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Application of the Water Cycle Algorithm to the Optimal Operation of Reservoir Systems

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Abstract: Water resources, in time, place, and quantity, are often poorly matched to the needs of humanity. This disparity tends to be more accentuated in arid and semiarid regions, predominant in Iran. Reservoir systems are a common means to control and manage water resources. Due to limited resources and the increasing demands for water, these systems must be optimally operated to maximize the efficiency of water use. Evolutionary optimization algorithms provide reliable and simple methods for solving complex optimization problems. One of those methods, the water cycle algorithm (WCA), is used in this paper to find optimal operation strategies for the Karon-4 reservoir and a four-reservoir system in Iran. The results demonstrate the high efficiency and reliability of the WCA in solving reservoir operation problems.
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Introduction

The ever-increasing growth of water needs in the agricultural, energy, industrial, and municipal sectors has stressed the water resources available to humanity. Moreover, aggravating factors as climate change phenomena have exacerbated the water stress in many parts of the world, especially in arid and semiarid climates. More profound present and future water shortages call for greater attention and more actions towards sustainable development. In many dry and semidry regions of the world, including most parts of Iran, the precipitation is not well-matched to the needs of water users in terms of quantity, time, and place. For this reason it is necessary to engage in management actions to fulfill consumer needs to achieve sustainable development and maximize benefits. Reservoirs are one of the most important management tools for the control and management of water resources in river basins. Considering the limited water resources in Iran, and the increase of water needs, using optimization methods in designing, implementing, and operating reservoirs is indispensable.

Various optimization methods are used in the operation of reservoirs; (1) linear programming (LP), and (2) dynamic programming (DP) could be mentioned as two classical optimization methods; e.g., Loáiciga and Mariño (1986). With the increase in the computational capacity of computers in recent years, new metaheuristic methods, which are mainly inspired by natural phenomena, have gained prominence relative to the classical methods. Metaheuristic and evolutionary methods have been proposed as a useful tool in the optimization of complex water resources systems in the past two decades. Using the deterministic dynamic programming (DDP) method, Karamouz and Houck (1982) generated monthly and annual reservoir operation rules. Karamouz and Houck (1987) made a comparison between DDP and stochastic dynamic programming (SDP) ability to generate operating rule of reservoirs. Using energy management and maintenance analysis (EMMA) and successive linear programming (SLP), Reznisek and Simonovic (1989) deduced different scenarios for the optimal operation of Manitoba's (Canada) compound power plant. The objective function maximized the system revenue and minimized its production costs. Their results showed that SLP had a better performance than the EMMA. Arunkumar and Jothiprakash (2012) developed an optimal operation for the Koyna reservoir, India that improved the hydroelectrical production and met agricultural needs by means of nonlinear programming (NLP). Wardlaw and Sharif (1999) applied the genetic algorithm (GA) to a four-reservoir system and demonstrated that real-value coding has better solution efficiency than binary coding. Jalali et al. (2007) added pheromone reinitiations (PRIs) and partial path replacement (PPR) to ant colony optimization (ACO), and successfully applied this algorithm to the optimization of single and multireservoir operation. Bozorg Haddad et al. (2011a) studied the efficiency of honey bee mating optimization (HBMO) algorithm in optimal operation of four-reservoir and 10-reservoir systems, in continuous and discrete domains. Their results indicated high capability of the HBMO algorithm in solving the optimization of reservoir operation.

Recently, many evolutionary and metaheuristic optimization algorithms have been developed and applied in all aspects of water resources systems such as reservoir operation (Bozorg Haddad et al. 2008b, c, 2009; Afshar et al. 2010; Fallah-Mehdipour et al. 2011b, 2012), cultivation rules (Moradi-Jalal et al. 2007; Noory et al. 2012), pumping scheduling (Bozorg Haddad and Mariño 2007; Rasoulzadeh-Gharibdousti et al. 2011; Bozorg Haddad et al. 2011b), water distribution networks (Bozorg Haddad et al. 2008a; Soltanjalili et al. 2011; Fallah-Mehdipour et al. 2011a; Seifollahi-Aghmiuni et al. 2011; Ghajarnia et al. 2011; Sabbaghpour et al. 2012), operation of aquifer systems (Bozorg Haddad and Mariño 2011), and site selection of infrastructures

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(Karimi-Hosseini et al. 2011). None of these works dealt with the application of the water cycle algorithm (WCA) to the optimal operation of water resources systems.

The WCA was introduced by Eskandar et al. (2012). They used it to solve several engineering optimization problems. They compared the results of the WCA with some other metaheuristic algorithms such as GA, particle swarm optimization (PSO) algorithm, harmony search, bee colony, and differential evolution. Their results showed that the WCA is a suitable method for solving constrained optimization problems and has the ability to compete with other metaheuristic algorithms. Eskandar et al. (2013) illustrated the application of the WCA by solving the problem of designing truss structures and compared the results with those of other metaheuristic algorithms such as GA, PSO, mine blast algorithm (MBA), and so on. The results of their comparison demonstrated the high capability of the WCA algorithm to find optimal solutions and its rapid convergence.

Because of its high capacity to find optimization solutions close to the global optimum efficiently, the WCA algorithm is implemented in the research reported in this paper to assess its performance in solving reservoir operation problems with applications to reservoir systems in Iran.

Water Cycle Algorithm

The WCA was described by Eskandar et al. (2012). The WCA is based on the water cycle of the real world. Streams and rivers flow downhill toward the sea. Water moves downhill in the form of streams and rivers, starting from high up in the mountains, and discharging to the sea and lakes. Streams and rivers collect water from rain and other streams on their way downhill. Some of the water forms rivers and lakes evaporate. Then, clouds are generated when the evaporated water is carried in the atmosphere. These clouds condense in the colder atmosphere and release the water back in the form of rain, creating new streams and rivers. The WCA is described in detail in the subsequent subsections.

Create the Initial Population

In order to solve an optimization problem using population-based metaheuristic methods, it is necessary that the values of the problem variables be formed as an array. In GA and PSO terminologies such array is called chromosome and particle position, respectively. In the WCA it is called raindrop for a single solution. In an N_{var} dimensional optimization problem (where N_{var} is the number of design variables), a raindrop is an array of size $1 \times N_{\text{var}}$. This array is defined as

$$\text{Raindrop} = [x_1, x_2, x_3, \ldots, x_{N_{\text{var}}}] \tag{1}$$

To start the optimization algorithm, a candidate representing a matrix of raindrops of size $N_{\text{pop}} \times N_{\text{var}}$ is generated (where N_{pop} is the number of populations of raindrops, initial population). Hence, the matrix $\mathbf{X}$ which is generated randomly is given as (rows and columns are the number of populations, and the number of design variables, respectively)

$$\text{Populations of raindrops} = \begin{bmatrix} \text{Raindrop}_1 \\ \text{Raindrop}_2 \\ \text{Raindrop}_3 \\ \cdot \\ \text{Raindrop}_{N_{\text{pop}}} \end{bmatrix} \tag{2}$$

Each of the decision variable values $(x_1, x_2, x_3, \ldots, x_{N_{\text{var}}})$ can be represented as floating point number (real values) or as a predefined set for continuous and discrete problems, respectively. The cost of a raindrop is obtained by the evaluation of a cost function (C) is

$$C_i = \text{cost}_i = f(x_1^i, x_2^i, x_3^i, \ldots, x_{N_{\text{var}}}^i) \qquad i = 1, 2, 3, \ldots, N_{\text{pop}} \tag{3}$$

In the first step, N_{pop} raindrops are created. A number N_{sr} [the summation of number of rivers (which is a user parameter) and a single sea as per Eq. (4)] from the best individuals (minimum values) are selected as sea and rivers. The raindrop which has the minimum value among others is considered as a sea. The rest of the population (raindrops form the streams which flow to the rivers or may directly flow to the sea) is calculated using Eq. (5)

$$N_{sr} = \text{Number of rivers} + \underbrace{1}_{\text{sea}} \tag{4}$$

$$N_{\text{raindrops}} = N_{\text{pop}} - N_{sr} \tag{5}$$

Eq. (6) is used to designate/assign raindrops to the rivers and sea depending on the intensity of the flow

$$NS_n = \text{round}\left(\left|\frac{\text{cost}_n}{\sum_{i=1}^{N_{sr}} \text{cost}_i}\right| \times N_{\text{raindrops}}\right), \qquad n = 1, 2, \ldots, N_{sr} \tag{6}$$

where round = function that rounds the value of the function within the parentheses to the closest integer number; and NS_n = number of streams which flow to the specific rivers or sea.

How Does a Stream Flow to the Rivers or Sea

The streams are created from the raindrops and join each other to form new rivers. Some of the streams may also flow directly to the sea. All rivers and streams discharge to the sea (best optimal point). A stream flows to a river along the connecting line between them using a randomly chosen distance is

$$X \in (0, C \times d), \qquad C > 1 \tag{7}$$

where C = value between 1 and 2 (near to 2; the best value for C may be chosen as 2); d = current distance between stream and river; and X = distributed random number (uniformly, or may be any appropriate distribution) between 0 and ($C \times d$). The value of C being greater than 1 enables streams to flow in different directions towards the rivers.

The concept behind Eq. (7) involving the flow of streams to rivers may also be used for the flow of rivers to the sea. Therefore, the new position for streams and rivers may be given as

$$X_{\text{stream}}^{i+1} = X_{\text{stream}}^{i} + [\text{rand} \times C \times (X_{\text{river}}^{i} - X_{\text{stream}}^{i})] \tag{8}$$

$$X_{\text{river}}^{i+1} = X_{\text{river}}^{i} + [\text{rand} \times C \times (X_{\text{sea}}^{i} - X_{\text{river}}^{i})] \tag{9}$$

where rand is a uniformly distributed random number between 0 and 1. If the solution given by a stream is better than its connecting river, the positions of river and stream are exchanged (i.e., stream becomes river and river becomes stream). Such exchange can similarly happen for rivers and sea.

Evaporation Condition

Evaporation is one of the most important factors that can prevent the algorithm from rapid convergence (immature convergence). As can be seen in nature, water evaporates from rivers and lakes. The evaporated water is carried into the atmosphere to form clouds which then condenses in the colder atmosphere, releasing the water back to earth in the form of rain. The rain creates the new streams and rivers, and the water cycle procedure continues. In the WCA, the evaporation process causes the sea water to evaporate as rivers/streams flow to the sea. This assumption is considered in order to avoid being trapped in local optima. The subsequent commands show how to determine whether or not a river flows to the sea

$$\text{If}|X^i_{\text{sea}} - X^i_{\text{river}}| < d_{\max} i = 1,2,3,\ldots,N_{sr} - 1$$
$$\text{Evaporation and raining process} \quad \text{End} \tag{10}$$

where $d_{\max}$ = small number (close to zero). Therefore, if the distance between a river and the sea is less than $d_{\max}$, it indicates that the river has reached/joined the sea. In this situation, the evaporation process is applied and as seen in nature, after sufficient evaporation the precipitation will start. A large value for $d_{\max}$ reduces the search while a small value encourages the search intensity near the sea. Therefore, $d_{\max}$ controls the search intensity near the sea (the optimum solution). The value of $d_{\max}$ decreases adaptively as

$$d_{\max^{i+1}} = d_{\max^i} - \frac{d_{\max^i}}{\max\ \text{iteration}} \tag{11}$$

Raining Process

After satisfying the evaporation process, the raining process is applied. In the raining process, the new raindrops form streams at different locations (acting similar to mutation operator in GA). For specifying the new locations of the newly formed streams

$$X^{\text{new}}_{\text{stream}} = LB + [\text{rand} \times (UB - LB)] \tag{12}$$

where LB and UB = lower and upper bounds defined by the given problem, respectively. Again, the best newly formed raindrop is considered as a river flowing to the sea. The rest of new raindrops are assumed to form new streams which flow to the rivers or may directly flow to the sea.

In order to enhance the convergence rate and computational performance of the algorithm for constrained problems, Eq. (13) is used only for the streams that flow directly to the sea. Eq. (13) encourages the generation of streams flow directly to the sea in order to improve the search near sea (the optimum solution) in the feasible region of constrained problems

$$X^{\text{new}}_{\text{stream}} = X_{\text{sea}} + [\sqrt{\mu} \times \text{rand}n(1, N_{\text{var}})] \tag{13}$$

where μ = coefficient which shows the range of the search region near the sea; and rand n = normally distributed random number. A larger value for μ increases the possibility to exit from the feasible region. On the other hand, the smaller value for μ leads the algorithm to search in smaller regions near the sea. A suitable value for μ is set to 0.1. From a mathematical viewpoint, the term $\sqrt{\mu}$ represents the SD and accordingly μ captures the concept of variance. Using these concepts, the generated individuals with variance μ are distributed around the best obtained optimum point (sea). The steps of WCA can be found at Eskandar et al. (2012). Fig. 1 shows the flowchart of WCA.

Evaluate the WCA with Mathematical Benchmark Functions

The (1) sphere, (2) Rosenbrock, and (3) Bukin6 benchmark functions (Bhattacharya 2010) were selected in the research reported in this paper with the purpose of evaluation the performance of the WCA. Table 1 shows details of these functions, where d represents the dimension of the functions; $d = 20$.

For comparison, the GA was also applied to obtain optimum solutions for the three selected benchmark functions. The evolutionary algorithms generally, and the WCA specifically, start from a set of random solutions, so an individual run does not show the capability or weakness of the algorithm. Therefore to test the effect of initial starting points, 10 different runs for both GA and WCA were performed in the research reported in this paper. Figs. 2(a–c) show the maximum and minimum of the benchmark functions [(1) sphere, (2) Rosenbrock, and (3) Buckin6] evaluated with the WCA. Figs. 3(a–c) depict the convergence rate of the three benchmark functions achieved with the GA and the WCA. Fig. 3 shows rapid convergence of the WCA in comparison with the GA. The WCA converges faster than the GA for all benchmark functions. Also it converges closer to global optima than the GA.

Figs. 4(a–c) show variables of best solution obtained by NLP (global optimum), WCA, and GA. Fig. 4 shows that values resulting from WCA are the closest to the global optimum. The GA results diverge from it. Table 2 shows summary results of the 10 different runs. The low values of SD indicate high reliability of the WCA compared to the GA.

Reservoir Operation

Fig. 5 shows the schematic of a reservoir and its variables. The storage in a reservoir is calculated as

$$S_{t+1} = S_t + Q_t - L_t - SP_t - R_t \tag{14}$$

in which S_{t+1} = reservoir storage at the start of period of $t + 1$; S_t = reservoir storage at the start of period of t; Q_t = inflow to reservoir in period t; L_t = net evaporation in period t, which can be calculated using Eq. (15) as per LoÆiciga (2002); SP_t = spill from reservoir in period t, which is defined by Eq. (16); and R_t = release from reservoir in period t. The variables appearing in Eq. (14) are volumetric

$$L_t = (E_t - P_t)A_t \tag{15}$$

$$SP_t = \begin{pmatrix} 0 & \text{if } S_t \le S_{\max} \\ S_t - S_{\max} & \text{if } S_t > S_{\max} \end{pmatrix} \tag{16}$$

in which E_t = evaporation depth in time period t; P_t = precipitation depth in period t; A_t = average area of reservoir lake in period t; and $S_{\max}$ = maximum of reservoir storage or reservoir capacity. Eqs. (15) and (16) the values of inflow, evaporation depth, and precipitation depth are available from historical data or simulation models. In multireservoir operation, inflow to each reservoir may include releases from upper reservoirs. Storage release is a decision variable in reservoir optimization. Reservoir storage is a state variable.

Constraints on releases are

$$R_{\min^i_t} \le R^i_t \le R_{\max^i_t} \tag{17}$$

Constraints on storages are

$$S_{\min^i_t} \le S^i_t \le S_{\max^i_t} \tag{18}$$

Carry over constraint is

$$S^i_1 = S^i_{T+1} \tag{19}$$

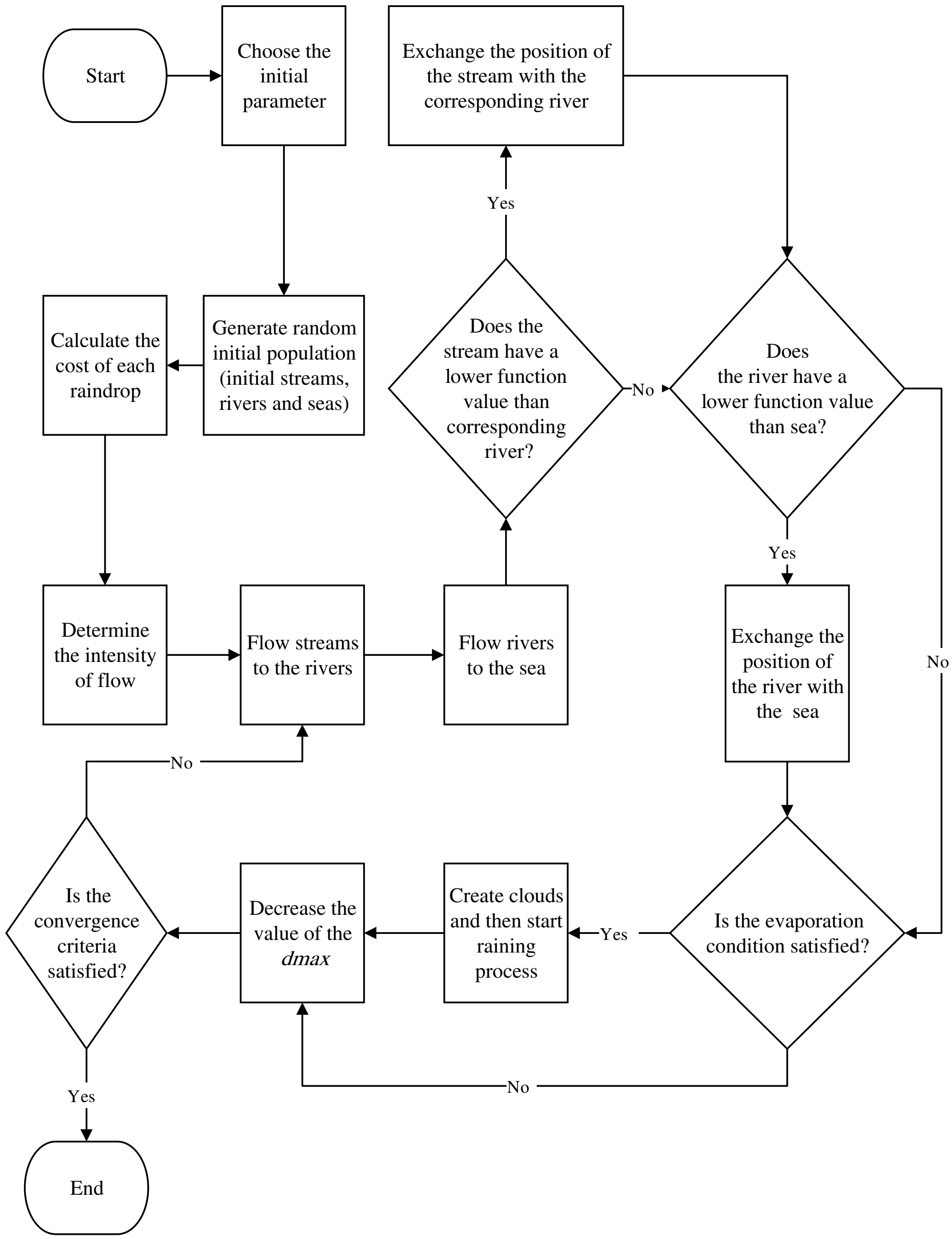

Fig. 1. Flowchart of the WCA

Table 1. Definition of the Mathematical Benchmark Functions

Row	Function name	Equation	Search space	Global optimum
1	Sphere	$f(x)=\sum_{i=1}^{d} x_i^2$	$-5.12 \le x_i \le 5.12$	$f_{\min}=0$ at $(0,0,\ldots,0)$
2	Rosenbrock	$f(x)=\sum_{i=1}^{d-1}[(1-x_i)^2+100(x_{i+1}-x_i^2)^2]$	$-5 \le x_i \le 10$	$f_{\min}=0$ at $(1,1,\ldots,1)$
3	Bukin6	$f(x,y)=100\sqrt{\lvert y-0.01x^2\rvert}+0.01\lvert x+10\rvert$	$-15 \le x \le -5$ $-3 \le y \le 3$	$f_{\min}=0$ at$(-10,1)$

in which $R_{\min_t^i}$ and $R_{\max_t^i}$ = minimum and maximum release from reservoir i in period t, respectively; R_t^i = release from reservoir i in period t; $S_{\min_t^i}$ and $S_{\max_t^i}$ = minimum and maximum storage of reservoir i at the start of period t, respectively; S_t^i = storage of reservoir i at the start of period t; S_1^i = storage of reservoir i at the start of the first period; S_{T+1}^i = storage of reservoir i at the start of period $T+1$; and T = total number of operation periods of the reservoir system. A fixed penalty is applied to the objective function in case of constraint violations.

Optimal Operation of the Karon-4 Reservoir System

The objective function of the reservoir optimization problem is defined by Eq. (20), which minimizes the sum of the squared

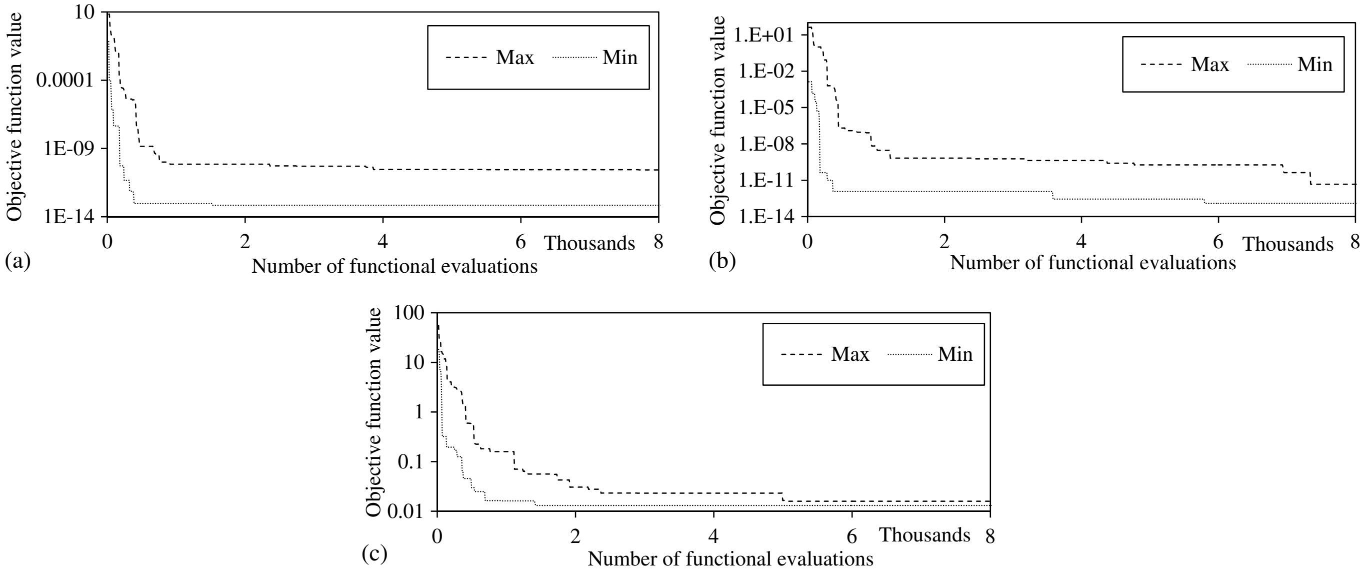

Fig. 2. Maximum and minimum of mathematical benchmark functions in each function evaluation by the WCA: (a) sphere; (b) Rosenbrock; (c) Bukin6

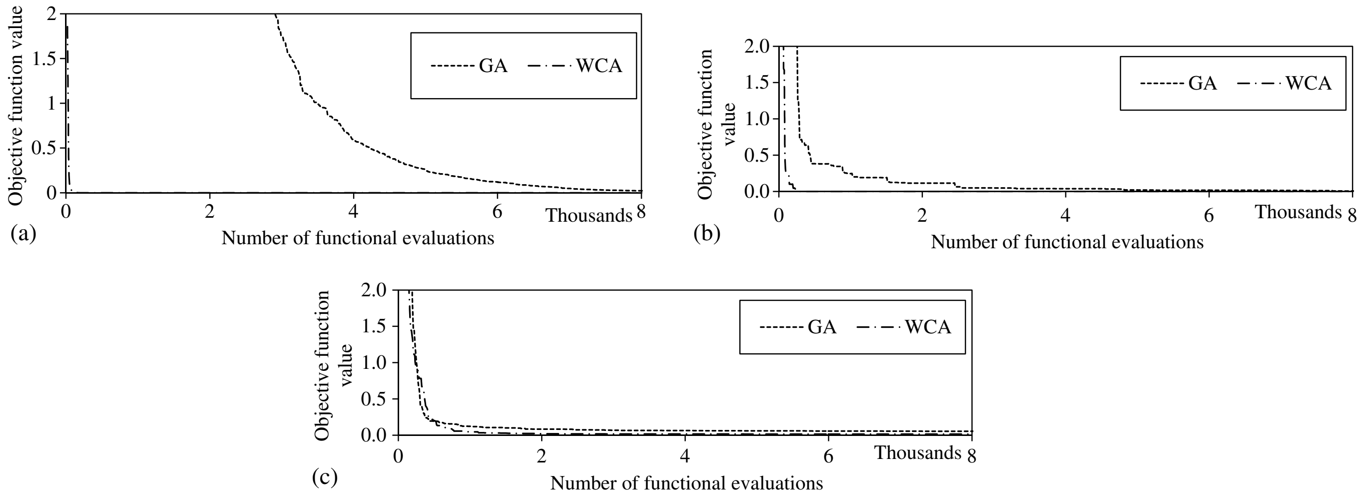

Fig. 3. Convergence rate of mathematical benchmark functions by the GA and the WCA: (a) sphere; (b) Rosenbrock; (c) Bukin6

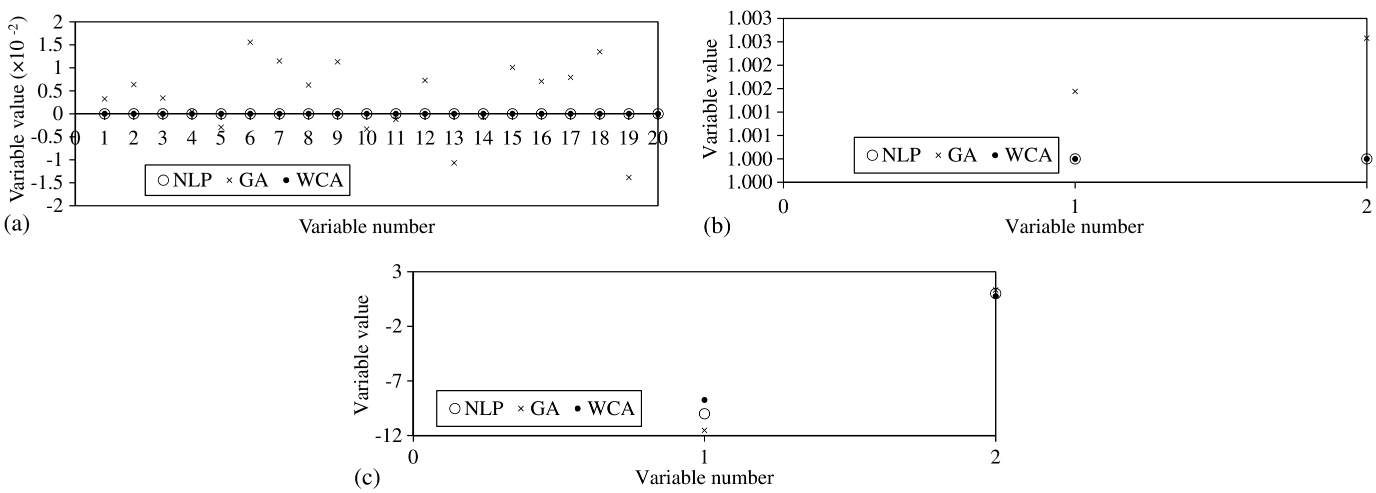

Fig. 4. Values of mathematical benchmark function variables by NLP, GA, and WCA: (a) sphere; (b) Rosenbrock; (c) Bukin6

Table 2. Summarized Results of Mathematical Benchmark Functions from 10 Different Runs of the GA and the WCA

	GA			WCA		
Statistics	Sphere	Rosenbrock	Bukin6	Sphere	Rosenbrock	Bukin6
Maximum	0.0458702	1.7×10^{-6}	0.016755	6.98976×10^{-14}	1.23286×10^{-13}	0.013086
Minimum	0.133336	0.00031	0.074314	2.70579×10^{-11}	4.72781×10^{-12}	0.015884
Average	0.08050554	0.000105	0.046036	3.26398×10^{-12}	1.29206×10^{-12}	0.014323
SD	0.027254698	0.000111	0.017238	8.39956×10^{-12}	1.58748×10^{-12}	0.000875

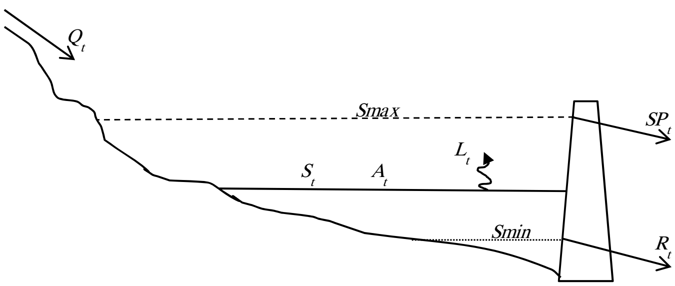

Fig. 5. Schematic view of reservoir storage and its variables

normalized differences between power generated and installed generating capacity, subject to constraints [Eqs. (14)–(19)] presented in the previous section

$$\text{Min } Z = \sum_{t=1}^{T} \left(1 - \frac{P_t}{\text{PPC}}\right)^2 \tag{20}$$

where Z = objective function; P_t = power generated by hydroelectric plant in period t (W) as defined by Eq. (21); and PPC = total capacity of hydroelectric plant (W) equal to 1,000 (10^6 W)

$$P_t = \text{Min}\left[\left(\frac{g \times \eta \times R_t}{PF}\right) \times \left(\frac{h_t}{1{,}000}\right), \text{PPC}\right] \tag{21}$$

$$h_t = \left(\frac{H_t + H_{t+1}}{2}\right) - \text{TWL} \tag{22}$$

where g = gravitational acceleration (9.81 m^2/s); η = efficiency of the hydroelectric plant; PF = plant factor; h_t = effective head of hydroelectric plant; H_t = elevation of water reservoir at the start of period t; H_{t+1} = elevation of water reservoir at the start of period $t+1$; and TWL = downstream (tailwater) elevation of the hydroelectric plant.

The global optimum of this problem was obtained with produced with NLP solver of *LINGO 8.0* and is equal to 1.2132. The WCA was applied to this problem with initial parameters value of 70, 35, 2.7, and 1,000 for N_{pop}, N_{sr}, $d_{\max}$, and max iteration, respectively. Therefore 70,070 function evaluations were performed. Table 3 presents results of 10 different runs of the WCA and the GA, showing of the maximum (worst), minimum (best), average, SD, and coefficient of variation of these 10 runs.

The best result obtained from the 10 different runs of the WCA produced optimum close to 97% of the global optimum. On the other hand, the GA converged to 79% of the global optimum. Fig. 6 shows the maximum and minimum of the objective function in each function evaluation employing the WCA. Fig. 7 shows the average solution over the 10 runs of the GA and WCA.

The best result obtained with the WCA was equal to 97% of the global optimum solution while GA algorithm provided a solution equal to 79%. The average achieved solution from the WCA is also closer to the global optimum solution than from the GA for the reservoir operation problem. In addition, the small amount of coefficient of variation obtained with the WCA demonstrates the superior ability of the algorithm to reach a solution near the global

Table 3. Results of 10 Different Runs of Karon-4 Problem Using the GA and the WCA

	Objective function value	
Number of run	GA	WCA
1	1.67	1.289
2	1.54	1.269
3	1.86	1.287
4	1.75	1.260
5	1.98	1.289
6	1.75	1.285
7	1.93	1.281
8	1.56	1.279
9	1.84	1.286
10	1.52	1.262
Global optimum	1.213	
Maximum, worst	1.98	1.289
Minimum, best	1.52	1.260
Average	1.74	1.279
SD	0.16	0.010
Coefficient of variation	0.09	0.008

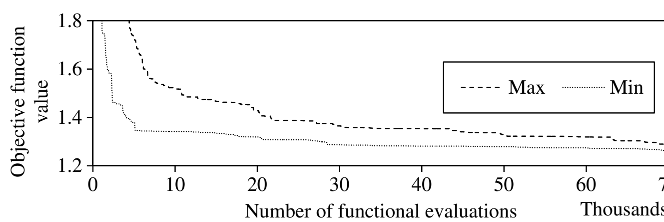

Fig. 6. Maximum and minimum of the objective function of the Karon-4 reservoir system in term of the number of functional evaluations by the WCA

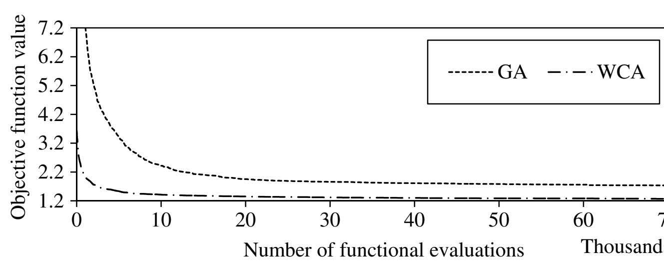

Fig. 7. Convergence rate of the objective function of Karon-4 reservoir system in terms of the number of functional evaluations by the GA and WCA

optimum solution of the reservoir problem. Fig. 6 shows the maximum, average, and minimum of the WCA objective function in terms of the number of functional evaluation, and Fig. 7 compares the convergence rates of the WCA and the GA. Fig. 6 shows that the maximum, minimum, and average values of the objective function converge to each other as with increasing number of functional evaluations, which demonstrates the high reliability of the WCA.

Figs. 8–10 show reservoir releases, reservoir storage, and power generated in each period of operation (month), respectively. Figs. 8–10 show that all defined constraints in the reservoir optimization problem have been satisfied. Moreover, the compatibility of the WCA results with the global optimum solution can be observed (Figs. 8–10). The GA results, on the other hand, show deviations from the global optimum solution. This finding is more pronounced for reservoir release and generated power (Figs. 8 and 10, respectively).

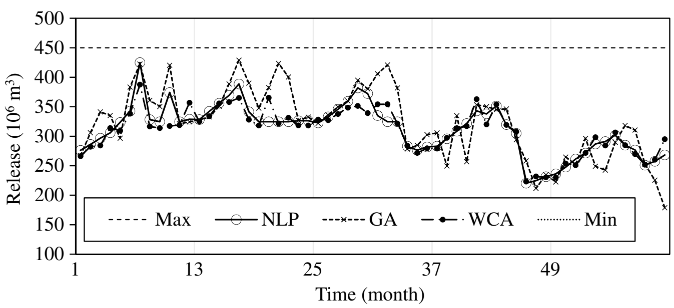

Fig. 8. Monthly optimal releases from Karon-4 reservoir

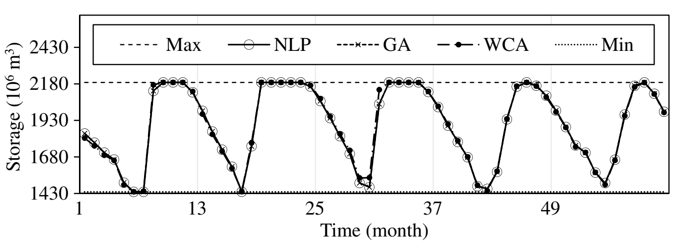

Fig. 9. Monthly optimal storage of the Karon-4 reservoir

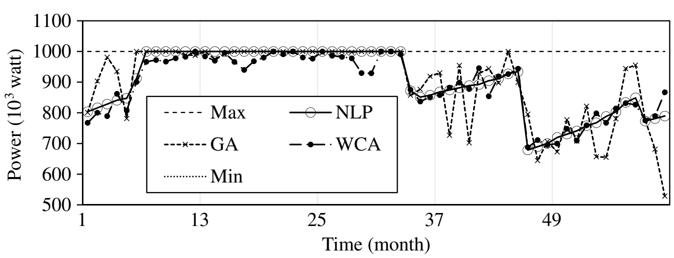

Fig. 10. Monthly optimal power generated by the Karon-4 reservoir

Optimal Operation of a Four-Reservoir System

The considered four-reservoir system was first formulated by Chow and Cortes-Rivera (1974), and was reutilized by Murray and Yakowitz (1979). A global optimum of 308.26 units was reported by Chow and Cortes-Rivera (1974). Fig. 11 shows the schematic of this problem. Water released from these reservoirs is utilized to meet hydropower and agricultural functions. The objective function of this problem is defined as

$$\text{Max } B = \sum_{i=1}^{n}\sum_{t=1}^{T} b_t^i \times R_t^i - P \tag{23}$$

where B = total benefits from all reservoirs; i = reservoir number; n = total number of reservoirs; b_t^i = benefit function in time period t for reservoir i; and P = penalty function as defined in Eqs. (24)–(26). Other parameters were defined previously

$$P = \sum_{i=1}^{n} C^i + \sum_{i=1}^{n}\sum_{t=1}^{T} SL_t^i \tag{24}$$

$$C^i = \begin{bmatrix} k_1(S_{T+1}^i - S_1^i)^2 & \text{for } \forall\ i = 1, \ldots, nS_1^i > S_{T+1}^i \\ 0 & \text{for } \forall\ i = 1, \ldots, nS_1^i \le S_{T+1}^i \end{bmatrix} \tag{25}$$

$$SL_t^i = \begin{bmatrix} k_2(S_{\min^i} - S_t^i)^2 & \text{for } \forall\ i = 1, \ldots, nS_t^i < S_{\min^i} \\ k_3(S_t^i - S_{\max^i})^2 & \text{for } \forall\ i = 1, \ldots, nS_t^i > S_{\max^i} \\ 0 & \text{for } \forall\ i = 1, \ldots, nS_{\min^i} \le S_t^i \ge S_{\max^i} \end{bmatrix} \tag{26}$$

where C_i = penalty of violation of Eq. (19); SL_t^i = penalty of violating minimum and maximum of reservoir storage; k_1 = constant of penalty C_i; k_2 is a constant of penalty for violating constraint on minimum reservoir storage; and k_3 = constant of penalty for violating constraint of maximum reservoir storage. The connectivity of releases between the reservoirs of this system (Fig. 11) is determined by the matrix $\mathbf{M}$, defined by Eq. (27). The constraint parameters for this problem are set to (1) $R_{\min} = 0.005$, $S_{\min} = 1$, and $S_{\max} = 10$ for all reservoirs; (2) $R_{\max} = 4.0$ for Reservoir 1, 4.5 for Reservoirs 2 and 3, and 8 for Reservoir 4; and (3) $S_1 = 6$ for Reservoirs 1–3, and 8 for Reservoir 4. In this problem, the penalty constant of k_1 was considered equal to 20, and penalty constants of k_2 and k_3 were considered equal to 40. A penalty constant of 40 was previously used by Heidari et al. (1971) and Bozorg Haddad et al. (2011a)

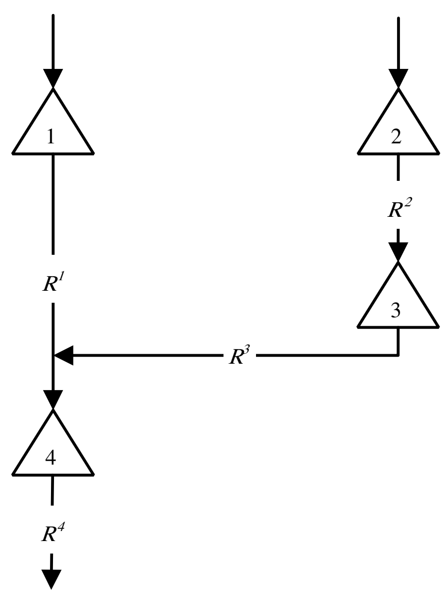

Fig. 11. Layout of the four-reservoir system

Table 4. Results of 10 Different Runs of the Four-Reservoir Problem Using the GA and the WCA

	Objective function value	
Number of run	GA	WCA
1	300.42	306.83
2	298.89	302.40
3	300.09	303.65
4	300.47	303.60
5	298.46	302.38
6	300.00	306.01
7	299.22	304.05
8	299.87	306.75
9	299.20	306.63
10	300.35	306.92
Global optimum	308.29	
Maximum, best	300.47	306.92
Minimum, worst	298.46	302.38
Average	299.70	304.92
SD	0.705	1.887
Coefficient of variation	0.002	0.006

$$\mathbf{M} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 1 & -1 \end{bmatrix} \quad (27)$$

For comparison purposes, the nonlinear programming solution of linear programming was also obtained with the *LINGO* software; the solution equals 308.29, which is similar to the solution reported by Bozorg Haddad et al. (2011a). Initial parameters value of 100, 50, 1×10^{-5}, and 5,000 for N_{pop}, N_{sr}, $d_{\max}$, and max iteration, respectively. were taken into account for solving the problem of optimal operation of the example four-reservoir system using the WCA. Table 4 shows the results obtained with the (1) GA, and (2) WCA. Table 4 summarizes the maximum, minimum, average, SD, and coefficient of variation from 10 different runs of the two algorithms.

The best solution achieved with the WCA is 306.920, which has a 0.4% difference with the global optimum solution of the problem. Fig. 12 summarizes the maximum, minimum, and average values obtained with the WCA in terms of the number of functional evaluations. The latter results and the small coefficient of variation resulted from the 10 different runs (0.006) establish that WCA-derived results are slightly dependent on the stochastically selected values of the initial population. Furthermore, the results are very near the global optimum solution. This trait is clearly a favorable strength of the WCA.

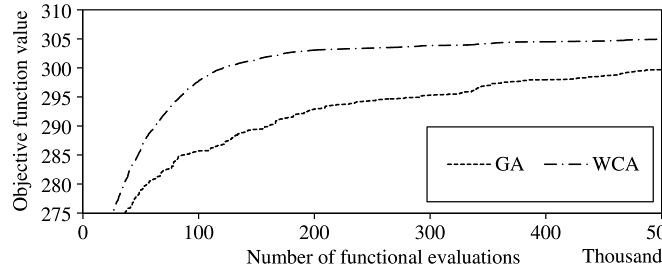

Fig. 13. Convergence rate of objective function of four-reservoir system terms of the number of functional evaluations by the GA and the WCA

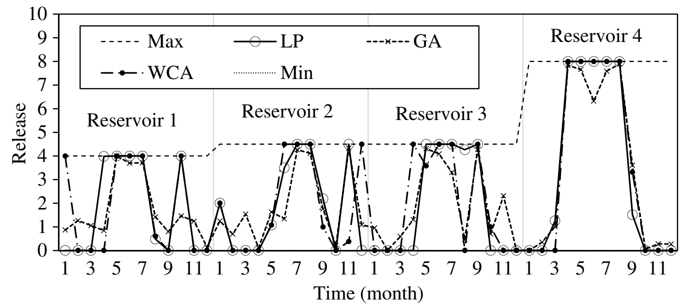

Fig. 14. Monthly optimal releases from the four-reservoir system

The average amount attained with the algorithm exhibits 1% difference with the global optimum solution of the problem, which indicates high accuracy of the WCA and its capacity to achieve near-optimal global solutions. Fig. 13 shows the comparison of convergence rates of the GA and WCA, which shows that the convergence of the WCA is faster than that of the GA.

This reservoir optimization problem was solved by Bozorg Haddad et al. (2011a) using the HBMO algorithm. On the one hand, 14 million functional evaluations yielded an objective function value equal to 308.24 units, which is very close to the global optimum. On the other hand, 1.1 million WCA functional evaluations yield a value of 307.50 units, which is even closer to the global optimum (306.920).

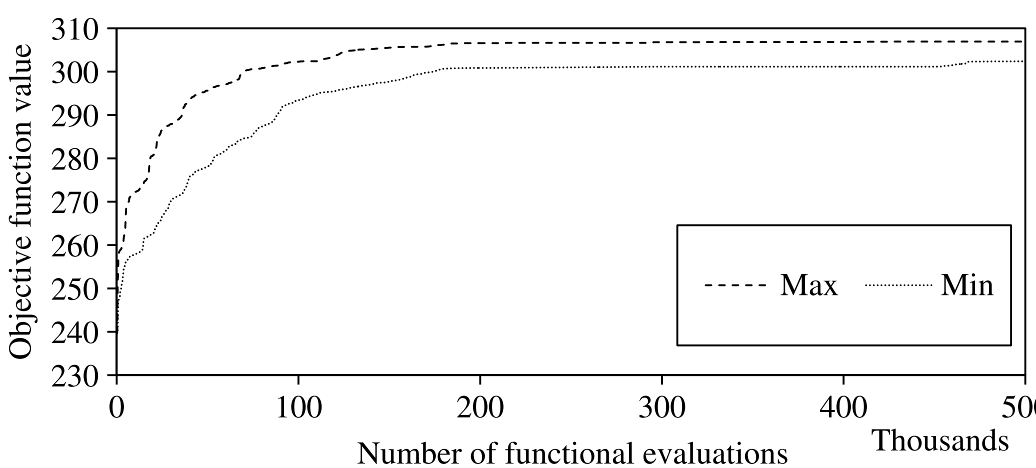

Fig. 12. Maximum and minimum of the objective function of the four-reservoir system in terms of the number of functional evaluations by the WCA

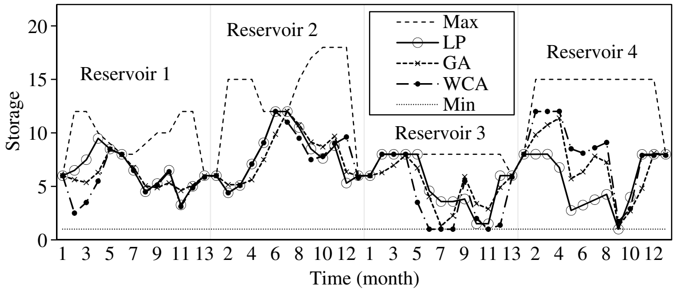

Fig. 15. Monthly optimal storage of the four-reservoir system

Figs. 14 and 15 show monthly reservoir releases and reservoir storages. The maximum and minimum were met by the optimized releases and storages. Additionally, reservoir storage is the same at the beginning and end of the operation period. Figs. 14 and 15 show the high compatibility of the WCA solution with the global optimum solution, whereas the GA results exhibits noticeable deviations from the optimal solution.

Concluding Remarks

The WCA has been advanced as a method for solving optimization problems. It provides a fast convergence rate to a near-optimal solution, as well. In the research reported in this paper, the application of this algorithm was discussed in the context of the solution of water resources management problems. First, the algorithm was assessed with several benchmark functions and its performance was examined. Afterwards, optimal operation of the Karon-4 reservoir and a four-reservoir system were solved using this algorithm. The results confirmed the high capacity of the WCA in solving the type of reservoir optimization problems considered in this paper. The algorithm averaged solutions that are 96 and 99% of the global optimum solution, in the Karon-4 reservoir and in the four-reservoir system, respectively. The performance of the WCA established its superior convergence to near-optimal solutions, faster convergence rate, and higher reliability than those exhibited by several leading competing solution algorithms.

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