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## Impedance of Accelerator Components

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Impedance of Accelerator Components\*

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# Impedance of Accelerator Components\*

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**Abstract.** As demands for high luminosity and low emittance particle beams increase, an understanding of the electromagnetic interaction of these beams with their vacuum chamber environment becomes more important in order to maintain the quality of the beam. This interaction is described in terms of the wake field in time domain, and the beam impedance in frequency domain. These concepts are introduced, and related quantities such as the loss factor are presented. The broadband  $Q=1$  resonator impedance model is discussed. Perturbation and coaxial wire methods of measurement of real components are reviewed.

## INTRODUCTION

At low beam currents the motion of a charged particle beam can be described by the optics in the accelerator - the beam experiences accelerating and focusing forces due to the external magnetic and electric fields purposely applied and controlled. In addition to this interaction with external fields, the beam also communicates with its surrounding vacuum chamber through electromagnetic fields generated by the beam itself. When the beam current is sufficiently large, the effects of the beam induced fields become more important, and can limit the performance of an accelerator. The various accelerator components, such as RF cavities, bellows, injection septa, dielectric walls, and even a smooth pipe of finite conductivity result in scattering or trapping of the beam-induced fields. These fields can last for long enough to be experienced by a charge following the exciting charge, causing perturbations to the energy or angle of the following particle's orbit. The dynamics of bunches of particles due to their interaction with the environment of the accelerator through the beam-induced electromagnetic field are generally described in terms of "collective effects" which may be highly disruptive to the beam. Problems may also be encountered in heating of accelerator components as a result of these scattered or trapped fields.

Prediction and measurement of the effects of the beam induced fields, in terms of beam impedance or wake function, is necessary for accurate assessment of machine performance, and a considerable formalism has been developed to

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describe the beam interaction with electromagnetic fields. Analytical estimates, computer simulation, test laboratory measurements, and measurements with beam in an accelerator are used to determine the beam impedance (1 - 4).

The effects of the self-fields of the beam may be analyzed in either time-domain or frequency-domain, and each has its advantages and drawbacks. For circular accelerators, the periodic nature of the beam signals makes the frequency domain approach generally more useful, whereas the time domain approach is more often applied to linear machines. In the time-domain the beam-induced electromagnetic field in an accelerator component may be described by *wake function*; in the frequency-domain by the *beam impedance* (sometimes known as the *coupling impedance*). The beam impedance is a complex quantity: the real part is associated with extraction of energy from the beam; the imaginary part with deformation of the beam profile. The wake function and impedance are equivalent, in the sense that the impedance is the Fourier transform of the wake function.

## BASIC CONCEPTS

The electric field vector for a charged particle moving in free space, in the laboratory frame, may be written (5):

$$\vec{E} = \frac{q \vec{r}_0}{4\pi\epsilon_0\gamma^2 r^2 \left(1 - \frac{v^2}{c^2} \sin^2\Psi\right)^{3/2}}$$

where  $\Psi$  is the angle between the observer at  $r_0$  and the particle velocity  $v$ .

The opening angle of the radial E-field is of the order  $1/\gamma$ . For ultra-relativistic particles the fields resemble plane waves -  $\mathbf{E}$  and  $\mathbf{B}$  are transverse to each other and lie in a disk transverse to the particle velocity. We have only radial E field and azimuthal H field, confined to a disk perpendicular to the direction of motion of the charge, producing a  $\delta$ -function distribution in the direction of motion. Outside this disk there are no fields, and consequently there would be no forces acting on a charge ahead of or following the particle. The situation remains the same for charges moving along the axis of an infinitely conducting smooth cylindrical pipe. The electric field lines are then terminated with surface charges on the inside wall.

For non-relativistic particles the situation is more complex.  $\gamma$  is determined by the energy of the particle beam divided by the rest energy of the constituent particles. For low- $\gamma$  beams the space-charge force which cancels out with the magnetic forces of ultra-relativistic charges cannot be neglected, and the fields associated with a charge are not confined to a disk around the charge. Here, we will deal mainly with the simpler case of ultra-relativistic charges, and ignore these latter complications.

## Longitudinal Wake Fields

The beam-induced electromagnetic fields are called the *wake fields*, since in the limiting case of charges moving at the speed of light, causality requires that the fields exist only behind the charge.

Consider a point charge  $q_1$  traveling with velocity  $v = \beta c$  at position  $z_1$ , and followed by another point “test” charge  $q$  traveling with equal velocity parallel to  $q_1$  and at position  $z$ . The time delay between the charges is  $\tau$ , and their longitudinal coordinates are given by  $z_1(t) = vt$ ,  $z(t) = v(t - \tau)$ . Figure 1 shows the coordinate system.

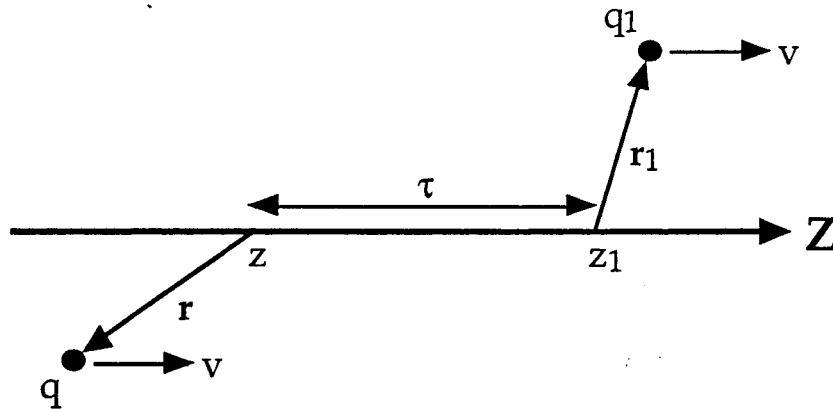


FIGURE 1. Coordinates of the point charges

The Lorentz force experienced by the test charge,  $q$ , due to fields created by the exciting charge  $q_1$ , is given by:

$$\mathbf{F} = q[\mathbf{E} + \mathbf{v} \times \mathbf{B}]$$

In general the wake fields and resulting force will have transverse and longitudinal components. The energy lost from the leading charge  $q_1$  is given by the work done against the electromagnetic fields:

$$\Delta U_{11} = - \int_{-\infty}^{\infty} \mathbf{F}(\mathbf{r}_1, z_1, t) \cdot d\mathbf{z} \quad ; t = \frac{z_1}{v}$$

and for the longitudinal component we find:



$$\Delta U_{11} = -q_1 \int_{-\infty}^{\infty} E_z(\mathbf{r}_1, z_1, t) \cdot dz \quad ; t = \frac{z_1}{v}$$

This accounts for the energy loss to resistive heating in the vacuum chamber walls, to fields scattered at discontinuities in the pipe, and to fields stored in irregular regions of the pipe.

The *loss factor*  $k(\mathbf{r}_1)$  is defined as the energy loss to the self-field per unit charge squared:

$$k(\mathbf{r}_1) = \frac{\Delta U_{11}(\mathbf{r}_1)}{q^2}$$

and has units of volt per Coulomb.

The test charge experiences an energy change due to the fields produced by the leading charge:

$$\Delta U_{21} = -q \int_{-\infty}^{\infty} E_z(\mathbf{r}, z, \mathbf{r}_1, z_1, t) dz \quad ; t = \frac{z_1}{v} + \tau$$

The *longitudinal wake function*  $w_z(\mathbf{r}, \mathbf{r}_1, t)$  may be defined as the energy lost by the trailing charge  $q$  per unit charge of both  $q_1$  and  $q$ :

$$w_z(\mathbf{r}, \mathbf{r}_1, \tau) = \frac{\Delta U_{21}(\mathbf{r}, \mathbf{r}_1, \tau)}{q_1 q}$$

or equivalently in terms of the longitudinal electric field:

$$w_z(\mathbf{r}, \mathbf{r}_1, \tau) = \frac{\int_{-\infty}^{\infty} E_z(\mathbf{r}, z, \mathbf{r}_1, z_1, t) dz}{q_1} \quad ; t = \frac{z_1}{v} + \tau$$

Like the loss factor, the wake function has units of volt per Coulomb.

We need to know the wake function for a distribution of particles in a bunch. By using the wake function as defined above for a point charge, also known as a *Green function*, we apply linear superposition to add the effects of a bunch of particles. Thus the wake field of an arbitrary charge distribution  $i_b(t)$ , where

$$q_1 = \int_{-\infty}^{\infty} i_b d\tau$$

is obtained by the convolution of the  $\delta$ -function wake function with the bunch distribution. We omit the transverse position dependence, and write the integral over the bunch coordinate  $\tau'$ :

$$W_z(\tau) = \frac{\int_{-\infty}^{\infty} i_b(\tau') w_z(\tau - \tau') d\tau'}{q_1} \quad (1)$$

Note that the wake function is zero for time  $\tau' > \tau$ , by causality - the distant tails of the exciting bunch cannot influence a test particle closer to the center of the bunch. For a unit test charge,  $q_1 = 1$ , the wake function is known as the *wake potential*  $V(\tau)$ .

The loss factor for a bunch may now be defined as:

$$K = \frac{\int_{-\infty}^{\infty} W_z(\tau) i_b(\tau) d\tau}{q_1} \quad (2)$$

## Longitudinal Beam Impedance

The frequency-domain or impedance representation may be related to the time-domain wake function by Fourier transform. For a point charge the *beam impedance* or *coupling impedance* is defined as the Fourier transform of the wake function:

$$Z(\omega) = \int_{-\infty}^{\infty} w_z(\tau) e^{-j\omega\tau} d\tau$$

and has units of Ohms.

The Fourier spectrum of the charge distribution,  $I(\omega)$ , is

$$I(\omega) = \int_{-\infty}^{\infty} i_H(\tau) e^{-j\omega\tau} d\tau$$

and by transforming equations (1) and (2) we find the wake function in terms of the impedance, and the frequency domain representation of the loss factor:

$$W_z(\tau) = \frac{\int_{-\infty}^{\infty} Z(\omega) I(\omega) e^{j\omega\tau} d\omega}{2\pi q_1}$$

$$K = \frac{\int_0^{\infty} Z_{\text{real}}(\omega) |I(\omega)|^2 d\omega}{\pi q_1^2}$$

where we have taken the real part of the impedance in calculating the loss factor.

For the case of a Gaussian charge distribution:

$$I(\omega) = q_1 e^{-\frac{(\omega\sigma_1)^2}{2}}$$

and the loss factor for a bunch is given by:

$$K = \frac{\int_0^{\infty} Z_{\text{real}}(\omega) e^{-(\omega\sigma_1)^2} d\omega}{\pi}$$

and it is apparent that the loss depends on the bunch length.

We may also write the wake potential as

$$V_z(\omega) = I(\omega) Z(\omega)$$

and we see that a convolution integral in time domain has become a simple product in frequency domain.

### *Longitudinal wake of a resonant cavity*

The wake function of a cavity resonant at frequency  $\omega_r$  may be found by modeling the cavity as a parallel RLC circuit, and calculating the response of the

circuit to a  $\delta$ -function pulse. The impedance of the RLC circuit is given by

$$\frac{1}{Z_{||}} = \frac{1}{R_s} + \frac{1}{j\omega L} + j\omega C$$

which gives

$$Z_{||} = \frac{R_s}{1 - jQ\left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r}\right)} \quad (3)$$

where

$$\omega_r = \frac{1}{\sqrt{CL}}, \quad Q = R_s \sqrt{\frac{C}{L}}$$

Charge  $q$  induces a voltage given by

$$\frac{q}{C} = q \frac{\omega_r R_s}{Q}$$

The energy stored in the capacitor can be related to the loss factor by:

$$\Delta U = \frac{q^2}{2C} = \frac{\omega_r R_s}{2Q} q^2 = kq^2$$

where we have introduced the loss factor for a point charge, or the *parasitic mode loss factor*  $k$  for a given cavity mode

$$k = \frac{\omega_r R_s}{2Q}$$

The wake function is given by the inverse Fourier transform of (3):

$$w(t) = 2q\alpha R_s e^{\alpha t} \left( \cos \bar{\omega} t + \frac{\alpha}{\bar{\omega}} \sin \bar{\omega} t \right)$$

where

$$\alpha = \frac{\omega_r}{2Q}, \quad \bar{\omega} = \sqrt{\omega_r^2 - \alpha^2}$$

and for high Q-value

$$w(t) \approx q \frac{\omega_r R_s}{Q} e^{\omega_r t \left( j - \frac{1}{2Q} \right)}$$

## Transverse Wake Fields

Consider the leading charge,  $q_1$ , transversely displaced from the axis. The charge can excite electromagnetic fields which can be expanded in multipole components (dipole, quadrupole, etc.); for small displacements the dipole component is usually dominant. The test charge  $q$  receives a momentum change (a kick) from the fields:

$$\Delta p_{21} = q \int_{-\infty}^{\infty} (\mathbf{E} + \mathbf{v} \times \mathbf{B})_{\perp} dz \quad ; t = \frac{z_1}{v} + \tau$$

which in general depends on the positions of both the exciting and trailing charges, and in general is not in the direction of the displacement of the leading charge. The *transverse wake function*,  $w_{\perp}(\tau)$ , is defined as the kick per unit of both charges and is given by:

$$w_{\perp}(\tau) = \frac{\Delta p_{21}(\tau)}{q_1 q}$$

and has units of volt per Coulomb. Here, the symbol  $\perp$  represents either the x or y direction. In analogy to the longitudinal case we define the *transverse loss factor*,  $k_{\perp}$ , as the transverse kick given to the charge by its own wake per unit charge squared:

$$k_{\perp} = \frac{\Delta p_{11}}{q_1^2}$$

and again has units of volt per Coulomb.

Usually the dipole component dominates, and this term is proportional to the transverse displacement of the exciting charge  $q_1$ . The *transverse dipole wake function*,  $w'_{\perp}(\tau)$ , is defined as the transverse wake function per unit transverse displacement:

$$w'_{\perp}(\tau) = \frac{w_{\perp}(\tau)}{r_1}$$

and has units volt per Coulomb per meter. Similarly the *transverse dipole loss factor* is given by:

$$k_{\perp}' = \frac{\Delta p_{11}}{q_1^2 r_1}$$

## Transverse Beam Impedance

The *transverse beam impedance*,  $Z_{\perp}$ , may be found from the Fourier transform of the transverse wake function:

$$Z_{\perp}(\omega) = j \int_{-\infty}^{\infty} w_{\perp}(\tau) e^{-j\omega\tau} d\tau$$

the units are Ohms.

Since the transverse dynamics is dominated by the dipole wake, we define the dipole transverse impedance :

$$Z_{\perp}'(\omega) = \frac{Z_{\perp}(\omega)}{r_1}$$

and the units are Ohms per meter. If the impedance is known, the transverse wake may be calculated from the inverse Fourier transform:

$$w_{\perp}(\tau) = \frac{j}{2\pi} \int_{-\infty}^{\infty} Z_{\perp}(\omega) e^{j\omega\tau} d\omega$$

## Relationship Between Transverse and Longitudinal Wake Fields and Impedance

If we differentiate the momentum kick experienced by a charge  $q$  with respect to time, we obtain:

$$\frac{\partial \Delta \mathbf{p}}{\partial t} = q \int_{t_a}^{t_b} \left( \frac{\partial \mathbf{E}}{\partial t} dt + \frac{\partial (\mathbf{v} \times \mathbf{B})}{\partial t} dt \right)$$

$$\frac{\partial \Delta \mathbf{p}}{\partial t} = q \int_{t_a}^{t_b} \left( \frac{\partial \mathbf{E}}{\partial t} dt + \mathbf{v} \times \frac{\partial \mathbf{B}}{\partial t} dt + \mathbf{B} \times \frac{\partial \mathbf{v}}{\partial t} dt \right)$$

For relativistic particles, of constant velocity

$$dz = \mathbf{v} dt$$

we have

$$\frac{\partial \Delta \mathbf{p}}{\partial t} = q \int_{t_a}^{t_b} \frac{\partial \mathbf{E}}{\partial t} dt + \int_a^b dz \times \frac{\partial \mathbf{B}}{\partial t} dt$$

Using Maxwell's equation

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

and the identity

$$dz \times \nabla \times \mathbf{E} = \nabla(dz \cdot \mathbf{E}) - (dz \cdot \nabla)\mathbf{E} = \nabla(dz \cdot \mathbf{E}) - \frac{\partial \mathbf{E}}{\partial z} dz$$

then we find

$$\frac{\partial \Delta \mathbf{p}}{\partial t} = q \int_{t_a}^{t_b} \frac{\partial \mathbf{E}}{\partial t} dt - \int_a^b \left( \nabla(dz \cdot \mathbf{E}) - \frac{\partial \mathbf{E}}{\partial z} dz \right)$$

$$\frac{\partial \Delta \mathbf{p}}{\partial t} = q \int_a^b \left( \nabla(dz \cdot \mathbf{E}) + 2d\mathbf{E} \right)$$

The transverse components are

$$\frac{\partial}{\partial t} (\Delta \mathbf{p}_\perp) = q \int_a^b \left[ -\nabla_\perp (dz \cdot \mathbf{E}) + 2d\mathbf{E}_\perp \right]$$

and noting that

$$\Delta E = q \int_a^b dz \cdot \mathbf{E}$$

we find

$$\frac{\partial}{\partial t} (\Delta \mathbf{p}_\perp) = -\nabla_\perp (\Delta E) + 2q[\mathbf{E}_\perp(b) - \mathbf{E}_\perp(a)]$$

The bracketed term we choose to extend over the region that the entry and exit fields  $\mathbf{E}_\perp(a)$ ,  $\mathbf{E}_\perp(b)$ , are zero. Then for fields with sinusoidal time variation we have the *Panofsky-Wenzel* theorem:

$$\frac{j\omega \Delta \mathbf{p}_\perp}{q} = -\frac{1}{q} \nabla_\perp (\Delta E) = -\nabla_\perp V$$

This theorem tells us that the transverse kick can be described purely in terms of the longitudinal electric field. There must be a longitudinal electric field component in order to produce a transverse momentum change in a particle traveling through a structure. The frequency dependence shows that the higher the frequency at which the deflecting fields are encountered, the less of a kick they impart.

## IMPEDANCE MEASUREMENTS

### Perturbation Measurements

For narrow-band impedances, e.g. cavity resonances, perturbation measurements are an accurate method for mapping fields and determining the beam impedance. The method involves the introduction of a small perturbing object into the fields of the resonator (6).

The change in resonant frequency upon introducing the object is proportional to the relative change in electric and magnetic stored energy:

$$\frac{\Delta \omega}{\omega} = \frac{\Delta U_E - \Delta U_M}{U}$$

For a small object, i.e. one for which the unperturbed field is roughly constant over the volume of the perturbing object, the perturbed energy is generally expressible as the product of the stored energy of the unperturbed field integrated over the volume of the perturbing object and a form factor which depends on the shape and orientation, and electromagnetic properties of the perturbing object. For



a small sphere of radius  $r$ , we find:

$$\frac{\Delta\omega}{\omega} = \frac{\Delta U}{U} = -\frac{\pi r^3}{U} \left[ \epsilon_0 \frac{\epsilon_r - 1}{\epsilon_r + 2} E_0^2 + \mu_0 \frac{\mu_r - 1}{\mu_r + 2} H_0^2 \right]$$

For a dielectric bead  $\mu_r = 1$ , and

$$\frac{\Delta\omega}{\omega} = -\frac{\pi r^3}{U} \left[ \epsilon_0 \frac{\epsilon_r - 1}{\epsilon_r + 2} E_0^2 \right]$$

For a metal bead  $\mu_r \rightarrow 0$  and  $\epsilon_r \rightarrow \infty$ , and

$$\frac{\Delta\omega}{\omega} = -\frac{\pi r^3}{U} \left[ \epsilon_0 E_0^2 - \frac{\mu_0}{2} H_0^2 \right]$$

For the case of a monopole mode, with zero magnetic field on axis, the electric field is:

$$|E| = \sqrt{\frac{\Delta\omega}{\omega_r} \frac{U}{\pi r^3 \epsilon_0}}$$

A metallic bead gives a large frequency shift, but the perturbation is sensitive to both E and H fields. A dielectric bead will only perturb the E field. Shaped perturbing objects, such as needles, can enhance the perturbation and offer directional sensitivity. For an ellipsoid, the enhanced perturbation can be calculated analytically; other objects can be calibrated in a known field.

The absolute fields may be calculated if the power and Q value are known; however the geometrical factor R/Q can be found from the longitudinal field distribution. By mapping the longitudinal distribution of  $E_{||}$  and integrating, the R/Q of the cavity mode can be measured:

$$\frac{R}{Q} = \frac{V^2}{2\omega_r U} = \frac{\left( \int E_{||} dz \right)^2}{2\omega_r U} = \frac{\left( \int \sqrt{\frac{\Delta\omega}{\omega_r} \frac{U}{\pi r^3 \epsilon_0}} dz \right)^2}{2\omega_r U} = \frac{\left( \int \sqrt{\Delta\omega} dz \right)^2}{2\pi\omega_r^2 r^3 \epsilon_0}$$

The *transit-time factor*, T, is defined as the ratio of energy received by a charge passing through the time-varying fields to that which would be received if the field everywhere along the path were at its time-maximum value:

$$T = \frac{\left| \int E_{\parallel} e^{j \frac{\omega}{c} z} dz \right|}{\int E_{\parallel} dz}$$

Consider a charge traveling at speed  $v=z/t$  through a field extending a length  $\pm g$ , where the field is:

$$E_{\parallel} = \frac{V_0}{g} \cos \omega t$$

Then the energy gain is given by:

$$\Delta E = \int_{-\frac{g}{2}}^{\frac{g}{2}} q E_{\parallel} dz = \int_{-\frac{g}{2}}^{\frac{g}{2}} \frac{e V_0}{g} \cos \frac{\omega z}{v} dz$$

$$\Delta E = 2qV_0 \frac{v}{\omega g} \sin \frac{\omega g}{2v}$$

$$\Delta E = qV_0 \frac{\sin \theta}{\theta}$$

The transit-time factor is given by

$$T = \frac{\sin \theta}{\theta}$$

and the transit angle  $\theta$  is

$$\theta = \frac{\omega g}{2v}$$

The transit-time-corrected shunt impedance  $RT^2$  of an even mode (symmetric in  $z$  about the center of the cavity) is obtained from:

$$\frac{RT^2}{Q} = \frac{\left( \int \sqrt{\Delta \omega} \left( \cos \frac{\omega_r z}{c} \right) dz \right)^2}{2\pi \omega_r^2 r^3 \epsilon_0}$$

## *Transverse Dipole Mode Impedance*

To determine the transverse impedance of a dipole mode, we measure the longitudinal impedance of the dipole mode, at a radial offset  $r$ , and use the Panofsky-Wenzel theorem to calculate the transverse impedance.

The transverse energy change is related to the transverse voltage by:

$$\Delta E_{\perp} = \beta c \Delta p_{\perp} = e V_{\perp}$$

and the dipole transverse impedance is:

$$R_{\perp} = j \frac{V_{\perp}}{I r}$$

Then, for ultra-relativistic particles

$$R_{\perp} = j \frac{c \Delta p_{\perp}}{e I r} = - \nabla_{\perp}(V_{\parallel}) \frac{c}{\omega I r}$$

For small radial displacements in dipole modes,  $E_{\parallel}$  is proportional to the radial offset  $r$ , and we may write:

$$\nabla_{\perp}(V_{\parallel}) = \frac{V_{\parallel}(r)}{r}$$

which gives:

$$R_{\perp} = \frac{V_{\parallel}(r) c}{\omega I r^2} = \frac{R_{\parallel}(r) c}{\omega r^2} = \frac{R_{\parallel}(r)}{k r^2}$$

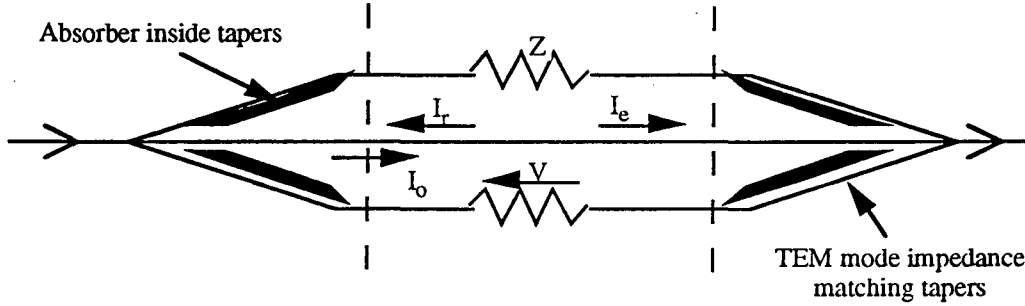
## **Coaxial Wire Measurements**

The coaxial wire impedance measurement uses a conducting rod placed along the beam axis in the vacuum chamber, forming the center conductor in a coaxial line system (7,8). Tapers at either end of this section allow for smooth impedance transformation from the 50  $\Omega$  lines used in common microwave measurement equipment, to the characteristic impedance of the vacuum chamber and center conductor, of the order of hundreds of Ohms.

A smooth vessel of the same entrance/exit cross-section and length as those of the device under test is used in a reference measurement. Resonances within the apparatus are difficult to avoid completely and require careful placing of absorptive material, manufacture of test and reference chambers, and assembly of apparatus.

## Frequency domain

Figure 2 shows a schematic of the measurement apparatus and the currents in the apparatus. Current  $I_0$  is applied upstream of the impedance to be determined,  $Z$ . The coaxial wire forms a line of characteristic impedance  $R$  with the vacuum chamber. A voltage  $V$  is generated at the impedance, inducing currents  $V/2R$  traveling equally upstream and downstream.



**Figure 2.** Coaxial wire impedance measurement apparatus. Absorptive material is placed in the impedance matching transformers at the ends of the device under test to avoid resonances of traveling waveguide modes within the apparatus. A reference vessel replaces the device under test, between the vertical dashed lines, to normalize to the losses in the tapers, cables etc.

For a localized impedance (small in extent compared with the wavelength of the applied current), the current that excites the voltage  $V$  in the impedance is:

$$I_e = I_0 - I_r$$

The perturbation in wire current is:

$$\Delta I = I_0 - I_e = \frac{V}{2R} = \frac{I_e Z}{2R}$$

and

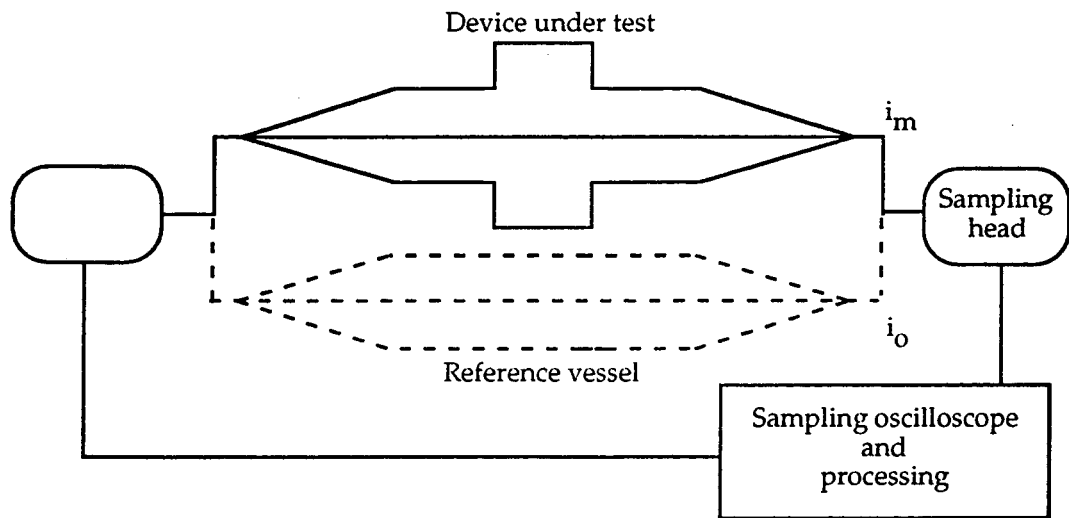
$$Z = \frac{2R(I_0 - I_e)}{I_e} = 2R \left( \frac{I_0}{I_e} - 1 \right)$$

$S_{21}$  measurements without the impedance  $Z$  (reference measurement) and with the impedance  $Z$  (object measurement) give:

$$Z = 2R \left( \frac{S_{21}^{\text{reference}}}{S_{21}^{\text{object}}} - 1 \right)$$

## *Time-domain*

The time-domain measurement gives the wake potential and the loss factor for a bunch simulated by a current pulse from a pulse generator. Figure 3 shows a schematic of the measurement apparatus. A reference pulse  $i_o$  is measured at the output of the reference line with characteristic impedance  $R$ , and a pulse through the device under test  $i_m$ .



**Figure 3.** Coaxial wire impedance measurement apparatus, time domain.

The energy in the “unperturbed” current pulse at the output of the reference line is:

$$U_o = \int R i_o^2(t) dt$$

For the perturbed pulse, at the output of the device under test line we have

$$i_m(t) = i_o(t) + \Delta i(t)$$

and the energy in the pulse at the output of the device under test line is

$$U_m \approx \int R i_o(t) [i_o(t) + 2\Delta i(t)] dt$$

The energy difference  $U_m - U_o$  can be interpreted as the energy lost by the pulse

traveling through the device under test. Then the loss from a bunch is given by:

$$\Delta U = kq^2 = k \left( \int i_o dt \right)^2 = 2R \int i_o(i_m - i_o) dt$$

the loss factor k is:

$$k = 2R \frac{\int i_o(i_m - i_o) dt}{\left( \int i_o dt \right)^2}$$

and the wake potential is given by

$$w(t) = 2R (i_o - i_m)$$

## Transverse Impedance Measurements

Transverse impedance measurements are made with two off-axis wires driven differentially with a hybrid, or a single wire and ground-plane. The technique is basically the same as the longitudinal measurements described above.

The transverse impedance is given by:

$$Z_{\perp} = -j \frac{\Delta V_{\perp}}{I \Delta x} = \frac{1}{I \Delta x} \frac{c}{\omega} \frac{\partial V_{\parallel}}{\partial x} = \frac{1}{k \Delta x} \frac{\partial}{\partial x} (Z_{\parallel})$$

$$Z_{\perp} = \frac{2 R_w}{k (\Delta x)^2} \left( \frac{S_{21}^{\text{reference}}}{S_{21}^{\text{object}}} - 1 \right)$$

where  $\Delta x$  is one half of the separation of the effective electrical centers (i.e. the location of the “point” wire which would give a cylindrical equipotential at the surface of the actual finite wire),  $R_w$  the impedance of the twin-wire line.

## Effects of Wakes or Impedance

The fields of a passing bunch, or train of bunches, induce image charges on the walls which terminate the field lines of the free charges in the beam. The associated image currents can cause heating on the surface of vacuum chamber

components, which may present problems for certain structures. In particular, if the beam current spectrum can excite resonances in vacuum chamber components, large wall currents can be induced and significant damage to the accelerator may result.

The type of impedance creating the wake fields may be useful in identifying potential problems. A broad-band, or low-Q, impedance, will decay rapidly and the heating will be well characterized by the single-pass calculation. Narrow-band, or high-Q, impedance will have a memory which may enhance the heating if the resonant frequency is close to a beam harmonic.

Resistive heating due to image currents in the resistive walls of the vacuum chamber may be calculated from the Fourier series of the beam current and the wall resistance, taking into account the skin-depth penetration of the fields. The Fourier coefficients of the beam current for bunches of length  $\sigma_t$  spaced by time  $T_b$ ,  $\omega_b = 2\pi/T_b$  are:

$$C_n = I_0 e^{-\frac{(n\omega_b\sigma_t)^2}{\gamma}}$$

For a circular pipe, of length L and radius r, conductivity  $\sigma_{dc}$ , the resistance at the frequency corresponding to the nth harmonic of the bunch frequency is given by:

$$R = \frac{L}{2\pi r} \sqrt{\frac{n\omega_b\mu_0}{2\sigma_{dc}}}$$

Then the total power is:

$$P = \sum_{n=-\infty}^{\infty} C_n^2 R_n = \frac{L}{2\pi r} \sqrt{\frac{\mu_0 \omega_b}{2\sigma_{dc}}} I_0^2 \sum_{n=1}^{\infty} e^{-(n\omega_b\sigma_t)^2} \sqrt{n}$$

The summation is approximated by an integral evaluated using the definition of the Gamma function and the final result is:

$$P = \frac{L}{8\pi^2 r} I_0^2 \Gamma\left(\frac{3}{4}\right) \sqrt{\frac{\mu_0}{2\sigma_{dc}}} \frac{T_b}{\sigma_t^2}$$

Single-pass power loss, due to short-range wake fields (or broad-band, low-Q impedance) that decay before the arrival of the next bunch, can be calculated from the loss factor:

$$P = k \frac{q_b^2}{T_b} = k T_b I_0^2$$

where  $N_b$  is the number of equally spaced bunches of charge  $q_b$ . This calculation assumes that the wake field has decayed in the time interval between bunches.

For wakes that persist until the arrival of the next bunch, the situation may be quite different. Depending on the phase of the wake field at the time of passage of the following bunch, there may be energy imparted to that bunch or extracted from the bunch. The multi-bunch losses may be more or less than the losses calculated from the loss factor for single-pass effects.

For the narrow-band case, an analytic expression may be derived in time domain (9), or the power lost from the beam can be calculated from the product of the beam current squared and the impedance, summing over all beam spectral lines. For a uniformly bunched beam, the current spectrum is given by:

$$I_b(\omega) = I_0 + 2I_0 e^{-\frac{(\omega\sigma_t)^2}{2}} \sum_{n=0}^{\infty} \delta(\omega - n\omega_b)$$

### Broadband Q=1 Impedance Model

When considering single-bunch collective effects, we are concerned with the wakefields over the bunch length. If we Fourier analyze these short-range wakefields, we find that the effective impedance sampled by a single bunch does not show the detail that may exist in the actual impedance of a structure; the short-range wake “smooths out” the impedance. The actual impedance of the many and varied components of an accelerator is often replaced with an effective impedance described by a low-Q resonator of the form of equation 3. The resonant frequency  $\omega_r$  is generally taken to be the TM-mode cut-off of the vacuum chamber, and the Q-value is taken to be unity. The shunt impedance may be estimated by making the energy loss to the low-Q resonator equal to the total loss of the individual component resonances in the accelerator:

$$k_{Q=1} = \sum k_{\text{components}}$$

For resonant modes, and Gaussian bunches:

$$k_{Q=1} = \sum_{\text{n modes}} \frac{\omega_n R}{2Q} e^{-(\omega_n \sigma_t)^2}$$

In the case of components, such as shallow cavities, with mostly inductive impedance, the situation may be better modeled by equating the low-frequency inductance of the resonator to the calculated inductance of the components. The imaginary part of the resonant impedance at low-frequencies is given by:



$$Z_i = j \frac{\omega R}{\omega_r Q}$$

A convenient measure of broadband impedance is  $|Z/n|$ , the magnitude of the resonant impedance divided by normalized frequency  $n$ ;  $n=\omega/\omega_0$ , the frequency divided by the revolution frequency of the ring.  $|Z/n|$  is approximately constant below the resonant frequency, and this makes the model attractive when calculating single-bunch collective effects in some cases. Approximate expressions dependent upon  $|Z/n|$  are often used to estimate single-bunch effects.

The  $Q=1$  resonator has limitations, particularly for very short bunches. In this case the beam power spectrum extends beyond the beam-pipe cut-off frequency, and the bunch resolves more of the detail of the beam impedance.

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