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Publication Date

1973-07-06

Submitted to
Phys. Fluids Research Note

LBL-2088
Preprint c.1

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Prepared for the U. S. Atomic Energy Commission
under Contract W-7405-ENG-48

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SPACE-TIME INTERACTION OF OPPOSED TRANSVERSE

WAVES IN A PLASMA*

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July 6, 1973

ABSTRACT

The spatial and temporal interaction of two opposed, intense electromagnetic waves, whose difference frequency approximately equals the local plasma frequency, is studied. The interaction results in plasma heating. In a uniform plasma, the leading edge of the lower frequency wave steepens. For a nonuniform plasma, such pulse modification and concomitant heating occur only in a limited resonance zone.

The nonlinear interaction of coherent electromagnetic waves (lasers) in an underdense plasma ($\omega_p < \omega$) is of interest because of the potential for heating when the difference frequency of the two waves is near the local plasma frequency.¹⁻⁶ When this is the case, energy is partially transferred from one transverse wave to the other and is partially deposited in the plasma in the form of electrostatic plasma oscillations. In the present note we extend the work of Kaufman and Cohen⁶ by including the temporal evolution of the interaction.

We examine the transfer of energy between two transverse waves of frequencies ω_0, ω_1 , with $\omega_0 > \omega_1$. The transfer is driven at the beat frequency $\Omega \equiv \omega_0 - \omega_1$ and at the beat wave number $K \equiv k_0 + k_1$ (for opposed wave vectors). The plasma is considered to be infinite in extent. We assume that the dissipation rate ν of the plasma wave and the mismatch $\Delta\omega(z) = \Omega - \omega_p(z)$ are small, so the interaction is nearly resonant.

The analysis follows that of Ref. 6, employing the longitudinal dielectric function and using the same notation. Both the temporal and the spatial evolution of the intensities of the electromagnetic waves are studied. The electrostatic potential in response to a vector potential $A(z,t)$ is $\phi(z,t) \propto [\epsilon^{-1} - 1] A^2(z,t) + \text{noise}$. The behavior of the transverse waves will be analyzed over time scales long compared to ν^{-1} ; thus any potential $\phi(z,t)$ present at either the initiation or termination of the laser pulses can be ignored as a transient whose decay occurs in a time of the order of ν^{-1} . The generalization of Kaufman and Cohen's⁶ nonlinear coupled mode equation for the action fluxes in natural units is

$$(\partial_z + c^{-1}\partial_t)J_0 = (\partial_z - c^{-1}\partial_t)J_1 = -2\Gamma J_0 J_1, \quad (1)$$

where $\Gamma \equiv -4\pi \text{Im} \epsilon^{-1}(\Omega, K)$ and $J_\ell \equiv k_\ell |A_\ell|^2 e^2 / (2\pi m^2 c^4)$. The group velocities of the two electromagnetic waves have been set equal to c , since we assume $\omega \gg \omega_p$. Because $\Gamma > 0$ the higher frequency wave loses action flux J_0 as it propagates to the right (increasing z). The action flux J_1 of the lower frequency wave increases as it propagates to the left. The conservation law for action flux, from (1), is $\partial_t(J_0 + J_1) + \partial_z(cJ_0 - cJ_1) = 0$.

For a uniform plasma the solution of (1), subject to conditions that the ω_0 -wave is incident from the left and the ω_1 -wave from the right with step function profiles $\theta(ct - z)$ and $\theta(ct + z)$ respectively, is straightforward. The mismatch $\Delta\omega$ and damping ν are constant. One transforms to new variables $ct - z$ and $ct + z$, the characteristics of the operators of (1). The equations are then solved using the method of Maier, Kaiser, and Giordmaine.⁷ In terms of the input action fluxes, J_0^{in} and J_1^{in} , let $\rho \equiv J_1^{\text{in}}/J_0^{\text{in}}$, $\eta \equiv (ct - z)\Gamma J_0^{\text{in}}$ and $\xi \equiv (ct + z)\Gamma J_0^{\text{in}}$. The solutions for the relative action fluxes are

$$\bar{J}_0(t, z) \equiv J_0(t, z)/J_0^{\text{in}} = e^{-\eta} [e^{-\eta} + e^{\rho\xi} - 1]^{-1}, \quad (2)$$

$$\bar{J}_1(t, z) \equiv J_1(t, z)/J_0^{\text{in}} = \rho e^{\rho\xi} [e^{-\eta} + e^{\rho\xi} - 1]^{-1},$$

valid for $\eta, \xi \geq 0$. For a cold plasma, $\epsilon = 1 - \omega_p^2/(\Omega^2 + i\nu\omega_{ei})$, so that $\Gamma \approx 4\pi(\nu_{ei}/\omega_p)[(\nu_{ei}/\omega_p)^2 + (2\Delta\omega/\omega_p)^2]^{-1}$. The energy density deposited in the longitudinal wave, $8\pi(mc^2/e)^2 J_0 J_1 |\epsilon^{-1}(\Omega, \kappa) - 1|^2$, is ultimately available for plasma heating.

We consider the example of CO_2 lasers ($\omega_0 \sim 2 \times 10^{14} \text{ sec}^{-1}$) with $\omega_p/\omega_0 = 0.1$, $\nu_{ei}/\omega_p = 2\nu/\omega_p = 0.01$, pulse lengths $\sim 60\nu_{ei}^{-1}$, and $\Delta\omega \sim \nu_{ei}$. Significant transfer of action occurs when the dimensionless space and time variables are of order unity:

$\eta, \xi \sim \Gamma J_0^{\text{in}} ct = \omega_0 \nu^{-1} \omega_p t (P^0/2.5 \times 10^{16} \text{ W/cm}^2) = \mathcal{O}(1)$. Hence appreciable interaction occurs for intensities $P^0 \geq 2 \times 10^{10} \text{ W/cm}^2$.

Figure 1(a) shows the solutions (2) for $\rho = 1$. Dimensionless time and space variables $t' = \Gamma J_0^{\text{in}} ct$ and $z' = \Gamma J_0^{\text{in}} z$ are employed. The pulse shape of the ω_1 -wave peaks and sharpens as the ω_0 -photons are turned around and converted to ω_1 -photons. The

ω_1 -photons "pile up" as the ω_1 -wave propagates through the ω_0 -wave. Energy transferred to the plasma then grows as the ω_1 -wave front progresses to the left. Since the interaction rate depends upon the product of the actions, we have chosen $\rho = 1$ to maximize the action transfer.

Analytic solutions to (1) for a uniform plasma but with linearly varying leading edges for the two pulses are obtained similarly. If we define $J'_\ell \equiv |\partial_z J_\ell^{\text{in}}|$, $\sigma \equiv (ct - z)(\Gamma J'_0)^{\frac{1}{2}}$, and $\tau \equiv (ct + z)(\Gamma J'_0)^{\frac{1}{2}}$, the action transfer between the two transverse waves is described by

$$\begin{aligned} \bar{J}_0(t, z) &\equiv J_0(t, z)(\Gamma/J'_0)^{\frac{1}{2}} = \\ &= \sigma \exp(-\sigma^2/2) [\exp(-\sigma^2/2) + \exp(\rho\tau^2/2) - 1]^{-1} \end{aligned} \quad (3)$$

$$\begin{aligned} \bar{J}_1(t, z) &\equiv J_1(t, z)(\Gamma/J'_0)^{\frac{1}{2}} = \\ &= \rho\tau \exp(\rho\tau^2/2) [\exp(-\sigma^2/2) + \exp(\rho\tau^2/2) - 1]^{-1}, \end{aligned}$$

valid for $\sigma, \tau \geq 0$. Again marked amplification and "pile-up" of the ω_1 -wave occurs just behind its leading edge [Fig. 1(b)].

We next examine the case of a linear density inhomogeneity. We define the density scale length $L \equiv |d \ln n/dz|^{-1}$. The frequency mismatch now depends upon position: $\Delta\omega(z) \equiv \Omega - \omega_p(z) \approx \Omega z(2L)^{-1}$. It follows that $\Gamma(z) \approx 4\pi\nu_{ei}\Omega[\nu_{ei}^2 + (\Omega z/L)^2]^{-1}$. For a nonuniform plasma, $\Gamma(z)$ is large (coupling is appreciable) only in a narrow zone around $z = 0$ [$\Omega = \omega_p(0)$]. The width of this resonance zone has been defined as $h = 2\pi L\nu_{ei}/\Omega$ by Mostrom et al.⁸ We assume that $k_0 h \gg 1$ to permit a WKB analysis.

Again we follow the evolution of step-function and linear initial pulse profiles. Since $\Gamma \propto \nu^{-1}$ at the exact resonance position $z = 0$, while the resonance width $h \propto \nu$, the total action transfer is only a weak function of the damping ν . In absolute units the action transfer is linear in both the scale length of the plasma and the laser intensity (for $\rho = 1$).⁶ Again $\rho = 1$ gives the maximum transfer efficiency. Figure 1(c) and Fig. 1(d) display the numerical solution of (1) for $\nu/\Omega = 0.01$. Figure 1(c) is the case $h = 0.0025 (4\pi J_0^{\text{in}})^{-1}$, while $h = 0.005 (4\pi J_0^{\text{in}})^{-\frac{1}{2}}$ in Fig. 1(d). Nearly all pulse profile modification occurs within the resonance zone. For the step-function profiles a steady state for the conversion of the ω_0 -wave to the ω_1 -wave is achieved at times greater than $6h/c$ for the dimensionless parameter $h' = h 4\pi J_0^{\text{in}}$ chosen. There is no such steady state in the case of linear pulse profiles, because the unperturbed wave intensities are linearly increasing functions of time.

These solutions to the coupled mode equations (1) are valid as long as the transverse waves overlap within the plasma. For a finite length plasma the interaction can be very different; for example, the possibility of relaxation oscillation arises.⁹ However, for a non-uniform plasma, the interaction is appreciable only in the resonance zone. In this case the solutions remain valid as long as the region over which the transverse waves overlap contains the resonance region, regardless of the finite extent of the plasma.

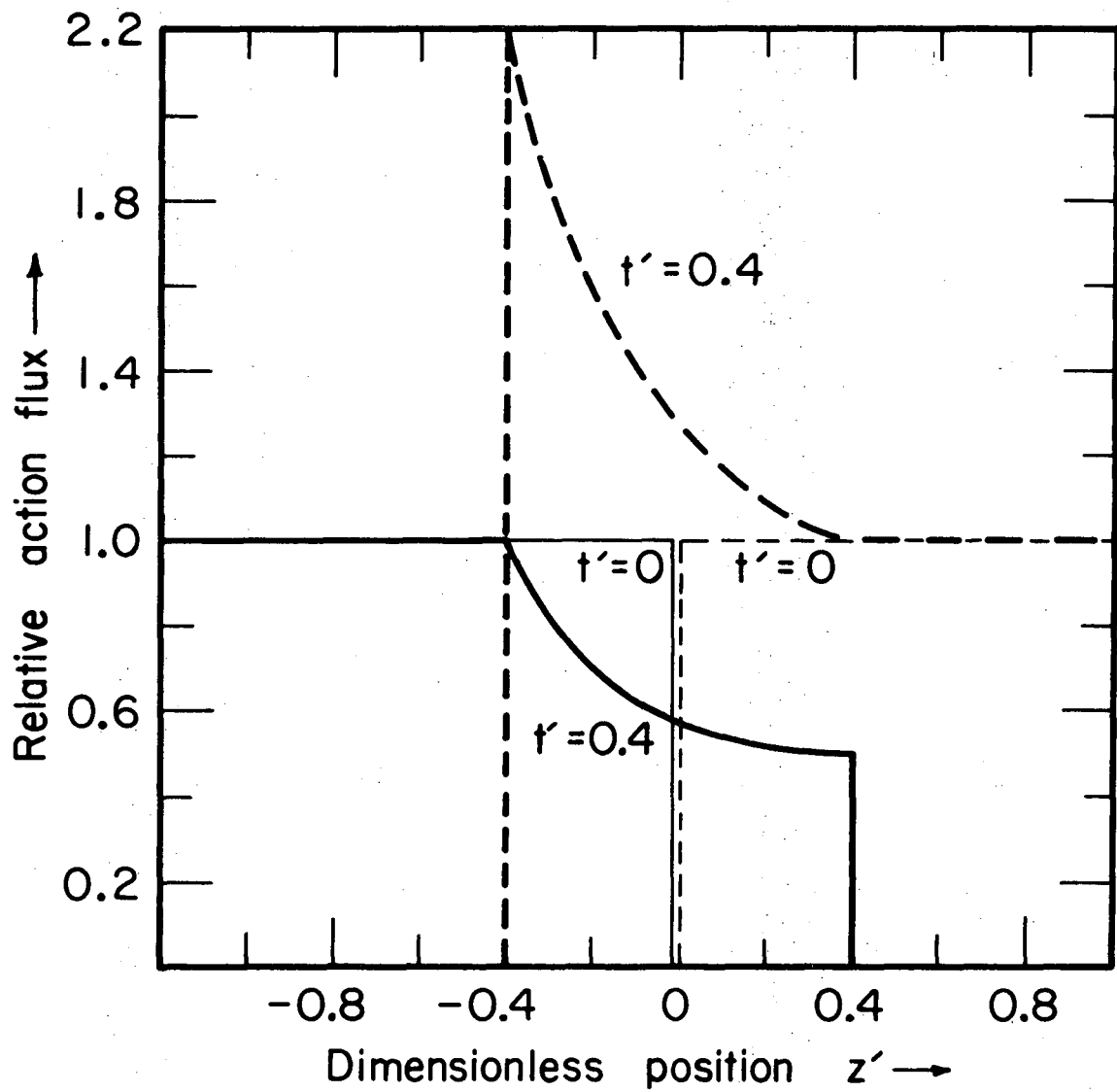
The author thanks A. N. Kaufman for his assistance and criticism and is indebted to D. Nicholson, C. Max, M. Mstrom, K. Watson, R. Pyle, W. Kunkel, Y.-R. Shen, and J. Dawson for their discussion and encouragement.

FOOTNOTES AND REFERENCES

- * Work supported by the U. S. Atomic Energy Commission, the U. S. Air Force Office of Scientific Research F44620-C-70-0028, and the National Science Foundation.
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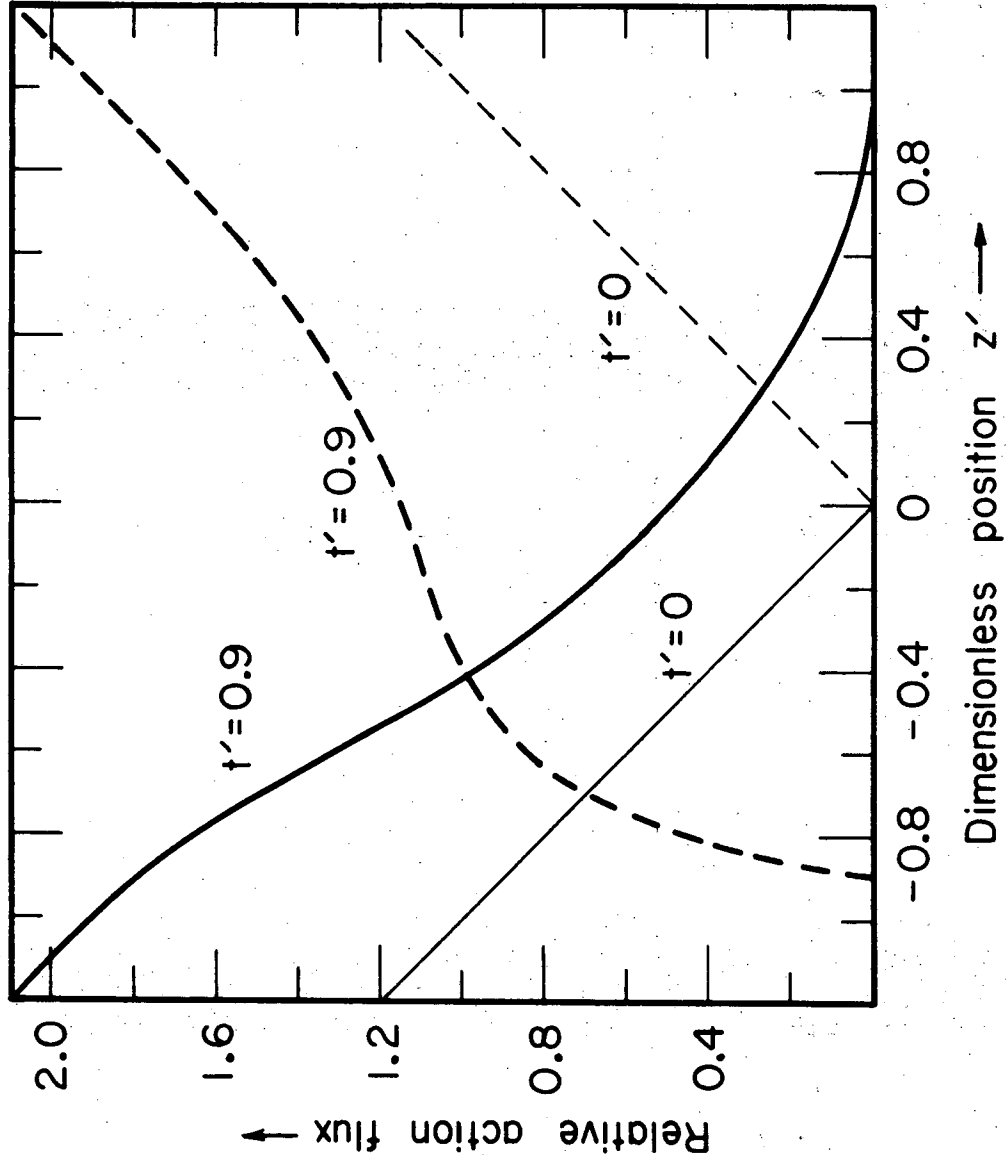
FIGURE CAPTION

Fig. 1. Dimensionless action fluxes $\bar{J}_0(t,z)$ ——, and $\bar{J}_1(t,z)$ ----, as functions of dimensionless time t' and position z' defined below, for input ratio $\rho = 1$. Case (a) shows step-function pulse profiles for a uniform plasma, with $t' = \Gamma J_0^{\text{in}} ct$ and $z' \equiv \Gamma J_0^{\text{in}} z$. Case (b) shows linear pulse profiles for a uniform plasma, with $t' \equiv (\Gamma J_0')^{\frac{1}{2}} ct$, and $z' \equiv (\Gamma J_0')^{\frac{1}{2}} z$. Case (c) shows step-function pulse profiles for a linear density variation, with $t' \equiv 4\pi J_0^{\text{in}} ct$ and $z' \equiv 4\pi J_0^{\text{in}} z$. Case (d) shows linear pulse profiles for a linear density variation, with $t' \equiv (4\pi J_0')^{\frac{1}{2}} ct$ and $z' \equiv (4\pi J_0')^{\frac{1}{2}} z$. In (a) and (b) for a uniform plasma, $\Gamma = 4\pi v \Omega [v^2 + (2\Delta\omega)^2]^{-1}$. In (c) and (d) for a linear variation of the plasma density, $h' = 0.0025$ and 0.005 respectively in the dimensionless units of z' .



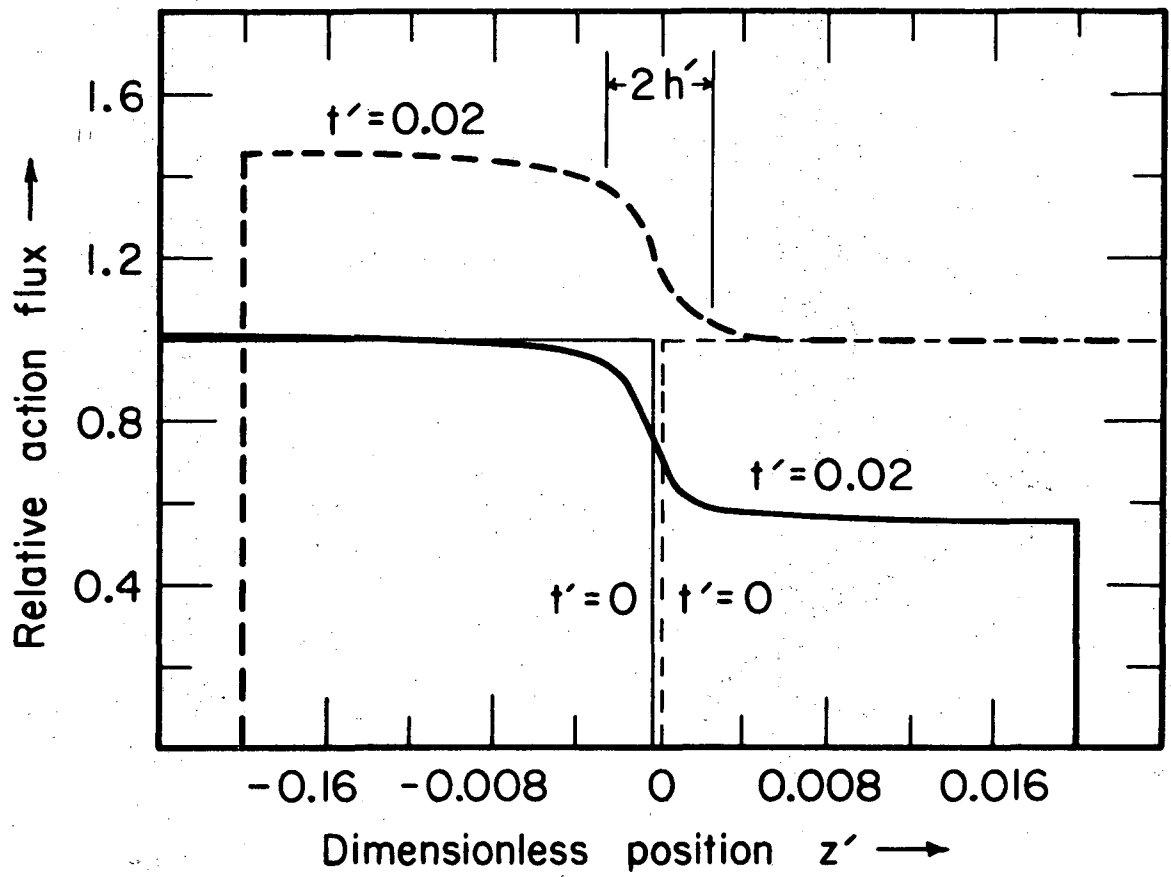
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Fig. 1a



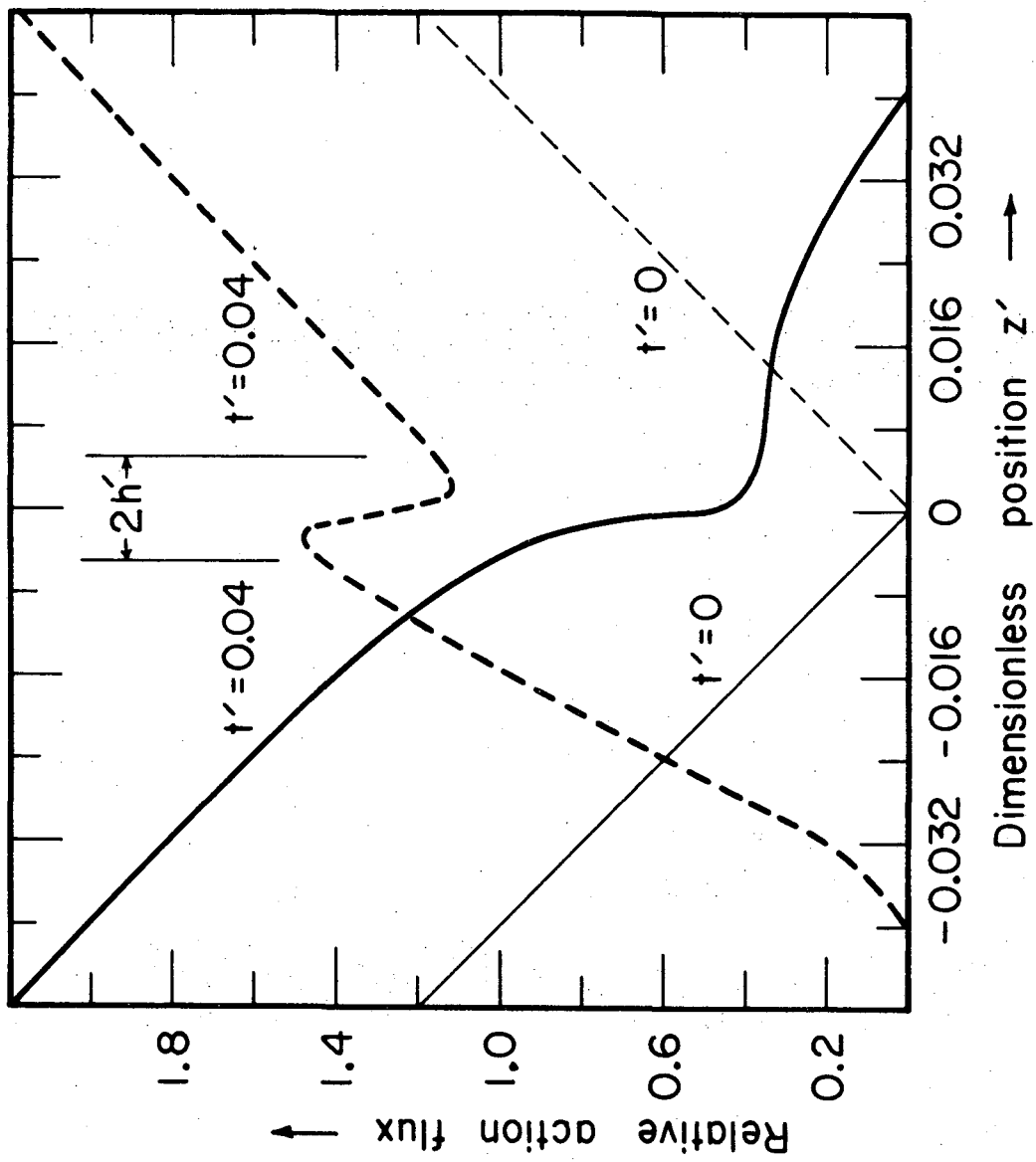
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Fig. 1b



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Fig. 1c



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Fig. 1d

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