Title

Predicting the frequency and cost of hot and cold complaints in buildlings

Author

Federspiel, C.

Publication Date

2000-10-01

VOL. 6, NO. 4 HVAC&R RESEARCH OCTOBER 2000

Predicting the Frequency and Cost of Hot and Cold Complaints in Buildings

Clifford C. Federspiel, Ph.D.
Associate Member ASHRAE

When building occupants become sufficiently hot or cold and have exhausted all coping behaviors available to alleviate their discomfort, they often complain to the facility manager. These complaint events trigger maintenance service calls. This paper focuses on predicting the frequency of hot and cold complaint events so that control policies and decisions that affect both energy utilization and comfort-related service calls can be formulated. A mathematical model of the mean frequency of hot and cold complaint events in buildings is developed that is based on the level-crossing theory of stochastic processes. The model quantitatively relates the statistical behavior and performance of the temperature control system to the mean complaint frequency. When combined with the labor rate and with estimates of the mean time to respond to complaints, the model becomes an estimate of the mean cost of service calls resulting from hot and cold complaints. Data from a commercial facility are used to determine parameters of the model. The relationship between this model and the Predicted Percent Dissatisfied (PPD) model is discussed. The economic consequences of operating buildings at the limits of the ASHRAE comfort range are illustrated. Examples illustrate how the model may be used to optimize the operational performance of buildings by balancing the energy-saving benefits of uncomfortable conditions with the cost of service calls caused by complaints arising from uncomfortable conditions, and for the cost-benefit analysis of retrofits.

INTRODUCTION

One focus of research on thermal comfort has been the analysis and prediction of subjective assessments of hot and cold from objective measures of the environment. The early efforts in this area were purely empirical. Nevins et al. (1966) and McNall et al. (1966) give examples of empirical predictions of thermal sensation ratings. Fanger (1972) describes a model-based, semi-empirical method of predicting thermal sensation ratings and for predicting the fraction of dissatisfied occupants. The indices he developed are called Predicted Mean Vote (PMV) and Predicted Percent Dissatisfied (PPD), respectively. PMV predicts the subjective thermal sensation rating of a large group based on six variables that affect the human heat balance, and PPD predicts the expected fraction of a large group that will make with a subjective assessment of hot or cold above an absolute PMV level of 1.5 scale units. Extensions of the model-based approach for predicting thermal sensation have been developed by others. Gagge et al. (1986) give an example of the efforts to extend the model-based approach.

Quantifying the economic cost of thermal discomfort in buildings has been an elusive concept for decades. Much of the focus in this area has been on the relationship between environmental conditions and worker productivity. The effects of the indoor environment on productivity are reviewed and discussed by Wyon (1993, 1996), and Sensharma and Woods (1997). While it has been possible to relate manual and cognitive task performance to environmental conditions,

---

Clifford C. Federspiel is a research specialist with the Center for Environmental Design Research, University of California, Berkeley.

there has been considerably less success in relating environmental conditions to objective measures of performance. Some success has been achieved at relating environmental conditions to task performance in industrial environments. In office environments, there has been considerably less success in relating work performance to environmental conditions. Since the relations between environmental conditions and productivity are numerous and complex, it has not yet been possible to formulate a predictive model relating the two. This fact makes it difficult for engineers and operations staff to make use of research results on productivity to optimize the operation of facilities.

Another aspect of the operational cost of buildings is the labor cost associated with service calls. Relatively little effort has focused on studying the relationship between the indoor environment and the cost of service calls. Federspiel (1998) describes an analysis of the conditions resulting in hot and cold complaints along with an analysis of the time required to service these complaints. It was shown that the cost avoidance potential of eliminating unsolicited hot and cold complaints in buildings is considerable.

In this paper, a mathematical model that predicts the frequency at which unsolicited hot and cold complaints occur is developed. The next section describes the mathematical basis of the model. Data from a complaint log are used to estimate parameters of the model. Qualitative model validation is performed by comparing the parameter estimates with analogous parameters in the PPD model. Examples that illustrate how the model may be used to optimize control policies and economic decisions regarding control system investments are included.

PROCESS MODEL

Building temperatures are influenced by uncontrollable and unpredictable disturbances. In the model developed in this work, the room air temperature is modeled as a random process. Hot and cold complaints are modeled as level-crossing events. Hot complaints occur when the building temperature crosses above a high-temperature level, and cold complaints occur when the building temperature crosses below a low-temperature level. Each level is associated with a group of people in a zone or space within a building, rather than with an individual. Unlike alarms levels, which are fixed and known, the levels associated with complaint events are assumed to be random processes because the perception of temperature will be influenced by uncontrollable and unpredictable changes in metabolism, clothing, posture, attention to the environment, and varying task requirements. The levels will also be affected by the group dynamics, which may vary with time and from group to group.

Figure 1 shows hypothetically how the three processes described above are related to complaint events. The figure shows two hot complaints and two cold complaints. If the temperature at which the level-crossings occur are measured, then after a large number of complaints have been recorded, the distribution of the level-crossing temperatures will look like the distributions on the right-hand side of the figure. The number of complaints at very high and very low temperatures will be low because the exposure to such high and low temperatures is low.

The mathematical results in this section are based on the assumptions that the three underlying processes (the building temperature, the high-temperature level, and the low-temperature level) are normally distributed and stationary. In practice, these processes may be non-stationary. For example, the hot and cold complaint levels may be non-stationary because of seasonal changes in clothing insulation. Seasonal load changes may cause the building temperature to be non-stationary depending on how the temperature controls are designed. Nevertheless, it is possible that valuable results could be derived from relying on the assumption of stationarity even if the processes are not stationary if the magnitudes of the non-stationary effects are small.

Figure 2 shows the temperature distribution obtained from measurements made in a large building and the normal distribution with the same mean and variance. The temperatures were

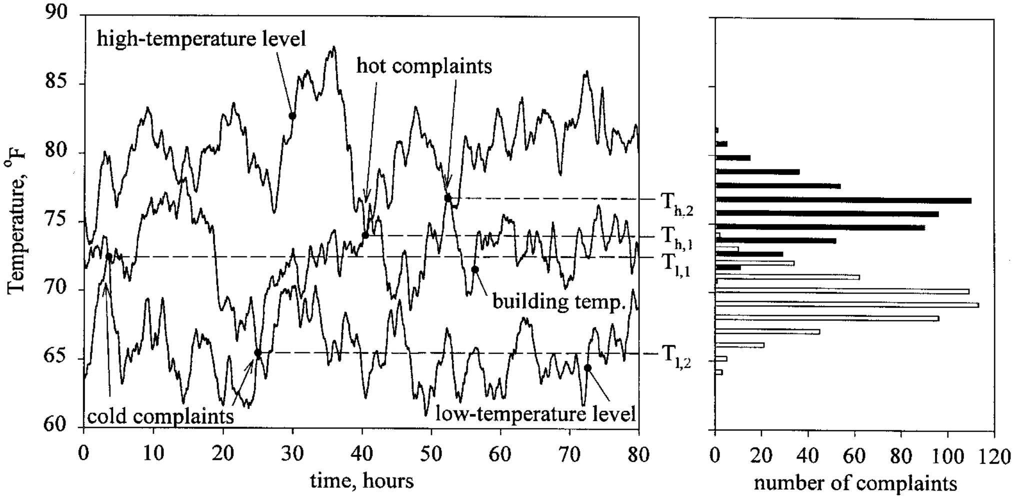

Figure 1. Underlying processes of complaint model, and hot and cold complaint temperature distributions

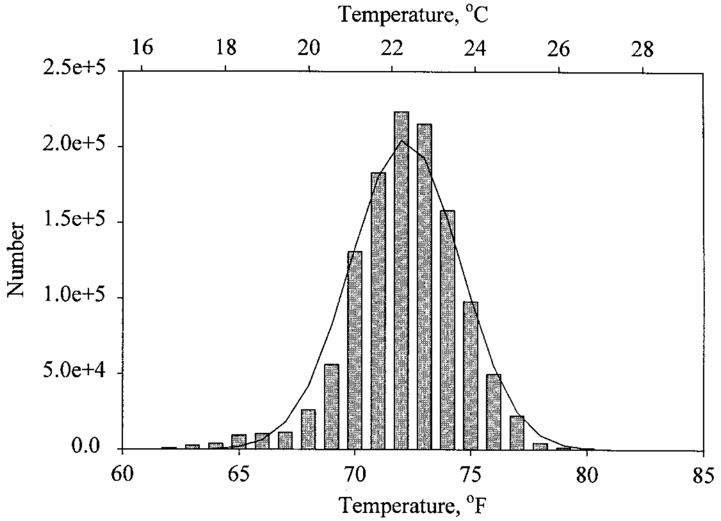

Figure 2. Temperature distribution in large building

recorded by 100 microdataloggers with an accuracy of 0.4°F (0.22°C). The close match indicates that the assumption of normality for building temperatures is valid. The high-temperature and low-temperature levels are similar to the dissatisfied levels associated with the PPD index developed by Fanger (1972). The probit analysis of PPD illustrated that the dissatisfied levels are normally distributed. Therefore, the assumption that high-temperature and low-temperature complaint levels are also normally distributed is reasonable.

The presence of a feedback controller does not mean that the building temperature is not random. The process is random because it is affected by random disturbances such as weather, movement of people within the building, and numerous other unpredictable load factors. These disturbances affect the process whether or not there is feedback. Feedback may reduce the variance of the controlled variable, and it may make low-frequency, non-stationary disturbances less apparent, but it does not make the process deterministic.

The high-temperature level at which a hot complaint occurs is T_H, the building temperature is T_B, and the low-temperature level at which a cold complaint occurs is T_L. These temperatures are assumed to be continuous functions of time with time-derivatives that exist. The parameters μ_{T_H}, σ_{T_H}, and $\sigma_{\dot{T}_H}$ are the mean, standard deviation, and standard deviation of the rate of change of T_H. The parameters μ_{T_B}, σ_{T_B}, and $\sigma_{\dot{T}_B}$ are the mean, standard deviation, and standard deviation of the rate of change of T_B. The parameters μ_{T_L}, σ_{T_L}, and $\sigma_{\dot{T}_L}$ are the mean, standard deviation, and standard deviation of the rate of change of T_L.

The standard level-crossing problem is one in which a random, normally-distributed process crosses a fixed level. The mathematical theory of the standard level-crossing problem was first developed by Rice (1945). Cramer and Leadbetter (1967) developed additional mathematical properties of the standard level-crossing problem as well as extensions of the theory to non-stationary processes. For the standard level-crossing problem, the mean frequency that a zero-mean, random process denoted as x crosses above a fixed level L is as follows

$$\nu_x = \frac{\sigma_x}{2\pi\sigma_x}\exp\left(-\frac{1}{2}\frac{L^2}{\sigma_x^2}\right) \tag{1}$$

where ν_x is the mean up-crossing frequency, σ_x is the standard deviation of x, and $\sigma_{\dot{x}}$ is the standard deviation of the rate of change of x. The mean upcrossing frequency is also the probability that a level-crossing will occur in a very small (infinitesimal) length of time. The expected number of up-crossings in a time period t is the product of the upcrossing frequency and the length of the time period and can be expressed as follows

$$E[n] = \frac{\sigma_{\dot{x}}}{2\pi\sigma_x}\exp\left(-\frac{1}{2}\frac{L^2}{\sigma_x^2}\right)t \tag{2}$$

The complaint event process model described above is not in the standard form because the levels are not fixed. It can be converted into the standard form by the following changes of variables

$$z_h = \frac{T_B - \mu_{T_H}}{(\sigma_{T_H}^2 + \sigma_{T_B}^2 - 2\sigma_{T_H}\sigma_{T_B}\rho_{T_H T_B})^{1/2}} \tag{3}$$

$$z_l = \frac{\mu_{T_L} - T_B}{(\sigma_{T_L}^2 + \sigma_{T_B}^2 - 2\sigma_{T_L}\sigma_{T_B}\rho_{T_L T_B})^{1/2}} \tag{4}$$

where $\rho_{T_H T_B}$ is the cross-correlation coefficient for T_H and T_B, and $\rho_{T_L T_B}$ is the cross-correlation coefficient for T_L and T_B. With these transformations, the mean number of hot complaints in a time period t is now the mean number of zero-level upcrossings of the variable z_h, and the mean number of cold complaints in a time period t is now the mean number of zero-level upcrossings of the variable z_l.

Most buildings are typically occupied only during the daytime. In this case, the number of complaints per day per zone will depend on the level crossing frequencies, and on the probability that when someone arrives in the morning a complaint condition already exists. Mathematically, the expected number of complaints per zone per day is as follows

$$E[n_h] = P_h + \nu_h t \tag{5}$$

$$E[n_l] = P_l + \nu_l t \tag{6}$$

where

$$P_h = \int_{-\infty}^{Z_h} \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}} dz \tag{7}$$

$$P_l = \int_{-\infty}^{Z_l} \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}} dz \tag{8}$$

$$Z_h = \frac{\mu_{T_B} - \mu_{T_H}}{(\sigma^2_{T_H} + \sigma^2_{T_B} - 2\sigma_{T_H}\sigma_{T_B}\rho_{T_H T_B})^{1/2}} \tag{9}$$

$$Z_l = \frac{\mu_{T_L} - \mu_{T_B}}{(\sigma^2_{T_L} + \sigma^2_{T_B} - 2\sigma_{T_L}\sigma_{T_B}\rho_{T_L T_B})^{1/2}} \tag{10}$$

$$\nu_h = \frac{1}{2\pi}\left(\frac{\sigma^2_{\dot{T}_H} + \sigma^2_{\dot{T}_B} - 2\sigma_{\dot{T}_H}\sigma_{\dot{T}_B}\rho_{\dot{T}_H\dot{T}_B}}{\sigma^2_{T_H} + \sigma^2_{T_B} - 2\sigma_{T_H}\sigma_{T_B}\rho_{T_H T_B}}\right)^{1/2} \exp\left(-\frac{1}{2}\,\frac{(\mu_{T_B} - \mu_{T_B})^2}{\sigma^2_{T_H} + \sigma^2_{T_B} - 2\sigma_{T_H}\sigma_{T_B}\rho_{T_H T_B}}\right) \tag{11}$$

$$\nu_l = \frac{1}{2\pi}\left(\frac{\sigma^2_{\dot{T}_L} + \sigma^2_{\dot{T}_B} - 2\sigma_{\dot{T}_L}\sigma_{\dot{T}_B}\rho_{\dot{T}_L\dot{T}_B}}{\sigma^2_{T_L} + \sigma^2_{T_B} - 2\sigma_{T_L}\sigma_{T_B}\rho_{T_L T_B}}\right)^{1/2} \exp\left(-\frac{1}{2}\,\frac{(\mu_{T_B} - \mu_{T_L})^2}{\sigma^2_{T_L} + \sigma^2_{T_B} - 2\sigma_{T_L}\sigma_{T_B}\rho_{T_L T_B}}\right) \tag{12}$$

and where t is the length of time each day that the building is occupied. The quantities P_h and P_l are the probabilities that a hot complaint condition and a cold complaint condition exist when the building is first occupied because a level-crossing may have occurred before the occupants arrived. In addition to being dependent on the mean and standard deviation of the three processes, the predicted complaint rate is dependent on the standard deviation of the rate of change of the three processes. This is evident from Equation (11) and Equation (12), which are similar to Equation (1).

Equation (5) and Equation (6) may be converted to a complaint cost function if the mean (expected) times to handle complaints (denoted as $E[\tau_h]$ and $E[\tau_l]$ for hot and cold complaints, respectively) are known and if the labor rate of the service technician, denoted as R, is known. $E[\tau_h]$ and $E[\tau_l]$ are the average labor times associated with hot and cold complaints, respectively. If a technician is dispatched immediately, then there are the times from when the complaint occurs until the complaint is resolved by the technician. The complaint cost function will be the cost of service calls resulting from hot and cold complaints. Mathematically, the complaint cost is as follows

$$C = \left(\sum_{i=1}^{n_h} \tau_{h_i} + \sum_{j=1}^{n_l} \tau_{l_j} \right) R \tag{13}$$

where R is the labor rate. The cost C is a random variable that depends on the random variables n_h, n_l, τ_h, and τ_l. Under the assumption that n_h is independent of τ_h and that n_l is independent of τ_l, the expected cost of complaints per zone per day is as follows:

$$E[C] = (E[n_h]E[\tau_h] + E[n_l]E[\tau_l])R \tag{14}$$

This cost function can be converted to an annual cost function by multiplying $E[C]$ by the number of occupied days and by the number of zones in the building.

Under the assumption that T_H, T_B, and T_L are normal and stationary, the temperatures at which complaints occur are also normally distributed. The temperatures at which hot complaints occur will be denoted as T_h, and the temperature at which cold complaints occur will be denoted as T_l. While T_H, T_B, and T_L are continuous random processes, T_h and T_l are discrete random sequences. When a hot complaint occurs, $T_H = T_B = T_h$. When a cold complaint occurs, $T_B = T_L = T_l$. The relation between the variances of T_H, T_h, and T_B is as follows

$$\sigma_{T_H}^2 = \frac{\sigma_{T_h}^2(2\sigma_{T_H}\sigma_{T_B}\rho_{T_HT_B} - \sigma_{T_B}^2) - \sigma_{T_H}^2\sigma_{T_B}^2\rho_{T_HT_B}^2}{\sigma_{T_h}^2 - \sigma_{T_B}^2} \tag{15}$$

The relation between the variances of T_L, T_l, and T_B is as follows

$$\sigma_{T_L}^2 = \frac{\sigma_{T_l}^2(2\sigma_{T_L}\sigma_{T_B}\rho_{T_LT_B} - \sigma_{T_B}^2) - \sigma_{T_L}^2\sigma_{T_B}^2\rho_{T_LT_B}^2}{\sigma_{T_l}^2 - \sigma_{T_B}^2} \tag{16}$$

The relation between the means and variances of T_H, T_h, and T_B is as follows

$$\mu_{T_H} = \frac{\mu_{T_h}(\sigma_{T_H}^2 + \sigma_{T_B}^2 - 2\sigma_{T_H}\sigma_{T_B}\rho_{T_HT_B}) + \mu_{T_B}(\sigma_{T_H}\sigma_{T_B}\rho_{T_HT_B} - \sigma_{T_H}^2)}{\sigma_{T_B}^2 - \sigma_{T_H}\sigma_{T_B}\rho_{T_HT_B}} \tag{17}$$

The relation between the means and variances of T_L, T_l, and T_B is as follows

$$\mu_{T_L} = \frac{\mu_{T_l}(\sigma_{T_L}^2 + \sigma_{T_B}^2 - 2\sigma_{T_L}\sigma_{T_B}\rho_{T_LT_B}) + \mu_{T_B}(\sigma_{T_L}\sigma_{T_B}\rho_{T_LT_B} - \sigma_{T_L}^2)}{\sigma_{T_B}^2 - \sigma_{T_L}\sigma_{T_B}\rho_{T_LT_B}} \tag{18}$$

ESTIMATING MODEL PARAMETERS FROM MEASURED DATA

The model described above accounts for the correlation between the building temperature and the high-temperature and low-temperature thresholds. It is assumed that the building temperature is uncorrelated with the high-temperature and low-temperature thresholds. Although building occupants may exhibit coping behaviors in response to varying building temperatures, the maximum extent to which they can cope with undesirable temperatures is not related to the temperature itself. Instead it is related to the maximum extent to which clothing insulation, metabolism, and tolerance can be varied. These maxima are affected by the clothing style, the task being performed, the mood and health of the occupants, and other factors all of which are independent of the temperature.

Under the assumption that the building temperature is uncorrelated with the high-temperature and low-temperature thresholds, the model contains six unknown parameters. They are μ_{T_H}, σ_{T_H}, $\sigma_{\dot{T}_H}$, μ_{T_L}, σ_{T_L}, $\sigma_{\dot{T}_L}$. These statistics cannot be estimated directly because the respective random variables can only be measured directly when a complaint occurs. Therefore the statistics must be estimated indirectly. When the statistics of T_B, T_h, and T_l are estimated from temperature logs and complaint logs, the six unknown parameters of the model can be computed explicitly using the following procedure:

1. Log building temperatures.
2. Log complaint temperatures
3. Compute μ_{T_B}, σ_{T_B}, $\sigma_{\dot{T}_B}$ from the building temperature log of Step 1.
4. Compute μ_{T_h}, σ_{T_h}, μ_{T_l}, σ_{T_l} from the complaint log of Step 2.
5. Compute σ_{T_H} and σ_{T_L} from Equations (15) and (16), respectively.
6. Compute μ_{T_H} and μ_{T_L} from Equations (17) and (18), respectively.
7. Compute $\sigma_{\dot{T}_H}$ and $\sigma_{\dot{T}_L}$ from Equations (11) and (12), respectively.

Because each building log contains sufficient information to determine all of the model parameters, it is possible to “tune” the model to a particular building. Therefore, decisions regarding complaint costs can be made based on information that is relevant to that building.

The data in a complaint log referred to as Log A was analyzed by Federspiel (1998) and used to determine appropriate parameters of the complaint model. The facility corresponding to Log A will be referred to as Facility A. Building temperatures at Facility A were not recorded, so μ_{T_B}, σ_{T_B}, $\sigma_{\dot{T}_B}$ could not be computed explicitly according to Step 3. Instead, the value of μ_{T_B} was computed from the temperatures in the complaint log that were associated with humidity and air motion complaints because the mean value of the complaint temperatures for humidity and air motion complaints was the same as the mean building temperature. The value of $\sigma_{\dot{T}_B}$ was computed from the complaint and resultant temperatures for humidity and air motion complaints and the difference between the complaint time and the resolution time. Only the complaints for which either no action was taken or no action could be taken by the time that the complaint was resolved were used. Using these methods, the estimated value of μ_{T_B} was 74.0°F (23.3°C), and the estimated value of $\sigma_{\dot{T}_B}$ was 0.91°F/hour (0.50 K/h).

Figure 3 shows the hot and cold complaint temperature distributions from Federspiel (1998). At the time that the occupants complained, they were asked to read the temperature on the nearest thermostat and state the reading. This temperature was recorded in the complaint log.

The data contain large spikes, which are attributed to recruitment errors resulting from the manual recording process. Recruitment errors arise from the tendency of humans to “round” a manual reading to the nearest scale marker on an indicator. Some texts on the design of quantitative displays recommend that users round the readings to the nearest scale marker when the

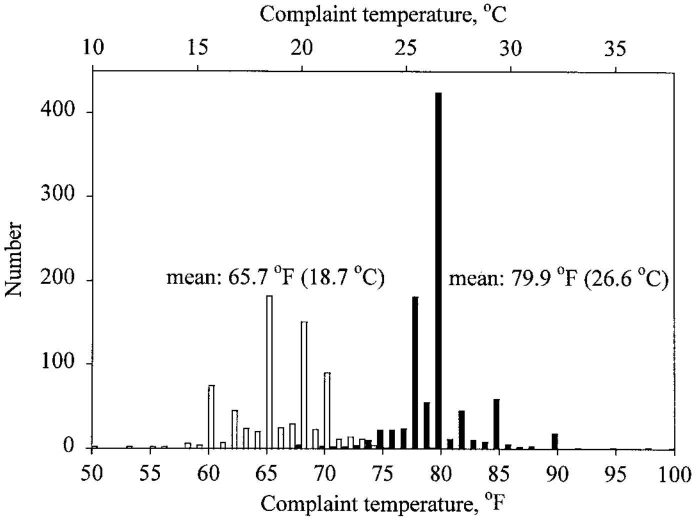

Figure 3. Hot and cold complaint temperature distributions from Log A

indicator lies between scale markers (Sanders and McCormick, 1987). Thermostats may have scale markers in increments of 2°F, 5°F, or 10°F (1°C, 2°C or 5°C).

Additionally, it was suspected that 80°F (26.7°C) has a "trigger" effect that causes the number of complaints at 80°F (26.7°C) to be even larger than can be explained by recruitment alone. In other words, when occupants feel too hot and they see that an indicator reads 80°F (26.7°C), then they complain. Had they not seen the indicator, they might not have complained until it the temperature was higher.

To eliminate the effects of recruitment and triggering on the estimates of the means and standard deviations of the hot and cold complaint temperature populations, the data were grouped into bins surrounding the recruitment temperatures, and the estimates that minimized the chi-squared norm were computed. The bins used for determining parameters of the model do not relate to the comfort zone, but are a mathematical technique used to improve the accuracy of the parameter estimates given the effects of recruitment and triggering observed in Figure 3. The bins used for the cold and hot complaint temperatures were:

Cold Complaint	Hot Complaints
T_l < 58.5°F (14.7°C)	T_h < 73.5°F (23.1°C)
58.5°F < T_l < 61.0°F (16.1°C)	73.5°F < T_h < 76.5°F (24.7°C)
61.0°F < T_l < 63.5°F (17.5°C)	76.5°F < T_h < 79.0°F (26.1°C)
63.5°F < T_l < 66.5°F (19.2°C)	79.0°F < T_l < 89.5°F (31.9°C)
66.5°F < T_l < 69.0°F (20.6°C)	T_l > 89.5°F (31.9°C)
69.0°F < T_l < 71.5°F (21.9°C)	
T_l > 71.5°F (21.9°C)	

The bins were selected so that each bin contained at least one recruitment spike. The large bin from 79 to 89.5°F (26.1 to 31.94°C) was used to capture complaints triggered at 80°F (26.7°C) . Using these bins, the estimated values of the means and standard deviations are as follows:

$$\mu_{T_h} = 81.0°\text{F}, \sigma_{T_h} = 3.70°\text{F}, \mu_{T_l} = 66.2°\text{F}, \sigma_{T_l} = 3.73°\text{F}$$

$$(\mu_{T_h} = 27.2°\text{C}, \sigma_{T_h} = 2.1°\text{C}, \mu_{T_l} = 19°\text{C}, \sigma_{T_l} = 2.1°\text{C})$$

The estimates of the standard deviations are affected by measurement errors. According to the 1997 *ASHRAE Handbook*, the uncertainty of a liquid-filled thermometer may be as high as 2°F (1°C). This value was taken as the mean absolute deviation of the measurement errors for a large sample of thermometers. The effect of the measurement errors on the estimated values of σ_{T_h} and σ_{T_l} were eliminated by subtracting the measurement error variance (6.25°F^2) from the square of the standard deviations shown above. This reduces the estimates to the following values: $\sigma_{T_h} = 2.73°\text{F}, \sigma_{T_l} = 2.77°\text{F}\ (\sigma_{T_h} = 1.51°\text{C}, \sigma_{T_l} = 1.54°\text{C})$.

To account for a trigger effect in the model, an additional term must be added to Equation (7). This term will contain a standard upcrossing frequency at 80°F (26.7°C), so that Equation (5) becomes

$$E[n_h] = P_h + (\nu_h + P_t \nu_{80})t \tag{19}$$

The parameter P_t is the probability that an up-crossing of 80°F (26.7°C), will trigger a complaint, and v_{80} is the mean up-crossing frequency of 80°F (26.7°C). It was estimated that 169 of the 412 complaints at 80°F (26.7°C), were caused by the trigger effect, and that $P_t = 0.0065$.

Because there was no temperature log that could be used to compute the value of σ_{T_B}, another method was used. This method relies on the fact that the model predicts that there will be more complaints at the beginning of the day than during the rest of the day. This is because there may have been a level crossing before the building was occupied that does not get identified as a complaint condition until the building becomes occupied. The probabilities P_h and P_l reflect this fact, and only contribute to the expected number of complaints when the building becomes occupied. Therefore, the estimated value of σ_{T_B} is chosen as the value that makes the estimated ratio of the complaints logged in the morning (prior to 10 a.m.) to the total number per day equal to the measured ratio. From the complaint log, it was determined that there were 2290 hot and cold complaints between 6 a.m. and 6 p.m., and that 1011 of these complaints occurred before 10 a.m. Therefore, σ_{T_B} was chosen as the value that makes the model predict that 1011 of the 2290 complaints occur before 10 a.m.

A Simplex type search method (Nelder and Mead 1965) was used to calculate the value of σ_{T_B}. Using this method, the estimated parameters are as follows:

$$\begin{aligned}&\sigma_{T_B} = 3.57°\text{F}, \mu_{T_H} = 91.0°\text{F}, \sigma_{T_H} = 4.24°\text{F}, \sigma_{\dot{T}_H} = 0.84°\text{F/hr},\\ &\mu_{T_L} = 54.5°\text{F}, \sigma_{T_L} = 4.39°\text{F}, \sigma_{\dot{T}_L} = 3.69°\text{F/hr}\end{aligned}$$

$$\left(\begin{aligned}&\sigma_{T_B} = 1.98°\text{C}, \mu_{T_H} = 32.8°\text{C}, \sigma_{T_H} = 2.36°\text{C}, \sigma_{\dot{T}_H} = 0.47\ \text{K/h},\\ &\mu_{T_L} = 12.5°\text{C}, \sigma_{T_L} = 2.43°\text{C}, \sigma_{\dot{T}_L} = 2.05\ \text{K/h}\end{aligned}\right)$$

The standard deviation of the rate of change of building temperature was $\sigma_{\dot{T}_B} = 0.91°\text{F/hr}$ $(\sigma_{\dot{T}_B} = 0.91\ \text{K/h})$.

Figure 4 shows the frequency of complaints as a function of the mean building temperature for different standard deviations. Also shown as the square in the figure is the estimated operating point for the facility from which the data used to determine the model parameters were taken. The figure shows that the complaint rate can be very low when the temperature is between 72°F (22.2°C) and 75°F (23.9°C). However, it will never be zero even if the variance of the building temperature is zero.

The procedure described above was applied separately to data acquired during the summer and winter. It was found that the mean values of the complaint levels were higher in the summer than in the winter, but the differences were less than one degree Fahrenheit. This indicates that seasonal, non-stationary behavior of the process exists, but that the magnitude of the non-sta-

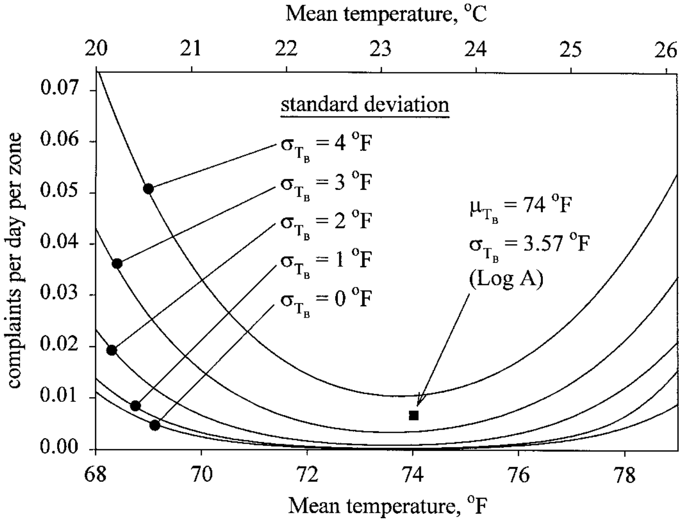

Figure 4. Complaint frequency as function of mean and standard deviation of temperature

tionary effect is small. Consequently, valuable results may be obtained using the assumption of stationarity.

QUALITATIVE MODEL VALIDATION

The values of P_h and P_l are the probabilities that a complaint condition exists given only the mean and standard deviation of the building temperature. When the standard deviation of the building temperature is zero, the sum of P_h and P_l is the expected fraction of zones in a complain condition given the temperature. This fraction is analogous to the Predicted Percent Dissatisfied (PPD) index.

Probit regression (Finney 1971) was used to formulated the PPD index. Probit is short for probability unit. Mathematically it is defined as the quantity Y in the following equation:

$$P = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Y-5} \exp\left(-\frac{1}{2}u^2\right) du$$

The value of five in the upper integrand was used to make the value of the probits nearly always positive.

Figure 5 shows the probit regression line for the data in Table 14 of Fanger (1972). The figure shows the standard deviations of the "too cool" and "too warm" dissatisfied populations. In the figure, the standard deviations are the magnitudes of the inverses of the slopes of the probit regression lines. The estimated values of σ_{T_L} and σ_{T_H} deviate from the standard deviations associated with PPD by only 8.7% and 12.8% respectively. In other words, the variability in the dissatisfied levels is comparable to the variability in the complaint levels. This fact lends credibility to the complaint model because dissatisfied conditions are qualitatively similar to complaint conditions.

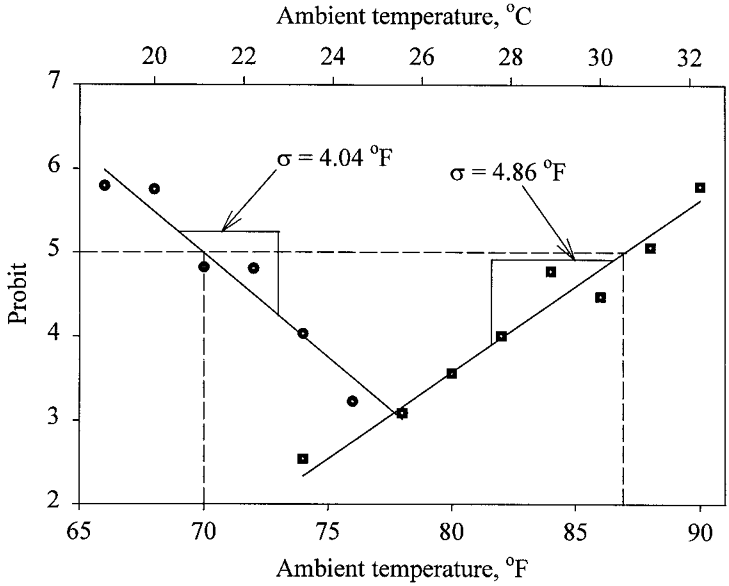

Figure 5. Maximum likelihood probit regression lines from data of Fanger (1972)

The difference between PPD and the sum of the conditional probabilities of the level-crossing complaint model is that the temperature level associated with a complaint is different from the level associated with the dissatisfied criteria. Figure 5 shows that the mean value of the "too cold" dissatisfied threshold is 70°F (21.1°C) and that mean value of the "too warm" dissatisfied threshold is 86.9°F (30.5°C). These values are the temperatures corresponding to a probit value of five. In the previous section, the mean value of the cold complaint threshold was determined to be μ_{T_L} = 54.5°F (12.5°C), and the mean value of the hot complaint threshold was determined to be μ_{T_H} = 91.0°F (32.8°C). The temperature that minimizes $P_h + P_l$ when μ_{T_B} = 0°F (–17.8°C) is 73.05°F (22.8°C). When the clothing insulation value is 0.75 clo, the metabolic rate is 1.29 met, the air velocity is 30 ft/min (9.2 m/s), the radiant temperature equals the dry bulb air temperature, and the relative humidity is 60%, the value of PMV will be zero and the value of PPD will be minimized at a temperature of 73.05°F (22.8°C). Using these values of clothing insulation, metabolic rate, air velocity and relative humidity, the values of μ_{T_L} = 54.5°F (12.5°C) and μ_{T_H} = 91.0°F (32.8°C) correspond to PMV = –2.55 and PMV = 2.55, respectively. These calculations indicate that complaint levels are, on average, one scale unit more intense than dissatisfied levels. If the PPD criterion were ±2.5 scale units instead of ±1.5 scale units, then near PMV = 0 the PPD curve would have the flattened shape of the zero-variance complaint curve shown in Figure 5.

EXAMPLES OF USE

The model described above may be used to assign economic cost to thermal discomfort, and therefore may form the basis for economic decisions regarding building operations which were not previously possible. In this section, an example is described which illustrates the following: (1) the economic impact of controlling building temperatures to the limits of the ASHRAE comfort zone, (2) use of the model for evaluating return on investment for a control system upgrade, and (3) optimizing building temperatures.

Energy calculations were made with a commercially-available energy analysis program and were based on a set of 60 buildings each with 50,000 ft^2 (4700 m^2) with a total square footage the same as Facility A. The HVAC systems in these fictitious buildings were 80% VAV and 20% constant volume and controlled to maintain the building at the limit of the ASHRAE comfort zone. Weather data typical of Houston was used in the energy analysis. For the service call costs, it was assumed that the set of 60 buildings contained 1358 zones which is the number of zones determined from analyzing the complaint log on which the parameter estimates are based. The standard deviation of the rate of change of the temperature was $\mu_{\dot{T}_B} = 0.91$ (0.51°C).

Figure 6 shows the annual cost of service calls triggered by hot and cold complaints as a function of the mean temperature and the standard deviation of the temperature. The complaint frequencies were converted to cost using the mean response times determined by Federspiel (1998) (1.6 and 2.1 hours for hot and cold, respectively) and a labor rate of $35/hour. These curves can be used as a cost function for evaluating the cost reduction potential of a control system upgrade that reduces the standard deviation of the temperature, and for determining the mean temperature that minimizes the cost of service calls triggered by hot and cold complaints.

The lower limit of the ASHRAE comfort zone is 68°F (20°C) for people clothed in typical winter attire and performing primarily sedentary activities (ASHRAE 1992). When they are clothed in typical summer attire and performing primarily sedentary activity, the upper limit of the ASHRAE comfort zone is 79°F (26.1°C). Figure 6 shows that there is a significant cost penalty when a building is controlled at the limit of the ASHRAE comfort zone. This penalty is the cost of responding to many more complaints than if the temperature had been less extreme. The penalty is dependent on the control performance of the system. When the control performance is poor (i.e., when the standard deviation is high), the absolute penalty is high. When the control performance is good, the relative penalty is high because the minimum complaint rate with good control performance is close to zero.

The mean building temperature will affect not only the complaint frequency but also the energy consumed by HVAC equipment. Therefore some decisions that affect the cost of service calls triggered by hot and cold complaints also affect energy cost. Figure 7 shows the predicted

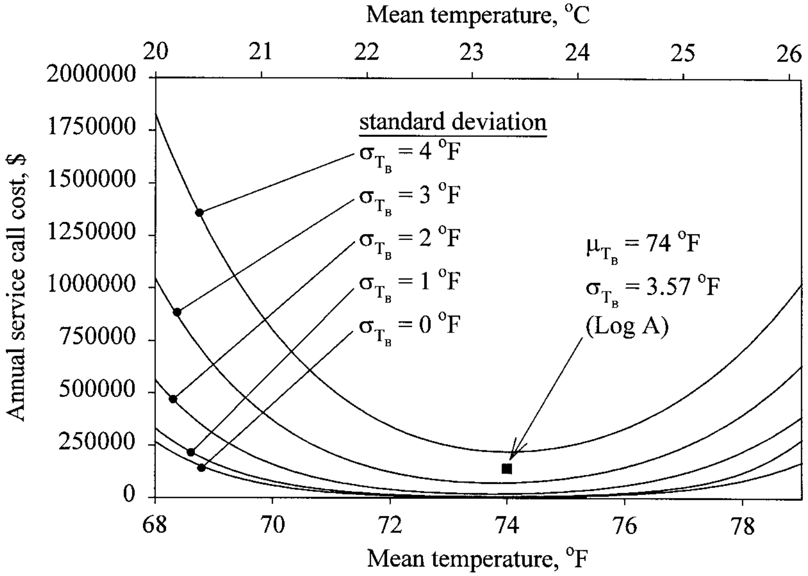

Figure 6. Costs of service calls triggered by hot and cold complaints (1358 zones)

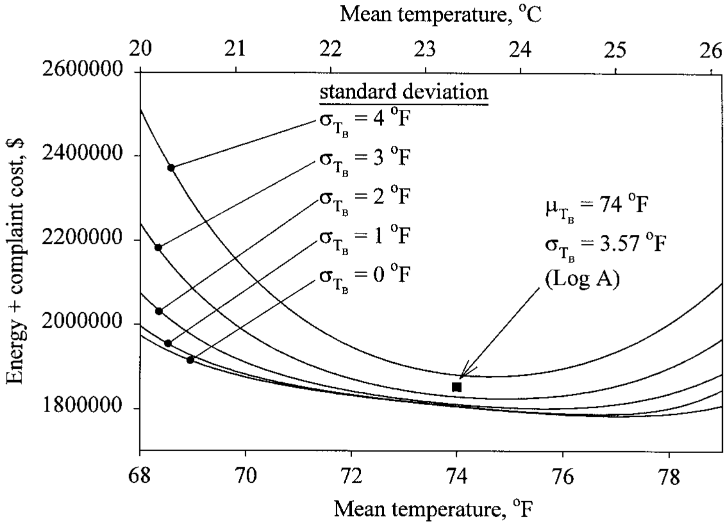

Figure 7. Energy plus complaint costs during summer
(1358 zones; 3 million square feet or 280 000 m^2)

cost of energy plus service calls resulting from complaints during the summer (May through September). The energy costs in the figure include all energy costs, gas and electric, and not just HVAC costs. The energy cost is only a function of the mean building temperature.

Figure 8 shows the relative magnitude of complaint costs to energy costs as a function of the mean and standard deviation of the building temperature. The figure illustrates that energy costs are generally much higher than complaint costs. Although the magnitude of energy costs are higher, the sensitivity of the energy cost to the mean temperature is lower than the sensitivity of complaint cost to the mean temperature when the mean temperature becomes extreme. For a given standard deviation of the temperature, there is always a mean temperature that will minimize the sum of the energy and complaint cost.

Figure 9 shows the cost effectiveness of the temperature controls as a function of the mean temperature and the standard deviation of the temperature. The cost effectiveness is defined as the minimum cost with perfect control ($\sigma_{T_B} = 0$) divided by the actual cost. This figure illustrates that even when the energy savings of raising the indoor temperature are considered, there is still a penalty associated with controlling building temperatures to the limit of the ASHRAE comfort zone. The magnitude of the penalty depends on the control performance. When σ_{T_B} = 3.57°F (1.98°C), the cost effectiveness of controlling these fictitious buildings at the limit of the ASHRAE comfort zone (i.e., at 79°F, 26.1°C) is 89.7%. In other words, there is a cost avoidance potential of 10.3%. When σ_{T_B} = 1.0°F (0.56°C), the cost effectiveness of controlling these fictitious buildings at the limit of the ASHRAE comfort zone (i.e., at 79°F) is 96.6% (cost avoidance potential of 3.4%). If weather data from climates milder than Houston had been used in the energy analysis then the cost effectiveness at the limit of the ASHRAE comfort zone would be less, and the cost avoidance potential more, because the energy costs would have been less dependent on the indoor temperature.

Figure 10 shows the optimal mean temperatures that minimize the energy plus service call cost and that minimize the service call cost during the summer. The independent variable is the standard deviation of the building temperature. Also shown in the figure is the point, marked A, at which Facility A was estimated to operate. From the location of point A it is clear that the

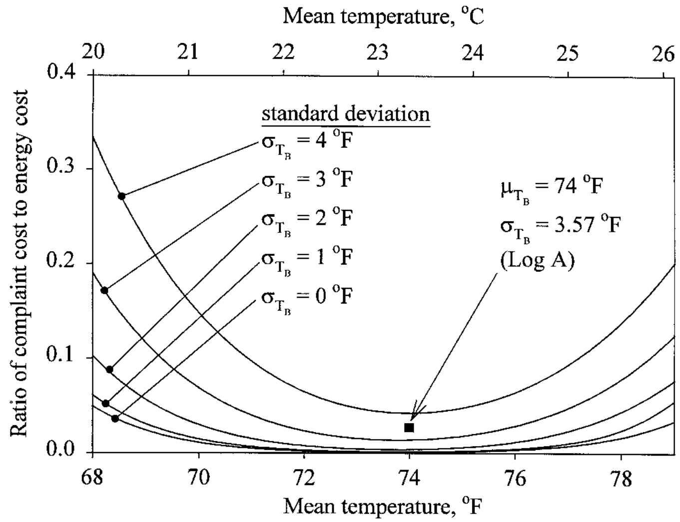

Figure 8. Cost of complaints relative to cost of energy during summer in Houston

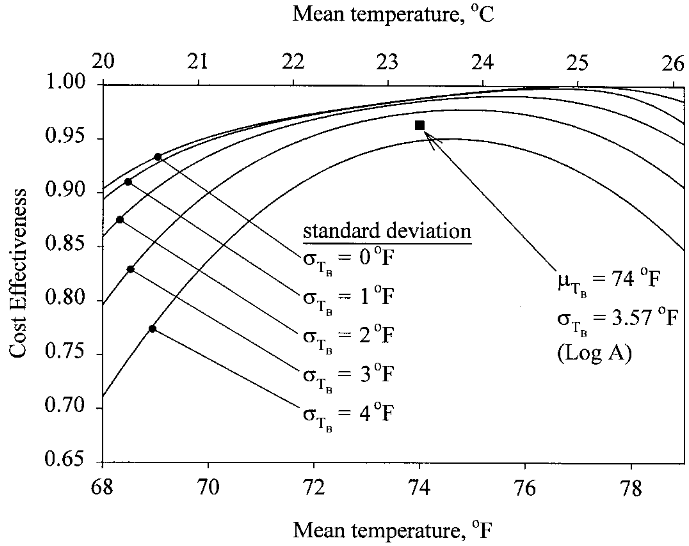

Figure 9. Cost effectiveness of temperature controls during summer in Houston

facility management has attempted to minimize complaints rather than to save energy by raising the building temperature. This analysis allows a quantitative determination of how much the building temperature should be raised in the summertime in order to optimize the energy plus complaint costs. As the figure shows, the amount that the temperature should be raised depends on the control system performance. If the standard deviation is low, then there is a larger potential to save energy without excessively increasing the frequency, and hence the cost, of complaints.

Some control systems use two different setpoints for heating and cooling to save energy, with the heating setpoint chosen lower than the cooling setpoint. Dual setpoints of this kind will make

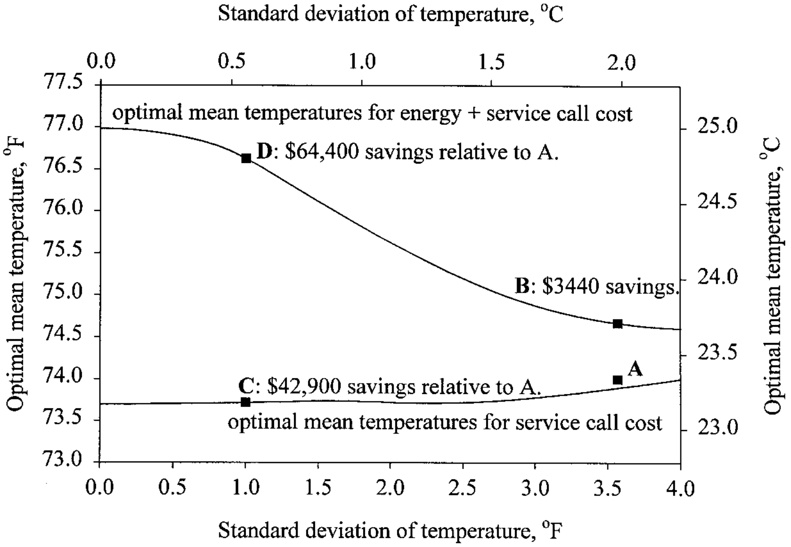

Figure 10. Optimal mean temperatures for two policies during summertime in Houston

the process non-stationary. The variance of the building temperature will be larger when the loads are nearly zero, and low loads will have a daily and seasonal dependence. Figure 10 does not account for the effect of dual setpoints because it is based on the assumption of normal processes. Although dual setpoints will save energy, the energy savings may be offset by the increased complaint cost caused by the increased variance of the building temperature.

Figure 10 also shows the cost savings of moving from operating point A to three other points marked B, C, and D. Point B could be achieved simply by raising the building temperature (i.e., change the temperature setpoints). Points C and D involve reducing the variance of the building temperature to 1.0 °F (0.55°C) in addition to changing the building temperature. A reduction in the variance might be achieved by upgrading the controls systems. The cost savings shown in the figure are for the four-month period of May through September for which the energy analysis was conducted. The figure illustrates that the standard deviation of the temperature has a greater impact on the cost savings than the mean value of the temperature. If the energy cost were independent of the mean temperature during the other eight months of the year which might be the case in Houston because the winter weather is mild, then the cost savings associated with reducing the standard deviation from 3.57°F (1.98°C) to 1.0°F (0.56°C) for that time period would be $92,400. The non-summer cost savings is greater than twice the savings of going from Point A to Point C because there is an energy penalty caused by the fact that the mean temperature at Point C is lower than at Point A. If the cost of the upgrade required to move from point A to point C or D were $500 per zone, then the return on investment of moving to point C would be 4.17 years, and the return on investment of moving to point D would be 3.7 years.

DISCUSSION

The model developed in this paper is based on the assumption that all three processes are stationary. If this assumption is not valid then the model may be extended to handle the non-stationary processes. However, the relations between the statistics of the complaint temperatures and the statistics of the three processes will no longer be valid, and it will be difficult to display the results of the model graphically. Additionally, estimating parameters of the model will become a more complicated process.

Controls that use the structure of a building as a thermal storage medium to shift the cooling loads off-peak to take advantage of lower off-peak energy rates have been investigated by Keeney and Braun (1997) and others. To get the most cost-saving benefit, these strategies require that the building temperature setpoints be adjusted so that the temperature is low at the beginning of a day and allow the temperature to rise through the course of the day. These variations will undoubtedly influence the frequency of hot and cold complaints. To date, the operational cost associated with complaints has not been included in the design of these control strategies. Instead, the strategies have been designed so that the temperatures are constrained to run against the limits of existing thermal comfort standards. The model described in this paper could be included in the total cost function to design dynamic building control strategies optimized for energy plus complaint cost.

The complaint rate model is consistent with related work on thermal comfort and sensation. A large data base is needed to evaluate the accuracy and precision of the model.

CONCLUSIONS

The complaint rate model predicts the following:

- Less temperature variability will lead to fewer complaints and lower operating cost.
- Operating buildings at the limits of the ASHRAE comfort zone incurs a significant cost penalty.
- The variances of the temperature levels at which occupants complaint that it is hot or cold are the same as the variances of temperatures associated with dissatisfied levels in the experiments on which the PPD index is based.
- Better control performance can lower both the cost of service calls triggered by complaints and the cost of energy.

ACKNOWLEDGMENTS

The author gratefully acknowledges Jim Kummer's assistance with the energy analysis, as well as numerous discussions with David Wyon regarding the implications of the theory.

NOMENCLATURE

C	complaint cost
$E[.]$	expected value
L	level
n	number of complaints
P	probability
R	labor rate
t	time period
T	temperature
$\dot{T}$	time rate of change of temperature
x	random process
z	standard normal deviate
Z	normal equivalent deviate
μ	mean value
ν	up-crossing frequency
ρ	correlation coefficient
σ	standard deviation
τ	complaint handle time

Subscripts

B	building
h	hot complaint
H	high (hot complaint) level
l	cold complaint
L	low (cold complaint) level
T	temperature
$\dot{T}$	time rate of change of temperature

REFERENCES

ASHRAE. 1992. Thermal Environmental Conditions for Human Occupancy. *Standard 55-92*. Atlanta: ASHRAE.

ASHRAE. 1997. *ASHRAE Handbook: Fundamentals*, p. 14.4.

Cramer, H., and M.R. Leadbetter. 1967. *Stationary and Related Stochastic Processes: Sample Function Properties and Their Application*. New York: John Wiley and Sons.

Fanger, P.O. 1972. *Thermal Comfort*. New York: McGraw-Hill.

Federspiel, C.C. 1998. Statistical Analysis of Unsolicited Thermal Sensation Complaints in Commercial Buildings. *ASHRAE Transactions* 104(1): 912-923.

Finney, D.J. 1971. *Probit Analysis*. Cambridge: The University Press.

Gagge, A.P., A.P. Fobelets, and L.G. Berglund. 1986. A Standard Predictive Index of Human Response to the Thermal Environment. *ASHRAE Transactions* 92(2B): 709-31.

Keeney, K.R. and J.E. Braun. 1997. Application of Building Precooling to Reduce Peak Cooling Requirements. *ASHRAE Transactions* 103(1): 463-69.

McNall, P.E., Jr., J. Jaax, F.H. Rohles, R.G. Nevins, and W. Springer. 1967. Thermal Comfort (Thermally Neutral) Conditions for Three Levels of Activity. *ASHRAE Transactions* 73. I.3.1-I.3.14.

Nelder, J.A. and R. Mead. 1965. A Simplex Method for Function Minimization. *The Computer Journal* 7: 308-13.

Nevins, R.G., F.H. Rohles, W. Springer, and A.M. Feyerherm. 1966. A Temperature Humidity Chart for the Thermal Comfort of Seated Persons. *ASHRAE Journal* 8(4): 55-61.

Rice, S.O. 1945. Mathematical Analysis of Random Noise. *Bell System Technology Journal* 24: 46-156.

Sanders, M.S., and E.J. McCormick. 1987. Human Factors in Engineering and Design. New York: McGraw-Hill, p. 122.

Sensharma, N.P., and J.E. Woods. 1997. An Extension of a Rational Model for Evaluation of Human Responses, Occupant Performance, and Productivity. *Proceedings of Healthy Buildings/IAQ '97*. 149-54.

Wyon, D.P. 1993. Healthy Buildings and Their Impact on Productivity. *Proceedings of Indoor Air '93*. 6: 3-13.

Wyon, D.P. 1996. Indoor Environmental Effects on Productivity. *Proceedings of IAQ '96*. Atlanta: ASHRAE.