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ELECTRICAL ENGINEERING REVIEW COURSE

LECTURE XVII

August 5, 1952

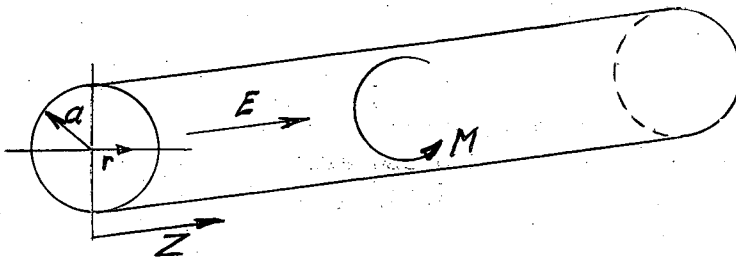
E. Martinelli

(Notes by: A. Chesterman)

I. Cavity - Right Circular Cylinder

A. Application, Linear Accelerators.

B. Excitation, TM_{010}



1. Define TM_{010}

TM, transverse magnetic

z axis has 0 nodes, electric field

r axis has 1 node, magnetic field

ϕ axis has 0 nodes

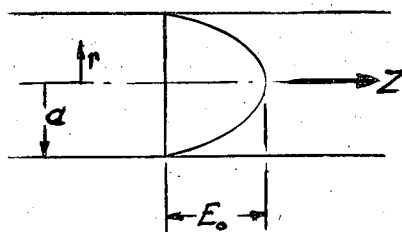
2. Resonance Frequency depends on radius, $\lambda = 2.61 a$

$$\frac{c}{\lambda} = \text{frequency}$$

c = velocity of light

C. Field distribution

1. Electrical

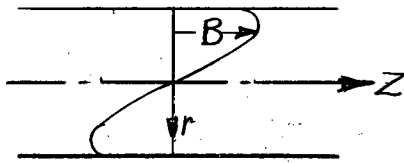


$$E_z = J_0\left(\frac{2.405r}{a}\right)e^{j\omega t}$$

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2. Magnetic

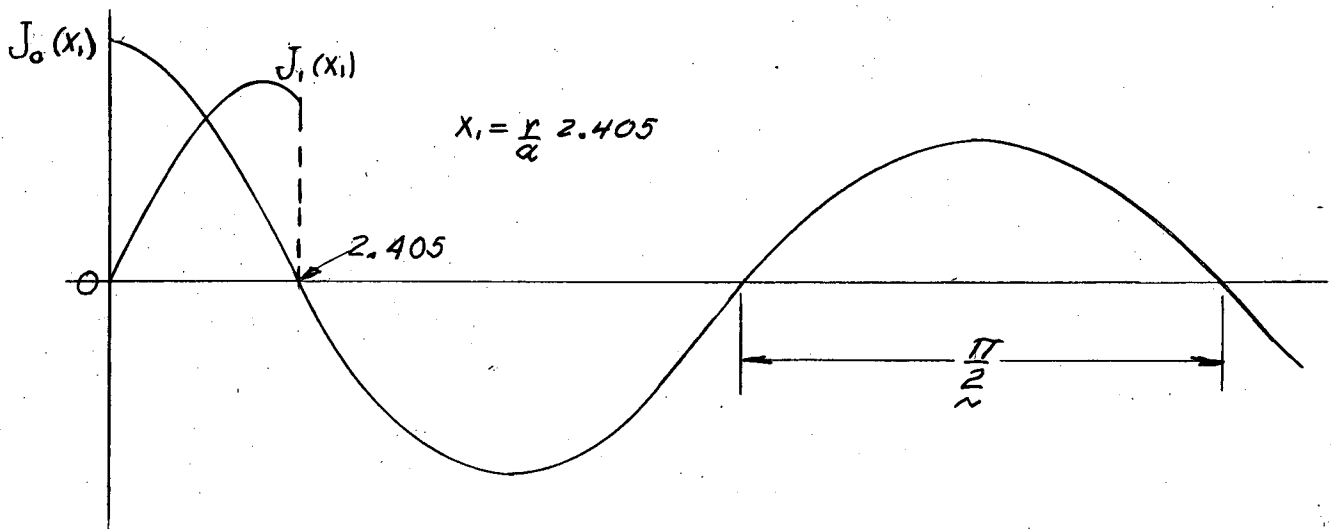


$$B_{\phi} = j \frac{2.405}{a} \frac{c}{w} J_1\left(\frac{2.405r}{a}\right) e^{j\omega t}$$

or

$$B_{\phi} = \frac{2.405}{a} \frac{c}{w} J_1\left(\frac{2.405r}{a}\right) e^{j\omega t + \frac{\pi}{2}}$$

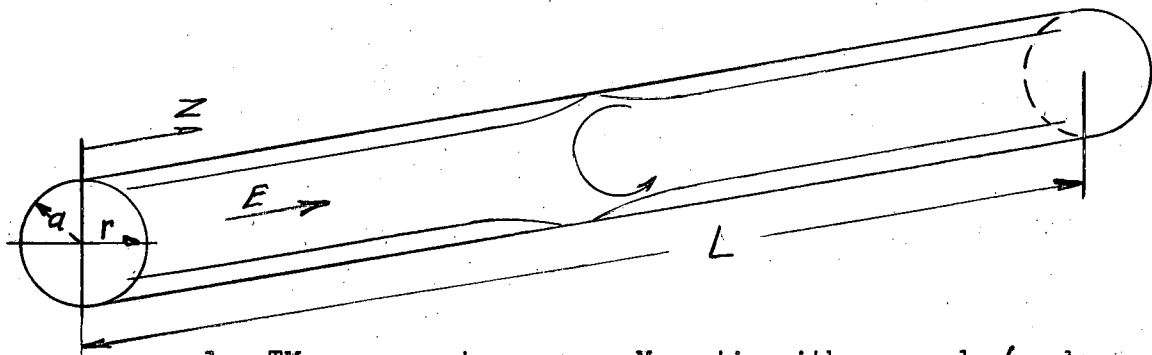
PLOT OF BESSEL'S FUNCTIONS, J_0 & J_1



- From expressions given for Electrical and Magnetic fields, one can see they have a 90° phase difference and in general the energy flows in and out radially.

The cavity can be fed at one point and the energy transferred axially is proportional to $\frac{c}{\sqrt{2Q}}$. Unless the cavity becomes quite long the length doesn't effect the frequency, however, for long cavities there is a tendency for a shift in resonance frequency to the next higher node.

D. Excitation in TM_{110} - Long Cavities



- TM_{110} means transverse Magnetic with one node (or bump as shown) along Z axis, one node along r axis, and no nodes along ϕ axis.

2. Resonant Frequency depends upon radius, however --

$$E = J_0 r \frac{2.405}{a} \cos(\beta_{on} Z) e^{j\omega t}$$

if $\beta_{on} = \frac{n\pi}{2L}$ we obtain successive nodes.

$$\beta_{on} = \sqrt{\left(\frac{\pi}{L}\right)^2 - \left(\frac{2.405}{a}\right)^2}$$

From solution of Differential Equations,

$$\lambda_{on} = 2.61a \frac{1}{\sqrt{(2.61a)^2 \left(\frac{n}{2L}\right)^2 + 1}} \approx \lambda_{00} \left[1 - \frac{1}{8} \left(\frac{\lambda_{00}^2}{L^2} \right) \right]$$

For example compare TM_{010} and TM_{110} (that is for $n = 0$ and $n = 1$) for 40 ft. (approximately 10 meters) Linear Accelerator, $\lambda = 1.5$ meters (Approximately).

$$\frac{\lambda_{01}}{\lambda_{00}} \approx 1 - \frac{1}{8} \frac{(1.5)^2}{100} \approx 1 - \frac{1}{400}$$

This means it is easy for the cavity to slip into the next mode since there is so little difference between λ_{01} and λ_{00} .

E. "Q" of Cavity

1. "Q" is defined as ratio of energy stored in the cavity to the energy lost per cycle.

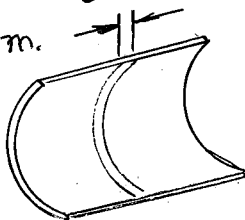
Energy Stored = $\omega \int_V \frac{B^2}{4\pi} dV$, per second for all of volume "V".

Energy Lost = $\int_S i^2 R ds$, per second for complete area "S"

$$Q = \frac{\omega \int_V \frac{B^2}{4\pi} dV}{\int_S i^2 R ds}$$

But $R = \frac{\rho}{\delta}$ per cm., where ρ = specific resistance
 δ = skin depth cm.

i in cm.



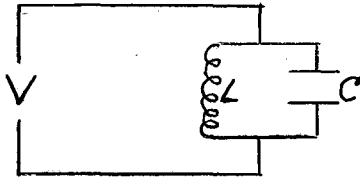
And $i = \frac{B_t}{4\pi}$, where B_t = Tangential Magnetic Field

B = Total Magnetic Field

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$$\text{Thus, } Q = \frac{\omega \delta}{4\pi} \frac{V \int B^2 dV}{\int_s B_r^2 ds}, \text{ Theoretical}$$

F. Shunt Impedance



Parallel circuit of resonance acts like a high impedance.

$$\frac{V}{I} = Z_s, \text{ shunt impedance}$$

For a cavity,



$$V = \int E_z dz \text{ along } z \text{ axis.}$$

$$P = \frac{V^2}{2Z_s}, \text{ power or } \int_s i^2 R ds$$

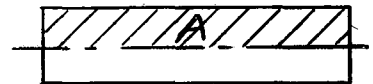
$$\text{Thus } Z_s = \frac{V^2}{2P} \text{ or } \frac{\left[\int E_z dz \right]^2}{\frac{2}{(4\pi)^2} \frac{1}{\delta} \int_s B_t^2 ds}$$

$$V = \int E_z dz = \int \dot{B}_\phi dA$$

Consider cavity as a transformer

$$V = \omega \int B_\phi dA$$

$$\text{And } Z_s = \frac{\left[\omega \int B_\phi dA \right]^2}{\frac{2}{(4\pi)^2} \frac{1}{\delta} \int_s B_t^2 ds} = \underset{\substack{\uparrow \\ \text{Theoretical}}}{Z_s} \times \underset{\substack{\uparrow \\ Q}}{\frac{Q}{Q}} =$$



Z_s is proportional to length L , i.e., P decreases with gradient squared.