$

[19] We need to assume some technical smoothness and integrability conditions [see, for example, Duffie (1988)].

[20] For example, for a zero coupon bond the payout rate, C, is zero, and its value at maturity is \$1. For a mortgage, there are constant scheduled monthly payments, the terminal value is zero, and we need in addition an optimal exercise condition for the embedded prepayment option.

[21] We shall be solving Equation 5 numerically on a rectangular grid of interest rate and time values. The finer the grid, the better will be our approximation to the solution of Equation 5. However, the processing time is proportional to each grid dimension. For a given grid size in the y direction, the denser the implied r values are in the range corresponding to observed interest rates (say 4% to 20%), the better will be our approximation. We can affect this density by our choice of the constant γ. The larger the value of γ, the more points on a given y grid correspond to values of r less than 20%. Conversely, the smaller the value of γ, the more points on a given y grid correspond to values of r greater than 4%. As a compromise between these two objectives, $\gamma = 12.5$ was used. The middle of the range, $y = 0.5$, corresponds to $r = 8\%$.

$$V_{rr} = U_y \frac{d^2y}{dr^2} + U_{yy} \left(\frac{dy}{dr} \right)^2, \tag{10}$$

to obtain

$$\frac{1}{2}\gamma^2 y^4 \sigma^2 r(y) U_{yy} + \left(-\gamma y^2 \left[\kappa\mu - (\kappa + q) r(y) \right] + \gamma^2 y^3 \sigma^2 r(y) \right) U_y + U_t - r(y) U + C = 0. \tag{11}$$

To value a single mortgage, we use the Crank-Nicholson finite difference algorithm[22] to solve Equation (11). Using this algorithm involves replacing the derivatives that appear in Equation (11) with equations involving the differences between the values of the asset at neighboring points on a discrete grid of y and t values. For convenience we use a time interval of one month, yielding a total of 360 intervals in the time dimension. The algorithm works backward to solve Equation (11) one period at a time to calculate the value of the mortgage borrower's liability, V^B.[23] This gives the value of the mortgage liability conditional on the prepayment option remaining unexercised, $V_u^B(y,t)$.[24] The value of the mortgage liability if the prepayment option is exercised is the amount repaid, including transaction costs,

$$F_t(1 + X).$$

It is optimal to refinance the mortgage if $V_u^B(y,t) > F_t(1+X)$. Otherwise it is optimal not to refinance.[25] The actual value, $V^B(y,t)$, is a weighted average of $V_u^B(y,t)$ and $F_t(1+X)$, the weight on $F_t(1+X)$ being the probability that the mortgage is prepaid in month t. This

[22]See McCracken and Dorn (1969) for a discussion of this algorithm.

[23]The value of the mortgage in month 360 is 0, since all principal has been repaid. Given known values for the asset or liability at month $t+1$, the algorithm calculates their values at every interest rate level on the grid at month t by discounting back a weighted average of their possible values at time $t+1$. This is analogous to the "binomial tree" option pricing algorithm. For a detailed discussion of the relationship between binomial methods, discounted expected values, explicit and implicit finite difference methods for the valuation of contingent claims, see Brennan and Schwartz (1978).

[24]Subscript u for "unexercised".

[25]To facilitate empirical implementation, we assume that the new contract obtained after refinancing is *not* subject to further refinancing costs.

probability is determined by the borrower horizon, τ, and the parameter λ. Define

$$\pi = 1 - e^{-\lambda/12}, \tag{12}$$

the probability of exogenous prepayment this month. The value of the mortgage liability is then

$$V^B(y,t) = \begin{cases} F_t(1+X) & \text{if } V_u^B \geq F_t(1+X), \\ V_u^B & \text{if } V_u^B < F_t(1+X) \text{ and } t < \tau, \\ (1-\pi)V_u^B + \pi\,[F_t(1+X)] & \text{if } V_u^B < F_t(1+X) \text{ and } t \geq \tau. \end{cases} \tag{13}$$

To determine the value of the lender's asset, V^L, the process is similar. When the prepayment option is exercised, the security owner receives the remaining principal balance on the mortgage, F_t. The value of the mortgage to the lender is thus

$$V^L(y,t) = \begin{cases} F_t & \text{if } V_u^B \geq F_t(1+X), \\ V_u^L & \text{if } V_u^B < F_t(1+X) \text{ and } t < \tau, \\ (1-\pi)V_u^L + \pi F_t & \text{if } V_u^B < F_t(1+X) \text{ and } t \geq \tau. \end{cases} \tag{14}$$

This parallels Equation (13) above with each V^B replaced by V^L, and with a different payoff if the mortgage is prepaid. The lender's asset value is less than the value of the mortgage holder's liability for all values of y and t if transaction costs are positive, since since the money paid out by the mortgage holder is always less than or equal to the amount received by the lender.

4.3 Results

The algorithm described above was applied to various term structures, interest rate processes, and distributions of borrower horizons. Figures 7 and 8 show two examples, using the term structures depicted in Figure 6, and three classes of borrower with horizons 10, 15, and 20 years respectively. The speed-of-moving parameter, $\lambda = 0.1$.[26] The transaction cost payable on refinancing, X, is 5% of remaining principal.

Figure 7 shows the results for the flat term structure. As before, the solid lines are the lender's zero profit lines and the dashed lines are borrower indifference curves. The solid line furthest to the right is the zero profit curve for a lender issuing a thirty year mortgage to a borrower with a 20 year horizon. From Proposition 4, the longest horizon borrower (the 20 year borrower) chooses a loan with the lowest possible coupon rate and highest possible points. We assume 10 points as a realistic maximum, corresponding to a coupon rate of 11.5%. As expected, the 15 and 10 year horizon borrowers select progressively higher coupon and lower point combinations, all three contracts yielding zero profit to the lender. Although the spread between the coupon rates on the loan taken by the 20 year borrower and that of the 10 year borrower is only 0.5%, with a corresponding difference of 2.3 points, these differences reveal very important information about the characteristics of the borrowers taking out the loans. Ignoring this may lead to significant errors in predicting prepayment, and hence to errors in valuing and hedging mortgages and MBS.

Figure 8 shows the corresponding results for the upward sloping term structure depicted in Figure 6. The general pattern is similar, but the differences between the contracts are more marked than for the flat term structure. The difference between the coupon rates of the loans selected by the 10 and 20 year borrowers is 1%, with a difference of 9 points.

In general, the exact contracts taken out in equilibrium depend on all of the factors of the model. However, of particular importance are:

1. *The initial shape of the yield curve* – The more downward sloping the yield curve, the closer together the zero profit lines become, as the increased option value for longer borrowers is somewhat made up for by the lower forward rates at long maturities.

2. *The volatility of interest rates* – The more volatile interest rates are, the greater the

[26] In other words, after reaching horizon τ, the probability that a borrower moves each year, conditional on not having previously moved, is $1 - e^{-\lambda} = 9.5\%$.

value of the embedded prepayment options, widening the gap between zero profit lines, but also the more likely are the options to be exercised, widening the spread between the zero profit lines and borrower indifference curves.

3. *Borrower transaction costs* – With zero costs, we know that no separating equilibrium can possibly exist. As costs increase, the value of borrowers' prepayment options falls, narrowing the gap between zero profit lines. In addition, as costs increase (at least initially), zero profit lines and indifference curves start to diverge, making the construction of the separating schedule easier. However, if the costs increase without limit, eventually no borrowers can ever afford to refinance their loans, and the loans effectively become non-prepayable. In this case, the shape of the zero profit lines is given by the values of non-prepayable bonds of different maturities, depending only on the current term structure. In addition, since borrowers will never refinance early, lender isoprofit lines and borrower indifference curves will again coincide, and no separating equilibrium is possible.

These results clearly indicate that offering a menu of contracts with differing points and coupon combinations can provide a mechanism for lenders to learn private information about borrower mobility. This is not achieved costlessly, however. All but the longest horizon borrowers refinance more often than they would in the absence of asymmetric information, incurring the deadweight costs associated with such refinancing. In the next section, we show that these deadweight costs can be reduced if lenders offer loans which differ in maturity as well as points/coupon rate.

5 Separation Using Loan Maturity

If all borrowers know *exactly* when they will move, and if it is costly to refinance, the optimal strategy for lenders is to offer each borrower a contract with a maturity which exactly matches his or her horizon. This permits costless separation between borrowers.

Even when borrowers do not know exactly when they will move (i.e. $\lambda < \infty$), lenders can reduce the deadweight costs of separation by issuing loans with different maturities. This is because, even with uncertainty about the actual time of moving, the likelihood of a long horizon borrower having to refinance a short horizon loan is still higher than the likelihood of a short horizon borrower refinancing the same loan.

Assume lenders may issue loans with any combination of coupon, points (as before) and maturity. All loans are amortized over 30 years (i.e. the monthly payments are the same as for a 30 year loan with the same coupon rate), with a final balloon payment of the remaining principal if the loan maturity is less than 30 years. As before, all loans are prepayable prior to maturity. Assume the current term structure is the upward sloping term structure depicted in Figure 6.

Consider first Figure 9. This shows all possible loans which can separate between a borrower with a horizon of 15 years and another with a horizon of 25 years. The parameters are $X = 0.05$ and $\lambda = 15$,[27] so in this case, borrowers are almost certain to move within a short time after τ. Each solid narrow line shows the set of zero-profit n-year contracts for a lender making loans to the 15 year borrower, where n takes on the values $15, 16, \ldots, 30$. The bold solid line connects the contracts on each zero-profit line that lie on the 25 year borrower's indifference surface through the 30 year loan that this borrower takes out in equilibrium. The lender can thus separate the two borrowers by offering either a 30 year loan with high coupon rate, and low points, or a 29 year loan with slightly lower coupon rate, and slightly higher points, and so on down to 15 years, where separation can be achieved by offering close to the first best loan for the 15 year borrower, a loan with low coupon rate (which is unlikely to be refinanced for interest rate reasons) and high points.

Figure 10 shows the set of separating contracts for the same transaction cost, 5%, but a lower value $\lambda = 0.1$.[28] Compared to Figure 9, the set of separating contracts is steeper,

[27] I.e. refinancing costs are 5% of remaining principal, and after borrower horizon time τ, the likelihood of moving is $1 - e^{-\lambda/12} = 71\%$ in the first month, $1 - e^{-\lambda/6} = 92\%$ in the first two months.

[28] This corresponds to a probability of $1 - e^{-\lambda/12} = 1\%$ in the first month, $1 - e^{-\lambda} = 9.5\%$ in the first year

dropping sharply at a higher loan maturity of 19 years. In this case, very short contracts (close to 15 years) are not as attractive to the short borrower as before, since there is a substantial probability that he or she will not move before the loan matures, thus incurring refinancing costs merely because the original loan has matured.

Finally, consider Figure 11. For this case we use the same mobility parameter, $\lambda = 0.1$, but a higher refinancing penalty, $X = 0.2$. Compared with Figure 10, the set of separating contracts is even steeper, dropping sharply at a maturity of 22 years – the higher costs make the 15 year borrower even keener to avoid refinancing by taking out a longer maturity loan.

In each case, the size of the deadweight costs increases as the loan maturity increases, since the higher coupon rate makes it more and more likely that the short term borrower will refinance the loan. Comparing the shorter loans with a 30 year loan, we find that the strategy of offering loans with shorter maturities can result in substantial reductions in deadweight costs compared with relying only on points vs. coupon rate to separate borrowers. For example, the deadweight cost (computed as the difference between the lender's and borrower's valuation of the mortgage) when $X = 0.05$ and $\lambda = .1$ (corresponding to Figure 10), is \$0.76 per hundred dollars of principal when separation between the borrowers is achieved by issuing 30 year contracts to both borrowers, and only \$0.55 per hundred dollars when the 15 year borrower takes out a 19 year contract. Similarly, for parameter values $X = 0.05$ and $\lambda = 15$ (corresponding to Figure 9), the deadweight costs are \$2.03 per hundred dollars in principal for a 30 year loan, compared with only \$0.94 per hundred dollars for a 16 year loan. These reductions in deadweight costs, looked at in the context of the huge size of the overall U.S. mortgage market, suggest that lenders' recent innovations in originating a broader menu of contract maturities are well justified.

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after horizon τ.

6 Summary

This paper shows that a menu of prepayable fixed rate mortgage contracts, differing in the tradeoff between points and coupon rate, can provide lenders with a mechanism for learning private information about potential borrowers' tenure. It develops a screening equilibrium, along the lines of Rothschild and Stiglitz (1976), in which competitive mortgage lenders make zero expected profit on each contract, and borrowers choose the optimal contract from a menu. Short horizon borrowers reveal their type by choosing a mortgage that they will, with significant probability, want to refinance at some time in the future. Since refinancing is costly, they would prefer, in the absence of asymmetric information, to take out loans with no likelihood of future refinancing. The construction of the equilibrium mortgage schedule integrates the literature on screening with that on contingent claims pricing, allowing valuation of different loans, as well as explicit predictions of borrower prepayment behavior as a function of the loan they take out.

Constructing the separating schedule requires the existence of some friction that drives a wedge between borrower and lender values assigned to the same loan. This provides an explanation for why mortgage contracts contain prepayment options, even though there are significant deadweight costs associated with these options. We also show that lenders can achieve the same separation, at lower deadweight cost, by offering loans with different maturities.

Finally, our results suggest that many commonly used empirical prepayment models are misspecified. If a prepayment model fails to take into account the selection of loans available at the time the loan was made, it may be overlooking a significant predictor of future prepayment behavior, leading in turn to errors in pricing and hedging mortgages and mortgage-backed securities, such as those behind the 1994 collapse of Askin Capital Management.

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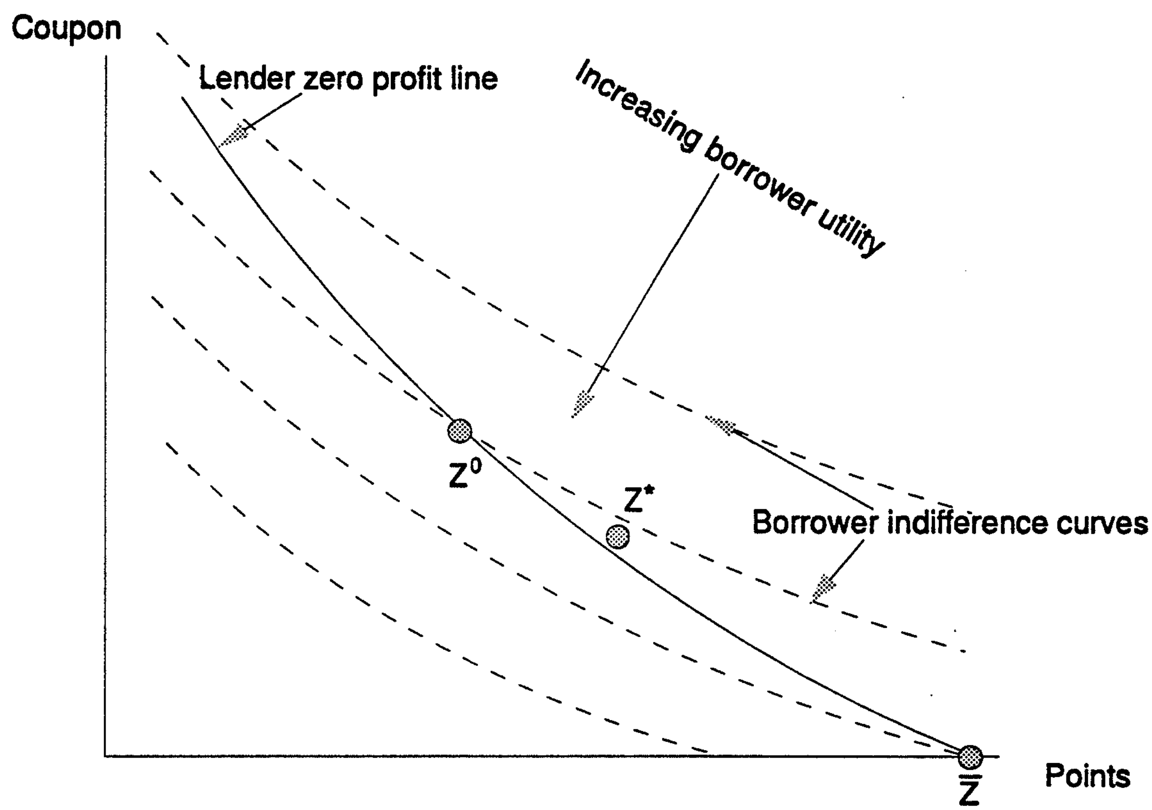
Table 1: Loans available, October 1993

Loan Life	Interest Rate	Points (Conforming)	Points (Jumbo)
30 yr	6.250%	3.000%	3.125%
	6.500%	1.375%	2.000%
	6.625%	0.750%	–
	6.750%	0.500%	0.875%
	7.000%	-0.625%	0.125%
	7.250%	-1.250%	-0.750%
	7.500%	–	-1.125%
15 yr	5.750%	2.625%	3.250%
	6.000%	1.750%	2.125%
	6.250%	0.875%	1.125%
	6.500%	-0.250%	0.375%
	6.750%	-0.750%	-0.250%
	7.000%	-1.500%	-0.875%

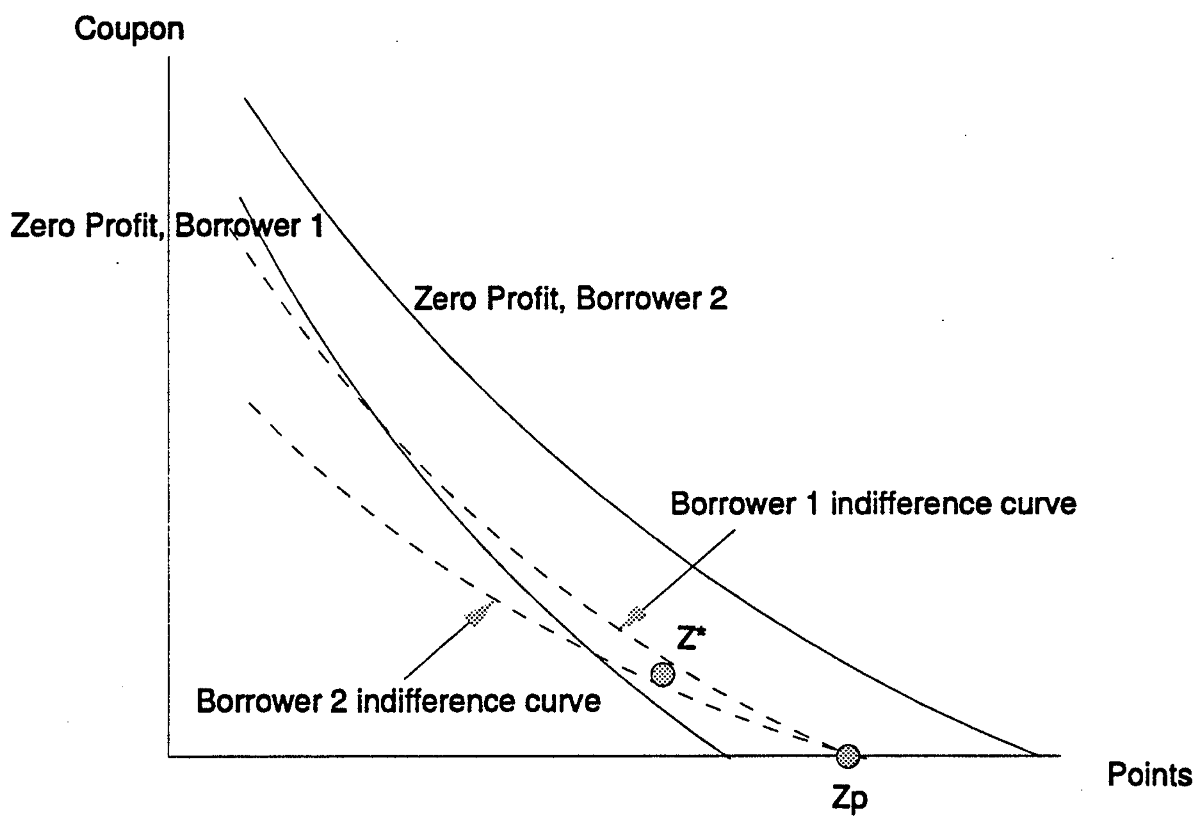
A selection of the loans available from one of the largest U.S. mortgage lenders on a single date in October 1993.

Figure 1: Loan choice without asymmetric information



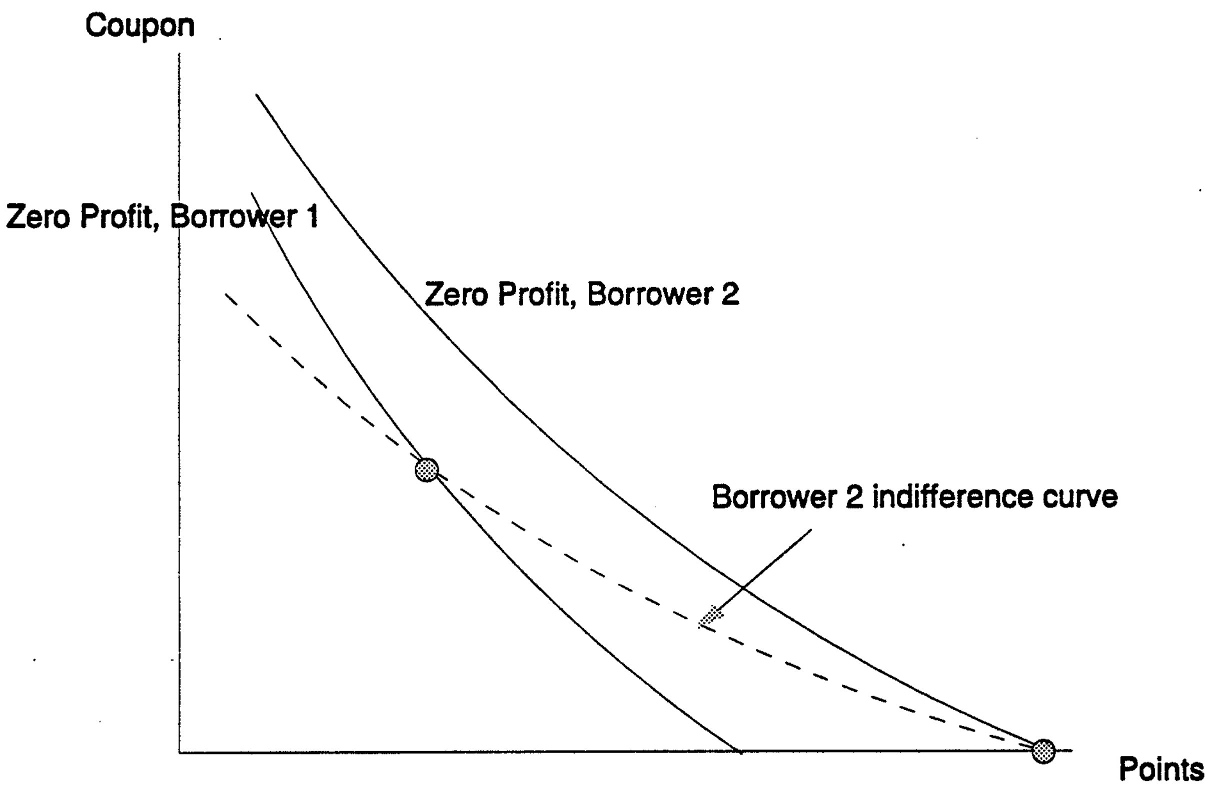
Solid line shows contracts yielding zero profit to the lender. Dashed lines are borrower indifference curves. Both lender and borrower prefer contract Z^* to contract Z^0.

Figure 2: Non-existence of pooling equilibrium



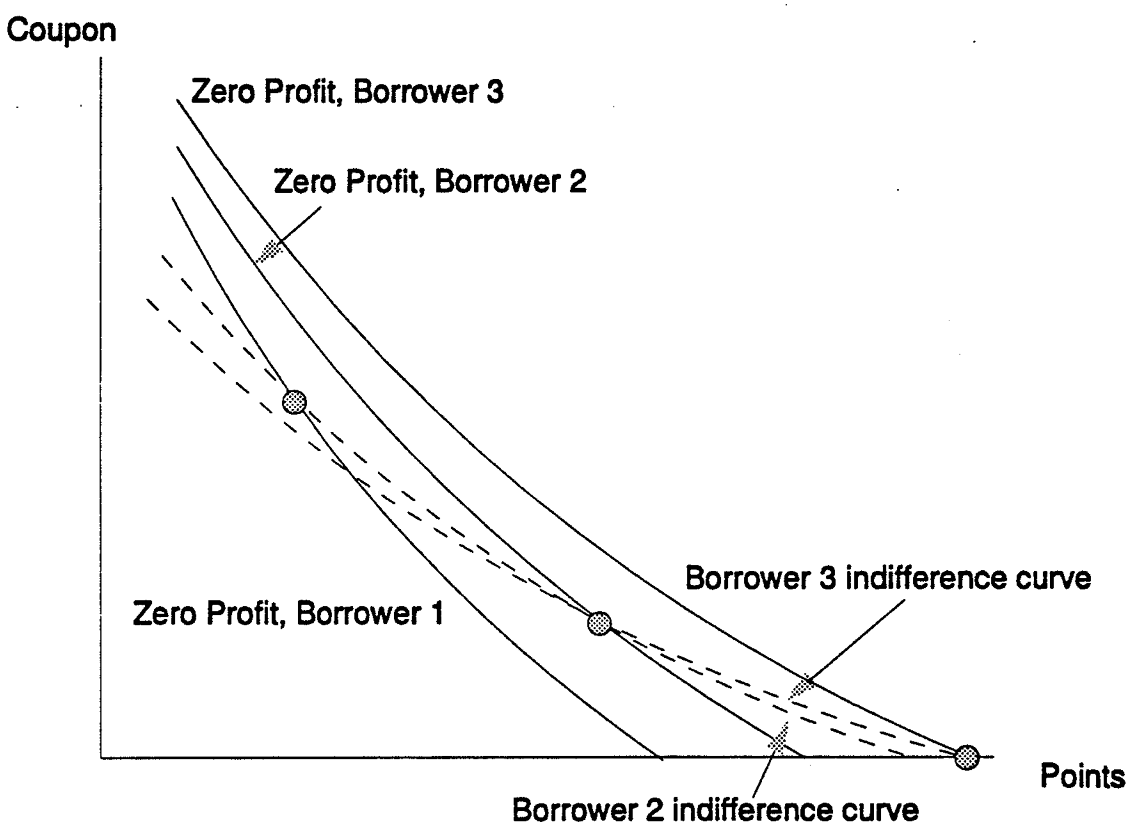
Solid lines show contracts yielding zero profit to the lender when taken out by borrower 1 (short horizon) and borrower 2 (long horizon) respectively. Dashed lines are borrower indifference curves. If pooling contract Z_P is offered, offering contract Z^* will attract only the (profitable) short horizon borrowers.

Figure 3: Separation with two borrowers



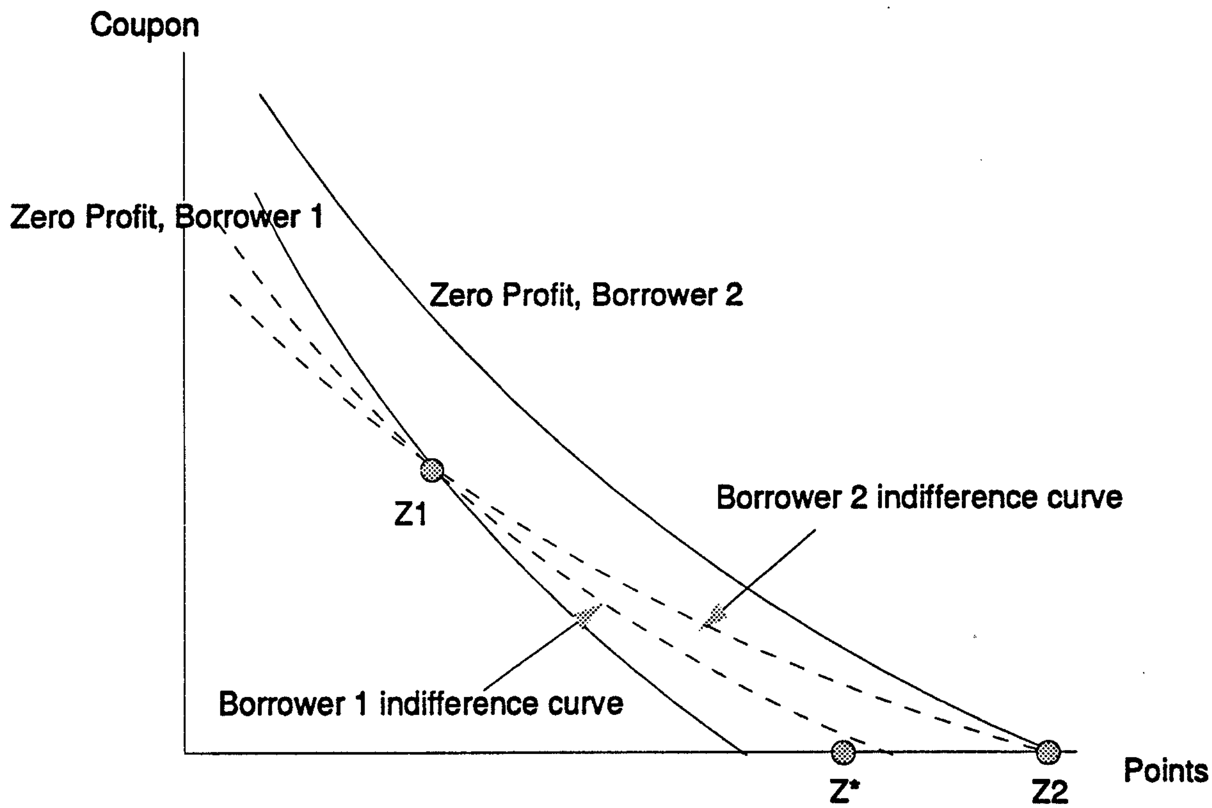
Solid lines show contracts yielding zero profit to the lender when taken out by borrower 1 (short horizon) and borrower 2 (long horizon) respectively. Dashed line is borrower 2's indifference curve. The short horizon borrower (borrower 1) selects loan that lies on intersection of lender's zero-profit line and borrower 2's indifference curve through borrower 2's first-best contract.

Figure 4: Separation with three borrowers



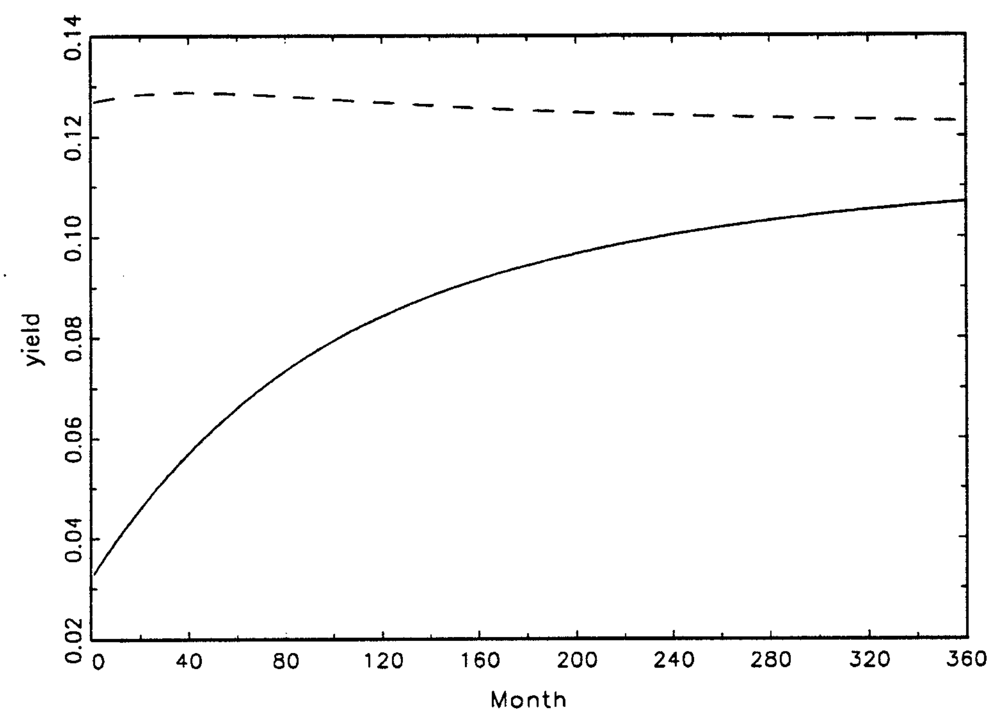
Solid lines shows contracts yielding zero profit to the lender when taken out by borrowers 1 (short horizon), borrower 2 (medium horizon), and borrower 3 (long horizon) respectively. Dashed lines are borrower indifference curves. The middle horizon borrower (borrower 2) selects loan that lies on intersection of lender's zero-profit line and borrower 3's indifference curve through borrower 3's first best contract. The short horizon borrower (borrower 1) selects loan that lies on intersection of lender's zero-profit line and borrower 2's indifference curve through borrower 2's equilibrium contract.

Figure 5: Importance of Reaction Condition



Solid lines show contracts yielding zero profit to the lender when taken out by borrower 1 (short horizon) and borrower 2 (long horizon) respectively. Dashed lines are borrower indifference curves. Loan Z^* is preferred by borrower 1 to loan Z_1, and by borrower 2 to loan Z_2. Depending on the relative proportions of borrowers of types 1 and 2, offering loan Z^* may yield positive profits to the lender.

Figure 6: Yield Curves

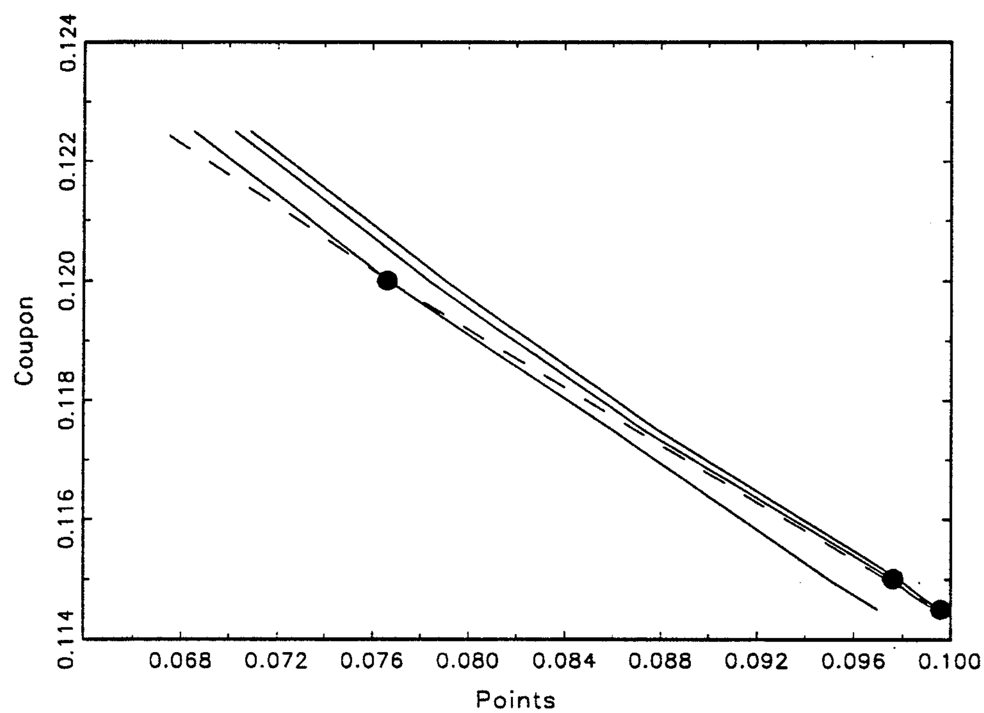


Flat and upward sloping yield curves generated using the Cox, Ingersoll and Ross (1985) interest rate model,

$$dr_t = \kappa\,(\mu - r_t)\,dt + \sigma\sqrt{r_t}dZ_t,$$

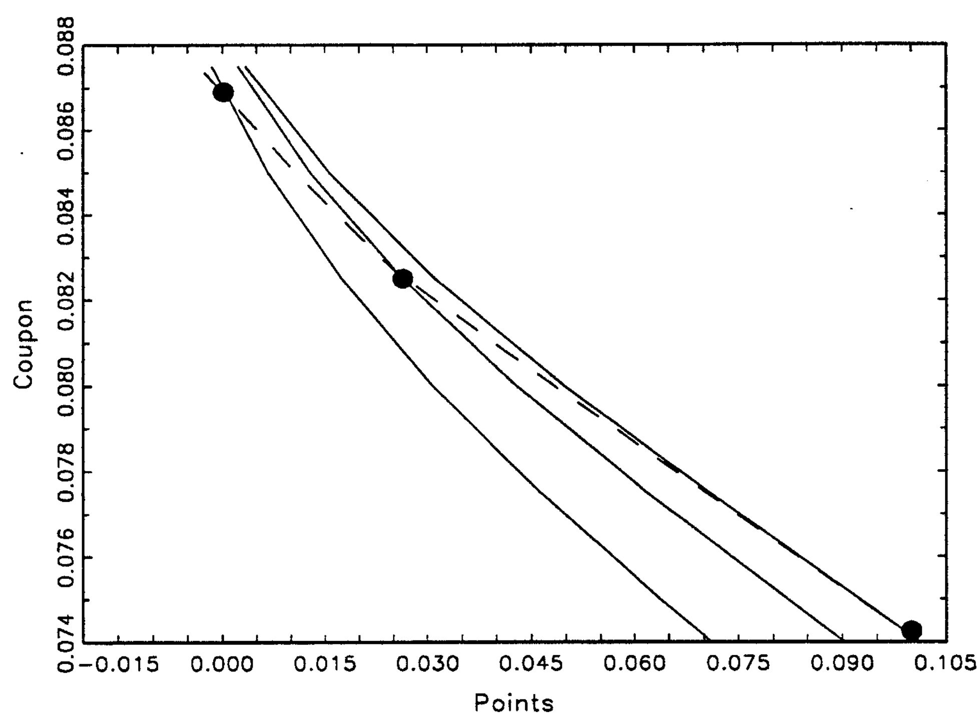
with parameters $\kappa = 0.29368$, $\mu = 0.07935$, $\sigma = 0.11425$, and risk aversion parameter $q = -0.12165$.

Figure 7: Separating equilibrium, flat yield curve



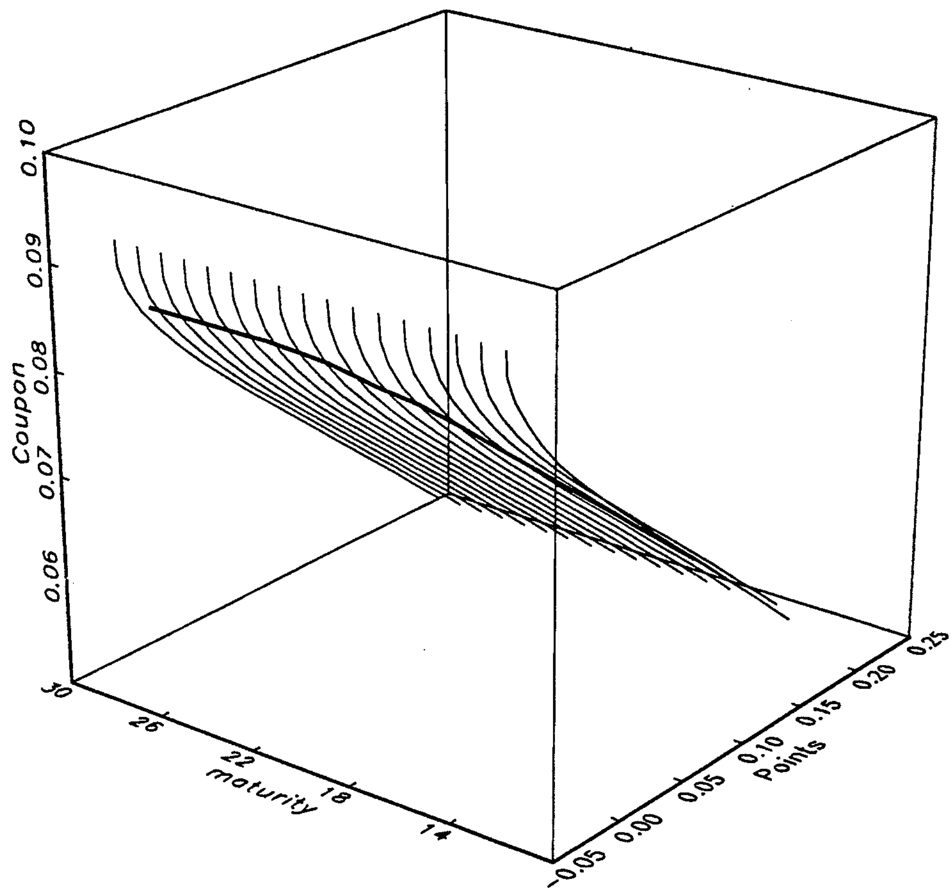
Separating loan schedule for three classes of borrower, with horizons 10, 15 and 20 years respectively. For all three types, speed-of-moving parameter $\lambda = 0.1$, and transaction cost X is 5% of the remaining principal on the loan.

Figure 8: Separating equilibrium, upward sloping yield curve



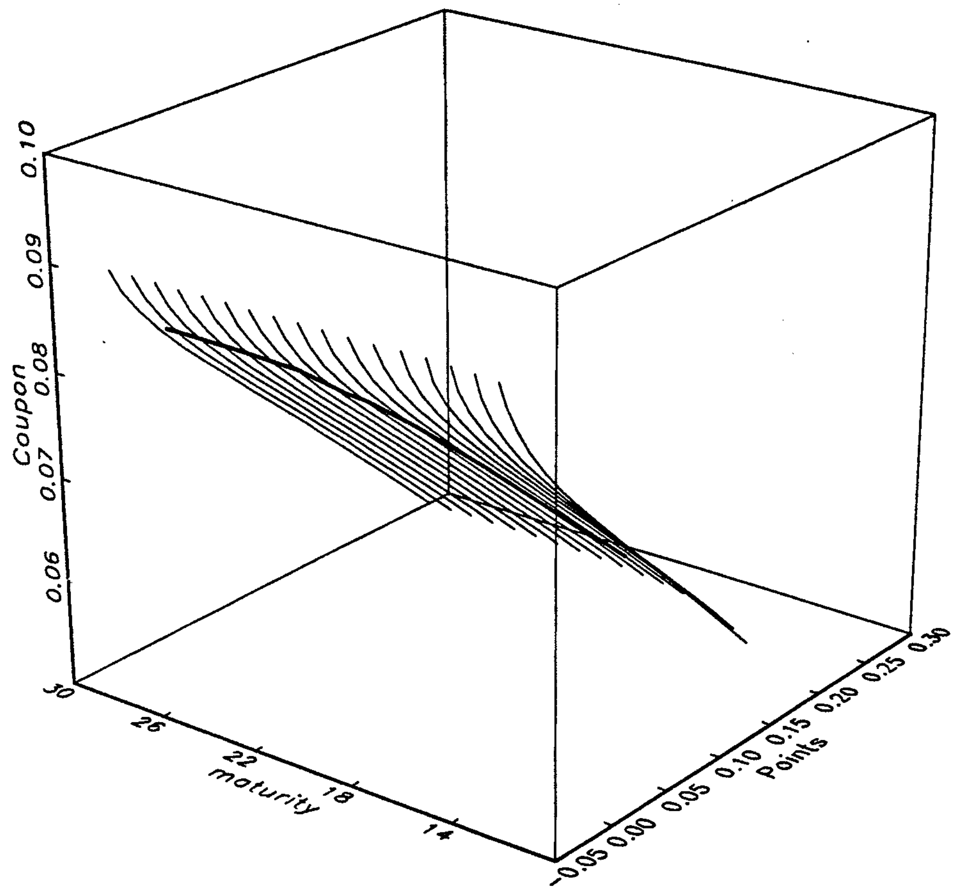
Separating loan schedule for three classes of borrower, with horizons 10, 15 and 20 years respectively. For all three types, speed-of-moving parameter $\lambda = 0.1$, and transaction cost X is 5% of the remaining principal on the loan.

Figure 9: Separation by points, coupon and maturity



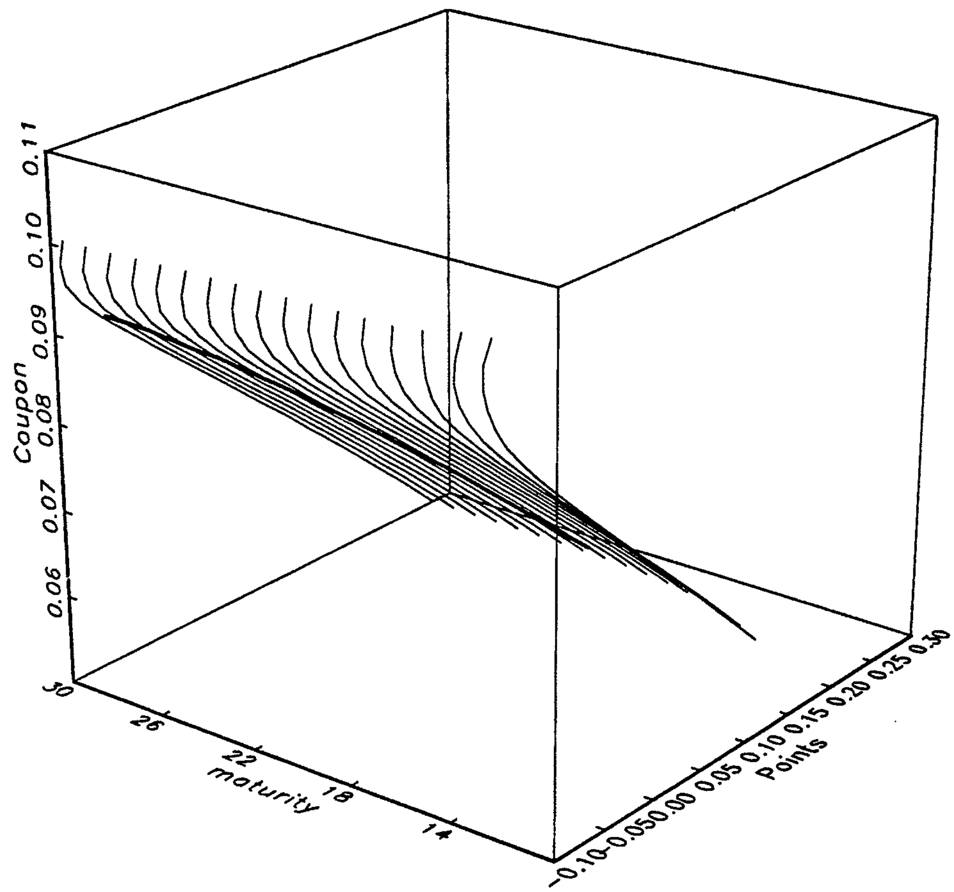
There are two classes of borrower, with horizons 15 and 25 years respectively. Narrow lines show contracts of different maturities yielding zero profits to the lender when accepted by the 15 year borrower. Solid line joins loans of each maturity which lie on 25 year borrower's indifference surface through his or her first-best loan. For both borrower types, speed-of-moving parameter $\lambda = 15$, and transaction cost X is 5% of the remaining principal on the loan.

Figure 10: Separation by points, coupon and maturity



There are two classes of borrower, with horizons 15 and 25 years respectively. Narrow lines show contracts of different maturities yielding zero profits to the lender when accepted by the 15 year borrower. Solid line joins loans of each maturity which lie on 25 year borrower's indifference surface through his or her first-best loan. For both borrower types, speed-of-moving parameter $\lambda = 0.1$, and transaction cost X is 5% of the remaining principal on the loan.

Figure 11: Separation by points, coupon and maturity



There are two classes of borrower, with horizons 15 and 25 years respectively. Narrow lines show contracts of different maturities yielding zero profits to the lender when accepted by the 15 year borrower. Solid line joins loans of each maturity which lie on 25 year borrower's indifference surface through his or her first-best loan. For both borrower types, speed-of-moving parameter $\lambda = 0.1$, and transaction cost X is 10% of the remaining principal on the loan.