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Essays on Corporate Capital Structure

by

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Abstract

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This dissertation studies capital structure decisions of levered and unlevered firms using the modeling framework of Leland (1994). The first chapter, *Cash Holdings and Financial Constraints*, focuses on optimal management of cash holdings by equity holders of a levered, financially constrained firm. I add financial constraints as a market friction to the traditional model. A financially constrained firm is not able to issue new equity to subsidize net operating losses and is subject to premature, costly default on its straight debt. The more constrained the firm is the less equity it is able to issue and the more likely it is to default. Equity holders mitigate the effects of financial constraints by managing a costly cash account, based on retained net operating profits. In the theoretical section, I show that firms that are more financially constrained optimally hold more cash but remain more likely to default compared to their less constrained counterparts. Hence, firms with higher cash holdings are riskier, and claims on their assets should trade at a premium. In the empirical section, I find evidence of this observation in straight debt and common equity markets. Firms with higher cash holdings are observed with higher yields on debt and higher returns on equity,

In the second chapter, *Contingent Capital Bonds (CCBs) and Capital Structure Decisions*, a joint work with Dwight Jaffee and Alexei Tchistyi, we provide a formal model of CCBs, a new instrument offering potential value as a component of corporate capital structures for all types of firms, as well as being considered for the reform of prudential bank regulation following the financial crisis of 2007-2008. CCBs are debt instruments that automatically convert to equity if and when the issuing firm reaches a specified level of financial distress. We develop closed form solutions for CCB value under three assumptions. First, the firm is allowed a tax deduction on its CCB

interest payments as long as the security remains outstanding as a bond. Second, we assume that adding CCBs to a firm's capital structure has no impact on the level of the firm's asset holdings. Third, we require that the CCB conversion to equity occurs at a time prior to any possible default by the firm on its straight debt. The key contribution of our work is that we provide a formal financial model in which the effects of alternative CCB contract provisions can be analytically evaluated. We show that a firm will always gain from including CCB in its capital structure as a result of the tax shield benefit. A firm creating a *de novo* capital structure, assuming it faces the regulatory constraint that the CCB can only replace a part of what would have been the optimal amount of straight debt, will always issue at least a small amount of CCB. The reduction in expected bankruptcy costs ensures a net gain, even if the tax shield benefits are reduced. We show that a firm will never add CCB to an existing capital structure, assuming that it faces the regulatory constraint that the CCB can only be introduced as part of a swap for a part of the outstanding straight debt. While the swap may increase the firm's value - the value of reduced bankruptcy costs may exceed any loss of tax shield benefits - the gain accrues only to the holders of the existing straight debt. As in a classic debt overhang problem, equity holders will not act to enhance the overall firm value. We show that for a Too-Big-To-Fail firm, for which the straight debt is risk free because the bond holders correctly assume they will protected from any potential insolvency, under a regulatory limitation on the amount of debt such a firm may issue, a CCB for straight debt swap reduces the value of the government subsidy by reducing the expected cost of bondholder bailouts. While this has a taxpayer benefit, the equity holders of such a firm would not voluntarily participate in such a swap. We demonstrate that CCBs create an incentive for market manipulation. CCB holders may have an incentive to manipulate the stock price to a lower value if the amount of equity they receive at conversion is sufficiently high. Equity holders may have an incentive to manipulate the stock price down if the amount of equity they give up at conversion is sufficiently low. We summarize, that the regulatory benefits of CCB issuance with respect to bank safety will generally depend on the CCB contract and issuance terms. Perhaps most importantly, the regulatory benefits vanish if banks simply substitute CCBs for capital, leaving the amount of straight debt unchanged. It is thus essential to require CCB issuance to substitute for straight debt (and not for equity).

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Chapter 1

Cash Holdings and Financial Constraints

1.1 Introduction

Most "traditional" structural models of equity and debt pricing (e.g., Black and Cox (1976), Leland (1994), Longstaff and Schwartz (1995), Collin-Dufresne and Goldstein (2001)) assume perfect capital markets. In these models, as long as the stock price remains positive, equity holders can always issue new equity to cover unexpected net operating losses and avoid inefficient closure. Firms are modeled to have unlimited access to costless outside financing, and there is no role for internal reserves of liquid assets, such as cash. This view is mirrored in empirical studies of corporate debt (e.g., Collin-Dufresne, Goldstein and Martin (2001), Duffee (1998), Schaefer and Strebulaev (2008)), that also ignore the role of cash holdings. Yet, the financial crisis of 2007-2009 magnified the importance of internal liquidity. Campello, Graham and Harvey (2010) survey 1,050 Chief Financial Officers worldwide and find that more than half of their respondents identified their firms as being financially constrained during 2008-2009. These firms faced difficulties with accessing external financing, paid higher borrowing costs, and burned through substantial amounts of cash reserves. Financially constrained firms not only bypassed attractive investment opportunities, but they also dramatically reduced employment, cut spending on technology and marketing, and sold more assets to fund ongoing operations.

This paper builds on the idea that rationing external financing increases the value of a marginal dollar of cash held internally and elevates the importance of cash management policies in the context of pricing claims on a firm's assets. I start with the structural model of Goldstein, Ju, and Leland (2001), in which dead-weight costs of bankruptcy and tax saving benefits of debt are the two market frictions. I introduce financial constraints as the third friction. A firm is financially constrained if equity holders are unable to issue new equity to finance net operating losses, despite the fact that, conditional on being to able fund *ongoing operations*, the stock price is positive. A financially constrained firm is subject to suboptimal default, resulting in ex-ante equity and total firm value losses. Higher financial constraints are associated with smaller net operating losses, for which equity holders are unable to raise financing, and higher losses of equity value. In an attempt to shield the firm, equity holders respond to financial constraints by managing a cash holding account. The account is based on retained earnings, which would otherwise be paid out as dividends. As noted by Opler, Pinkowitz, Stulz, and Williamson (1999), accumulating cash inside the firm is costly due to the liquidity premium and tax disadvantages. The cost of hoarding cash becomes the fourth market friction in the model.

I consider a levered, financially constrained firm with a fixed amount of perpetual debt at time $t > 0$. Independent of asset value realization, equity holders do not re-leverage the firm. This is consistent with the solution approach of Goldstein et al. (2001)[1]: they solve for an optimal capital structure of an unlevered firm at time $t = 0$, conditional on the coupon rate remaining fixed at any $t > 0$ before default. As in the baseline setup, in the model with financial constraints and costly cash, equity holders of a levered firm solve for the level of asset value at which to declare bankruptcy. Additionally, at each $t > 0$ before default, equity holders target an optimal cash inventory amount by trading the costs and benefits of hoarding cash. To equity holders, the marginal value of one dollar in cash below the target is more than one, as it shields an amount of equity value that exceeds unity. The marginal value of a dollar above the target is one, as it is being instantaneously paid out in dividends. At the target, equity holders are indifferent between retaining cash inside the firm and paying it out as dividends.

I argue two points. First, higher observable levels of cash holdings *proxy* for higher unobservable individual financial constraints. Higher expected losses of equity value, due to higher financial constraints, incentivize equity holders to retain more cash inside the firm. Second, since hoarding cash is costly, even with optimal cash inventories, firms that are more financially constrained are expected to default sooner

[1]Leland (1994) shows that continuous readjustment of the coupon rate may be blocked by existing equity and debt holders. Equity holders will always resist decreasing the coupon rate via small open market repurchases of current debt financed by new equity. Debt holders will always resist increasing the total coupon payments through additional debt issuance.

than their less constrained counterparts. On average, in addition to paying more for cash (due to higher cash reserves), equity holders of more constrained firms collect fewer dividends and bond holders collect fewer coupon payments. In sum, *higher observable levels of cash holdings should correspond to lower values of both equity and debt.*

By assumption, net operating profits are the primary source of cash[2] for the firm. If operating cash flows are sufficiently low, equity holders' optimal cash management policy, based on evaluating long-term costs and benefits of hoarding cash, will no longer be feasible. For instance, in the case of severe net operating losses, equity holders of a financially constrained firm with the marginal value of cash above unity instead of hoarding cash will be burning through their reserves in an attempt to avoid immediate default. In general, low realizations of operating cash flows will erode the role of cash holdings as a proxy for the intensify of financial constraints. As the probability of running into financial constraints increases substantially, more cash will translate into lower default risk and, consequently, higher ex-ante values of equity and debt. A positive relationship between cash reserves and values of claims on a distressed firm's assets is consistent with the "traditional" view that higher cash holding firms are safer.

Based on the above discussion, there are two main testable implications for corporate debt. First, for firms with sufficiently high operating cash flows, cash holdings should be negatively correlated with debt value. Or, considering that the risk-free rate is modeled as a constant, cash holdings should be positively correlated with corporate-Treasury yield spreads. Second, for firms with sufficiently low operating cash flows, the above relationship between cash holdings and yield spreads should be either weaker or reversed. I confirm both implications in the data for straight corporate debt[3]. Additionally, I find that, for firms with sufficiently high operating cash flows, the correlation between cash holdings and corporate-Treasury yield spreads gradually increases with default risk.

My empirical analysis of equity is focused on exploring the impact of financial constraints on stock *returns*. I use the methodology developed by Lamont, Polk, and Saa-Requejo (2001) and repeated in Whited and Wu (2006). Lamont et al.

[2]In the model, the interest earned on the assets in the cash account is the other, implicit source of cash.

[3]I exclude financial and regulated firms from all empirical tests, as their capital structure decisions are guided by principals different from the ones modeled in this paper. Cash holdings are represented by variable *CASH*, defined as book value of cash and short-term investments divided by net market value of assets. Operating cash flows are represented by EBITDA divided by net market value of assets. For more details, please, see Section 1.3.

(2001) construct an index of financial constraints based on regression coefficients on observable firm characteristics estimated in Kaplan and Zingales (1997). Whited et al. (2006) build an index of financial constraints, also based on observable characteristics, via generalized methods of moments estimation of an investment Euler equation. I distinguish between more and less financially constrained firms purely based on the level of their cash holdings. The main issue, inherited from Lamont et al. (2001) and Whited et al. (2006), is whether financial constraints represent a source of priced risk. The issue is addressed from both a cross-sectional and a time-series prospective.

Cross-sectional regressions of stock returns on cash holdings with controls for size, book-to-market equity, an interaction term between size and book-to-market equity, and momentum, indicate that more financially constrained (i.e., higher cash holding) firms earn higher returns. The average coefficient on cash holdings is positive and statistically significant. The observed positive relationship between returns and the level of financial constraints is robust to splitting the data into sub-samples. Additionally, the magnitude and statistical significance of beta coefficients on control variables in regression specifications with and without cash holdings are almost the same. Hence, the financial constraint risk premium is not an artifact of size, book-to-market equity or momentum effects.

I continue the cross-sectional analysis of stock returns by sorting firms into groups by the level of their operating cash flows / likelihood of default. Each group represents a certain area of the cash flow distribution. I re-run the above regressions for each group separately. The focus of the analysis is exclusively on average values of cash holding coefficients. I observe that, first, cash holding betas gradually increase as I move from the highest five percent to the second lowest five percent of operating cash flow firms. When the risk of default is low, firms target higher cash reserves as the probability of facing financial constraints increases. At the same time, due to costly hoarding of cash, even in the presence of optimal cash holdings, higher risk of running into financial constraints translates into higher expected returns. This dual dynamic results in an observed correlation between cash holdings and stock returns that increases as the cash holdings fall. Second, the cash holding beta drops significantly as I move to the firms in the lowest five percent of operating cash flows. Financially distressed firms are burning though their cash reserves. More cash translates into relative safety, as the firm is being able to avoid immediate default, and safer firms offer lower expected returns. The positive correlation between cash holdings and returns weakens substantially.

My analysis of the time-series of stock returns employs portfolios, constructed based on independent ranking of firms by size, book-to-market equity and cash holdings. First, using monthly time-series of returns on these portfolios, I find that equity returns on high cash holding firms positively covary with the returns on other high cash holding firms. I conclude, that there is common variation in stock returns associated with cash holdings. Second, using the technique of Fama-French (1993), I construct a systematic risk factor, represented by a zero-cost portfolio that is long portfolios of high cash holding firms and short portfolios of low cash holding ones. The factor is neutral with respect to size and book-to-market equity characteristics. Sizable variation in the factor portfolio cannot be explained by the traditional factors, including the three Fama-French and the momentum factor. The factor portfolio earns a positive, statistically significant risk premium of 5.99%-7.48% on an annual basis, which is comparable to the premiums on the traditional risk factors. Given the role of cash holdings as a proxy for the intensity of unobservable financial constraints, I attribute the systematic risk associated with cash holdings to financial constraints.

Several of my empirical results are similar to the ones from the concurrent papers of Acharya, Davydenko and Strebulaev (2011) and Palazzo (2011). In the model of Acharya et al. (2011) a levered firm at time zero can either invest its cash in a long-term project or retain it as a buffer against future cash flow shortfalls when the firm's short-term debt becomes due. Higher cash reserves reduce the probability of default due to financial constraints in the short run, but may increase it in the long run. Optimal investments and cash reserves are determined simultaneously. I identify positive correlation between cash holdings and corporate-Treasury spreads in the full debt sample. Although our data sources and time periods are different, my finding matches the main empirical result of Acharya et al. (2011). In the model of Palazzo (2011), a firm expects an investment opportunity in the future and needs to decide whether to hoard cash, and thus earn a lower return than the opportunity cost of capital, or to distribute dividends today, and thus increase the expected cost of future investment. This trade-off determines the current period's optimal cash reserves. The assumption that cash flows are correlated with an aggregate risk drives riskier (higher correlation) firms to save more. There is no explicit role for financial constraints. I identify a positive correlation between cash holdings and stock returns. Here, again, although our data samples are different, my result is similar to the one of Palazzo (2011).

There are three key differences between my work and the work of the above authors. First, in my paper the source of corporate precautionary savings is associated exclusively with funding existing operations when the firm incurs net operating losses. The implicit argument of my paper is that the one can explain the observed empirical results without modeling investment decisions. Second, while Acharya et al. (2011) study debt and Palazzo (2011) analyses equity, I am able to tie cash holding decisions to the values of both types of claims simultaneously. My paper is more general. Finally, Acharya et al. (2011) and Palazzo (2011) employ discrete modeling setups. The continuous-time model in my paper allows me to explain the observed non-linear correlation between cash holdings and yield spreads / equity returns as a function of realized operating cash flows.

My paper is also related to a small literature studying the determinants of corporate cash holdings and their time-series properties[4]. Opler et al. (1999) document that cash holdings are negatively correlated with size and book-to-market equity and positively correlated with capital expenditures, payouts, research and development expenditures, and cash flow volatility. Consistent with Opler et al. (1999), Han and Qiu (2007) find that the volatility of cash flows has a positive impact on a firm's precautionary motive for holding cash, which is significant only for financially constrained firms. Almeida, Campello and Weisbach (2004) hypothesize that the effect of financial constraints is captured by the firm's propensity to save cash out of cash flows. They find that constrained firms display significantly positive cash flow sensitivity of cash, while unconstrained firms do not. Acharya, Almeida and Campello (2007) examine both the propensity to save out of cash flow and the propensity to issue debt. Their analysis suggests that cash should not be viewed as negative debt in the presence of financing frictions.

The rest of the paper is organized as follows. Section 1.2 presents the model. Section 1.3 describes the data that feeds the debt and equity samples. Section 1.4 presents the empirical analysis of corporate-Treasury yield spreads. Section 1.5 presents the empirical analysis of equity returns. Section 1.6 concludes. All proofs are detailed in the appendix.

1.2 Model

I use the modeling framework of the traditional trade-off theory of capital structure, based on Goldstein, Ju, and Leland (2001). I start with summarizing the main

[4]Other studies on the determinants of cash holdings are Huberman (1984), Asvanunt, Broadie, and Sunderson (2010), Bolton, Chen, and Wang (2011), Dittmar and Mahrt-Smith (2007), Hartford, Mansi, Maxwell (2008).

assumptions and selected results of the baseline model. Then, I introduce financial constraints to the traditional setting and analyze their impact on equity and debt values. Finally, I show how holding cash allows equity holders to reduce the negative effect of financial constraints on equity value and how it impacts debt value.

Let at each time t the operating assets of the firm generate after-tax cash flows at the rate δ_t. The tax rate is fixed at $\theta \in (0,1)$. I call δ the *operating cash flow.* By assumption, δ follows a diffusion process governed by equation

$$\frac{d\delta_t}{\delta_t} = \mu dt + \sigma dB_t^Q,$$

where μ and σ are constant, and B^Q defines a standard Brownian motion under the risk-neutral measure. All agents are risk-neutral and discount future cash flows at a fixed market interest rate r. At any time t the market value of assets is defined as the discounted value of all future cash flows,

$$A_t = \mathbb{E}_t^Q\left[\int_t^\infty e^{-r(s-t)}\delta_s ds\right] = \frac{\delta_t}{r-\mu}, \tag{1.1}$$

which is finite if $\mu < r$. The dynamics for A_t are $dA_t = \mu A_t dt + \sigma A_t dB_t^Q$.

The firm issues equity and a bond (debt). The bond is a *consol*, meaning that it pays coupons at a constant rate $c > 0$, continually in time, until liquidation (default) time τ. The firm issues only one class of debt, public debt, and it is assumed that, as in Gertner and Scharfstein (1991), the dispersion of public creditors prevents renegotiation outside formal bankruptcy when the firm finds itself in distress. At default fraction $\alpha \in [0,1]$ of the firm's assets is lost as a frictional cost. The value of the remaining assets is assigned to debt holders.

For a given constant $K \in (0, A_t)$, the market value of a security that claims one unit of account at the hitting time $\tau(K) = \inf\{s : A_s \leq K\}$ is, at any $t < \tau(K)$,

$$\mathbb{E}_t^Q\left[e^{-r(\tau(K)-t)}\right] = \left(\frac{A_t}{K}\right)^{-\gamma}, \tag{1.2}$$

where $\gamma = \frac{1}{\sigma^2}\left[\left(\mu - \frac{\sigma^2}{2}\right) + \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right] > 0.$

The firm is operated by its equity holders. At any time t, given the obligation to pay c and the current asset level A_t, the liquidation policy of the firm maximizes the value of equity $E(A_t;c)$. That is,

$$E(A_t;c) \equiv \sup_{\tau\in\mathcal{T}} \mathbb{E}_t^Q \left[\int_t^\tau e^{-r(s-t)} \left[\delta_s - (1-\theta)c\right] ds\right], \tag{1.3}$$

where $\mathcal{T}$ is the set of stopping times. The optimal liquidation time is the first time $\tau(A_B) = \inf\{s : A_s \le A_B\}$ that the asset level falls to some sufficiently low boundary $A_B > 0$. The value of the optimal default boundary, at any time $s < \tau(A_B)$, is

$$A_B = \beta(1-\theta)c, \tag{1.4}$$

where $\beta = \frac{\gamma}{(\gamma+1)r}$.

Given some asset value realization, A_t, the coupon rate, c, and the optimal-default boundary, A_B, market values of equity and debt in closed-form are as follows:

- the value of equity

$$E(A_t;c) = A_t - \frac{c(1-\theta)}{r}\left(1 - \left(\frac{A_t}{A_B}\right)^{-\gamma}\right) - A_B\left(\frac{A_t}{A_B}\right)^{-\gamma} \tag{1.5}$$

- the value of debt

$$D(A_t;c) = \frac{c}{r}\left(1 - \left(\frac{A_t}{A_B}\right)^{-\gamma}\right) + \left(\frac{A_t}{A_B}\right)^{-\gamma}(1-\alpha)A_B. \tag{1.6}$$

In the spirit of Morellec and Nikolov (2009), the present value of the perpetual stream of operating cash flows minus the after-tax coupon payments (i.e., *net operating cash flows* or net operating gains/losses) starting at t is:

$$\begin{aligned}\Pi(A_t;c) &= \mathbb{E}^Q\left[\int_t^\infty e^{-r(s-t)}\left[\delta_s - c(1-\theta)\right] ds\right] = \frac{\delta_t}{r-\mu} - \frac{c(1-\theta)}{r} \\ &= A_t - \frac{c(1-\theta)}{r}. \end{aligned} \tag{1.7}$$

It follows from (1.7) and (1.5) that the value of equity at time t can be expressed as:

$$E(A_t;c) = \Pi(A_t;c) - \Pi(A_B;c)\left(\frac{A_t}{A_B}\right)^{-\gamma}. \tag{1.8}$$

Note that, based on (2.3), $\Pi(A_B;c) < 0$:

$$\Pi(A_B;c) \;=\; A_B - \frac{c(1-\theta)}{r} = c(1-\theta)\left(\beta - \frac{1}{r}\right) = \frac{c(1-\theta)}{r}\left(\frac{\gamma}{\gamma+1} - 1\right) < 0.$$

Therefore, the value of equity is the sum of two terms. The first one, $\Pi(A_t;c)$, is the value of perpetual entitlement to net operating cash flows at time t. And the second one, $-\Pi(A_B;c)\left(\frac{A_t}{A_B}\right)^{-\gamma}$, is the value of equity holders' option to abandon the assets early (i.e., default), which they optimally do at A_B. The option to default creates a *positive* surplus, $-\Pi(A_B;c) > 0$, discounted back to time t by $\left(\frac{A_t}{A_B}\right)^{-\gamma}$, based on (1.2).

If the default option had no value, equity holders would be following the NPV rule and abandoning the firm as soon as the value of assets dropped to A_{NPV}, such that $\Pi(A_{NPV};c) = 0$. Equity holders would be defaulting as soon as their net worth approached zero. Based on equation (1.7), $A_{NPV} = \frac{c(1-\theta)}{r}$, and, as expected, based on equation (2.3) and $\beta = \frac{\gamma}{r(1+\gamma)}$,

$$A_{NPV} \;=\; \frac{c(1-\theta)}{r} = c(1-\theta)\beta\frac{1+\gamma}{\gamma} = A_B\frac{1+\gamma}{\gamma} > A_B.$$

The firm realizes a net operating loss if the operating cash flow, δ_t, is not sufficient to cover the after-tax cost of debt: $\delta_t < c(1-\theta)$. The value of the after-tax coupon payment, $c(1-\theta)$, marks the break-even point and, based on equation (2.2), corresponds to asset value

$$\bar{A}_D \;=\; \frac{c(1-\theta)}{r-\mu}. \tag{1.9}$$

In the baseline model of Goldstein et al. (2001) equity holders have an unlimited access to costless external financing. When $A_t \in (A_B, \bar{A}_D)$ they raise capital to cover net operating losses by issuing new equity. Because $A_B < A_{NPV} < \bar{A}_D$, even an NPV-positive firm might need access to external funds.

The tax rate, θ, and the permanent asset value loss due to default, α, are the two market frictions of the baseline structural model. I add financial constraints as the third friction.

Definition 1. *The firm is financially constrained by an exogenously set financial-constraint boundary A_D, such that $A_D \in (A_B, \bar{A}_D)$, if equity holders have no access to outside funds when $A_t \leq A_D$.*

A financially constrained firm, that is not able to access external financing and does not have any other means to cover operating losses, defaults prematurely when asset value, A_t, hits A_D. In expectation, higher A_D's translate into fewer dividends collected by equity holders and result in larger losses of equity value. Equity holders may retain net operating profits to build up cash reserves. (In the model of Goldstein et al. (2001) net operating profits are entirely paid out as dividends at the time they are realized.) Following Morellec and Nikolov (2009), I assume that, if a financially constrained firm holds an amount m in cash, its new default boundary is given by

$$\hat{A}_D(m) \quad = \quad \frac{m}{k+m}A_B + \frac{k}{k+m}A_D, \tag{1.10}$$

where $k > 0$ is a positive constant. This specification implies that, as the level of cash holdings increases, $\hat{A}_D(m)$ gets closer to the optimal default boundary, A_B. Specifically, $\hat{A}_D(0) = A_D$, $\frac{\partial \hat{A}_D(m)}{\partial m} < 0$, $\lim_{m\to\infty} \hat{A}_D(m) = A_B$ and $\lim_{m\to 0} \hat{A}_D(m) = A_D$.

Hoarding cash inside the firm is costly. As noted by Opler, Pinkowitz, Stulz, and Williamson (1999), the cost of accumulating cash includes a lower rate of return on these assets because of the liquidity premium and tax disadvantages. (In the current setup this rate would be lower than the risk-free rate, r.) The cost of retaining cash is the fourth market friction in the model. I assume that the marginal cost of the x-*th* dollar is $\zeta\frac{x}{1+x}$. The first term, $\zeta > 0$, is a positive constant that increases with the difference in the rates of return on liquid assets outside and inside the firm. The second term, $\frac{x}{1+x}$, is motivated by the fact that every extra dollar in the cash account reduces $\hat{A}_D(m)$ and, on average, earns a lower rate of return over a relatively longer time period. Given the above marginal cost, the total cost of holding m dollars of cash is:

$$\psi(m) = \zeta \int_0^m \frac{x}{1+x}dx = \zeta(m - \log(1+m)). \tag{1.11}$$

Note, that $\psi(0) = 0$.

Proposition 1. *If the firm is financially constrained by $A_D \in (A_B, \bar{A}_D)$ and at time t holds an amount m in cash, then*

- *the value of equity*[5]

$$E(A_t, m; c, A_D) = A_t - \frac{c(1-\theta)}{r}\left(1 - \left(\frac{A_t}{\hat{A}_D(m)}\right)^{-\gamma}\right) - \hat{A}_D(m)\left(\frac{A_t}{\hat{A}_D(m)}\right)^{-\gamma} - \psi(m) \quad (1.12)$$

- *and the value of debt*

$$D(A_t, m; c, A_D) = \frac{c}{r}\left(1 - \left(\frac{A_t}{\hat{A}_D(m)}\right)^{-\gamma}\right) + \left(\frac{A_t}{\hat{A}_D(m)}\right)^{-\gamma}(1-\alpha)\hat{A}_D(m). \quad (1.13)$$

For each asset value realization, A_t, equity holders identify a target level of cash holdings, $\bar{m}(A_t)$, by weighting the costs and benefits of retaining cash inside the firm. Since hoarding cash is costly, when cash holdings are high, $\bar{m}(A_t) < m$, equity holders are better off paying out the extra cash, $m - \bar{m}(A_t)$, as dividends. At time t the marginal benefit of any dollar of cash above $\bar{m}(A_t)$ is one. The marginal benefit of any dollar of cash below $\bar{m}(A_t)$ exceeds one, as it recovers an amount of equity value that exceeds unity. At $\bar{m}(A_t)$ equity holders are indifferent between retaining and distributing cash:

$$E_m(A_t, \bar{m}(A_t); c, A_D) = 1. \quad (1.14)$$

By assumption, the firm has two sources of cash: the net operating profits and the interest earned on funds in the cash account. Net operating profits represent the primary source of cash. Therefore, on average, the higher the realization of operating cash flow, δ_t, (i.e., the higher the realization of asset value, A_t) the easier it is for the firm to achieve its target level of cash inventory, $\bar{m}(A_t)$.

Proposition 2. *If the firm is financially constrained by $A_D \in (A_B, \bar{A}_D)$, then at any time t*

- *the target amount of cash holdings increases in intensity of financial constraints,*

[5] I slightly change notations for equity and debt values to include the financial-constraint boundary, A_D, and the current amount of cash holdings, m.

A_D: $\frac{\partial \bar{m}}{\partial A_D} > 0$

- *the target amount of cash holdings decreases in asset value, A_t: $\frac{\partial \bar{m}}{\partial A_t} < 0$*
- *the default boundary, $\hat{A}_D(\bar{m})$, increase in intensity of financial constraints, A_D: $\frac{\partial \hat{A}_D(\bar{m})}{\partial A_D} > 0$.*

The key implication of the first result of Proposition 2 is that *target cash holdings can be viewed as a proxy for the intensity of financial constraints.* The intuition is as follows. As discussed above, higher levels of financial constraints are associated with larger expected losses in equity value due to premature default. Larger values of equity that are up for protection increase the value of a marginal dollar of cash in the hands of equity holders and incentivize them to retain more liquid assets inside the firm. Hence, the target amount of funds in the cash account goes up.

The second result of the proposition says that, as the value of assets increases, equity holders aim at holding less cash. The intuition is that, since the financial-constraint boundary, A_D, and the amount of debt, c, are fixed, higher realizations of asset value translate into lower default risk. If the probability of facing net operating losses is low, equity holders are less inclined to hoard costly cash.

The third result is less intuitive. Based on specification (1.10), $\frac{\partial \hat{A}_D(m)}{\partial A_D} = \frac{k}{k+m} A_D > 0$, which means that higher financial constraints force the default boundary, $\hat{A}_D(\bar{m})$, up. On the other hand, since $\frac{\partial \bar{m}}{\partial A_D} > 0$, higher financial constraints result in higher target levels of cash holdings, which, based on $\frac{\partial \hat{A}_D(m)}{\partial m} = \frac{k(A_B - A_D)}{(k+m)^2} < 0$, force $\hat{A}_D(\bar{m})$ down. Proposition 2 says that, even with higher cash holdings, firms that are more financially constrained are expected to default sooner. The intuition is as following. Firms that are less financially constrained are able to reach the default boundaries, $\hat{A}_D(\bar{m})$'s, of their more constrained counterparts with less cash. Additionally, they spend *some* of the "saved" cash to push their own default boundaries even further down.

Proposition 3. *If the firm is financially constrained by $A_D \in (A_B, \bar{A}_D)$, then at any time t*

- *equity value decreases in target cash holdings*

$$\frac{\partial E(A_t, \bar{m}; c, A_D)}{\partial \bar{m}} < 0 \tag{1.15}$$

- *and, for* $A_D < \frac{\bar{m}(\alpha+\theta-\alpha\theta)+k}{(1-\alpha)k} A_B$*, debt value decreases in target cash holdings*

$$\frac{\partial D(A_t, \bar{m}; c, A_D)}{\partial \bar{m}} \;<\; 0. \tag{1.16}$$

Given Proposition 2, the intuition behind the results of Proposition 3 is as follows. First, since $\frac{\partial \bar{m}}{\partial A_D} > 0$, higher cash holdings represent firms that are more financially constrained. Since $\frac{\partial \hat{A}_D(\bar{m})}{\partial A_D} > 0$, on average, these firms default earlier. Their equity holders collect fewer dividends, which, consistent with closed-form solution (1.12), results in lower values of equity. Additionally, holding more cash is expensive, which further reduces the values of equity.

Second, based on equation (1.13), debt holders' value is comprised of two parts. The first one is the expected present value of coupon payments collected before default. As above, firms targeting higher levels of cash holdings are more constrained and, on average, default earlier. Their debt holders collect fewer coupon payments, which reduces the value of debt. However, the second part of debt value is the present value of recovered assets at default. Since α is fixed, debt holders might benefit from premature default if the gains from obtaining larger assets at default dominate the losses due to uncollected coupon payments. Debt holders lose value if $A_D < \frac{\bar{m}(\alpha+\theta-\alpha\theta)+k}{(1-\alpha)k} A_B$ (i.e., if the firm is not severely constrained). Consistent with this explanation, since $\frac{\partial}{\partial \alpha}\left[\frac{\bar{m}(\alpha+\theta-\alpha\theta)+k}{(1-\alpha)k} A_B\right] = \frac{k+m}{k(1-\alpha)^2} A_B > 0$, lower α's (i.e., smaller shares of asset values lost at default) reduce the level of financial constraints below which debt holders lose value.

In summary, based on the second result of Proposition 2, target cash holdings increase as the operating cash flows fall. The intuition is that equity holders aim at higher cash reserves in response to a higher probability of facing financial constraints. However, the proof behind this result hinges heavily on optimality condition (1.14). Since net operating cash flows are the primary source of a firm's cash holdings, on average, the lower are the cash flows the harder it is for equity holders to stay closer to their cash holding targets, dictated by condition (1.14). For instance, if the firm experiences a significant net operating loss, it will burn through its cash reserves in order to avoid immediate default, despite the fact that the marginal value of retained cash might exceed unity. The overall expectation is that, for sufficiently low realizations of operating cash flows, cash reserves will decline as the cash flows fall. One could anticipate a "humped" shape of cash holdings as a function of operating cash flows. Proposition (3) suggests that, if the firm is not severely constrained, one should anticipate a negative correlation between target cash reserves and the values of equity and debt. As above, the proof of the proposition is based on condition (1.14). When the cash flows are sufficiently low, one should observe either a weaker negative or a positive relationship between cash holdings and the values of equity and debt. On average, if operating losses are significant, higher cash reserves will translate into relative safety and higher values of equity and debt. In general, high default risk will erode the role of cash holdings as a proxy for the intensity of financial constraints.

1.3 Data

I use two separate samples of data to conduct empirical studies of corporate debt and equity in Sections 1.4 and 1.5, correspondingly. In this section I construct variables that are based on financial statement information from COMPUSTAT Fundamentals Quarterly database. The resulting series will feed both samples. The reader may skip to Section 1.4 and use the current section as a reference.

I obtain quarterly accounting data from COMPUSTAT for the time period from 1980:1 to 2010:6. I delete observations with missing, negative or zero total assets (item ATQ), total liabilities (item LTQ), cash and short-term investments (item CHEQ), sales (item SALEQ), cost of goods sold (item COGSA), equity price (item PRCCQ), and common shares outstanding (item CSHOQ). I delete firms with missing Standard Industrial Classification (SIC) codes. I further remove firms from regulated industries (SIC 4900-4999), non-operating establishments (SIC 9995), and financial firms (SIC 6000-6999), as their capital structure decisions are guided by principals different from the ones modeled in this paper. For instance, the potential too-big-to-fail (TBTF) implicit government guarantee is one reason to remove the financials. In the extreme case, a TBTF institution would be willing to issue an infinite amount of debt, pay out the proceeds as dividends and default immediately. The market would be willing to buy this debt as, due to the guarantee, it would be risk-free.

I start constructing the main variables of the sample by computing book equity, BE, as total assets (item ATQ) minus total liabilities (item LTQ) minus preferred stock (item PSTKQ) plus deferred taxes and investment tax credit (item TXDITCQ). I calculate market equity as equity price (item PRCCQ) times shares outstanding (item CSHOQ) and net market assets as total assets (item ATQ) minus book equity plus market equity minus cash and short-term investments (item CHEQ). I use net market assets to deflate firm-level variables, detailed below. The literature uses a deflator in order to reduce heteroscedasticity in the data. Often book value of total assets (item ATQ) is the deflator. Instead, I use the market value of assets, as I want the corresponding variables to incorporate more recent market information. I also subtract cash and short-term investments (item CHEQ) from the market value of assets to stay consistent with the theoretical model from Section 1.2, which separates cash holdings from the value of operating assets.

In my empirical analysis cash holdings are represented by *CASH*, defined as cash and short-term investments (item CHEQ) divided by net market assets. Operating cash flows are represented by CF, defined as EBITDA divided by net market assets, where EBITDA is sales (item SALEQ) minus cost of goods sold (item COGSQ) minus selling, general and administrative expenses (item XSGAQ). *Leverage* is the ratio of the sum of short- (item DLCQ) and long-term (item DLTTQ) debt to total assets (item ATQ). *Market-to-Book Assets* is the ratio of net market assets to total assets (item ATQ) less cash and short-term investments (item CHEQ).

In order to limit the influence of outliers, I winsorize all the variables defined in the previous paragraph, except for *CASH* and CF, at the 5th and 95th percentiles of their polled distributions across all firm-quarters. That is, I replace any observation below the 5th percentile with the 5th percentile and any observation above the 95th percentile with the 95th percentile. *CASH* and CF are winsorized at the 1st and 99th percentiles, as my analysis focuses on the tail of the distributions of these variables.

Next, I augment the data with Standard & Poor domestic long-term credit ratings from COMPUSTAT and construct two additional variables. *CF Sigma*, is the prior twelve-quarter volatility of CF, including the current quarter. *Ln*(*Real Assets*) is the natural logarithm of net market assets in 2002 dollars.

Lastly, to be consistent with the theoretical model, I drop the observations with zero *Leverage* (about 9.3% of all entries). The final data series consist of 1,200,680 firm-quarters representing the time period from 1980:1 to 2010:9.

1.4 Empirical Analysis of Corporate Debt Yields

In this section I present my empirical analysis of corporate debt data. The model from Section 1.2 suggests that, first, higher cash holdings should be observed with lower values of debt. Since the risk-free rate is fixed, this implies a positive correlation between corporate-Treasury yield spreads and cash holdings. Second, for sufficiently high realizations of operating cash flows (i.e, when default risk is low), the above positive correlation should strengthen as the cash flows decline; for sufficiently low realizations of operating cash flows (i.e., when default risk high), it should weaken or reverse. I will test both implications. I will be consistent with the model in analyzing only straight debt.

1.4.1 Debt Sample

I start with building the debt sample. I explain how I obtain and refine the transaction data on corporate bonds and how I match it with the variables based on quarterly accounting data constructed in Section 1.3.

The sample is based on trade information, which includes transaction dates, yields and volumes, from FINRA's Trade Reporting and Comprehensive Engine (TRACE) database. TRACE was introduced in July of 2002. It consolidates over-the-counter transactions for corporate bonds, including investment grade, high yield and convertible debt. By using 9-digit CUSIPs, I link TRACE data with issue information from the SDC Platinum database. Issue information allows me to separate the trades on straight bonds from the rest of the transactions. I limit the influence of outliers and winsorize the yields at the 5th and 95th percentiles of the polled distribution across all issue-dates.

The corporate debt market is very illiquid, so issues are often observed with only several trades per month. For each issue tracked in TRACE I construct one volume-weighted yield per month based on all the available transactions for that month. I record the volume-weighted yields as of the end of the corresponding months. Consolidating all the yields for the month in one number allows me to have more consistent (monthly) yield data, further reducing the impact of outliers and smoothing the difference in yields across multiple dealers.

I fit 9-th degree polynomial regressions with independent variables based on maturity to Treasury yield curve rates obtained from the Yield Book as of the end of each month in the sample. The rates are available at monthly frequency for maturities of up to 30 years. Then, at the end of each month for each issue I use the corresponding estimated polynomial coefficients and the remaining time to maturity to obtain an approximate Treasury rate. I construct corporate-Treasury yield spreads by subtracting these rates from the corresponding volume-weighted yields.

Lastly, I link the data series from Section 1.3 with the corporate-Treasury yield spreads from above. Every quarter, for each issuer I match the (quarterly) variables from the series with the yield-spreads two months after the end of the quarter, if available. I use the two-month gap to ensure that accounting variables are known before I observe the spreads they are used to explain. If a firm has yield spreads on several outstanding issues, I create an entry (with repeated data from the series) for every available yield spread. All entries with missing data from the series or yield spreads are thrown out. The resulting debt sample is an unbalanced panel of 5,556 issue-quarters representing 184 different issuers and 458 issues during the time period from 2002:8 to 2010:9.

Debt Sample Summary Statistics

I continue with summarizing some of the main properties of the debt sample. Table 1.1 shows that there are seven major industries, as defined by the two-digit SIC codes, represented in the sample. More than half of the firms, 105 out of 184 total, are from the manufacturing sector.

Table 1.1: Debt Sample Major Industries

The table lists the number of unique debt sample firms in each major industry sector, identified by two-digit SIC codes. The debt sample is based on over-the-counter transaction data for corporate straight bonds from TRACE and quarterly accounting variables from COMPUSTAT. It is an unbalanced panel of 5,556 issue-quarters representing the time period from 2002:8 to 2010:9. Financial firms (SIC 6000-6999), utilities (SIC 4900-4999), non-operating establishments (SIC 9995) are excluded.

Industry	Two-Digit SIC Codes Begin	Issuers
Mining	10	14
Construction	15	5
Manufacturing	20	105
Transportation, Communications, Electric, Gas, and Sanitary Services	40	20
Wholesale Trade	50	6
Retail Trade	52	22
Services	70	12
		Total: 184

Table 1.2 describes the distribution of the outstanding principal amounts in million dollars across issue-quarters. (All distributions presented in this section reflect the influence of cross-sectional and time-series variation.) The main observation is that the debt sample is represented by a wide range of principal amounts. The distribution, however, is slightly skewed towards smaller issues.

Table 1.2: Debt Sample Outstanding Principal Amounts

The table presents characteristics of the polled distribution of outstanding principal amounts (in million dollars) across all debt sample issues. The debt sample is based on over-the-counter transaction data for corporate straight bonds from TRACE and quarterly accounting variables from COMPUSTAT. It is an unbalanced panel of 5,556 issue-quarters representing the time period from 2002:8 to 2010:9. Financial firms (SIC 6000-6999), utilities (SIC 4900-4999), non-operating establishments (SIC 9995) are excluded.

Minimum	Maximum	Mean	S.D.	25%	Median	75%	Issues
39.4	3,750.0	373.0	413.6	150.0	250.0	400.0	458

I group the observations in the sample by S&P domestic long-term credit ratings. Table 1.3 presents the characteristics of the issues in each major credit rating category. There are several key observations. First, the sample is well represented by both, investment grade ('BBB' and above) and speculative grade ('BB' and below) bonds. Second, in each rating category the issues are relatively similar in terms of average maturities and mean outstanding principal amounts. Third, the highest credit quality 'AAA' bonds are poorly diversified across issuers - they are represented only by three firms.

Table 1.4 presents distributional characteristics of the independent variables used in the regression analysis of corporate-Treasury yield spreads in the Section 1.4.2. Panel A is based on the data in the debt sample. Panel B is based on a broader sample that matches the time period of the debt sample but uses all Section 1.3 series firms, including the ones without TRACE data. The statistics in this table are based on firm-quarters as opposed to issue-quarters used in Tables 1.2 and 1.3. Due to missing historical cash flows, not all entries have cash flow volatility values. The key observations are that, based on the distributional means, the debt sample firms hold less cash, generate larger and slightly less volatile cash flows, are slightly more leveraged, are bigger in asset size and have lower ratios of market-to-book assets compared to the firms in the broader sample.

1.4.2 Regression Analysis

As discussed at the beginning Section 1.4, the focus of my empirical analysis is on the relationship between corporate-Treasury yield spreads and observable cash holdings. Before turning to the regressions, I motivate the main results of this section by discussing some additional characteristics of the sample.

Table 1.3: Debt Sample Summary Statistics by Credit Rating

The table presents summary statistics for the debt sample issues, grouped by the major Standard & Poor domestic long-term credit ratings. The corporate-Treasury yield spread is the difference between the bond yield and the Treasury yield curve rate, that matches the remaining time to maturity of the bond on the corresponding trade date. Characteristics of the yield spread distribution (column 2-6), mean years to maturity (column 7) and mean principal (column 8) are based on all issue-quarters in the corresponding credit rating category. Issues (column 9) is the number of unique bonds and issuers (column 10) is the number of unique firms in each credit rating category across all issue-quarters. The debt sample is based on over-the-counter transaction data for corporate straight bonds from TRACE and quarterly accounting variables from COMPUSTAT. It is an unbalanced panel of 5,556 issue-quarters representing the time period from 2002:8 to 2010:9. Financial firms (SIC 6000-6999), utilities (SIC 4900-4999), non-operating establishments (SIC 9995) are excluded.

S&P Credit	Corporate-Treasury Yield Spreads (bps)					Mean Yrs	Mean Princ.	Issues	Firms	Issue-Qtrs
Rating	Mean	S.D.	25%	Median	75%	to Mat.	($ million)			(5,556)
AAA	61.8	41.4	33.1	48.5	81.7	10.7	452.5	11	3	220
AA	72.8	74.8	31.2	50.0	84.4	6.1	562.9	54	13	574
A	121.7	99.3	59.0	92.1	148.7	8.12	399.4	258	105	2,097
BBB	204.6	167.2	91.9	152.2	261.1	8.3	326.0	284	132	1,626
BB	427.0	356.3	201.3	343.7	544.6	7.2	245.5	122	70	537
B	897.3	784.3	456.2	603.9	1,101.7	10.4	317.8	79	33	439
CCC-D, NR†	2,393.7	1,563.3	984.3	2,031.6	4,093.2	9.8	272.6	39	13	63

†: not rated

Table 1.4: Independent Variables

The table reports distributional characteristics of independent variables used in regression analysis of corporate debt. Panel A represents the debt sample. Panel B represents all levered COMPUSTAT firms during the time period of the debt sample, including the ones without TRACE data and excluding regulated firms (SIC 4900-4999), financial firms (SIC 6000-6999) and non-operating establishments (SIC 9995). The characteristics are based on values pooled across firm-quarters. The variables are defined in Section 1.3. The debt sample is based on over-the-counter transaction data for corporate straight bonds from TRACE and quarterly accounting variables from COMPUSTAT. It is an unbalanced panel of 5,556 issue-quarters representing the time period from 2002:8 to 2010:9. Financial firms (SIC 6000-6999), utilities (SIC 4900-4999), non-operating establishments (SIC 9995) are excluded.

Variable	Mean	S.D.	25%	Median	75%	Firm-Quarters
Panel A: Debt Sample						
CASH	0.062	0.078	0.016	0.037	0.076	2,676
CF	0.022	0.017	0.017	0.022	0.027	2,676
CF Sigma	0.007	0.006	0.003	0.005	0.008	2,431
Leverage	0.291	0.131	0.196	0.267	0.372	2,676
Ln(*Real Assets*)	8.741	0.429	8.828	8.874	8.949	2,676
Mkt-to-Book Assets	1.754	0.886	1.154	1.498	2.066	2,676
Panel B: Broader Sample						
CASH	0.151	0.272	0.020	0.062	0.161	101,929
CF	0.013	0.033	0.005	0.019	0.031	101,929
CF Sigma	0.013	0.012	0.005	0.009	0.017	101,929
Leverage	0.245	0.187	0.0.083	0.217	0.366	101,929
Ln(*Real Assets*)	6.179	1.899	4.751	6.289	7.726	101,929
Mkt-to-Book Assets	2.053	1.668	1.019	1.432	2.303	101,929

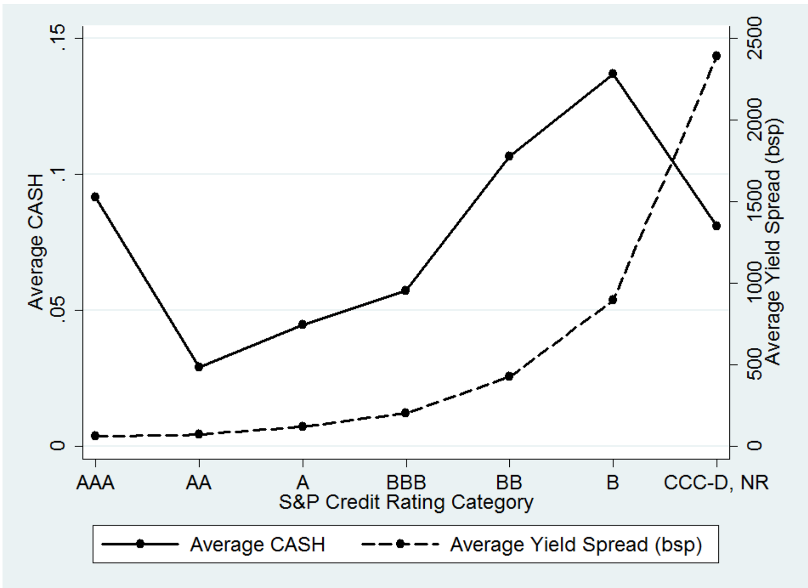
Figure 1.1 plots the average cash holdings and the average corporate-Treasury yield spreads for each major S&P long-term domestic credit rating category. Both, the average cash holdings and the average yield spreads increase as the debt credit quality falls from ’AA’ to ’B’. The common trend in the values of these variables is very explicit.

However, the average cash holdings for the firms in two credit categories, ’AAA’ and ’CCC-NR’[6], stand out. While, as expected, the average ’AAA’ corporate-Treasury yield spread is the lowest among the average spreads for all categories, on average, the firms with the highest credit rating hold more cash than their lower ranked ’AA’, ’A’ and ’BBB’ counterparts. I do not have a model-based explanation for why the observed trend in the relationship between the average cash and the average spreads

[6] NR is for ’not rated’.

Figure 1.1: Average Cash Holdings and Yield Spreads by Credit Rating

The figure plots cash holdings and corporate-Treasury yield spreads averaged across debt sample issue-quarters, grouped by the major Standard & Poor domestic long-term credit ratings. The corporate-Treasury yield spread is the difference between the bond yield and the Treasury yield curve rate, that matches the remaining time to maturity of the bond on the corresponding trade date. *CASH* is defined in Section 1.3. The debt sample is based on over-the-counter transaction data for corporate straight bonds from TRACE and quarterly accounting variables from COMPUSTAT. It is an unbalanced panel of 5,556 issue-quarters representing the time period from 2002:8 to 2010:9. Financial firms (SIC 6000-6999), utilities (SIC 4900-4999), non-operating establishments (SIC 9995) are excluded.



for 'AA'-'B' firms breaks for 'AAA' firms. Although, as noted in Section 1.4.1, the debt sample is represented by only three 'AAA' firms, so the inconsistency with the trend could be specific to these firms. As for the lowest rated and not rated firms, represented by category 'CCC-NR', again, as expected, their average corporate-Treasury yield spread is the highest among the average spreads for all the major credit rating categories. However, on average, these firms hold less cash than the ones rated 'BB' and 'B'. I view the 'CCC-NR' firms as the ones with significant default risk. Consistent with an earlier discussion, for these firms more cash translates into lower default risk. Therefore, the correlation between yield spreads and cash holdings weakens.

Next, I present my formal analysis of the relationship between cash holdings and corporate-Treasury yield spreads. I apply a pooled cross-sectional time-series OLS regression model to the debt sample data. I regress the yield spreads on the cash holdings plus additional independent variables. In all regressions parameter variances are adjusted in order to control for the dependence on unobservables within *firm* clusters. The idea is to account for the fact that in the sample large firms might be represented by multiple issues.

Table 1.5 shows the results for the full debt sample. Column 2 corresponds to the variant of the model with the cash holdings, *CASH*, as the only independent variable. The beta coefficient on *CASH* is positive and statistically distinct from zero at the 1% level.

Including additional firm and issue control characteristics (column 3), industry and year dummy variables (column 4), and a quadratic term, $CASH^2$, (column 5) improves the fit of the regression model in terms of higher R^2 values. Two additional observations consistent with the intuition based on the structural model are revealed. First, based on the negative coefficients on CF, lower realizations of operating cash flows (asset values) result in wider corporate-Treasury yield spreads. Second, based on the positive coefficients on $Leverage$, wider spreads correspond to higher leverage. Both, lower operating cash flow realizations and higher leverage increase the chance of default. On average, both result in fewer coupons payments collected by debt holders and lead to lower debt values / higher corporate-treasury yield spreads. I perform a robustness check of the above results by splitting the debt sample into equal-length halves and re-running the regression model for the two separate sub-samples. The results are presented in columns 6-13 of Table 1.5. The observations made for the whole sample are confirmed in the sub-samples.

To emphasize, the main observation is that corporate-Treasury yield spreads are positively correlated with observable cash holdings. This relationship is consistent and statistically significant across all regression specifications. The result confirms the fist implication based on the theoretical model. It holds without conditioning on the level of operating cash flows, which suggests that the role of cash as a proxy for the intensity of financial constraints dominates corporate debt data.

I move to the second implication, motivated by the theoretical discussion. First, I split the debt sample into four groups. The first group includes observations with non-positive realizations of the operating cash flow, CF. The next three groups include observations in the lowest, middle and bottom thirds of the CF distribution, based on observations with strictly positive operating cash flows. I rerun the above regression model, as specified in column 3 of Table 1.5, based on observations in each of the four groups. The results are presented in Table 1.6.

Table 1.5: Regression Analysis of Corporate-Treasury Yield Spreads

The table presents the results of the pooled cross-sectional time-series OLS regression model applied to the debt sample. Parameter variances are adjusted in order to control for the dependence on unobservables within (individual) firm clusters. The corporate-Treasury yield spread is the difference between the bond yield and the Treasury yield curve rate, that matches the remaining time to maturity of the bond on the corresponding trade date. *Maturity* is time to maturity of the bond on the corresponding trade date. The remaining independent variables are defined in Section 1.3. The debt sample is based on over-the-counter transaction data for corporate straight bonds from TRACE and quarterly accounting variables from COMPUSTAT. It is an unbalanced panel of 5,556 issue-quarters representing the time period from 2002:8 to 2010:9. Financial firms (SIC 6000-6999), utilities (SIC 4900-4999), non-operating establishments (SIC 9995) are excluded.

Dependent variable: *Corporate-Treasury Yield Spread*

Model (pooled, OLS, clustered)			w/t dummy for				w/t dummy for				w/t dummy for	
Independent variable			*industry, year*				*industry, year*				*industry, year*	
	full sample: 2002:8 - 2010:9				sub-sample: 2002:8 - 2006:8				sub-sample: 2006:9 - 2010:9			
Intercept	0.015	0.086	n/a	n/a	0.010	0.20	n/a	n/a	0.023	0.101	n/a	n/a
	(6.09)**	(1.39)			(4.34)**	(0.45)			(6.76)**	(1.24)		
CASH	0.159	0.124	0.118	0.269	0.120	0.117	0.116	0.206	0.154	0.105	0.092	0.262
	(3.50)**	(3.34)**	(3.29)**	(4.17)**	(2.29)*	(2.70)**	(2.67)**	(2.51)*	(3.25)**	(2.32)*	(2.25)*	(3.25)**
CF		-0.567	-0.571	-0.532		-0.870	-0.844	-0.778		-0.382	-0.386	-0.373
		(3.00)**	(3.03)**	(3.01)**		(2.00)*	(2.00)*	(1.97)*		(1.89)	(1.96)*	(2.03)*
CF Sigma		0.034	0.117	0.133		0.824	0.756	0.698		-0.093	0.199	0.140
		(0.09)	(0.32)	(0.38)		(1.85)	(1.64)	(1.55)		(0.19)	(0.48)	(0.35)
Leverage		0.083	0.081	0.084		0.078	0.078	0.080		0.089	0.085	0.088
		(2.59)*	(2.65)**	(2.78)**		(1.69)	(1.70)	(1.71)		(2.40)*	(2.44)*	(2.71)**
Ln(Real Assets)		-0.008	-0.009	-0.013		-0.001	-0.002	-0.003		-0.008	-0.009	-0.014
		(1.17)	(1.50)	(2.10)*		(0.25)	(0.44)	(0.75)		(1.00)	(1.09)	(1.92)
Market-to-Book Assets		-0.007	-0.007	-0.007		-0.004	-0.004	-0.004		-0.011	-0.012	-0.011
		(4.10)**	(4.31)**	(4.32)**		(2.72)**	(2.72)**	(2.66)**		(3.60)**	(4.12)**	(4.34)**
Maturity		0.001	0.001	0.001		0.001	0.001	0.001		0.001	0.001	0.001
		(2.38)*	(2.29)*	(1.95)		(1.87)	(1.97)*	(1.87)		(1.24)	(2.48)*	(1.32)
$CASH^2$				-0.468				-0.332				-0.480
				(3.40)**				(1.83)				(2.85)**
R^2	0.066	0.229	0.426	0.440	0.032	0.217	0.341	0.345	0.071	0.234	0.513	0.529
F-test	12.27	12.19	26.81	25.62	5.24	5.19	11.28	10.46	10.54	12.76	36.26	35.94
Observations	5,556	5,045	5,045	5,045	2,981	2,692	2,692	2,692	2,575	2,353	2,353	2,353

**,*: statistically distinct from zero at the 1 and 5 % level, respectively

Table 1.6: Regression Analysis of Corporate-Treasury Yield Spreads

The table presents the results of the pooled cross-sectional time-series OLS regression model applied to the debt sample split into four groups, based on the values of the operating cash flow, CF. Parameter variances are adjusted in order to control for the dependence on unobservables within (individual) firm clusters. The corporate-Treasury yield spread is the difference between the bond yield and the Treasury yield curve rate, that matches the remaining time to maturity of the bond on the corresponding trade date. $Maturity$ is time to maturity of the bond on the corresponding trade date. The remaining independent variables are defined in Section 1.3. The debt sample is based on over-the-counter transaction data for corporate straight bonds from TRACE and quarterly accounting variables from COMPUSTAT. It is an unbalanced panel of 5,556 issue-quarters representing the time period from 2002:8 to 2010:9. Financial firms (SIC 6000-6999), utilities (SIC 4900-4999), non-operating establishments (SIC 9995) are excluded.

Dependent variable: *Corporate-Treasury Yield Spread*
Model (pooled, OLS, clustered; with dummy for industry, year)
Independent variable

Full sample: 2002:8 - 2010:9

	$CF \leq 0$	$CF > 0$		
		bottom 1/3	middle 1/3	top 1/3
CASH	-0.057	0.152	0.125	0.082
	(0.87)	(2.76)**	(2.30)*	(2.18)*
CF	-0.345	-3.370	0.309	-0.297
	(1.41)	(1.18)	(1.05)	(1.54)
CF Sigma	0.284	-0.903	0.620	1.26
	(0.28)	(1.72)	(1.31)	(2.23)*
Leverage	0.258	0.113	0.050	0.051
	(2.09)*	(2.66)**	(2.27)*	(1.38)
Ln(Real Assets)	-0.011	-0.013	-0.014	-0.006
	(0.76)	(1.86)*	(1.93)	(0.89)
Market-to-Book Assets	-0.041	-0.006	-0.004	-0.004
	(0.93)	(3.63)**	(2.90)**	(3.13)**
Maturity	-0.001	0.001	0.001	0.001
	(0.83)	(1.40)	(1.93)	(0.86)
R^2	0.791	0.469	0.487	0.347
F-test	28.00	20.22	25.07	45.18
Observations	182	1,640	1,656	1,577
Issues	85	320	335	312
Firms	31	128	138	134

**,*: statistically distinct from zero at the 1 and 5 % level, respectively

There are three key observations. First, when operating cash flows are positive the beta coefficient on *CASH* is positive and statistically significant (columns 3-5). When default risk is low, the correlation between corporate-Treasury yield spreads and cash holdings is positive. Second, the beta coefficients increase as one moves from observations with the highest CF to the ones with the lowest, positive CF. When default risk is low, the correlation between corporate-Treasury yield spreads and cash holdings increases and the cash flows fall. Finally, although not statistically significant, the beta coefficient on *CASH* when CF is non-positive is negative. When default risk is high, the positive correlation between corporate-Treasury yield spreads and cash holdings reverses.

Technology Firms and Bank Lines of Credit

I repeat the empirical analysis of corporate-Treasury yield spreads based on a reduced debt sample that does not include technology firms. Bardhan, Jaffee and Kroll (2004) identify firms with the following SIC codes as the ones representing high tech: 3571, 3572, 3575, 3577, 3578, 3661, 3671, 3672, 3674-3679, 3825, 7371-7376, 7379. Excluding this kind of firm from the analysis is motivated by a well-discussed phenomenon in the popular press that large technology firms hold substantial amounts of cash as a percentage of their total assets. Some may argue that, in addition to holdings a lot of cash, technology firms are subject to higher market risk. Therefore, the observed correlation between cash holdings and yield spreads and equity returns (Section 1.5.2) cannot be attributed to financial constraints. My work shows that excluding technology firms from consideration has almost no effect on the corresponding results.

According to theoretical literature on bank lines of credit (e.g., Boot, Thakor, and Udell (1987), Holmstrom and Tirole (1998), Martin and Santomero (1997)), credit lines should resolve the capital market frictions that motivate firms to hold cash as a liquidity buffer. Additionally, Kashyap, Rajan and Stein (2002) and Gatev and Strahan (2006) argue that banks are the most efficient liquidity providers in the economy, which also suggests that firms should rely on lines of credit over internal cash. I augment the debt sample with data on unused credit line capacity, based on information collected directly from annual 10-K SEC fillings. I repeat the analysis of corporate-Treasury yield spreads by replacing cash holdings with the amount of available credit line capacity divided by net market value of assets. My findings indicate that there is no correlation between the levels of unused credit line capacity and corporate-Treasury yield spreads.

In summary, the results presented in Section 1.4 as a whole are consistent with the intuition motivated by the theoretical discussion. In the presence of financial constraints, when operating cash flows are sufficiently high and default risk is low, firms that are more financially constrained hoard more cash. Since hoarding cash is costly, even optimal cash reserves cannot completely eliminate the exposure to financial constraints. Firms that are more constrained target higher cash reserves but remain riskier. Therefore, on average, one observes a positive correlation between corporate-Treasury yield spreads and cash holdings. As the operating cash flows fall, the probability of running into financial constraints increases. This results in both higher default risk and a stronger response on behalf of equity holders in terms of hoarding more cash. The correlation between yield spreads and cash holdings increases. When default risk becomes significantly high, the correlation weakens. Constrained firms are forced to burn through their cash reserves in order to avoid immediate closure. Higher cash holdings represent safer, high default risk firms.

1.5 Empirical Analysis of Equity Returns

In Section 1.4 I have shown that financial constraints are priced in corporate debt markets: financial constraints, proxied by cash holdings, are correlated with corporate debt yields. In this section I explore whether financial constraints are priced in equities. In other words, do financial constrains affect stock returns? And if so, is exposure to financial constraint risk diversifiable? I inherit these questions from a small literature on financial constraint risk, represented by Lamont et al. (2001) and White et al. (2006). I structure my analysis similar to the way it is done in these two papers. The only difference between my work and the work of the above authors is that I use cash holdings, instead of more comprehensive indexes, to proxy for the intensity of exposure to financial constraints. More specifically, I investigate whether cash holdings are positively related to average returns on common equity in the cross-section. Additionally, I analyze whether cash holdings proxy for a source of *common* time-series variation in stock returns. In the terminology of asset pricing, I analyze whether there is a risk factor in stock returns associated with financial constraints, as represented by cash holdings.

1.5.1 Equity Sample

I start with building the equity sample. I explain how I merge the series constructed in Section 1.3 based on quarterly accounting information from COMPUSTAT with monthly data from the Center for Research in Security Prices (CRSP), introduce new variables, and summarize some of the main properties of the sample.

First, when applicable, I change the dates on the entries in Section 1.3 series so that they appear as of the end of the nearest calendar quarter. For instance, all entries as of 2000:11 are reassigned to 2000:12. Second, I identify the firms from COMPUSTAT with the ones from CRSP by using permanent number codes from the linking table of the CRSP/COMPUSTAT Merged Database. Lastly, I merge Section 1.3 quarterly variables with monthly prices, returns (simple returns that include dividends) and shares outstanding for ordinary common equity from CRSP. I assign the values of the variables to the three months immediately following the end of the corresponding (calendar) quarter. For example, I assign values representing the quarter ending in March of any sample year to the months of April, May and June of the same year. Then, I use the codes from the second step to link these data with the corresponding monthly stock data from CRSP. This matching approach insures that variables based on accounting data are known before the returns they are used to explain.

I introduce additional variables. Size, ME, is measured by market capitalization (equity price times shares outstanding) on a quarterly basis and matches the timing of the corresponding accounting variables. Book-to-market equity, BE/ME, is the ratio of book equity, BE, to (quarterly) market capitalization. Following Campbell, Hilscher and Szilagyi (2008), I replace negative book-to-market ratios (about 1.5% of all firm-months) with $1 to make sure that the values are in the right (not left) tail of the distribution. The interaction term between size and book-to-market equity, $ME/BEME$, is the ratio of size to book-to-market equity. $Momentum$ is defined by the prior twelve-month mean return, excluding the latest month. As before, in order to limit the influence of the outliers I winsorize the newly constructed variables at the 5th and 95th percentiles of their pooled distributions across all firm-months. The definitions of size and book-to-market equity are consistent with Fama and French (1992), the interaction term is consistent with Daniel and Titman (1997), and $Momentum$ - with Carhart (1997).

The final equity sample spans the months from 1980:1 to 2010:9 and includes 1,200,680 firm-month observations. Table 1.7 reports selected summary statistics for the sample. The focus is on cash holdings, *CASH*, for which Figure 1.2 graphs the evolution of mean and median values, summarizing the cross-sectional and the time series dimensions for the corresponding years. Consistent with the observation of Bates, Kahle and Stulz (2009), the cash holdings have grown significantly since 1980. In the equity sample the average *CASH* has more than doubled from 0.0730 in 1980 to 0.1658 in 2010. At the same time, other variables, including $Leverage$, CF and $CF\,Sigma$, that play key roles in equity pricing have remained relatively stable over the years. Another observation is that *CASH* peaked during the most recent recessions of 2001:3-2001:10 and 2007:12-2009:6. It has shown a significant uprise during the sub-prime mortgage crisis years of 2007-2008.

Table 1.7: Equity Sample Summary Statistics

The table reports the number of unique firms and summaries of their selected characteristics for each year in the equity sample. Distributional characteristics of *CASH* and means of *Leverage*, *CF* and *CF Sigma* are based on values pooled across all firm-quarters for the corresponding year. The variables are defined in Section 1.3. The equity sample is based on monthly stock returns, equity prices and common shares outstanding from CRSP and quarterly accounting variables from COMPUSTAT. Only firm-quarters with positive book values of total debt are included. Financial firms (SIC 6000-6999), utilities (SIC 4900-4999), non-operating establishments (SIC 9995) are excluded. The final sample represents the time period from 1980:1 to 2010:9.

Year	*CASH*					*Leverage*	*CF*	*CF Sigma*	Firms
	Mean	S.D.	25%	Median	75%	Mean	Mean	Mean	
1980	0.0730	0.1210	0.0176	0.0378	0.0836	0.2682	0.0338	0.0148	1,785
1981	0.0756	0.1305	0.0165	0.0385	0.0860	0.2557	0.0311	0.0150	1,746
1982	0.0971	0.1782	0.0153	0.0416	0.1083	0.2612	0.0223	0.0153	2,386
1983	0.0911	0.1610	0.0147	0.0429	0.1058	0.2551	0.0171	0.0158	3,124
1984	0.1069	0.1895	0.0153	0.0474	0.1229	0.2484	0.0180	0.0156	3,478
1985	0.0999	0.1838	0.0140	0.0412	0.1088	0.2638	0.0153	0.0163	3,585
1986	0.1018	0.1875	0.0137	0.0427	0.1134	0.2690	0.0123	0.0166	3,535
1987	0.1057	0.1942	0.0133	0.0424	0.1176	0.2753	0.0127	0.0170	3,667
1988	0.1180	0.2295	0.0130	0.0413	0.1227	0.2819	0.0152	0.0172	3,847
1989	0.1013	0.2071	0.0106	0.0341	0.1049	0.2876	0.0149	0.0170	3,673
1990	0.1036	0.2192	0.0104	0.0335	0.1036	0.2904	0.0154	0.0169	3,602
1991	0.0986	0.1989	0.0105	0.0354	0.1040	0.2831	0.0144	0.0170	3,540
1992	0.1047	0.2017	0.0113	0.0391	0.1127	0.2601	0.0152	0.0164	3,577
1993	0.0966	0.1824	0.0109	0.0367	0.1063	0.2489	0.0142	0.0156	3,661
1994	0.1012	0.1894	0.0103	0.0368	0.1127	0.2431	0.0144	0.0144	4,028
1995	0.0914	0.1881	0.0097	0.0310	0.0957	0.2485	0.0146	0.0139	4,302
1996	0.0927	0.1805	0.0099	0.0330	0.1029	0.2463	0.0139	0.0140	4,414
1997	0.1039	0.2028	0.0104	0.0353	0.1104	0.2469	0.0118	0.0140	4,855
1998	0.1164	0.2337	0.0101	0.0359	0.1177	0.2610	0.0105	0.0141	4,974
1999	0.1107	0.2290	0.0104	0.0338	0.1083	0.2730	0.0127	0.0147	4,725
2000	0.1072	0.2302	0.0113	0.0354	0.1017	0.2605	0.0121	0.0155	4,530

1.5.2 Correlation between Cash Holdings and Stock Returns

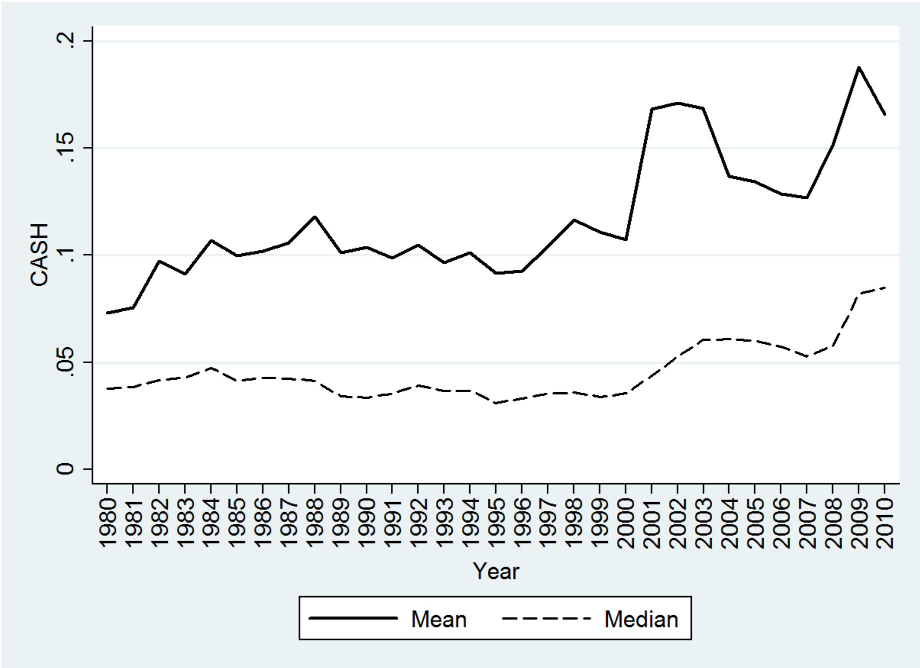
I analyze the relationship between cash holdings and returns on common equity in two steps. First, I investigate how individual cash holdings are associated with average stock returns on a cross-sectional basis. Then, I form portfolios of firms, grouped by different areas of the cash holding distribution, and study how average returns on these portfolio are related to average cash holdings of the corresponding firms.

Cross-Sectional Analysis of Equity Returns

I examining whether any variation in average stock returns is related to *CASH* and whether higher cash holding firms earn higher returns on a cross-sectional basis. I follow Whited et al. (2006) and regress individual equity returns in excess of one-month

Figure 1.2: Evolution of *CASH*

The figure plots values of *CASH*, defined in Section 1.3. The equity sample represents the time period from 1980:1 to 2010:9.



Treasury Bill yield lagged one month on size, book-to-market equity, momentum, the interaction term between size and book-to-market equity, and *CASH*. Excess returns are regressed directly on firm characteristics instead of the corresponding betas. As argued by Whited et al. (2006), the advantage of using the characteristics is that, compared to the betas, they are much more precisely measured. The disadvantage is that it is more difficult to assign economic meaning to the estimated coefficients. However, for the purpose of the analysis in this section it is sufficient to know the signs and the statistical significance of the corresponding coefficients.

The choice of independent variables is justified by the literature. Fama and French (1992, 1996) examine multiple characteristics, including size, leverage, past returns, dividend-yield, earnings-to-price ratios and book-to-market equity, and conclude that, with the exception of momentum, the cross-sectional variation in expected returns can be explained by only two of these characteristics, size and book-to-market equity. Beta, the traditional CAPM measure of risk, explains almost none of the cross-sectional dispersion in expected returns once size is taken into account. The use of the interaction term is motivated by Daniel and Titman (1997). They argue that a simple linear or log-linear regression of excess returns on size and book-to-market equity might not be sufficient to characterize the observed stock returns. The likely sign on the interaction is negative. Smaller-sized or higher book-to-market equity firms are expected to earn higher returns.

The Fama and MacBeth (1973) technique is used to compute the means of the time series of the regression coefficients. Positive, statistically significant loadings on *CASH* would mean that, on average, firms with higher cash holdings earn higher returns and the difference in returns for high and low cash holding firms is statistically significant.

I run cross-sectional regressions of excess returns on the above firm characteristics for each month in the equity sample, 1980:1-2010:9. Table 1.8 reports the time series means and t-statistics for the estimated coefficients for the full sample and two sub-samples, 1980:1-1995:4 and 1995:5-2010, representing the equal-length halves of the full sample.

Model 1 is a simple regression of excess returns on size. The sign of the beta coefficient on size is positive, although, the estimated value is not statistically significant with a t-statistic of only 0.01. Model 2 is a regression of excess returns on book-to-market equity. Here the coefficient is positive and statistically significant. As expected, on average, firms with higher values of book-to-market equity earn higher returns. In Model 3 excess returns are regressed on both, size and book-to-market equity. The results are similar to the ones for the univariate regressions. Adding momentum in Model 4 reveals that the momentum effect is positive although not statistically significant with a t-statistic of 0.18. Model 5 includes the interaction term between size and book-to-market equity. Contrary to what was expected, the coefficient on the interaction term is positive and statistically significant.

The focus of the analysis is on model specifications 6 and 7, which add *CASH* to regressions 4 and 5. The loadings on cash holdings are positive and statistically significant, independent of whether the interaction term is included or not. Adding *CASH* does not significantly change neither the coefficients for the other variables nor their statistical significance. I re-run models 6 and 7 for the two the sub-samples. The observed differences in the values of the average coefficient estimates and their statistical significance between the full sample and the two sub-samples are relatively small. They could be viewed as an artifact of differences in coefficient stability within each of the time periods.

Table 1.8: Cross-Sectional Regressions of Excess Returns on Firm Characteristics

The table reports results for month-by-month cross-sectional regressions of excess returns on firm characteristics based on equity sample data. The Fama and MacBeth (1973) technique is used to compute the means of the time-series of regression coefficients. The time-series t-statistics are reported in parentheses. Excess returns are stock returns in excess of one-month Treasury Bill yields lagged one month. Size, ME, is measured by market capitalization (equity price times shares outstanding). Book-to-market equity, BE/ME, is the ratio of book equity to market capitalization. The interaction term between size and book-to-market equity, $ME/BEME$, is the ratio of size to book-to-market equity. $Momentum$ is the prior twelve-month mean return, excluding the latest month. The equity sample spans the months from 1980:1 to 2010:9.

Specification	Size (ME)	Book-to-Market (BE/ME)	$ME/BEME$	$Momentum$	$CASH$
Full Sample: January, 1980-September, 2010					
Model 1	0.0001 (0.01)				
Model 2		0.0083 (8.34)**			
Model 3	0.0001 (0.54)	0.0083 (8.27)**			
Model 4	0.0001 (0.59)	0.0076 (9.68)**		0.0042 (0.18)	
Model 5	-0.0001 (1.24)	0.0077 (9.77)**	0.0001 (2.52)*	0.0040 (0.18)	
Model 6	0.0001 (0.79)	0.0070 (8.36)**		0.0043 (0.19)	0.0114 (5.67)**
Model 7	-0.0001 (1.01)	0.0071 (8.44)**	0.0001 (2.39)*	0.0042 (0.19)	0.0114 (5.67)**
Sub-Sample: January, 1980-April, 1995					
Model 6	0.0001 (0.83)	0.0090 (8.06)**		0.0218 (0.89)	
Model 7	-0.0001 (0.94)	0.0092 (8.20)**	0.0001 (2.31)*	0.0215 (0.87)	0.0114 (7.00)**
Sub-Sample: May, 1995-September, 2010					
Model 6	-0.0001 (0.03)	0.0051 (4.07)**		-0.0129 (0.35)	
Model 7	-0.0001 (0.42)	0.0051 (4.07)**	0.0001 (1.45)	-0.0129 (0.35)	0.0115 (3.09)**

**,*: statistically distinct from zero at the 1 and 5 % level, respectively

The main observation, based on the results from Table 1.8, is that the average coefficient estimates for *CASH* in the corresponding specifications are positive and strongly statistically significant. Based on the cross-sectional analysis of equity returns, cash holdings are positively correlated with average excess returns. Based on the entire equity sample, firms with higher cash holdings earn a positive, significantly different from zero premium.

Next, each month I rank the firms in the equity sample by the level of their operating cash flows, CF, and, based on these rankings, sort the firms into different groups. The groups contain stocks in the percentiles of 0 to 5, 5 to 10, 10 to 20, 20 to 30, 30 to 40, 40 to 50, 50 to 60, 60 to 70, 70 to 80, 80 to 90, 90 to 95, and 95 to 100 of the cash flow distribution. I rerun the above regressions, as specified in model 7, using the firms in each of these groups separately. Table 1.9 reports the corresponding average beta coefficients on *CASH*. Figure 1.3 plot these coefficients.

There are several observations. First, the average betas on cash holdings for the firms with the highest operating cash flows, in CF9095-CF9500, are not significantly different from zero. For highly profitable firms the probability of facing financial constraints is minimal. The link associated with financial constraint risk that ties together cash holding decisions and equity returns is missing. Second, the average betas increase as the cash flows fall from CF9095 to CF1020. Declining operating cash flows increase the probability of facing financial constraints. Equity holders respond by building higher cash reserves. But, since hoarding cash is costly, even in the presence of optimal cash holdings, the default risk goes up and leads to higher risk premiums. This dual effect strengthens the correlation between cash holdings and stock returns. Third, the average beta on cash holdings drops significantly for the stocks with the lowest cash flows, in CF0005. Among the firms with high levels of default risk, the ones that hold more cash are safer and offer a relatively lower risk premium. This effect reduces the correlation between cash and equity returns.

Finally, I repeat the empirical analysis of the cross-section of equity returns based on a reduced equity sample that excludes technology firms, as identified in Section 1.4.2. As in the case of corporate-Treasury yield spreads, my work shows that excluding tech firms from consideration has no effect on the corresponding results.

Average Returns and Firm Characteristics for *CASH*-Ranked Portfolios

I study the relationship between average returns on portfolios of firms, representing different areas of the cash holdings distribution, and average values of *CASH* and other characteristics of the corresponding firms.

Table 1.9: Average Loadings on *CASH* Characteristic

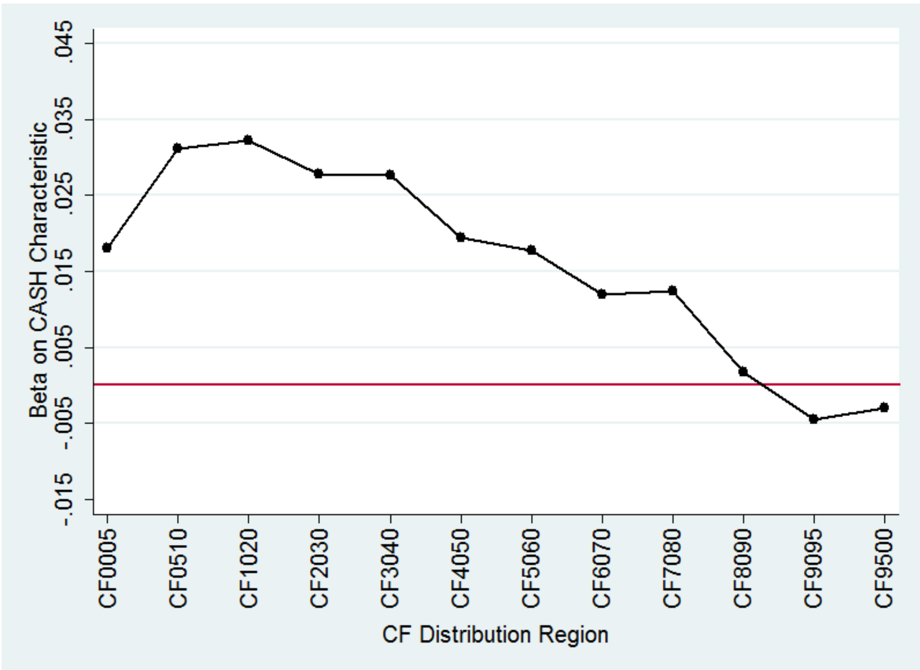
The table reports average beta coefficients on cash holdings, *CASH*, for month-by-month cross-sectional regressions of excess stock returns on firm characteristics, including size, book-to-market equity, an interaction term between size and book-to-market equity, momentum and cash holdings. The coefficients are reported for firms in different areas of the cash flow distribution based on monthly rankings. For instance, CF0005 is represented by firms with the lowest 5 percent of operating cash flows, CF. The Fama and MacBeth (1973) technique is used to compute the means of the time-series of regression coefficients. The time-series t-statistics are reported in parentheses. Excess returns are stock returns in excess of one-month Treasury Bill yields lagged one month. *CASH* and CF are defined in Section 1.3. Size, ME, is measured by market capitalization (equity price times shares outstanding). Book-to-market equity, BE/ME, is the ratio of book equity to market capitalization. The interaction term between size and book-to-market equity, $ME/BEME$, is the ratio of size to book-to-market equity. $Momentum$ is the prior twelve-month mean return, excluding the latest month. The equity sample spans the months from 1980:1 to 2010:9.

CF Distrib. Region	CF0005	CF0510	CF1020	CF2030	CF3040	CF4050	CF5060	CF6070	CF7080	CF8090	CF9095	CF9000
Beta on *CASH*	0.0180	0.0312	0.0322	0.0278	0.0276	0.0194	0.0178	0.0119	0.0124	0.0017	-0.0045	-0.0030
	(5.82)**	(6.15)**	(7.62)**	(6.79)**	(5.75)**	(3.91)**	(3.56)**	(2.64)*	(3.22)**	(0.42)	(1.19)	(1.46)
Average CF	-0.0823	-0.0416	-0.0151	0.0030	0.0125	0.0185	0.0234	0.0282	0.0336	0.0424	0.0573	0.0662

**,*: statistically distinct from zero at the 1 and 5% level, respectively

Figure 1.3: Average Loadings on *CASH* Characteristic

The figure plots the average beta coefficients on cash holdings, *CASH*, presented in Table 1.9. The coefficients are based on month-by-month cross-sectional regressions of excess stock returns on firm characteristics, including size, book-to-market equity, an interaction term between size and book-to-market equity, momentum and cash holdings. The coefficients are reported for firms in different areas of the cash flow distribution based on monthly rankings. For instance, CF0005 is represented by firms with the lowest 5 percent of operating cash flows, CF. The Fama and MacBeth (1973) technique is used to compute the means of the time-series of regression coefficients. The time-series t-statistics are reported in parentheses. Excess returns are stock returns in excess of one-month Treasury Bill yields lagged one month. *CASH* and CF are defined in Section 1.3. Size, ME, is measured by market capitalization (equity price times shares outstanding). Book-to-market equity, BE/ME, is the ratio of book equity to market capitalization. The interaction term between size and book-to-market equity, $ME/BEME$, is the ratio of size to book-to-market equity. $Momentum$ is the prior twelve-month mean return, excluding the latest month. The equity sample spans the months from 1980:1 to 2010:9.



Each month I rank the firms in the equity sample by *CASH*. (Since cash holdings change on a quarterly basis, given the way the equity sample was constructed, the rankings remain the same for three consecutive months immediately after the end of each quarter.) Then, I form 12 equal-weighted and value-weighted portfolios of stocks that fall into different regions of the cash holding distribution. Consistent with the literature, the value-weighted portfolios are based on market capitalizations as of the end of the previous months. The portfolios contain stocks in the percentiles of 0 to 5, 5 to 10, 10 to 20, 20 to 30, 30 to 40, 40 to 50, 50 to 60, 60 to 70, 70 to 80, 80 to 90, 90 to 95, and 95 to 100 of the cash holding distribution. This portfolio construction procedure pays a greater attention to the tails of the distribution (i.e., to the firms with the least and the most cash). The equal-weighted portfolios correspond to the equal roles of firms in *CASH* rankings, while the value-weighted portfolios correspond to more realistic investment opportunities.

Tables 1.10 and 1.11 present the results for return analysis of the above portfolios. Panels A report average returns in excess of one month Treasury Bill yields lagged one month and alphas with respect to the CAPM, the three-factor Fama and French (1993), and the four-factor Carhart (1997) models. Panels B report estimated factor loadings of the excess returns on the three Fama-French (1993) factors. The mean excess returns and alphas (only) are presented in annualized percentage points. The corresponding t-statistics are shown in the parentheses below the estimates. The models are estimated using monthly return series for the standard factor-mimicking portfolios for the market, $R_M - R_f$, size, SMB, value, HML, and momentum, UMD, from the website of Professor Kenneth French. Panels C report standard deviations and skewness of the portfolio excess returns. Finally, Figures 1.4-1.5 graphically summarize the behavior of the mean excess returns, alphas and factor loadings across the corresponding 12 portfolios.

The average excess returns for the equal-weighted portfolios in Table 1.10 and Figure 1.4 are strongly and almost monotonically increasing in cash holdings starting with the portfolio of stocks in the second lowest 5% of *CASH* (0510). The average excess return for this portfolio is 5.53% per year and the average excess return for the portfolio of stocks in the highest 5% of *CASH* (9900) is 22.55% per year.

An uptrend in average excess returns for the corresponding value-weighted portfolios in Table 1.11 and Figure 1.6 is also noticeable. Although, the increase in returns is not monotonic and the difference in mean excess returns for portfolios 0510 and 9500 is smaller, with 5.57% and 9.89% per year, correspondingly. The overall positive relationship between mean excess returns and cash holdings is weaker for the value-weighted portfolios because of the mismatch between equal-weighting of stocks in *CASH* rankings and value-weighting of their returns in the computations of portfolio returns.

Table 1.10: Returns on *CASH*-Sorted, Equal-Weighted Stock Portfolios

The table reports mean excess returns; alphas for the CAPM, Fama-French (1993) three-factor and Carhart (1997) four-factor models; and, loading on the market, $R_M - R_f$, size, SMB, and value, HML, factors of the Fama-French (1993) three-factor model for 12 *CASH*-sorted, equal-weighted portfolios. The mean excess returns and alphas are presented in annualized percentage points. T-statistics are reported in parentheses. At the end of each calendar quarter all equity sample stocks are sorted by *CASH* and divided into portfolios based on percentile cutoffs. For instance, 0510 is a portfolio of firms with cash holdings in percentiles 5 to 10 of the *CASH* distribution. The equity sample spans the months from 1980:1 to 2010:9.

Portfolios	0005	0510	1020	2030	3040	4050	5060	6070	7080	8090	9095	9000
Panel A: Portfolio Excess Returns and Alphas (%, annualized)												
Mean Ex.	5.53	2.03	5.11	4.38	7.13	8.37	9.54	11.01	12.84	15.03	18.46	22.55
Return	(1.38)	(0.53)	(1.35)	(1.16)	(1.81)	(2.06)*	(2.28)*	(2.50)*	(2.71)**	(2.97)**	(3.33)**	(3.98)**
CAPM	-1.26	-4.67	-1.96	-2.70	-0.33	0.59	1.48	2.53	3.81	5.70	8.25	12.85
Alpha	(0.50)	(2.08)*	(0.99)	(1.40)	(0.18)	(0.29)	(0.70)	(1.12)	(1.52)	(1.95)*	(2.54)*	(3.38)**
3-Factor	-3.05	-7.59	-4.64	-5.05	-2.59	-1.36	-0.13	1.23	3.23	5.44	8.50	13.31
Alpha	(1.50)	(4.74)**	(3.32)**	(3.66)**	(1.83)	(0.97)	(0.10)	(0.86)	(1.93)	(2.83)**	(3.77)**	(4.43)**
4-Factor	-0.39	-6.38	-3.09	-2.97	-0.46	0.76	1.97	3.57	6.31	9.24	12.65	19.62
Alpha	(0.20)	(3.98)**	(2.27)*	(2.31)*	(0.35)	(0.59)	(1.56)	(2.68)**	(4.21)**	(5.47)**	(6.18)**	(7.18)**
Panel B: 3-Factor Regression Coefficients												
$R_M - R_f$	0.945	1.001	1.044	1.043	1.062	1.072	1.078	1.104	1.130	1.112	1.175	1.066
	(24.13)**	(32.13)**	(38.63)**	(38.93)**	(38.83)**	(40.24)**	(41.86)**	(40.57)**	(36.19)**	(31.33)**	(28.56)**	(19.42)**
SMB	0.806	0.843	0.736	0.723	0.772	0.816	0.870	0.909	0.947	1.081	1.116	1.048
	(14.20)**	(18.52)**	(18.79)**	(18.62)**	(19.49)**	(21.13)**	(23.31)**	(23.06)**	(20.92)**	(21.02)**	(18.72)**	(13.17)**
HML	0.205	0.420	0.376	0.322	0.287	0.228	0.158	0.097	-0.033	-0.105	-0.193	-0.214
	(3.48)**	(8.88)**	(9.26)**	(8.00)**	(6.97)**	(5.70)**	(4.09)**	(2.38)*	(0.71)	(1.98)*	(3.12)**	(2.59)**
Panel C: Additional Distributional Characteristics												
Sigma	0.062	0.060	0.058	0.059	0.061	0.062	0.064	0.067	0.071	0.075	0.081	0.82
Skewness	0.022	-0.391	-0.565	-0.546	-0.546	-0.566	-0.518	-0.4358	-0.255	-0.021	0.153	0.743

**,*: statistically distinct from zero at the 1 and 5% level, respectively

Figure 1.4: Mean Excess Returns and Alphas of *CASH*-Sorted, Equal-Weighted Portfolios

The figure plots annualized mean excess returns and alphas for the CAPM, Fama-French (1993) three-factor and Carhart (1997) four-factor models for 12 *CASH*-sorted, equal-weighted portfolios. At the end of each calendar quarter all equity sample stocks are sorted by *CASH* and divided into portfolios based on percentile cutoffs. For instance, 0510 is a portfolio of firms with cash holdings in percentiles 5 to 10 of the *CASH* distribution.

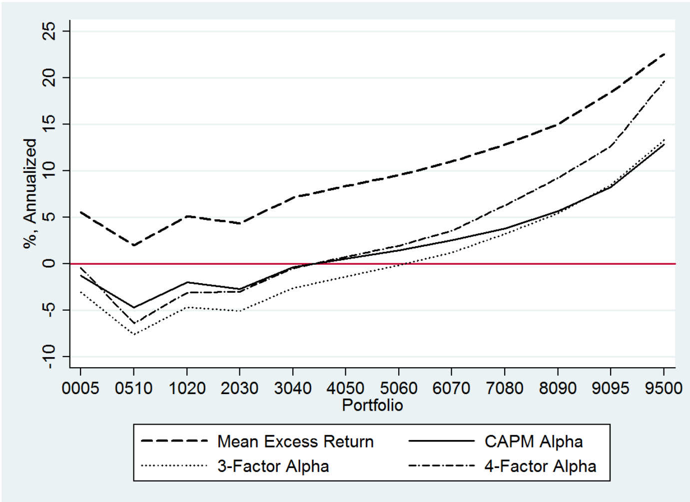
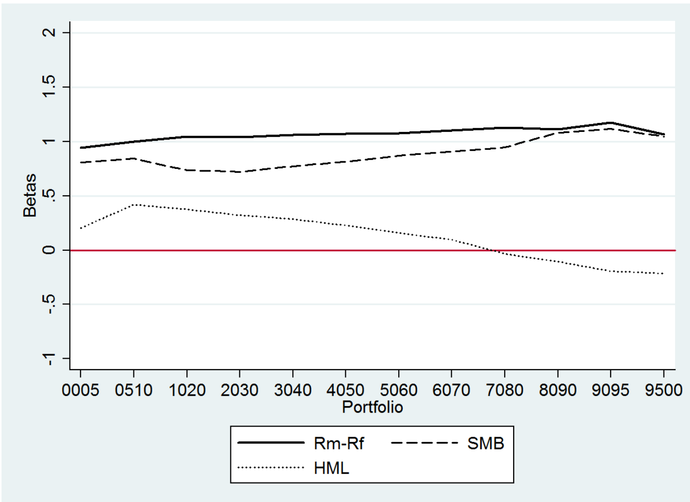


Figure 1.5: Factor Loadings of *CASH*-Sorted, Equal-Weighted Portfolios

The figure plots the loadings on the market, $R_M - R_f$, size, SMB, and value, HML, factors of the Fama-French (1993) three-factor model for 12 *CASH*-sorted, equal-weighted portfolios.



When I correct for risk using either the CAPM, the three-factor Fama and French (1993) or the four-factor Carhart (1997) models, the observed trends in alphas parallel the ones in the corresponding mean excess returns for both, the equal-weighted and the value-weighted portfolios. Risk-adjusting the excess returns does not eliminate the relatively poor performance of the low cash holding firms compared to the high cash holding ones.

The Fama-French (1993) three-factor model betas, reported in Tables 1.10 and 1.11 and graphed in Figures 1.5 and 1.7, are relatively similar across the corresponding portfolios. The loadings on the size factor, SMB, are slightly bigger and positive and the ones on the value factor, HML, are smaller and negative for the portfolios consisting of high cash holding firms.

1.5.3 Cash Holding - Based Risk Factor

I turn to testing whether there is a cash holding-based risk factor in stock returns. I follow the approach introduced by Lamont et al. (2001) and repeated by Whited et al. (2006). Both papers build factor portfolios by using firm rankings based on the values of the corresponding financial constraint indexes. I rank the firms by cash holdings.

Cash Holding Zero Cost Portfolio

I build a zero cost portfolio related to cash holdings, FCASH, using a method analogous to that of Fama and French (1993). FCASH is based on a set of *CASH*-ranked portfolios (different from the ones analyzed in Section 1.5.2) and is constructed to be neutral with respect to size, ME, and book-to-market equity, BE/ME.

Each month I rank the firms in the equity sample independently by *CASH*, ME and BE/ME. Then, I sort the firms based on cash holdings into the bottom 30% (*Low*), middle 40% ($Medium$) and top 30% ($High$). I split all the firms based on the NYSE median size into two groups, *Small* and *Big*. And, I sort the firms based on the NYSE breakpoints for the bottom 30% (*Low*), middle 40% ($Medium$) and top 30% ($High$) of book-to-market equity.

I am consistent with Fama and French (1993) in grouping the firms based on the NYSE breakpoints for size and book-to-market equity, as opposed to the breakpoints based on the firms in the sample, and sorting the firms in two ME and three BE/ME groups.

Table 1.11: Returns on *CASH*-sorted, Value-Weighted Stock Portfolios

The table reports mean excess returns; alphas for the CAPM, Fama-French (1993) three-factor and Carhart (1997) four-factor models; and, loading on the market, $R_M - R_f$, size, SMB, and value, HML, factors of the Fama-French (1993) three-factor model for 12 *CASH*-sorted, value-weighted portfolios. The mean excess returns and alphas are presented in annualized percentage points. T-statistics are reported in parentheses. At the end of each calendar quarter all equity sample stocks are sorted by *CASH* and divided into portfolios based on percentile cutoffs. For instance, 0510 is a portfolio of firms with cash holdings in percentiles 5 to 10 of the *CASH* distribution. The equity sample spans the months from 1980:1 to 2010:9.

Portfolios	0005	0510	1020	2030	3040	4050	5060	6070	7080	8090	9095	9000
Panel A: Portfolio Excess Returns and Alphas (%, annualized)												
Mean Ex.	5.37	2.64	6.82	4.53	7.25	6.85	7.97	12.06	8.26	10.01	11.78	9.89
Return	(1.49)	(0.84)	(2.28)*	(1.49)	(2.30)*	(2.25)*	(2.36)*	(2.98)**	(1.96)*	(2.09)*	(2.34)*	(2.11)*
CAPM	-1.22	-3.38	0.62	-1.83	0.55	0.37	0.93	3.79	-0.26	0.70	2.07	1.26
Alpha	(0.61)	(2.28)*	(0.55)	(1.84)	(0.55)	(0.39)	(0.75)	(2.18)*	(0.15)	(0.30)	(0.81)	(0.48)
3-Factor	-1.78	-3.59	0.21	-1.66	0.87	1.04	2.09	5.62	1.45	2.55	4.18	3.10
Alpha	(0.88)	(2.39)*	(0.19)	(1.66)	(0.93)	(1.10)	(1.74)	(3.55)**	(0.94)	(1.40)	(1.99)*	(1.41)
4-Factor	-2.18	-4.40	-0.15	-2.18	0.33	0.62	2.80	6.36	3.31	4.63	6.87	6.05
Alpha	(1.06)	(2.92)**	(0.14)	(2.16)*	(0.35)	(0.65)	(2.30)*	(3.97)**	(2.19)*	(2.59)**	(3.38)**	(2.85)**
Panel B: 3-Factor Regression Coefficients												
$R_M - R_f$	1.007	0.950	0.953	0.970	1.000	0.952	0.983	1.073	1.150	1.182	1.212	1.073
	(25.98)**	(32.80)**	(45.13)**	(50.52)**	(56.71)**	(52.93)**	(43.51)**	(36.76)**	(39.27)**	(34.57)**	(31.09)**	(26.03)**
SMB	0.056	-0.064	-0.0816	-0.061	-0.074	-0.087	0.046	0.192	0.223	0.554	0.551	0.590
	(1.00)	(1.55)	(2.78)**	(2.20)*	(2.90)**	(3.36)**	(1.42)	(4.54)**	(5.27)**	(11.19)**	(9.76)**	(9.88)**
HML	0.089	0.045	0.081	-0.20	-0.052	-0.103	-0.204	-0.329	-0.326	-0.392	-0.430	-0.392
	(1.53)	(1.04)	(2.57)*	(0.70)	(1.99)*	(3.84)**	(6.01)**	(7.50)**	(7.41)**	(7.61)**	(7.34)**	(6.33)**
Panel C: Additional Distributional Characteristics												
Sigma	0.056	0.049	0.046	0.047	0.048	0.047	0.052	0.061	0.064	0.073	0.076	0.071
Skewness	-0.683	-0.551	-0.638	-0.714	-0.831	-0.768	-0.5787	-0.224	-0.570	-0.228	-0.288	-0.052

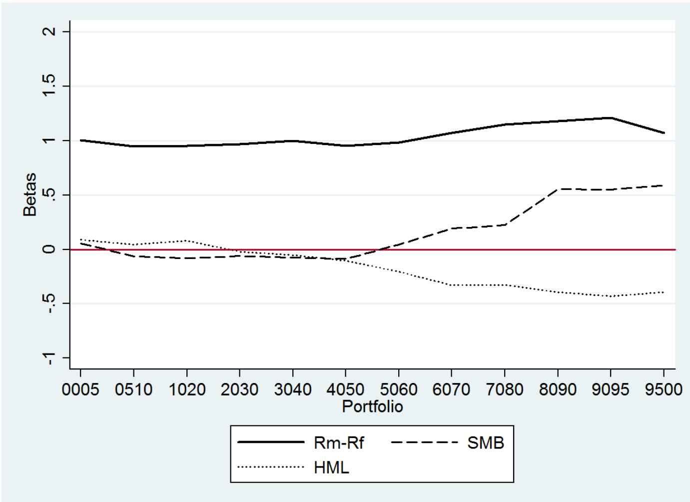
**,*: statistically distinct from zero at the 1 and 5% level, respectively

Figure 1.6: Mean Excess Returns and Alphas of *CASH*-Sorted, Value-Weighted Portfolios

The figure plots annualized mean excess returns and alphas for the CAPM, Fama-French (1993) three-factor and Carhart (1997) four-factor models for 12 *CASH*-sorted, value-weighted portfolios. At the end of each calendar quarter all equity sample stocks are sorted by *CASH* and divided into portfolios based on percentile cutoffs. For instance, 0510 is a portfolio of firms with cash holdings in percentiles 5 to 10 of the *CASH* distribution.

Figure 1.7: Factor Loadings of *CASH*-Sorted, Value-Weighted Portfolios

The figure plots the loadings on the market, $R_M - R_f$, size, SMB, and value, HML, factors of the Fama-French (1993) three-factor model for 12 *CASH*-sorted, value-weighted portfolios.



I sort firms into three (not two) *CASH* groups based on the strong role of cash holdings in average stock returns observed in Section 1.5.2. Lamont et al. (2001) and Whited et al. (2006) rank firms by financial constraint index values and size, sort firms into three ME groups, and use sample-based ME breakpoints.

Next, I build 18 portfolios cross-sorted on size, book-to-market equity and cash holdings. Every month I split all the firms into groups, each identified by a size (small (S), big (B)), a book-to-market equity (low (L), middle (M), high (H)) and a cash holding (low (L), middle (M), high (H)) category. I calculate value-weighted and equal-weighted average monthly portfolios returns using individual stock returns of the firms in the corresponding groups. As before, the value-weighted returns are based on market capitalizations as of the end of the previous month.

I form three more portfolios as linear combinations of the 18 portfolios from above. The first portfolio, HighCASH, is the equal-weighted average of the six highest cash holding portfolios in each of the size and book-to-market categories:

$$\text{HighCASH} = (\text{SLH} + \text{SMH} + \text{SHH} + \text{BLH} + \text{BMH} + \text{BHH})/6.$$

The second one, LowCASH, is the equal-weighted average of the six lowest cash holding portfolios in each of the size and book-to-market categories:

$$\text{LowCASH} = (\text{SLL} + \text{SML} + \text{SHL} + \text{BLL} + \text{BML} + \text{BHL})/6.$$

The third portfolio, FCASH, is the difference between the other two:

$$\text{FCASH} = \text{HighCASH - LowCASH}.$$

It represents a monthly time series of returns on a zero-cost portfolio mimicking the risk factor in stock returns associated with cash holdings. FCASH is a stream of returns that one would get by buying levered, high cash holding firms and shorting levered, low cash holding ones. By forcing the long, HighCASH, and short, LowCASH, portfolios to represent small and big, and low, medium, and high book-to-market equity firms equally, I ensure that no class of firms dominates the returns on FCASH. Controlling for size and book-to-market equity results in a time series of returns that is associated with the differences in cash holdings and is not due to differences in size or book-to-market equity.

Table 1.12 reports average monthly returns and characteristics for the 18 portfolios, as well for HighCASH, LowCASH and FCASH. I obtain the values of portfolio characteristics by time-averaging monthly equal-weighted characteristics of the corresponding individual firms. The returns in Table 1.12 are annualized.

The fifth column reports the average monthly number of firms in each portfolio. The fact that most of the firms are classified as small is an artifact of splitting the firms into *Small* and *Big* by the NYSE median size. Most of the stocks in the equity sample are traded on AMEX and NASDAQ and, generally, represent smaller firms. All portfolios, however, are fairly well diversified.

There are no consistent, noticeable differences in average CF, $CF\ Sigma$ and size, measured by the natural logarithm of ME in 2002 dollars, between high cash holding firms and low cash holding ones within size / book-to-market equity categories. High cash holding firms have slightly higher average values of book-to-market equity. In line with the corresponding observation from Section 1.5.2, low cash holding firms are observed with higher average *Leverage*. Finally, on average, high cash holdings firms earn higher excess returns.

A key observation is that the difference in monthly returns on the equal-weighted HighCASH and LowCASH portfolios averages 7.48% on an annual basis and is statistically distinct from zero at the 1% level. Under value weighting, the average difference is slightly lower, with 5.99% on an annual basis, but is still distinct from zero at the 1% level. Therefore, based on FCASH, on average, high cash holding firms earn a positive and statistically significant premium. It is comparable to the risk premiums (presented in Table 1.13) on the traditional risk factors, including the market, $R_M - Rf$, size, SMB, value, HML, and momentum, UMD.

Time Series Tests of Common Variation

I follow Lamont et al. (2001) and Whited et al. (2006) and turn to the central issue of Section 1.5.3: testing for a source of common variation in returns of high cash holdings firms.

I start by testing whether stock returns move together by controlling for other, known sources of common variation, including the market and size factors. I regress the returns on each of the 18 portfolios, shown in Table 1.14, on three reference portfolios. The first reference portfolio is a proxy for the market factor, the second - for the size factor, and the third - for FCASH.

The market and size proxies are constructed using the portfolios from Table 1.14. The proxy for the overall market consists of portfolios of big firms with cash holdings in the lower and medium categories: BIG = (BLL + BLM + BML + BMM + BHL + BHM)/6. The proxy for size consists of portfolios of small firms with lower and medium cash holdings: SMALL = (SLL + SLM + SML + SMM + SHL + SHM)/6.

Table 1.12: Portfolio Characteristics and Returns

The table reports summary statistics for 18 equal- and value-weighted portfolios, formed by ranking equity sample firms *independently* by size, ME, book-to-market equity BE/ME, and cash holdings, *CASH*, at the end of each calendar quarter. Based on the NYSE median size, firms are either Small or Big. Based on the NYSE breakpoints for book-to-market equity, firms are either Low (low 30%), Medium (middle 40%) or High (top 30%) in book-to-market equity. Based on sorting the equity sample by *CASH*, firms are either Low (low 30%), Medium (middle 40%) or High (top 30%) in cash holdings. Portfolios contain firms that are in a given size (S/B), book-to-market equity (L/M/H) and cash holding (L/M/H) category. Value-weighted portfolios are based on market capitalizations as of the end of previous month. HighCASH = (SLH+SMH+SHH+BLH+BMH+BHH)/6, LowCASH = (SLL+SML+SHL+BLL+BML+BHL)/6, FCASH = HighCASH-LowCASH. Column 'Excess Returns' presents the means of portfolio monthly returns in excess of one-month Treasury Bill yields lagged one month in annualized percentage points. Column N presents the average number of firms across all sample months. Columns *CASH*, *Leverage*, CF, CF *Sigma*, *Ln*(*Real* ME) and BE/ME present mean values based on averaging across both, firms and time. *CASH* and CF are defined in Section 1.3. CF *Sigma* is the prior twelve-quarter volatility of CF, including the current quarter. *Leverage* is the ratio of the sum of short- (item DLCQ) and long-term (item DLTTQ) debt to total assets (item ATQ). Size, ME, is measured by market capitalization (equity price times shares outstanding). *Ln*(*Real* ME) is the natural logarithm of size in 2002 dollars. Book-to-market equity, BE/ME, is the ratio of book equity to market capitalization. The equity sample spans the months from 1980:1 to 2010:9.

Portfolios		Excess Return (%, annualized)		Mean						
		Equal-Wght.	Value-Wght.	N	*CASH*	*Lev.*	CF	CF *Sig.*	*Ln*(*R.* ME)	BE/ME
Panel A: Size / Book-to-Market Equity / *CASH* Categories										
SMALL FIRMS (ME, bottom half)										
Low-BE/ME, Low-CASH	SLL	-3.09	0.20	410	0.008	0.353	0.007	0.015	4.138	0.249
Low-BE/ME, Mid-CASH	SLM	3.57	5.16	398	0.047	0.271	0.005	0.016	4.365	0.249
Low-BE/ME, High-CASH	SLH	11.23	10.22	406	0.204	0.182	-0.002	0.018	4.520	0.287
Mid-BE/ME, Low-CASH	SML	6.82	6.36	294	0.008	0.311	0.021	0.013	4.495	0.688
Mid-BE/ME, Mid-CASH	SMM	10.45	9.16	285	0.046	0.265	0.020	0.014	4.518	0.693
Mid-BE/ME, High-CASH	SMH	15.19	11.73	339	0.304	0.158	0.011	0.018	4.368	0.692
High-BE/ME, Low-CASH	SHL	13.76	7.28	319	0.008	0.323	0.019	0.017	3.788	1.531
High-BE/ME, Mid-CASH	SHM	17.02	10.19	319	0.047	0.307	0.019	0.017	3.794	1.583
High-BE/ME, High-CASH	SHH	25.59	14.50	385	0.426	0.196	0.012	0.022	3.682	1.628

continued on next page

Table 1.12: continued from previous page

Portfolios		Excess Return (%, annualized) Equal-Wght.	Excess Return (%, annualized) Value-Wght.	Mean N	Mean *CASH*	Mean *Lev.*	Mean CF	Mean CF *Sig.*	Mean $Ln(R.\ ME)$	Mean BE/ME
Panel A: Size / Book-to-Market Equity / *CASH* Categories										
BIG FIRMS (ME, top half)										
Low-BE/ME, Low-CASH	BLL	12.02	11.40	149	0.009	0.290	0.022	0.006	8.228	0.278
Low-BE/ME, Mid-CASH	BLM	14.25	13.10	155	0.048	0.204	0.020	0.006	8.288	0.275
Low-BE/ME, High-CASH	BLH	15.94	15.72	80	0.152	0.196	0.019	0.009	7.843	0.302
Mid-BE/ME, Low-CASH	BML	12.42	11.04	96	0.010	0.294	0.027	0.007	8.044	0.652
Mid-BE/ME, Mid-CASH	BMM	16.09	13.54	86	0.046	0.242	0.028	0.008	8.007	0.676
Mid-BE/ME, High-CASH	BMH	16.83	14.69	50	0.191	0.193	0.027	0.010	7.792	0.705
High-BE/ME, Low-CASH	BHL	15.08	12.72	41	0.010	0.315	0.028	0.008	7.943	1.272
High-BE/ME, Mid-CASH	BHM	16.72	13.68	42	0.048	0.286	0.031	0.010	7.937	1.401
High-BE/ME, High-CASH	BHH	20.53	20.56	32	0.278	0.208	0.035	0.012	7.794	1.654
Panel B: *CASH*-Based Portfolios										
HighCASH		20.47	17.45		0.294	0.181	0.009	0.018	4.648	0.836
LowCASH		12.15	10.86		0.009	0.323	0.073	0.013	5.023	0.718
FCASH†		7.48	5.99		0.285	-0.141	-0.008	0.005	-0.375	0.118
(FCASH t-stat, monthly)		(3.85)**	(3.06)**							

†: column Excess Returns reports returns

**: statistically distinct from zero at the 1% level

Table 1.13: Factor Risk Premiums

The table presents average monthly returns on the value-weighted financial constraint risk factor, FCASH, and four traditional factors, including the market, R_M-R_f, size, SMB, value, HML, and momentum, UMD. FCASH is long levered, high cash holding firms and short levered, low cash holding firms; it is neutral with respect to size and book-to-market equity characteristics. The construction of FCASH is detailed in Table 1.12. The sample period is from 1980:1 to 2010:9.

Risk Factor	Average Monthly Return (%, annualized)
UMD	8.05
R_M-R_f	6.71
FCASH	5.99
HML	5.17
SMB	0.37

I want to regress each of the 18 portfolio returns on the measures of market, size, and cash holding-based factors. Simply using BIG, SMALL and FCASH in these regressions would lead to spurious results due to the same return series being included in the dependent and independent variables. Hence, for each of the 18 portfolios I customize the three benchmark portfolios by excluding the left-side variable from the construction of the right-side variables. For instance, in the regression where SLL is the dependent variable SMALL does not include SLL. Additionally, in order to facilitate comparison of results across the corresponding regressions, I make the definitions of FCASH constant within size / book-to-market equity groups. For example, in the case of small / low book-to-market equity firms, represented by SLL, SLM, and SLH, portfolios SLL and SLH are excluded from the construction of FCASH.

Table 1.14 shows the results for the 18 regressions. First, as expected, large firms have higher loadings on BIG, and small firms have higher loadings on SMALL.

The focus, however, is on the cash holding-based factor. The loadings on FCASH are lower for the firms in the bottom 30% and higher for the firms in the top 30% of *CASH*. This observation is strongly consistent across all size / book-to-market equity groups.

Table 1.14 demonstrates that stock returns on high cash holding firms positively covary with the returns on other high cash holdings firms. There is a common risk factor in stock returns associated with cash holdings, and FCASH is the corresponding risk factor portfolio.

Table 1.14: Portfolio Covariance Tests

The table reports regression results for the 18 value-weighted portfolios, described in Table 1.12. Monthly excess returns on each portfolio are regressed on returns on three reference portfolios: a market proxy, a size factor proxy and FCASH. The proxies are constructed using the 18 portfolios as follows. The market proxy is the return on a portfolio of large firms with low/medium/high book-to-market equity and low/medium cash holdings, BIG=(BLL+BML+BHL+BLM+BMM+BHM)/6, in excess of one-month Treasury Bill yields lagged one month. The size proxy is the return on a portfolio of small firms with low/medium/high book-to-market equity and low/medium cash holdings, SMALL=(SLL+SML+SHL+SLM+SMM+SHM)/6, in excess of one-month Treasury Bill yields lagged one month. FCASH = (SLL+SML+SHL+BLL+BML+BHL)/6 -(SLH+SMH+SHH+BLH+BMH+BHH)/6. In each regression, portfolio that is the dependent variable is omitted from the construction of the corresponding independent variable, representing BIG, SMALL or FCASH. In the case of FCASH, the matching portfolio on the short side is also omitted. T-statistics are reported in parentheses. The sample period is from 1980:1 to 2010:9.

	Regression Results					Variable Definition		
Portfolios	Alpha	BIG	SMALL	FCASH	R^2	BIG	SMALL	FCASH
SMALLER FIRMS (ME, bottom half)								
SLL	-0.006	0.023	0.962	0.064	0.87	BIG	(SLM+SML+	(SMH+SHH+BLH+BMH+BHH)/5-
	(5.10)**	(0.46)	(23.94)**	(1.51)			SMM+SHL+SHM)/5	(SML+SHL+BLL+BML+BHL)/5
SLM	-0.004	0.112	0.906	0.680	0.86	BIG	(SLL+SML+	same as for SLL
	(3.30)**	(1.94)	(19.84)**	(14.58)**			SMM+SHL+SHM)/5	
SLH	-0.002	0.028	1.001	1.210	0.86	BIG	SMALL	same as for SLL
	(1.35)	(0.40)	(17.62)**	(19.88)**				
SML	0.001	0.064	0.858	-0.231	0.93	BIG	(SLL+SLM+	(SLH+SHH+BLH+BMH+BHH)/5-
	(1.04)	(2.06)*	(35.28)**	(9.09)**			SMM+SHL+SHM)/5	(SLL+SHL+BLL+BML+BHL)/5
SMM	0.002	0.093	0.891	-0.052	0.95	BIG	(SLL+SLM+SML	same as for SML
	(4.11)**	(3.21)**	(39.15)**	(2.21)*			SHL+SHM)/5	
SMH	0.001	0.094	0.905	0.693	0.93	BIG	SMALL	same as for SML
	(0.67)	(2.42)*	(29.30)**	(22.02)**				
SHL	0.001	0.151	0.902	-0.159	0.86	BIG	(SLL+SLM+SML+	(SLH+SMH+BLH+BMH+BHH)/5-
	(1.02)	(2.96)**	(22.06)**	(4.02)**			SMM+SHM)/5	(SLL+SML+BLL+BML+BHL)/5
SHM	0.003	0.089	0.971	-0.112	0.86	BIG	(SLL+SLM+SML	same as for SHL
	(2.99)**	(1.71)	(23.37)**	(2.64)**			SMM+SHL)/5	
SHH	0.004	0.198	0.819	0.406	0.88	BIG	SMALL	same as for SHL
	(3.34)**	(3.87)**	(20.05)**	(10.40)**				

**,*: statistically distinct from zero at the 1 and 5 % level, respectively

continued on next page

Table 1.14: continued form previous page

Portfolios	Regression Results					Variable Definition		
	Alpha	BIG	SMALL	FCASH	R^2	BIG	SMALL	FCASH
LARGER FIRMS (ME, top half)								
BLL	-0.002	0.767	0.082	0.021	0.73	(BLM+BML+	SMALL	(SLH+SMH+SHH+BMH+BHH)/5-
	(0.18)	(14.28)**	(1.90)	(0.45)		BMM+BHL+BHM)/5		(SLL+SML+SHL+BML+BHL)/5
BLM	-0.001	0.727	0.061	0.381	0.77	(BLL+BML+	SMALL	same as for BLL
	(0.15)	(14.61)**	(1.51)	(9.05)**		BMM+BHL+BHM)/5		
BLH	-0.004	0.833	0.171	1.238	0.84	BIG	SMALL	same as for BLL
	(2.63)	(12.97)**	(3.39)**	(22.36)**				
BML	0.001	0.827	0.106	-0.228	0.86	(BLL+BLM+	SMALL	(SLH+SHH+BLH+BMH+BHH)/5-
	(0.25)	(22.17)**	(3.57)**	(7.42)**		BMM+BHL+BHM)/5		(SLL+SHL+BLL+BML+BHL)/5
BMM	0.001	0.912	0.032	0.112	0.85	(BLL+BLM+BML+	SMALL	same as for BML
	(0.63)	(21.72)**	(0.98)	(3.26)**		BHL+BHM)/5		
BMH	-0.001	0.903	0.060	0.739	0.85	BIG	SMALL	same as for BML
	(1.53)	(17.40)**	(1.48)	(17.65)**				
BHL	0.001	0.930	0.124	-0.205	0.76	(BLL+BLM+BML+	SMALL	(SLH+SMH+BLH+BMH+BHH)/5-
	(0.61)	(15.66)**	(2.67)**	(4.58)**		BMM+BHM)/5		(SLL+SML+BLL+BML+BHL)/5
BHM	0.001	0.838	0.260	0.003	0.77	(BLL+BLM+BML+	SMALL	same as for BHL
	(0.35)	(13.56)**	(5.45)	(0.07)		BMM+BHL)/5		
BHH	0.002	0.990	0.047	0.567	0.77	BIG	SMALL	same as for BHL
	(1.78)	(14.52)**	(0.88)	(10.93)**				

**,*: statistically distinct from zero at the 1 and 5 % level, respectively

Portfolios of Stayers and Switchers: Variances and Covariances

In this section I conduct Daniel and Titman's (1997) test as refined by Lamont et al. (2001). Daniel and Titman (1997) hypothesize that common variation in stock returns related to the book-to-market equity characteristic, documented by Fama and French (1993), is spuriously reflecting other factors. To test this hypothesis they sort stocks into portfolios based on book-to-market equity values at year $t-1$[7] and examine if covariances *within* these portfolios rise between years $t-5$ and t. Daniel and Titman (1997) conjecture that firms that have similar book-to-market equity in year $t-1$ could be firms that always covary together, even in years when they do not have similar book-to-market equity. The key premise of the test is that book-to-market equity changes substantially between year $t-5$ and t. The refinement of Lamont et al. (2001), explained below, accounts for the fact that the main characteristic of interest might be changing relatively slowly over time.

In Section 1.5.3 I have shown common variation in stock returns for high cash holding firms after controlling for the market and size effects. These controls, however, may not have exhausted the list of potential confounding characteristics in stock returns. The goal in this section is to provide evidence that there is indeed common variation in returns associated *strictly* with cash holdings.

Following the approach of Lamont et al. (2001), I start with the sample of firms that as of month t are in FCASH portfolio and have data going back five quarters. (Remember, that in the equity sample variables based on financial statement information, such, for example, as BE/ME and *CASH*, are updated on a quarterly basis. Therefore, consistent with Whited et al. (2006), I examine changes in stock returns over the last six quarters, including the current quarter, as opposed to the last six years.) The fact that these firms are in FCASH means that at t they rank either in the top or bottom 30% of CASH. I divide this sample into stayers and switchers. Stayers are the firms that have stayed in the same top or bottom 30% of CASH for the last six quarters, from $t-5q$ through t. Switchers are the firms that switched to the opposite 30% of CASH from $t-5q$ to t. For example, a switcher could be in the top 30% of CASH at $t-5q$ and in the bottom 30% of CASH at t. I use these firms to build two additional cash holding portfolios: FCASH_{STAY} and FCASH_{SWITCH}. Both portfolios are long firms in the top 30% of CASH and short firms in the bottom 30% of CASH as of t. However, FCASH_{STAY} is based on stayers, and FCASH_{SWITCH} is based on switchers.

[7] Fama and French (1993) examine returns on portfolios formed at the end of June of year t based on book-to-market equity values as of the end of year $t-1$.

I examine the returns on six FCASH_{STAY} and FCASH_{SWITCH} portfolios that correspond to different formation quarters. To ensure good diversification, firms in formation quarter j portfolios are stratified by the values of size and book-to-market equity as of $t - jq$ in the same manner as it was done for FCASH. Additionally, individual stock returns are value-weighed based on market capitalizations as of the end of the month preceding $t - jq$. I emphasize that, independent of the formation quarter, FCASH_{STAY} and FCASH_{SWITCH} are long high cash holding firms and short low cash holding firms based on *CASH* rankings as of t.

As explained in Lamont et al. (2001), FCASH_{STAY} and FCASH_{SWITCH} allow a distinction between two hypotheses that have different implications about covariances and variances of returns on switchers and stayers. First, under the hypothesis that *CASH* spuriously reflects other factors and that firms in FCASH portfolio always covary, both switchers and stayers should always covary. Switchers should covary with other switchers as well as with stayers.

Second, under the hypothesis that the covariance in stock returns is related to cash holdings, switchers should covary less with each other and with stayers when their *CASH* status is less similar and more when their *CASH* status is more similar.

Table 1.15 summarizes the results for FCASH_{STAY} and FCASH_{SWITCH}. The second column, 'Percent Switching', shows the average percent of FCASH_{SWITCH} firms (representing both the long and the short side) at $t-jq$ that will end up in the opposite 30% of *CASH* at t. By construction, the percentage of firms switching to the opposite *CASH* ranking group moves from 100 at $t - jq$ to zero at t.

If cash holdings proxy for a distinct source of common variation, the covariance between returns on FCASH_{SWITCH} and FCASH_{STAY} should rise from $t-5q$ to t. As reported in column 7, 'Covariance', the covariance rises from 0.71 to 0.81. The coefficient on FCASH_{STAY} from univariate regressions of FCASH_{SWITCH} on FCASH_{STAY} also increases from 0.61 for $t - 5q$ to 0.79 for t. The increase in covariances accompanies the increase in similarities along the cash holdings dimension.

Next, I examine the variance of FCASH_{SWITCH} returns. Column 5, 'Variance', shows that the variance rises from 14.59 at $t-5q$ to 25.25 at t. Column 6, 'Standard Deviation', reports that the standard deviation rises from 3.82 at $t - 5q$ to 5.05 at t. These increases are significant. In contrast, the standard deviation of FCASH_{STAY} returns, reported in column 4, 'Standard Deviation', declines from 5.18 to 4.44. The composition of FCASH_{SWITCH} becomes more homogeneous from $t - 5q$ to t, which results in the increased variance. In other words, the covariance across individual stock returns is higher when the cash holdings status is more similar.

Table 1.15: Portfolios of Stayers and Switchers: Variances and Covariances

The table presents time-series properties of monthly returns on portfolios FCASH_{STAY} and FCASH_{SWITCH}. FCASH is described in Table 1.12. FCASH_{STAY} and FCASH_{SWITCH} are constructed from the sample of firms that (a) are in FCASH at month t and (b) have data going back five quarters from t. The fact that these firms are in FCASH means that at t they rank either in the top or bottom 30% of *CASH*. Stayers are the firms that have remained in the same top or bottom 30% of *CASH* for the last six quarters, from $t-5q$ to t. Switchers are the firms that are in the opposite 30% of *CASH* at $t-5q$ and t. For example, a switcher could be in the top 30% of *CASH* at $t-5q$ and in the bottom 30% of *CASH* at t. FCASH_{STAY} and FCASH_{SWITCH} are build from switchers and stayers, correspondingly, in the same way as it was done to construct FCASH in Table 1.12. Both portfolios are long firms in the top 30% of *CASH* and short firms in the bottom 30% of *CASH* as of t. Both portfolios are value-weighted based on market capitalizations as of the end of the month preceding $t-jq$ and are stratified by size and book-to-market equity as of $t-jq$. 'Percent Switching' at $t-jq$ shows the percentage of firms in portfolio FCASH_{SWITCH} that are not in the same top or bottom 30% of *CASH* at $t-5q$ and t. 'Variance' and 'Standard Deviation' report monthly return variances and standard deviations for the corresponding portfolios. 'Covariance' shows covariances of monthly returns on FCASH_{SWITCH} and FCASH_{STAY}. 'Coefficient on FCASH_{STAY}' reports FCASH_{STAY} coefficients (with t-statistics in parentheses) from univariate regressions of FCASH_{SWITCH} on FCASH_{STAY}. 'R^2' reports adjusted R^2's for the above regressions. The sample period is from 1981:6 to 2010:9.

	FCASH_{SWITCH}			FCASH_{STAY}		$\text{FCASH}_{SWITCH}/\text{FCASH}_{STAY}$ Regression Results		
	Percent Switching	Variance	Stand. Deviat.	Variance	Stand. Deviat.	Covariance	Coefficient on $FCASH_{STAY}$	R^2
$t-5q$	100	14.59	3.82	26.83	5.18	0.71	0.61 (18.7)	0.49
$t-4q$	94	18.58	4.31	25.70	5.07	0.73	0.69 (19.9)	0.53
$t-3q$	87	19.54	4.42	24.50	4.95	0.76	0.73 (21.9)	0.57
$t-2q$	56	21.25	4.61	21.99	4.69	0.78	0.72 (23.6)	0.61
$t-1q$	31	21.81	4.67	20.25	4.50	0.78	0.73 (23.3)	0.61
t	0	25.50	5.05	19.71	4.44	0.81	0.79 (26.2)	0.66

Relating FCASH to Known Empirical Factors

In this section I examine whether the cash holding-based factor, FCASH, reflects *only* known empirical factors, including the market, $R_M - Rf$, size, SMB, value, HML, and momentum, UMD.

The focus of the analysis is on the intercepts and R^2's from time series regressions of the cash holding-based factor on the other factors. First, if the known factors correctly price the cash holding-based factor, the intercepts should be zero. Second, the R^2's measure how much variation in the cash holding-based factor can be explained by the known factors. Therefore, low R^2's would mean that the cash holding-based factor measures sources of variation independent of the other factors.

Table 1.16 presents the results for the time series regressions of the cash holding-based factor, both value- and equal-weighted, on the three Fama-French (1993) and the momentum factor. The results are shown for the full equity sample, 1980:1-2010:9, and two sub-samples, 1980:1-1995:4 and 1995:5-2010.

The first and the third rows in the full sample section of Table 1.16 show how well the cash holding-based factor can be explained by the Fama-French (1993) three-factor model. Both, the equal-weighted and the value-weighted factors are mispriced with alphas of 60 and 50 basis points per month, correspondingly. The alphas are statistically distinct from zero at the 1% level. Based on the R^2 values, only 42% of the variation of the equal-weighted cash holding-based factor and only 39% of the variation of the value-weighted factor can be explained by the three-factor model.

Table 1.16: Relating FCASH to R_M-R_f, SMB, HML, and UMD

The table reports results from asset pricing tests of the equal- and value-weighted FCASH portfolios. FCASH is constructed in Table 1.12. The Carhart (1997) four-factor model is the asset pricing model. The factors include the market, R_M-R_f, size, SMB, value, HML, and momentum, UMD. Monthly FCASH returns in excess of one-month Treasury Bill yields lagged one month are regressed on factor returns. T-statistics are reported in parentheses. The sample period is from 1980:1 to 2010:9.

Dependent Variable	Alpha	R_M-R_f	SMB	HML	UMD	Adj. R^2
Full Sample: January, 1980-September, 2010						
Equal-Weighted FCCASH	0.006	0.115	0.172	-0.457		0.42
	(5.58)**	(4.19)**	(4.33)**	(11.05)**		
Equal-Weighted FCCASH	0.008	0.083	0.179	-0.511	-0.146	0.47
	(6.84)**	(3.12)**	(4.71)**	(12.60)**	(5.99)**	
Value-Weighted FCCASH	0.005	0.125	0.193	-0.419		0.39
	(4.26)**	(4.40)**	(4.67)**	(9.75)**		
Value-Weighted FCCASH	0.006	0.094	0.199	-0.471	-0.143	0.44
	(5.40)**	(3.38)**	(5.02)**	(11.13)**	(5.60)**	
Sub-Sample: January, 1980-April, 1995						
Equal-Weighted FCCASH	0.006	0.021	0.023	-0.063		0.03
	(7.86)**	(1.02)	(0.67)	(1.08)		
Equal-Weighted FCCASH	0.006	0.023	0.020	-0.068	-0.022	0.03
	(7.84)**	(1.12)	(0.59)	(1.92)	(0.89)	
Value-Weighted FCCASH	0.005	0.016	0.058	-0.033		0.01
	(5.44)**	(0.73)	(1.51)	(0.85)		
Value-Weighted FCCASH	0.004	0.015	0.060	-0.028	-0.016	0.01
	(5.12)**	(0.65)	(1.55)	(0.73)	(0.61)	
Sub-Sample: May, 1995-September, 2010						
Equal-Weighted FCCASH	0.005	0.265	0.158	-0.681		0.65
	(3.35)**	(6.95)**	(3.13)**	(12.75)**		
Equal-Weighted FCCASH	0.007	0.198	0.180	-0.733	-0.149	0.69
	(4.21)**	(5.16)**	(3.77)**	(14.27)**	(4.91)**	
Value-Weighted FCCASH	0.004	0.286	0.171	-0.642		0.63
	(2.59)*	(7.22)**	(3.27)**	(11.58)**		
Value-Weighted FCCASH	0.005	0.217	0.193	-0.696	-0.153	0.67
	(3.39)**	(5.44)**	(3.90)**	(13.02)**	(4.85)**	

**,*: statistically distinct from zero at the 1 and 5 % level, respectively

The results are similar for the four-factor Carhart (1997) model. Full-sample regression statistics are shown in the second and in the fourth rows of the full-sample section of Table 1.16. The equal-weighted and the value-weighted cash holding-based factors remain mispriced with alphas of 80 and 60 basis points per month, correspondingly. The alphas are strongly statistically significant. Compared to the previous case, the R^2 values increase slightly, but still only 47% of the variation of the equal-weighted and only 44% of the variation of the value-weighted factor can be explained by the four-factor model.

The main results for the first and the second halves of the full equity sample reported in Table 1.16 are similar to the ones above. The mispricing is statistically significant and big portions of the variation of the cash holding-based factor remain unexplained. In the first sub-sample, 1980:1-1995:4, the values of R^2 are very low - not higher than 3%. This suggests a larger independent role for the cash holding-based factor. The values of R^2, however, increase significantly and get close to 70% in the second sub-sample, 1995:5-2010:9.

In Table 1.16 the loadings on the market, $R_M - R_f$, and size, SMB, factors are positive, while the loadings on the value, HML, and the momentum, UMD, factors are negative. Time-series returns on the cash holding-based factor are positively correlated with the returns on the market and size factors and negatively correlated with the returns on the value and momentum factors. These results suggest that high cash holding firms are smaller and have higher CAPM betas, lower book-to-market ratios, and lower momentum.

In summary, the results of the empirical analysis of equity returns are consistent with the theoretical discussion. There is evidence of a positive, robust correlation between cash holdings and equity returns in the cross-section. When default risk is low, declining operating cash flows strengthen the relationship between cash reserves and stock returns reflecting an increasing effect of financial constraints on both cash policies and equity value. When default risk is high, this relationship weakens reflecting a change in the role of cash reserves from a proxy for the level financial constraints to the means of avoiding immediate closure. Given the results of the time-series analysis of equity returns and the model-motivated role of *CASH* as a proxy for the level of financial constraints, the cash holding-based factor, FCASH, can be interpreted as the factor mimicking portfolio related to non-diversifiable *financial constraint risk*. The factor has earned a sizable, statistically significant premium.

1.6 Conclusion

In this study, I add exogenous financial constraints as another market friction to the structural model of Goldstein, Ju, and Leland (2001). A financially constrained firm is not able to issue new equity to fund unexpected net operating losses and faces the possibility of premature default. In order to shield their value and reduce the effect of suboptimal default, equity holders manage a costly cash account, based on retained net operating profits.

I demonstrate that target cash holdings increase in the level of financial constraints, which suggests that cash can be viewed as a proxy for the intensity of unobservable financial constraints. I also prove that, even in the presence of an optimally managed cash account, the default boundary increases in intensity of financial constraints. I show that higher cash holdings are associated with lower values of debt (i.e., higher corporate-Treasury yield spreads) and equity.

I find confirmation of the relationship between cash and corporate-Treasury yield spreads in corporate debt data. The evidence for straight bonds is such, that there is positive, statistically significant correlation between cash holdings and the yield spreads. Additionally, this correlation changes with the level of operating cash flows. When default risk is low, the correlation increases as the cash flows fall. When default risk is high, the correlation reverses from positive to negative (although not statistically significant).

Cross-sectional regressions of stock returns for levered firms on cash holdings and other firm characteristics indicate that firms with more cash earn higher equity returns. This relationship changes with the level of operating cash flows. When default risk is low, as the cash flows fall the correlation between cash holding and equity returns increases. When default risk is high, the correlation drops significantly.

Time-series tests indicate that stock returns on levered, high cash holding firms positively covary with each other. This evidence of common variation points to a systematic risk factor in stock returns. A significant portion of the variation in the associated factor mimicking portfolio, based on ranking firms by cash holdings, cannot be explained by the Fama-French factors and the momentum factor. Given the role of cash as a proxy for unobservable financial constraints, I interpret the cash holding-based factor as the systematic financial constraint risk factor.

In sum, I add financial constraints a market friction to a traditional structural model of capital structure and demonstrate a strong role of cash policies on equity and debt pricing. I motivate the idea of cash holdings proxying for the level of financial constraints, construct a financial constraint risk factor mimicking portfolio for stocks, and find empirical evidence of the financial constraint risk being priced in both, debt and equity markets.

Chapter 2

Contingent Convertible Bonds and Capital Structure Decisions

2.1 Introduction

This paper provides a formal model of contingent convertible bonds (CCBs), a new instrument offering potential value as a component of corporate capital structures for all types of firms, as well as being considered for the reform of prudential bank regulation following the recent financial crisis. CCBs are debt instruments that automatically convert to equity if and when the issuing firm or bank reaches a specified level of financial distress. While qualitative discussions of CCBs are available in the literature, this is the first paper to develop a complete and formal model of their properties[1]. The paper provides analytic propositions concerning CCB attributes and develops implications for structuring CCBs to maximize their general benefits for corporations and their specific benefits for prudential bank regulation.

CCBs are receiving attention as a new instrument for prudential banking regulation because they have the potential to avoid the bank bailouts that occurred during the subprime mortgage crisis when banks could not raise sufficient new capital and bank regulators feared the consequences if systemically important banks failed[2]. A

[1]Raviv (2004) analyzes CCBs using a contingent claims approach, while Pennacchi (2010) analyzes CCBs using a jump process to create default. Neither paper includes tax shield benefits and Pennacchi (2010) has no bankruptcy costs. In contrast, we adapt a Leland (1994) model that integrates the tax shield benefit and bankruptcy cost effects of debt. This allows us to analyze the impact of various CCB contract terms for a range of issues including regulatory benefits, tax shield costs, and incentives for risk shifting and equity price manipulation.

[2]The bank bailouts during the subprime crisis reflect a failure of the regulatory principles created under the 1991 Federal Deposit Insurance Corporation Improvement Act (FDICIA) and specifically

more standard proposal for bank regulatory reform is to raise capital requirements since, if set high enough, they can achieve any desired level of bank safety. Very high capital ratios, however, impose significant costs on banks and thereby inhibit financial intermediation; and/or the capital requirements will be circumvented through regulatory arbitrage[3]. There have also been proposals to focus a component of the capital requirements on systemic risk (Adrian and Brunnermeier (2009)), or to prohibit banks outright from risky activities that are not fundamental to their role as financial intermediaries (Volcker (2010)). While these proposals could well improve prudential bank regulation, they do not directly address the issue of how distressed banks can raise new capital in order to preclude the need for government bailouts.

In this setting, CCBs have been proposed by academics (Flannery (2002, 2009a, 2009b), Duffie (2009), Squam Lake Working Group on Financial Regulation (2009), and McDonald (2010)) and endorsed for further study by bank regulators (Bernanke (2009), Dudley (2009), and Flaherty (2010))[4]. Furthermore, both the House and Senate 2010 financial reform bills require studies of CCBs for regulatory applications and provide regulatory approval for their use[5]. The Financial Stability Board of the G20 and the Bank for International Settlement are also studying CCBs. In fact, Lloyd's bank issued the first £7 billion ($11.6 billion) CCBs (CoCo bonds)[6] in 2009.

CCBs initially enter a bank's capital structure as debt instruments, thus providing the debt-instrument benefits of a tax shield and a control on principal-agent conflicts between bank management and shareholders. If and when the bank reaches

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of its requirement that bank regulators take "prompt corrective action" (PCA) in response to declining bank capital ratios. Eisenbeis and Wall (2002) provide a detailed discussion of FDICIA and its PCA requirements. As examples of the PCA requirements, "significantly undercapitalized" banks are to raise new equity or promptly merge into a well capitalized bank, and "critically undercapitalized" banks are to be placed under a receiver within 90 days of attaining that status. The subprime crisis also revealed that subordinated debt holders failed to "discipline" the banks, while the government bank bailouts protected these debt holders from the major losses they would have otherwise faced in a bankruptcy.

[3]High capital requirements limit the use of debt tax shields, impose a high cost on existing shareholders when raising new capital due to the debt overhang problem, and accentuate important principal-agent inefficiencies within the banks; see Kashyap, Rajan, and Stein (2008), Squam Lake Working Group on Financial Regulation (2009), Dudley (2009), and Flannery (2009) for further discussion of these issues.

[4]There have also been proposals for contingent capital instruments that are not bonds. Kashyap, Rajan, and Stein (2008) propose an insurance contract that provides banks with capital when certain triggering events occur and Zingales and Hart (2009) focus on the use of credit swaps. Wall (2009) provides a survey of this evolving literature.

[5]The House bill is 111th Congress, first session, H.R. 4173. The Senate bill is 111th Congress, second session, S.3217.

[6]Source: Financial Times from November 5, 2009.

the specified degree of financial distress, however, the debt is automatically converted to equity. The conversion recapitalizes the bank without requiring any ex-post action by banks to raise new equity or the government to bail them out. The automatic recapitalization feature of CCBs thus offers a relatively low-cost mechanism to avoid the costs that otherwise arise with the threatened bankruptcy of systemically important banking firms.

The existing CCB proposals-see especially Flannery (2009a) and McDonald (2010)-provide a list of issues that must be settled in formulating any specific plan for implementation:

- The trigger must be designed to avoid accounting manipulation, and the resulting conversion of CCB to equity must be automatic and inviolable. In fact, an accounting trigger in the Lloyd's bank 2009 CCB issue has already raised serious concern; see Duffie (2009). Most proposals instead recommend a trigger based on a market measure of each bank's solvency[7]. We model the case when equity value is used as a trigger. We also analyze the issue of market manipulation of the equity value that may arise with a market-value trigger.

- The CCB to equity conversion terms applied after the trigger is activated must be specified. A key question is how the value of the equity shares received at conversion compare to the value of the converting bonds; see Flannery (2009a and 2009b) and McDonald (2010). We consider the general case in which the ratio of the equity conversion value to the CCB face value is a contract parameter (λ) to be chosen. Among other effects, we analyze the impact this contract parameter may have on the incentive to manipulate the market value of the bank's equity shares. In all cases, however, we assume the CCB to equity conversion has no income tax consequences.

- The CCB contract could impose a dynamic sequence in which specified amounts of CCB convert at different thresholds. Flannery (2009a), furthermore, proposes a regulatory requirement whereby converted CCB must be promptly replaced in a bank's capital structure. While we comment on the possible advantages

[7]The Squam Lake Working Group and McDonald proposals require two triggers to be activated before conversion occurs. One trigger is based on each bank's own financial condition while the second trigger is based on an aggregate measure of banking system distress. This means that individual banks can become insolvent prior to CCB conversion if the aggregate trigger is not activated. For this reason, Flannery (2009a) argues for a single, bank-based, trigger. In this paper, we formally model only this single trigger case.

of such dynamic contract features, our formal model covers only the case of a one-time and complete conversion.

- The adoption of CCBs by banks could be voluntary or a required component of their capital requirements[8]. We consider both possibilities.

The key contribution of the current paper is to provide a formal financial model in which the effects of alternative CCB contract provisions can be analytically evaluated. We develop closed form solutions for CCB value by adapting the Leland (1994) model[9]. Our results apply equally well to the addition of CCBs to the capital structure of corporations generally, as well as for their specific application as a tool for prudential bank regulation. We make three assumptions throughout the paper regarding a firm's use of CCBs:

1. The firm is allowed a tax deduction on its CCB interest payments as long as the security remains outstanding as a bond. This would be the likely case for banks if CCBs were to become a formal and established component of prudential banking regulation. At the same time, this means that the public cost of the CCB tax shield must be included when evaluating the possible role of CCBs for prudential bank regulation. For corporations more generally, we acknowledge that the tax deductibility of CCB interest payment will likely require further IRS rulings, including possible legal challenges and new legislative actions.

2. In all cases, we assume that adding CCBs to a firm's capital structure has no impact on the level of the firm's asset holdings (A). In other words, we assume the addition of CCBs must take the form of either a CCB for equity swap (with the CCB proceeds paid out as a dividend to equity holders) or as a CCB for straight debt swap (with the CCB proceeds used to retire existing straight

[8]For example, Flannery (2009a) provides an illustrative example in which banks are required to choose between satisfying their capital requirements by (i) holding equity equal to 6% of an asset aggregate or (ii) holding equity equal to 4% of the asset aggregate and CCBs equal to 4% of the asset aggregate. This suggests a regulatory tradeoff in which 4 percentage points of CCBs are the equivalent of 2 percentage points of equity.

[9]The Leland model has been successful applied in recent studies of other fixed-income debt security innovations, although none analyzes the case of a bond conversion triggered by financial distress. Bhanot and Mello (2006) study corporate debt that includes a rating trigger such that a rating downgrade requires the equity holders to compensate the bondholders with early debt redemption or other benefits. Manso, Strulovici, and Tchistyi (2009) study a class of debt obligations where the required interest payments depend on some measure of the borrower's performance. This could include the extreme case in which the debt interest coupons reach zero at some level of financial distress. This case would provide some of the same benefits in reducing or eliminating bankruptcy costs as provided by CCBs in the current paper.

debt). The CCB for straight debt swap appears to be the most important case for regulatory applications, while we acknowledge that the future study of assets effects could be important for more general corporate finance applications.

3. Our analysis is carried out under a No Prior Default Condition - Condition 1 - that requires the CCB conversion to equity to occur at a time prior to any possible default by the firm on its straight debt. This condition constrains the set of feasible CCB contracts that are considered in our analysis. This constraint is a necessary, and sensible, requirement if CCBs are to have the desired property of reducing the bankruptcy costs associated with a bond default.

The following is a summary of our main questions and results. We first consider a firm that has a new opportunity to include CCBs within its existing capital structure in a setting where no regulatory restrictions are imposed on CCB issuance (with the exception of the contract constraints created by the No Prior Default Condition 1 as just described).

Q1. Will a firm include CCBs in its capital structure if it is freely allowed to do so?

A1. A firm will always gain from including CCB in its capital structure as a result of the tax shield benefit. This is true whether or not the firm also includes straight debt in its optimal capital structure; in fact, the optimal amount of straight debt is unaffected by the addition of CCB. Given that total assets are unchanged by assumption, in effect the CCB are being swapped for, i.e. replacing, equity in the capital structure. Since the asset-value default threshold on any existing straight debt is unchanged, adding CCB in this manner provides no benefits for regulatory safety, while taxpayers pay the cost of the additional tax shield. The addition of CCB in this form may also magnify the firm's incentive for asset substitution (to expand its asset risk).

We thus next consider a firm that operates under the regulatory constraint that it may issue CCB only as a part of a swap that retires an equal amount of straight debt. This constraint is implicit or explicit in various proposals to use CCB for prudential bank regulation; see Flannery (2009a). The result depends on whether the firm is creating a *de novo* optimal capital structure or is adding CCB to an already existing capital structure.

Q2. Will a firm add CCBs to a *de novo* optimal capital structure, assuming it faces the regulatory constraint that the CCB can only replace a part of what would have been the optimal amount of straight debt?

A2. A bank creating a *de novo* capital structure under the regulatory constraint will always include at least a small amount of CCB in its optimal capital structure. The reduction in expected bankruptcy costs ensures a net gain, even if the tax shield benefits are reduced, at least for small additions of CCB. The addition of CCBs also has the effect of reducing the incentive for asset substitution. The bottom line is that CCBs in this form provide an unambiguous benefit for regulatory safety.

Q3. Will a firm add CCBs to an existing capital structure, assuming it faces the regulatory constraint that the CCB can only be introduced as part of a swap for a part of the outstanding straight debt?

A3. Assuming the initial amount of straight debt equals or exceeds the optimal amount, the existing equity holders will not voluntary enter into the proposed swap of CCB for straight debt. While the swap may increase the firm's value–the value of reduced bankruptcy costs may exceed any loss of tax shield benefits–the gain accrues only to the holders of the existing straight debt. This is thus a classic debt overhang problem in which the equity holders will not act to enhance the overall firm value. To be clear, this result depends in part on our assumption that the straight debt has the form of a consol with indefinite maturity. If the straight debt has finite maturities, then the CCB could be swapped only for maturing debt, thus reducing the debt overhang cost.

Q4. How can CCBs be designed to provide a useful regulatory instrument for expanding the safety and soundness of banks that are acknowledged to be too big to fail (TBTF)?

A4. We assume a TBTF bank is one for which its straight debt is risk free because the bond holders correctly assume they will protected from any potential insolvency. We also assume a regulatory limitation on the amount of debt such a bank may issue. Under this limitation, a CCB for straight debt swap reduces the value of the government subsidy because it reduces the expected cost of bondholder bailouts. While this has a taxpayer benefit, the equity holders of such a bank would not voluntarily participate in such a swap.

Q5. May CCBs create an incentive for market manipulation?

A5. CCB may potentially create an incentive for either the CCB holders or the bank's equity holders to manipulate the bank's stock price to a lower value in order to force a CCB for equity conversion. The incentive for CCB holders to manipulate the equity price exists only if the ratio of equity conversion value to CCB value (λ in the model) is sufficiently high to make the conversion profitable for the CCB holders. The incentive for bank equity holders to manipulate the equity price exists, comparably, only if the ratio of equity conversion value to CCB value (λ) is sufficiently low to make the forced conversion profitable for the equity holders.

Q6. May restrictions on CCB contract and issuance terms be useful in maximizing the regulatory benefits of bank safety?

A6. The regulatory benefits of CCB issuance will generally depend on the CCB contract and issuance terms. Perhaps most importantly, the regulatory benefits vanish if banks simply substitute CCBs for capital, leaving the amount of straight debt unchanged. It is thus essential to require CCB issuance to substitute for straight debt (and not for equity). In addition, the higher the threshold for the conversion trigger the greater the regulatory safety benefits. The conversion ratio of equity for CCBs may also determine the incentive for CCB holders or equity holders to manipulate the stock price.

The structure of the paper is as follows. Part 2 develops the formal model. Part 3 applies the model to determine the role CCBs play in a bank's optimal capital structure. Part 4 analyzes bank issuance of CCBs when regulators require that the CCBs provide a net addition to bank safety. Part 5 applies the model to the role of CCBs when banks are too big to fail (TBTF). Part 6 provides our discussion of market manipulation involving CCBs. Part 7 investigates the effects of CCBs on asset substitution efficiency. Part 8 provides a summary and policy conclusions.

2.2 Model

We use the traditional capital structure modeling framework based on Leland (1994). A firm has productive assets that generate after-tax cash flows with the following dynamics

$$\frac{d\delta_t}{\delta_t} = \mu dt + \sigma dB_t^Q, \tag{2.1}$$

where μ and σ are constant, and B^Q defines a standard Brownian motion under the risk-neutral measure. The risk-free rate, r, is constant and, by assumption, is such that $\mu < r$. The tax rate $\theta \in (0, 1)$. Interest payments are tax deductible.

At any time t the market value of assets A_t is defined as the value of all future cash flows. Given (2.1), that is

$$A_t = E_t^Q \left[\int_t^\infty e^{-r(s-t)} \delta_s ds \right] = \frac{\delta_t}{r - \mu}.$$

The dynamics for A_t are: $dA_t = \mu A_t dt + \sigma A_t dB_t^Q$.

The firm can issue equity and either a straight bond (straight debt) or both a straight bond and a CCB. Both bonds are *consol* type, meaning they are annuities with infinitive maturity. Straight debt pays coupon c_b, continually in time, until default. At default, fraction $\alpha \in [0,1]$ of the firm assets is lost.

CCB pays coupon c_c, continually in time, until conversion. CCB converts into equity the first time the value of assets drops to a pre-determined level A_C. The time of conversion is denoted by $\tau(A_C) = \inf\{s : A_s \leq A_C\}$. At $\tau(A_C)$ CCB *fully* converts into equity - bond holders receive equity valued at its market price in the amount of $\left(\lambda \frac{c_c}{r}\right)$. The coefficient λ is the CCB contract term that determines the ratio of the market value of equity relative to the face value of debt, $\frac{c_c}{r}$[10]. With $\lambda = 1$, CCB converts into a market value of equity equal to the face value of CCB. With $\lambda < 1$ ($\lambda > 1$), the market value of equity received is at a discount (premium) relative to the face value of CCB.

A_C corresponds to some equity value W_C, so, in some parts of the paper, we will explicitly use W_C and an equity-value-based conversion rule. However, within the modeling framework of the paper it is easier to work with the value of assets and the asset-value-based trigger, A_C.

The following Condition 1 is assumed to hold **always**.

No Prior Default Condition 1: c_b, c_c, A_C and λ are such that the firm does not default prior to or at CCB conversion.

We will discuss Condition 1 in Section 2.2.2, after we obtain closed-form solutions for the values of different claims to the cash flows of the firm.

There are two results from the existing financial structure literature that will be used later in the paper. First, as in Duffie (2001), for a given constant $K \in (0, A_t)$, the market value of a security that claims one unit of account at the hitting time $\tau(K) = \inf\{s : A_s \leq K\}$ is, at any $t < \tau(K)$,

$$E_t^Q\left[e^{-r(\tau(K)-t)}\right] = \left(\frac{A_t}{K}\right)^{-\gamma}, \tag{2.2}$$

where $\gamma = \frac{m+\sqrt{m^2+2r\sigma^2}}{\sigma^2}$ and $m = \mu - \frac{\sigma^2}{2}$.

Second, also as in Duffie (2001), the default-triggering asset level that corresponds to the optimal default time $\tau(A_B)$ for the case when the capital structure of the firm

[10] $\frac{c_c}{r}$ is the risk-free value of CCB and, therefore, we call it the face value.

includes only equity and straight debt is, at any $t < \tau(A_B)$,

$$A_B = \beta(1-\theta)c_b, \tag{2.3}$$

where $\beta = \frac{\gamma}{r(1+\gamma)}$.

Lemma 1. *Let the capital structure of the firm include equity and straight debt. If at any time t before default the firm decides to issue CCB without changing the existing amount of straight debt, the optimal default boundary A_B will remain the same.*

Proof. We assume that Condition 1 holds. Therefore, there is no default before or at conversion. At conversion CCB holders become equity holders. The value of assets does not change. After conversion the maximum-equity-valuation problem of equity holders (including the ones that became equity holders as the result of conversion) is the same as in the case when the capital structure includes only equity and straight bond. Hence, the same A_B. □

2.2.1 Closed-Form Solutions

Our goal in this subsection is to derive closed-form solutions for the values of claims associated with the capital structure when the firm issues equity, straight debt and CCB.

At any time t the following budget equation holds:

$$\begin{aligned} A_t + TB(A_t; c_b, c_c) &= W(A_t; c_b, c_c) + U^B(A_t; c_b, c_c) + \\ & \quad U^C(A_t; c_b, c_c) + BC(A_t; c_b, c_c), \end{aligned} \tag{2.4}$$

where $TB(A_t; c_b, c_c)$ is the expected present value of tax benefits, $W(A_t; c_b, c_c)$ is the value of equity, $U^B(A_t; c_b, c_c)$ is the value of straight debt, $U^C(A_t; c_b, c_c)$ is the value of CCB and $BC(A_t; c_b, c_c)$ is the expected present value of bankruptcy costs[11].

The total value of the firm, $G(A_t; c_b, c_c)$, is the sum of the market values of equity and debt

$$G(A_t; c_b, c_c) = W(A_t; c_b, c_c) + U^B(A_t; c_b, c_c) + U^C(A_t; c_b, c_c). \tag{2.5}$$

[11]Regarding our notations, in the future $c_c = 0$ will mean that CCB is *not* used. For instance, $U^B(A_t; c_b, 0)$ is the market value of straight debt at time t for a firm that issued only equity and straight debt with coupon c_b.

Based on (2.5), this can be re-written as

$$G(A_t; c_b, c_c) \;=\; A_t + TB(A_t; c_b, c_c) - BC(A_t; c_b, c_c). \qquad (2.6)$$

Proposition 1. *Let the capital structure of the firm include equity, straight debt and CCB. Then, for any* $t < \tau(A_C)$

$$\begin{aligned}
G(A_t; c_b, c_c) &= A_t + \frac{\theta c_b}{r}\left(1 - \left(\frac{A_t}{A_B}\right)^{-\gamma}\right) + \frac{\theta c_c}{r}\left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right) - \\
&\quad \alpha A_B \left(\frac{A_t}{A_B}\right)^{-\gamma}, \\
W(A_t; c_b, c_c) &= A_t - \frac{c_b(1-\theta)}{r}\left(1 - \left(\frac{A_t}{A_B}\right)^{-\gamma}\right) - \frac{c_c(1-\theta)}{r}\left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right) - \\
&\quad A_B \left(\frac{A_t}{A_B}\right)^{-\gamma} - \left(\lambda \frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma}, \\
U^B(A_t; c_b, c_c) &= \frac{c_b}{r}\left(1 - \left(\frac{A_t}{A_B}\right)^{-\gamma}\right) + \left(\frac{A_t}{A_B}\right)^{-\gamma}(1-\alpha)A_B, \\
U^C(A_t; c_c) &= \frac{c_c}{r}\left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right) + \left(\frac{A_t}{A_C}\right)^{-\gamma}\left(\lambda \frac{c_c}{r}\right), \\
TB(A_t; c_b, c_c) &= \frac{\theta c_b}{r}\left(1 - \left(\frac{A_t}{A_B}\right)^{-\gamma}\right) + \frac{\theta c_c}{r}\left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right), \\
BC(A_t; c_b, c_c) &= \alpha A_B \left(\frac{A_t}{A_B}\right)^{-\gamma}.
\end{aligned}$$

We note that a full solution must also incorporate the No Prior Default Condition 1 stated above.

It follows from Proposition 1 that the value of straight debt, $U^B(A_t; c_b, c_c)$, and the cost of bankruptcy, $BC(A_t; c_b, c_c)$, are not affected by the presence of CCB[12].

The total value of tax benefits, $TB(A_t; c_b, c_c)$, includes two parts:

1. the benefits associated with straight bond

$$TB^B(A_t; c_b, c_c) \;=\; \frac{\theta c_b}{r}\left(1 - \left(\frac{A_t}{A_B}\right)^{-\gamma}\right)$$

[12] Although $U^B(A_t; c_b, c_c)$ and $BC(A_t; c_b, c_c)$ do not depend on c_c, we use it in our notations when the capital structure of the firm includes CCB.

2. and the benefits associated with CCB

$$TB^C(A_t; c_b, c_c) \;=\; \frac{\theta c_c}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right). \qquad (2.7)$$

2.2.2 CCB Parameter Choice under the No Prior Default Condition 1

In this section we use the closed-form solution for the value of equity from Proposition 1 to analyze Condition 1.

We stat with noting that one can generally interpret Condition 1 as a requirement to choose A_C so that to avoid default before or at conversion. The firm defaults when the value of equity becomes negative. Therefore, to satisfy this condition the level of assets should remain sufficiently high and the value of equity should stays positive before A_t hits A_C. The lowest level of A_C that satisfies Condition 1 can be defined as follows:

$$A_{CL} \;=\; \inf\{A_C : W(A_s; c_b, c_c) \geq 0,\ for\ any\ s \leq \tau(A_C)\}. \qquad (2.8)$$

Based on Proposition 1, in addition to A_C, the value of equity depends on c_b, c_c and λ. Therefore, these parameters are part of Condition 1 as well.

Next we use the closed-form solution for $W(A_t; c_b, c_c)$ from Proposition 1 and consider several numerical examples. The focus is on A_C.

Let $r = 5.00\%$, $\mu = 1.00\%$, $\theta = 35.0\%$, $\alpha = 50.0\%$ and the *current* value of assets be at \$100.0. Figure 2.1 shows how the value of equity, $W(A_t; c_b, c_c)$, depends on *future* realizations of the value of assets, A_t. (The above parameters are shared across all subfigures in Figure 2.1. Other, specific parameters are shown in the descriptions of the corresponding subfigures.)

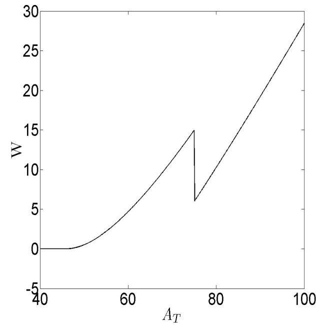
We start with the top three subfigures: (a)-(c). What distinguishes these subfigures among each other are the values of A_C: \$75.0, \$66.9 and \$60.0.

In subfigure (a) the conversion-triggering asset level is set high, $A_C = \$75$. Condition 1 is satisfied and it does not bind. In subfigure (b) $A_C = \$66.9$. Condition 1 is satisfied and it binds. For the chosen set of parameters this is the lowest possible level of A_C that satisfies Condition 1. Finally, in subfigure (c), which corresponds to $A_C = \$60$, Condition 1 is violated. The value of equity becomes negative and the firm defaults before conversion.

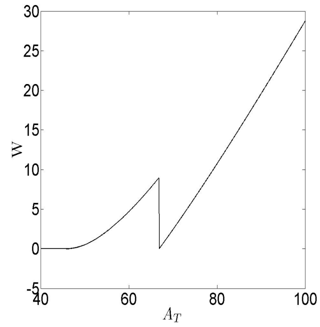
'Very' low values of λ are the main features of subfigures (d) and (e) (compared

Figure 2.1: Equity Value: Different Conversion-Triggering Asset Levels

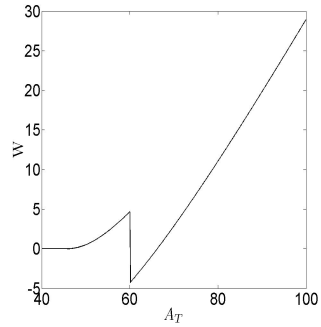
The figure shows how the value of equity, $W(A_t; c_b, c_c)$, based on the closed-form solution from Propositions 1, depends on asset value realizations, A_t. The corresponding parameter values, in addition to the ones shown under the subfigures, are as follows: $r = 5.00\%$, $\mu = 1.00\%$, $\theta = 35.0\%$, $\alpha = 50.0\%$.



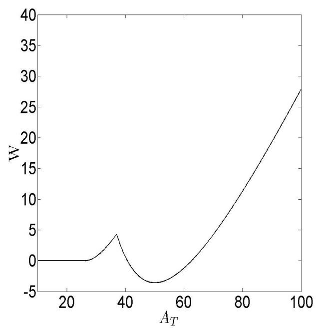
(a) A_C=\$75.0, c_c=\$0.5, A_B = \$45.85, c_b=\$5.24, λ=0.9



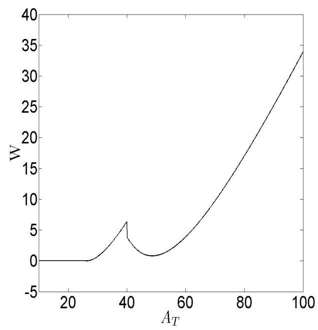
(b) A_C=\$66.9, c_c=\$0.5, A_B = \$45.85, c_b=\$5.24, λ=0.9



(c) A_C=\$60.0, c_c=\$0.5, A_B = \$45.85, c_b=\$5.24, λ=0.9



(d) A_C=\$37.0, c_c=\$3.0, A_B = \$26.23, c_b=\$3.0, λ=0



(e) A_C=\$40.0, c_c=\$2.5, A_B = \$26.23, c_b=\$3.0, λ=0.05

to subfigures (a)-(c)). In subfigure (d) the value of equity becomes negative before A_t hits $A_C = \$37.0$ which violates Condition 1. In subfigure (e) the conversion-triggering asset level is set higher, $A_C = \$40.0$, and Condition 1 holds and binds. In both subfigures, however, low values of λ lead to non-monotonicity of equity values as functions of A_t before conversion.

When required in the future we will explicitly avoid non-monotonicity of equity values in A_t by imposing Condition 2.

Condition 2: c_b, c_c, A_C and λ are such that equity value, $W(A_t; c_b, c_c)$, is strictly increasing in asset level, A_t, for $A_t \geq A_C$.

Given Condition 2, since $W(A_t; c_b, c_c)$ is strictly increasing in A_t, (with no asymmetric information) there is a one-to-one correspondence between equity and assets values for $A_t \geq A_C$. Therefore, the conversion condition for CCB can be formulated in terms of equity values. The CCB debt converts into equity if the value of equity drops to $W_C = W(A_C; c_b, c_c)$. This becomes important since equity prices are observable while asset values are not. We will use A_C and W_C interchangeably - A_C is more convenient for analytical calculations and W_C is better for implementations.

2.3 Optimal Capital Structure

Assume that at time t the firm has no debt but is planning to leverage up by issuing both straight and CCB. The owners (either equity holders or the original owners of the private firm) *fix the amount of CCB they plan to issue by setting A_C, c_c and λ first.* Then, they maximize the total value of the firm by finding an optimal amount of straight debt. We look at how the resulting capital structure compares to the optimal capital structure without CCB.

Theorem 1. *Assuming that the amount of CCB is sufficiently low and parameters c_b, c_c, A_C and λ satisfy Condition 1, the optimal amount of straight debt in a capital structure that includes CCB, equity and straight debt equals the amount of straight debt in the optimal capital structure that includes only equity and straight debt. The coupon on straight debt is the same for both cases*

$$c_b^* \quad = \quad \frac{A_t}{\beta(1-\theta)} \left(\frac{\theta}{r}\right)^{\frac{1}{\gamma}} \left[(\gamma+1)(\frac{\theta}{r} + \alpha\beta(1-\theta))\right]^{-\frac{1}{\gamma}}. \tag{2.9}$$

The firm does not default before or at conversion and, therefore, as in the proof of Lemma 1, after conversion the maximum-equity-valuation problem of equity holder is the same as in the case when the capital structure includes only equity and straight debt. This leads to the same optimal amount of straight debt. Note, that c_b^* depends neither on c_c nor on A_C or λ.

Proposition 2. *If the firm chooses a capital structure that includes CCB, equity and the optimal amount of straight debt then, compared to the case when it chooses the optimal capital structure that includes only equity and straight debt,*

1. *the total value of the firm will be higher by the amount of tax savings associated with c_c*

$$G(A_t; c_b^*, c_c) \quad = \quad G(A_t; c_b^*, 0) + TB^C(A_t; c_b^*, c_c)$$

2. adjusted for tax benefits, equity will be crowded by CCB one-to-one

$$W(A_t; c_b^*, c_c) \quad = \quad W(A_t; c_b^*, 0) - U^C(A_t; c_b^*, c_c) + TB^C(A_t; c_b^*, c_c) \quad (2.10)$$

3. the total value of tax benefits will be higher by the amount of savings associated with c_c

$$TB(A_t; c_b^*, c_c) \quad = \quad TB(A_t; c_b^*, 0) + TB^C(A_t; c_b^*, c_c)$$

4. and, the values of straight debt and bankruptcy costs will be the same

$$\begin{aligned} U^B(A_t; c_b^*, c_c) &= U^B(A_t; c_b^*, 0), \\ BC(A_t; c_b^*, c_c) &= BC(A_t; c_b^*, 0). \end{aligned}$$

The owners of the firm will issue CCB as it increases the total value of the firm by the amount of additional tax savings. The amount of straight debt does not change as CCB is issued on top of the optimal amount of straight debt. Therefore, allowing firms to introduce CCB to their capital structures in the way described above will create extra social costs in the form of additional tax subsidies. The cost of bankruptcy and timing of default will remain the same so the quality of straight debt will not improve.

2.3.1 Leveraged Firm with Suboptimal Amount of Straight Debt

We started this section with the firm being unlevered. Assume instead that the firm has a capital structure that includes equity and straight debt paying $\tilde{c}_b$ (not necessarily equal to c_b^*) and decides to issue CCB without changing the amount of straight debt. Based to Lemma 1, the default boundary does not change and, therefore, issuing CCB does not affect the values of straight debt and bankruptcy costs. The total value of the firm increases by the value of tax benefits associated with CCB, $G(A_t; \tilde{c}_b, c_c) = G(A_t; \tilde{c}_b, 0) + TB^C(A_t; \tilde{c}_b, c_c)$. And, based on budget equation (2.5), the new value of equity is

$$W(A_t; \tilde{c}_b, c_c) \quad = \quad W(A_t; \tilde{c}_b, 0) - \left[U^C(A_t; \tilde{c}_b, c_c) - TB^C(A_t; \tilde{c}_b, c_c)\right].$$

These results are similar to the ones from Proposition 2. Equity holders are willing to issue CCB as it increases their overall value. Although they experience a drop in the value of their holdings in the amount of $U^C(A_t; \tilde{c}_b, c_c) - TB^C(A_t; \tilde{c}_b, c_c)$, they collect dividends in the amount of $U^C(A_t; \tilde{c}_b, c_c)$. Extra social costs are created. The quality of straight debt remains the same.

2.4 CCB Instead of Straight Debt

In Section 2.3 CCB was issued *on top* of straight debt. We move now to cases when CCB *replaces* a portion of straight debt that is either to be newly issued (in the optimal amount) by the firm when it has no debt or is already part of a capital structure that includes equity and straight debt (not necessarily in the optimal amount). We study the effect of debt replacement on the values of different claims associated with the capital structure of the firm.

2.4.1 Initial Choice of Capital Structure Under Regulatory Constraint

We start with the case when CCB replaces a portion of the optimal amount of straight debt that is to be newly issued. Assume that at time t the firm has no debt but is planning to leverage up. Instead of issuing an optimal amount of straight debt, $U^B(A_t, A_B^*; c_b^*, 0)$[13], it has an option to issue both straight and CCB under a regulatory constraint. Regulators fix the amount of straight debt, $U^B(A_t, \bar{A}_B; \bar{c}_b, c_c)$, and the amount of CCB, $U^C(A_t, \bar{A}_B; \bar{c}_b, c_c)$, so that

$$U^B(A_t, \bar{A}_B; \bar{c}_b, c_c) + U^C(A_t, \bar{A}_B; \bar{c}_b, c_c) \quad = \quad U^B(A_t, A_B^*; c_b^*, 0). \qquad (2.11)$$

The total amount of debt equals the optimal amount of straight debt when the capital structure of the firm includes only equity and straight debt.

The same amount of CCB can be issued with different coupons and conversion-triggering asset levels. The firm, for instance, can pick A_C and find c_c by solving

[13] We slightly change notations in order to keep track of the corresponding default boundaries. For instance, in the case of $U^B(A_t, A_B^*; c_b^*, 0)$ the default boundary is A_B^*.

(2.11) as shown below.

$$
\begin{aligned}
&U^C(A_t, \bar{A}_B; \bar{c_b}, c_c) = U^B(A_t, A_B^*; c_b^*, 0) - U^B(A_t, \bar{A}_B; \bar{c_b}, c_c) \\
&\frac{c_c}{r}\left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right) + \left(\frac{A_t}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right) = U^B(A_t, A_B^*; c_b^*, 0) - U^B(A_t, \bar{A}_B; \bar{c_b}, c_c) \\
&c_c = \frac{U^B(A_t, A_B^*; c_b^*, 0) - U^B(A_t, \bar{A}_B; \bar{c_b}, c_c)}{\frac{1}{r}\left(1 - (1-\lambda)\left(\frac{A_t}{A_C}\right)^{-\gamma}\right)}. \qquad (2.12)
\end{aligned}
$$

We investigate if the firm would prefer the optimal capital structure that includes only equity and straight debt to the one that includes equity, straight debt and CCB but is subject to regulatory constraint (2.11).

Proposition 3. *If instead of the optimal capital structure that includes equity and straight debt an unlevered firm chooses its capital structure based on regulatory constraint (2.11) then*

1. *the change in the total value of the firm will equal the difference in the corresponding values of equity*

$$
\begin{aligned}
&G(A_t, \bar{A}_B; \bar{c_b}, c_c) - G(A_t, A_B^*; c_b^*, 0) = W(A_t, \bar{A}_B; \bar{c_b}, c_c) - W(A_t, A_B^*; c_b^*, 0) = \\
&(\theta + \alpha - \theta\alpha)\left(\left(\frac{A_t}{A_B^*}\right)^{-\gamma} A_B^* - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} \bar{A}_B\right) - \theta\left(\frac{A_t}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right) \qquad (2.13)
\end{aligned}
$$

2. *if coupon c_c is sufficiently small, the total value of the firm will be higher: there exists $\bar{c}_1$ such that $G(A_t, \bar{A}_B; \bar{c_b}, c_c) > G(A_t, A_B^*; c_b^*, 0)$ for any $c_c \in (0, \bar{c}_1)$*

3. *the cost of bankruptcy will be lower, $BC(A_t, \bar{A}_B; \bar{c_b}) < BC(A_t, A_B^*; c_b^*)$.*

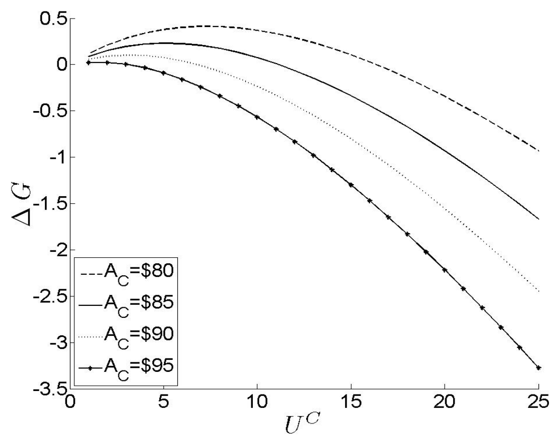
The key result is that the owners of the firm gain from replacing a small amount ($c_c \in (0, \bar{c}_1)$) of straight debt with CCB. The intuition is that the tax savings associated with coupon payments decrease due to $\tau(A_C) < \tau(A_B^*)$, but the firm benefits from reducing its bankruptcy costs due to a smaller amount of straight debt after the replacement. For small amounts of CCB the benefits exceed the lost tax savings.

The amount of CCB that the firm can issue is set by regulators exogenously, via constraint (2.11). Therefore, for the firm to be willing to replace straight debt with CCB, regulators need know how to set the constraint so that c_c does not exceed $\bar{c}_1$.

We continue by numerically analyzing the effects of issuing CCB instead of straight debt. We denote $G(A_t, \bar{A}_B; \bar{c_b}, c_c) - G(A_t, A_B^*; c_b^*, 0)$ by ΔG and use (2.13) to show how the total value of the firm changes depending on how much of the optimal amount of straight debt is being replaced with CCB.

Figure 2.2: The Effect of Replacing Straight Debt with CCB on the Firm Value - *De Novo* Capital Structure

The figure shows changed in the total value of the firm, $\Delta G = G(A_t, \bar{A}_B; \bar{c_b}, c_c) - G(A_t, A_B^*; c_b^*, 0)$, as the result of issuing CCB instead of straight debt, so that regulatory condition 2.11 holds. The corresponding parameter values are as follows: $A_t = \$100$, $r = 0.05$, $\mu = 0.01$, $\sigma = 15\%$, $\theta = 35\%$, $\alpha = 50\%$, $\lambda = 1$; $c_b^* = \$5.24$ and $U^B(A_t, A_B^*; c_b^*, 0) = \$88.36$.



Consider an unlevered firm with parameters in the description of Figure 2.2. The optimal amount of straight debt that can be issued without the regulatory constraint is $U^B(A_t, A_B^*; c_b^*, 0) = \$88.36$. The amount of straight debt that is being replaced subject to constraint (2.11) ranges from \$1 to \$25. As mentioned above, the same amount of CCB can be issued with different conversion-triggering asset levels. For each value of U^C we consider four A_C values: \$80, \$85, \$90, and \$95.

There are three main observations based on Figure 2.2. First, only a portion of the optimal amount of straight debt can be replaced with CCB without lowering the total value of the firm. As U^C gets above (roughly) \$20, ΔG becomes negative and keeps decreasing for all A_C values. Losses in tax benefits due to significant reductions in the amount of straight debt lead to lower total values of the firm. Second, lower A_C values result in higher total values of the firm. (This is consistent with part (**??**) of Proposition **??**.) Curves that correspond to lower A_C values lie strictly above the ones that correspond to higher A_C values. Lower A_C values translate into later conversions and lead to higher tax savings associated with coupon c_c.

Finally, changes in the total value of the firm are non-monotonic in U^C. For lower A_C values they first increase and then decrease. By gradually replacing straight debt with CCB starting with very small amounts, the firm reduces its bankruptcy costs and increases its total value. But, as the amount of CCB keeps increasing, reduced tax savings start dominating the benefits of lower bankruptcy costs which causes the total value of the firm to go down.

In summary, the firm in the above example would prefer a capital structure based on regulatory constraint (2.11) to the optimal capital structure that includes only equity and straight debt.

The main economic result of this section is that letting unlevered firms replace straight debt with CCB in their new, leveraged capital structures creates benefits without additional costs. Total firm values increase and bankruptcy costs decrease. The total amount of debt in the economy remains the same, so there are no extra costs of additional tax subsidies.

2.4.2 Partially Replacing Existing Straight Debt with CCB

We continue with the case when CCB replaces a portion of already existing (not necessarily in the optimal amount) straight debt. Assume that at time t the capital structure of the firm consists of equity and straight debt paying coupon $\hat{c_b}$ (not necessarily equal to c_b^*). The firm wants to issue CCB and swap it for a portion of straight debt in order to reduce $\hat{c_b}$ to $\bar{c_b}$, where $\bar{c_b} < \hat{c_b}$. Once the announcement is made, the market value of straight debt, that is still paying $\hat{c_b}$, will rise from $U^B(A_t, \hat{A}_B; \hat{c_b}, 0)$ to $U^B(A_t, \bar{A}_B; \hat{c_b}, 0)$ to reflect a lower default boundary due to a lesser amount of straight debt after the swap. For the straight debt holders to be indifferent between exchanging their holdings for CCB and continuing to hold straight debt the following budget equation should be true

$$U^B(A_t, \bar{A}_B; \hat{c_b}, 0) \quad = \quad U^C(A_t, \bar{A}_B; \bar{c_b}, c_c) + U^B(A_t, \bar{A}_B; \bar{c_b}, c_c). \qquad (2.14)$$

The updated value of existing straight debt *post* announcement should equal the value of CCB plus the value of straight debt that remains after the swap.

Coupon $\bar{c_b}$ is set exogenously and, as before, the same amount of CCB can be issued with different coupons and conversion-triggering asset levels. The firm, for example, could pick $\bar{c_b}$ and A_C first and then find c_c by solving (2.14) as shown below.

$$
\begin{aligned}
& U^C(A_t, \bar{A}_B; \bar{c}_b, c_c) = U^B(A_t, \bar{A}_B; \hat{c}_b, 0) - U^B(A_t, \bar{A}_B; \bar{c}_b, c_c) \\
& \frac{c_c}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right) + \left(\frac{A_t}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right) = \frac{\hat{c}_b}{r}\left(1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) + \qquad (2.15) \\
& \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}(1-\alpha)\bar{A}_B - \frac{\bar{c}_b}{r}\left(1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}(1-\alpha)\bar{A}_B \\
& \frac{c_c}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right) + \left(\frac{A_t}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right) = \frac{(\hat{c}_b - \bar{c}_b)}{r}\left(1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) \qquad (2.16) \\
& c_c = \frac{(\hat{c}_b - \bar{c}_b)\left(1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right)}{1-(1-\lambda)\left(\frac{A_t}{A_C}\right)^{-\gamma}}
\end{aligned}
$$

We analyze if equity holders would be willing to replace some of the existing straight debt with CCB and what effect this replacement would have on the total value of the firm.

Proposition 4. *If a leveraged firm with a capital structure that includes equity and straight debt replaces a portion of straight debt with CCB then*

1. *the value of equity decreases,* $W(A_t, \bar{A}_B; \bar{c}_b, c_c) - W(A_t, \hat{A}_B; \hat{c}_b, 0) < 0$
2. *the change in the total value of the firm is such that*

$$
\begin{aligned}
\text{(a) } G(A_t, \bar{A}_B; \bar{c}_b, c_c) - G(A_t, \hat{A}_B; \hat{c}_b, 0) \; &= \; \frac{\hat{c}_b\theta}{r}\left(\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) \qquad (2.17) \\
& \quad + \alpha\hat{A}_B\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \alpha\bar{A}_B\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} \\
& \quad - \theta\left(\frac{A_t}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right) \\
\text{(b) } G(A_t, \bar{A}_B; \bar{c}_b, c_c) - G(A_t, \hat{A}_B; \hat{c}_b, 0) \; &> \; W(A_t, \bar{A}_B; \bar{c}_b, c_c) - W(A_t, \hat{A}_B; \hat{c}_b, 0)
\end{aligned}
$$

   (c) if $\hat{c}_b \geq c_b^*$ *and* $\lambda \geq 2 - \left(\frac{A_t}{A_C}\right)^{\gamma}$, *then there exists* $\bar{\bar{c}}_1$ *such that* $G(A_t, \bar{A}_B; \bar{c}_b, c_c) > G(A_t, \hat{A}_B; \hat{c}_b, 0)$ *for* $c_c \in (0, \bar{\bar{c}}_1)$

3. *the cost of bankruptcy decreases,* $BC(A_t, \bar{A}_B; \bar{c}_b) < BC(A_t, \hat{A}_B; \hat{c}_b)$.

If the firm is leveraged optimally or over-leveraged compared to its optimal capital structure ($\hat{c_b} \geq c_b^*$) it could benefit in terms of its *total value* from replacing a certain amount ($c_c \in (0, \bar{\bar{c}}_1)$) of straight debt with CCB (issued with $\lambda \geq 2 - \left(\frac{A_t}{A_C}\right)^{\gamma}$[14]). There are two forces at work here. First, as before, replacing straight debt with CCB pushes the tax savings down, but the firm benefits from reducing the cost of bankruptcy. For certain amounts of CCB the benefits will dominate the lost tax savings.

Second, although debt becomes less risky due to $\bar{A}_B < \hat{A}_B$ the total amount of debt increases by the difference between the value of straight debt post announcement, $U^B(A_t, \bar{A}_B; \hat{c_b}, 0)$, and the value of straight debt pre announcement, $U^B(A_t, \hat{A}_B; \hat{c_b}, 0)$. By increasing the total amount of debt while reducing the cost of bankruptcy the firm benefits from relatively higher (compared to the case when the total amount of debt did not change as in Section 2.4.1) tax savings[15]. The presence of these relative benefits is independent of the amount of CCB. The new tax savings for the firm, though, might (if not compensated by the reduction in tax savings due to the use of CCB) translate into additional social costs in the form of extra tax subsidies. As the amount of CCB debt increases ($c_c > \bar{\bar{c}}_1$) lost tax benefits become larger and can turn all the gains, including the ones from lower bankruptcy costs and the additional tax savings, into losses.

Although the total value of the firm could increase, equity holders will not replace voluntarily any amount of existing straight debt with CCB as their value decreases. All the potential gains in the total value of the firm plus a portion the value of equity are passed on to debt holders. The observed effect is due to debt overhang inefficiency.

We analyze the results of Proposition 4 numerically. We plot values of $G(A_t, \bar{A}_B; \bar{c_b}, c_c) - G(A_t, \hat{A}_B; \hat{c_b}, 0)$, denoted by $\Delta\hat{G}$ and computed based on the formula in part (a) of item (2) of the proposition, for a range of CCB values and several conversion-triggering asset levels.

We use the firm from Section 2.4.1. The assumption is that it has issued straight debt in the optimal amount and now is replacing some of this debt with CCB. Coupon $\hat{c_b}$ is set equal to c_b^* and values of the rest of the parameters, including the market value of assets, are exactly the same as in Section 2.4.1 (all shown in the description of Figure 2.3).

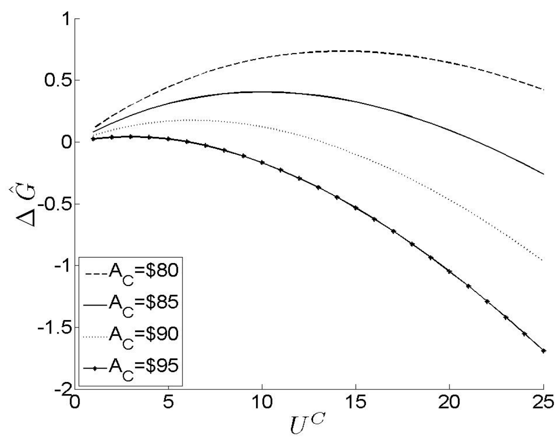
Figures 2.2 and 2.3 are comparable. Figure 2.2 shows how the total value of the firm changes depending on the amount of CCB the firm uses when it leverages

[14]Note, that $2 - \left(\frac{A_t}{A_C}\right)^{\gamma} \leq 1$ for $A_t \geq A_C$.

[15]Based on this, one would expect that, everything else being equal, $\bar{\bar{c}}_1 > \bar{c}_1$.

Figure 2.3: The Effect of Replacing Straight Debt with CCB on the Firm Value - Existing Capital Structure

The figure shows changed in the total value of the firm, $\Delta\hat{G} = G(A_t, \bar{A}_B; \bar{c_b}, c_c) - G(A_t, \hat{A}_B; \hat{c_b}, 0)$, when the firm is already leveraged and replaces straight debt with CCB, so that market constraint (2.14) holds. The corresponding parameter values are as follows: $A_t = \$100$, $r = 0.05$, $\mu = 0.01$, $\sigma = 15\%$, $\theta = 35\%$, $\alpha = 50\%$, $\lambda = 1$; $\hat{c_b} = c_b^* = \$5.24$ and $U^B(A_t, \hat{A}_B; \hat{c_b}, 0) = U^B(A_t, A_B^*; c_b^*, 0) = \$88.36$.



up for the first time based on regulatory constraint (2.11). Figure 2.3 shows how the total value of the firm changes depending on the same amounts of CCB when the firm is already leveraged and replaces straight debt with CCB so that market constraint (2.14) holds. In both cases the firm uses CCB to replace portions of the same (optimal) amount of straight debt.

The general logic of the observations based on Figure 2.2 applies to Figure 2.3, so we are not going to repeat the related discussions from Section 2.4.1.

There is an important difference between the two figures, though. Notice that, if plotted together, the curves from Figure 2.3 would lie above their counterparts from Figure 2.2. Changes in total values of the firm for the current exercise exceed the corresponding changes in total values for the exercise in Section 2.4.1. This is due to relatively larger total amounts of debt post replacement and, correspondingly, higher tax savings.

In summary, in the above exercise all the benefits for the firm and the economy overall from replacing straight debt with CCB discussed in Section 2.4.1 remain.

The key economic result of Section 2.4.2 is that if the firm decided to partially replaces existing straight debt with CCB the total value of the firm would increase while bankruptcy costs together with the total amount of risky straight debt would decrease. Equity holders, however, will never initiate this kind of debt replacement on their own due to debt overhang inefficiency.

2.5 TBTF Firms

In this section we look at firms that are 'too big' for the government to let them fail as they pose systemic risk. In our model the government bails out a TBTF firm by assuming control over its assets and taking over its obligation to make payments to straight debt holders at the point of bankruptcy. We study the possible reduction in the government's TBTF subsidy that may be achieved by requiring partial replacement of straight debt with CCB in the capital structure of a TBTF firm.

Consider a firm with a capital structure that includes equity and straight debt, paying coupon c_b. In the structural model we have used so far the firm reaches bankruptcy when the value of assets, A_t, hits the default boundary level, A_B, for the first time. At that point, if the government decides to step in to prevent bankruptcy, it will obtain assets worth A_B and an obligation to pay c_b forever with the risk-free value of $\frac{c_b}{r}$. Therefore, the value of the government subsidy at the time of bankruptcy is $\frac{c_b}{r} - A_B$.

Given (2.2), at any time t before bankruptcy, the value of subsidy is[16]

$$S(A_t; c_b, 0) = \left(\frac{c_b}{r} - A_B\right)\left(\frac{A_t}{A_B}\right)^{-\gamma}. \qquad (2.18)$$

By definition, a government subsidy prevents the firm from going into default. Therefore, it eliminates bankruptcy costs, $BC(A_t; c_b) = 0$, and makes straight debt default-free, $U^B(A_t; c_b, 0) = \frac{c_b}{r}$.

The optimal time to default $\tau(A_B)$ solves the maximum-equity-valuation problem of equity holders. A government subsidy kicks in at time $\tau(A_B)$ and covers only straight debt obligations. Therefore, it affects neither the timing of default nor the value of $A_B = \beta(1-\theta)c_b$. This implies that, provided that the capital structure does not change, the value of equity remains the same. The government guarantee benefits only the debt holders and does not subsidize equity.

Based on Lemma 1, the time of default and the value of assets at the time of default do not depend on whether the capital structure of the firm includes equity and straight debt or equity, *the same amount of* straight debt and CCB. Therefore,

[16]We return to our initial notations. The default boundary is tied to the corresponding coupon on straight debt based on (2.3).

given (2.18), for a fixed amount of straight debt, c_b, the value of subsidy is the same whether CCB is present or not,

$$S(A_t; c_b, c_c) \quad = \quad S(A_t; c_b, 0). \tag{2.19}$$

Proposition 5. *Let a firm have a capital structure that includes equity and straight debt, paying coupon c_b. If at any time t the government issues a guarantee for the straight debt of the firm, then*

1. *the larger is the amount of outstanding straight debt the larger is the subsidy,* $\frac{dS(A_t;c_b,0)}{dc_b} > 0$
2. *the total value of the firm will increase to*

$$G(A_t; c_b, 0) \quad = \quad A_t + \frac{\theta c_b}{r}\left(1 - \left(\frac{A_t}{A_B}\right)^{-\gamma}\right) + \left(\frac{c_b}{r} - A_B\right)\left(\frac{A_t}{A_B}\right)^{-\gamma}. \tag{2.20}$$

.

2.5.1 Replacing Straight Debt with CCB

Assume that a TBTF firm is currently unlevered but is considering leveraging up. We try to understand how the value of government subsidy would depend on whether the firm chooses to issue straight debt or both straight debt and CCB under a regulatory limit for the total amount of debt.

Based on equation (2.20), the total value of the firm strictly increases in the size of government subsidy[17], which, based on Proposition 5 (part 1), increases with the amount of straight debt. Therefore, equity holders of an unlevered firm would try to issue as much straight debt as possible, collect the proceeds as dividends and default immediately after the issuance[18]. Knowing this, the government could set limits on how much debt a TBTF firm could issue. It could set a maximum straight debt coupon c_b^g for the capital structure that includes equity and straight debt. It could also offer the firm an alternative to issue straight debt with coupon $\bar{c_b}$ and CCB with coupon c_c such that the total amount of debt is fixed

$$U^B(A_t; c_b^g, 0) \quad = \quad U^C(A_t; \bar{c_b}, c_c) + U^B(A_t; \bar{c_b}, c_c). \tag{2.21}$$

[17] The last term on the right-hand side of equation (2.20) equals the value of subsidy from equation (2.18).

[18] Although the market would know that the firm is going default, due to the presence of the government guarantee, the firm would still be able to sell this debt.

In the presence of a government guarantee, straight debt is risk-free. Therefore, given the closed-form solution for $U^C(A_t;\bar{c_b},c_c)$ from Proposition 1, equation(2.21) can be re-written as

$$\frac{c_b^g}{r} = \frac{c_c}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right)+\lambda\frac{c_c}{r}\left(\frac{A_t}{A_C}\right)^{-\gamma}+\frac{\bar{c_b}}{r}.$$

This leads to

$$\bar{c_b} = c_b^g - c_c\left(1-(1-\lambda)\left(\frac{A_t}{A_C}\right)^{-\gamma}\right). \tag{2.22}$$

Note, that $c_b^g > \bar{c_b}$. Based on equation (2.19) and Proposition 5 (part 1),

$$S(A_t;c_b^g,0) > S(A_t;\bar{c_b},c_c). \tag{2.23}$$

Replacing straight debt with CCB reduces the value of subsidy. Also, based on equation (2.22), the higher is c_c the lower is $\bar{c_b}$. The larger is the portion of CCB, the smaller is the amount of the remaining straight debt and, therefore, the lower is the value of the government subsidy.

A government subsidy eliminates bankruptcy costs, $BC(A_t;\hat{c_b},0)=0$, and, similar to equation (2.6), the total value of the firm for the case when it does not issue CCB is

$$G(A_t;c_b^g,0) = A_t + TB(A_t;c_b^g,0)+S(A_t;c_b^g,0).$$

Equivalently, the total value of the firm when it does issue CCB is

$$G(A_t;\bar{c_b},c_c) = A_t + TB(A_t;\bar{c_b},c_c)+S(A_t;\bar{c_b},c_c).$$

Consider the difference

$$G(A_t;c_b^g,0)-G(A_t;\bar{c_b},c_c) = TB(A_t;c_b^g,0)-TB(A_t;\bar{c_b},c_c)+S(A_t;c_b^g,0)- S(A_t;\bar{c_b},c_c). \tag{2.24}$$

Based on equation (2.23), the last difference on the right-hand side of the above equation is strictly positive, $S(A_t;c_b^g,0)-S(A_t;\bar{c_b},c_c)>0$. In the appendix (see proof of Proposition 6) we also show that $G(A_t;c_b^g,0)-G(A_t;\bar{c_b},c_c)>0$. The reduction in the value of government subsidy is the main driver behind the reduction in the total value of the firm when straight debt is replaced with CCB.

The above arguments can be summarized in a proposition.

Proposition 6. *Replacing straight debt with CCB in a new capital structure of an unlevered TBTF firm subject to a regulatory limit on how much debt it is allowed to issue*

1. *reduces the value of the government subsidy,* $S(A_t; c_b^g, 0) > S(A_t; \bar{c}_b, c_c)$

2. *reduces the total value of the firm (i.e., the gains of equity holders from leveraging up the firm),* $G(A_t; c_b^g, 0) > G(A_t; \bar{c}_b, c_c)$.

We started this subsection by considering an unlevered TBTF firm. Had the firm been leveraged with straight debt, replacing its existing straight debt with CCB would have led to results similar to the ones captured in Proposition 6.

The key observation is that, due to the government subsidy, the straight debt of a TBTF firm is risk-free, so, when the firm announces the swap of a portion of straight debt for CCB, the straight debt does not appreciate. In other words, there is no debt overhang effect similar to the one we observed in section 2.4.2. Therefore, the regulatory constraint (2.21) is the same whether the firm replaces existing or to-be-newly-issued straight debt with CCB.

Proposition 7. *Replacing straight debt with CCB in a capital structure of a TBTF firm that includes equity and straight debt*

1. *reduces the value of the government subsidy*

2. *reduces the value of equity holders.*

Propositions 6 and 7 suggest that equity holders of a TBTF firm will oppose issuing CCB.

2.6 Multiple Equilibrium Equity Prices and Market Manipulations

Our goal in this section is to present two equity-value issues that may arise when firms use CCB. First, we show the existence of multiple equilibrium equity prices, and, second, we look at the incentives of market participants to manipulate the equity market. We distinguish between manipulations by CCB holders and the ones by equity holders.

We assume Condition 2 and, therefore, the value of equity is strictly increasing in the value of assets before and at conversion. This allows us to formulate a conversion rule for CCB directly in terms of equity value - CCB converts into equity when the equity value drops to $W_C = W(A_C; c_b, c_c)$. W_C corresponds to A_C in the earlier analysis.

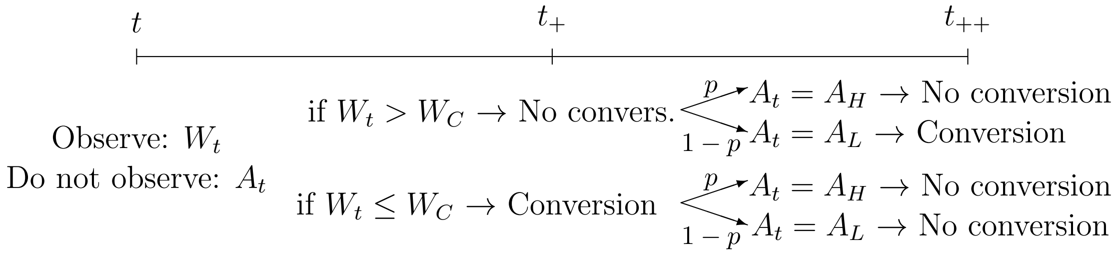
2.6.1 Two Equilibrium Equity Prices

We start by showing the existence of two different equilibrium prices for the equity of a firm that issues straight debt, equity and CCB.

As it is captured in Figure 2.4, consider three time instances t, t_+ and t_{++}, where $t < t_+ < t_{++}$.

At time t the market value of assets, A_t, is uncertain. The uncertainty is resolved at t_{++} when A_t takes either the value of A_H with probability p or the value of A_L with probability $(1-p)$. A_H, A_L and A_C are such that $A_L < A_C < A_H$.

Figure 2.4: Equity Price, Asset Value and CCB Conversion Decisions
The figure shows two equilibria at time t_+: equity price, W_t, above the conversion level, W_C, with no conversion, and equity price, W_t, at or low the conversion level, W_C, with conversion.



The sequence of possible events is as follows. The market value of equity, W_t, is observed at time t. If $W_t \leq W_C$, then at time t_+ CCB converts into equity. Otherwise, there is no conversion at t_+. The realized market value of assets, A_t, is observed at t_{++}. If the realization is A_L and there was no conversion at t_+ CCB converts into equity at time t_{++}. Otherwise, there is no conversion at t_{++}.

Consider the case when there is no conversion (i.e., $W_t > W_C$) at time t_+. If the realization of A_t at time t_{++} is A_H, then the value of old equity at time t_{++} is $W(A_H; c_b, c_c)$. This value reflects the fact that, due to $A_C < A_H$, A_H does not trigger conversion at t_{++}. There are no new equity holders.

On the other hand, if the realization of A_t at time t_{++} is A_L, then the value of old equity at time t_{++} is $W(A_L; c_b, 0) - \lambda \frac{c_c}{r}$. In this case, due to $A_L < A_C$, A_L does trigger conversion at t_{++}. The value of old equity equals the value of total (old and new) equity post conversion minus the value of new equity issued to replace CCB.

We denote the value of old (observed) equity at time t_+ when there is no conversion at t_+ by $\bar{W}_t$. It can be calculated as the expected value of old equity at time t_{++}:

$$\bar{W}_t = pW(A_H; c_b, c_c) + (1-p)\left(W(A_L; c_b, 0) - \lambda\frac{c_c}{r}\right). \tag{2.25}$$

Now consider the case when at time t_+ CCB does convert into equity (i.e., $W_t \leq W_C$). We denote the value of old (observed) equity at time t_+ when there is conversion at t_+ by $\hat{W}_t$. Then, the value of total (old and new) equity post conversion at time t_+ is $\hat{W}_t + \lambda\frac{c_c}{r}$. The values of total equity at time t_{++} for A_H and A_L are $W(A_H; c_b, 0)$ and $W(A_L; c_b, 0)$, correspondingly. And, the value of total equity at time t_+ is its expected value at time t_{++}:

$$\hat{W}_t + \lambda\frac{c_c}{r} = pW(A_H; c_b, 0) + (1-p)W(A_L; c_b, 0).$$

This leads to

$$\hat{W}_t = pW(A_H; c_b, 0) + (1-p)W(A_L; c_b, 0) - \lambda\frac{c_c}{r}. \tag{2.26}$$

Based on (2.25), (2.26) and Proposition 1, the difference in the observed values of equity for the two cases is

$$\begin{aligned}
\bar{W}_t - \hat{W}_t &= p\left(W(A_H; c_b, c_c) - W(A_H; c_b, 0)\right) - \frac{\lambda c_c}{r} - (1-p)\frac{\lambda c_c}{r} \\
&= p\left(-\frac{c_c(1-\theta)}{r}\left(1-\left(\frac{A_H}{A_C}\right)^{-\gamma}\right) - \frac{\lambda c_c}{r}\left(\frac{A_H}{A_C}\right)^{-\gamma} + \frac{\lambda c_c}{r}\right) \\
&= p\frac{c_c}{r}\left(1-\left(\frac{A_H}{A_C}\right)^{-\gamma}\right)(\lambda+\theta-1).
\end{aligned}$$

If $(\lambda + \theta) > 1$, $\bar{W}_t > \hat{W}_t$.

We proved the following theorem.

Theorem 2. *If $(\lambda + \theta) > 1$, then there could be two market equilibria*

1. *a relatively high equity price at time t and no conversion of CCB into equity at time t_+*

2. *and, a relatively low equity price at time t and conversion of CCB into equity*

at time t_+.

Note that the proof above is based on the assumption that W_C is such that $\hat{W}_t < W_C < \bar{W}_t$. Since $\bar{W}_t$ is strictly higher than $\hat{W}_t$, we can find values of W_C that satisfy the above condition by picking certain values for c_c, p, etc.[19]

The intuition behind Theorem 2 is as follows. Early conversion leads to a guaranteed loss of tax benefits associated with CCB. This leads to a lower value of equity, $\hat{W}_t$. If, on the other hand, the debt is not converted before the uncertainty about the value of assets is resolved, the tax benefits are lost only with probability $(1-p)$. This corresponds to a higher value of equity, $\bar{W}_t$.

2.6.2 Equity Market Manipulations

We continue with analyzing the conditions under which market participants might be willing to manipulate the equity market.

Manipulation by CCB Holders

We start with the case when CCB holders attempt the manipulation.

The motivation is as follows. Market participants might profit by buying CCB when the stock price of the firm is above the conversion-triggering level W_C, driving the price down (by spreading negative news, short selling equity, etc.) in order to trigger conversion and then selling the equity obtained as the result of conversion when the price corrects.

Assume that, as in Section 2.6.1, at time t the market value of assets, A_t, is uncertain. At some future time the uncertainty is resolved and A_t can take the value of A_H with probability p or A_L with probability $(1-p)$. p reflects correct beliefs about realizations A_H and A_L. Also, A_H and A_L are such that, when the true value of A_t is realized, only A_L triggers conversion (i.e., $A_L < A_C < A_H$).

As before, we also assume that the market value of equity is observed before the uncertainty about A_t is resolved.

We model CCB holder manipulations as actions that drive down the equity price by convincing the market that the probability of A_t reaching A_H is p' and the probability of A_t reaching A_L is $(1-p')$, where $p' < p$.

Assume that, if the market believes that the probability of A_t reaching A_H is p (and the probability of A_t reaching A_L is $(1-p)$), the value of equity is above W_C

[19]One can try to specify more explicitly the corresponding values (or ranges of values) for these parameters by using (2.25), (2.26) and Proposition 1.

and, therefore, there is no conversion. On the other hand, if the market believes that the probability is p', CCB does convert into equity.

In expectation the *post* manipulation (i.e., after the stock price is driven down, CCB is converted into equity and the correct belief p is restored) value of equity is

$$\tilde{W}_t = pW(A_H; c_b, 0) + (1-p)W(A_L; c_b, 0).$$

When the market is manipulated, at the point of conversion, the value of equity is

$$\tilde{\tilde{W}}_t = p'W(A_H; c_b, 0) + (1-p')W(A_L; c_b, 0).$$

CCB holders receive equity in the amount of $\lambda\frac{c_c}{r}$. As the market belief corrects, the value of equity changes from $\tilde{\tilde{W}}_t$ to $\tilde{W}_t$. Therefore, the expected value of the payoff to (former) CCB holders after the market corrects is

$$\Pi'_t = \lambda\frac{c_c}{r}\frac{pW(A_H; c_b, 0) - (1-p)W(A_L; c_b, 0)}{p'W(A_H; c_b, 0) - (1-p')W(A_L; c_b, 0)}.$$

If there is no manipulation that triggers conversion, the expected value of the payoffs to CCB holders is

$$\Pi_t = pU^C(A_H; c_b, c_c) + (1-p)\lambda\frac{c_c}{r}.$$

Consider the difference in these two values

$$\Pi'_t - \Pi_t = \lambda\frac{c_c}{r}\frac{(p-p')(W(A_H; c_b, 0) - W(A_L; c_b, 0))}{p'W(A_H; c_b, 0) - (1-p')W(A_L; c_b, 0)} - p\left(U^C(A_H; c_b, c_c) - \lambda\frac{c_c}{r}\right).$$

By using the closed-form solution for $U^C(A_H; c_b, c_c)$ from Proposition 1 and rearranging terms, we get

$$\begin{aligned}\Pi'_t - \Pi_t &= \lambda\frac{c_c}{r}\frac{(p-p')(W(A_H; c_b, 0) - W(A_L; c_b, 0))}{p'W(A_H; c_b, 0) - (1-p')W(A_L; c_b, 0)} - \\ &\quad p(1-\lambda)\frac{c_c}{r}\left(1 - \left(\frac{A_H}{A_C}\right)^{-\gamma}\right) \\ &= \lambda\frac{c_c}{r}\frac{p-p'}{p' + \frac{W(A_L;c_b,0)}{W(A_H;c_b,0)-W(A_L;c_b,0)}} - p(1-\lambda)\frac{c_c}{r}\left(1 - \left(\frac{A_H}{A_C}\right)^{-\gamma}\right). \quad (2.27)\end{aligned}$$

It's easy to see that if $\lambda = 0$, based on equation (2.27), $\Pi'_t < \Pi_t$ and, therefore, CCB holders do not have an incentive to manipulate the market.

Also from (2.27), $\Pi'_t - \Pi_t$ is strictly increasing in λ and the value of λ for which the difference in the two payoffs is zero is

$$\lambda^* = \frac{p\left(1-\left(\frac{A_H}{A_C}\right)\right)^{-\gamma}}{(p-p')\left(p'+\frac{W(A_L;c_b,0)}{W(A_H;c_b,0)-W(A_L;c_b,0)}\right)^{-1}+p\left(1-\left(\frac{A_H}{A_C}\right)\right)^{-\gamma}}. \tag{2.28}$$

Clearly, $\lambda^* > 0$. Also, since $(p-p')\left(p'+\frac{W(A_L;c_b,0)}{W(A_H;c_b,0)-W(A_L;c_b,0)}\right)^{-1} > 0$, $\lambda^* < 1$.

The above arguments are summarized in Theorem 3.

Theorem 3. *There exists $\lambda^* \in (0,1)$ such that $\Pi_t - \Pi'_t = 0$ and*

1. *if $\lambda \leq \lambda^*$ CCB holders will not manipulate the equity market*
2. *and, if $\lambda > \lambda^*$ CCB holders will manipulate the equity market.*

The intuition for why small values of λ should prevent manipulation is as follows. At conversion, CCB holders give up a stream of future coupon payments for the value of $\lambda\frac{c_c}{r}$. Small values of λ mean that, even after we account for the appreciation of equity post conversion, the value CCB holders receive is too small compared to the value of future coupon payments they need to give up. Therefore, CCB holders will not try to force conversion.

Based on equation (2.28), there are two major drivers behind the value of λ^*. The first one is the distance between the probabilities p and p'. The bigger is the difference $(p-p')$ the lower is λ^*. The interpretation is that the greater the benefit of a manipulation, the lower should the conversion ratio be in order to avoid manipulation.

The second driver is the difference between equity values for asset realizations A_H and A_L. Here, again, the bigger is the difference $(W(A_H;c_b,0) - W(A_L;c_b,0))$ the lower is λ^*. That is, the greater the possible asset range, the greater the benefit of a manipulation, and the lower should the conversion ratio be.

Manipulation by Equity Holders

We turn to the case when equity holders might attempt to manipulate the market.

The motivation is that equity holders might increase the value of their holdings by manipulating the equity price down to W_C, triggering conversion, and then correcting the market belief. The potential value increase arises if the obligation to pay the CCB holders c_c is removed through conversion at below the market value.

The value of (old) equity holders before they attempt to manipulate the market is $W(A_t;c_b,c_c)$. At the point of conversion the value of equity is $W(A_C;c_b,0)$. As the market belief corrects, the value of equity rises to $W(A_t;c_b,0)$. The new value of (old) equity holders equals the difference between $W(A_t;c_b,0)$ and the value of (new) equity that belongs to former CCB holders, $\left(\lambda\frac{c_c}{r}\right)\frac{W(A_t;c_b,0)}{W(A_C;c_b,0)}$. (Old) Equity holders will not manipulate the market if

$$W(A_t;c_b,c_c) - \left[W(A_t;c_b,0) - \lambda\frac{c_c}{r}\frac{W(A_t;c_b,0)}{W(A_C;c_b,0)}\right] \;\geq\; 0.$$

Based on Proposition 1

$$W(A_t;c_b,c_c) - \left[W(A_t;c_b,0) - \lambda\frac{c_c}{r}\frac{W(A_t;c_b,0)}{W(A_C;c_b,0)}\right] =$$
$$-\frac{c_c(1-\theta)}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right) - \frac{\lambda c_c}{r}\left(\frac{A_t}{A_C}\right)^{-\gamma} + \frac{\lambda c_c}{r}\frac{W(A_t;c_b,0)}{W(A_C;c_b,0)}. \quad (2.29)$$

Clearly, when $\lambda = 0$ equity holders will manipulate the market as the right-hand side of equation (2.29) is negative.

For $\theta = 0$ and $\lambda = 1$ the right-hand side of equation (2.29) is strictly positive:

$$-\frac{c_c}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right) - \frac{c_c}{r}\left(\frac{A_t}{A_C}\right)^{-\gamma} + \frac{c_c}{r}\frac{W(A_t;c_b,0)}{W(A_C;c_b,0)} =$$
$$\frac{c_c}{r}\left(\frac{W(A_t;c_b,0)}{W(A_C;c_b,0)} - 1\right) > 0.$$

As θ increases the right-hand side of equation 2.29 only becomes larger. Therefore, for any feasible θ, if $\lambda = 1$, the difference in the equity values is going to be positive. This means that no market manipulations will be taking place.

It is also clear that the right-hand side of equation (2.29) is strictly increasing in λ and the value of λ for which the difference in the equity values is zero is

$$\lambda^{**} \;=\; \frac{(1-\theta)\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right)}{\frac{W(A_t;c_b,0)}{W(A_C;c_b,0)} - \left(\frac{A_t}{A_C}\right)^{-\gamma}}. \quad (2.30)$$

For values of λ higher or equal to λ^{**} the right-hand side of equation (2.29) is going to be non-negative and equity holders will not have an incentive to manipulate the market. On the other hand, for values of λ lower than λ^{**} they will manipulate the market. Note, that λ^{**} is a decreasing function of the asset value A_t. Therefore, for the above observation to be true for any realization of A_t before or at conversion, we choose the highest

$$\lambda^{**} \quad \leq \quad \frac{(1-\theta)\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right)}{\frac{W(A_C;c_b,0)}{W(A_C;c_b,0)}-\left(\frac{A_t}{A_C}\right)^{-\gamma}} = (1-\theta). \tag{2.31}$$

We proved the following theorem.

Theorem 4. *If $\lambda \geq (1-\theta)$ equity holders will not manipulate the equity market, and if $\lambda < (1-\theta)$ equity holders will manipulate the equity market.*

It is intuitive that equity holders will not manipulate the market when λ values are above λ^{**}, since the cost of the obligation to pay c_c is 'too' high.

There maybe additional costs associated with manipulations (by both equity and CCB holders) that we did not take into account. These could include implementation costs, potential penalties, legal fees, etc. The additional costs would make manipulations harder to implement.

2.7 CCB and Asset Substitution Inefficiency

In this section we investigate if including CCB in the capital structure of a firm would make equity owners more willing to switch to riskier technologies by choosing higher asset volatility parameters. There are three cases. First, we consider including CCB in a capital structure as part of a CCB for equity swap that leaves the amount of straight debt unchanged as discussed in Section 2.3. Second, we return to the assumptions of Section 2.4.1 and look at the case when CCB is included in a de novo capital structure as a CCB for straight debt swap under regulatory constraint (2.11). Third, we consider adding CCB to an existing capital structure as a CCB swap for existing straight debt under constraint (2.14), following the assumptions of Section 2.4.2.

2.7.1 CCB Introduced as a CCB for Equity Swap in a *De Novo* Capital Structure

We start by returning to the conditions of Section 2.3, where we found that an unconstrained firm would add CCB to a de novo capital structure as part of a CCB for equity swap. We now consider whether adding CCB to the capital structure of a firm in this manner creates an incentive for the firm to change the riskiness of its technology (i.e. by changing the parameter σ). We have the following result.

Proposition 8. *Assume that the firm has set its capital structure, which includes CCB, equity, and the optimal amount of straight debt. If $(\lambda + \theta) > 1$ ($(\lambda + \theta) < 1$) then, compared to the case of the optimal capital structure that includes only equity and straight debt, the dollar gains of equity holders from switching to riskier technologies will be lower (higher).*

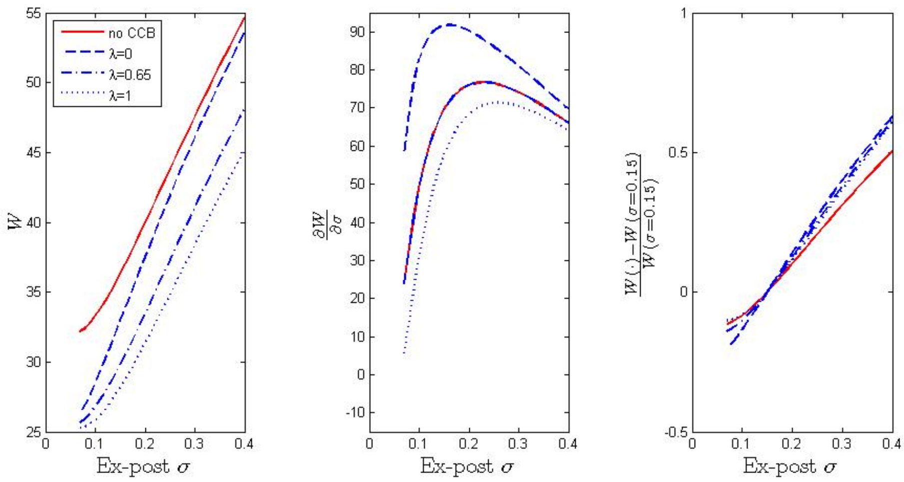
The intuition for Proposition 8 follows from understanding that a higher asset volatility increases the likelihood that the CCB trigger will be reached. From our discussion above, conversion is costly for existing shareholders when $(\lambda + \theta) > 1$, and thus the equity holders in a firm with such a CCB contract will prefer safer technologies. In contrast, CCB conversion is profitable for the existing shareholders when $(\lambda + \theta) < 1$ and the equity holders in a firm with such a CCB contract would then prefer the higher risk technologies.

The applicability of Proposition 8 also requires that the dollar gain in equity value be the proper criterion for assessing asset substitution effects. This is an issue because, for the case in this section, the firm adds CCB to its capital structure through a CCB for equity swap that leaves the CCB firm with a lower equity value than a comparable firm with only straight debt. As long as the firm's shareholders participate proportionately in the CCB for equity swap, the dollar gain in equity remains the proper criterion. On the other hand, if a set of shareholders were to gain a larger proportionate share of the firm as a result of the CCB for equity swap, they could make the asset substitution decision based on the percentage increase in their equity value. This might be true, in particular, for managers with stock options who would be unable to participate in the CCB for equity swap. We now use two numerical exercises to demonstrate that a percentage change in equity value criterion expands the conditions under which the incentive for asset substitution is greater for a CCB firm.

Consider an unlevered firm at time t with the market value of assets $A_t = \$100$ and a constant instantaneous volatility of changes in the value of assets firm $\sigma = 15.0\%$.

Figure 2.5: Ex-Post Equity Values: Single Firm - *De Novo* Capital Structure, No Constraints

The figures show the effect of changes in asset volatility on the value of equity after an unlevered firm sets its capital structure. CCB is issued on top of straight debt. The corresponding parameter values are as follows: $A_t = \$100$, (initial) $\sigma = 15.0\%$, $c_b^* = \$5.24$, $A_C = \$70$, $c_c = \$0.5$, $r = 0.05$, $\mu = 0.01$, $\theta = 35\%$, and $\alpha = 50\%$.

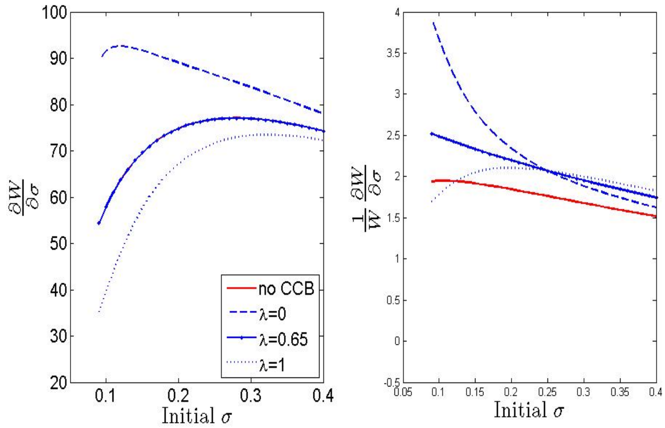


The firm sets an optimal capital structure by leveraging up. Four cases are considered. In the base case the firm issues no CCB. In the remaining three cases the firm issues CCB with $A_C = \$70$, $c_c = \$0.5$ and three different conversion values: $\lambda = 0$, $\lambda = 0.65$, and $\lambda = 1$. Since we set $\theta = 0.35$, the three λ cases correspond to $(\lambda + \theta) < 1$, $(\lambda + \theta) = 1$, and $(\lambda + \theta) > 1$, respectively. Straight debt is issued with coupon $c_b^* = \$5.24$ based on equation (2.9). Values for the remaining parameters are shown in the description of Figure 2.5.

Once the straight debt and CCB are issued, equity holders might attempt to raise the value of their holdings by increasing or decreasing σ. Figure 2.5 helps analyze how the value of equity depends on changes in volatility. The subfigure on the left-hand side graphs the values of equity as functions of ex-post σ values. The subfigure in the middle graphs the slopes of the functions on the left-hand side. Both are consistent with the results of Proposition 8. In particular, in the middle panel, the sensitivity of the equity value to σ is inversely related to the λ value of the respective curves. In addition, the curve for $\lambda = 0.65$ (that is, $(\lambda + \theta) = 1$) coincides exactly with the curve for no CCB. The plot on the right-hand side of Figure 2.5 shows the values of $\frac{W(\sigma)-W(\sigma=0.15)}{W(\sigma=0.15)}$. The key observation is that for ex-post σ values (roughly) above

Figure 2.6: Local Rate of Change and Percentage Change in Ex-Post Equity Values: Multiple Firms - *De Novo* Capital Structure, No Constraints

The figure plots ex-post values of $\frac{\partial W}{\partial \sigma}$ (left panel) and $\frac{1}{W}\frac{\partial W}{\partial \sigma}$ (right panel) for each σ (that is, each firm) and for no CCB and the three λ cases. CCB is issued on top of straight debt. The corresponding parameter values are as follows: $A_t = \$100$, individual initial σ values, $c_b^* = \$5.24$, $A_C = \$70$, $c_c = \$0.5$, $r = 0.05$, $\mu = 0.01$, $\theta = 35\%$, and $\alpha = 50\%$.



20%, independent of λ and θ, relative changes in the value of equity in the presence of CCB are higher than the ones for the base case. The conclusion is that equity holders are generally inclined to increase the volatility ex-post if CCB is part of the capital structure.

Our second exercise looks at a range of firms (each represented by a different initial σ) and evaluates the effects of local changes in volatility. This contrasts with the first exercise in which we considered a single firm for a range of different ex-post σ outcomes. We let the values of A_t, A_C, r, c_c, μ, θ and α be the same as before and the same across all firms. Similar to the first exercise, we consider a base case - when firms issue only straight debt and equity - and three more cases - when the firm includes CCB in its capital structure and we evaluate three different λ values: 0 ($(\lambda + \theta) < 1$), 0.65 ($(\lambda + \theta) = 1$), and 1 ($(\lambda + \theta) > 1$).

Figure 2.6 plots ex-post values of $\frac{\partial W}{\partial \sigma}$ (left panel) and $\frac{1}{W}\frac{\partial W}{\partial \sigma}$ (right panel) for each σ (that is, each firm) and for no CCB and the three λ cases. The left panel shows, again, that the sensitivity of equity to σ is higher the lower the λ value and that the curve with $\lambda = 0.65$ coincides exactly with the no CCB curve. The right panel shows that the percentage change in equity value with respect to σ is always higher for the capital structures with CCB except for the lower end of the $\lambda = 1$ curve. This suggests, that in the presence of CCB, equity holders will generally switch to riskier technologies for the above choice of parameters across all initial σ values (i.e., across all firms). This reinforces the conclusion that CCBs tend to magnify asset substitution inefficiency.

2.7.2 CCB Introduced as a CCB for Straight Debt Swap in a *De Novo* Capital Structure under Regulatory Constraint

We continue with the question of asset substitution inefficiency, but now for a firm that introduces CCB to its capital structure by swapping CCB for straight debt following regulatory constraint (2.11). We repeat the two exercises from Section 2.7.1. Everything remains the same, including the amounts of CCB and the optimal amounts of straight debt that are issued. The only difference is that now CCB replaces a portion of straight debt based on equation (2.11) instead of being added to a capital structure that also includes straight debt.

Figure 2.7: Ex-Post Equity Values: Single Firm - *De Novo* Capital Structure, Regulatory Constraint

The figures show the effect of changes in asset volatility on the value of equity after an unlevered firm sets its capital structure. A portion of straight debt is swapped for CCB. The corresponding parameter values are as follows: $A_t = \$100$, initial $\sigma = 15.0\%$, $c_b^* = \$5.24$, $A_C = \$70$, $c_c = \$0.5$, $r = 0.05$, $\mu = 0.01$, $\theta = 35\%$, and $\alpha = 50\%$.

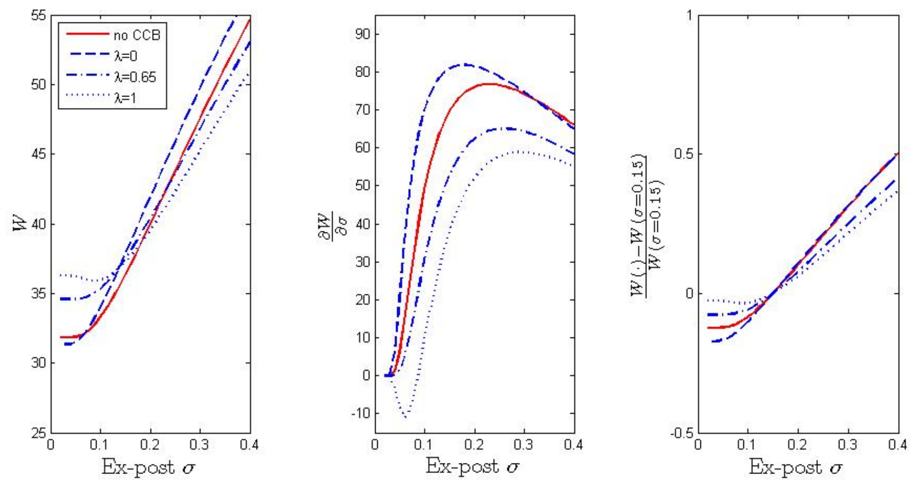


Figure 2.7 helps to analyze how the value of equity changes depending on what level is set to ex-post. The values are shown for a single company (represented by the

initial $\sigma = 15.0\%$) and a range of ex-post σ values. As in Figure 2.5, the level of the graphs in Figure 2.7 are always inversely related to the λ value. The curve with no CCB, however, is now higher, while the addition of CCB now systematically reduces the incentive for asset substitution except for some instances with the extreme value of $\lambda = 0$. The intuition is that compared with the base case of no CCB, the firm now has less straight debt, the effect of which is to reduce the incentive for asset substitution.

Figure 2.8: Local Rate of Change and Percentage Change in Ex-Post Equity Values: Multiple Firms - *De Novo* Capital Structure, Regulatory Constraint

The figure plots ex-post values of $\frac{\partial W}{\partial \sigma}$ (left panel) and $\frac{1}{W}\frac{\partial W}{\partial \sigma}$ (right panel) for each σ (that is, each firm) and for no CCB and the three λ cases. A portion of straight debt is swapped for CCB. The corresponding parameter values are as follows: A_t = \$100, individual initial σ values, $c_b^* =$ \$5.24, $A_C =$ \$70, $c_c =$ \$0.5, $r = 0.05$, $\mu = 0.01$, $\theta = 35\%$, and $\alpha = 50\%$.

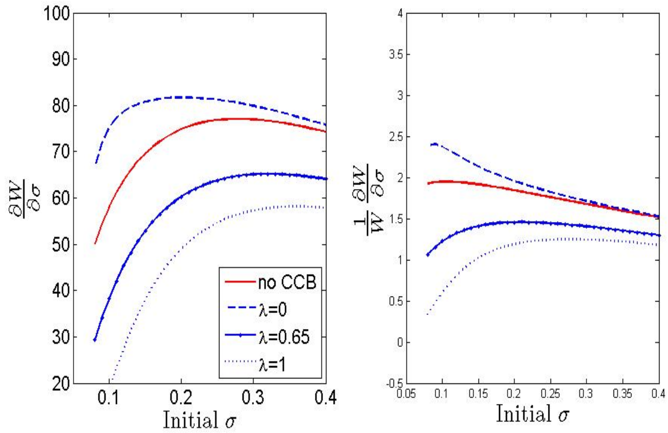


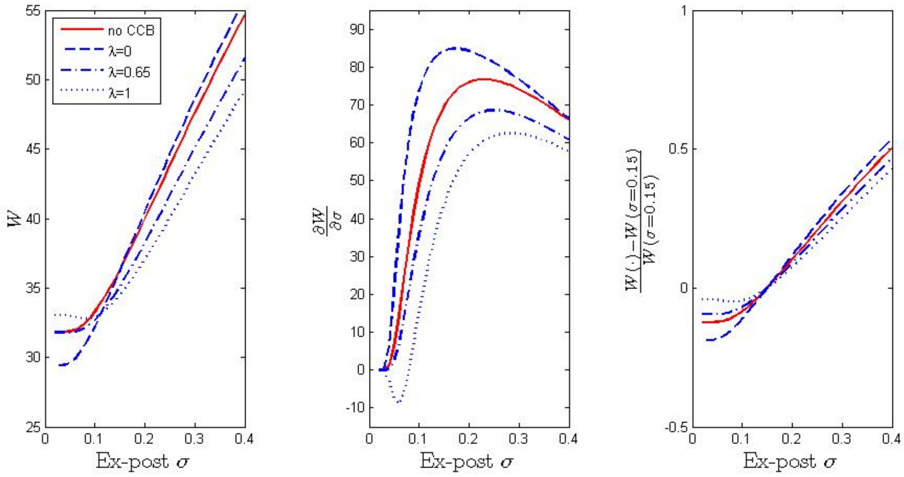
Figure 2.8 shows how equity values respond in relative terms to local changes in volatility for a range of firms (each represented by a different initial σ). These graphs confirm the conclusion of Figure 2.7, namely that a swap of CCB for straight debt reduces the incentive for asset substitution except for some extreme cases with $\lambda = 0$. Thus quite generally, we find that introducing CCB to the capital structures of a firm based on regulatory constraint (2.11) reduces the effect of asset substitution inefficiency. This is different from what we saw in Section 2.7.1.

2.7.3 CCB Introduced as a CCB for Straight Debt Swap in an Existing Capital Structure under Market Constraint

We continue with the question of asset substitution, but now for a firm that swaps CCB for straight debt in an already existing capital structure. We repeat the two exercises from Sections 2.7.1 and 2.7.2. Coupon $\hat{c_b}$ is set equal to c_b^* so the amount of straight debt is optimal and the same as before. Everything else, including the amounts of CCB, is also the same. The only difference is that CCB replaces straight debt that has already been issued so that the swap constraint (2.14) holds. The CCB swap for straight debt now has the additional effect, however, of increasing the value of the straight debt, so that this firm will have a somewhat greater value of outstanding straight debt than in Section 2.7.2.

Figure 2.9: Ex-Post Equity Values: Single Firm - Existing Capital Structure, Market Constraint

The figures show the effect of changes in asset volatility on the value of equity after a levered firm swaps a portion of straight debt for CCB. The corresponding parameter values are as follows: $A_t = \$100$, initial $\sigma = 15.0\%$, $\hat{c_b} = \$5.24$, $A_C = \$70$, $c_c = \$0.5$, $r = 0.05$, $\mu = 0.01$, $\theta = 35\%$, and $\alpha = 50\%$.

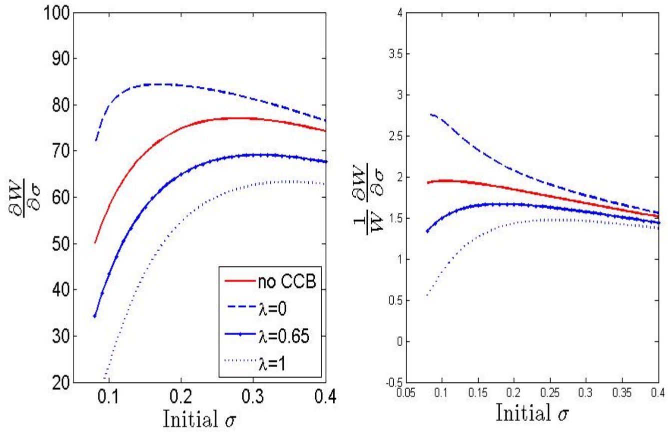


Figures 2.9 and 2.10 show the same experiment as illustrated in Figures 2.7 and 2.8. Indeed, the graphs are very similar, and the conclusion is again that the introduction of CCB in the capital structure under the condition of a CCB for debt swap decreases the incentive for asset substitution. Careful examination, however, indicates that the no CCB curve is slightly lower in Figures 2.9 and 2.10, indicating that the disincentive for asset substitution is slightly weaker in this case. The intuition is that the value of

the straight debt in the capital structure is slightly greater in this case, which slightly magnifies the incentive for asset substitution.

Figure 2.10: Local Rate of Change and Percentage Change in Ex-Post Equity Values: Multiple Firms - Existing Capital Structure, Market Constraint

The figure plots ex-post values of $\frac{\partial W}{\partial \sigma}$ (left panel) and $\frac{1}{W}\frac{\partial W}{\partial \sigma}$ (right panel) for each σ (that is, each firm) and for no CCB and the three λ cases. A portion of straight debt is swapped for CCB. The corresponding parameter values are as follows: $A_t = \$100$, individual initial σ values, $\hat{c_b} = \$5.24$, $A_C = \$70$, $c_c = \$0.5$, $r = 0.05$, $\mu = 0.01$, $\theta = 35\%$, and $\alpha = 50\%$.



The main take-away of this section is that issuing CCB as part of a CCB for equity swap, keeping the amount of straight debt unchanged, could magnify the effect of asset substitution inefficiency. On the other hand, issuing CCB as part of a CCB for straight debt swap could lead to reduced asset substitution.

2.8 Summary and Policy Conclusions

This paper has provided a formal model of CCBs. The results of the formal model are summarized in Tables 2.1 and 2.2. Table 2.1 summarizes the primary effects of CCB issuance on firm and equity value as a function of the firm's capital structure status and any imposed constraints. Table 2.2 provides our primary results showing how the conversion ratio λ affects the incentives for CCB holders and equity holders to manipulate the firm's stock price in order to trigger conversion.

In terms of prudential bank regulation, we have shown that CCBs provide a new instrument that allows banks or firms to recapitalize in an automatic and dependable fashion whenever their capital reaches a distressed level. In other words, CCBs gen-

Table 2.1: Effects of CCB Issuance on the Capital Structure of the Firm

This table summarizes the primary effects of CCB issuance on firm and equity value as a function of the firm's capital structure status and any imposed constraints.

Firm	**Constraint**	**Firm Value**	**Equity Holders' Value**	**Default Risk**	**Asset Substitution**	**Tax Savings**	**Other Effects**	**Firm Decision**
Unleveraged	Sufficiently small amount of CCB	↑	↑	↔	↑	↑	n/c	Issue CCB on top of optimal amount of SD*
Leveraged with SD	Sufficiently small amount of CCB	↑	↑	↔	↑	↑	n/c	Issue CCB on top of existing amount of SD
Unleveraged	Total amount of debt is fixed	↑	↑	↓	↓	∼	n/c	Replace some SD with CCB
Leveraged	Total amount of debt is fixed	↑	↓	↓	↓	∼	Debt overhang	Do not issue CCB
TBTF (Leveraged/ Unleveraged)	Total amount of debt is fixed	↓	↓	↓	n/c	∼	Reduced subsidy	Do not issue CCB

*SD: straight debt; TBTB: Too-big-to-fail; n/c: not considered; ↑: increase; ↓: decrease; ↔: no change; ∼: no effect or insignificant increase/decrease

Table 2.2: Incentives of CCB Holders and Equity Holders to Manipulate the Stock Market

This table provides our primary results showing how the conversion ratio λ affects the incentives for CCB holders and equity holders to manipulate the firm's stock price in order to trigger conversion.

Conversion Ratio	**Action**	**Intuition**
$0 < \lambda^* < \lambda$	CCB holders want to drive the stock price down to trigger conversion	If λ is high CCB holders receive a large amount of undervalued equity at conversion
$\lambda \leq \lambda^*$	CCB holders do not want to trigger conversion	If λ is low CCB holders are poorly compensated at conversion
$\lambda < 1 - \theta$	Equity holders want to drive the stock price down to trigger conversion	If λ is low equity holders can cheaply get rid of the obligation to pay c_c
$1 - \theta \leq \lambda$	Equity holders do not want to trigger conversion	If λ is high conversion is costly to equity holders

erally have the potential to provide most of the tax shield benefits of straight debt while providing the same protection as equity capital against bankruptcy costs. For CCBs to be effective in this role, however, it is important that the banks be required to substitute CCBs for straight debt, and not for equity, in their capital structure. The regulatory benefits of CCBs for bank safety also are greater the higher the trigger at which conversion occurs.

We conclude with comments on important topics for future research. We first comment on three extensions that would generalize assumptions in the current paper. One useful extension would fully determine the firm's optimal capital structure in the presence of CCBs. In particular, our analysis has been static in the sense that we assume the firm's entire CCB issue is converted into equity at a single point when the trigger is activated. We suspect, however, that the CCB benefits would expand further if the bonds could be converted in a sequence of triggers and/or that banks committed to issue new CCBs as soon as the existing bonds were converted. A second factor is that our analysis has assumed that both the CCBs and straight debt have an unlimited maturity in the fashion of a consol. We expect that an analysis with finite maturity bonds would find lower debt overhang costs of swapping CCBs for straight debt. A third extension would allow the geometric Brownian motion of asset dynamics to include jumps. This would have an impact on the valuation of all claims in the model. A related assumption is that we have not allowed the firm to use CCBs to purchase additional assets. While we do not expect this will change our basic results, this should be confirmed.

We conclude with two topics for future research concerning the use of CCBs for prudential bank regulation. As one topic, Flannery (2009a) has suggested that banks be presented with the choice of raising their capital ratio by a given amount or of raising their capital ratio by a smaller amount as long as it is combined with a specified amount of CCBs. The regulatory parameters in such a menu determine the tradeoff between regulatory benefits and bank costs. A calibrated version of our model could potentially measure the terms of this tradeoff. A second regulatory topic concerns the amount of tax shield benefit allowed CCB. As just one example, it would be useful to explore the effects of allowing full deduction for interest payments that correspond to the coupon on similar straight bank debt, but to exclude any part of the CCB coupon that represents compensation for the conversion risk.

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Chapter 3

Appendix

3.1 First Chapter

3.1.1 Proof of Proposition 1

Proof. I start with deriving the solutions for the values of debt, $D(A_t, m; c, A_D)$, tax benefits, $TB(A_t, m; c, A_D)$, and bankruptcy costs $BC(A_t, m; c, A_D)$. Values of these claims are viewed as expected present values of the corresponding cash flows. Equation (1.2) is applied repeatedly with $K = \hat{A}_(m)$.

$$
\begin{aligned}
D(A_t, m; c, A_D) &= E_t^Q \left[\int_t^{\tau(\hat{A}_D(m))} e^{-r(s-t)} c ds + e^{-r(\tau(\hat{A}_D(m))-t)} (1-\alpha)\hat{A}_D(m) \right] \\
&= E_t^Q \left[\frac{c}{r} \left(1 - e^{-r(\tau(\hat{A}_D(m))-t)} \right) + e^{-r(\tau(\hat{A}_D(m))-t)} (1-\alpha)\hat{A}_D(m) \right] \\
&= \frac{c}{r} \left(1 - \left(\frac{A_t}{\hat{A}_D(m)} \right)^{-\gamma} \right) + \left(\frac{A_t}{\hat{A}_D(m)} \right)^{-\gamma} (1-\alpha)\hat{A}_D(m). \\
TB(A_t, m; c, A_D) &= E_t^Q \left[\int_t^{\tau(\hat{A}_D(m))} e^{-r(s-t)} \theta c ds + \right] = \frac{\theta c}{r} \left(1 - \left(\frac{A_t}{\hat{A}_D(m)} \right)^{-\gamma} \right). \\
BC(A_t, m; c, A_D) &= E^Q \left[\int_0^{\tau(\hat{A}_D(m))} e^{-r(s-t)} \alpha \hat{A}_D(m) ds \right] = \alpha \hat{A}_D(m) \left(\frac{A_t}{\hat{A}_D(m)} \right)^{-\gamma}
\end{aligned}
$$

$\hat{A}_D(m)$ in the above expressions is defined by equation (1.10).

As for the value of equity, $E(A_t, m; c, A_D)$, notice, that the following budget equation should hold at any t:

$$\begin{aligned} E(A_t, m; c, A_D) + D(A_t, m; c, A_D) &= A_t + TB(A_t, m; c, A_D) - \\ &\quad BC(A_t, m; c, A_D) - \psi(m), \end{aligned}$$

where $\psi(m)$ is defined in equation 1.11. This leads to

$$\begin{aligned} E(A_t, m) &= A_t + TB(A_t, m; c, A_D) - BC(A_t, m; c, A_D) - D(A_t, m; c, A_D) - \psi(m) \\ &= A_t + \frac{\theta c}{r}\left(1 - \left(\frac{A_t}{\hat{A}_D(m)}\right)^{-\gamma}\right) - \alpha \hat{A}_D(m)\left(\frac{A_t}{\hat{A}_D(m)}\right)^{-\gamma} - \\ &\quad \frac{c}{r}\left(1 - \left(\frac{A_t}{\hat{A}_D(m)}\right)^{-\gamma}\right) - \left(\frac{A_t}{\hat{A}_D(m)}\right)^{-\gamma}(1-\alpha)\hat{A}_D(m) - \psi(m) \\ &= A_t - \frac{c(1-\theta)}{r}\left(1 - \left(\frac{A_t}{\hat{A}_D(m)}\right)^{-\gamma}\right) - \hat{A}_D(m)\left(\frac{A_t}{\hat{A}_D(m)}\right)^{-\gamma} - \psi(m). \end{aligned}$$

□

3.1.2 Proof of Proposition 2

Proof. Given the closed-form solution for the value of equity (1.12) and the assumed form for $\hat{A}_D(m)$ (1.10), condition (1.14) leads to the following:

$$\frac{\partial}{\partial m}[A_t - \frac{c(1-\theta)}{r}\left(1 - \left(\frac{A_t}{\hat{A}_D(m)}\right)^{-\gamma}\right) - \hat{A}_D(m)\left(\frac{A_t}{\hat{A}_D(m)}\right)^{-\gamma}$$
$$-\psi(m)]|_{m=\bar{m}} = 1$$
$$\frac{\partial}{\partial m}[\frac{c(1-\theta)}{r}\frac{1}{A_t^\gamma}\left(\frac{m}{k+m}A_B + \frac{k}{k+m}A_D\right)^\gamma - \frac{1}{A_t^\gamma}\left(\frac{m}{k+m}A_B + \frac{k}{k+m}A_D\right)^{\gamma+1}$$
$$-\zeta(m - \log(1+m))]|_{m=\bar{m}} = 1$$
$$\frac{c(1-\theta)}{r}\frac{1}{A_t^\gamma}\frac{\gamma k(A_B - A_D)}{(k+\bar{m})^2}\left(\frac{\bar{m}A_B + kA_D}{k+\bar{m}}\right)^{\gamma-1} - \frac{1}{A_t^\gamma}\frac{(\gamma+1)k(A_B - A_D)}{(k+\bar{m})^2} \times$$
$$\left(\frac{\bar{m}A_B + kA_D}{k+\bar{m}}\right)^\gamma - \frac{\zeta\bar{m}}{1+\bar{m}} - 1 = 0$$
$$\frac{1}{A_t^\gamma}\frac{k(A_B - A_D)}{(k+\bar{m})^2}\left(\frac{\bar{m}A_B + kA_D}{k+\bar{m}}\right)^{\gamma-1}\left[\frac{\gamma c(1-\theta)}{r} - (\gamma+1)\frac{\bar{m}A_B + kA_D}{k+\bar{m}}\right] -$$
$$\frac{\zeta\bar{m}}{1+\bar{m}} - 1 = 0$$

$$\frac{1}{A_t^\gamma}\frac{k(A_B - A_D)}{(k+\bar{m})^2}\left(\frac{\bar{m}A_B + kA_D}{k+\bar{m}}\right)^{\gamma-1}\left[(\gamma+1)\frac{\gamma c(1-\theta)}{(\gamma+1)r} - (\gamma+1)\frac{\bar{m}A_B + kA_D}{k+\bar{m}}\right] - \frac{\zeta\bar{m}}{1+\bar{m}} - 1 = 0.$$

Next, since $\beta = \frac{\gamma}{(\gamma+1)r}$ and $A_B = \beta(1-\theta)c$,

$$\frac{1}{A_t^\gamma}\frac{k(A_B - A_D)}{(k+\bar{m})^2}\left(\frac{\bar{m}A_B + kA_D}{k+\bar{m}}\right)^{\gamma-1}\left[(\gamma+1)\beta c(1-\theta) - (\gamma+1)\frac{\bar{m}A_B + kA_D}{k+\bar{m}}\right] - \frac{\zeta\bar{m}}{1+\bar{m}} - 1 = 0$$

$$\frac{(\gamma+1)}{A_t^\gamma}\frac{k(A_B - A_D)}{(k+\bar{m})^2}\left(\frac{\bar{m}A_B + kA_D}{k+\bar{m}}\right)^{\gamma-1}\left[A_B - \frac{\bar{m}A_B + kA_D}{k+\bar{m}}\right] - \frac{\zeta\bar{m}}{1+\bar{m}} - 1 = 0$$

$$\frac{(\gamma+1)}{A_t^\gamma}\frac{k^2(A_B - A_D)^2}{(k+\bar{m})^3}\left(\frac{\bar{m}A_B + kA_D}{k+\bar{m}}\right)^{\gamma-1} - \frac{\zeta\bar{m}}{1+\bar{m}} - 1 = 0 \tag{3.1.1}$$

Equation (3.1.1) can be rewritten in general functional form: $F(A_t, \bar{m}, A_D, \sigma, \zeta) = 0$. Based on the Implicit Function Theorem, $\frac{\partial\bar{m}}{\partial A_D} = -\frac{F_{A_D}}{F_{\bar{m}}}$. From (3.1.1),

$$\begin{aligned}
F_{A_D} &= \frac{(\gamma+1)k^2}{A_t^\gamma}\left(\frac{\bar{m}A_B + kA_D}{k+\bar{m}}\right)^{\gamma-2}(A_B - A_D)(((\gamma-1)k - 2\bar{m})A_B - \\
&\quad (\gamma+1)kA_D) \\
&= \frac{(\gamma+1)k^2}{A_t^\gamma}\left(\frac{\bar{m}A_B + kA_D}{k+\bar{m}}\right)^{\gamma-2}(A_B - A_D)((\gamma k - k - 2\bar{m})A_B - \\
&\quad (\gamma+1)kA_D) \\
&= \frac{(\gamma+1)k^2}{A_t^\gamma}\left(\frac{\bar{m}A_B + kA_D}{k+\bar{m}}\right)^{\gamma-2}(A_D - A_B)((-\gamma k + k + 2\bar{m})A_B + \\
&\quad (\gamma+1)kA_D) \\
&= \frac{(\gamma+1)k^2}{A_t^\gamma}\left(\frac{\bar{m}A_B + kA_D}{k+\bar{m}}\right)^{\gamma-2}(A_D - A_B)(\gamma k(A_D - A_B) + \\
&\quad (k + 2\bar{m})A_B + kA_D)
\end{aligned} \tag{3.1.2}$$

Since, $A_B < A_D$, all product terms in (3.1.2) are positive. Therefore, $F_{A_D} > 0$.

Also, from (3.1.1),

$$F_{\bar{m}} = -\frac{(\gamma+1)}{A_t^{\gamma}}\frac{k^2(A_B-A_D)^2((-\gamma k+k+3\bar{m})A_B+(\gamma+2)kA_D)}{(k+\bar{m})^3(\bar{m}A_B+kA_D)^2}\times \left(\frac{\bar{m}A_B+kA_D}{k+\bar{m}}\right)^{\gamma}-\frac{\zeta}{(1+\bar{m})^2}. \quad (3.1.3)$$

Note, that, since $A_B < A_D$,

$$(-\gamma k+k+3\bar{m})A_B+(\gamma+2)kA_D > A_B(-\gamma k+k+3\bar{m}+\gamma k+2k) = A_B(3\bar{m}+3k) > 0.$$

This means that the first term on the right-hand side of equation (3.1.3) is always negative. Since $\zeta > 0$, the second term is also negative. Based on (3.1.3), $F_{\bar{m}} < 0$ and, consequently, $\frac{\partial \bar{m}}{\partial A_D} > 0$. This the first statement of the proposition.

Regarding the second statement, based on the Implicit Function Theorem, $\frac{\partial \bar{m}}{\partial A_t} = -\frac{F_{A_t}}{F_{\bar{m}}}$. I have shown above, that $F_{\bar{m}} < 0$. Based on (3.1.1), since $\gamma > 0$,

$$F_{A_t} = -\frac{\gamma(\gamma+1)}{A_t^{\gamma+1}}\frac{k^2(A_B-A_D)^2}{(k+\bar{m})^3}\left(\frac{\bar{m}A_B+kA_D}{k+\bar{m}}\right)^{\gamma-1} < 0.$$

Therefore, $\frac{\partial \bar{m}}{\partial A_t} < 0$.

As for the third statement of the proposition, $\frac{\partial \hat{A}_D(\bar{m})}{\partial \bar{m}} = \frac{\partial \hat{A}_D(\bar{m})}{\partial A_D}\frac{\partial A_d}{\partial \bar{m}}$. Based on specification (1.10), $\frac{\partial \hat{A}_D(m)}{\partial A_D} = \frac{k}{k+m}A_D > 0$. Based on the Implicit Function Theorem, $\frac{\partial A_D}{\partial \bar{m}} = -\frac{F_{\bar{m}}}{F_{A_D}}$. I have shown above, that $F_{A_D} > 0$ and $F_{\bar{m}} < 0$. Therefore, $\frac{\partial \hat{A}_D(\bar{m})}{\partial \bar{m}} > 0$. □

3.1.3 Proof of Proposition 3

Proof. Based on closed-form solution (1.12), the value of equity can be represented as

$$\begin{aligned} E(A_t,\bar{m};c,A_D) &= A_t - \frac{c(1-\theta)}{r}\left(1-\left(\frac{A_t}{\hat{A}_D(\bar{m})}\right)^{-\gamma}\right) - \hat{A}_D(\bar{m})\left(\frac{A_t}{\hat{A}_D(\bar{m})}\right)^{-\gamma} - \psi(\bar{m}) \\ &= S(A_D) - \psi(\bar{m}), \end{aligned}$$

where $S(A_D) = A_t - \frac{c(1-\theta)}{r}\left(1 - \left(\frac{A_t}{\hat{A}_D(\bar{m})}\right)^{-\gamma}\right) - \hat{A}_D(\bar{m})\left(\frac{A_t}{\hat{A}_D(\bar{m})}\right)^{-\gamma}$ and $\hat{A}_D(\bar{m})$ is defined by equation (1.10). Given this representation,

$$\frac{\partial E(A_t, \bar{m}; c, A_D)}{\partial A_D} = \frac{\partial S(A_D)}{\partial A_D}\frac{\partial A_D}{\partial \bar{m}} - \frac{\partial \psi(\bar{m})}{\partial \bar{m}}. \qquad (3.1.4)$$

Since $\zeta > 0$, the last term in equation (3.1.4) is always negative, $-\frac{\partial \psi(\bar{m})}{\partial \bar{m}} = -\frac{\zeta}{1+\bar{m}} < 0$. $\frac{\partial S(A_D)}{\partial A_D} < 0$, because, since $A_B < \hat{A}_D(\bar{m})$, where $A_B = \beta(1-\theta)c$ and $\beta = \frac{\gamma}{(\gamma+1)r}$,

$$\begin{aligned}
\frac{\partial S(A_D)}{\partial A_D} &= \left[A_t - \frac{c(1-\theta)}{r}\left(1 - \left(\frac{A_t}{\hat{A}_D(\bar{m})}\right)^{-\gamma}\right) - \hat{A}_D(\bar{m})\left(\frac{A_t}{\hat{A}_D(\bar{m})}\right)^{-\gamma}\right]' \\
&= \left[\frac{c(1-\theta)}{r}(A_t)^{-\gamma}(\hat{A}_D(\bar{m}))^{\gamma} - (A_t)^{-\gamma}(\hat{A}_D(\bar{m}))^{\gamma}\right]' \\
&= [\frac{c(1-\theta)}{r}(A_t)^{-\gamma}\left(\frac{\bar{m}}{k+\bar{m}}A_B + \frac{k}{k+\bar{m}}A_D\right)^{\gamma} - \\
&\quad (A_t)^{-\gamma}\left(\frac{\bar{m}}{k+\bar{m}}A_B + \frac{k}{k+\bar{m}}A_D\right)^{\gamma+1}]' \\
&= (A_t)^{-\gamma}\left(\frac{\bar{m}}{k+\bar{m}}A_B + \frac{k}{k+\bar{m}}A_D\right)^{\gamma-1}\frac{k}{k+\bar{m}} \times \\
&\quad \left[\frac{\gamma c(1-\theta)}{r} - (\gamma+1)\left(\frac{\bar{m}}{k+\bar{m}}A_B + \frac{k}{k+\bar{m}}A_D\right)\right] \\
&= (A_t)^{-\gamma}\left(\frac{\bar{m}}{k+\bar{m}}A_B + \frac{k}{k+\bar{m}}A_D\right)^{\gamma-1}\frac{k}{k+\bar{m}} \times \\
&\quad \left[(\gamma+1)\frac{\gamma c(1-\theta)}{(\gamma+1)r} - (\gamma+1)\left(\frac{\bar{m}}{k+\bar{m}}A_B + \frac{k}{k+\bar{m}}A_D\right)\right] \\
&= (A_t)^{-\gamma}\left(\frac{\bar{m}}{k+\bar{m}}A_B + \frac{k}{k+\bar{m}}A_D(\gamma+1)\right)^{\gamma-1}\frac{k}{k+\bar{m}}\left[A_B - \hat{A}_D(\bar{m})\right] < 0.
\end{aligned}$$

As in the proof of Proposition 2, by representing equation (3.1.1) in general functional form, $F(A_t, \bar{m}, A_D, \sigma, \zeta) = 0$, and applying the Implicit Function Theorem, $\frac{\partial A_D}{\partial \bar{m}} = -\frac{F_{\bar{m}}}{F_{A_D}}$. (I remind the reader, that equation (3.1.1) followed from equation (1.14).) In the proof of Proposition 2, I have shown that $-\frac{F_{A_D}}{F_{\bar{m}}} > 0$. Therefore, $-\frac{F_{\bar{m}}}{F_{A_D}} > 0$ and $\frac{\partial A_D}{\partial \bar{m}} > 0$. Both terms on the right-hand side of equation (3.1.4) are negative. Therefore, $\frac{\partial E(A_t, \bar{m}; c, A_D)}{\partial A_D} < 0$.

As for the value of debt, $\frac{\partial D(A_t, \bar{m}; c, A_D)}{\partial \bar{m}} = \frac{\partial D(A_t, \bar{m}; c, A_D)}{\partial A_D}\frac{\partial A_D}{\partial \bar{m}}$. I have shown above, that $\frac{\partial A_D}{\partial \bar{m}} > 0$. Regarding $\frac{\partial D(A_t,\bar{m};c,A_D)}{\partial A_D}$, given closed-form solution (1.13),

$$\begin{aligned}
\frac{\partial D(A_t,\bar{m};c,A_D)}{\partial A_D} &= \left[\frac{c}{r}\left(1-\left(\frac{A_t}{\hat{A}_D(\bar{m})}\right)^{-\gamma}\right)+\left(\frac{A_t}{\hat{A}_D(\bar{m})}\right)^{-\gamma}(1-\alpha)\hat{A}_D(\bar{m})\right]' \\
&= [-\frac{c}{r}(A_t)^{-\gamma}\left(\frac{\bar{m}}{k+\bar{m}}A_B+\frac{k}{k+\bar{m}}A_D\right)^{\gamma}+ \\
&\quad (1-\alpha)(A_t)^{-\gamma}\left(\frac{\bar{m}}{k+\bar{m}}A_B+\frac{k}{k+\bar{m}}A_D\right)^{\gamma+1}]' \\
&= -\frac{c}{r}(A_t)^{-\gamma}\gamma\left(\frac{\bar{m}}{k+\bar{m}}A_B+\frac{k}{k+\bar{m}}A_D\right)^{\gamma-1}\frac{k}{k+\bar{m}}+ \\
&\quad (1-\alpha)(A_t)^{-\gamma}(\gamma+1)\left(\frac{\bar{m}}{k+\bar{m}}A_B+\frac{k}{k+\bar{m}}A_D\right)^{\gamma}\frac{k}{k+\bar{m}} \\
&= (A_t)^{-\gamma}\left(\frac{\bar{m}}{k+\bar{m}}A_B+\frac{k}{k+\bar{m}}A_D\right)^{\gamma-1}\frac{k}{k+\bar{m}}\times \\
&\quad \left[-\frac{c}{r}\gamma+(1-\alpha)(\gamma+1)\left(\frac{\bar{m}}{k+\bar{m}}A_B+\frac{k}{k+\bar{m}}A_D\right)\right] \\
&= (A_t)^{-\gamma}(\hat{A}_D(\bar{m}))^{\gamma-1}\frac{k}{k+\bar{m}}\times \\
&\quad \left[-\frac{\gamma+1}{1-\theta}\frac{c(1-\theta)\gamma}{(\gamma+1)r}+(1-\alpha)(\gamma+1)\left(\frac{\bar{m}A_B+kA_D}{k+\bar{m}}\right)\right] \\
&= (A_t)^{-\gamma}(\hat{A}_D(\bar{m}))^{\gamma-1}\frac{k}{k+\bar{m}}(\gamma+1)\times \\
&\quad \left[-\frac{A_B}{1-\theta}+(1-\alpha)\left(\frac{\bar{m}}{k+\bar{m}}A_B+\frac{k}{k+\bar{m}}A_D\right)\right] \\
&= (A_t)^{-\gamma}(\hat{A}_D(\bar{m}))^{\gamma-1}\frac{k}{k+\bar{m}}(\gamma+1)\times \\
&\quad \left[-\frac{A_B}{1-\theta}+(1-\alpha)\left(\frac{\bar{m}}{k+\bar{m}}A_B+\frac{k}{k+\bar{m}}A_D\right)\right] \\
&= (A_t)^{-\gamma}(\hat{A}_D(\bar{m}))^{\gamma-1}\frac{k}{k+\bar{m}}(\gamma+1)\times \\
&\quad \left[\frac{(1-\alpha)k}{k+\bar{m}}A_D-\frac{\bar{m}(\alpha+\theta-\alpha\theta)+k}{k+\bar{m}}A_B\right].
\end{aligned}$$

Therefore, if $A_D < \frac{\bar{m}(\alpha+\theta-\alpha\theta)+k}{(1-\alpha)k}A_B$, $\frac{\partial D(A_t,\bar{m};c,A_D)}{\partial A_D} > 0$ and $\frac{\partial D(A_t,\bar{m};c,A_D)}{\partial \bar{m}} > 0$. □

3.2 Second Chapter

3.2.1 Proof of Proposition 1

Proof. Except for the solutions for $G(A_t; c_b, c_c)$ and $W(A_t; c_b, c_c)$ all the other ones are derived as present values of the corresponding cash flows. Equation (2.2) is applied repeatedly with different values for K.

$$\begin{aligned}
U^B(A_t; c_b, c_c) &= E_t^Q\left[\int_t^{\tau(A_B)} e^{-r(s-t)} c_b ds + e^{-r(\tau(A_B)-t)}(1-\alpha)A_B\right] \\
&= E_t^Q\left[\frac{c_b}{r}\left(1-e^{-r(\tau(A_B)-t)}\right) + e^{-r(\tau(A_B)-t)}(1-\alpha)A_B\right] \\
&= \frac{c_b}{r}\left(1-\left(\frac{A_t}{A_B}\right)^{-\gamma}\right) + \left(\frac{A_t}{A_B}\right)^{-\gamma}(1-\alpha)A_B. \\
U^C(A_t; c_c) &= E_t^Q\left[\int_t^{\tau(A_C)} e^{-r(s-t)} c_c ds + e^{-r(\tau(A_C)-t)}\left(\lambda\frac{c_c}{r}\right)\right] \\
&= E_t^Q\left[\frac{c_c}{r}\left(1-e^{-r(\tau(A_C)-t)}\right) + e^{-r(\tau(A_C)-t)}\left(\lambda\frac{c_c}{r}\right)\right] \\
&= \frac{c_c}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right) + \left(\frac{A_t}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right). \\
TB(A_t; c_b, c_c) &= E_t^Q\left[\int_t^{\tau(A_B)} e^{-r(s-t)}\theta c_b ds + \int_t^{\tau(A_C)} e^{-r(u-t)}\theta c_c du\right] \\
&= \frac{\theta c_b}{r}\left(1-\left(\frac{A_t}{A_B}\right)^{-\gamma}\right) + \frac{\theta c_c}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right). \\
BC(A_t; c_b) &= E^Q\left[\int_0^{\tau(A_B)} e^{-r(s-t)}\alpha A_B ds\right] = \alpha A_B\left(\frac{A_t}{A_B}\right)^{-\gamma}.
\end{aligned}$$

Based on the budget equation (2.5)

$$W(A_t; c_b, c_c) = A_t + TB(A_t; c_b, c_c) - U^B(A_t; c_b, c_c) - U^C(A_t; c_c) - BC(A_t; c_b).$$

Therefore,

$$W(A_t; c_b, c_c) = A_t + \frac{\theta c_b}{r}\left(1-\left(\frac{A_t}{A_B}\right)^{-\gamma}\right) + \frac{\theta c_c}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right)$$

$$
\begin{aligned}
& -\frac{c_b}{r}\left(1-\left(\frac{A_t}{A_B}\right)^{-\gamma}\right)-\left(\frac{A_t}{A_B}\right)^{-\gamma}(1-\alpha)A_B \\
& -\frac{c_c}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right)-\left(\frac{A_t}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right)-\alpha A_B\left(\frac{A_t}{A_B}\right)^{-\gamma} \\
= \;& A_t+\frac{c_b(\theta-1)}{r}\left(1-\left(\frac{A_t}{A_B}\right)^{-\gamma}\right)+\frac{c_c(\theta-1)}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right)- \\
& A_B\left(\frac{A_t}{A_B}\right)^{-\gamma}-\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma}.
\end{aligned}
$$

Finally, based on equation (2.5)

$$
G(A_t;c_b,c_c) = A_t+\frac{\theta c_b}{r}\left(1-\left(\frac{A_t}{A_B}\right)^{-\gamma}\right)+\frac{\theta c_c}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right)-\alpha A_B\left(\frac{A_t}{A_B}\right)^{-\gamma}
$$

□

3.2.2 Lemma 2 and Proof of Lemma 2

Lemma 2. *If $(\lambda+\theta)>1$, then the value of equity, $W(A_t;c_b,c_c)$, is a strictly increasing function of A_t for $A_t \geq A_C$. Moreover, if $(\lambda+\theta)>1$, the lowest asset level A_{CL} that satisfies definition (2.8) is such that*

$$
A_{CL}\in\left(A_B+\lambda\frac{c_c}{r},\lambda\frac{c_c}{r}+\frac{c_b(1-\theta)}{r}\right).
$$

Proof of Lemma 2. Based on Proposition 1

$$
\begin{aligned}
\frac{\partial W(A_t;c_b,c_c)}{\partial A_t} = \;& 1+\frac{c_b(1-\theta)}{r}(-\gamma)\frac{1}{A_t}\left(\frac{A_t}{A_B}\right)^{-\gamma}+\frac{c_c(1-\theta)}{r}(-\gamma)\frac{1}{A_t}\left(\frac{A_t}{A_C}\right)^{-\gamma}- \\
& (-\gamma)A_B\frac{1}{A_t}\left(\frac{A_t}{A_B}\right)^{-\gamma}-\frac{\lambda c_c}{r}(-\gamma)\frac{1}{A_t}\left(\frac{A_t}{A_C}\right)^{-\gamma}.
\end{aligned}
$$

Since by design $A_t \geq A_B$ it follows that

$$
\frac{\partial W(A_t;c_b,c_c)}{\partial A_t} \geq 1+\frac{c_b(1-\theta)}{r}(-\gamma)\frac{1}{A_B}+\frac{c_c(1-\theta)}{r}(-\gamma)\frac{1}{A_t}\left(\frac{A_t}{A_C}\right)^{-\gamma}-
$$

$$
\begin{aligned}
&(-\gamma) - \frac{\lambda c_c}{r}(-\gamma)\frac{1}{A_t}\left(\frac{A_t}{A_C}\right)^{-\gamma} \\
= \; & 1 - \gamma\left(\frac{c_b(1-\theta)}{r}\frac{1}{A_B} - 1\right) - \frac{c_c(1-\theta)\gamma}{r}\frac{1}{A_t}\left(\frac{A_t}{A_C}\right)^{-\gamma} + \frac{\lambda c_c \gamma}{r}\frac{1}{A_t}\left(\frac{A_t}{A_C}\right)^{-\gamma} \\
= \; & 1 - \gamma\left(\frac{1}{r\beta} - 1\right) - \frac{c_c\gamma}{r}\frac{1}{A_t}\left(\frac{A_t}{A_C}\right)^{-\gamma}(1-\theta-\lambda) \\
= \; & 1 - \gamma\left(\frac{\gamma+1}{\gamma} - 1\right) - \frac{c_c\gamma}{r}\frac{1}{A_t}\left(\frac{A_t}{A_C}\right)^{-\gamma}(1-\theta-\lambda) \\
= \; & -\frac{c_c\gamma}{r}\frac{1}{A_t}\left(\frac{A_t}{A_C}\right)^{-\gamma}(1-\theta-\lambda).
\end{aligned}
$$

Finally, if $(\lambda+\theta) > 1$, then $\frac{\partial W(A_t;c_b,c_c)}{\partial A_t} > 0$. This completes the proof of the first statement.

Given that $(\lambda+\theta) > 1$ (meaning that $W(A_t;c_b,c_c)$ is monotone in A_t for $A_t \geq A_C$) and based on definition (2.8), $W(A_{CL};c_b,c_c) = 0$. Based on Proposition 1, this leads to

$$
\begin{aligned}
A_{CL} \; - \; & \frac{c_b(1-\theta)}{r}\left(1 - \left(\frac{A_{CL}}{A_B}\right)^{-\gamma}\right) - \frac{c_c(1-\theta)}{r}\left(1 - \left(\frac{A_{CL}}{A_{CL}}\right)^{-\gamma}\right) - \\
& A_B\left(\frac{A_{CL}}{A_B}\right)^{-\gamma} - \left(\lambda\frac{c_c}{r}\right)\left(\frac{A_{CL}}{A_{CL}}\right)^{-\gamma} = 0 \\
A_{CL} \; = \; & \frac{c_b(1-\theta)}{r}\left(1 - \left(\frac{A_{CL}}{A_B}\right)^{-\gamma}\right) + A_B\left(\frac{A_{CL}}{A_B}\right)^{-\gamma} + \left(\lambda\frac{c_c}{r}\right). \qquad (3.2.1)
\end{aligned}
$$

The right-hand side in (3.2.1) equals

$$
\begin{aligned}
\frac{c_b(1-\theta)}{r} + \left(A_B - \frac{c_b(1-\theta)}{r}\right)\left(\frac{A_{CL}}{A_B}\right)^{-\gamma} + \left(\lambda\frac{c_c}{r}\right) \; &= \\
\frac{c_b(1-\theta)}{r} + \left(\frac{c_b(1-\theta)}{r}\beta - \frac{c_b(1-\theta)}{r}\right)\left(\frac{A_{CL}}{A_B}\right)^{-\gamma} + \left(\lambda\frac{c_c}{r}\right) \; &= \\
\frac{c_b(1-\theta)}{r} + \frac{c_b(1-\theta)}{r}(\beta-1)\left(\frac{A_{CL}}{A_B}\right)^{-\gamma} + \left(\lambda\frac{c_c}{r}\right) \; &< \; \frac{c_b(1-\theta)}{r} + \left(\lambda\frac{c_c}{r}\right).
\end{aligned}
$$

The last inequality is based on $\beta < 1$. This proves that $A_{CL} < \frac{c_b(1-\theta)}{r} + \left(\lambda\frac{c_c}{r}\right)$.

Also, based on equation (3.2.1)

$$\begin{aligned} A_{CL} &= \frac{c_b(1-\theta)}{r}\left(1-\left(\frac{A_{CL}}{A_B}\right)^{-\gamma}\right) - A_B + A_B\left(\frac{A_{CL}}{A_B}\right)^{-\gamma} + A_B + \left(\lambda\frac{c_c}{r}\right) \\ &= \left(\frac{c_b(1-\theta)}{r} - A_B\right)\left(1-\left(\frac{A_{CL}}{A_B}\right)^{-\gamma}\right) + A_B + \left(\lambda\frac{c_c}{r}\right) > A_B + \left(\lambda\frac{c_c}{r}\right). \end{aligned}$$

This completes the proof of the second statement. □

3.2.3 Proof of Theorem 1

Proof. The capital structure that maximizes the market value, $G(A_0; c_b, c_c)$, received by the *initial owners* from the sale of a straight bond, equity and CCB is based on the straight debt coupon c_b^* that solves

$$\max_{c_b \geq 0} G(A_0; c_b, c_c) \equiv \max_{c_b \geq 0} \left[A_0 + TB(A_0; c_b, c_c) - BC(A_0; c_b)\right].$$

Since A_0 at $t = 0$ is constant, we get a new maximization problem:

$$\max_{c_b \geq 0} \left[TB(A_0; c_b, c_c) - BC(A_0; c_b)\right].$$

From Proposition 1 and (2.3)

$$\begin{aligned} \max_{c_b \geq 0} \quad & [\frac{\theta c_b}{r}\left(1-\left(\frac{A_0}{\beta(1-\theta)c_b}\right)^{-\gamma}\right) + \frac{\theta c_c}{r}\left(1-\left(\frac{A_0}{A_C}\right)^{-\gamma}\right) - \\ & \alpha\beta(1-\theta)c_b\left(\frac{A_0}{\beta(1-\theta)c_b}\right)^{-\gamma}]. \end{aligned}$$

FOCs:

$$\begin{aligned} &\left(\frac{\theta c_b}{r}\left(1-\left(\frac{\beta(1-\theta)}{A_0}\right)^{\gamma} c_b^{\gamma}\right) + \frac{\theta c_c}{r}\left(1-\left(\frac{A_C}{A_0}\right)^{\gamma}\right) - \alpha\frac{(\beta(1-\theta))^{\gamma+1}}{A_0^{\gamma}} c_b^{\gamma+1}\right)' = 0 \\ &\left(\frac{\theta c_b}{r} - \frac{\theta}{r}\left(\frac{\beta(1-\theta)}{A_0}\right)^{\gamma} c_b^{\gamma+1} + \frac{\theta c_c}{r}\left(1-\left(\frac{A_C}{A_0}\right)^{\gamma}\right) - \alpha\frac{(\beta(1-\theta))^{\gamma+1}}{A_0^{\gamma}} c_b^{\gamma+1}\right)' = 0 \\ &\frac{\theta}{r} - \frac{\theta(\gamma+1)}{r}\left(\frac{\beta(1-\theta)}{A_0}\right)^{\gamma} c_b^{\gamma} - \alpha(\gamma+1)\frac{(\beta(1-\theta))^{\gamma+1}}{A_0^{\gamma}} c_b^{\gamma} = 0 \end{aligned}$$

$$c_b = \left[\frac{\theta}{r}(\gamma+1)^{-1}\left(\frac{\beta(1-\theta)}{A_0}\right)^{-\gamma}\left(\frac{\theta}{r}+\alpha\beta(1-\theta)\right)^{-1}\right]^{\frac{1}{\gamma}}.$$

Finally,

$$c_b^*(A_0;c_c) = \frac{A_0}{\beta(1-\theta)}\left(\frac{\theta}{r}\right)^{\frac{1}{\gamma}}\left[(\gamma+1)(\frac{\theta}{r}+\alpha\beta(1-\theta))\right]^{-\frac{1}{\gamma}}.$$

We can see that the optimal coupon on the straight debt does not depend on any characteristics of CCB. One can repeat the above calculations for the case when the capital structure includes only equity and straight debt to show formally that $c_b^*(A_0;c_c) = c_b^*(A_0;0)$. □

3.2.4 Proof of Proposition 2

Proof. We start with the last statement and work our way up to the first one in a consecutive order.

One can closely follow the math in Appendix A to derive results similar to the ones in Proposition 1 for the case when the firm uses only straight debt and equity[1]. Item (iv) will follow.

As for item (iii), based on Proposition 1 and (2.7)

$$\begin{aligned} TB(A_0;c_b^*,c_c) &= \frac{\theta c_b^*}{r}\left(1-\left(\frac{A_0}{A_B}\right)^{-\gamma}\right)+\frac{\theta c_c}{r}\left(1-\left(\frac{A_0}{A_C}\right)^{-\gamma}\right) \\ &= TB(A_0;c_b^*,0)+TB^C(A_0;c_b^*,c_c). \end{aligned}$$

$A_C < A_0$ (by design) and $\gamma > 0$ (as shown above), so $TB^C(A_0;c_b^*,c_c) > 0$. The tax savings increase in c_c and decline in A_C. Higher c_c raises the deductibles which leads to higher savings. But, higher levels of A_C trigger sooner conversions and, therefore, reduce the amount of time during which the firm can deduct c_c.

By re-grouping the terms in the formula for the value of equity from Proposition 1, we get

$$W(A_0;c_b^*,c_c) = A_0 - \frac{c_b^*(1-\theta)}{r}\left(1-\left(\frac{A_0}{A_B}\right)^{-\gamma}\right) - A_B\left(\frac{A_0}{A_B}\right)^{-\gamma} -$$

[1]These results have been derived in previous literature (Leland (1994)). One can also use Proposition 1 to try to recognize what the corresponding values should be when CCB is not used.

$$\left[\frac{c_c(1-\theta)}{r}\left(1-\left(\frac{A_0}{A_C}\right)^{-\gamma}\right)+\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_0}{A_C}\right)^{-\gamma}\right]+\frac{\theta c_c}{r}\left(1-\left(\frac{A_0}{A_C}\right)^{-\gamma}\right)$$
$$= W(A_0;c_b^*,0)-U^C(A_0;c_c)+TB^C(A_0;c_b^*,c_c)$$

as in item (ii).

Finally, based on equation (2.5),

$$\begin{aligned}
G(A_0;c_b^*,c_c) &= A_0+TB(A_0;c_b^*,c_c)-BC(A_t,c_b)\\
&= \left[A_0+\frac{\theta c_b}{r}\left(1-\left(\frac{A_t}{A_B}\right)^{-\gamma}\right)-\alpha A_B\left(\frac{A_t}{A_B}\right)^{-\gamma}\right]+\\
&\quad \frac{\theta c_c}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right),\\
&= G(A_0;c_b^*,0)+TB^C(A_0;c_b^*,c_c).
\end{aligned}$$

This proves item (i). □

3.2.5 Proof of Proposition 3

Proof. Based on budget equation (2.5) and regulatory constraint (2.11)

$$\begin{aligned}
G(A_0,\bar{A}_B;\bar{c_b},c_c)-G(A_0,A_B^*;c_b^*,0) &= W(A_0,\bar{A}_B;\bar{c},c_c)+U^C(A_0,\bar{A}_B;\bar{c_b},c_c)+\\
&\quad U^B(A_0,\bar{A}_B;\bar{c_b},c_c)-W(A_0,A_B^*;c_b^*,0)-\\
&\quad U^B(A_0,A_B^*;c_b^*,0)\\
&= W(A_0,\bar{A}_B;\bar{c_b},c_c)-W(A_0,A_B^*;c_b^*,0).
\end{aligned}$$

The change in the total value of the firm equals the change in the value of equity.

Denote $W(A_0,\bar{A}_B;\bar{c_b},c_c)-W(A_0,A_B;c_b^*,0)$ by ΔW. Based on budget equation (2.5)

$$\begin{aligned}
\Delta W &= TB(A_0,\bar{A}_B;\bar{c_b},c_c)-U^B(A_0,\bar{A}_B;\bar{c_b},c_c)-U^C(A_0,\bar{A}_B;c_c)-\\
&\quad BC(A_t,\bar{A}_B;\bar{c_b})-TB(A_0,A_B^*;c_b^*,0)+U^B(A_0,A_B^*;c_b^*,0)+BC(A_0,A_B^*;c_b^*)\\
&= TB(A_0,\bar{A}_B;\bar{c_b},c_c)-TB(A_0,A_B^*;c_b^*,0)+BC(A_0,A_B^*;c_b^*)-BC(A_t,\bar{A}_B;\bar{c_b})
\end{aligned}$$

Next, based on closed-form solutions from Proposition 1

$$\begin{aligned}\Delta W &= \frac{\theta\bar{c_b}}{r}\left(1-\left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma}\right)+\frac{\theta c_c}{r}\left(1-\left(\frac{A_0}{A_C}\right)^{-\gamma}\right)-\frac{\theta c_b^*}{r}\left(1-\left(\frac{A_0}{A_B^*}\right)^{-\gamma}\right)+\\ &\quad \alpha A_B^*\left(\frac{A_0}{A_B^*}\right)^{-\gamma}-\alpha\bar{A}_B\left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma}.\end{aligned}$$

Add and subtract terms and use Proposition 1 again

$$\begin{aligned}\Delta W &= \theta\{\frac{\bar{c_b}}{r}\left(1-\left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma}\right)+\left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma}(1-\alpha)\bar{A}_B+\frac{c_c}{r}\left(1-\left(\frac{A_0}{A_C}\right)^{-\gamma}\right)+\\ &\quad \left(\frac{A_0}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right)-\frac{c_b^*}{r}\left(1-\left(\frac{A_0}{A_B^*}\right)^{-\gamma}\right)-\left(\frac{A_0}{A_B^*}\right)^{-\gamma}(1-\alpha)A_B^*-\\ &\quad \left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma}(1-\alpha)\bar{A}_B-\left(\frac{A_0}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right)+\left(\frac{A_0}{A_B^*}\right)^{-\gamma}(1-\alpha)A_B^*\}+\\ &\quad \alpha A_B^*\left(\frac{A_0}{A_B^*}\right)^{-\gamma}-\alpha\bar{A}_B\left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma}\\ &= \theta\{U^B(A_0,\bar{A}_B;\bar{c_b},c_c)+U^C(A_0,\bar{A}_B;c_b,c_c)-U^B(A_0,A_B^*;c_b^*,0)-\\ &\quad \left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma}(1-\alpha)\bar{A}_B-\left(\frac{A_0}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right)+\left(\frac{A_0}{A_B^*}\right)^{-\gamma}(1-\alpha)A_B^*\}+\\ &\quad \alpha A_B^*\left(\frac{A_0}{A_B^*}\right)^{-\gamma}-\alpha\bar{A}_B\left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma}.\end{aligned}$$

Based on (2.11) and by re-grouping terms

$$\begin{aligned}\Delta W &= -\theta\left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma}(1-\alpha)\bar{A}_B-\theta\left(\frac{A_0}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right)+\theta\left(\frac{A_0}{A_B^*}\right)^{-\gamma}(1-\alpha)A_B^*+\\ &\quad \alpha A_B^*\left(\frac{A_0}{A_B^*}\right)^{-\gamma}-\alpha\bar{A}_B\left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma}.\end{aligned}$$

Finally,

$$\begin{aligned}\Delta W &= -(\theta(1-\alpha)+\alpha)\left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma}\bar{A}_B+(\theta(1-\alpha)+\alpha)\left(\frac{A_0}{A_B^*}\right)^{-\gamma}A_B^*-\\ &\quad \theta\left(\frac{A_0}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right)\end{aligned}$$

$$= (\theta+\alpha-\theta\alpha)(\left(\frac{A_0}{A_B^*}\right)^{-\gamma} A_B^* - \left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma} \bar{A}_B) - \theta \left(\frac{A_0}{A_C}\right)^{-\gamma} \left(\lambda \frac{c_c}{r}\right),$$

which completes the proof of the first part of the proposition.

Denote $G(A_0, \bar{A}_B; \bar{c_b}, c_c) - G(A_0, A_B^*; c_b^*, 0)$ by ΔG. We know that $\Delta G = \Delta W$ and, therefore,

$$\Delta G = (\theta+\alpha-\theta\alpha)(\left(\frac{A_0}{A_B^*}\right)^{-\gamma} A_B^* - \left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma} \bar{A}_B) - \theta \left(\frac{A_0}{A_C}\right)^{-\gamma} \left(\lambda \frac{c_c}{r}\right).$$

This leads to

$$\frac{\partial \Delta G}{\partial \bar{c_b}} = -(\theta+\alpha-\theta\alpha)(\gamma+1) \left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma} \frac{\partial \bar{A}_B}{\partial \bar{c_b}} - \theta \left(\frac{A_0}{A_C}\right)^{-\gamma} \frac{\lambda}{r} \frac{\partial c_c}{\partial \bar{c_b}}. \quad (3.2.2)$$

Based on (2.12)

$$\begin{aligned}
\frac{\partial c_c}{\partial \bar{c_b}} &= \frac{-\frac{\partial U^B(A_t, \bar{A}_B; \bar{c_b}, 1)}{\partial \bar{c_b}}}{\frac{1}{r}\left(1-(1-\lambda)\left(\frac{A_t}{A_C}\right)^{-\gamma}\right)} \\
&= -\frac{\frac{1}{r}\left(1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - \frac{\bar{c_b}}{r}\gamma\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\frac{1}{\bar{A}_B}\frac{\partial \bar{A}_B}{\partial \bar{c_b}} + (1-\alpha)(\gamma+1)\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\frac{\partial \bar{A}_B}{\partial \bar{c_b}}}{\frac{1}{r}\left(1-(1-\lambda)\left(\frac{A_t}{A_C}\right)^{-\gamma}\right)}.
\end{aligned}$$

Given that $\bar{A}_B = \beta(1-\theta)\bar{c_b}$ and $\frac{\partial \bar{A}_B}{\partial \bar{c_b}} = \beta(1-\theta)$ and by plugging the above into (3.2.2), we get

$$\begin{aligned}
\frac{\partial \Delta G}{\partial \bar{c_b}} &= -(\theta+\alpha-\theta\alpha)(\gamma+1)\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\beta(1-\theta) + \theta\left(\frac{A_t}{A_C}\right)^{-\gamma}\frac{\lambda}{r}\times \\
&\quad \frac{\frac{1}{r}\left(1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - \frac{1}{r}\gamma\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} + (1-\alpha)(\gamma+1)\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\beta(1-\theta)}{\frac{1}{r}\left(1-(1-\lambda)\left(\frac{A_t}{A_C}\right)^{-\gamma}\right)} \\
&= -(\theta+\alpha-\theta\alpha)(\gamma+1)\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\beta(1-\theta) + \\
&\quad \frac{\theta\lambda\left[\frac{1}{r}\left(1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - \frac{1}{r}\gamma\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} + (1-\alpha)(\gamma+1)\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\beta(1-\theta)\right]}{\left(\frac{A_t}{A_C}\right)^{\gamma} - 1 + \lambda}.
\end{aligned}$$

Since $\frac{\lambda}{\left(\frac{A_t}{A_C}\right)^{\gamma}-1+\lambda} \leq 1$,

$$
\begin{aligned}
\frac{\partial \Delta G}{\partial \bar{c_b}} &\leq -(\theta+\alpha-\theta\alpha)(\gamma+1)\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\beta(1-\theta)+\theta \times \\
&\left[\frac{1}{r}\left(1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right)-\frac{1}{r}\gamma\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}+(1-\alpha)(\gamma+1)\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\beta(1-\theta)\right] \\
&= -\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\left[(\gamma+1)\beta(1-\theta)\alpha+\frac{\theta\gamma}{r}\right]+\frac{\theta}{r}\left(1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) \\
&= -\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\left[(\gamma+1)\left(\alpha\beta(1-\theta)+\frac{\theta}{r}\right)\right]+\frac{\theta}{r}.
\end{aligned}
$$

Based on the above

$$
\left.\frac{\partial \Delta G}{\partial \bar{c_b}}\right|_{\bar{c_b}=c_b^*} \leq -\left(\frac{A_t}{A_B^*}\right)^{-\gamma}\left[(\gamma+1)\left(\alpha\beta(1-\theta)+\frac{\theta}{r}\right)\right]+\frac{\theta}{r}. \tag{3.2.3}
$$

From (2.9)

$$
\begin{aligned}
\frac{\beta(1-\theta)c_b^*}{A_t} &= \left(\frac{\theta}{r}\right)^{\frac{1}{\gamma}}\left[(\gamma+1)(\frac{\theta}{r}+\alpha\beta(1-\theta))\right]^{-\frac{1}{\gamma}} \\
\left(\frac{A_t}{A_B^*}\right)^{-\gamma} &= \left(\frac{\theta}{r}\right)\left[(\gamma+1)(\frac{\theta}{r}+\alpha\beta(1-\theta))\right]^{-1}
\end{aligned}
$$

By using this in (3.2.3)

$$
\left.\frac{\partial \Delta G}{\partial \bar{c_b}}\right|_{\bar{c_b}=c_b^*} \leq -\frac{\theta}{r}+\frac{\theta}{r}=0.
$$

This means that there exists $\bar{c}_1$ such that, for any $c_c \in (0, \bar{c}_1)$, $\Delta G \leq 0$. Given that $G(A_0, A_B^*; c_b^*, 0)$ is fixed, for any $c_c \in (0, \bar{c}_1)$, $G(A_0, \bar{A}_B; \bar{c_b}, c_c) \geq G(A_0, A_B^*; c_b^*, 0)$. This proves the second part of the proposition.

As for the bankruptcy costs, based on equation (2.5), $A_B^* = \beta(1-\theta)c_b^*$ and $\bar{A}_B = \beta(1-\theta)\bar{c_b}$. Given the closed-form solutions from Proposition 1

$$
BC(A_0, \bar{A}_B; \bar{c_b}) - BC(A_0, A_B^*; c_b^*) = \alpha\bar{A}_B\left(\frac{A_0}{\bar{A}_B}\right)^{-\gamma} - \alpha A_B^*\left(\frac{A_0}{A_B^*}\right)^{-\gamma}
$$

$$= \alpha\bar{c_b}\beta(1-\theta)\left(\frac{\bar{c_b}\beta(1-\theta)}{A_0}\right)^{\gamma} - \alpha c_b^*\beta(1-\theta)\left(\frac{c_b^*\beta(1-\theta)}{A_0}\right)^{\gamma}$$
$$= (\bar{c_b}^{\gamma+1} - (c_b^*)^{\gamma+1})\alpha\frac{(\beta(1-\theta))^{\gamma+1}}{A_0^{\gamma}}.$$

Since $\bar{c_b} < c_b^*$, the last term is strictly negative. $BC(A_0, \bar{A}_B; \bar{c_b}) < BC(A_0, A_B^*; c_b^*)$. □

3.2.6 Proof of Proposition 4

Proof. Denote $W(A_t, \bar{A}_B; \bar{c_b}, c_c) - W(A_t, \hat{A}_B; \hat{c_b}, 0)$ by $\Delta\hat{W}$. When the capital structure of the firm includes only equity and straight debt the closed-form solution for the value of equity is

$$W(A_t, \hat{A}_B; \hat{c_b}, 0) = A_t - \frac{\hat{c_b}(1-\theta)}{r}\left(1 - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) - \hat{A}_B\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}. \quad (3.2.4)$$

Based on (3.2.4) and the closed-form solution for $W(A_t, \bar{A}_B; \bar{c_b}, c_c)$ from Proposition 1

$$\begin{aligned}\Delta\hat{W} &= A_t - \frac{\bar{c_b}(1-\theta)}{r}\left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - \frac{c_c(1-\theta)}{r}\left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right) - \\ &\quad \bar{A}_B\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - \left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma} - A_t + \frac{\hat{c_b}(1-\theta)}{r}\left(1 - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) + \\ &\quad \hat{A}_B\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} \\ &= \frac{(\hat{c_b} - \bar{c_b})(1-\theta)}{r}\left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - \frac{c_c(1-\theta)}{r}\left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right) - \\ &\quad \left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma} + \frac{\hat{c_b}(1-\theta)}{r}\left(\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) + \hat{A}_B\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \\ &\quad \bar{A}_B\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}.\end{aligned}$$

Multiply both sides of (2.16) by $(1-\theta)$ and use the result to reduce the first three terms after the equal sign above to get

$$\Delta\hat{W} = -\theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma} + \frac{\hat{c_b}(1-\theta)}{r}\left(\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) +$$

$$\hat{A}_B \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \bar{A}_B \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}. \tag{3.2.5}$$

Continue with showing that $\Delta\hat{W} < 0$. From (3.2.5)

$$\begin{aligned}
\Delta\hat{W} &= \frac{\hat{c}_b(1-\theta)}{r}\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - \bar{A}_B\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - \\
&\left(\frac{\hat{c}_b(1-\theta)}{r}\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) - \theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma} \\
&= -(H(\hat{A}_B) - H(\bar{A}_B)) - \theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma}, \qquad (3.2.6)
\end{aligned}$$

where $H(X) \equiv \frac{\hat{c}_b(1-\theta)}{r}\left(\frac{A_t}{X}\right)^{-\gamma} - X\left(\frac{A_t}{X}\right)^{-\gamma}$. $H(X)$ is such that

$$\begin{aligned}
H'(X) &= \gamma\frac{\hat{c}_b(1-\theta)}{r}\left(\frac{A_t}{X}\right)^{-\gamma}\frac{1}{X} - (1+\gamma)\left(\frac{A_t}{X}\right)^{-\gamma} \\
&= \left(\frac{A_t}{X}\right)^{-\gamma}\frac{1}{X}\left(\gamma\frac{\hat{c}_b}{r}(1-\theta) - (1+\gamma)X\right).
\end{aligned}$$

Since $\hat{A}_B = \frac{\gamma(1-\theta)\hat{c}_b}{r(1+\gamma)}$

$$\begin{aligned}
H'(\hat{A}_B) &= \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\frac{1}{\hat{A}_B}\left(\gamma\frac{\hat{c}_b}{r}(1-\theta) - (1+\gamma)\frac{\gamma(1-\theta)\hat{c}_b}{r(1+\gamma)}\right) \\
&= \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\frac{1}{\hat{A}_B}\left(\gamma\frac{\hat{c}_b}{r}(1-\theta) - (1-\theta)\gamma\frac{\hat{c}_b}{r}\right) \\
&= 0.
\end{aligned}$$

It it also clear from above that if $0 < X < \hat{A}$, then $H'(X) > 0$. $H(X)$ is an increasing function of X on $(0, \hat{A}_B)$. Since $0 < \bar{A}_B < \hat{A}_B$, $H(\hat{A}_B) > H(\bar{A}_B)$ and, based on (3.2.6), $\Delta\hat{W} < 0$. The value of equity always decreases. This proves item (1).

Denote $G(A_t, \bar{A}_B; \bar{c}_b, c_c) - G(A_t, \hat{A}_B; \hat{c}_b, 0)$ by $\Delta\hat{G}$. When the capital structure of the firm includes only equity and straight debt the closed-form solution for the total value of the firm is

$$G(A_t, \hat{A}_B; \hat{c}_b, 0) = A_t + \frac{\hat{c}_b\theta}{r}\left(1 - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) - \alpha\hat{A}_B\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}. \tag{3.2.7}$$

Given (3.2.7) and the closed-form solution for $G(A_t, \bar{A}_B; \bar{c}_b, c_c)$ from Proposition 1

$$\begin{aligned}
\Delta\hat{G} &= A_t + \frac{\theta\bar{c}_b}{r}\left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) + \frac{\theta c_c}{r}\left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right) - \alpha\bar{A}_B\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - \\
&\quad A_t - \frac{\hat{c}_b\theta}{r}\left(1 - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) + \alpha\hat{A}_B\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} \\
&= \frac{\theta\bar{c}_b}{r}\left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) + \frac{\theta c_c}{r}\left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right) - \alpha\bar{A}_B\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - \\
&\quad \frac{\hat{c}_b\theta}{r}\left(1 - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) + \alpha\hat{A}_B\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}.
\end{aligned}$$

Multiply both sides of (2.16) by θ and replace $\frac{\theta c_c}{r}\left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right)$ above to get

$$\begin{aligned}
\Delta\hat{G} &= \frac{\theta\bar{c}_b}{r}\left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) + \frac{\theta(\hat{c}_b - \bar{c}_b)}{r}\left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - \theta\left(\frac{A_t}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right) - \\
&\quad \frac{\hat{c}_b\theta}{r}\left(1 - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) + \alpha\left(\hat{A}_B\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \bar{A}_B\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) \\
&= \frac{\hat{c}_b\theta}{r}\left(\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) + \alpha\left(\hat{A}_B\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \bar{A}_B\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - \\
&\quad \theta\left(\frac{A_t}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right).
\end{aligned}$$

This proves the first part of item (2) of the proposition.

We continue with showing that $\Delta\hat{G} > \Delta\hat{W}$. Based on (3.2.5) and the above result

$$\begin{aligned}
\Delta\hat{G} - \Delta\hat{W} &= \frac{\hat{c}_b}{r}\left(\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - (1-\alpha)\times \\
&\quad \left(\hat{A}_B\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \bar{A}_B\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) \\
&= \frac{\hat{c}_b}{r}\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - (1-\alpha)\hat{A}_B\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \\
&\quad \left(\frac{\hat{c}_b}{r}\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - (1-\alpha)\bar{A}_B\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right)
\end{aligned} \tag{3.2.8}$$

$$= \quad F(\hat{A}_B) - F(\bar{A}_B), \tag{3.2.9}$$

where $F(X) \equiv \frac{\hat{c}_b}{r}\left(\frac{A_t}{X}\right)^{-\gamma} - (1-\alpha)X\left(\frac{A_t}{X}\right)^{-\gamma}$. $F(X)$ is such that

$$\begin{aligned} F'(X) &= \gamma\frac{\hat{c}_b}{r}\left(\frac{A_t}{X}\right)^{-\gamma}\frac{1}{X} - (1-\alpha)(1+\gamma)\left(\frac{A_t}{X}\right)^{-\gamma} \\ &= \left(\frac{A_t}{X}\right)^{-\gamma}\frac{1}{X}\left(\gamma\frac{\hat{c}_b}{r} - (1-\alpha)(1+\gamma)X\right). \end{aligned}$$

Note, that $\hat{A}_B = \frac{\gamma(1-\theta)\hat{c}_b}{r(1+\gamma)}$ and, therefore,

$$\begin{aligned} F'(\hat{A}_B) &= \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\frac{1}{\hat{A}_B}\left(\gamma\frac{\hat{c}_b}{r} - (1-\alpha)(1+\gamma)\frac{\gamma(1-\theta)\hat{c}_b}{r(1+\gamma)}\right) \\ &= \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\frac{1}{\hat{A}_B}\left(\gamma\frac{\hat{c}_b}{r} - (1-\alpha)(1-\theta)\gamma\frac{\hat{c}_b}{r}\right) \\ &= \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\frac{1}{\hat{A}_B}\gamma\frac{\hat{c}_b}{r}\left(1-(1-\alpha)(1-\theta)\right). \end{aligned}$$

By assumption, $\alpha \in [0,1]$ and $\theta \in (0,1)$, so $(1-(1-\alpha)(1-\theta)) > 0$. It follows that $F'(\hat{A}_B) > 0$ for all $0 < X \leq \hat{A}_B$, and, since $0 < \bar{A}_B < \hat{A}_B$, $F(\hat{A}_B) > F(\bar{A}_B)$. Finally, based on (3.2.9), $\Delta\hat{G} > \Delta\hat{W}$.

We continue with proving the last statement of item (2).

$$\begin{aligned} \frac{\partial\Delta\hat{G}}{\partial\bar{c}_b} &= \left(-\gamma\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\frac{1}{\hat{A}_B}\frac{\hat{c}_b\theta}{r} - \alpha(1+\gamma)\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right)\frac{\partial\bar{A}_B}{\partial\bar{c}_b} - \\ &\quad \theta\left(\frac{A_t}{A_C}\right)^{-\gamma}\frac{1}{r}\frac{-\left(1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - (\hat{c}_b-\bar{c}_b)\gamma\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\frac{1}{\bar{A}_B}\frac{\partial\bar{A}_B}{\partial\bar{c}_b}}{1-(1-\lambda)\left(\frac{A_t}{A_C}\right)^{-\gamma}} \end{aligned}$$

where $\frac{\partial\bar{A}_B}{\partial\bar{c}_b} = \beta(1-\theta)$. Based on the above

$$\left.\frac{\partial\Delta\hat{G}}{\partial\bar{c}_b}\right|_{\bar{c}_b=\hat{c}_b} = -\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\left(\frac{\gamma}{\hat{A}_B}\frac{\bar{c}_b\theta}{r} + \alpha(1+\gamma)\right)\beta(1-\theta)+$$

$$\frac{\theta}{r}\left(\frac{A_t}{A_C}\right)^{-\gamma}\frac{1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}}{1-(1-\lambda)\left(\frac{A_t}{A_C}\right)^{-\gamma}}$$

$$= -\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\left(\frac{\gamma\theta}{r}+\alpha(1+\gamma)\beta(1-\theta)\right)+\frac{\theta}{r}\left(\frac{A_t}{A_C}\right)^{-\gamma}\frac{1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}}{1-(1-\lambda)\left(\frac{A_t}{A_C}\right)^{-\gamma}}$$

$$= -\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\left(\frac{\gamma\theta}{r}+\alpha(1+\gamma)\beta(1-\theta)\right)+\frac{\theta}{r}\frac{1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}}{\left(\frac{A_t}{A_C}\right)^{\gamma}-(1-\lambda)}.$$

For $\left(\frac{A_t}{A_C}\right)^{\gamma}-(1-\lambda)\geq 1$ or $\lambda\geq 2-\left(\frac{A_t}{A_C}\right)^{\gamma}$

$$\left.\frac{\partial\Delta\hat{G}}{\partial\bar{c_b}}\right|_{\bar{c_b}=\hat{c_b}} \leq -\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\left(\frac{\gamma\theta}{r}+\alpha(1+\gamma)\beta(1-\theta)\right)+\frac{\theta}{r}\left(1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right)$$

$$= -\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}(1+\gamma)\left(\frac{\theta}{r}+\alpha\beta(1-\theta)\right)+\frac{\theta}{r}. \tag{3.2.10}$$

Next, assume that $\hat{c_b}\geq c_b^*$. Then, based on (2.9)

$$\hat{c_b} \geq \frac{A_t}{\beta(1-\theta)}\left(\frac{\theta}{r}\right)^{\frac{1}{\gamma}}\left[(\gamma+1)(\frac{\theta}{r}+\alpha\beta(1-\theta))\right]^{-\frac{1}{\gamma}}$$

$$\frac{\beta(1-\theta)\hat{c_b}}{A_t} \geq \left(\frac{\theta}{r}\right)^{\frac{1}{\gamma}}\left[(\gamma+1)(\frac{\theta}{r}+\alpha\beta(1-\theta))\right]^{-\frac{1}{\gamma}}.$$

Based on (2.3)

$$\frac{\hat{A}_B}{A_t} \geq \left(\frac{\theta}{r}\right)^{\frac{1}{\gamma}}\left[(\gamma+1)(\frac{\theta}{r}+\alpha\beta(1-\theta))\right]^{-\frac{1}{\gamma}}$$

$$\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} \geq \frac{\theta}{r}\left[(\gamma+1)(\frac{\theta}{r}+\alpha\beta(1-\theta))\right]^{-1}$$

$$-\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} \leq -\frac{\theta}{r}\left[(\gamma+1)(\frac{\theta}{r}+\alpha\beta(1-\theta))\right]^{-1}. \tag{3.2.11}$$

By using (3.2.11) in (3.2.10), we get

$$\left.\frac{\partial\Delta\hat{G}}{\partial\bar{c_b}}\right|_{\bar{c_b}=\hat{c_b}} \leq -\frac{\theta}{r}+\frac{\theta}{r}=0.$$

This means that there exists $\bar{\bar{c}}_1$ such that, for any $c_c \in (0, \bar{\bar{c}}_1)$, $\Delta\hat{G} \leq 0$. Given that $G(A_0, A_B^*; c_b^*, 0)$ is fixed, for any $c_c \in (0, \bar{\bar{c}}_1)$, $G(A_0, \bar{A}_B; \bar{c_b}, c_c) \geq G(A_0, A_B^*; c_b^*, 0)$.

Finally, we prove item (3) of the proposition. Since $\hat{c_b} > \bar{c_b}$, based on (2.5), the optimal default-triggering boundary drops from $\hat{A}_B = \beta(1-\theta)\hat{c_b}$ to $\bar{A}_B = \beta(1-\theta)\bar{c_b}$. Given this and the closed-form solution for the cost of bankruptcy from Proposition 1

$$BC(A_t, \bar{A}_B; \bar{c_b}) - BC(A_t, \hat{A}_B; \hat{c_b}) = \alpha\bar{A}_B \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma} - \alpha\hat{A}_B \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}$$

$$= \alpha\bar{c_b}\beta(1-\theta)\left(\frac{\bar{c_b}\beta(1-\theta)}{A_t}\right)^{\gamma} - \alpha\hat{c_b}\beta(1-\theta)\left(\frac{\hat{c_b}\beta(1-\theta)}{A_t}\right)^{\gamma}$$
$$= (\bar{c_b}^{\gamma+1} - (\hat{c_b})^{\gamma+1})\alpha\frac{(\beta(1-\theta))^{\gamma+1}}{A_t^{\gamma}}.$$

Since $\bar{c_b} < \hat{c_b}$, the last term above is strictly negative. Therefore, $BC(A_t, \bar{A}_B; \bar{c_b}) < BC(A_t, \hat{A}_B; \hat{c_b})$. □

3.2.7 Proof of Proposition 5

Proof. We can use (2.3) to re-write (2.18) as

$$S(A_t; c_b, c_c) = \left(\frac{c_b}{r} - c_b(1-\theta)\beta\right)\left(\frac{A_t}{c_b(1-\theta)\beta}\right)^{-\gamma} = c_b\left(\frac{1}{r} - (1-\theta)\beta\right) \times \left(\frac{c_b(1-\theta)\beta}{A_t}\right)^{\gamma}.$$

Given that $\beta = \frac{\gamma}{r(1+\gamma)}$,

$$\frac{dS(A_t; c_b, 0)}{dc_b} = \left(\frac{1+\gamma}{r} - (1+\gamma)(1-\theta)\beta\right)\left(\frac{A_t}{A_B}\right)^{-\gamma} = \frac{1}{r}(1+\gamma\theta)\left(\frac{A_t}{A_B}\right)^{-\gamma} > 0.$$

Since $BC(A_t; \hat{c_b}, 0) = 0$ and based on equation (2.6), the total value of the firm in the presence of government subsidy when it does not issue CCB is

$$G(A_t; c_b, 0) = A_t + TB(A_t; c_b, 0) + S(A_t; c_b, 0).$$

Based on the closed-form solution for $TB(A_t; c_b, 0)$ and equation (2.18)

$$G(A_t; c_b, 0) \quad = \quad A_t + \frac{\theta c_b}{r}\left(1 - \left(\frac{A_t}{A_B}\right)^{-\gamma}\right) + \left(\frac{c_b}{r} - A_B\right)\left(\frac{A_t}{A_B}\right)^{-\gamma}.$$

Given budget equation (2.6), it is easy to see that $G(A_t; c_b, 0)$ from above is higher than the total value of the firm before the guarantee was issued by

$$S(A_t; c_b, 0) + BC(A_t; c_b) = \left(\frac{c_b}{r} - A_B\right)\left(\frac{A_t}{A_B}\right)^{-\gamma} + \alpha A_B \left(\frac{A_t}{A_B}\right)^{-\gamma} > 0.$$

□

3.2.8 Proof of Proposition 6

Proof. The only statement that remains to be proved: $G(A_t; c_b^g, 0) - G(A_t; \bar{c}_b, c_c) > 0$.

Denote $G(A_t; c_b^g, 0) - G(A_t; \bar{c}_b, c_c)$ by $\Delta\tilde{G}$. When the capital structure of the firm includes only equity and straight debt the closed-form solution for the value of tax benefits is

$$TB(A_t; c_b^g, 0) \quad = \quad \frac{\theta c_b^g}{r}\left(1 - \left(\frac{A_t}{A_B^g}\right)^{-\gamma}\right). \tag{3.2.12}$$

Given (2.24), (3.2.12), the closed-form solution for the value of tax benefits from Proposition 1, and equation (2.18) for the value of government subsidy

$$\begin{aligned}\Delta\tilde{G} \quad = \quad & \frac{\theta c_b^g}{r}\left(1 - \left(\frac{A_t}{A_B^g}\right)^{-\gamma}\right) - \frac{\theta \bar{c}_b}{r}\left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - \frac{\theta c_c}{r}\left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right) + \\ & \left(\frac{c_b^g}{r} - A_B^g\right)\left(\frac{A_t}{A_B^g}\right)^{-\gamma} - \left(\frac{\bar{c}_b}{r} - \bar{A}_B\right)\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}.\end{aligned} \tag{3.2.13}$$

By multiplying both side of equation (2.21) by θ and using the closed-form solutions for the values of straight and CCB, we get

$$\frac{c_b^g\theta}{r}\left(1 - \left(\frac{A_t}{A_B^g}\right)^{-\gamma}\right) + \left(\frac{A_t}{A_B^g}\right)^{-\gamma}(1-\alpha)A_B^g\theta = \frac{\bar{c}_b\theta}{r}\left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) +$$

$$\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}(1-\alpha)\bar{A}_B\theta+\frac{c_c\theta}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right)+\theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma}.$$

By rearranging terms

$$\frac{c_b^g\theta}{r}\left(1-\left(\frac{A_t}{A_B^g}\right)^{-\gamma}\right)-\frac{\bar{c}_b\theta}{r}\left(1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right)-\frac{c_c\theta}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right)=$$
$$\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}(1-\alpha)\bar{A}_B\theta+\theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma}-\left(\frac{A_t}{A_B^g}\right)^{-\gamma}(1-\alpha)A_B^g\theta. \quad (3.2.14)$$

Now we can use (3.2.14) in (3.2.13) to get

$$\begin{aligned}\Delta\tilde{G} &= \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}(1-\alpha)\bar{A}_B\theta+\theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma}-\left(\frac{A_t}{A_B^g}\right)^{-\gamma}(1-\alpha)A_B^g\theta+\\ &\quad \left(\frac{c_b^g}{r}-A_B^g\right)\left(\frac{A_t}{A_B^g}\right)^{-\gamma}-\left(\frac{\bar{c}_b}{r}-\bar{A}_B\right)\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\\ &= \theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma}+\left(\frac{c_b^g}{r}\left(\frac{A_t}{A_B^g}\right)^{-\gamma}-\frac{\bar{c}_b}{r}\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right)+\\ &\quad \left((1-\alpha)\theta+1\right)\bar{A}_B\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}-\left((1-\alpha)\theta+1\right)A_B^g\left(\frac{A_t}{A_B^g}\right)^{-\gamma}.\end{aligned}$$

Given (2.3) and $\beta=\frac{\gamma}{r(1+\gamma)}$,

$$\begin{aligned}\Delta\tilde{G} &= \theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma}+\left(\frac{c_b^g}{r}\left(\frac{A_t}{A_B^g}\right)^{-\gamma}-\frac{\bar{c}_b}{r}\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right)-\\ &\quad \left((1-\alpha)\theta+1\right)(1-\theta)\frac{\gamma}{(1+\gamma)}\left(\frac{c_b^g}{r}\left(\frac{A_t}{A_B^g}\right)^{-\gamma}-\frac{\bar{c}_b}{r}\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right)\\ &= \theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma}+\\ &\quad \left(1-\left((1-\alpha)\theta+1\right)(1-\theta)\frac{\gamma}{(1+\gamma)}\right)\left(\frac{c_b^g}{r}\left(\frac{A_t}{A_B^g}\right)^{-\gamma}-\frac{\bar{c}_b}{r}\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right).\end{aligned}$$

Above the first term on the right-hand side is positive, $\theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma}>0$. Also, for

$c_b^g > \bar{c_b}$, $\left(\frac{c_b^g}{r}\left(\frac{A_t}{A_B^g}\right)^{-\gamma} - \frac{\bar{c_b}}{r}\left(\frac{A_t}{A_B}\right)^{-\gamma}\right) > 0$. Finally,

$$1 - ((1-\alpha)\theta + 1)(1-\theta)\frac{\gamma}{(1+\gamma)} > 1 - (\theta+1)(1-\theta)\frac{\gamma}{(1+\gamma)} =$$
$$1 - (1-\theta^2)\frac{\gamma}{(1+\gamma)} > 0.$$

All the terms on the right-hand side of equation (3.2.15) are positive. Therefore, $\Delta\tilde{G} > 0$. □

3.2.9 Proof of Proposition 7

Proof. Consider a TBTF firm with a capital structure that includes equity and straight debt paying coupon c_b. We try to analyze the effect of replacing a portion of straight debt with CCB on the values of government subsidy and equity.

In the presence of government subsidy at any time t the following budget equation holds

$$A_t + TB(A_t;\hat{c_b},0) + S(A_t;\hat{c_b},0) \quad = \quad W(A_t;\hat{c_b},0) + U^B(A_t;\hat{c_b},0). \tag{3.2.15}$$

There are no bankruptcy costs, $BC(A_t;\hat{c_b},0) = 0$, and debt is risk-free, $U^B(A_t;\hat{c_b},0) = \frac{\hat{c_b}}{r}$.

The firm is to replace a portion of its current straight debt with CCB paying c_c. The remaining straight debt will be paying coupon $\bar{c_b}$, such that $\bar{c_b} < \hat{c_b}$. The government guarantee remains in place, so straight debt will still be risk-free, $U^B(A_t;\bar{c_b},c_c) = \frac{\bar{c_b}}{r}$. As before, straight debt holders should be indifferent between exchanging their holdings for CCB and continuing to hold straight debt

$$U^B(A_t;\hat{c_b},0) = U^C(A_t;\bar{c_b},c_c) + U^B(A_t;\bar{c_b},c_c). \tag{3.2.16}$$

Equation (3.2.16) is equivalent to equation (2.14) in Section 2.4.2. The key difference, though, is that after a TBTF firm announces its decision to replace straight debt with CCB the value of existing straight debt does not change. Debt is risk-free and, therefore, contrary to what we had before, the announcement does not affect its default boundary.

After the firm replaces a portion of its straight debt with CCB for any time t the following budget equation will hold

$$A_t + TB(A_t;\bar{c_b},c_c) + S(A_t;\bar{c_b},c_c) \;=\; W(A_t;\bar{c_b},c_c) + U^B(A_t;\bar{c_b},c_c) + U^C(A_t;\bar{c_b},c_c).$$

Given (3.2.15), (3.2.16) and above,

$$\begin{aligned} W(A_t;\hat{c_b},0) - W(A_t;\bar{c_b},c_c) \;=\; & TB(A_t;\hat{c_b},0) - TB(A_t;\bar{c_b},c_c) + \\ & S(A_t;\hat{c_b},0) - S(A_t;\bar{c_b},c_c). \end{aligned} \qquad (3.2.17)$$

Since $\hat{c_b} > \bar{c_b}$, based on Proposition 5 (part 1), $S(A_t;\hat{c_b},0) - S(A_t;\bar{c_b},c_c) > 0$. We proved the first part of the proposition.

Denote $W(A_t;\hat{c_b},0) - W(A_t;\bar{c_b},c_c)$ by $\Delta\tilde{W}$. When the capital structure of the firm includes only equity and straight debt the closed-form solution for the value of tax benefits is

$$TB(A_t;\hat{c_b},0) \;=\; \frac{\theta\hat{c_b}}{r}\left(1 - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right). \qquad (3.2.18)$$

Given (3.2.17), (3.2.18), the closed-form solution for the value of tax benefits from Proposition 1, and equation (2.18) for the value of government subsidy

$$\begin{aligned} \Delta\tilde{W} \;=\; & \frac{\theta\hat{c_b}}{r}\left(1 - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) - \frac{\theta\bar{c_b}}{r}\left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) - \frac{\theta c_c}{r}\left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right) + \\ & \left(\frac{\hat{c_b}}{r} - \hat{A}_B\right)\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma} - \left(\frac{\bar{c_b}}{r} - \bar{A}_B\right)\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}. \end{aligned} \qquad (3.2.19)$$

By multiplying both side of equation (3.2.16) by θ and using the closed-form solutions for the values of straight and CCB, we get

$$\begin{aligned} & \frac{\hat{c_b}\theta}{r}\left(1 - \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right) + \left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}(1-\alpha)\hat{A}_B\theta = \frac{\bar{c_b}\theta}{r}\left(1 - \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right) + \\ & \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}(1-\alpha)\bar{A}_B\theta + \frac{c_c\theta}{r}\left(1 - \left(\frac{A_t}{A_C}\right)^{-\gamma}\right) + \theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma}. \end{aligned}$$

By rearranging terms

$$\frac{\hat{c}_b\theta}{r}\left(1-\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}\right)-\frac{\bar{c}_b\theta}{r}\left(1-\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right)-\frac{c_c\theta}{r}\left(1-\left(\frac{A_t}{A_C}\right)^{-\gamma}\right)=$$
$$\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}(1-\alpha)\bar{A}_B\theta+\theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma}-\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}(1-\alpha)\hat{A}_B\theta. \quad (3.2.20)$$

Now we can use (3.2.20) in (3.2.19) to get

$$\begin{aligned}\Delta\tilde{W} &= \left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}(1-\alpha)\bar{A}_B\theta+\theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma}-\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}(1-\alpha)\hat{A}_B\theta+\\ &\quad\left(\frac{\hat{c}_b}{r}-\hat{A}_B\right)\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}-\left(\frac{\bar{c}_b}{r}-\bar{A}_B\right)\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\\ &= \theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma}+\left(\frac{\hat{c}_b}{r}\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}-\frac{\bar{c}_b}{r}\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right)+\\ &\quad\left((1-\alpha)\theta+1\right)\bar{A}_B\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}-\left((1-\alpha)\theta+1\right)\hat{A}_B\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}.\end{aligned}$$

Given (2.3) and $\beta=\frac{\gamma}{r(1+\gamma)}$,

$$\begin{aligned}\Delta\tilde{W} &= \theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma}+\left(\frac{\hat{c}_b}{r}\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}-\frac{\bar{c}_b}{r}\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right)-\\ &\quad\left((1-\alpha)\theta+1\right)(1-\theta)\frac{\gamma}{(1+\gamma)}\left(\frac{\hat{c}_b}{r}\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}-\frac{\bar{c}_b}{r}\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right)\\ &= \theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma}+\\ &\quad\left(1-\left((1-\alpha)\theta+1\right)(1-\theta)\frac{\gamma}{(1+\gamma)}\right)\left(\frac{\hat{c}_b}{r}\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}-\frac{\bar{c}_b}{r}\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right).\end{aligned}$$

Above the first term on the right-hand side is positive, $\theta\left(\lambda\frac{c_c}{r}\right)\left(\frac{A_t}{A_C}\right)^{-\gamma}>0$. Also, for $\hat{c}_b>\bar{c}_b$, $\left(\frac{\hat{c}_b}{r}\left(\frac{A_t}{\hat{A}_B}\right)^{-\gamma}-\frac{\bar{c}_b}{r}\left(\frac{A_t}{\bar{A}_B}\right)^{-\gamma}\right)>0$. Finally,

$$1-\left((1-\alpha)\theta+1\right)(1-\theta)\frac{\gamma}{(1+\gamma)}>1-(1-\theta^2)\frac{\gamma}{(1+\gamma)}>0.$$

All the terms on the right-hand side of equation (3.2.21) are positive. Therefore, $\Delta\tilde{W} > 0$. This proves the second part of the proposition. □

3.2.10 Proof of Proposition 8

Proof. By differentiating both sides of (2.10) relative to σ, we get

$$\frac{\partial W(A_0; c_b^*, c_c)}{\partial\sigma} = \frac{\partial W(A_0; c_b^*, 0)}{\partial\sigma} - \frac{\partial\left[U^C(A_0; c_c) - TB^C(A_0; c_b^*, c_c)\right]}{\partial\sigma} \tag{3.2.21}$$

Based on Proposition 1 and (2.7)

$$\begin{aligned}
U^C(A_0; c_c) - TB^C(A_0; c_b^*, c_c) &= \frac{c_c}{r}\left(1 - \left(\frac{A_0}{A_C}\right)^{-\gamma}\right) + \left(\frac{A_0}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right) - \\
&\quad \frac{\theta c_c}{r}\left(1 - \left(\frac{A_0}{A_C}\right)^{-\gamma}\right) \\
&= \frac{c_c}{r}(1-\theta)\left(1 - \left(\frac{A_0}{A_C}\right)^{-\gamma}\right) + \left(\frac{A_0}{A_C}\right)^{-\gamma}\left(\lambda\frac{c_c}{r}\right) \\
&= \frac{c_c}{r}(1-\theta) + \frac{c_c}{r}(\lambda + \theta - 1)\left(\frac{A_0}{A_C}\right)^{-\gamma}
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{\partial\left[U^C(A_0; c_c) - TB^C(A_0; c_b^*, c_c)\right]}{\partial\sigma} &= \frac{\partial\left[\frac{c_c}{r}(1-\theta) + \frac{c_c}{r}(\lambda+\theta-1)\left(\frac{A_0}{A_C}\right)^{-\gamma}\right]}{\partial\sigma} \\
&= \frac{c_c}{r}(\lambda+\theta-1)\frac{\partial\left[\left(\frac{A_0}{A_C}\right)^{-\gamma}\right]}{\partial\sigma} \\
&= \frac{c_c}{r}(\lambda+\theta-1)\frac{\partial\left[e^{-\gamma\log\left(\frac{A_0}{A_C}\right)}\right]}{\partial\sigma} \\
&= \frac{c_c}{r}(\lambda+\theta-1)e^{-\gamma\log\left(\frac{A_0}{A_C}\right)}\left(-\log\left(\frac{A_0}{A_C}\right)\right)\frac{\partial\gamma}{\partial\sigma} \\
&= -\frac{c_c}{r}(\lambda+\theta-1)e^{-\gamma\log\left(\frac{A_0}{A_C}\right)}\log\left(\frac{A_0}{A_C}\right)\frac{\partial\gamma}{\partial\sigma}.
\end{aligned}$$

By plugging this into (3.2.21), we get

$$\frac{\partial W(A_0; c_b^*, c_c)}{\partial \sigma} = \frac{\partial W(A_0; c_b^*, 0)}{\partial \sigma} + \frac{c_c}{r}(\lambda + \theta - 1)e^{-\gamma \log\left(\frac{A_0}{A_C}\right)} \log\left(\frac{A_0}{A_C}\right)\frac{\partial \gamma}{\partial \sigma}.$$

It is easy to show that for $\sigma > 0$: $\frac{\partial \gamma}{\partial \sigma} < 0$[2]. If $(\lambda + \theta) > 1$, then the last term in (2.10) is strictly positive and $\frac{\partial W(A_0; c_b^*, c_c)}{\partial \sigma} < \frac{\partial W(A_0; c_b^*, 0)}{\partial \sigma}$. If $(\lambda + \theta) > 1$, then it is strictly negative and $\frac{\partial W(A_0; c_b^*, c_c)}{\partial \sigma} > \frac{\partial W(A_0; c_b^*, 0)}{\partial \sigma}$. The interpretation is that, if $(\lambda + \theta) > 1$, then the equity value holders of the firm that issued CCB, straight debt and equity compared to the equity holders of the firm that issued only straight debt and equity will gain less from switching to riskier technologies. On the other hand, if $(\lambda + \theta) > 1$, the presence of CCB makes the shareholders gain more from taking extra risk. □

[2]Show that or $\sigma > 0$: $\frac{\partial \gamma}{\partial \sigma} < 0$