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A New Look at the Structural and Magnetic Properties of Potassium Neptunate K_2NpO_4 Combining XRD, XANES Spectroscopy, and Low-Temperature Heat Capacity

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S [Supporting Information](#page-11-0)

ABSTRACT: The physicochemical properties of the potassium neptunate K_2NpO_4 have been investigated in this work using X-ray diffraction, X-ray absorption near edge structure $(XANES)$ spectroscopy at the Np- $L₃$ edge, and low-temperature heat capacity measurements. A Rietveld refinement of the crystal structure is reported for the first time. The $Np(VI)$ valence state has been confirmed by the XANES data, and the absorption edge threshold of the XANES spectrum has been correlated to the Mössbauer isomer shift value reported in the literature. The standard entropy and heat capacity of K_2NpO_4 have been derived at 298.15 K from the low-temperature heat capacity data. The latter suggest the existence of a magnetic ordering transition around 25.9 K, most probably of the ferromagnetic type.

■ INTRODUCTION

The alkali and alkaline-earth ternary oxides of uranium, neptunium, and plutonium have attracted interest since the 1960s because of their exciting and intriguing electronic and magnetic properties.^{[1](#page-11-0),[2](#page-11-0)} In these systems with $\text{[Rn]}6d^15f^n$ (uranium and neptunium) and $[\text{Rn}]$ 6d⁰5fⁿ (plutonium) electronic configurations, the 5f valence shell electrons have a large spatial extension and are close in energy to the 6d electrons, making them prone to chemical bonding, in contrast with the 4f electrons of the lanthanides, which are more core-like. This character leads to a wide range of oxidation states, between +3 and +7, and the occurrence of magnetic ordering behavior. The theoretical description of these systems appears extremely challenging, however, as the crystal field interaction is usually of the same order of magnitude as the spin−orbit coupling interaction and electronic repulsion.[3](#page-11-0) The crystal-field interaction cannot be treated as a small perturbation of the electronic energy levels as is done for the $[Xe]$ 4fⁿ rare earths.^{[3](#page-11-0)} In the case of $[{\rm Rn}]$ 5f¹ and $[{\rm Rn}]$ 5f⁰ electronic configurations, however, the contribution from electronic repulsion is removed, which greatly simplifies the interpretation.

A number of studies have recently been reported on several sodium actinide phases because of their relevance for the safety assessment of sodium-cooled fast reactors $(SFRs).⁴⁻⁷$ $(SFRs).⁴⁻⁷$ $(SFRs).⁴⁻⁷$ $(SFRs).⁴⁻⁷$ $(SFRs).⁴⁻⁷$ Those studies have revealed intriguing magnetic properties for the α -Na₂Np^{VI}O₄^{[7](#page-11-0)} and Na₄Np^{VI}O₅^{[6](#page-11-0)} compositions and have stressed the need to re-evaluate the $Np(VI)$ crystal-field ground state.

The physical and chemical properties of K_2NpO_4 have been investigated in the present work in an attempt to bring new insights into the complex behavior of $Np(VI)$ phases. Nectoux et al. performed Mössbauer spectroscopy and magnetic susceptibility measurements on this phase in 1981 and reported intriguing results.^{[8](#page-11-0)} The authors suggested the occurrence of a first-order magnetic transition at $19.5(5)$ K as the Mössbauer spectra showed hyperfine splitting below that temperature, with an associated magnetic hyperfine field of 122 T, corresponding to an ordered moment of about 0.6 μ_B . However, their magnetic susceptibility data did not show any sign of an anomaly around 20 K, as could be expected from the Mö ssbauer results.

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Electronic structures can also be probed using X-ray absorption spectroscopy (XAS) .^{[9](#page-11-0)} Coupling XAS measurements in the high-energy-resolution fluorescence-detection (HERFD) mode with theoretical calculations using the Anderson impurity model, Butorin et al.^{[9](#page-11-0)} have recently estimated the crystal field parameters and 5f occupancy in pentavalent $\mathrm{NaUO}_{3}\left([\mathrm{Rn}]\mathrm{5f}^1 \right)$ and hexavalent $Pb_3UO_6^T([Rn] 5f^0)$, revealing a significant covalent character of the chemical bond. The XAS data available on solid actinide compounds with a valence state higher than IV is still very scarce, however, and the relationship between XAS features and electronic density is not fully understood.^{[10](#page-11-0)}

In this work, we have synthesized K_2NpO_4 and report for the first time a Rietveld refinement of its crystal structure and XANES spectrum collected at the $Np-L_3$ edge. The relationship between local coordination environment and shape of the XANES spectrum is discussed, as well as the correlation between the edge absorption threshold of the neptunium XANES spectrum and the isomer shift value measured by ²³⁷Np Mössbauer spectroscopy.^{[8](#page-11-0)} Moreover, low-temperature heat capacity measurements have been performed to solve the discrepancy regarding the existence of magnetic ordering in this compound.

EXPERIMENTAL METHODS

Sample Preparation and Characterization. K_2NpO_4 and $K₂UO₄$ were synthesized under oxygen flow and air by reaction between accurately weighed samples of neptunium dioxide $(^{237}\text{NpO}_2)$ ORNL, Oak Ridge) or uranium dioxide $(^{238} \text{UO}_{2.10}$, JRC-Karlsruhe stocks) and potassium carbonate $(K_2CO_3, 99\%$, Baker). The stoichiometric mixtures were heated with intermediate regrinding steps at 1093 K for 22 h and 1073 K for 5 h for the neptunium (green color) and uranium (orange color) compounds, respectively. The purity of the obtained materials was analyzed by X-ray diffraction. No secondary

²³⁸U is an α emitter with a very long half-life (4.47 billion years),¹¹ making it only weakly radioactive. ²³⁷Np decays to ²³³Pa by α emission with a half-life of 2.14 million years. The ²³³Pa daughter product is a $β$ ⁻ emitter with a very short half-life (27 days) and significant $γ$ dose rate (1.335 \times 10⁻⁴ (mSv/h)/MBq).^{[11](#page-11-0)} The handling of those materials, requiring considerable safety precautions, was therefore done with limited quantities in α gloveboxes.

The X-ray diffraction measurements were carried out using a Bruker D8 X-ray diffractometer mounted in the Bragg−Brentano configuration with a copper tube (40 kV, 40 mA) and a curved Ge monochromator (111), equipped with a LinxEye position-sensitive detector. The data were collected by step scanning in the angle range $10^{\circ} \leq 2\theta$ ≤ 120° over a period of about 8 h. Structural analysis at room temperature was performed by the Rietveld method with the Fullprof2k suite.^{[12](#page-11-0)}

XANES. XANES measurements were performed at the Rossendorf BeamLine $(ROBL)^{13}$ $(ROBL)^{13}$ $(ROBL)^{13}$ of the European Synchrotron Radiation Facility (ESRF, Grenoble, France) on the K_2NpO_4 material. Small amounts (5−10 mg) of powdered sample were mixed with boron nitride (BN) in an argon-filled α glovebox and pressed into pellets for the measurements. The storage ring operating conditions were 6.0 GeV and 170−200 mA. A double-crystal monochromator mounted with a Si(111) crystal coupled to collimating and focusing Rh-coated mirrors was used.

XANES spectra were collected at room temperature in transmission mode at the $Np-L_3$ edge. A step size of 0.5 eV was used in the edge region. The energy E_0 of the edge absorption threshold position was taken at the first inflection point of the spectrum by using the first node of the second derivative. The position of the white-line maximum was selected from the first node of the first derivative. Several acquisitions were performed on the same sample and summed up to improve the signal to noise ratio. Before the scans were averaged, each spectrum was aligned using the XANES spectrum of a metallic yttrium

(17038 eV) reference foil located between the second and the third ionization chambers and measured at the same time as the sample. The ATHENA software (Version $0.9.20$)^{[14](#page-12-0)} was used to remove the background and to normalize the spectra.

Low-Temperature Heat Capacity of K_2NpO_4 and K_2UO_4 . Lowtemperature heat capacity measurements were performed using thermal relaxation calorimetry^{[15](#page-12-0)} with a PPMS (Physical Property Measurement System, Quantum Design) instrument at applied magnetic fields $B = 0$ and 9 T in the temperature ranges $T = 2.1 - 298.4$ K for K₂NpO₄ and T = 2.0–312.4 K for K₂UO₄, respectively. The measurements were carried out on 22.6(5) mg of K_2NpO_4 material encapsulated in Stycast 2850 FT, and the heat capacity contribution of the Stycast was subtracted from the recorded data. A more detailed description of the experimental procedure, which is particularly well adapted to the study of radioactive materials, was given in ref [16](#page-12-0). The measurement of K_2UO_4 was done on 33.2(5) mg of material without additional encapsulation in Stycast. The contributions of the sample platform, wires, and grease were deduced by a separate measurement of an addenda curve. Considering the accuracy of the PPMS instrument as estimated by Lashley et al ,^{[15](#page-12-0)} the reproducibility of the measurements, and the error introduced by the encapsulation procedure in Stycast of the radioactive neptunium material, 16 the final uncertainty was estimated to be about 1−2% in the middle range of acquisition (10−100 K) and reach about 3% at the lowest temperatures and near room temperature. Self-heating effects coming from the radioactive decay of 237 Np were considered but appeared negligible. The use of Stycast is the main contributor to the uncertainties on the heat capacity and entropy values quoted hereafter. The final uncertainty for the uranium material is estimated to be about 1% from 100 to 300 K and reach about 3% at the lowest temperatures.

■ RESULTS AND DISCUSSION

Structural Refinement. K_2NpO_4 is isostructural with $K₂UO₄$ and crystallizes with tetragonal symmetry in space group I4/mmm. The structure of K_2UO_4 was refined recently on the basis of single-crystal data.^{[17](#page-12-0)} The refined cell parameters obtained in the present study, $a = 4.3322(3)$ Å and $c =$ 13.1881(13) Å, are in good agreement with the literature.^{[17](#page-12-0)} The refined atomic positions are given in the [Supporting](http://pubs.acs.org/doi/suppl/10.1021/acs.inorgchem.7b00462/suppl_file/ic7b00462_si_003.pdf) [Information.](http://pubs.acs.org/doi/suppl/10.1021/acs.inorgchem.7b00462/suppl_file/ic7b00462_si_003.pdf) A Rietveld refinement for the K_2NpO_4 phase is reported for the first time in this work ([Figure 1\)](#page-3-0). The cell parameters obtained are $a = 4.2973(4)$ Å and $c = 13.144(12)$ Å. The refined atomic positions are given in [Table 1](#page-3-0) and selected bond lengths in [Table 2.](#page-3-0) In this structure, the neptunium cations are 6-fold coordinated, and the octahedra are connected by their equatorial vertices, forming sheets in the (ab) plane [\(Figure 2](#page-4-0)a). The potassium cations, in 9-fold coordination, are located between the sheets, holding them together. The NpO_6 octahedra show a neptunyl type of coordination, with two short Np−O1 bonds at 1.84(1) Å in the axial direction and four long Np−O2 bonds at 2.15(1) Å in the equatorial plane [\(Figure 2b](#page-4-0)). The presence of a neptunyl configuration is quite common for hexavalent alkali-metal actinide oxide phases. It has been reported already for α -Na₂NpO₄ (Np−2O_I = 1.762(5) Å, $Np-4O_{II} = 2.086(5)$ Å),^{[7](#page-11-0)} β -Na₂NpO₄ (Np−2O_I = 1.90 Å, $Np-2O_{II}$ = 2.16 Å, $Np-2O_{III}$ = 2.17 Å)^{[18](#page-12-0)} (orthorhombic in space groups Pbam and Pbca, respectively), and $BaNpO₄$ $(Np-2O_I = 1.89 \text{ Å}, Np-2O_{II} = 2.10 \text{ Å}, Np-2O_{III} = 2.20 \text{ Å})^{19}$ $(Np-2O_I = 1.89 \text{ Å}, Np-2O_{II} = 2.10 \text{ Å}, Np-2O_{III} = 2.20 \text{ Å})^{19}$ $(Np-2O_I = 1.89 \text{ Å}, Np-2O_{II} = 2.10 \text{ Å}, Np-2O_{III} = 2.20 \text{ Å})^{19}$ (orthorhombic in space group Pbcm). The unit cell volume in K_2NpO_4 (242.7 Å³) is smaller than that for K_2UO_4 (247.5 Å³), which can be related to the decreasing ionic radius along the series of the actinide elements.

XANES. The XANES spectrum of K_2NpO_4 collected at the $Np-L_3$ edge is shown in [Figure 3](#page-4-0) together with those of NpO_2 , $Na_3Np^VO_4$, α -Na₂Np^{VI}O₄, Na₄Np^{VI}O₅, and Na₅Np^{VI}O₆ reference materials.¹⁰ The valence states of the sodium neptunates

Figure 1. Comparison between the observed $(Y_{obs}$ in red) and calculated (Y_{cal} in black) X-ray diffraction patterns of K₂NpO₄ collected at room temperature (T = 295 \pm 2 K). $Y_{obs} - Y_{calc}$ in blue, is the difference between the experimental and calculated intensities. The Bragg reflection angular positions are marked in green. The inset shows an enlargement of the refinement in the angle range $2\theta = 70-120^{\circ}$. Measurements were carried out with $\lambda = Cu K \alpha_1$ radiation.

Table 1. Refined Atomic Positions in K_2NpO_4 Derived from the Analysis of the X-ray Diffraction Data Collected at Room Temperature $(T = 295 \pm 2 \text{ K})^a$

atom	oxidn state	Wyckoff	$\boldsymbol{\mathcal{X}}$		\boldsymbol{z}	$B_0(\AA^2)$
Np	$+6$	2a	Ω	Ω		0.78(1)
K	$+1$	4e	0.5	0.5	0.1520(2)	1.91(6)
O ₁	-2	4e	0	0	0.1415(5)	1.5(2)
O ₂	-2	4c	0.5	0	0	2.1(2)
	${}^{a}R_{wp} = 13.5$, $R_{exp} = 5.37$, $\chi^2 = 6.29$.					

Table 2. Selected Bond Lengths in M_2AnO_4 (An = U, Np; $M = Na$, K)^a

were confirmed by ²³⁷Np Mössbauer spectroscopy from the values of their isomer shifts,^{[5](#page-11-0)−[7](#page-11-0)} while the corresponding XANES spectra were reported in ref [10](#page-11-0). The values of the measured inflection points and white lines are reported in [Table 3](#page-4-0). The investigated series covered a wide range of oxidation states (IV−VII) and a variety of local coordination geometries around the neptunium cation, i.e., neptunyl (Na_3NpO_4) and α -Na₂NpO₄), "reverse" neptunyl (Na₄NpO₅), and distorted NpO_6 octahedra (Na₅NpO₆), which has allowed us to correlate the shape of the XANES spectra with the local structural environments,^{[10](#page-11-0)} as described later in this paper.

The inflection point position of K_2NpO_4 , corresponding to the absorption edge threshold E_0 for the 2p \rightarrow 6d transitions, is well aligned with those of α -Na₂Np^{VI}O₄ and Na₄Np^{VI}O₅ [\(Table 3\)](#page-4-0). These results confirm that neptunium is exclusively in the oxidation state VI in K_2NpO_4 , corresponding to a $[Rn]$ 5f¹ electronic configuration. The Np ion in this structure is therefore a Kramers ion with a ${}^{2}F_{5/2}$ ground state manifold, and a ${}^{2}F_{7/2}$ first excited state arising from spin–orbit coupling.

Smith et al. reported a linear correlation of the absorption edge threshold E_0 determined by XANES versus the Mössbauer isomer shift for the series of sodium neptunates.^{[10](#page-11-0)} The value for K_2NpO_4 fits very well with this trend [\(Figure 4\)](#page-4-0) when the isomer shift value determined by Nectoux et al. is used: i.e., δ_{IS} = −56.9(6) mm s⁻¹ at 4.2 K relative to the standard NpAl₂ absorber.^{[8](#page-11-0)} The linear variation between E_0 and δ_{IS} can be understood from the fact that both quantities result from the Coulomb interaction with the surrounding electrons. The 5f shells produce a shielding effect on the electronic charge density of the $s_{1/2}$ and $p_{1/2}$ inner shells, which affects the isomer shift: $\delta_{\text{IS}} = \alpha [\Delta \rho_{\text{e}}(0)]$ (α being a calibration constant and $\Delta\rho_e(0)$ the difference in electronic charge density between the source material and the absorber at the nuclear origin). $\rho_e(0)$ increases with the removal of 5f electrons. Moreover, the increase in formal valence state produces a decrease in Coulomb energy in the final state between the 5f and 6d electrons and the $2p_{3/2}$ core hole,^{[20](#page-12-0)−[22](#page-12-0)} which leads to a shift to higher energy of the absorption edge threshold E_0 .

In addition, the XANES spectrum of K_2NpO_4 shows the typical double-peaked white lines (WL) of $Np(V)$, $Np(VI)$, and $Np(VII)$ compounds,^{[10](#page-11-0),[20](#page-12-0),[21](#page-12-0)} while the tetravalent $Np^{IV}O_2$ compound exhibits a single WL peak. The double peak consists of the main white line at 17620.2(5) eV and a shoulder and reduced peak amplitude about 15 eV above the Np edge. This feature has traditionally been attributed to localized multiplescattering resonance of the neptunyl configuration. However, studies 10 on the sodium uranates and neptunates have shown that the correlation between the shape of the XANES spectra and the local coordination geometries is probably more intricate. The isostructural compounds α -Na₂U^{VI}O₄ and α -Na₂Np^{VI}O₄, presenting a neptunyl type of configuration, do not show the expected secondary shoulder and reduced peak amplitude, whereas $\text{Na}_{4}\text{U}^{\text{VI}}\text{O}_{5}$, presenting a "reverse" neptunyl type of configuration, does. 10 Other factors could play a role, and effects of the degree of localization of the $5f$ electrons^{[10](#page-11-0)} and core-ionized final states with different 5f occupancies^{[20](#page-12-0),[21](#page-12-0)} have been suggested.

The absorption edge threshold E_0 is finally slightly higher (0.3 eV) for K_2NpO_4 than for α -Na₂Np(VI)O₄, which could be related to the neptunyl bond distances (Table 2), degree of covalency, or (in other words) degree of localization of the 5f electrons in both compounds. The shorter the Np−O bonds, the more localized the 5f electrons, and the greater the Coulomb energy, making it easier to eject an electron from the 2p_{3/2} core−shell. However, it is not possible to conclude definitively, given the experimental uncertainties (0.5 eV) of the measurements at the Np-L₃ edge, and this would require performing high-energy-resolution fluorescence-detected (HERFD) XANES measurements at the M_4 edge.

Heat Capacity Measurements. The heat capacity data of K_2NpO_4 and K_2UO_4 measured at low temperatures in the absence of a magnetic field are shown in [Figure 5](#page-4-0) and given in [Tables 5](#page-6-0) and [6](#page-8-0) of the [Appendix](#page-11-0). The two curves cross above $T = 150$ K, although one would expect them to become equal, corresponding to the same lattice contribution at high temperatures for the uranium and neptunium compounds. However, this discrepancy can be related on the one hand to the uncertainty on our experimental results, which increases toward

Figure 2. (a) Crystal structure of K₂NpO₄ (K atoms in purple, O atoms in red, NpO₆ octahedra in gray) showing the sheets of corner-sharing NpO₆ octahedra in the (ab) plane. (b) $NpO₆$ octahedron in $K₂NpO₄$ with the neptunyl type of coordination.

Figure 3. Normalized XANES spectrum of K_2NpO_4 (present work) together with those of NpO_2 , Na_3NpO_4 , α -Na₂NpO₄, Na₄NpO₅, and $Na₅NpO₆$ reference materials.^{[10](#page-11-0)}

Table 3. Energies of the Inflection Points and White Lines of the Np-L₃ XANES Spectra^{a}

			white line (eV)	
compound	inflection point (eV)			ref
K_2NpO_4	17614.5(5)	17620.2(5)	17631.5(5)	this work
$Np^{IV}O_{2}$	17612.1(5)	17618.3(5)		10
$Na_3Np^VO_4$	17613.4(5)	17618.5(5)		10
α -Na ₂ Np ^{VI} O ₄	17614.2(5)	17624.3(5)		10
$Na_4Np^{VI}O_5$	17614.1(5)	17618.8(5)	17625.6(5)	10
$Na_{s}Np^{VII}O_{6}$	17615.1(5)	17619.0(5)	17626.9(5)	10
	^a The secondary white line, if present, is given in italics.			

high temperatures using the PPMS technique, and on the other hand to the fact that one compound was measured with Stycast and the other without. The uncertainty on the neptunium data corrected for the Stycast contribution is around 3% at room temperature, whereas that of the uranium data measured without Stycast is around 1%. The heat capacities reach values that are about 17−24 J $\rm{K^{-1}}$ mol $^{-1}$ below the classical Dulong– Petit limit ($C_{\text{lat}} = 21R \approx 174.6 \text{ J K}^{-1} \text{ mol}^{-1}$ for the seven atoms in the formula unit) as the temperature approaches 298.15 K.

The collected data for K_2NpO_4 show a small anomaly at $T = 25.9$ K, which is almost unaffected by the application of a 9 T magnetic field apart from a small decrease in the amplitude. This feature could be interpreted at first as an indication of the presence of $NpO₂$ impurity within the investigated material. Indeed, neptunium dioxide shows a sharp λ anomaly at

Figure 4. Absorption edge threshold E_0 relative to NpO_2 versus isomer shift measured by Mö ssbauer spectroscopy.

Figure 5. Heat capacity of K₂NpO₄ (black \circ) and K₂UO₄ (blue \Box) measured in zero magnetic field and the numerical fit to the neptunium (red line) and uranium (blue dotted line) data.

 $T = 25.7 K^{23,24}$ $T = 25.7 K^{23,24}$ $T = 25.7 K^{23,24}$ due to rank 5 triacontadipolar order as described in the studies of Santini et al., ^{[25](#page-12-0)} with a similar behavior upon application of a magnetic field. However, the X-ray diffraction data did not reveal any secondary $NpO₂$ phase. Moreover, the shape of the anomaly in K_2NpO_4 does not match that of $NpO₂$, although the critical temperatures are very close. The anomaly is very symmetrical in K_2NpO_4 , in contrast with NpO_2 , showing an asymmetrical profile. On the basis of its amplitude, the amount of $NpO₂$ impurity would correspond to 26.9 \pm 1.0%, which should be detected easily by the X-rays.

Figure 6. C_p/T for K_2NpO_4 (black O) and K_2UO_4 (blue \square) measured in zero magnetic field and the numerical fit to the neptunium (red line) and uranium (blue dotted line) data.

We conclude that the anomaly is an intrinsic property of K_2NpO_4 . The corresponding magnetic contribution was derived as $S_{\text{mag}} = 3.1 \pm 0.1 \int K^{-1} \text{ mol}^{-1}$ after subtraction of the lattice heat capacity contribution (Figure 7). The latter was approximated with the heat capacity of K_2UO_4 (which has electronic configuration $[{\rm Rn}]$ $5f^0$, as the two compounds are isostructural and have very similar atomic masses. The derived magnetic entropy value represents about 53.8% of the expected order-disorder entropy for such a Kramers system (S_{mag} = Rln2). It is worth pointing out that a low magnetic entropy, i.e. 0.19Rln2, has also been reported for α -Na₂NpO₄.^{[7](#page-11-0)}

Interestingly, no anomaly was observed around $19.5(5)$ K, as could be expected from the Mö ssbauer results of Nectoux et al.^{[8](#page-11-0)} An X-ray diffraction pattern collected after the lowtemperature heat capacity measurement moreover confirmed that the sample had retained its integrity during the experiment. Since the magnetic susceptibility measurements of the authors also did not show any anomaly around 20 K ^{[8](#page-11-0)}, the existence of a first-order magnetic phase transition at the latter temperature is unlikely. To explain the origin of the magnetic hyperfine

Figure 7. Electronic contribution to the heat capacity in K_2NpO_4 obtained by subtracting the data for K_2UO_4 .

splitting reported below $19.5(5)$ K, we could suggest the occurrence of a slow electron spin relaxation phenomenon in this paramagnetic system.[26](#page-12-0) However, this is doubtful, as it would require a sudden collapse of the relaxation time at the critical temperature. The cell parameters reported by for K₂NpO₄ ($a = 4.26$ Å and $c = 13.01$ Å) are lower than those found in this study. A contamination of their sample with a magnetic impurity is possible although unlikely, as they reported a single-phase material, but they give very little detail on phase preparation and purity. The magnetic hyperfine splitting effect observed by the authors probably corresponds to the anomaly observed herein at 25.9 K, with a somewhat lower critical temperature. It should be pointed out that differences in critical temperatures of about 3−8 K have been reported in the literature between Mössbauer and magnetic susceptibility results of the uranium–neptunium mixed oxides $U_{1-x}Np_xO_2$.^{[27](#page-12-0)} For the magnetic susceptibilty results of Nectoux et al., a clear deviation from the Curie−Weiss law is observed below about 40 K, which could suggest ferromagnetic ordering below the latter temperature. The hypothesis of a ferromagnetic transition is moreover in good agreement with the low-temperature heat

Table 4. Summary of Fitting Parameters of the Heat Capacities of K_2NpO_4 and K_2UO_4

Debye and Einstein fit		harmonic lattice model		
		K_2NpO_4		
temp range/K	$7.71 - 298.4$	temp range/K	$2.1 - 8.0$	
$n_{\rm D}$ /mol	2.3637	δ /mJ mol ⁻¹ K ⁻¹	22.25	
$\Theta_{\rm D}/K$	154.41	B_3 /mJ mol ⁻¹ K ⁻⁴	-5.00553×10^{-1}	
$n_{\rm E1}$ /mol	1.9806	B_5 /mJ mol ⁻¹ K ⁻⁶	6.48526×10^{-2}	
Θ_{E1}/K	588.19	B_7/mJ mol^{-1} K^{-8}	-1.13475×10^{-3}	
n_{E2}/mol	2.5292	B_9 /mJ mol ⁻¹ K ⁻¹⁰	7.32029×10^{-6}	
$\theta_{\rm E2} / \rm{K}$	287.56			
$n_{\rm D}$ + $n_{\rm E1}$ + $n_{\rm E2}$ mol	6.87			
		K_2UO_4		
temp range/K	$19.8 - 312.4$	temp range/K	$2.0 - 20.3$	
n_D/mol	2.0522	δ /mJ mol ⁻¹ K ⁻¹		
$\Theta_{\rm D}/K$	157.08	B_3 /mJ mol ⁻¹ K ⁻⁴	6.43253×10^{-1}	
$n_{\rm E1}$ /mol	2.8798	B_{5}/m J mol ⁻¹ K ⁻⁶	7.77235×10^{-3}	
Θ_{F1}/K	256.28	B_7/mJ mol^{-1} K^{-8}	-4.50504×10^{-5}	
n_{E2}/mol	2.1601	B_9 /mJ mol ⁻¹ K ⁻¹⁰	1.03519×10^{-7}	
Θ_{E2}/K	610.54	B_{11}/mJ mol ⁻¹ K^{-12}	-8.70422×10^{-11}	
$n_{\rm D}$ + $n_{\rm E1}$ + $n_{\rm E2}$ /mol	7.09			

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T (K) $C_{p,m}$ (J K⁻¹ mol⁻¹)

Table 5. Experimental Heat Capacity Data for K_2NpO_4

 T (K) $C_{p,m}$ (J K⁻¹ mol⁻¹)

l,

Table 5. continued

capacity data, showing a slight decrease of the anomaly at $T = 25.9$ K upon application of a magnetic field. The negative value of the Curie constant, i.e., $\Theta_{\rm P}$ = −150 K, derived from the Curie–Weiss analysis of the data^{[8](#page-11-0)} is rather surprising, however. It suggests a more complex order, possibly with a canting of the ferromagnetically coupled moments or with strong antiferromagnetic interactions.

In the present work, the thermodynamic functions of K_2NpO_4 and K_2UO_4 were derived at 298.15 K by fitting the experimental data to theoretical functions below $T = 8.0$ K and $T = 20.0 \text{ K}^{28}$ $T = 20.0 \text{ K}^{28}$ $T = 20.0 \text{ K}^{28}$ respectively, and a combination of Debye and Einstein heat capacity functions^{[29](#page-12-0)−[31](#page-12-0)} at $T = 7.8 - 298.4$ K and at T = 20.0−312.4 K, respectively. The fitted data are shown with solid and dotted lines in [Figures 5](#page-4-0) and [6.](#page-5-0) The heat capacity values at 298.15 K were obtained by interpolation, yielding $C_{p,m}^{\circ}$ ^o(K₂NpO₄, cr, 298.15 K) = 152.7 ± 4.5 J K⁻¹ mol⁻¹ and $C_{p,m}^{N}$ °(K₂UO₄, cr, 298.15 K) = 156.5 ± 1.6 J K⁻¹ mol⁻¹ (in both cases, the quoted uncertainty corresponds to the standard uncertainty). The experimental standard entropies at 298.15 K were determined by numerical integration of $C_{p,m}/T = f(T)$ using the aforementioned fitted functions and including the magnetic entropy contribution, yielding $S_m^{\circ}(K_2NpO_4)$, cr, 298.15 K) = 209.3 \pm 4.9 J K⁻¹ mol⁻¹ and S_m^o(K₂UO₄, cr, 298.15 K) = 210.1 \pm 2.7 J K⁻¹ mol⁻¹, respectively. The values obtained for K_2NpO_4 are slightly lower than that of K_2UO_4 , whereas the inverse behavior would be expected. However, this is due to the uncertainty introduced by the use of Stycast as mentioned before and the crossing of the two curves. When adding the derived magnetic entropy to the lattice contribution of K_2UO_4 , one derives 213.2 J K⁻¹ mol⁻¹ for the standard entropy of K_2NpO_4 , which remains within the uncertainty of the present measurement.

Fitting of the Lattice Contribution above $T = 8.0$ K (K_2NpO_4) and $T = 20.0 K (K_2UO_4)$. At very low temperatures where the thermal expansion is negligible, the heat capacity at constant pressure can be approximated to the heat capacity at constant volume $C_{p,m} \approx C_{V,m}$, which comprises lattice vibra-tions, electronic, and magnetic contributions^{[32](#page-12-0)}

The lattice contribution dominates at temperatures above about $T = 8-20$ K and can be modeled using a combination of Debye and Einstein functions, as shown in eq 1. Two Einstein functions were used in this work to fit the data. Fitting with a single Einstein function was attempted but could not reproduce accurately the high-temperature region:

$$
C_{p,m} = n_{D}D(\Theta_{D}) + n_{E1}E(\Theta_{E1}) + n_{E2}E(\Theta_{E2})
$$
\n(1)

where R is the universal gas constant equal to 8.3144621 J K^{-1} mol⁻¹, $D(\Theta_D)$, $E(\Theta_{E1})$, and $E(\Theta_{E2})$ are the Debye and Einstein functions, respectively, as written in eqs 2 and 3. $\Theta_{\rm D}$, $\Theta_{\rm E1}$, and Θ_{E2} are the characteristic Debye and Einstein temperatures. n_D , n_{E1} , and n_{E2} are adjustable parameters, whose sum $n_D + n_{E1}$ + n_{E2} should be approximately equal to the number of atoms in the formula unit (i.e., 7 in this case).

$$
D(\Theta_{D}) = 9R\left(\frac{1}{x}\right)^{3} \int_{0}^{x} \frac{e^{x}x^{4}}{\left[e^{x} - 1\right]^{2}} dx \quad x = \frac{\Theta_{D}}{T}
$$
 (2)

$$
E(\Theta_{\rm E}) = 3Rx^2 \frac{e^x}{\left[e^x - 1\right]^2} \quad x = \frac{\Theta_{\rm E}}{T} \tag{3}
$$

The fitted parameters are listed in [Table 4.](#page-5-0) The sum $n_D + n_{E1}$ + n_{E2} is very close to 7.

Fitting below $T = 8.0$ K (K₂NpO₄) and $T = 20.0$ K (K_2UO_4) . At very low temperatures $(T < 20 K)$, the phonon contribution is well-represented using a harmonic lattice model, 28 28 28 as expressed by the polynomial function (4), where the number of required terms augments the high-temperature limit of the fit:

$$
C_{\text{latt}} = \sum B_n T^n \quad n = 3, 5, 7, 9, 11 \dots \tag{4}
$$

The electronic contribution of the conduction electrons at the Fermi surface are represented with the linear term γT .^{[33](#page-12-0)} For insulating materials such as K_2NpO_4 and K_2UO_4 , the electronic specific heat is 0. However, a linear term was reported in materials such as α -FeOOH,^{[28](#page-12-0)} Fe₃(P₂O₇)₂,^{[34](#page-12-0)} and Sr₂TiSi₂O₈,^{[35](#page-12-0)} which was related to departure from stoichiometry, oxygen vacancies, or defects within the material.^{[28](#page-12-0)}

The heat capacity of K_2NpO_4 was fitted with the harmonic model using four terms over the temperature range T = 2.1–8.0 K. That of K_2UO_4 was fitted with five terms over

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Table 6. Experimental Heat Capacity Data for $K_2₂U_{0₄}}$

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Table 6. continued

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Table 6. continued

the temperature range $T = 2.0 - 20.3$ K. The corresponding coefficients are given in [Table 4](#page-5-0). In addition, the use of a linear δT term appeared necessary to describe the experimental curve of K_2NpO_4 . More recently, the occurrence of such a linear term was also reported in $\text{Na}_4\text{NpO}_5^{36}$ $\text{Na}_4\text{NpO}_5^{36}$ $\text{Na}_4\text{NpO}_5^{36}$ which was related to the presence of defects within the material and an asymmetric peak profile shape in opposite directions for successive hkl reflections clearly visible on the X-ray diffraction pattern. The X-ray diffraction data of K_2NpO_4 do not show such features, however. Self-heating effects coming from the radioactive decay of $237Np$ were considered but appeared negligible. Moreover, departure

from stoichiometry is unlikely according to the present Np-L3 XANES results and Mö ssbauer data of ref [8](#page-11-0). Hence, the physical origin of this feature remains unclear. The appearance of a nuclear Schottky effect arising from the magnetic hyperfine splitting interaction between the unpaired 5f electron and the magnetic moment at the Np nucleus $(I = 5/2)$ was suggested for $Na₂NpO₄$, as the corresponding data showed a reincrease below 3.7 K³⁷ K₂NpO₄ might show similar behavior ([Figure 7\)](#page-5-0), but we cannot conclude in the absence of data below 2.0 K, which would require complementary measurements using a ³He refrigerator.

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■ CONCLUSION **Inorganic Chemistry Article** Article **Article** 2014 **Article** 2014 **Article** 2014 **Article**

A Rietveld refinement of the crystal structure of K_2NpO_4 , tetragonal in space group I4/mmm, is reported for the first time in the present work. The refined cell parameters and bond lengths are in good agreement with the trend of decreasing ionic radii along the actinide series. XANES data have also been collected at the $Np-L_3$ edge, which have confirmed the hexavalent state of neptunium in this compound and therefore the assigned stoichiometry. The measured absorption edge threshold E_0 fits very well the linear correlation observed for the sodium neptunates between E_0 and the isomer shift value $\delta_{\rm IS}$ measured by Mössbauer spectroscopy. Moreover, double-peak white lines have been observed for K_2NpO_4 , which are usually attributed to multiple scattering resonances of the actinyl compounds, but the interpretation could be more intricate. Electronic density calculations are needed to obtain more insight into those complex features.

Low-temperature heat capacity data have been collected in the temperature range $T = 2.1 - 298.4$ K for K₂NpO₄ and $T = 2.0 - 312.4$ K for K₂UO₄, and the standard entropy and heat capacity of both compounds have been derived at 298.15 K. The latter data have revealed the presence of an anomaly at 25.9 K with an associated magnetic entropy $S_{\text{mag}} = 3.1 \pm$ 0.1 J K[−]¹ mol [−]¹ , which most probably corresponds to the magnetic hyperfine splitting event observed in the literature by Mö ssbauer spectroscopy at a slightly lower temperature: i.e., $T = 19.5(5)$ K.⁸ Both the present low-temperature heat capacity data and the magnetic susceptibility measurements of Nectoux et al. 8 are consistent with the hypothesis of a ferromagnetic ordering transition around $T = 25.9$ K. Complementary studies involving repeated Mössbauer spectroscopy and magnetic susceptibility measurements, as well as neutron diffraction measurements on a well-characterized material, would allow confirmation of those results. Finally, the amplitude of the anomaly at 25.9 K is smaller than expected for this Kramers system $(S_{\text{mag}} = R \ln 2)$, but similar results have also been reported for α -Na₂NpO₄. The low values of the ordered moment derived from the Mössbauer data (\sim 0.6 $\mu_{\rm B}$), of the paramagnetic effective moment derived from the magnetic susceptibility data $(\mu_{\text{eff}} = 1.37 \mu_{\text{B}})$, and of the magnetic entropy $(S_{\text{mag}} = 0.538R \ln 2)$, are not unusual for 5f¹ systems. Further investigations involving spectroscopy measurements at low energy and theoretical calculations are clearly required to get further insight into the Np(VI) crystal-field ground state and magnetic behavior of the alkali and alkaline-earth neptunates.

■ APPENDIX

Experimental heat capacity data for K_2NpO_4 and K_2UO_4 are given in [Tables 5](#page-6-0) and [6.](#page-8-0)

■ ASSOCIATED CONTENT

S Supporting Information

The Supporting Information is available free of charge on the [ACS](http://pubs.acs.org) [Publications website](http://pubs.acs.org) at DOI: [10.1021/acs.inorgchem.7b00462](http://pubs.acs.org/doi/abs/10.1021/acs.inorgchem.7b00462).

X-ray crystallographic file of K_2NpO_4 [\(CIF](http://pubs.acs.org/doi/suppl/10.1021/acs.inorgchem.7b00462/suppl_file/ic7b00462_si_001.cif)) X-ray crystallographic file of K_2UO_4 [\(CIF](http://pubs.acs.org/doi/suppl/10.1021/acs.inorgchem.7b00462/suppl_file/ic7b00462_si_002.cif)) X-ray diffraction pattern and refined atomic positions for $K₂UO₄$ ([PDF\)](http://pubs.acs.org/doi/suppl/10.1021/acs.inorgchem.7b00462/suppl_file/ic7b00462_si_003.pdf)

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Notes

The authors declare no competing financial interest.

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