Optimization and Path-following

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Path-following Performance

The ability to follow a path between two points is crucial for any higher organism, and many different types of test have been devised to study human path-following.

For example, in the Trail Making Test (TMT), the subject has to draw a continuous line, connecting 25 randomly distributed circles, containing numbers and/or letters, in numerical or alphanumerical order.

A second type is the Zahlen-Verbindungs-Test (ZVT), in which the subject has to connect consecutive numbers in a 9x10 matrix in which successive numerals are positioned irregularly, but in immediately adjacent cells.

Both tests suffer two main disadvantages. First, since test construction is unprincipled, there has been no theoretically-driven study of path-following performance. Second, there is a shortage of multiple, measurably equivalent test forms.

This paper argues that the generation of such tests may be viewed as a problem of combinatorial optimization.

Algorithms for Test Generation

Our approach follows the application of neural networks to such problems, and, in particular, to the Traveling Salesman Problem (TSP). This can be formulated as: Given n cities, and a specified cost (distance) in moving between any two, devise an itinerary, such that (a) each city is visited exactly once, and (b) the total cost of the itinerary is a minimum.

Elastic Nets and the TMT

If the TMT is regarded as a self-avoiding, near-optimal solution to a Euclidean TSP, then a useful network for generating pathways is provided by the 'elastic net' algorithm of Durbin and Willshaw (1987). In addition to generating near-optimal tours, or circuits, this method can be modified by substituting a line for the initial ring structure, by fixing a starting point only, or by specifying both starting and finishing points (Vickers & Lee, in press).

Hopfield-style Networks and the ZVT

In the case of the ZVT matrices, the implied pattern of costs cannot be embedded in any metric space. However, if the cost of traveling between non-connected cells is assumed to be infinite, and the total cost of a path is not required to be a minimum, then the ZVT can be regarded as a Hamiltonian path. We used a Hopfield-style network for the generation of Hamiltonian paths which pass through each of n arbitrarily interconnected points exactly once (Lee, Brown, & Vickers, in press).

Optimization, Perception, and Cognitive Efficiency

Both the TMT and the ZVT are reputedly sensitive indicators of cognitive efficiency, and show moderate to high correlations with intelligence. Since the construction of TMT and ZVT pathways can be thought of as the realisation of optimization processes, it may be useful to consider path-following in turn as mediated by the efficiency with which humans achieve near-optimal solutions to such problems.

Data are presented which indicate that most subjects spontaneously perceive a random visual array as structured in a way that constitutes a near-optimal pathway. Data from a study of human performance at solving visually presented TSP arrays are also presented, which show a modest correlation with intelligence.

These findings suggest that the optimization methods which we advocate for the generation of path-following test forms may also provide a useful conceptual framework for understanding and investigating human path-following, as well as variations in perceptual and cognitive efficiency.

References

Durbin, R., & Willshaw, D. (1987). An analogue approach to the Traveling Salesman problem using an elastic net method. *Nature*, *326*, 689-691.

Lee, M.D., Brown, M., & Vickers, D. (in press). Neural network and tree search algorithms for the generation of path-following (trail-making) tests. *Journal of Intelligent Systems*.

Vickers, D., & Lee, M.D. (in press). Never cross the path of a traveling salesman: The neural network generation of Halstead-Reitan trail-making tests. *Behavior Research Methods, Instruments, & Computers*.