The complementary roles of knowledge and strategy in insight problem-solving

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Abstract

Two main classes of theory have been proposed to account for insight problem-solving performance; those that invoke the overcoming of constraints arising from prior knowledge as the source of insight, and those that propose strategic search for moves that make progress towards a hypothesized goal state. An experiment using matchstick algebra problems assessed the contributions of each source. Results indicate that, while prior knowledge creates the conditions under which matchstick algebra problems are more or less difficult to solve, search for moves that make the most apparent progress towards a hypothesized goal provides the key to eventual solution.

Keywords: insight problem-solving; restructuring; prior knowledge; strategic search; representational change; progress monitoring.

Introduction

A seminal paper by Knoblich, Ohlsson, Haider and Rheinus (1999) heralded a wave of research into insight problem-solving and introduced a new theoretical framework, Representational Change Theory (RCT). RCT builds upon previous work of Ohlsson (1984 a & b, 1992), in which he emphasized the role of the initial problem representation in eliciting from memory prior knowledge that might prove useful to solution or might be unhelpful, resulting in impasse. In the latter case, reaching a solution requires re-representation of the problem to activate potentially more useful knowledge for solution.

In RCT, the negative effects of prior knowledge can be overcome by constraint relaxation, a process described as "one of the mind's responses to persistent failure" (Knoblich et al, 1999, p. 1535). The ease with which a constraint of knowledge can be relaxed is a function of its scope, that is, how much an individual's mental representation would be affected by changing that knowledge, the narrower the scope the more likely being its relaxation. In addition to constraint relaxation, RCT proposes that the chunks in which prior knowledge is organized can restrict the solution of insight problems. To solve some problems requires chunk decomposition, a process also triggered by persistent failure. The ease of chunk decomposition, according to RCT, is a function of how 'tight' the chunk is bound together, where tightness typically relates to perceptual organization (e.g., where components are linked together as a single object).

Knoblich et al tested the predictions of RCT in a series of innovative experiments involving matchstick algebra problems, in which the task is to correct an arithmetic sum shown as Roman numerals made up of individual matchsticks by moving only a single matchstick (see Fig. 1 for examples). Problems in which the scope of knowledge to be relaxed applies widely across mathematics (e.g., the fact that formulae typically have an $x = F(y)$ structure rather than the tautologous structure required for some solutions, as exemplified by problems e and f, Fig.1) were solved less often than those where mathematical knowledge applies more locally (e.g., changing an operator from division to multiplication, as in problems a and b, Fig.1). Similarly, problems requiring decomposition of a tight chunk (e.g., decomposing a + to make a -, as in problem c, Fig.1) were solved less often than those requiring decomposition of a loose chunk (e.g., decomposing III to make a II, as in problems a and b, Fig.1).

A number of other studies have confirmed the role that scope and tightness of constraints play in mediating problem difficulty (e.g., Jones, 2003; Zhang et al., 2015; Öllinger et al., 2017), and there seems little doubt that prior knowledge is a source of difficulty in solving matchstick algebra problems. Otherwise, each match within a problem would presumably have a similar probability of selection, notwithstanding differences in perceptual salience. What is less certain, however, is the mechanism through which relaxation and decomposition occur, and also how more useful knowledge can become activated so that solutions are eventually found. The notion that processes of constraint relaxation and decomposition are a natural response to persistent failure implies that there is a mechanism for switching off unwanted knowledge, at least temporarily, and another for discovering and activating more useful knowledge. Yet, while repeated failed attempts may act in an inhibitory fashion, the current problem representation is presumably continually re-activating that inappropriate knowledge. Moreover, if inappropriate knowledge is somehow switched off to enable solution, presumably there needs to be some mechanism that switches it back on again once the problem-solving episode is over, otherwise general mathematical performance might be impaired. Even if the influence of inappropriate knowledge can somehow be temporarily suspended, this begs the question as to how the right knowledge gets activated in its place.

	Low value	High value
Loose chunk	a) VIII = IV / III Solution: VIII = IV × II	b) L = XXV / III Solution: L = XXV × II
Tight chunk	c) X + II = IV / II Solution: X − II = IV × II	d) C = XXV / III Solution: L = XXV X III
Standard tautology	e) II = II − III Solution: II = II = II	f) II = LI − II Solution: II = II = II
Reverse tautology	g) IV = II = II Solution: IV = II + II	h) C = L = L Solution: C = L + L

Figure 1. Matchstick algebra problems used in the experiments. The solution to each problem must be found by moving one match only to make the equation mathematically correct.

MacGregor, Ormerod & Chronicle (2001) proposed a theory of progress monitoring in insight problem-solving, which has subsequently been referred to as Criterion for Satisfactory Progress Theory (CSPT – Ormerod & MacGregor, 2017). CSPT offers an alternative account of difficulty and success in insight problem-solving to that of RCT. In CSPT, the source of difficulty in solving insight problems is not the imposition of inappropriate prior knowledge, but is instead the selection of move attempts that appear to make the most progress towards a hypothesized goal, but that do not lie on the solution path. For example, the nine-dot problem requires four straight lines to be drawn connecting nine dots arranged as a square grid, without removing one's pen from the paper. According to CSPT, the problem is hard because individuals can find many ways of making satisfactory progress in drawing the first three lines, so they fail to consider moves that make less initial progress but might allow them to discover the complete solution.

Insight occurs, according to CSPT, not when constraints of prior knowledge are relaxed, but when 'criterion failure' arises, that is, when no further moves can be discovered that meet a criterion of satisfactory progress (in the case of the nine-dot problem, the ratio of dots remaining to lines available), at which point the problem space is expanded to search for novel moves that do meet the criterion for satisfactory progress. The role of progress-monitoring in insight problem-solving has also been confirmed in other studies (e.g., Jones, 2003; Ormerod, MacGregor & Chronicle, 2002; Ormerod et al, 2013; Nakano, 2017).

CSPT has little to offer in explaining the initial difficulty of matchstick algebra problems: the scope and tightness of chunks of mathematical knowledge act independently of the perceived progress towards solution that moving any one match might make. However, it does offer a mechanism for discovering solutions, and it is one that does not require the suspension of knowledge to allow new knowledge to surface. Under CSPT, the search for moves in solving a matchstick algebra problem is guided by an estimation of the progress towards solution that any move might make, that is, the extent to which a move is likely to reduce the disparity between the sides of the expression. Where a move makes the most apparent progress towards solution, it is more likely to be selected than moves that make less progress. If this maximizing move also coincides with the correct solution, the problem will be solved.

This hypothesis is tested in two experiments reported below. The first experiment explores the effects of varying the size of the numerical values, to act as a signal for moves that make the most progress in reducing the equation disparity. This hypothesis is contrasted against a manipulation of chunk tightness, which according to RCT should determine solution rates. The second experiment compares the solution of problems requiring the discovery of tautologous expressions, the most difficult class of problem identified by Knoblich et al (1999), with problems presented in the form of a reverse tautology that, according to RCT, should be easier to solve because the constraint of knowledge scope is removed in the latter problems. The experiment also examines how manipulating numerical values moderates the effects of scope, as predicted by CSPT.

Experiment 1

In this experiment, participants attempted to solve matchstick algebra problems requiring the construction of a new operator to solve. The problems differed according to chunk tightness: RCT predicts that problems that require the decomposition of the loose chunk 'III' to solve (problems a and b in Fig. 1) will be solved faster and more often than those requiring the decomposition of the tight chunks (the '+' in problem c and the '[' representing 100 in problem d in Fig 1). A second prediction that follows from RCT is that participants' initial attempts should focus more on matches that are loosely chunked (operationalized here as any match that is not physically connected to any other, with the exception of the division operator, which because of its relatively high scope as an operator is counted as a tight chunk). CSPT predicts no effect of Chunking.

Problems also differed according to the numerical value of their components. According to CSPT, because the difference between numerical values in low value equations is relatively small, considering a change to the numbers is worthy of exploration, thereby reducing focus on changing the operator, which is the move required for solution. In high value equations, changing the numbers will not yield sufficient progress, increasing the likelihood that they will be quickly passed over and the operator will be focused upon. Thus, high-value equations should be solved more often than low-value equations. A further prediction that follows from CSPT is that participants' initial attempts to solve high-value problems should focus more on matches that are part of operators or high value numbers (e.g., 50 or 100) than matches that are part of low value numbers, with the opposite pattern of initial attempts being found with low-value problems. As a consequence, value acts in opposition to chunking, decreasing selections of matches from loose chunks when value is high. RCT predicts no effect of Value on solution rates or on initial attention focus, which should always be on number before operator.

Method

Participants One hundred and twenty nine adults participated on a voluntary basis during a series of undergraduate student recruitment open days at the University of Sussex. Five participants attempted fewer than 50 of the pre-test items to identify alphanumeric equivalents of Roman numerals, and 3 correctly identified fewer than 90% of those they attempted. This left 121 participants: 38 males (mean age = 26.9) and 83 females (mean age = 25.4).

Materials and Design The problems used as task stimuli in this experiment are problems a-d as shown in Fig.1. Each problem was composed of 14 matches, and was presented 10 times on a problem sheet to allow repeated attempts to be recorded. Participants were assigned to either high-value problem or low-value problem groups, and within each of those groups were further assigned to loose-chunk or tight-chunk groups, yielding a fully between-subjects design. Dependent variables comprised the frequency of solutions to each problem, and the initial attempts of participants classified in terms of the frequency of initial attempts involving loosely-chunked matches.

Procedure Participants solved the problems individually in groups of approximately 30 people during introductory Psychology workshops. To reduce the likelihood of collusion, participants sitting adjacent to each other were assigned to different experimental groups so that their problems differed. Participants were each given a booklet containing an ethical consent form, a Roman numerals training/pre-test, a matchstick algebra problem that varied according to experimental group, and a study debrief sheet.

Following the method of Knoblich et al (1999, Expt. 1a), participants received a training phase in which they first read a description of the structure and nomenclature of Roman numerals. They then received a sheet showing 100 numbers of values between 1 and 200 written as Roman numerals, and were required to provide the alphanumeric equivalent for as many as they could within 5 minutes.

After the pre-test, participants were then told to attempt the matchstick algebra problem. Before commencing to attempt the problem, they were told to record their very first solution idea, regardless of whether it led to a correct solution or not, by circling the match to be moved and then writing down the resulting outcome next to the problem statement, and were reminded to do so after 30s had elapsed. On completion of a 5-minute solving period, the booklets were collected, solutions to problems were revealed, and participants were debriefed as to the purpose of the study.

Results and Discussion

The frequencies with which participants' first moves involved matches that were loosely or tightly chunked are also shown in Table 1. For participants attempting low-value problems, solution rates were 77% (23/30) for the loose chunk problem and 43% (13/30) for the tight chunk problem. For participants attempting high-value problems, solution rates were 90% (28/31) for the loose chunk problem and 77% (24/31) for the tight chunk problem. A logistic regression using Chunking (loose, tight) and Value (low, high) and the interaction between these factors as predictors yielded a significant model, $\chi^2(3, N = 121) = 21.6$, $p = .001$, with Chunking (Wald = 8.12, p =.004), and Value (Wald = 8.65, p =.003) significant predictors in the model. The interaction between Chunk and Value was not significant in the model.

The frequencies with which participants' first moves involved matches that were loosely or tightly chunked are also shown in Table 1. A logistic regression using Chunking, Value, and the interaction between these factors as predictors on the yielded a significant model, $\chi^2(3, N = 121) = 15.03$, $p = .002$, with Value (Wald = 8.86, p =.003) and Chunking (Wald = 4.71, p =.030) significant predictors in the model. The interaction between Chunking and Value (Wald = .666, p =.414) was not significant in the model.

Table 1. Solution frequencies and 'loose chunk' first move selections in Experiment 1 (%s in brackets)

	Low value		High value	
	No. Correct	1st move loose	No. Correct	1st move loose
Loose chunk	23/30 (77)	28/30 (93)	28/31 (90)	21/31 (63)
Tight chunk	11/29 (40)	22/29 (76)	24/31 (77)	16/31 (52)

The results confirm the prediction derived from RCT that chunk tightness determines difficulty of these matchstick algebra problems, with more solutions for the loose-chunk than the tight-chunk problem. The first move data partially confirm this finding: with low-value problems, participants' initial attempts focused upon loosely chunked matches. However, the results also confirm the prediction derived from CSPT: Participants were more likely to solve high-value problems. Moreover, the value of equations also influenced the selection of first moves, with fewer loose-chunk first moves chosen for the high-value problem. Thus, it appears that a large disparity in values on each side of the equation serves as a cue to seeking moves that maximize the reduction of this difference, and this effect of value ameliorates to some extent the effects of chunk tightness.

Experiment 2

The results of Experiment 1 indicate roles for both prior knowledge and progress monitoring in the relative difficulty of different matchstick algebra problems. However, the experiment examined only a limited number of relatively simple problems. Moreover, the two 'tight chunk' problems differed in terms of the nature of chunk to be decomposed, the low-value problem requiring decomposition of the operator '+', the high-value problem requiring decomposition of the numeral '['. This difference introduced a confound, since the scope of an operator is greater than that of a numeral, so RCT would predict it would be more difficult to decompose the former than the latter. Thus, the hypothesized effects require a further test.

In the second experiment, some participants attempted to solve a matchstick algebra problem requiring a tautologous expression for solution (problems e and f in Fig.1), the hardest type explored by Knoblich et al. Others attempted 'reverse' tautologies, in which the tautologous expression was presented as the problem statement (problems g and h in Fig.1). The prediction that follows from RCT is that reverse tautology problems ought to be solved more readily than standard tautology problems, since the constraint imposed by prior knowledge of typical mathematical functions is undone by the presentation of the tautology itself. CSPT offers no prediction regarding the effect of Tautology.

As in Experiment 1, problems also varied in terms of the numerical value of their components. Again, according to CSPT, high values in equations signal move attempts that make greater progress in reducing disparities. Thus, high-value equations should be solved more often than low-value equations. Again, RCT offers no predictions regarding the effect of Value. In Experiment 2, no analysis of first moves was made, since chunk tightness was not manipulated systematically (e.g., there are no tight chunks apart from the operators in equation e, compared with equation h, which consists only of tight chunks).

Method

Participants A different sample of 144 adults participated on a voluntary basis during a series of undergraduate student recruitment open days at the University of Sussex. Seven participants attempted fewer than 50 of the pre-test items to identify alphanumeric equivalents of Roman numerals, and 4 correctly identified fewer than 90% of those they attempted, and these were excluded from the sample. This left 133 participants: 47 males (mean age = 26.3) and 86 females (mean age = 25.5).

Materials and Design The problems used as task stimuli in this experiment are problems e-h as shown in Fig.1. Each standard tautology problem comprised 10 matches and each reverse tautology problem comprised 11 matches. Presentation of the study materials to participants was as in Experiment 1.

Procedure The procedure was identical to that of the first experiment.

Results and Discussion

For participants attempting standard tautology problems, solution rates were 12% (4/32) for the low-value problem and 62% (21/34) for the high-value problem. For participants attempting the reverse tautology problems, solution rates were 72% (23/32) for the low-value problem and 77% (28/35) for the high-value problem. A logistic regression using Tautology (standard, reverse) and Value (low, high) and the interaction between these factors as predictors yielded a significant model, $\chi^2(3, N = 133) = 39.30$, $p < .001$, with Tautology (Wald = 19.33, $p < .001$), Value (Wald = 11.12, p =.001), and the interaction between Tautology and Value (Wald = 5.26, p =.022) all significant predictors in the model.

The results again provide support for RCT, in this case showing the effects of scope of mathematical knowledge on problem difficulty: As predicted, the standard tautology problems were solved considerably less frequently than the reverse tautology problems. However, again this effect was moderated by the value of numbers in the equation. In the case of the standard tautology, solution frequencies approached those of the reverse tautology problems. It appears that, with standard tautology problems, participants sought to overcome the debilitating effects of scope by searching for solution ideas prompted by moves that would make the greatest change in value. In equation f (Fig. 1), the numeral LI (51) is clearly the main source of the equation disparity, and so participants are orientated towards changing this value, a move that happens to lie on the solution path. In equation e (Fig. 1), there is little to differentiate the loose chunks of matches contained in the IIs and the III, so participants receive little or no hint from the problem itself as to what moves might make most progress. In the case of the reverse tautologies (g and h, Fig. 1), in both instances the relative values are the same (both having a structure 2x = x= x), and so the number values do not give a clear hint to which match to move, hence the similarity in solution rates for these problems.

General Discussion

In two experiments that employed matchstick algebra problems as insight puzzle stimuli, the relative impacts of manipulating constraints of prior knowledge and of perceived progress towards solution were manipulated, as tests of Knoblich et al's (1999) Representational Change Theory (RCT) and MacGregor et al's (2001) Criterion for Satisfactory Progress Theory (CSPT), respectively.

In Experiment 1, the constraint of prior knowledge that arises from the tightness with which chunks of knowledge are bound was examined. Results show that problems requiring the decomposition of tight chunks (e.g., operators) are more difficult than those requiring decomposition of loose chunks (e.g., II and III). This result is consistent with the predictions of RCT, and is confirmed by the main effect of Chunking found with first move selections. However, the effect of chunk tightness was all but eradicated by manipulating the numerical value of the presented expression. Large values orientated participants to seek moves that might offer the greatest reduction in disparity between the two sides of the equation, thereby providing a cue to a matchstick move that lay on the solution path. This result is confirmed in the first move data: With high-value problems, participants were more likely to select first moves from tightly chunked problem elements if they signaled a large value change.

In Experiment 2, the constraint of prior knowledge that arises from the scope with which mathematical knowledge applies was examined. Participants found the reverse tautology problems considerably easier to solve than the standard tautology problems, again a result predicted by RCT. It appears that participants do not have ready access to the concept that a mathematical expression can be correct while being tautologous. However, solution rates for the standard tautology problem which contained a high-value disparity were much higher, approaching those of the reverse tautologies. Again, it appears that the presence of a number that causes a large value disparity acts as a cue for participants to select a move that lies on the solution path.

Our contention is, then, that although prior knowledge undoubtedly creates the conditions under which matchstick algebra problems vary in difficulty, there is no need to propose the relaxation of prior knowledge to explain eventual solution. Instead, we argue, when participants enter a state of impasse, they begin a search for alternative moves that have not been considered before. Given that the problems used here contain between 10 and 14 matches that might be moved, this search is non-trivial, since each moved match can have many potential resting places. We propose that participants narrow the search by seeking to test moves that make the most apparent progress towards solution. In the experiments reported here, we believe participants solved when they did, not because they were able to 'switch off' erroneous knowledge and thereby access more useful knowledge, but because they picked up on cues to progress that happened to lie on the solution path.

In some respects, the search process of CSPT has similarities with the concept of 'detecting invariants' proposed by Kaplan & Simon (1990), in which they suggest that individuals search for problem features that do or do not vary, and use this search to provide cues for move attempts. Where the accounts differ, however, is that invariant detection examines only the idea generation component, whereas CSPT explains both sources of difficulty and eventual solution. In CSPT, it is cues to progress that provide both the source of difficulty, as participants select moves that seem to make progress but that do not lie on the solution path, and also the source of solution ideas, as participants respond to impasse by searching for novel moves that still seem to make progress.

Perhaps the most important aspect of the current findings is that, for the first time, a process for achieving insight is proposed in which the concept of constraint relaxation is rendered unnecessary. It may be possible for proponents of RCT and similar knowledge-based accounts to develop an implemented model that combines spreading activation and inhibition processes in a neurally plausible way to model contemporaneous activation and de-activation of mathematical knowledge (see Ohlsson, 2011, for a partial specification of such a model). We suggest, however, that an account which has no need for relaxation of prior knowledge to discover solutions will benefit from greater parsimony. In our view, the mind's 'response to persistent failure' is not to try to forget what it knows; it is to try to find something better.

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References

Hedne, M. R., Norman, E., & Metcalfe, J. (2016). Intuitive feelings of warmth and confidence in insight and noninsight problem solving of magic tricks. *Frontiers in Psychology*, *7*, 1314.

Jones, G. (2003). Testing two cognitive theories of insight. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *29*(5), 1017-1027.

Kaplan, C. & Simon, H. A. (1990). In search of insight. *Cognitive Psychology*, 22, 374-419.

Knoblich, G., Ohlsson, S., Haider, H., & Rhenius, D. (1999). Constraint relaxation and chunk decomposition in insight problem solving. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *25*(6), 1534-1555.

MacGregor, J. N., Ormerod, T. C., & Chronicle, E. P. (2001). Information processing and insight: a process model of performance on the nine-dot and related problems. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *27*(1), 176-201.

Nakano, Y. (2017). Cognitive and Attentional Process in Insight Problem Solving of the puzzle game "Tangram". *Proceedings of the Cognitive Science Society Conference*, July 2017, London, UK. pp. 2778-2783.

Ohlsson, S. (2011). *Deep Learning: How the Mind Overrides Experience*. Cambridge: CUP.

Öllinger, M., Fedor, A., Brodt, S., & Szathmáry, E. (2017). Insight into the ten-penny problem: guiding search by constraints and maximization. *Psychological Research*, *81*(5), 925-938.

Ormerod, T.C. and MacGregor, J.N. (2017) Enabling spontaneous analogy through heuristic change. *Cognitive Psychology*, 99. pp. 1-16. ISSN 0010-0285.

Ormerod, T. C., MacGregor, J. N. & Chronicle, E. P. (2002) Dynamics and Constraints in Insight Problem Solving. *Journal of Experimental Psychology Learning, Memory, and Cognition*, Vol. 28, No. 4, 791-799.

Ormerod, T.C., MacGregor, J.N., Chronicle, E.P., Dewald, A.D., & Chu, Y. (2013). Act first, think later: Effects of maximization, minimization and lookahead on inferential planning in problem-solving. *Memory & Cognition*, 41, pp. 1096-1108. doi: 10.3758/s13421-013-0318-5.

Zhang, Z., Xing, Q., Li, H., Warren, C. M., Tang, Z., & Che, J. (2015). Chunk decomposition contributes to forming new mental representations: An ERP study. *Neuroscience Letters*, *598*, 12-17.