Title
E.E. REVIEW COURSE - LECTURE XVII.

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ELECTRICAL ENGINEERING REVIEW COURSE

LECTURE XVII
August 5, 1952
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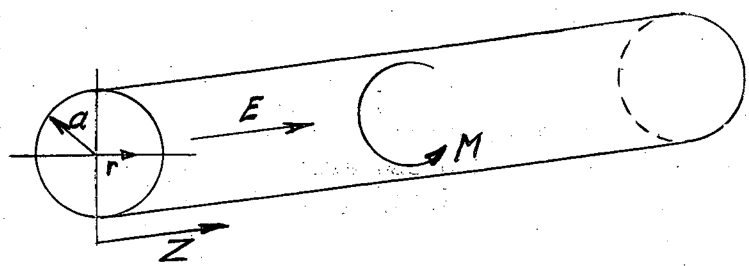
(Notes by: A. Chesterman)

I. Cavity - Right Circular Cylinder

A. Application, Linear Accelerators.

B. Excitation, TM_{010}

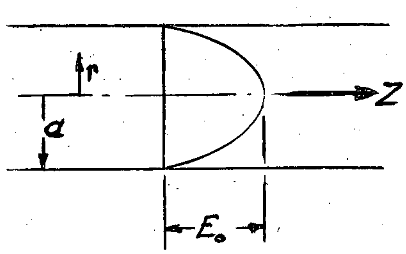


1. Define TM_{010}
   TM, transverse magnetic
   z axis has 0 nodes, electric field
   r axis has 1 node, magnetic field
   ϕ axis has 0 nodes

2. Resonance Frequency depends on radius, $\lambda = 2.61\ a$
   $\frac{c}{\lambda}$ = frequency c = velocity of light

C. Field distribution

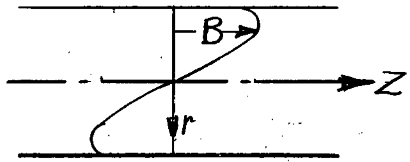
1. Electrical



$$E_z = J_o\left(\frac{2.405r}{a}\right)e^{j\omega t}$$

DISCLAIMER

2. Magnetic

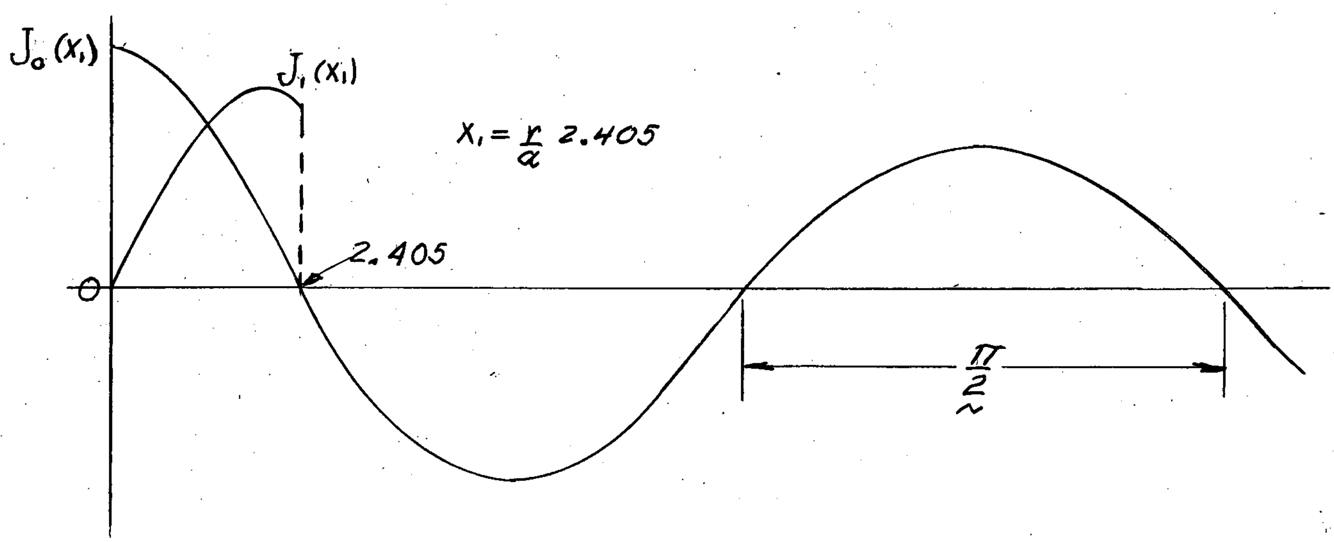


$$B_\phi = j\frac{2.405}{a}\,\frac{c}{w}J_1\left(\frac{2.405r}{a}\right)e^{jwt}$$

or

$$B_\phi = \frac{2.405}{a}\,\frac{c}{w}J_1\left(\frac{2.405r}{a}\right)e^{\left(jwt+\frac{\pi}{2}\right)}$$

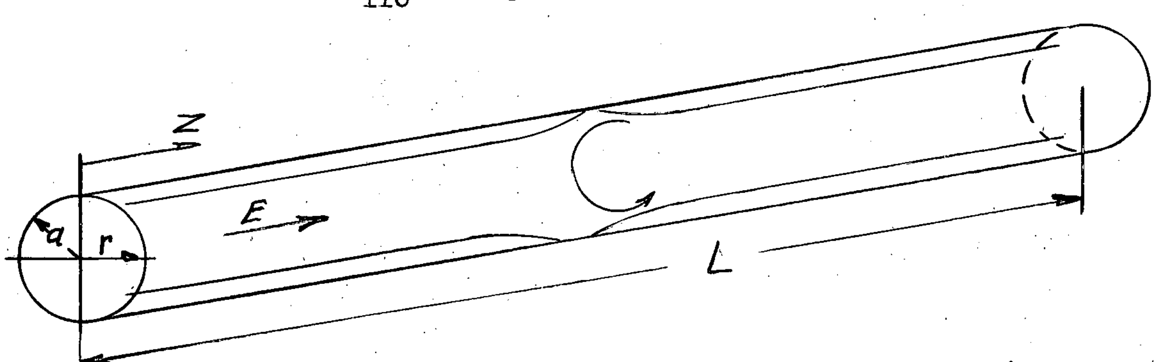
PLOT OF BESSEL'S FUNCTIONS, J_0 & J_1



3. From expressions given for Electrical and Magnetic fields, one can see they have a 90° phase difference and in general the energy flows in and out radially.

The cavity can be fed at one point and the energy transferred axially is proportional to $\frac{c}{\sqrt{2Q}}$. Unless the cavity becomes quite long the length doesn't effect the frequency, however, for long cavities there is a tendancy for a shift in resonance frequency to the next higher node.

D. Excitation in TM_{110} - Long Cavities



1. TM_{110} means transverse Magnetic with one node (or bump as shown) along Z axis, one node along r axis, and no nodes along ϕ axis.

2. Resonant Frequency depends upon radius, however --

$$E = J_0\, r\, \frac{2.405}{a} \,\mathrm{Cos}\left(\beta_{on} Z\right) e^{j\omega t}$$

if $\beta_{on} = \frac{n\pi}{2L}$ we obtain successive nodes.

$$\beta_{on} = \sqrt{\left(\frac{W}{L}\right)^2 - \left(\frac{2.405}{a}\right)^2}$$

From solution of Differential Equations,

$$\lambda_{on} = 2.61a \,\frac{1}{\sqrt{(2.61a)^2\left(\frac{n}{2L}\right)^2 + 1}} \simeq \lambda_{00}\left[1 - \frac{1}{8}\left(\frac{\lambda^2_{00}}{L^2}\right)\right]$$

For example compare TM_{010} and TM_{110} (that is for n = 0 and n = 1) for 40 ft. (approximately 10 meters) Linear Accelerator, λ = 1.5 meters (Approximately).

$$\frac{\lambda_{01}}{\lambda_{00}} \simeq 1 - \frac{1}{8}\,\frac{(1.5)^2}{100} \simeq 1 - \frac{1}{400}$$

This means it is easy for the cavity to slip into the next mode since there is so little difference between λ_{01} and λ_{00}.

E. "Q" of Cavity

1. "Q" is defined as ratio of energy stored in the cavity to the energy lost per cycle.

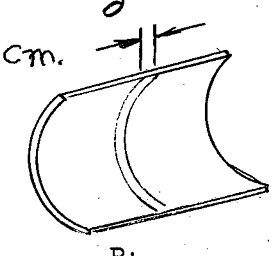
Energy Stored = $\omega \int_V \frac{B^2}{4\pi}\, dV$, per second for all of volume "V".

Energy Lost = $\int_s i^2 R\, ds$, per second for complete area "S"

$$Q = \frac{\omega \int_V \frac{B^2}{4\pi} dV}{\int_s i^2 R\, ds}$$

But $R = \frac{\rho}{\delta}$ per cm., where ρ = specific resistance
δ = skin depth cm.

l in cm.

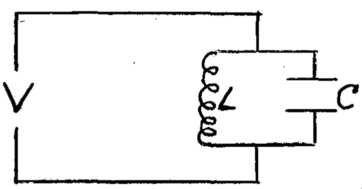


And $i = \frac{B_t}{4\pi}$, where B_t = Tangential Magnetic Field

B = Total Magnetic Field

Thus, $Q = \frac{\omega\delta}{4\pi\rho}\frac{\int_v B^2 dV}{\int_s B_r^2 ds}$, Theoretical

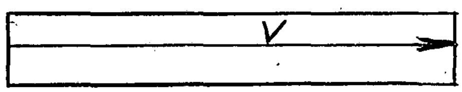
F. Shunt Impedance



Parallel circuit of resonance acts like a high impedance.

$\frac{V}{I} = Z_s$, shunt impedance

For a cavity,



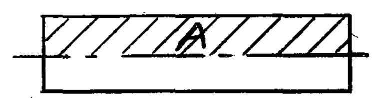
$V = \int E_z dz$ along z axis.

$P = \frac{V^2}{2Z_s}$, power or $\int_s i^2 R ds$

Thus $Z_s = \frac{V^2}{2P}$ or $\frac{\left[\int E_z dz\right]^2}{\frac{2\rho}{(4\pi)^2}\, 2\, \frac{1}{\delta}\int_s B_t^2 ds}$

$V = \int Ezdz = \int \dot{B}_\phi dA$

Consider cavity as a transformer



$V = \omega\int B_\phi dA$

And $Zs = \frac{\left[\omega\int B_\phi dA\right]^2}{\frac{2\rho}{(4\pi)^2\delta}\int_s B_t^2 ds} = \underset{\uparrow}{Zs} \times \frac{Q}{\underset{\uparrow}{Q}} =$

Theoretical

Zs is proportional to length L, i.e., P decreases with gradient squared.