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Tilted Cranking Classification of Multibandspectra

S. Frauendorf and F.R. May

June 1992

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Tilted Cranking Classification of Multibandspectra *

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June 19, 1992

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Abstract

The existence of TDHF-solutions rotating uniformly about a non-principal axis of the deformed axial potential is demonstrated. The solutions represent ΔI=1 bands. Selfconsistency and symmetry are discussed. The transformation of experimental spectra to the rotating frame of reference is introduced. Excitation spectra at high spin are calculated and found to agree well with recent data on ^{163}Er and ^{174}Hf.

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Tilted Cranking Classification of Multibandspectra

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1. Introduction

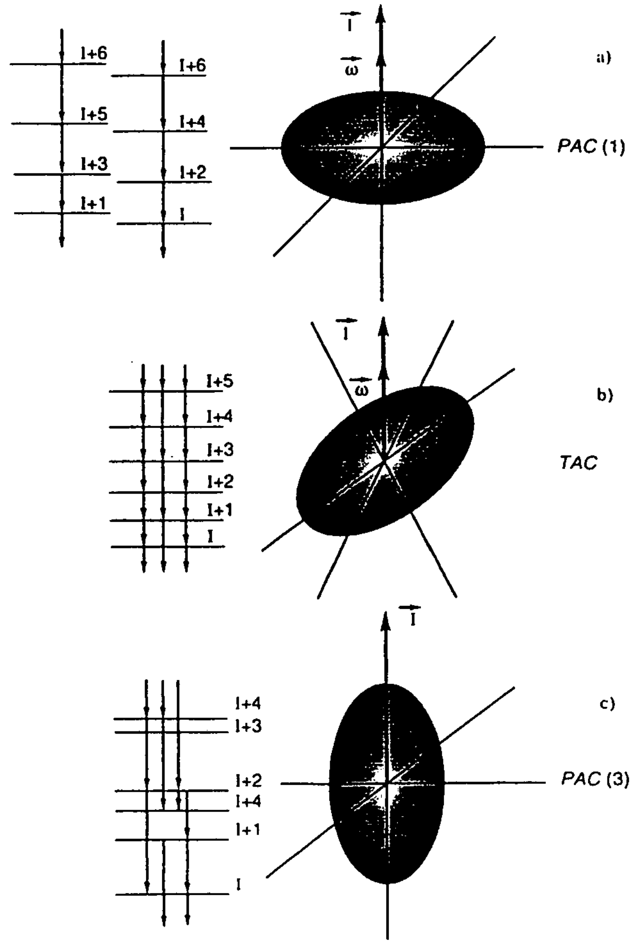

Fig. 1 Types of Rotation

Excited bands above the Yrast line can be sucessfully classified as quasiparticle configurations (conf.) in a deformed potential rotating uniformely about one of its principle axes. In the case of collective rotation this analysis is called CSM [1]. As sketched in fig. 1 a the bands are grouped into $\Delta I = 2$ sequences connected by fast stretched E2 transtions, each of which is interpreted as a quasiparticle conf. of fixed signature. In the case of rotation about the symmetry axis of the potential the method is referred to the as "sloping Fermisurface" [1]. As shown in fig. 1c, it describes the yrast spectrum of individual high spin states not forming bands. The states are irregularely spaced, connected by noncollective transitions and are often isomeric (yrast traps). They are the heads of of $\Delta I = 1$ bands of the type shown in fig. 1 b. In addition to the stretched E2 crossover transitions there are strong stretched M1 and $\Delta I = 1$ E2 transitions. We shall demonstrate that these very common bands may be interpreted as quasiparticle conf.s in a deformed potential that rotates *uniformely* about an axis that is tilted with respect to its principal axes PA. If the rotational axis is a principle axis we shall speak of principle axis cranking (PAC) if not of tilted axis cranking (TAC). The calculation of electromagnetic transition probabilities by means of TAC has been discussed in ref. [2]. We restrict ourselve to the energies of the $\Delta I = 1$ bands above the yrast line.

2. Selfconsistency and Symmetry

The meanfield solutions in a frame rotating uniformely about the z-axis are obtained by applying the HF approximation to the routhian

$$H' = H - \omega_z j_z = h_{sp} - \frac{\kappa}{2}\sum_\lambda q_\lambda q^*_\lambda - \omega_z j_z \qquad (1)$$

where we take the separable QQ-interaction. Then , the meanfield routhian is

$$h' = h_{sp} - \kappa \sum_\lambda Q^*_\lambda q_\lambda - \omega_z j_z \qquad (2)$$

and selfconsistency is reduced to the five relations for the components of the quadrupole tensor

$$Q_\lambda = < q_\lambda >, \qquad (3)$$

Here $| >$ denotes the single particle configuration. We define the PA 1,2,3 as the frame in which $Q'_1 = Q'_{-1} = 0$ and $Q'_2 = Q'_{-2}$, i. e. the quadrupole tensor is given by the two intrinsic moments Q'_0 and Q'_2 and the the angles ϑ and φ that fix the orientation of the PA relative to the z-axis,

$$Q_\lambda = D^{*2}_{0\lambda}(0,\vartheta,\varphi)Q'_0 + (D^{*2}_{2\lambda}(0,\vartheta,\varphi) + D^{*2}_{-2\lambda}(0,\vartheta,\varphi))Q'_2 \qquad (4)$$

There always exist PAC solutions for which the PA coincide with the lab axes x,y,z (fig. 1 a). However, there may be additional TAC solutions of lower energy whose PA are different from the lab axes (fig. 1 b).

The selfconsistent solutions $| >$ are stationary, i. e. $\delta E' = 0$. Consider the special variations generated by infinitesimal rotations about the x- and y-axes

$$\delta E' = i\delta\phi < [H', j_x] > = \delta\phi < j_y > = 0 \qquad (5)$$

$$\delta E' = i\delta\phi < [H', j_y] > = -\delta\phi < j_x > = 0 \qquad (6)$$

which show that for the selfconsistent solutions

$$I_x =< j_x >= 0, \quad I_y =< j_y >= 0, \quad I = I_z =< j_z > \tag{7}$$

Hence, selfconsistency ensures that the axis of rotation $\vec{\omega}$ and the expectation value of angular momentum $\vec{I}$ are parallel, what is the well known condition for uniform rotation in classical mechanics. The stationarity of $|\ >$ implies also the familiar canonical relations

$$\frac{dE'}{d\omega} = -I, \quad \frac{dE}{dI} = \omega, \quad E' = E - \omega I \tag{8}$$

Considering $E' =< H' >$ as a function of the Q_λ, its extrema are the the solutions of the selfconsistency eqs. (3). According to eq. (4) we may find the minima in two steps: For given Q'_0, Q'_2 we minimize E' with respect to the orientation angles ϑ, φ. Then, $E'(Q'_0, Q'_2)$ is minimized with respect to the deformation parameters Q'_0 and Q'_2. In the following we consider stiff nuclei, i. e we assume that the deformation parameters Q'_0 and Q'_2 are not changed by rotation. Hence, we keep them fixed to the values found for the ground state and to minimizes E' only with respect to the orientation angles. Restricting ourselve to axial nuclei the deformed meanfield routhian reads in the PA system

$$h' = h_{sp} - \kappa Q'_0 q_0 - \omega(sin\vartheta j_1 + cos\vartheta j_3) \tag{9}$$

where ϑ measures the angle between the symmetry and the rotational axes. We choose h_{sp} such that h becomes the Nilsson hamiltonian with a monopole pairfield. The equilibrium angles are found by minimizing

$$E'(\omega, \vartheta) =< h' > \tag{10}$$

with respect to ϑ. The condition for uniform rotation, $\vec{I} \| \vec{\omega}$, is fullfilled at these points (c. f. ref. [2] for a more explicite discussion). A sequence of equilibrium confs. $|\omega >$ is associated with a rotational band.

Due to the symmetry of the deformed potential the parity π is a good quantum number independent of the orientation of the rotational axis. The PAC solutions can additionally be classified by the signature α, which is the eigenvalue of the rotation $\mathcal{R}_1(\pi) = e^{i\alpha\pi}$. The fact that $\mathcal{R}_1(\pi)$ defines a subgroup of the full rotational rotational group has the consequence that $I = \alpha \ mod\ 2$. Each quasiparticle configuration represents a ΔI =2 band with given π (c. f. fig. 1 a) Since TAC solution are not eigenfunctions of $\mathcal{R}_1(\pi)$ they cannot be classified by signature and there is no spin selection rule. Each qp conf. considered as function of the frequency ω represents a ΔI =1 rotational band of given π (c. f. fig. 1 b) The absence of signature splitting is the experimental evidence for the observation of a TAC solution.

3. The TAC Spectrum

Fig. 2 shows the single neutron spectrum obtained by diagonalizing the routhian (9) with a zero pairfield. The occupation corresponds to ^{174}Hf which has 102 neutrons. In the left part the nucleus is spinned up from $\omega = 0$ to $\hbar\omega = 0.3 MeV$ along the symmetry axis 3. When levels cross the Fermilevel, the yrast conf. changes. The different conf.s encountered may be characterized by the angular momentum component $I_3 = K$, which is a good quantum number. The yrast sqeunce is : $I_3^\pi = 0^+,\ 6^-,\ 10^-$ and 15^-.

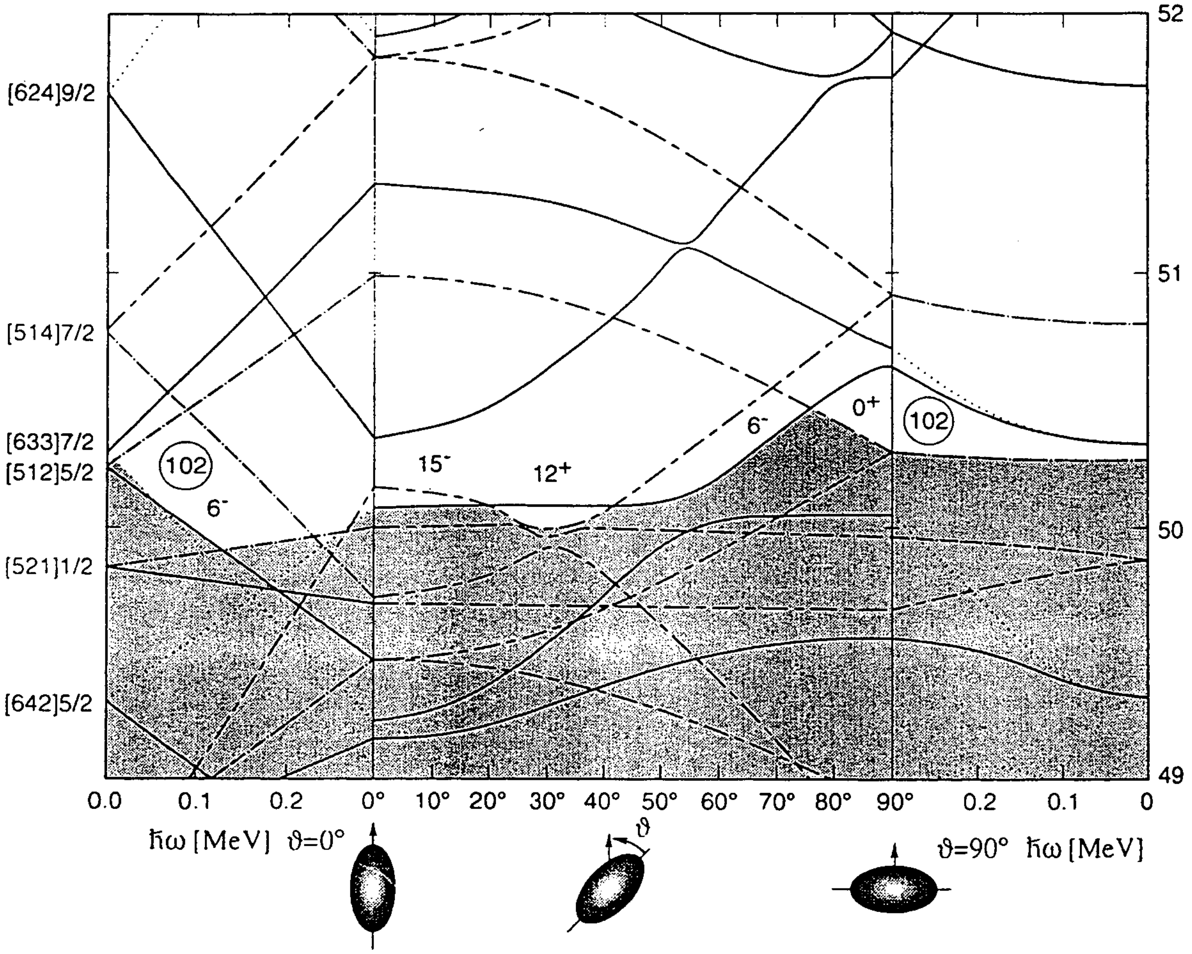

Fig. 2 Single neutron routhians. In the left and right parts the levels are classified with respect to (π, α) using the convention (+,1/2) full, (+,-1/2) dotted, (-,1/2) long dash short dash, (-,-1/2) dash dotted. In the middle full lines indicate positve dashed negative parity. The slopes of the lines are $-i_3$, $-\omega i_\perp$ and i_1 in the left, middle and right part, respectively. The shaded area indicates the occupied levels for N=102 and I_3^π (rounded to integers) is given. The deformations are $\varepsilon = 0.26$ and $\varepsilon_4 = 0.035$.

In the middle part the rotational axis turns from the 3-axis ($\vartheta = 0°$) to the collective 1-axis ($\vartheta = 90°$). Again the Fermi level crosses with other levels resulting in changes of the yrast conf. . Now I_3 is no longer conserved. Nevertheless it is a good way to characterize the conf.s. The yrast sequence is 15^-, 12^+, 6^- and 0^+, where I_3 is *rounded to to the next integer* . In the right part the nucleus rotates about the collective 1-axis where $\hbar\omega$ decreases from $0.3 MeV$ back to 0. The yrast conf. has always 0^+ and has the structure of the neutron s-conf. (Although $I_3 = 0$ exactly, it is not a good quantum number.)

Obviously there exist several minima of $E'(\omega, \vartheta)$ with respect to ϑ. They are easily found by minimizing the routhians of the p-h confs. generated from the levels connected diabatically through the crossings . The physical interpretation is that each minimum at $\vartheta > 0$ represents a rotational band. If $\vartheta = 90^O$ it is a PAC solution with good signature restricting the spin of the $\Delta I = 2$ band to $I = \alpha + 2n$. If $\vartheta < 90°$ the minimum is a TAC solution corresponding to a $\Delta I = 1$ band. [1]

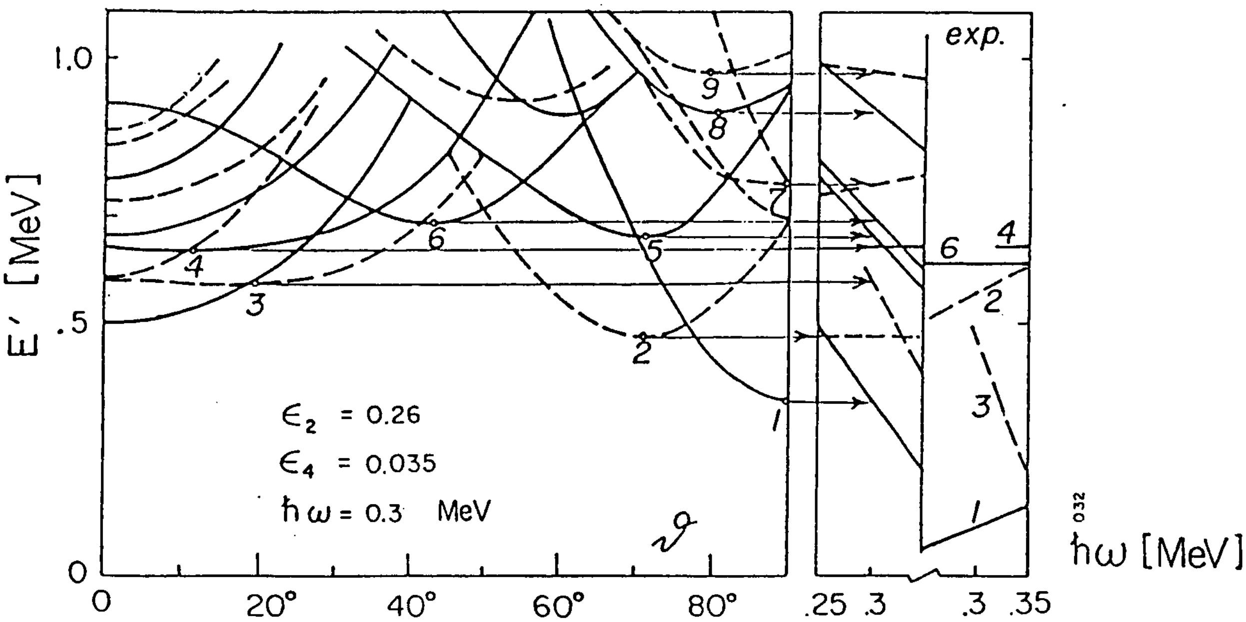

Fig. 3 Total routhians in ^{174}Hf as functions of the tilting angle ϑ (left) and the excitation spectrum as function of the frequency ω (right). Full lines indicate positve dashed negative parity. The energy zero is chosen such that the calculated band 4 agrees with the experimental one. All slopes are reduced by $I = 16\hbar$. The data is from N. Gjorup et al. [4].

Fig. 3 shows the lowest minima in ^{174}Hf obtained by combining the neutron

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[1] As for any symmetry breaking, the restoration of the signature for $\vartheta \rightarrow 90^O$ cannot completely be described by the meanfield approach used here. A more elaborate discussion of this question is given in ref. [2]

conf.s generated from the levels in fig. 2 with the lowest proton conf.s. For $\hbar\omega \geq 0.3 MeV$ the neutron excitatation spectrum is qualitatively reproduced assuming zero pairing (as in fig. 2). However one must include the paired g-conf. into the proton spectrum, since it is still the lowest one. [2] In addition to it only the 8^- proton conf. becoming the $9/2[514], 7/2[404]^{-1}$ at $\vartheta = 0$ comes into the play. Let us charcterize of the equilibrium conf.s by $(I_3^\pi)_\pi(I_3^\pi)_\nu$. The enumerated minima have the structure: $1: 0^+0^+$, $2: 0^+6^-$, $3: 8^-12^+$, $4: 8^-10^-$, $5: 0^+7^+$, $6: 8^-6^-$, $7: 0^+0^-$, $8: 0^+3^+$, $9: 0^+3^-$.

The minima at $\vartheta = 0$ do not belong to a band since the wavefunction does not depend on ω and consequently no angular momentum is gained if ω grows. The band starts at the frequency where the curvature becomes zero and the minimum moves to positive values of ϑ. This defines the bandhead.

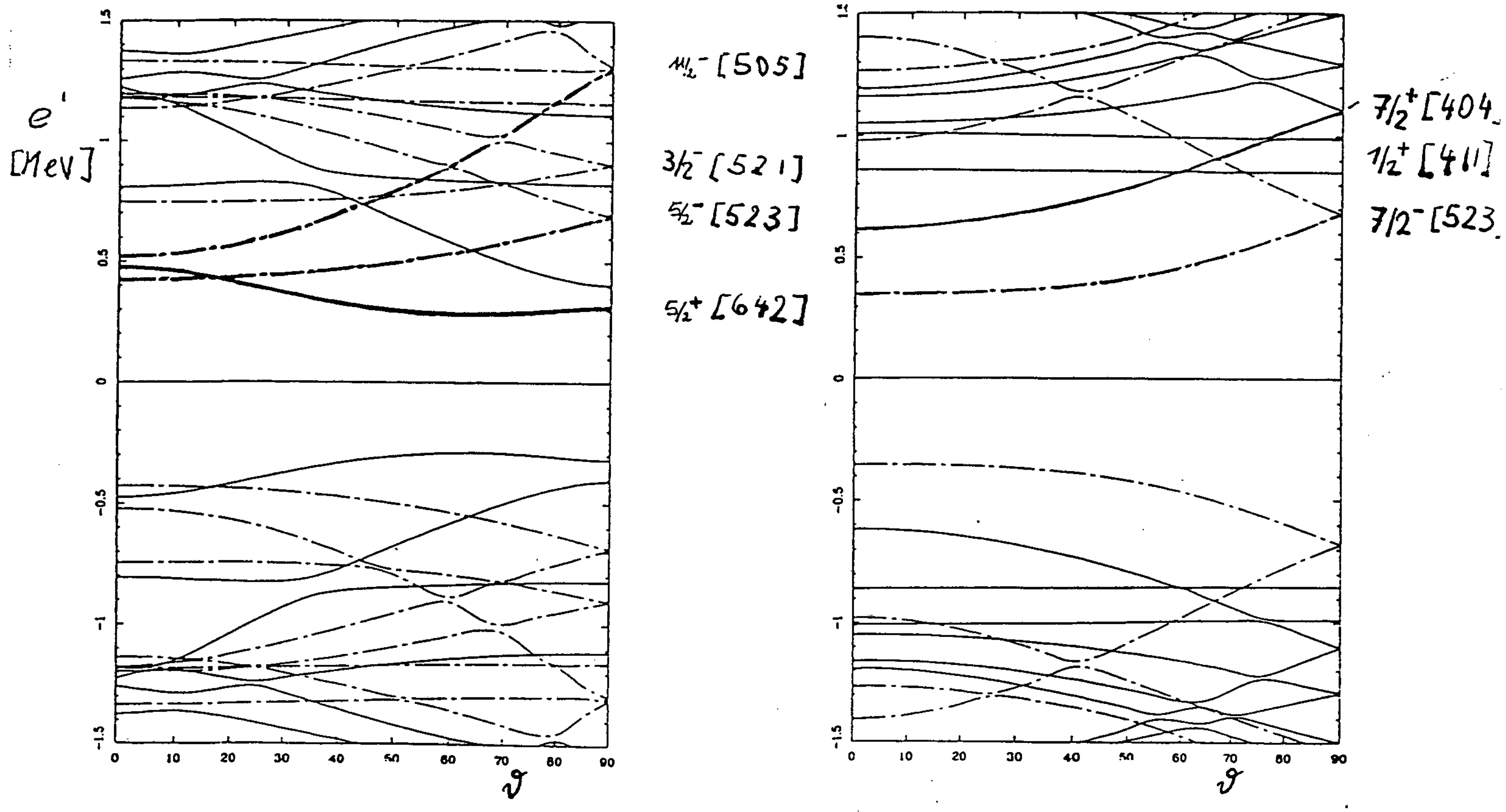

Fig. 4 Quasiparticle routhians for N=95 (left) and Z=68 (right). Full lines correspond to positive dashed dotted to negative parity. The parameters are $\hbar\omega = 0.15 MeV$, $\varepsilon = 0.252$, $\varepsilon_4 = 0$, $\Delta_\nu = 0.80 MeV$ and $\Delta_\pi = 0.87 MeV$.

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[2] It is calculated with $\Delta = 0.8\Delta_{eo}$ fixing the energy zero by identifying the s-conf. with the unpaired 0^+ yrast conf. at $\vartheta = 90^o$.

Fig.4 shows the quasiparticle levels as functions of the tilting angle for strong pairing. The parameters are appropriate for ^{163}Er. At the relatively low frequency of $\hbar\omega = 0.15 MeV$ the 0^+ g-conf. is always lowest. The spectrum in the N=95 Z=68 system is generated by putting 1,3, ... quasinuetrons (qn) on the neutron and 0, 2, ... quasiprotons (qp) on the proton levels. Again we construct diabatic confs., which we minimize with respect to ϑ. The routhians resulting from the minimizations at $\hbar\omega = 0.15$ and $0.30 MeV$ are given by the arrows in fig. 5 , which point into the direction of the rotational axis. It is seen how ϑ approaches 90^o along a band. The structure of the lowest configurations is:

- Band 1 PAC, 0qp 1qn on $\alpha = 1/2$, $i_{13/2}$
- Band 2 PAC, 0qp 1qn on $\alpha = -1/2$, $i_{13/2}$
- Band 3 TAC, 0qp 1qn on $5/2^-[523]$
- Band 4 TAC, 0qp 1qn on $11/2^-[505]$
- Band 5 TAC, 2qp on $7/2^-[523]$, $7/2^-[404]$, 1qn on the lowest $i_{13/2}$
- Band 5 TAC, 2qp on $7/2^-[523]$, $7/2^-[404]$, 1qn on $5/2^-[523]$

4. Experimental routhians

The canonical relations imply how to obtain experimental frquencies and routhians:

$$\hbar\omega(\bar{I}+1/2) = (E(I+\Delta I) - E(I))/\Delta I, \quad \bar{I} = I + \Delta I/2 \tag{11}$$

$$E'(\bar{I}+1/2) = \frac{1}{2}(E(I+\Delta I) + E(I)) - \omega(\bar{I}+1/2)(\bar{I}+1/2) \tag{12}$$

One may always use $\Delta I = 2$. For well developed TAC bands with no signature splitting $\Delta I = 1$ is better since the discrete datapoints are twice as dense and ω is just equal to the transition energy. Note, the procedure differs from the standart PA CSM [1]: There, the component ω_1 is calculated, here we determine the total ω. Fig. 5 shows the experimental routians $E'(\omega)$ obtained from the recent ^{163}Er data measured with the NORDBALL [3].

The excitation spectrum at a fixed ω can directly be compared with the $E'(\omega)$ calculated for the different minima relative to the lowest minimum. This is done in fig. 5 and the right part of fig.3, which is based on a plot similar to fig. 5 calculated from the recent NORDBALL data on ^{174}Hf [4] (c. f. abstract B3). In addition to the positions the slopes can also be calculated. According to eq. (7) the negative slope is the total spin $I + 1/2 = \sqrt{I_1^2 + I_3^2}$. Making the assignments to the experimental bands we use the parity, the ΔI characteristics (1 for TAC, 2 for PAC), the positions E', the slopes I, and the bandhead spins. (The last are often close to I_3., However, as discussed below, this needs not to be the case if easily alignable quasiparticles like $i_{13/2}$ are involved .) Both for ^{163}Er and ^{174}Hf the calculations account fairly well for the observed spectra. In particular the bandhead frequencies of bands 3 and 4 in ^{174}Hf , which happen to start inside the calculated window, are predicted right. The strongest disagreement occurs for the PAC band 1 in ^{174}Hf , which is the combination $g_\pi s_\nu$. This is not surprising since we assume zero pairing for the s_ν-conf..

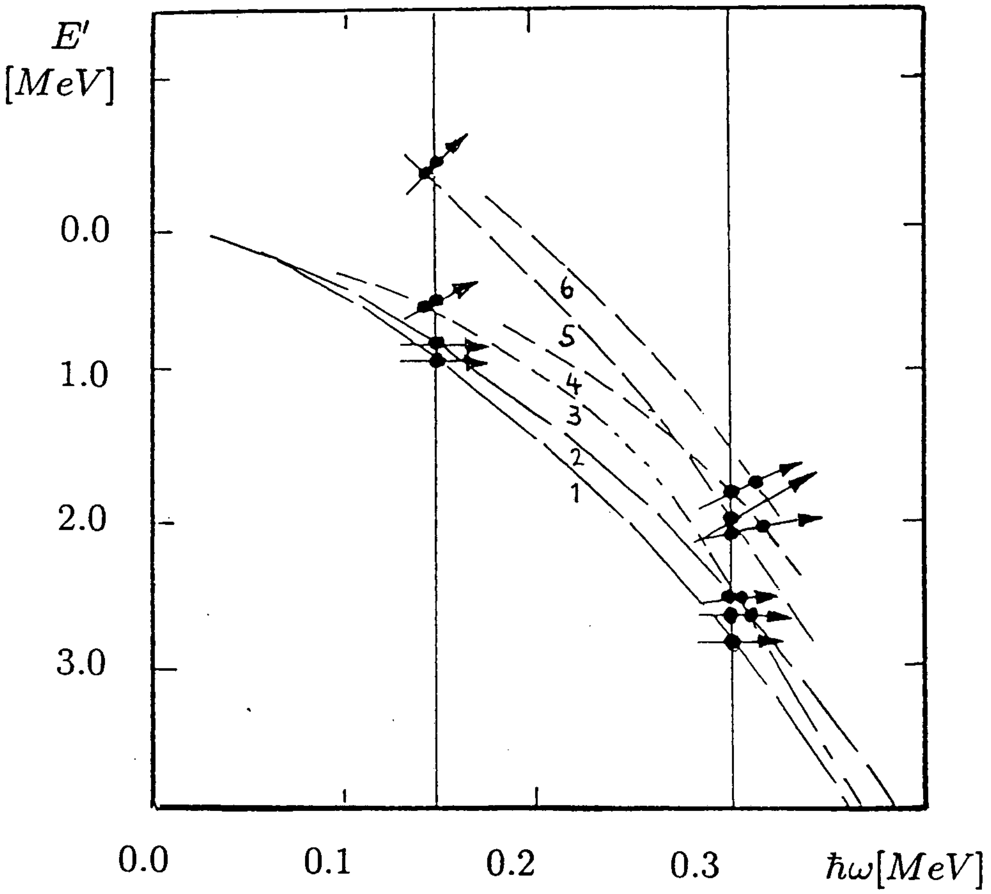

Fig. 5 Experimental routhians in ^{163}Er as calculated by eqs. (11) and (12). The data points lie at the spaces between the lines. Bands 1 and 2 are $\pi = +$ PAC bands. The other are $\pi = -$ TAC bands. The data is from A. Brøksted et al. [3]. The arrows represent the results of calculations at $\hbar\omega = 0.15$ and $0.30 MeV$. The dot at the vertical line gives the excitation energy. The energy zero is chosen such that the calculated band 1 agrees with the experimental one. The dot at the intersection with the experimental curve indicates the band the arrow is assigned to. The angle between the arrow and the vertical line is the tilting angle ϑ.

It should also be kept in mind that the scale in the right part of fig. 3 is very blown up. The differences in slope cover only $4\hbar$. Note the TAC band 3 in ^{174}Hf , which both in the calculations and in experiment becomes yrast slightly above $\hbar\omega = 0.35MeV$ [4].

5. Response of the Particles to Tilting

If the excited quasiparticles are strongly coupled to the deformed field, the tilted cranking becomes the semiclassical version of the well known rotational model for bands with finite K [5] . The levels $11/2^-[505]$, $7/2^+[404]$, $7/2^-[523]$ are examples. Their routhians change with the strong coupling function $-K cos\vartheta$ since the quasiparticle angular momentum is aligned with the deformation axis (DAL). The progess achieved by our method consist in the equally simple description of confs. that contain quasiparticles reacting sensitivly to the inertial forces. The $i_{13/2}$ qn are the well known example in the rare earth region. In fig. 4 the second $\pi = +$ level has a its minimum at $\vartheta = 90^\circ$, i. e. it is rotational aligned (RAL) although it is based on the $5/2^+[642]$ state at zero ω. The lowest $\pi = +$ level has its minimum at about 60^o meaning that its angular momentum vector points in this direction. It is an example of the Fermialigned coupling (FAL) suggested in ref. [6]. The 1qn conf. 1 is a PAC band since the collective rotation of the vacuum (core) drives strongly towards 90^o. In band 5 the same qn is combined with the 2 DAL qps resulting in a TAC solution which has quasiparticle angular momentum components along *both* the 1- and the 3-axes. This is demonstrated in a dramatic way by the fact that band 5 has a large value of ϑ already at the bandhead (c. f. fig. 5). It is an example of a band that does not start with $I = K$ as expected for strong coupling. A further combination of FAL and DAL quasiparticles are the the t-bands [7] becoming yrast in N=105, Z=74,76 nuclei (c. f. abstract B12). The M1-transition rates depend sensitivly on the orientation of the quasiparticle angular momentum [2]. They may be used to explore the geometry of the angular momentum coupling.

Another conspicuous feature is the complete decoupling of the pseudospin seen for the $\tilde{\Lambda} = 0$ states $1/2^-[521]$ in fig. 2 and $1/2^+[411]$ in fig. 4. In both cases the trajectories are horizontal , i. e. $i_\perp = 0$ and $\vec{s}$ follows $\vec{I}$ when it moves towards 90^o along the band. The consequences of this behaviour remain to be explored.

6. Summary

Tilted Cranking is a microscopic theory which treats the alignment of quasiparticle angular momentum with both the rotational and symmetry axis on equal footing. Compared with the standart CSM treatment of K [1] it *calculates* I_3 instead of making assumptions. Experimental routhians are defined which depend *only* on the experimental energies. High K- bands lying low above the yrast line have also low lying routhians. The disadvantage compared to the standart CSM is that each conf. has a different tilting angle (selfconsistency in orientation) and, hence, one quasiparticle diagram is not enough to classify the excitation spectrum. We demonstrate that the theory quantitatively accounts for the experimental excitation spectrum using some low band as a reference. Allowing for selfconsistency in the other important dergrees of freedom (Δ, ε, γ) one may also calculate the total routhians. In addition to energies the theory provides the electromagnetic as well as other intraband transition matrixelents. New structures are found combining quasiparticles that are strongly coupled to the deformed potential with ones reacting sensitivly to the inertial forces.

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