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Optimal maintenance and repair policies in infrastructure management under uncertain facility deterioration rates: an adaptive control approach

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Abstract

In this paper we present two adaptive control formulations that explicitly include uncertainty in characterizing a facility's deterioration rate in the process of developing maintenance and repair policies. The formulations use condition information that stems from the operation of a facility to improve the characterization over a finite planning horizon. We discuss issues related to the implementation and computational complexity of the formulations. Through a computational study, we show that the economic benefits can be achieved by implementing the adaptive control formulations. These benefits are most significant in situations where the initial expected deterioration rate of the facility is not a good representation of its actual deterioration rate. © 2002 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Infrastructure management systems support agencies in developing efficient policies to monitor, maintain and repair deteriorating facilities in infrastructure networks. Some examples of infrastructure facilities include pavement segments, bridge spans and pipeline segments. The successful implementation of infrastructure management systems has made them popular among agencies

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responsible for managing infrastructure networks and has warranted a significant amount of research in the field of infrastructure management. For example, Arizona's pavement management system saved the state's Department of Transportation $14 million in the first year it was implemented (fiscal year 1980–1981) and over $100 million in the first four years (Golabi et al., 1982; Kulkarni, 1984).

In this paper, we consider the problem of developing maintenance and repair policies for individual infrastructure facilities that deteriorate over time. A facility's deterioration process depends on factors such as environmental conditions, design characteristics and utilization level. As a facility deteriorates, its condition worsens and operating costs increase. In highway pavements, for example, the operating costs consist of the users' vehicle operating costs. By applying maintenance and repair actions, planning agencies can mitigate and even reverse the effects of deterioration, and, consequently decrease current and future operating expenses. These benefits may, however, be offset by the costs associated with performing maintenance and repair actions. The cost of applying maintenance and repair actions increases with the severity of the action, though severe actions result in more significant improvements in a facility's condition. As a result, when planning agencies develop maintenance and repair policies for infrastructure facilities they must weigh the trade-off between reduced operating costs and increased maintenance and repair costs.

The development of maintenance and repair policies requires methods for policy evaluation and policy selection. Policy evaluation involves predicting the total discounted costs to implement each element of a set of feasible policies. Policy selection involves choosing a policy that minimizes total discounted costs. Evaluating a maintenance and repair policy after observing a facility's deterioration process under the policy is straightforward. In practice, however, planning agencies must evaluate maintenance and repair policies before a facility's deterioration process can be observed. Furthermore, it is impossible to predict a facility's deterioration process under a given maintenance and repair policy with certainty due to the unknown variability caused by factors such as level of utilization, the environment, and differences in design and materials among facilities.

The model-based approach to perform policy evaluation and policy selection involves modeling a facility's deterioration process. Policies are evaluated by computing the expected discounted costs to implement policies to an ideal facility that deteriorates exactly like a performance model predicts. Thus, the expected model-based costs are used to estimate the actual costs. Policy selection is performed by finding policies that minimize the expected costs of managing the ideal facility.

State-of-the-art infrastructure management systems use a model-based approach to policy evaluation and policy selection. The problem of finding optimal maintenance and repair policies is formulated as a finite-state, discrete-time Markov decision process (MDP). In this framework, a facility's condition is represented by one of a finite number of states. Generally, it is assumed that at the end of each period the agency reviews the condition of a facility and applies a maintenance and repair action to the facility. We use the random variable X_t with finite state-space $\mathscr{S}$ to denote a facility's state at the end of period t, and the variable A_t in the finite set of actions $\mathscr{A}$ to denote the action taken at the end of period t. The underlying assumption in an MDP formulation is that facility deterioration is a Markovian process. This implies that the state of the facility in each period only depends on the state of the facility and the action applied to it in the preceding period. These dynamics can be represented with transition equations $f_t(\cdot)$ of the form

$$X_{t+1} = f_t(X_t, A_t, \eta_t), \quad t = 0, 1, 2, \ldots \tag{1}$$

The random variables η_t, $t = 0, 1, 2, \ldots$, aggregate the effect of all factors that affect a facility's deterioration process but that are not explicitly included in the deterioration model. Given the probability distribution of the variables η_t, $t = 0, 1, 2, \ldots$, a facility's deterioration can alternatively be represented with a set of transition probabilities of the form

$$P(X_{t+1} = j | X_t = i, A_t = a) \equiv \pi^t_{ij}(a) \quad \forall a \in \mathscr{A}, \; i, j \in \mathscr{S}, \text{ and } t = 0, 1, 2, \ldots \tag{2}$$

It is usually assumed that facility deterioration is a stationary process, i.e., a time-homogeneous process. Under this assumption the MDP formulation can be simplified by omitting the super-index t from π^t_{ij}. In the transition equation representation, the sub-index t is omitted from $f_t(\cdot)$ and η_t, $t = 0, 1, 2, \ldots$, and thus the dynamics are represented with a transition equation of the form $f(\cdot)$ and a random variable η.

1.1. Motivation and overview of the proposed framework

As stated previously, a facility's deterioration process depends on several factors. The effect of these factors is reflected in a set of parameters and an error term η that specify the model of a facility's deterioration. Any differences between actual observations of a facility's condition and predicted observations (based on the deterioration model) are attributed to randomness (and collected in η). Unfortunately, this approach does not capture the uncertainty that exists in generating the set of parameters to specify a deterioration model such as in the MDP framework described earlier. The uncertainty in generating a set of parameters can be attributed to the following:

(a) Exogenous factors such as the environment and level of utilization. The uncertainty in predicting these factors produces uncertainty in the parameters of the deterioration model.
(b) Endogenous factors such as facility design and materials. Unknown variability in these factors can make similar facilities respond differently to the same exogenous conditions.
(c) Statistical factors. Typically, the data sets used to generate deterioration models are of limited size and variability. They are often complemented by a planning agency's experience with similar facilities.

We address this limitation by presenting a framework that characterizes a facility's deterioration process with multiple deterioration models, i.e., multiple parameter sets. Each deterioration model explains a facility's deterioration under a given set of exogenous factors. For example, the transition equation $f^1(\cdot)$ might be used to represent a facility's deterioration under high utilization and $f^2(\cdot)$ might be used under low utilization. A set of deterioration models can also be used to capture the deterioration process of facilities with different values of the endogenous factors. In the proposed framework, the differences between actual observations of a facility's condition and expected observations are attributed not only to randomness, but also to the uncertainty in choosing a set of parameters to explain a facility's deterioration process. We use the random variable Y_t, $t = 0, 1, 2, \ldots$, to capture this uncertainty. Y_t is the probability that the planning agency assigns to the appropriateness of each parameter set in explaining the facility's deterioration process. For example, at the start of the horizon (end of year 0) a planning agency might assign equal probabilities to the transition equations corresponding to high and low utilization. In this example, $P(Y_0 = \text{high}) = P(Y_0 = \text{low}) = 0.5$.

We formulate the problem of finding optimal policies to manage a facility as an adaptive control (AC) problem. The random variable Y_t is explicitly included in the information available to planning agencies in the decision-making process at the end of period t. In addition, the methodology allows planning agencies to use the condition information they gather while managing a facility to improve the characterization of a facility's deterioration process by updating the probability mass function of Y_t, $t = 1, 2, 3, \ldots$ The methodology is attractive because it provides a systematic approach to include the information gathered while managing a facility to develop more efficient maintenance and repair policies.

Two AC formulations are presented a *closed-loop* control formulation and an *open-loop-optimal feedback* control formulation. In open-loop-optimal feedback control, the maintenance and repair action selected in each period depends on the facility's condition and on the planning agency's expectations about how a facility will deteriorate in current and future periods. That is, at the end of period t, the optimal action A_t depends on the facility's condition X_t, the agency's expectations about $X_{t+1}, X_{t+2}, \ldots, X_T$ and the random variable Y_t. In closed-loop control the action selected also accounts for the fact that a planning agency's expectations about future deterioration can change as new condition information becomes available. That is, A_t also depends on the agency's expectations about $Y_{t+1}, Y_{t+2}, \ldots, Y_{T-1}$.

1.2. Outline

The remainder of the paper is organized as follows. In Section 2 we review the literature on infrastructure management systems. In Section 3 we present AC formulations for the problem of finding optimal maintenance and repair policies for an infrastructure facility where there is uncertainty in the deterioration rate. In Section 4 we present a computational study where we compare the performance of policies obtained under the AC formulations to the performance of state-of-the-art infrastructure management systems. We conclude in Section 5.

2. Review of infrastructure management systems

In Section 3 we present a framework that explicitly addresses the uncertainty that exists in characterizing facility deterioration. The framework is presented in the context of developing maintenance and repair policies for individual infrastructure facilities under a finite-state, discrete-time, stationary Markovian deterioration model. In spite of this, our contribution is general as uncertainty in characterizing facility deterioration also applies to frameworks that encompass networks of facilities and frameworks where non-MDP deterioration models are used.

In order to provide additional background, we discuss and review relevant MDP formulations for the problem of finding optimal maintenance and repair policies for facilities and networks.

2.1. MDP formulations for facilities

The problem of developing maintenance and repair policies for individual infrastructure facilities has been modeled as a finite-state, discrete-time, stationary MDP. For each period, an optimal policy specifies a maintenance and repair action for every possible condition-state of a facility. For example, Madanat and Ben-Akiva (1994) present a finite-horizon, dynamic programming formulation in the context of pavement management, where the true condition state of a facility is revealed at the end of each period. In the model, the criterion used to evaluate a policy is the expected discounted cost of implementing the policy given the initial condition of the facility. Costs in this model include a component due to the application of actions to the facility and a component that reflects the costs incurred by the users of the facility which is directly related to its condition state. This formulation is the foundation of the models we present in Section 3.

As stated earlier, MDP deterioration models are specified with a single set of transition probabilities and therefore, do not capture the uncertainty in characterizing a facility's deterioration process. Deterioration models represent the expected deterioration rate of an arbitrary facility in an infrastructure network. These probabilities are not updated to account for condition information collected while managing the facility over the planning horizon. The policies that are implemented are referred to as open-loop control policies because they ignore the feedback, i.e., the state-to-state transitions, that results from applying actions to a facility. This feedback is referred to as inter-temporal feedback.

Carnahan (1988) recognizes that a single set of transition probabilities may not provide an adequate deterioration model. He presents empirical evidence that shows that similar facilities may respond differently to a given set of conditions. He proposes a framework that explicitly accounts for these differences in the process of developing maintenance and repair policies. He uses multiple models of a facility's deterioration process (each specified with a set of transition probabilities) but assumes that the planning agency knows which model is appropriate for each facility. Carnahan et al. (1987) present a linear programming model where the condition state of the system is characterized by the pavement condition index (PCI).

2.2. MDP formulations for networks

The problem of finding optimal maintenance and repair policies for networks of facilities has also been formulated as a finite-state, discrete-time, stationary MDP. Network-level frameworks assume centralized planning for a network of infrastructure facilities and account for administrative restrictions that link together the facilities in the network. At the end of each period, the condition state of the system is represented as the fraction of the infrastructure network which is in each of a finite number of possible states. The implication of the Markov assumption described in Section 1 extends to each of the facilities in the infrastructure network. Therefore, the dynamics of the deterioration process for the network are described by the Chapman–Kolmogorov difference equation:

$$\sum_{b\in\mathscr{A}} w_{jb}^{t+1} = \sum_{i\in\mathscr{S}} \sum_{a\in\mathscr{A}} w_{ia}^{t} \cdot \pi_{ij}(a), \quad j \in \mathscr{S}, \ \ t = 0, 1, 2, \ldots, \tag{3}$$

where w_{ia}^t fraction of infrastructure network in condition state i that receives action a at the end of period t.

Usually, the criterion used to evaluate policies in network-level formulations is the sum of expected discounted costs incurred when the policy is implemented. The costs include action and operating costs. Operating costs include a portion of the costs incurred by the network's users. Network-level administrative restrictions include constraints that reflect the availability of resources and constraints that reflect short-term and long-term strategic objectives. Budget constraints are a typical example of the first type of constraints. Requiring the network to meet a particular service level is an example of the second type of constraints.

Network-level formulations specify the deterioration for all facilities that comprise the network with a single set of transition probabilities. Typically, the policies that are implemented ignore inter-temporal feedback and are therefore open-loop control policies. Even the most sophisticated systems do not explicitly model the uncertainty in characterizing facility deterioration. However, a few infrastructure management systems, such as Arizona's pavement management system (Golabi et al., 1982; Kulkarni, 1984; Cambridge Systematics, 1991; Golabi et al., 1991), periodically update the deterioration model to account for inter-temporal feedback. This is done in part to compensate for the omission of the uncertainty in characterizing facility deterioration from the deterioration model. In these systems, the problem of obtaining the optimal policy for the remaining periods is solved when the deterioration model is updated. The policy that is implemented depends on the sequence of observations of the network's condition but it does not account for future updates. This type of policy is referred to as an open-loop-optimal feedback control policy and is similar in spirit to the policy that results from the formulation presented in Section 3.2.

Golabi et al. (1982) and Kulkarni (1984) describe the framework used in Arizona's pavement management system. In the framework, the objective is to minimize the cost of performing maintenance actions. The decision variables correspond to the fraction of facilities in each possible condition state that will receive a particular action at the end of each period. The resulting policy ensures that at the end of every period the fraction of the network's facilities in "unacceptable" condition states does not exceed a prespecified threshold and that the fraction of the network's facilities in "acceptable" condition states exceeds a prespecified threshold. The framework consists of two sub-problems that are formulated as linear programs. The infinite-horizon sub-problem seeks to find a policy that minimizes the average annual cost for a network that has reached its steady state. The finite-horizon sub-problem seeks to find a policy that minimizes the cost of operating the network over a prespecified planning horizon. At the end of the planning horizon the condition state of the network is constrained to reach the optimal condition for steady-state operation.

Gendreau and Duclos (1989) present a linear programming formulation where the state of each facility is extended to include information about the district where the facility is located. Madanat et al. (1999) develop a latent MDP approach to account for uncertainty in measuring the condition state of the facilities that comprise the network.

3. Adaptive control formulations

In this section we present two finite-horizon AC formulations for the problem of finding optimal maintenance and repair policies where there is uncertainty in choosing a set of parameters to represent a facility's deterioration process. We present an optimal closed-loop control formulation and a sub-optimal open-loop-optimal feedback control formulation. The AC methodology was selected because it allows for the explicit inclusion of the uncertainty in choosing a deterioration model in the decision-making process. In addition, the methodology uses inter-temporal feedback to improve the characterization of a facility's deterioration process over the planning horizon.

We consider a planning agency that manages a facility over a finite horizon T. The planning agency characterizes the facility's deterioration process with R Markovian models. Each model, referred to as a deterioration rate, is determined by a single set of parameters. Equivalently, each deterioration rate, $r = 1, \dots, R$, is determined by a single set of transition probabilities, $\pi^r_{ij}(a)$ $\forall a \in \mathscr{A}, i, j \in \mathscr{S}$. The planning agency in charge of managing the facility is uncertain about which deterioration rate is appropriate in explaining the facility's deterioration process. The uncertainty is captured by the random variable Y_t, $t = 0, 1, \dots, T$, with probability mass function $\vec{Q}^t$, $t = 0, 1, \dots, T$. The rth component of the vector $\vec{Q}^t$, denoted Q^t_r, is the probability that at the end of period t, the agency believes the facility deteriorates according to rate r, i.e., $Q^t_r \equiv P(Y_t = r)$. In the AC formulations the state of the system at the end of period t is given by the condition of the facility, $X_t \in \mathscr{S}$, and by the probability mass function $\vec{Q}^t \in \{[0,1]^R : \sum_{r=1}^R Q_r = 1\}$. The probability mass function $\vec{Q}^t$ is determined by the sequence of state-to-state transitions observed from 0 to $t-1$. An optimal maintenance and repair policy specifies an action for every possible state of the system at the end of every period. This implies that the action taken at the end of a period depends on the prior sequence of transitions. The objective is to minimize the sum of expected discounted costs over the planning horizon. The discount factor is denoted δ. The sum of the costs to apply action $a \in \mathscr{A}$ to a facility in condition $i \in \mathscr{S}$ and to operate a facility in state i for one period is denoted $g(i, a)$. The salvage value of a facility in condition i at the end of the horizon is $s(i)$. We assume that the deterioration process is stationary. The formulations are presented below.

3.1. Closed-loop control formulation

Under the closed-loop control scheme the optimal value function, denoted $v^{\mathrm{cl}}_t(X_t, \vec{Q}^t)$, represents the minimum expected sum of discounted costs until the end of T, given that the state of the system at the end of t is $(X_t, \vec{Q}^t)$. The optimal value function is given by

$$\begin{aligned} &\forall i \in \mathscr{S}, \quad \vec{Q} \in \left\{[0,1]^R : \sum_{r=1}^R Q_r = 1\right\}, \quad t = 0, \dots, T-1, \\ &v^{\mathrm{cl}}_t(i, \vec{Q}) = \min_{a \in \mathscr{A}} \left\{ g(i,a) + \delta \sum_{r=1}^R Q_r \left[\sum_{j \in \mathscr{S}} \pi^r_{ij}(a) \cdot v^{\mathrm{cl}}_{t+1}\left(j, \vec{Q'}(i,j,a,\vec{Q}) \right) \right] \right\}, \end{aligned} \tag{4}$$

where

$$Q'_r(i,j,a,\vec{Q}) = P\Big(Y_{t+1} = r \,|\, X_t = i,\ X_{t+1} = j,\ A_t = a,\ \vec{Q}^t = \vec{Q}\Big),$$

which is given by the Bayesian updating formula

$$Q'_r(i,j,a,\vec{Q}) = \frac{\pi^r_{ij}(a) \cdot Q_r}{\sum_{k=1}^R \pi^k_{ij}(a) \cdot Q_k}, \quad r = 1, \dots, R. \tag{5}$$

The boundary conditions are as follows:

$$v_T^{\mathrm{cl}}(i, \vec{Q}) = -s(i) \quad \forall i, \vec{Q}. \tag{6}$$

The expected discounted cost-to-go at the start of the horizon for a facility in condition, X_0, and probability mass function over the set of possible deterioration rates, $\vec{Q}^0$, is denoted $v_0^{\mathrm{cl}}(X_0, \vec{Q}^0)$. In the optimal value function (Equation set (4)) the first term is for the costs incurred in the current period and the second term is for the expected discounted sum of costs in the remaining periods. The minimum is taken over all actions in the set of actions $\mathscr{A}$. Equation set (5) is used to update the agency's beliefs about the deterioration rate of the facility in accordance with Bayes' law and based on the last observed state-to-state transition (from i at the end of t to j at the end of $t+1$), the action taken at the end of t, A_t, and the a priori probability mass function, $\vec{Q}^t = \vec{Q}$. The updating process allows agencies to use inter-temporal feedback to improve the characterization of facility deterioration. Equation set (6) assigns salvage values to a facility for each possible, terminal state of the facility.

The optimal policy is computed by evaluating Equation set (4) recursively from period $T-1$ to period 0 (by using dynamic programming). For each period the recurrence relation is evaluated for every possible state of the system. The computational complexity to determine the optimal policy is $\mathrm{O}(G^{R-1}|\mathscr{A}||\mathscr{S}|^2(R+T))$, where G is the number of points used to discretize the [0,1] probability space. This improvement over the brute-force implementation, whose complexity is $\mathrm{O}(G^{R}|\mathscr{A}||\mathscr{S}|^2RT)$, is obtained by precomputing the expected state-to-state transition probabilities for each action under every possible probability mass function ($\mathrm{O}(G^{R-1}|\mathscr{A}||\mathscr{S}|^2R)$) and by taking advantage of the fact that the probability mass functions have $R-1$ degrees of freedom. The remaining operations are necessary to evaluate the recurrence relation (Equation set (4)) from period $T-1$ to 0.

The closed-loop control formulation is optimal under the assumption of a "correct deterioration model". A deterioration model is "correct" when the actual deterioration rate of a facility is drawn (from the set of deterioration rates $\{1, \ldots, R\}$) according to the probability mass function $\vec{Q}^0$. This implies that for all $r = 1, \ldots, R$, the probability that the agency initially assigns to the deterioration rate r is equal to the probability that a facility's actual deterioration rate is r.

3.2. Open-loop-optimal feedback control formulation

The open-loop-optimal feedback control scheme is sub-optimal (under the assumption of a correct model) because the recurrence relation does not account for the fact that the probability mass functions of $Y_1, Y_2, \ldots, Y_{T-1}$ are updated at the end of each period. The recurrence relation used to obtain the policy is given by

$$v_t^{\mathrm{olof}}(i, \vec{Q}) = \min_{a \in \mathscr{A}} \left\{ g(i,a) + \delta \sum_{r=1}^{R} Q_r^t \left[\sum_{j \in \mathscr{S}} \pi_{ij}^r(a) \cdot v_{t+1}^{\mathrm{olof}}\left(j, \vec{Q}\right) \right] \right\} \quad \forall i, \vec{Q}. \tag{7}$$

Under the open-loop-optimal feedback control scheme the probability mass functions are still updated according to Equation set (5) and the boundary conditions are still given by Equation set (6). At the end of period t, the action specified by the open-loop-optimal feedback control formulation is obtained by solving an open-loop control problem over the remaining $T-t$ periods. In solving each of the open-loop control problems the agency assumes that the probability mass function will remain unchanged until the end of the horizon. $v_t^{\mathrm{olof}}(i,\vec{Q})$ represents the expected discounted cost of managing the facility until the end of the horizon given the initial state of the system $(i,\vec{Q})$ and that an open-loop control policy will be followed. This implies that the actual costs of implementing an open-loop-optimal feedback control policy from t to $T-1$ are bounded above by $v_t^{\mathrm{olof}}(i,\vec{Q})$. The mathematical proof is presented in Bertsekas (1995). The difference between $v_t^{\mathrm{olof}}(i,\vec{Q})$ and $v_t^{\mathrm{cl}}(i,\vec{Q})$ is that in the latter, the sum of expected, discounted costs in the remaining periods is computed while accounting for subsequent updates in the probability mass functions.

Instead of evaluating the recurrence relation for every state and for every period (complexity$\mathrm{O}(G^{R-1}|\mathscr{A}||\mathscr{S}|^2(R+T)))$, this control scheme can be implemented on-line with a computational complexity of $\mathrm{O}(|\mathscr{A}||\mathscr{S}|^2T(R+T))$. The complexity is a polynomial function of the number of deterioration rates R. In the on-line implementation, the total number of operations to find the expected state-to-state transition probabilities for the given mass function in all periods is $\mathrm{O}(|\mathscr{A}||\mathscr{S}|^2RT)$. The remaining operations are for solving $T-1$ open-loop control problems to find the actions to be applied at the end of each period. The computational complexity to compute the optimal policy for an open-loop control problem with $T-t$ periods remaining until the end of the planning horizon is $\mathrm{O}(|\mathscr{A}||\mathscr{S}|^2(T-t))$.

4. Computational study

In Section 3 we presented two AC formulations for the problem of finding optimal maintenance and repair policies where there is uncertainty in the deterioration rate of a facility. In this section, we present a computational study to compare the AC formulations presented in Sections 3.1 and 3.2 with the current state-of-the-art in infrastructure management systems which are based on open-loop control formulations. In Section 4.2 we compare the performance of maintenance and repair policies obtained under the three control schemes and under perfect information in situations where the deterioration model is correct. In Section 4.3 we compare the performance of policies obtained under incorrect deterioration models.

In both parts of the computational study, the open-loop control policy is obtained with respect to the initial, expected deterioration process. This amounts to using all of the information that is available at the start of the planning horizon. Under the initial expected deterioration process, the transition probability from state i to state j when action a is applied is equal to the initial expected probability that the transition occurs. Henceforth, this probability is denoted $\pi_{ij}(a)$ and is given as follows:

$$\pi_{ij}(a) = \sum_{r=1}^{R} Q_r^0 \cdot \pi_{ij}^r(a), \quad \forall a \in \mathscr{A},\ i,j \in \mathscr{S}. \tag{8}$$

4.1. Problem specification

The purpose of the study is to illustrate the performance of each of the control schemes to an application in the context of pavement management. The data that we use are taken from empirical studies presented in the literature on pavement management. We consider a discount rate

$((1/\delta) - 1)$ of 5% and a planning horizon (T) of 25 years. As in (Carnahan et al., 1987), the condition of the pavement segment is given by a PCI rating which is discretized into eight states (state 1 being a failed pavement and state 8 being a new pavement). The planning agency has seven maintenance and repair actions available in every period, and for every possible condition of the pavement segment. The actions considered are: (1) do nothing, (2) routine maintenance, (3) 1-in overlay, (4) 2-in overlay, (5) 4-in overlay, (6) 6-in overlay, and (7) reconstruction. The costs of performing actions are taken from Carnahan et al. (1987). The operating costs considered in the study are meant to represent the users' vehicle operating costs that are associated with the condition of the pavement segment. We restrict the terminal state of the pavement segment to any one of the four best states to ensure that the pavement provides high level-of-service until the end of the planning horizon. We use the salvage values for each state of the pavement segment to enforce this restriction. The costs are presented in Table 1 and are expressed in $ per lane-yard.

We assume that the deterioration process of the pavement segment is governed by one of the three rates ((1) slow, (2) medium, or (3) fast). In this instance, a deterioration rate, r, is characterized by a set of seven (one for each action) transition probability matrices, $\mathbf{\Pi}^r(a)$, $a \in \mathscr{A}$, where the (i, j) entry of the matrix $\mathbf{\Pi}^r(a)$ is the transition probability $\pi^r_{ij}(a)$. The transition probability matrices were generated using truncated normal distributions as in (Madanat and Ben-Akiva, 1994). We assume that:

(a) each maintenance and repair action has an associated mean effect on the state-to-state transition from the current state to the state that is observed in the year after the action is applied;
(b) for faster deterioration rates of the pavement segment, the same action is less effective in improving the state of the pavement segment;
(c) faster deterioration processes are characterized by higher variances in prediction than slower ones.

The mean effects of applying maintenance and repair actions and the standard deviations associated with each deterioration rate are presented in Table 2.

The steady-state optimal policies obtained with this data set are presented in Table 3. The policies specify the actions to be taken for a pavement segment in a given condition and assuming

Table 1
Costs ($/lane-yard)

Pav. state	Maintenance and repair actions							User costs	Salvage value
	1	2	3	4	5	6	7		
1	0.00	6.90	19.90	21.81	25.61	29.42	25.97	∞	$-\infty$
2	0.00	2.00	10.40	12.31	16.11	19.92	25.97	25.00	$-\infty$
3	0.00	1.40	8.78	10.69	14.49	18.30	25.97	22.00	$-\infty$
4	0.00	0.83	7.15	9.06	12.86	16.67	25.97	14.00	$-\infty$
5	0.00	0.65	4.73	6.64	10.43	14.25	25.97	8.00	0.00
6	0.00	0.31	2.20	4.11	7.91	11.72	25.97	4.00	0.00
7	0.00	0.15	2.00	3.91	7.71	11.52	25.97	2.00	0.00
8	0.00	0.04	1.90	3.81	7.61	11.42	25.97	0.00	0.00

Table 2
Mean and standard deviation of action effects on the state of a pavement segment

Action	Deterioration rate		
	Slow	Medium	Fast
	Mean effects		
1	−0.25	−0.75	−1.75
2	0.50	0.00	−0.50
3	1.75	1.00	0.25
4	3.00	2.00	1.00
5	4.25	3.00	1.75
6	5.50	4.00	2.50
7	8.00	6.00	4.00
	Standard deviation		
	0.30	0.50	0.70

that the deterioration rate is known. For example, we note that for a pavement segment in condition 3, the optimal steady-state policies specify that a 4-inch overlay (action 5) be applied if the deterioration rate is known to be slow, that a 6-inch overlay (action 6) be applied if the deterioration rate is known to be medium and that reconstruction (action 7) be applied if the deterioration rate is known to be fast. We note that the optimal steady-state actions to be taken for a pavement segment in a given condition depend on the agency's beliefs about which deterioration rate is appropriate. This is a necessary condition for the AC schemes to strictly outperform the open-loop control scheme.

The code for the computational study was written in C. In the implementation of the AC schemes the [0,1] probability space was discretized with a grid of 0.02, i.e., $G = 50$.

4.2. Results for policies obtained with correct deterioration models

In this section we present numerical results that illustrate the effect of uncertainty on the total expected discounted costs of managing a facility. We assume that the facility's actual deterioration

Table 3
Steady-state optimal policies for known deterioration rates

Condition state	Deterioration rate		
	Slow	Medium	Fast
1	7	7	7
2	6	7	7
3	5	6	7
4	4	5	6
5	4	4	5
6	3	4	4
7	2	3	4
8	2	2	3

rate is drawn from the initial probability mass function, $\vec{Q}^0$, i.e., the deterioration model is correct. We consider an instance of the problem specified in Section 4.1 and compare the expected discounted costs under the optimal closed-loop control scheme, clc, under the sub-optimal open-loop-optimal feedback control scheme, olofc, and under the open-loop control scheme, olc. The results of the study are presented in Tables 4 and 5.

Table 4 presents the expected discounted costs of managing the facility under perfect information for initial conditions (2)–(8) of a facility. For example, $\vec{Q}^0 = (1, 0, 0)$ means that the deterioration rate is slow with probability 1. Under perfect information, there is no difference in the policies dictated by the three control schemes and therefore the expected discounted costs under the three control schemes are equal. These cases are presented to illustrate the effect of uncertainty in the deterioration rates and to benchmark the performance of the three control schemes.

Table 5 presents the expected discounted costs under the three control schemes for every possible initial condition of the facility, in situations where there is uncertainty about the deterioration rates. We consider deterioration rates that are drawn according to the three probability mass functions: $(0.10, 0.80, 0.10)$, $(0.33, 0.34, 0.33)$, $(0.40, 0.20, 0.40)$. For example, $\vec{Q}^0 = (0.10, 0.80, 0.10)$ means that the probability that the deterioration rate is slow is 0.10; the probability

Table 4
Expected discounted costs ($/lane-yard)

Initial state	Initial probability mass function $\vec{Q}^0$		
	(1,0,0)	(0,1,0)	(0,0,1)
2	47.05	55.50	81.66
3	39.76	48.02	71.92
4	27.01	34.58	57.52
5	15.09	22.36	43.80
6	7.21	12.64	32.06
7	4.23	8.53	26.15
8	0.30	4.57	21.64

Table 5
Expected discounted costs ($/lane-yard)

Initial state	Initial probability mass function $\vec{Q}^0$											
	(0.10,0.80,0.10)				(0.33,0.34,0.33)				(0.40,0.20,0.40)			
	clc	olofc	olc	Gap (%)	clc	olofc	olc	Gap (%)	clc	olofc	olc	Gap (%)
2	57.94	57.94	58.15	0	62.63	62.63	63.32	1	64.48	64.48	65.32	1
3	50.29	50.29	50.53	0	54.52	54.52	55.33	1	56.32	56.32	57.29	2
4	36.63	36.63	36.84	1	40.37	40.37	41.08	2	41.99	42.00	42.83	2
5	24.22	24.22	24.39	1	27.53	27.53	28.14	2	28.99	29.00	29.70	2
6	14.46	14.46	14.58	1	17.72	17.72	18.18	3	19.12	19.14	19.62	3
7	10.24	10.24	10.35	1	13.27	13.27	13.69	3	14.58	14.60	15.03	3
8	6.18	6.18	6.27	1	9.00	9.00	9.36	4	10.23	10.23	10.61	4

that it is medium is 0.80; and the probability that it is fast is 0.10. The gap refers to the percentage difference between the expected discounted costs of managing the given facility under an open-loop control policy and the costs under a closed-loop control policy.

The main observations are that:

(a) As the variance in the probability mass function from which the deterioration rates are drawn increases, that is, as the uncertainty about the deterioration rate increases, the expected discounted costs increase.
(b) As the uncertainty increases, the gap increases.
(c) For the data presented in Section 4.1, the sub-optimal open-loop-optimal feedback control scheme consistently performs as well as the optimal closed-loop control scheme. However, there are small differences as the uncertainty increases.

4.3. Results for policies obtained with incorrect deterioration models

In this section of the computational study we present results for situations where the deterioration rate is not drawn from the initial probability mass function $\vec{Q}^0$. Specifically, we illustrate how the three control schemes perform in cases where initially, the deterioration rate is either underestimated or overestimated.

To consider cases where the deterioration rate is overestimated, we consider the initial probability mass functions: $(0.05, 0.05, 0.90)$, $(0.33, 0.34, 0.33)$. We compare the expected discounted costs for initial conditions of a facility that range from 2 to 8. We consider a facility that deteriorates according to the slow rate ($r = 1$). To consider cases where the deterioration rate is underestimated we consider the probability mass functions $(0.9, 0.05, 0.05)$, $(0.33, 0.34, 0.33)$ and a facility that deteriorates according to the fast rate ($r = 3$). We also present the expected discounted costs under perfect information in order to benchmark the performance of the control schemes. The results are presented in Fig. 1.

The main observations are that:

(a) The AC schemes always perform better than the open-loop control scheme.
(b) As the initial error in estimation of the actual deterioration rate increases, the penalty for not employing an AC scheme increases. This is more critical in cases where the actual deterioration rate is overestimated.

In the next part of the computational study we address the following questions: (i) When do the AC schemes produce substantial benefits over the open-loop control scheme? and (ii) When does the closed-loop control scheme perform substantially better than the open-loop-optimal feedback control scheme?

In order to address these questions we conducted a simulation study where we consider a facility initially in condition-state 5 that deteriorates according to rate 2 (medium). We consider probability mass functions $\vec{Q}^0$ that span the possible state space (by letting Q_1^0 and Q_2^0 vary between 0 and 0.90 in increments of 0.05). For each realization of $\vec{Q}^0$ we generate 30 random instances of the problem specified in Section 4.1. We present the number of times that the policy computed under the closed-loop control scheme results in at least a 10% reduction in expected

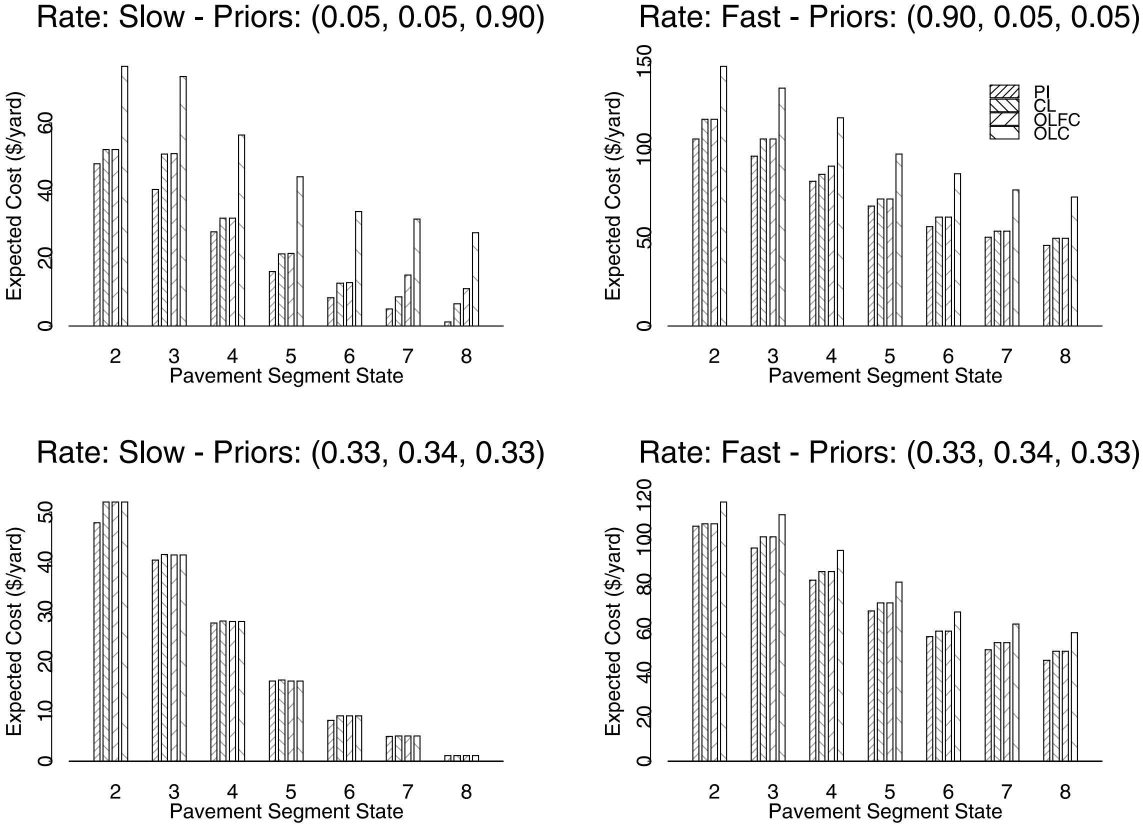

Fig. 1. Expected discounted costs ($/lane-yard).

discounted costs over the policy computed under the open-loop control scheme and over the policy computed under the open-loop-optimal feedback control scheme. The results are presented in Fig. 2. Each entry in the tables corresponds to the number of times (out of 30) that the implementation of the closed-loop control policy resulted in at least 10% reduction in costs over the implementation of either the open-loop or the open-loop-optimal feedback control policy.

The main observation is that the optimal, closed-loop control scheme is preferred when the initial error in the estimation of the actual deterioration rate is large. This is particularly true when the deterioration rate is overestimated. These cases are those for which $P(Y_0 = 1) + P(Y_0 = 2)$ is a small number, and lie close to the origin in Fig. 2.

4.4. An example

We present an example that illustrates why the closed-loop control policy performs better than the open-loop and open-loop-optimal feedback control policies, even in cases when the deterioration model is incorrect. We consider a random instance of the problem specified in Section 4.1 where a pavement segment that is in condition-state 7 at the start of the horizon will deteriorate slowly ($r = 1$) over the planning horizon. The planning agency in charge of managing the pave-

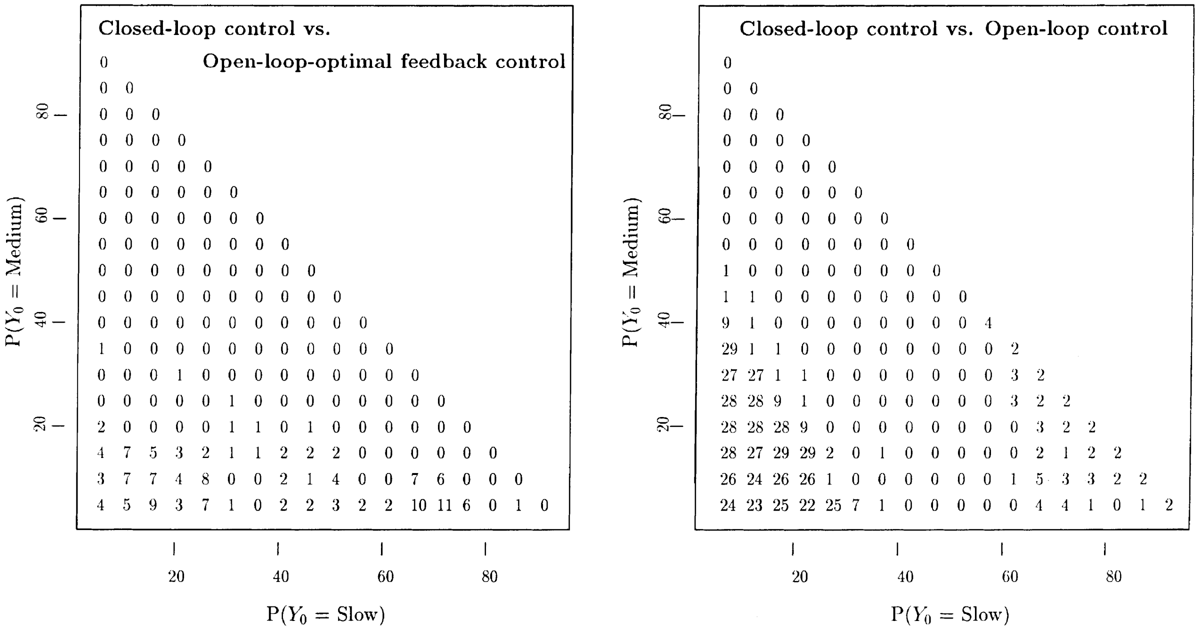

Fig. 2. Performance of control schemes.

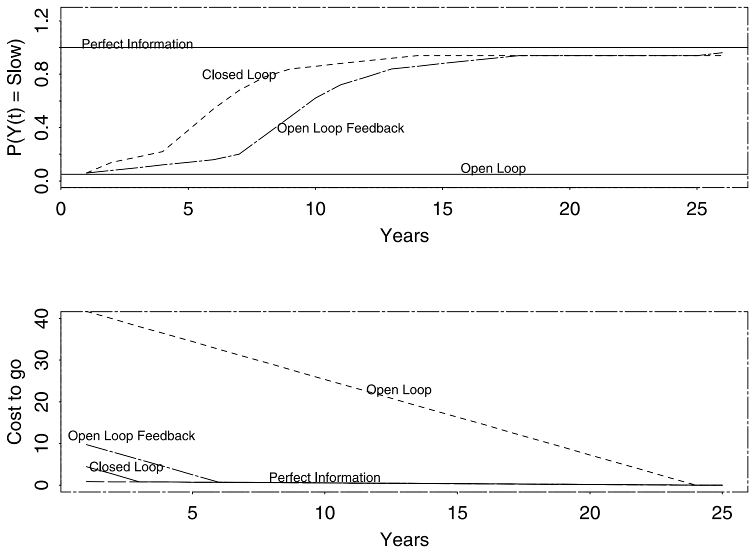

Fig. 3. Operation of control schemes.

ment initially believes that the deterioration rate for the pavement is drawn from ($\vec{Q}^0 = (0.05, 0.05, 0.90)$). That is, initially the agency has overestimated the pavement's deterioration rate. Fig. 3 illustrates how the planning agency's beliefs are updated over time under the two AC schemes and how the costs-to-go under the three control schemes converge to the costs-to-go in the perfect information case. As expected, the convergence in the beliefs is faster for the closed-loop control than the open-loop-optimal feedback control. Furthermore, the convergence in the cost-to-go is fastest for the closed-loop control and slowest for the open-loop control.

5. Conclusion

An important class of infrastructure management systems uses an MDP-based framework to characterize facility deterioration. The underlying deterioration process of a facility is represented with a single set of transition probabilities. This framework cannot explicitly account for the uncertainty in characterizing a facility's deterioration rate. We address this limitation by presenting a framework that characterizes a facility's deterioration under multiple deterioration rates. Each rate is represented with a set of transition probabilities. We present two AC formulations that use this deterioration model and that explicitly account for the uncertainty in characterizing a facility's deterioration rate. The formulations use infrastructure-condition-related information that stems from the actual operation of an infrastructure facility (inter-temporal feedback) to improve the characterization of a facility's deterioration over time. We present two AC formulations, a closed-loop control formulation and an open-loop-optimal feedback control formulation, and discuss their implementation and computational complexity. Through a computational study in the context of pavement management we show that economic benefits can be achieved by explicitly accounting for the uncertainty in characterizing a facility's deterioration rate. In the future, we plan to extend our AC formulations to network-level problems.

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