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DEFINITION AND REALIZATION OF CRITICAL FOCUSING CONDITIONS OF $\pi\sqrt{2}$ MAGNETIC FIELD AT HIGH RESOLUTION*

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ABSTRACT

An analysis is given of the conditions necessary for achieving a favorable solid angle at high resolution in a $\pi\sqrt{2}$ beta-spectrometer in the case where certain imperfections in the magnetic field shape are present. It is shown that one can achieve satisfactory radial focusing under field conditions less narrowly restricted than those implied by the theoretical field. This allows a fairly simple way to correct for minor imperfections in a spectrometer field. An application to the Berkeley 50 cm iron-free $\pi\sqrt{2}$ beta spectrometer is described.

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1. Introduction

In any practical effort to produce the magnetic field appropriate to a certain type of beta spectrometer, one is always left with some imperfections in the field shape which, at sufficiently high resolution, reduce the transmission relative to the value that is theoretically possible. An accurate description of these field imperfections may require a large number of parameters, and, correspondingly, a rather involved correction procedure may be necessary for their complete removal. The purpose of this note is to point out the existence of field shape conditions more general that those implied by the theoretical field which, although slightly departing from the theoretical field, retain the essential radial focusing properties at high resolution. These more general field shape conditions are much more easily arrived at from a given imperfect field, and thus they provide a practical means to correct for minor imperfections in a spectrometer field.

The following discussion will assume the well-known $\pi\sqrt{2}$ type of magnetic field, although most of the arguments are more generally applicable. In Sec. 2 the relevant field conditions for satisfactory performance at high resolution with respect to radial aberrations and entrance aperture are analyzed. In Sec. 3 the results of this analysis are applied to a simple field correction of an $r_o = 50$ cm iron-free spectrometer.

2. Basic Features of the Radial Aberration Pattern

We assume the "tall-aperture" case of the $\pi\sqrt{2}$ field[1]), i.e., in the series expansion of the field component B_z in the symmetry plane $z = 0$,

$$B_z = B_o \left[1 - 1/2 \left(\frac{r-r_o}{r_o} \right) + a_2 \left(\frac{r-r_o}{r_o} \right)^2 \ldots\ldots \right] ,$$

the second order field coefficient a_2 is chosen to be 3/8. In this case the radial aberration ρ is in second order independent of the axial aperture angle ϕ_z , and exhibits a parabolic dependence on the radial aperture angle ϕ_r. Figure 1(a) illustrates the radial aberration in the case of a perfectly realized field. The spectrometer performs favorably at high resolution (i.e. small values of $|\rho|$) because it is possible, for a considerable range in ϕ_z, to utilize a relatively large interval of ϕ_r around the flat vertex part of the aberration parabolas. Because of the quadratic dependence of ρ on ϕ_r and its independence of ϕ_z the solid angle (proportional to the product $\phi_r \phi_z$) decreases only as $\sqrt{|\rho|}$ when the radial aberration width $|\rho|$ is reduced.

When imperfections in the field are present, the aberration pattern of fig. 1(a) is converted to something like that illustrated by the solid lines in fig. 1(b). The aberration parabolas for different ϕ_z no longer coincide. In the general case both the ρ value and the ϕ_r value of the vertices vary with ϕ_z. The solid angle at high resolution then suffers because in order to select a narrow interval in the aberration ρ it is necessary, for most ϕ_z values, to select ϕ_r values that are far from the vertex points. The rapid variation with ϕ_r of the aberration functions at such ϕ_r values drastically limits the possible radial aperture.

A full correction of the imperfections in the field would involve restoring the unsymmetric aberration pattern (solid lines in fig. 1(b)) to that of fig. 1(a). In order to do this, the aberration parabolas would have to be repositioned in both the ρ and ϕ_r directions.

It is possible to make a partial correction of the field that is essentially equivalent to the full correction with respect to the relation between solid angle and radial aberration, although it contains a residual asymmetry. This partial correction consists in moving the aberration parabolas in the ρ-direction until the vertices for all ϕ_z are located at $\rho = 0$ and in this procedure, allowing the ϕ_r positions of the vertices to become whatever they may. This situation is illustrated by the dotted lines in fig. 1(b). By suitably designing the entrance aperture so that the allowed ϕ_r values vary with ϕ_z in the manner required by the residual asymmetry, a favorable situation is obtained because the flat vertex regions of the aberration parabolas are utilized for all ϕ_z values and hence the entrance aperture can be fairly wide.

Clearly the latter type of field correction is much easier to accomplish since the controlled repositioning of the aberration parabolas is confined to one direction only. The residual asymmetry in the field will manifest itself primarily in a somewhat larger axial aberration than in the symmetric case, but in most applications this should not be disturbing.

It is apparent from the above that the restoration to ϕ_z - independence of the radial aberration from a slightly distorted field is not necessary for achieving a favorable relationship between solid angle and radial aberration. The essential point is that the vertex positions of the aberration parabolas should fall at the same radius regardless of the axial aperture angle ϕ_z.

3. Application to the Berkeley Iron-free $\pi\sqrt{2}$ Spectrometer

The Berkeley spectrometer is in its basic features identical to the Uppsala 50 cm iron-free $\pi\sqrt{2}$ instrument described by Siegbahn et al.[2]. Certain imperfections of the type shown in fig. 1(b) (solid lines) in the field of this machine have been known to exist. A partial compensation for this had been achieved by using special shapes for the entrance apertures, empirically found from a study of the iso-aberration contours of the field[3]. As is clear from the discussion above such a measure can never fully restore the essential features of the theoretical tall-aperture field. Therefore we have attempted to correct the field in the limited but sufficient manner outlined above.

For probing the features of the field shape we have placed an electron gun at the source position and a fluorescent screen at the detector position. The aperture angles ϕ_r and ϕ_z of the narrow pencil of electrons from the gun were continuously variable from outside the spectrometer. The spectrometer current and the gun voltage (around 2.5 kV) were adjusted such that for $\phi_z = 0$ the vertex of the aberration parabola fell at the radial position of the detector slit. If the ϕ_r angle is varied the vertex point is easily identified as the radially stationary position of the image point. The fluorescent screen was viewed by means of a telescope with an eyepiece scale that allowed a reading of the ρ value of the vertex point to about 0.1 mm. Figure 2(a) shows the radial position of the vertex points as a function of the axial aperture angle ϕ_z without any correction applied to the field. Within the interval $|\phi_z| \lesssim 0.15$ the radial positions of the vertex points vary by about 4.8 mm, or 0.24% in focused momentum.

Assume that one wishes to alter the radial vertex positions ρ_ν for n different values of ϕ_z and that this is to be done by means of a number, m, of auxiliary correction coils with current I_μ. Let $\partial\rho_\nu/\partial I_\mu$ be the sensitivity of the radial vertex position for the ν^{th} ϕ_z values to a change of the current in the μ^{th} correction coil. Within the accuracy of the assumption that the sensitivities $\partial\rho_\nu/\partial I_\mu$ are independent of the correction currents I_1, I_2 ...... a determination can be made of the appropriate correction currents from the linear equation system

$$-\rho_\nu = \sum_{\mu=1}^{m} \frac{\partial\rho_\nu}{\partial I_\mu} I_\mu \qquad \text{(n equations)} \qquad (1)$$

If $m \geq n$ the equations can in principle always be satisfied. If the choice of geometry for the correction coils has been particularly unfavorable the solution may, however, involve inconveniently large currents I_μ and may result in large magnitudes of ρ for ϕ_z values between the selected n values. We tried first a set of five correction coils placed symmetrically around the outer tank wall, but it proved to be advantageous to add a sixth coil between the central coil and the one below it. The sensitivities $\partial\rho_\nu/\partial I_\mu$ were determined by observing the movement of the image spot of the vertex when a given current was passed through the μ^{th} coil. The eq. system (1) (with $m = 6$; $n = 5$) was then used to obtain a rough first approximation for the correction currents I_μ. The final current distribution was obtained from this by trial and error. Figure 2(b) shows the radial position of the vertex points after correction. Within the same axial aperture range $|\phi_z| \lesssim 0.15$ the radial spread is now ~20 times smaller than in fig. 2(a) and the radial vertex positions are constant to within 0.01% in momentum.

Table I gives some data for the correction coils. The coils are circular and placed in planes parallel with the spectrometer midplane. The number of turns in the final coils were chosen such that all six coils have the same current, which is obtained from a separate power supply, controlled by the main spectrometer current regulator.

The entrance aperture shapes for different maximum radial aberrations were obtained by means of a second fluorescent screen which could be moved to intercept the electron beam in the plane of the entrance aperture (at 54° from the source). For a given ϕ_z, the ϕ_r -parameter was adjusted until the desired radial aberration was observed on the fluorescent screen in the detector plane. The position of the orbit in the entrance aperture plane was then recorded by means of the movable fluorescent screen in that plane. Figure 3 shows entrance aperture shapes for 0.03% and 0.12% radial aberration (in $B\rho$) obtained in this way. The unsymmetric shape of the entrance apertures reveals the unsymmetric character of the corrected field.

In the theoretical tall-aperture case with the second field coefficient $a_2 = 3/8$ the radial aberration is in lowest order given by $\rho = -4/3\,\phi_r^2$ ref 2). A closer analysis of the apertures shown in fig. 3 for different aberration magnitudes reveals that the radial aperture permitted in the partly corrected field is within approximately 15% of that given by the expression for the theoretically ideal field. Within the axial aperture $|\phi_z| \leq 0.15$ the performance is therefore near to that of the ideal case. Outside this ϕ_z-range the radial aberration pattern deteriorates rapidly both without and with the correction field. Although, therefore, the usable axial aperture is smaller than in some $\pi\sqrt{2}$ instruments designed with particular emphasis on tall aperture, e.g. the Bartlett iron-core spectrometer[4]) which allows $|\phi_z| \leq 0.6$, the possible ϕ_z range at high resolution is roughly equal to e.g. that of the well-known Chalk River spectrometer[5]).

In the corrected field, the axial aberration within each aperture was found to increase rapidly towards the extreme values of ϕ_z, and the total image height of a point source is ~ 30 mm. In most applications this is a minor disadvantage, only necessitating the use of a correspondingly high counter slit.

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Table 1. Some data for the correction coils

Coil position (distance in cm from spectrometer midplane)	Number of turns
17	2(-)
8.5	1(-)
0	18
-4.3	48(-)
-8.5	6
-17	15

The coils have a diameter of 133.5 cm and are coaxial with the main coils. The correcting coil current amounts to 0.022 of the main spectrometer current. A negative sign in parenthesis means that the particular coil gives a field opposite to that of the spectrometer field.

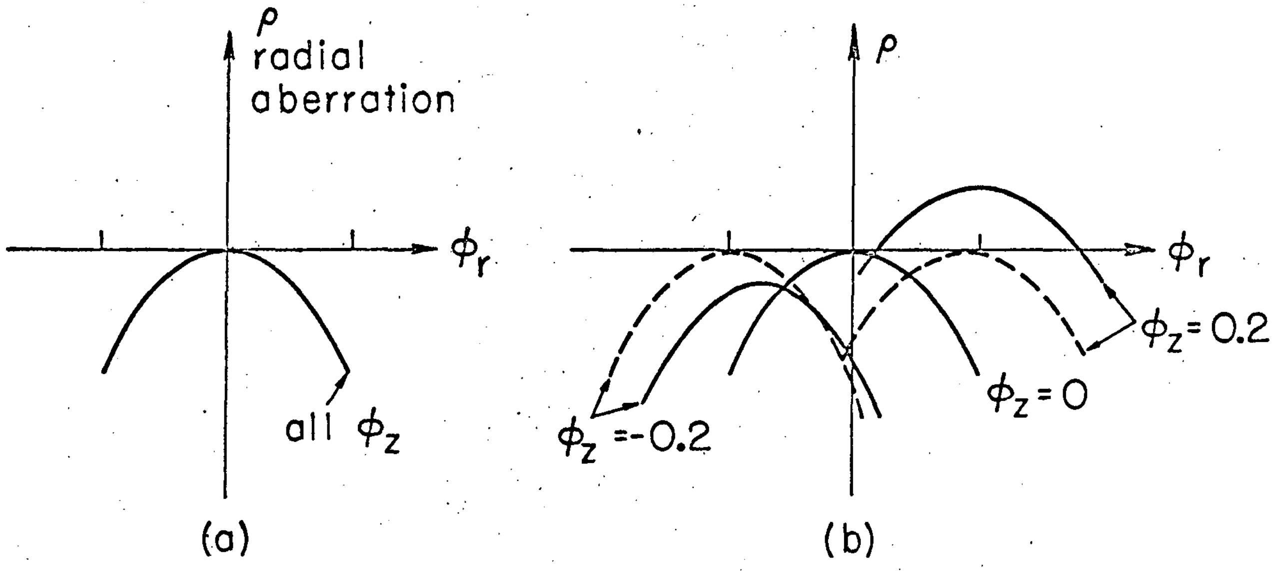

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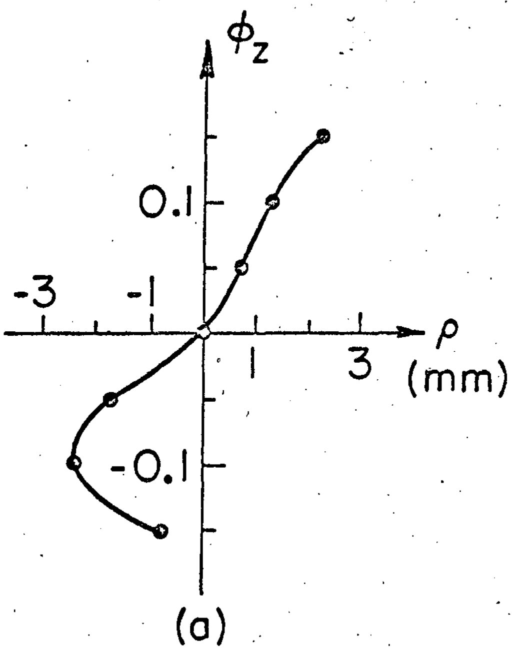
Fig. 1



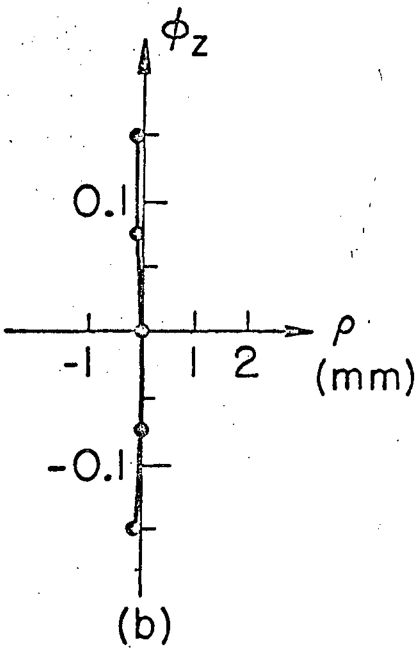


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Fig. 2

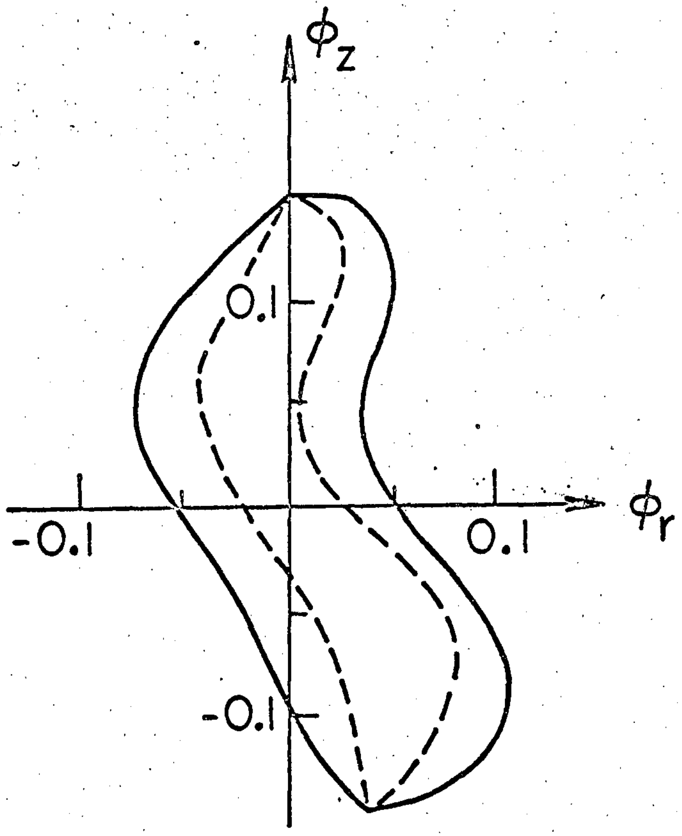

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Fig. 3