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About thirty-five nuclear moments in the rare earth region have been determined from paramagnetic resonance and atomic beam data. With the exception of europium, these determinations rest entirely on hyperfine-structure data and on assumptions made concerning the electronic fields at the nucleus. These assumptions can be summarized briefly as follows: (a) For the free atom, that the angular part of the wave function is amply described by assuming pure Russell-Saunders coupling among the electrons of the configuration $(4f)^n$ to the Hund's Rule ground term. (b) That the degree of configurational admixture is small, and its effect is less than about 5%. (c) The value of $\langle 1/r^3 \rangle$ for 4f electrons.

The evidence for assumption (a) comes from the observation that the free-atom g_J values lie very close to the pure Russell-Saunders values.[1] The theoretical analysis of the measured g_J values has yielded a set of parameters that permits accurate calculation of the correction to the hyperfine fields due to the breakdown of Russell-Saunders coupling.[2] These calculations have been performed for praseodymium[3] and promethium,[4] and (by us) for erbium. In all cases, the correction is of order 1%. Lindgren has neglected this effect in a recent recalculation of rare-earth moments.[5]

The effect of configurations involving unpaired s electrons on the hyperfine fields of triply ionized rare earths has been investigated by Bleaney.[6] He estimates that the correction for all rare earths is small, the maximum being about

6% for terbium. Lindgren believes that the effect should be even smaller for neutral rare earth atoms, and assigns an error of 5% in his calculations for this effect.

There is disagreement in the literature concerning the correct values of $\langle 1/r^3 \rangle$. Bleaney has derived values for the triply ionized rare earths on the basis of hydrogenic functions with an estimated uncertainty of about 5%. Values have since been obtained for both the triply ionized and neutral atoms, by use of modified hydrogenic functions.[2,5] These values differ from Bleaney's results by about 15 to 25%, and are also estimated to be uncertain to within about 5%. Hence the nuclear moments of an isotope determined from hyperfine-structure data can differ by as much as 25%, depending on whose values are used. The most recently reported work is that of Freeman and Watson, who have made nonrelativistic Hartree-Fock calculations for rare earth ions.[7] They report values lying within 5% of Bleaney's, but they conclude that rather substantial uncertainties are to be associated with the moments derived from any theoretical $\langle 1/r^3 \rangle$ values.

To obtain experimental information about these questions we have undertaken to measure by atomic beam magnetic resonance the hyperfine-structure constant, A, and the magnetic moment, μ_I, for Er^{169}. We find A = 725.46(31)Mc and, for the directly measured moment, μ_I=0.513(25)nm, where the nuclear moment has been corrected for diamagnetic shielding. From the A constant and the value of $\langle 1/r^3 \rangle$ derived from modified hydrogenic functions, we obtain μ_I = 0.504(50) nm. This agreement with the directly measured value is excellent, and is interpreted by us as an important verification of Bleaney's idea that configuration admixing is not appreciable in rare earths, and of the $\langle 1/r^3 \rangle$ values of Judd and Lindgren.[2]

Prior work on erbium-169 had determined the ground-state spin (I = 1/2)

and the electronic angular momentum (J = 6). The hyperfine structure of such a system is indicated in Fig. 1, and the energy levels are described by the Breit-Rabi Hamiltonian:

$$\mathcal{H} = A\mathbf{I}\cdot\mathbf{J} - g_J\mu_o\,\mathbf{J}\cdot\mathbf{H} - g_I\mu_o\,\mathbf{I}\cdot\mathbf{H}.$$

The effect of the term in g_I on the erbium transition frequencies is estimated to be a few parts in 10^5, but is not negligible with an apparatus of sufficient precision. Our ability to determine g_I for erbium rests on the good line width of our apparatus and on the ability to form very stable erbium beams which give highly reproducible data with good signal-to-noise ratio. A sample resonance curve is shown in Fig. 2.

Data were taken on the two transitions, which are observable by the flop-in method and are indicated in Fig. 1. The magnetic field was calibrated by observing the flop-in potassium resonance. It was found that, with this method of calibrating, the pulsating magnetic field of the nearby Bevatron caused small frequency shifts. All our data were obtained when the Bevatron was off.

The best fit to our data was obtained by a least-squares calculation on the IBM 709. The parameters best fitting our data are: A=725.46(31)Mc, $g_J = -1.1638(1)$, and $g_I = 5.55(27)\times 10^{-4}$, where the error given for A and g_I is twice the standard deviation. Frequencies calculated by use of these parameters are compared with the observed frequencies in Table I.

The observed g_I value must be increased by a factor 1.0078 to correct for diamagnetic shielding.[8] This gives the true nuclear moment $\mu_I = 0.513(25)$nm.

We have investigated the possibility that there are contributions to the measured g_I value that arise from other sources than the true nuclear moment. Let us write, for the nuclear moment term, $g_I(1+a)\mathbf{I}\cdot\mathbf{H}$, where g_I is the true

nuclear gyromagnetic ratio and a arises from possible perturbations. The possible contributions to a that we have considered are (1) systematic errors, which cause the true magnetic field to be different from the measured field with an error proportional to the field (such an error arises, for example, if the field seen by the calibrating isotope differs from the field traversed by the erbium beam by an amount proportional to the field); (2) mixing of the fine-structure levels of the ^{3}H term by the hyperfine structure of erbium in such a way as to introduce a pseudo $\Gamma \cdot \bar{H}$ term in the Hamiltonian.

The possibility of systematic errors in the field has been investigated by us and is found to be completely negligible (i.e. $< 10^{-6}$). We have calculated fine-structure mixing and find it to be less than 1%. Hence to within 1% our measured g_I is felt to be the true value.

From the A constant, the magnetic moment can be inferred from the relation

$$\mu_I = -IJA/\langle \bar{H} \rangle_{J, m_J = J},$$

where $\langle \bar{H} \rangle$ is the expectation value for the magnetic field at the nucleus and is given by

$$\bar{H} = -2\mu_o \left\langle \frac{1}{r^3} \right\rangle \left\langle \sum_i \left(\bar{l} - \bar{s} - \frac{3\bar{r}(\bar{s}\cdot\bar{r})}{r^2} \right)_i \right\rangle.$$

For $\langle 1/r^3 \rangle$ we have used the value of 9.84 atomic units given in reference 5. The evaluation of $\sum_i$ depends on the assumptions made about the angular part of the wave function. If the ground configuration is assumed to be $(4f)^{12}$, only ^{3}H and ^{1}I can give states with $J = 6$, and the ground-state wave function will be of the form

$\psi = \left[1-a^2\right]^{1/2} \left|{}^3H_6\right\rangle + a\left|{}^2I_6\right\rangle$. The value of a can be determined by diagonalizing the Coulomb and spin-orbit energies for J = 6. The Coulomb energy will be characterized by three Slater radial integrals, F_2, F_4, and F_6, while the spin-orbit energy is characterized by the parameter a_{4f}. These parameters have been reliably evaluated for erbium by Judd and Lindgren,[2] and we use their results. We obtain for a the value - 0.094. The contributions to $\left\langle \sum_i \right\rangle$ come from three terms: $\left\langle \sum_i \right\rangle = (1-a^2)\left\langle {}^3H_6 \left| \sum_i \right| {}^3H_6 \right\rangle + 2a\left\langle {}^3H_6 \left| \sum_i \right| {}^1I_6 \right\rangle + a^2 \left\langle {}^1I_6 \left| \sum_i \right| {}^1I_6 \right\rangle$. The first term represents the contribution to the field from assumption (a) of the first paragraph lowered by an amount proportional to a^2. The other terms give the corrections due to the breakdown of Russell-Saunders coupling. The net value of $\left\langle \sum_i \right\rangle$ is only about 1% different from that obtained from assumption (a), owing to fortuitous canceling in the last two terms. Therefore the moment inferred from the A value is μ_I = 0.504 nm.

The value of the Er^{169} moment can be compared to a predicted value of 0.7 based on the Bohr-Mottelson Nuclear Model. We have assumed that the state of the 101^{st} neutron is characterized by [5 2 1 1/2] and a deformation parameter $\delta \approx 0.3$.

REFERENCES

1. A. Y. Cabezas, I. Lindgren, and R. Marrus, Phys. Rev. 122, 1796 (1961).
2. B. R. Judd and I. Lindgren, Phys. Rev. 122, 1802 (1961).
3. A. Y. Cabezas, I. Lindgren, R. Marrus, and W. A. Nierenberg, Phys. Rev. 126, 1004 (1962).
4. Burton Budick, (thesis), Lawrence Radiation Laboratory Report UCRL-10245, May 23, 1962 (unpublished).
5. I. Lindgren, Nuclear Phys. 32, 151 (1962).
6. B. Bleaney, Proc. Phys. Soc. A68, 937 (1955).
7. R. E. Watson and A. J. Freeman, J. Appl. Phys. Suppl. 33, 1086 (1962).
8. W. E. Lamb, Phys. Rev. 60, 817 (1941).

Table I. Comparison of the observed frequencies with the frequencies predicted from the Hamiltonian (in Mc.) $725.46\,\vec{I}\cdot\vec{J} + 1.1638\,\mu_o\,\vec{J}\cdot\vec{H} - 5.55\times10^{-4}\,\mu_o\,\vec{I}\cdot\vec{H}$.

Field (gauss)	Predicted frequency (Kc.)	Observed frequency (Kc.)	Residual (Kc.)	Transition
467.595	704496.5	704499.0(4.0)	2.5	F = 13/2
504.329	760020.2	760020.0(4.0)	-0.2	F = 13/2
540.903	815336.0	815332.0(4.0)	-4.0	F = 13/2
540.903	815336.0	815336.0(2.4)	0.0	F = 13/2
613.698	925539.9	925533.0(4.0)	-6.9	F = 13/2
649.962	980996.1	980995.0(2.4)	-1.1	F = 13/2
686.158	1035387.7	1035388.0(2.4)	0.3	F = 13/2
722.298	1090232.4	1090237.0(2.4)	4.6	F = 13/2
722.298	1090232.4	1090235.0(4.0)	2.6	F = 13/2
758.390	1145044.8	1145044.0(2.4)	-0.8	F = 13/2
794.442	1199836.1	1199835.0(2.4)	-1.1	F = 13/2
196.279	344409.4	344411.0(5.6)	1.6	F = 11/2
317.719	557484.5	557480.0(5.6)	-4.5	F = 11/2
467.595	820293.9	820290.0(4.0)	-3.9	F = 11/2
467.595	820293.9	820292.0(8.0)	-1.9	F = 11/2
504.329	884665.7	884666.0(2.4)	0.3	F = 11/2
540.903	948736.7	948735.0(5.6)	-1.7	F = 11/2
577.351	1012564.7	1012567.0(2.4)	2.3	F = 11/2
613.698	1076191.7	1076190.0(2.4)	-1.7	F = 11/2
649.962	1139648.9	1139650.0(2.4)	1.1	F = 11/2
686.158	1202960.0	1202959.0(2.4)	-1.0	F = 11/2
722.298	1266142.9	1266143.0(2.4)	0.1	F = 11/2
794.442	1392177.4	1392180.0(4.0)	2.6	F = 11/2

FIGURE LEGENDS

1. Hyperfine structure of the system $I = 1/2$, $J = 6$, showing the observable transitions.
2. Sample resonance in the $F = 13/2$ level.

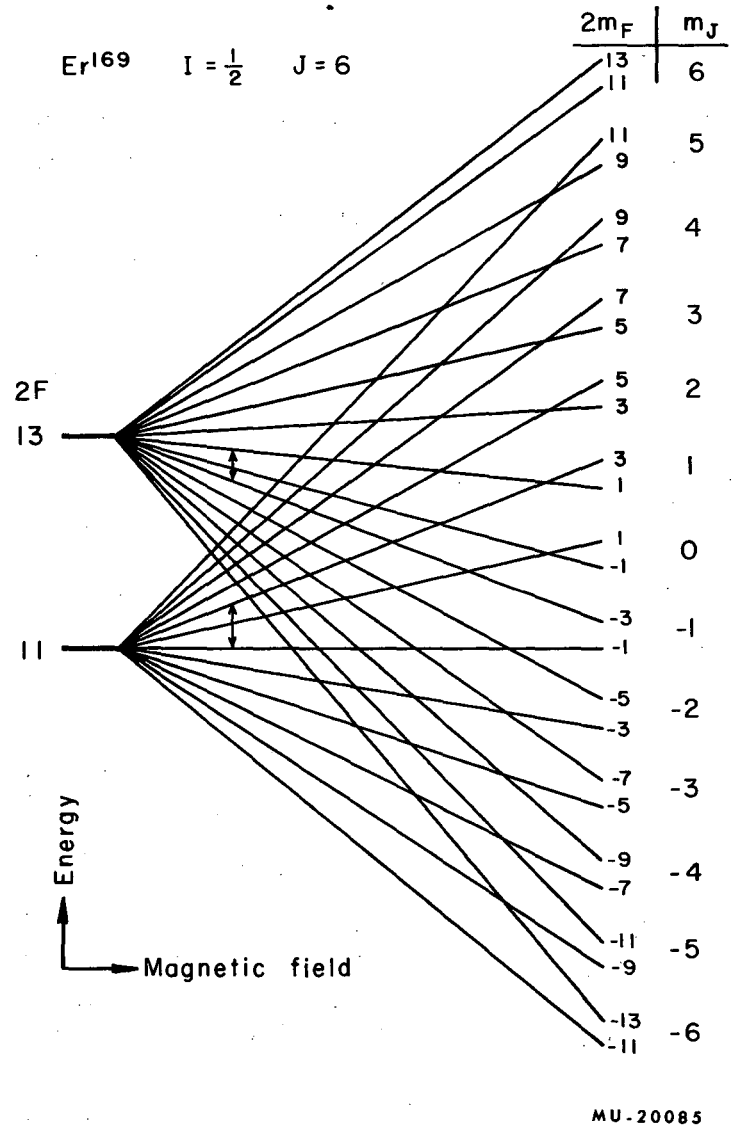
Er169 I = 1/2 J = 6
2mF
mJ
13
11
11
9
9
7
7
5
5
3
3
1
1
-1
-3
-1
-5
-3
-7
-5
-9
-7
-11
-9
-13
-11
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
2F
13
11
Energy
Magnetic field
MU-20085

Fig. 1

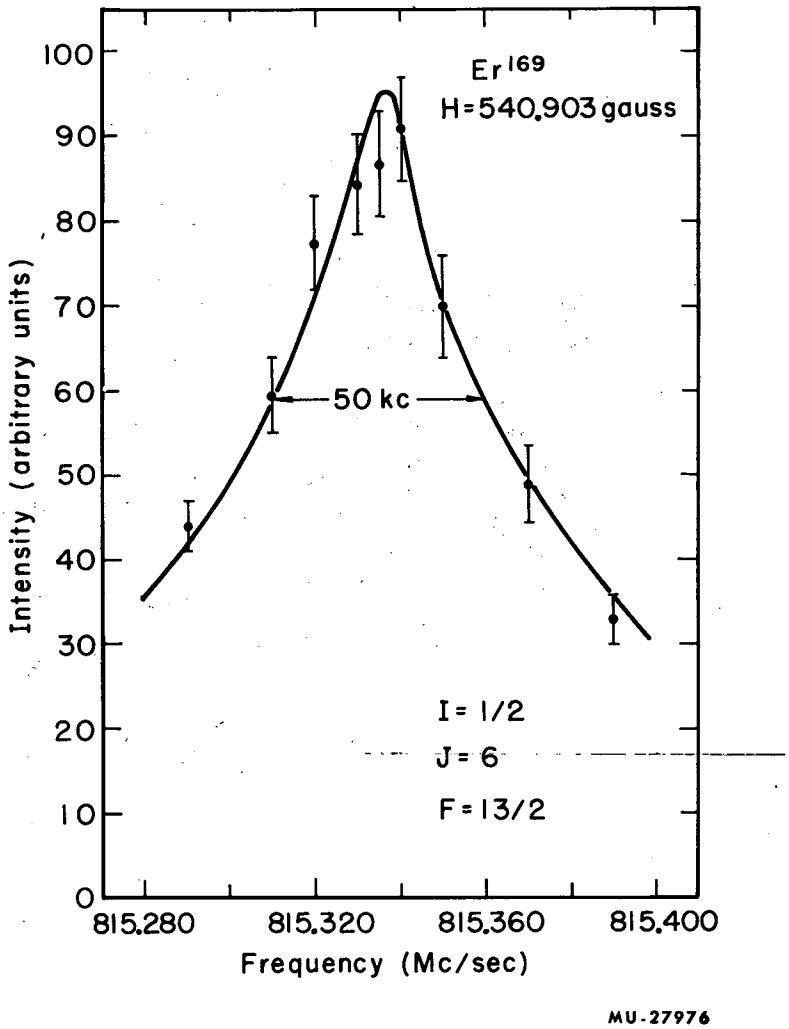
Er169
H=540.903 gauss
50 kc
I= 1/2
J= 6
F= 13/2
Intensity (arbitrary units)
Frequency (Mc/sec)
0
10
20
30
40
50
60
70
80
90
100
815.280
815.320
815.360
815.400
MU-27976

Fig. 2