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Numerical development in preschool: Examining potential mediators of children's approximate number system and mathematics achievement.

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We have no conflicts of interest to declare.

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Abstract

There is evidence of a relation between the approximate number system (ANS) and later mathematics achievement. Researchers have proposed various mediators of this relation, including executive functioning (EF), numeral knowledge, and mathematical language. The goal of the present study was to determine which factors mediate the relation between preschoolers' ANS and the change in their mathematics achievement over a 5-month period. We collected data from 125 preschoolers (*Mean age* = 4.2 years) in the fall and spring. We tested mediation models using path analysis models, controlling for children's fall mathematics achievement, age, sex, and parent education. EF was not a statistically significant predictor of mathematics achievement when controls were included. Numeral knowledge was not a significant mediator, but mathematical language was a significant mediator in both individual mediation and multiple mediation models. Although children's ANS predicts their later mathematics achievement, the relation is fully mediated by children's mathematical language.

Keywords: approximate number system; mathematics achievement; executive function; numeral knowledge; mathematical language

Educational Impact and Implications Statement

We found that preschoolers’ ability at the beginning of the school year to tell which of two sets of objects had more predicted their mathematics achievement at the end of the school year, however, that predictive relation was fully explained by children’s understanding of mathematical language (e.g., words like *many*, *most*, *few*). These results suggest that early childhood educators should include opportunities for preschoolers to practice mathematical language to support their mathematics learning.

Numerical development in preschool: Examining potential mediators of children's ANS and mathematics achievement

Young children can tell at a glance which of two cookies has more chocolate chips if one cookie has six and the other has 12. That ability - key to choosing the most delicious cookie - also predicts children's concurrent and later mathematics achievement (Schneider et al., 2017). As early as infancy humans are believed to possess the ability to tell the difference between sets with different numbers of items, such as a set of eight dots and a set of 16 dots (Cordes & Brannon, 2008; Izard et al., 2009; Xu & Spelke, 2000). This imprecise number estimation is termed Approximate Number Sense (ANS). Over time, children and adults become more accurate in their judgments: Infants aged 49 hours to 6 months old can discriminate between sets that differ by ratios of 1:3 or 1:2, while 4- and 5-year-old children can discriminate between sets in ratios of 4:5 or 5:6, and some adults can reliably discriminate ratios of 9:10 (Halberda & Feigenson, 2008; Izard et al., 2009; Xu & Spelke, 2000). Notably, people of the same age show significant individual variation in their ANS (Halberda et al., 2012; Halberda & Feigenson, 2008). Meta-analytic reviews have consistently found that children and adults with higher ANS have higher average performance on mathematics assessments (Chen & Li, 2014; Fazio et al., 2014; Schneider et al., 2017).

Why might children's ANS predict their mathematics achievement? A number of theoretical and empirical studies have addressed this topic in the past two decades (e.g., Dehaene, 2001; Feigenson et al., 2004; Halberda et al., 2008; Leibovich & Ansari, 2016; Lyons et al., 2016; Mussolin et al., 2016; Piazza, 2010; Reynvoet & Sasanguie, 2016; Siegler, 2016). Some researchers have suggested that the ANS grounds children's developing understanding of symbolic numbers (e.g., number words and numerals, Dehaene, 2001; Feigenson et al., 2004;

Piazza, 2010). This implies that observed individual differences in the ANS underlie individual differences in symbolic number understanding – a critical facet of children's early mathematics achievement (Siegler, 2016).

In support of this perspective, behavioral data have shown that performance on ANS tasks and symbolic number comparison tasks are both characterized by ratio effects (e.g., Halberda & Feigenson, 2008; Holloway & Ansari, 2009); longitudinal and meta-analytic studies have consistently shown a significant relation between the ANS and mathematics achievement (Chen & Li, 2014; Fazio et al., 2014; Schneider et al., 2017); some experimental training studies have shown evidence that improving children's ANS can lead to improvements in their mathematics achievement (Hyde et al., 2014; Wang et al., 2020); and data from neuroimaging studies have implicated the same region of the brain, the Intraparietal Sulcus (IPS), in the processing of approximate and exact, symbolic numbers (Eger et al., 2009; Nieder & Dehaene, 2009). However, this perspective has been challenged by other researchers, who suggest that children's understanding of exact symbolic numbers emerges separately from their ANS (Leibovich & Ansari, 2016; Lyons & Ansari, 2015; Reynvoet & Sasanguie, 2016). Proponents of this viewpoint have different explanations for the relation between children's ANS and mathematics achievement, but typically argue that this relation can be explained entirely by shared variability with children's cognitive control ability or alternatively argue that children develop their approximate and exact representations of number in parallel and over time gain the ability to map between the two forms of representation.

Although correlational studies of children's ANS suggest that it predicts concurrent and later mathematics achievement, researchers have asked whether experimentally training children's ANS can lead to improvements in their mathematics achievement. The results of ANS training studies with young children are mixed. For example, Honoré and Noël (2016) randomly assigned preschool children to play ten sessions of an approximate number comparison game and an approximate number line estimation game or a parallel condition with symbolic number games. Children in the approximate number training condition did not significantly improve their performance on an exact arithmetic task; however, children in the symbolic number training did significantly improve their arithmetic skills. This suggests improvements in preschool children's ANS do not translate directly to improvements in general mathematics performance.

In contrast, Park and colleagues (2016) trained preschoolers across ten sessions using an approximate number arithmetic game. Children were shown a target set of objects, followed by the addition of a second set (addition problem) or the removal of some of the target set (subtraction problem). They were then asked to indicate either which of two sets represented the total quantity (match problem) or whether the total quantity was more or less than another quantity (comparison problem). Children who received the approximate arithmetic training significantly improved their mathematics achievement scores relative to children assigned to a control picture memory training. However, the authors theorized that asking children to mentally manipulate the approximate quantities was key to improving their mathematics achievement, as opposed to training that focused solely on refining children's ANS representations.

More recently, a series of experiments by Wang and colleagues (2020) found that experimentally manipulating children's ANS precision led to significant effects on their mathematics achievement, but only for children aged 4.5 years and older. In sum, previous experimental studies suggest that although it is possible to improve preschoolers' mathematical achievement by training their ANS, the success of the trainings may depend on the specific approach and the age of participants.

Although the topic has been the subject of much debate, there is growing consensus that there are mediating skills that support the connection between children's ANS and mathematics achievement (Reynvoet & Sasanguie, 2016; Siegler, 2016). From a developmental perspective, understanding the interplay between the ANS, mathematics achievement, and potential mediating skills is key to understanding how young children learn mathematics. In the present study, we draw from models proposed by Dehaene (2001), LeFevre and colleagues (2010), and Purpura and colleagues (2013) to investigate whether three skills (executive functioning, numeral knowledge, and mathematical language) mediate the longitudinal relation between ANS and mathematics achievement (Figure 1). We focus on executive functioning (EF) because it allows children to focus, organize, and hold critical information from their approximate number understanding while completing mathematical tasks; numeral knowledge because it links exact quantities to their symbolic representations that are necessary for successful performance in formal mathematics tasks; and mathematical language because it may help children refine approximate numerical quantities into exact numerical quantities. We review each of the potential mediators in turn.

Executive Functioning

Executive functioning abilities (EF) refer to cognitive control processes including cognitive flexibility, inhibitory control, and working memory (Diamond, 2013). Although there is historical debate as to how various cognitive control processes represent EF, there is a general consensus that cognitive flexibility, inhibitory control, and working memory represent the core EFs (Diamond, 2013), with some degree of conceptual and measurement overlap between each of these core processes in early childhood (Bull & Lee, 2014). EF abilities relate to children's school readiness skills and academic achievement throughout schooling, including higher mathematics achievement (e.g., Blair & Razza, 2007; Bull & Lee, 2014; Clark et al., 2010; Cragg & Gilmore, 2014; Duncan et al., 2007; Morrison et al., 2010). EF skills may allow children to concentrate, pay attention to, and learn from mathematical information in their surroundings (Diamond, 2013). Furthermore, EF skills may directly affect children's ability to inhibit incorrect responses, keep relevant information in mind, and shift their attention to the relative features of the tasks that assess their mathematical understanding (Bull & Lee, 2014).

Beyond the broader relations between children's EF and mathematics skills, there is evidence of a specific relation between EF skills and the ANS. Completing a standard ANS task taps children's EF abilities. In typical ANS tasks, preschoolers see two sets presented briefly (e.g., 2 seconds) and are asked to quickly press a button or otherwise identify the set with more objects (Halberda & Feigenson, 2008). Children must avoid choosing the set with larger surface area but fewer total objects, as well as any impulse to select one option repeatedly (e.g., always picking their favorite color). Successfully completing the task engages children's cognitive control processes, specifically their abilities to inhibit irrelevant information (e.g., the color of the dots or the button they pressed on the previous trial), shift their attention to the important aspects of the task (e.g., the number of dots on each side), and store and manipulate information in their working memory (e.g., the task instructions, the approximate quantities of each set of dots).

Previous research on ANS and EF abilities with samples of preschoolers has had mixed findings, with some studies reporting positive, significant effects (e.g., Fuhs & McNeil, 2013; Keller & Libertus, 2015, Experiment 2) and others reporting no concurrent relation (e.g., Keller & Libertus, 2015, Experiment 1; Purpura & Simms, 2018). For example, Fuhs and McNeil (2013) found that 3- to 5-year-old low-income children's ANS on more difficult comparison trials (where the side with a larger quantity of dots had a smaller total surface area than the side with fewer dots) significantly correlated with their inhibitory control skills ($r[85] = .31$, $p < .01$). In addition, children's ANS on the same comparison trials predicted their mathematics achievement, but the association was no longer significant when children's inhibitory control skills were added to the model. In contrast, Keller and Libertus (2015, Experiment 1) found that middle-income 5- and 6-year-olds' inhibition skills were not related to their ANS when using a validated behavioral measure of interference control ability ($r[37] = .23$, $p = .17$). However, when the authors conducted a second experiment with a larger sample of 3- to 6-year-olds using a computerized inhibitory control task, they did find a significant association between children's ANS and inhibition skills ($r[158] = .23$, $p < .01$; Keller & Libertus, 2015, Experiment 2). Moreover, Gilmore and colleagues (2013) found that with a broad age range of children (4-12 years old), EF skills related to children's ANS and significantly mediated the relation between children's ANS and mathematics achievement.

Numeral Knowledge

Numeral knowledge refers to a child's ability to correctly label numerals with their verbal number word ("2" means "two") and also the quantity they represent ("5" means "▲▲▲▲▲"). Most mathematics achievement measures for preschool and elementary school students involve understanding exact, symbolic numbers (e.g., number words and numerals). Indeed, preschool children's numeral knowledge significantly predicts their later mathematics achievement (Chard et al., 2005; Clarke & Shinn, 2004; Purpura et al., 2013).

Numeral knowledge may also serve as the translation between informal number experiences (such as counting objects) and formal number experiences (such as formal arithmetic tasks in school; Purpura et al., 2013). In other words, numeral knowledge may help children to link their non-symbolic, more abstract representations of numerical magnitude to school-based mathematics tasks such as simple arithmetic. Purpura and colleagues (2013) found that children's informal number skills, including a dot set comparison task assessing the ANS, predicted their knowledge of written numerals. Moreover, children's numeral knowledge fully mediated the relation between their informal number skills and mathematics achievement. In other studies with preschool participants, ANS measures correlate significantly with tasks assessing numeral knowledge (e.g., *r*s = .16 – .36; Merkley & Ansari, 2016; Mussolin et al., 2012; vanMarle et al., 2014). Indeed, one previous study found that 3-to-5-year-old children's numeral knowledge (along with verbal counting and cardinality understanding) mediated the relation between their fall ANS and spring mathematics achievement (vanMarle et al., 2014).

Mathematical Language

Children's knowledge of mathematical language may also partially explain the relation between their ANS and mathematics achievement. Mathematical language refers to words describing the mathematical concepts of numerical quantity or spatial relations, including the words *more* and *less* (Purpura et al., 2019). Mathematical language is conceptually distinct from traditional productive and receptive language skills in that it focuses specifically on children's understanding of mathematically relevant words (Purpura & Reid, 2016). Mathematical language is also distinct from specific mathematical content, such as the meaning of individual number words (e.g., cardinality; Peng et al., 2020; Purpura et al., 2019). Previous research has shown that children's mathematical language performance relates to their numerical skills and broader mathematics achievement in preschool (Chan et al., under review; Hassinger-Das et al., 2015; Hornburg et al., 2018; Purpura & Logan, 2015). For example, Hornburg and colleagues (2018) showed that 3-to-6-year-old preschoolers' mathematical language significantly predicted their performance on tasks of verbal counting, numeral knowledge, cardinality, and story problems, controlling for age, gender, and parental education.

Children's mathematical language also relates to their approximate number sense (Negen & Sarnecka, 2015; Purpura & Logan, 2015). Researchers theorize that children's early language skills help them to refine their conceptual understanding of individual numbers and the relations between them (e.g., Purpura et al., 2019; Spelke, 2003). For instance, if a child understands that the word *more* can be used to compare two numbers (e.g., "seven is *more* than three"), they may be able to use that information to refine their approximate understanding of the numbers seven and three before they have an exact knowledge of both numbers (Purpura et al., 2019). Moreover, understanding mathematical language may also support children's correct interpretation of task instructions. Negen and Sarnecka (2015) demonstrated that preschoolers' limited mathematical language may prevent them from understanding what they are being asked to do in typical ANS comparison tasks. Specifically, the authors found that young children do not always understand that the instruction to select the group with *more dots* means numerically more rather than greater surface area. However, after completing a brief training to clarify the numerical meaning of the term *more*, children's ANS was no longer predictive of their symbolic number knowledge. Similarly, Purpura and Logan (2015) found that the predictive relation of the ANS on children's mathematics achievement was dampened when a measure of their mathematical language was included.

The Present Study

The goal of the present study is to unpack the developmental relations between preschoolers' ANS, mathematics achievement, and potential mediating skills across one school year. We operationalized children's mathematics achievement with a standardized assessment of their numeracy skills, the Preschool Early Numeracy Skills Test–Brief Version (PENS-B; Purpura et al., 2015). Specifically, we aimed to test the hypotheses that executive functioning, numeral knowledge, and mathematical language mediate the relation between preschool children's ANS and mathematics achievement. We controlled for children's age, gender, and parent education, as previous research has shown significant relations between each of these demographic characteristics and children's mathematics achievement (e.g., Garon-Carrier et al., 2018; Hutchison et al., 2019; Jordan et al., 2009; Leahey & Guo, 2001; Merz et al., 2014). Based on the previous literature, we hypothesized that:

1. Children's fall ANS will significantly predict their spring math achievement, controlling for fall math achievement, child age, gender, and parent education.
2. Executive functioning, numeral knowledge, and math language will each significantly and positively mediate the relation between children's fall ANS and spring math achievement in individual, single-mediator models. Specifically:
    a. EF will mediate the relation between ANS and math, controlling for fall math achievement, child age, gender, and parent education.
    b. Numeral knowledge will mediate the relation between ANS and math, controlling for fall math achievement, child age, gender, and parent education.
    c. Math language will mediate the relation between ANS and math, controlling for fall math achievement, child age, gender, and parent education.
3. In a full model with EF, numeral knowledge, and math language included as mediators of the relation between children's ANS and math achievement, math language will be a statistically significant mediator.

Identifying mediating pathways is key to understanding the mechanisms underlying the observed relations between ANS and mathematics achievement and provides insight for future interventions to support children at risk of poor mathematics achievement. If the hypothesized mediators explain the variability between ANS and mathematics achievement, it suggests that research should focus on supporting these skills which may be more effective than training the ANS to improve symbolic mathematics (e.g., Honoré & Noël, 2016; Park et al., 2016; Wang et al., 2020). We focus on executive functioning, numeral knowledge, and mathematical language as potential mediators for two reasons. First, previous theoretical and empirical models of children's numerical development highlight the role for executive functioning (e.g., Fuhs & McNeil, 2013; Gilmore et al., 2013; Keller & Libertus, 2015; LeFevre et al., 2010), numeral knowledge (e.g., Mussolin et al., 2012; Purpura et al., 2013; vanMarle et al., 2014), and mathematical language (e.g., Dehaene, 2001; LeFevre et al., 2010; Negen & Sarnecka, 2015; Purpura & Logan, 2015) in supporting children's number skills over time. Second, when compared to related skills such as general intelligence and broader receptive language skills, our selected mediators show more robust relations to mathematics achievement (e.g., Bull & Lee, 2014; Purpura & Reid, 2016). Specifically, reviews of previous research have shown that EF skills including working memory explain more variance in mathematics performance than measures of intelligence (Bull & Lee, 2014; Raghubar et al., 2010). Moreover, studies contrasting the predictive effects of language on children's mathematics achievement have demonstrated that while children's general language skills significantly predict their number knowledge, general language skills are no longer predictive when domain-specific mathematical language is included in the model (Purpura & Reid, 2016). This pattern of results suggests that mathematical language is a more proximal predictor of children's mathematics achievement than their general language skills.

Although previous research has identified each of the proposed mediators as potentially related to children's ANS and mathematics achievement, most previous studies have focused on individual skills which makes it difficult to distinguish the signal from the noise in the broader literature (Geary & vanMarle, 2016). To address this gap in the previous literature, the present study examines multiple mediating pathways in both single-mediation models and a combined multiple mediation model within a single sample of preschoolers. This approach allows us to go beyond identifying significant mediators to compare the predictive power of each – a critical step to both understanding the mechanisms supporting the relations between ANS and mathematics achievement as well as identifying which mediating skill(s), if any, should be the focus of future educational interventions. We specifically hypothesized that math language would be a statistically significant mediator in the multiple mediation model given the relative strength of the bivariate relations between math language, the ANS, and children's mathematics achievement in previous research (e.g., Purpura & Logan, 2015).

Method

Participants

Participants were 125 3- to 5-year-old preschoolers (M = 4.2 years, SD = 7 months; 46% female; 70% White, 8% Asian, 3% African American/Black, 3% Hispanic/Latino, 15% Multiracial). We recruited children from 12 private preschools in the Midwestern United States serving children from a range of socioeconomic backgrounds as part of a larger study on early mathematical development. The highest level of education completed by either of the child's parents ranged from receiving a GED (1.6%), high school diploma (6.4%), some college but no degree (17.6%), an associate's degree (10.4%), a bachelor's degree (21.6%), a master's degree (19.2%), and a doctoral or postgraduate degree (23.3%). A total of 136 parents of preschool children completed consent forms. We excluded children who left school before the initial data collection (*n* = 4), or refused to provide verbal assent to participate (*n* = 7), from the analyses.

Procedure

Trained experimenters assessed preschool children at two time points (fall and spring) of the school year. The average time between children's fall and spring assessments was 5 months. Children completed measures of their mathematical, verbal, and executive functioning skills individually with experimenters in three to four 20-to-30 min sessions, at times and in areas of children's schools designated by school administrators and teachers. Data from these measures formed a larger battery of assessments administered at both the fall and spring sessions, however, for the purposes of the present study we focus on the fall administration of the ANS, executive functioning, numeral knowledge, and mathematical language assessments, and the fall and spring administration of the mathematics achievement assessment. The experimenters were research assistants who had completed or were working towards the completion of a bachelor's degree in psychology, speech/language and hearing sciences, or human development. Prior to administering the assessments with study participants, each experimenter completed two 2-3-hour training sessions, individual and group practice sessions, and a mock assessment session with lead project staff to ensure that they were able to administer the assessments in a fluent and reliable manner.

Measures

Approximate number system (ANS). Experimenters assessed children's ANS in the fall with Panamath software displayed on a laptop computer (https://panamath.org/; Halberda et al., 2008). Children saw a set of yellow dots presented on the left side of the screen and a set of blue dots simultaneously presented on the right side of the screen for 2.5 seconds and asked to indicate which side had more dots. The program was set to 5 minutes and 4 years old as the default across all children. The default settings included four ratio bins (1.25 to 1.46, 1.46 to 1.71, 1.78 to 2.09, and 2.71 to 3.18) from which equal distributions of items were selected. The number of dots presented on each side ranged from 5 to 21. The Panamath program counterbalances the magnitudes for side of presentation and controls for dot area and density. We scored children's performance as percent correct (accuracy). Previous research has shown that this measure has split-half reliabilities ranging from 0.65 to 0.72 for preschoolers (Libertus et al., 2013), and test-retest reliability for this sample across fall and spring measurements was 0.71 ($p < .001$).

Executive functioning (EF). Experimenters administered three measures of EF in the fall, representing the three core components of EF (cognitive flexibility, inhibition, working memory).

Cognitive flexibility was assessed with a modified Dimensional Change Card Sort task (adapted from Zelazo, 2006). Experimenters asked children to sort cards based on color, shape, and size in 24 items. Children were then asked to sort cards by color or size depending on whether the card had a black border or not. We scored children's performance as the number of correct trials. Internal consistency for this sample was Cronbach's $\alpha = 0.93$.

Inhibition was assessed with a modified Stroop task (adapted from Gerstadt et al., 1994). Children were shown a page with 30 pictures of suns and moons in a 5 x 6 layout and asked to label each picture as "moon" or "sun". Next, experimenters asked children to label the same pictures with the opposite labels, by saying "moon" if the picture was a sun and "sun" if the picture was a moon. We scored children's performance as the total number of labels made within 45 seconds of the opposite trial. We could not calculate internal consistency for this task because it is time-limited and the number of items per child varies. However, test-retest reliability for this sample across fall and spring measurements was 0.55 ($p < .001$).

Working memory (WM) was assessed using the listening recall task from the Automated Working Memory Assessment (Alloway, 2007). Children listened to sentences, were asked whether each sentence was correct, and then had to recall the last word of each sentence. We scored children's performance as the total number of last words they correctly remembered. Previous research has shown that this test has a test–retest reliability of 0.82 (Alloway, 2007).

Numeral knowledge. Experimenters assessed children's numeral knowledge in the fall with two tasks, numeral identification and sets-to-numerals (Purpura & Lonigan, 2015).

Numeral identification. Children were presented with flashcards of nine numerals (1, 2, 3, 7, 8, 10, 12, 14, and 15) and asked, "What number is this?". We scored children's performance as percent correct. Internal consistency for this sample was Cronbach's $\alpha = 0.88$.

Set-to-numerals. Children were presented with a numeral at the top of the page and five sets of dots below ($n = 3$ trials) or a set of dots at the top of the page and five numerals below ($n = 2$ trials) and asked which of the bottom options meant the same thing as the top numeral or set of dots. We scored children's performance as percent correct. Internal consistency for this sample was Cronbach's $\alpha = 0.80$.

Mathematical language. Experimenters assessed children's mathematical language in the fall using a 16-item measure (Purpura & Logan, 2015). Experimenters asked children questions relating to their knowledge of comparative language (e.g., *more*, *less*, *take away*, *first*, *a little bit*, *most*, *fewest*, *last*) and spatial language (e.g., *near*, *far*, *before*, *under*, *front*, *middle*, *end, below*). Children's performance was scored as the total number of items answered correctly. Internal consistency for this sample was Cronbach's α = 0.85.

Mathematics achievement. Experimenters assessed children's mathematic achievement in the fall and spring with the Preschool Early Numeracy Skills Test–Brief Version (PENS-B; Purpura et al., 2015). The PENS-B has 24 items that include set comparison, numeral comparison, one-to-one correspondence, number order, identifying numerals, ordinality, and number combinations. We scored children's performance as the total number of items answered correctly. Internal consistency for this sample was Cronbach's α = 0.93.

Analytic Approach

The second author and his colleagues collected and analyzed the data for previous publications. However, the first author preregistered all planned analyses for the present study prior to gaining access to the data. Neither the second author nor his colleagues with access to the data had previously run any of the proposed analyses.

Statistical models. We tested each hypothesis using path analysis models in Mplus Version 8 (Muthén & Muthén, 1998-2017). We used Full Information Maximum Likelihood (FIML) to estimate model parameters based on all observed data. Models 1 and 2a – 2c were just-identified, meaning the number of free parameters equaled the number of known values and cannot be evaluated with standard absolute model-fit criteria. However, just-identified models do generate path estimates of the magnitude and significance of the modeled relations. Model 3 was over-identified, meaning the number of free parameters exceeded the number of known values, and could be evaluated using standard absolute model-fit indices. We used the Standardized Root Mean Squared Residual (SRMR) and Comparative Fit Index (CFI) as model fit indices because they are considered appropriate for use with smaller sample sizes (less than 250 participants; Hu

& Bentler, 1998, 1999). In addition, we interpreted the magnitude and significance of each parameter estimate to determine whether there is support for the proposed structural relations (e.g., children's ANS predicting EF skills). We report standardized path coefficients between all constructs to aid interpretability.

We ran five models to test our hypothesized mediations. All models controlled for children's fall mathematics achievement, age, gender, and parent education. Specifically:

1. A path analysis model to test for a significant direct effect of children's fall ANS scores on their spring mathematics achievement scores.[1]
2. Three separate path analysis models to test for significant effects of individual mediators:
    a. A model with EF skills mediating the relation between ANS and mathematics achievement.
    b. A model with numeral knowledge mediating the relation between ANS and mathematics achievement.
    c. A model with mathematical language mediating the relation between ANS and mathematics achievement.
3. A full measured variable path analysis model with EF, numeral knowledge, and mathematical language as mediators of the relation between children's ANS and mathematics achievement.

Results

Preliminary Analyses

We present descriptive statistics and bivariate correlations in Table 1. Bivariate correlations revealed that children's gender was not related to their performance across the

[1] Note that Purpura and Logan (2015) present a version of this first analysis with additional covariates. The key mediation analyses were novel to the present study.

measures in our sample, thus we ran all analyses presented below without controlling for child gender and the pattern of results remains the same. We used principal component analysis (PCA) using SPSS Statistics Version 25 to create two composite variables that summarize children's performance across tasks. Unlike averaging standardized task scores, PCA explains the total variation in the observed variables, which can result in unequal weights for each task. We conducted the PCA for executive functioning on the cognitive flexibility, response inhibition, and working memory measures, with component loadings of .807, .815, and .606, respectively. The EF component explained 56% of the total variance. We conducted the PCA for numeral knowledge on the numeral identification and sets-to-numerals tasks (Table 2). The numeral knowledge component explained 83% of the total variance. We generated regression scores for participants from each PCA and used them in the models.

ANS Predicting Mathematics Achievement (Model 1)

We tested whether children's fall ANS accuracy predicted their spring mathematics achievement, controlling for fall mathematics achievement, child age, gender, and parent education (Figure 2). Children's fall ANS accuracy was a significant predictor of their spring mathematics achievement ($\beta = 0.27$, $p = .001$), controlling for their fall mathematics achievement ($\beta = 0.41$, $p < .001$), age ($\beta = 0.17$, $p = .056$), sex ($\beta = -0.04$, $p = .517$), and parent education ($\beta = 0.01$, $p = .975$).

Individual Mediator Analyses

EF as mediator (Model 2a). We tested whether children's fall EF skills mediated the relation between their ANS accuracy and spring mathematics achievement (Figure 3a). Although children's EF and spring mathematics achievement skills were significantly correlated (Table 1), EF skills were not a significant predictor of spring mathematics achievement when ANS, fall mathematics achievement, child age, sex, and parent education were also included in the model (β = 0.15, p = .161). However, the direct path between ANS and spring mathematics achievement remained significant (β = 0.24, p = .003). We conducted additional posthoc exploratory analyses by running three versions of this model with the individual measures of children's EF skills (cognitive flexibility, inhibition, and working memory) rather than the EF composite. None of the individual measures significantly predicted children's spring mathematics achievement when ANS, fall mathematics achievement, child age, sex, and parent education were also included in the model (cognitive flexibility β = 0.08, p = .391; inhibition β = 0.11, p = .191; working memory β = 0.01, p = .943).

Numeral knowledge as mediator (Model 2b). We tested whether children's fall numeral knowledge mediated the relation between their ANS accuracy and spring mathematics achievement (Figure 3b). Numeral knowledge significantly predicted spring mathematics achievement (β= 0.40, p < .001) and the path from ANS to numeral knowledge was marginally significant (β = 0.15, p = .051). The direct path from ANS to spring mathematics achievement remained significant (β = 0.21, p = .007) and the indirect path via numeral knowledge was not significant (β = 0.06, p = .072), which suggests numeral knowledge did not mediate the relation between children's ANS and spring mathematics achievement.

Mathematical language as mediator (Model 2c). We tested whether children's mathematical language skills mediated the relation between their ANS accuracy and spring mathematics achievement (Figure 3c). Mathematical language significantly predicted spring mathematics achievement (β = 0.32, p < .001) and ANS significantly predicted mathematical language (β = 0.35, p < .001). The direct path from ANS to spring mathematics achievement was not significant (β = 0.14, p = .099), although the indirect path via mathematical language was significant (β = 0.12, p = .005), which suggests mathematical language significantly mediated the relation between children's ANS and spring mathematics achievement.

EF, Numeral Knowledge, and Mathematical Language as Mediators (Model 3)

We tested whether children's EF, numeral knowledge, and mathematical language mediated the relation between their ANS accuracy and spring mathematics achievement (Figure 4). The model was over-identified and model fit was satisfactory (SRMR = 0.04, CFI = 0.94). Numeral knowledge (β = 0.36, p < .001) and mathematical language (β = 0.27, p = .003) were significant predictors of spring mathematics achievement; EF was not a significant predictor of spring mathematics achievement (β = 0.01, p = .951). The direct path from ANS to spring mathematics achievement was not significant (β = 0.11, p = .164), however the indirect effect via mathematical language was significant (β = 0.10, p = .015). The indirect effects of children's ANS on spring mathematics achievement via their EF (β = 0.01, p = .951) and numeral knowledge (β = 0.06, p = .077) were not significant, although the total indirect effect combined across mediators was significant (β = .15, p = .001). This pattern of findings suggests that children's numeral knowledge predicted (but did not mediate) their spring mathematics achievement, however children's mathematical language significantly mediated the relation between children's ANS and spring mathematics achievement.

Discussion

The present study tested three potential mediators of the association between preschoolers' ANS and later mathematics achievement: executive functioning, numeral knowledge, and mathematical language. Like previous research (e.g., Libertus et al., 2013; Purpura & Simms, 2018), we found that preschoolers' ANS in the fall of the school year was a significant predictor of their spring mathematics achievement with a small effect size, controlling for fall mathematics achievement, child age, gender, and parent education. Contrary to our hypothesis, we found that children's EF skills did not significantly predict their spring mathematics achievement, nor did they mediate the relation between children's ANS and mathematics achievement. Children's numeral knowledge did predict their spring mathematics achievement, though it was not a significant mediator of the relation between children's ANS and mathematics achievement. However, mathematical language skills fully mediated the relation between ANS and mathematics achievement. This suggests that children's mathematical language skills can account completely for the shared variability of their ANS and mathematics achievement, highlighting the important role of mathematical language relative to the ANS in early childhood mathematical development.

The Curious Case of Executive Functioning and Mathematics Achievement

Although we hypothesized that children's executive functioning skills would mediate the association between their ANS and mathematics achievement, our results showed that EF skills were not a significant predictor of mathematics achievement. EF skills were unrelated to children's mathematics achievement when analyzed as a composite of cognitive flexibility, inhibition, and working memory as well as when each measure was analyzed separately. Previous studies have found a consistent positive association between preschooler's EF skills, particularly working memory, and mathematics achievement on both numerical tasks and standardized mathematics measures (see Bull & Lee, 2014 for a review).

One possible explanation for our discrepant finding is our analytic decision to control for children's fall mathematics achievement in all models. Controlling for the auto-regressive effects of children's fall mathematics achievement on their spring mathematics achievement may have explained much of the shared variance between children's fall executive functioning skills and spring mathematics achievement. Indeed, children's EF skills were highly correlated with both fall and spring mathematics achievement. Recent research by Nguyen and colleagues (2019) using large-scale datasets and meta-analytic estimates found that young children's performance on EF tasks were more correlated with mathematics achievement measures than with other EF tasks, suggesting considerable overlap between the two constructs in early childhood. Thus, if previous research that has shown that EF predicts children's later mathematics achievement did not control for prior mathematics performance (e.g., Clark et al., 2010), it may explain why we did not find the hypothesized relation between EF and mathematics achievement in our more conservative model.

Although the relation between EF and mathematics achievement is well-established in the previous literature, several studies have also shown mixed results as to the relation between children's ANS, EF, and mathematics achievement (e.g., Fuhs & McNeil, 2013; Gilmore et al., 2013; Keller & Libertus, 2015; Purpura & Simms, 2018). In light of these mixed findings, we hypothesize that if children's EF skills truly mediate the relation between their ANS and mathematics achievement at the population level, it is likely to be a partial mediation with a small effect size at most. However, this is an empirical question that remains unanswered. Future research could consider quantifying the significance and effect size of this potential mediating relation using meta-analytic techniques.

Numeral Knowledge Relates but not Mediates

Although children's numeral knowledge was a significant predictor of their spring mathematics achievement, numeral knowledge was not a significant mediator of the association between children's ANS and mathematics achievement as hypothesized. In order to successfully complete typical mathematics achievement measures, children must have some understanding of symbolic numbers. Much of the previous theoretical and empirical research on the ANS in early childhood has investigated this symbol-grounding problem, in attempt to unpack how children's approximate, non-symbolic understanding of numbers connects to their understanding of number symbols, if at all (e.g., De Smedt et al., 2013; Leibovich & Ansari, 2016; Mussolin et al., 2016; Reynvoet & Sasanguie, 2016). Although the present study does not shed light on how young children learn about symbolic numbers, it is possible that their understanding of written symbolic numerals may mediate later processes linking children's informal-exact number knowledge and their formal-exact number knowledge (Purpura et al., 2013). In contrast, the present study focused on mediators of the informal-approximate number knowledge (e.g., the ANS) and exact number knowledge (e.g., mathematics achievement measure), which may have obscured the more nuanced developmental process at play.

Mathematical Language as a Full Mediator

In accordance with our hypothesis, we found that children's mathematical language skills fully mediated the relation between their fall ANS and spring mathematics achievement. In a broad sense, children who do not understand the mathematical language in the ANS task instructions (e.g., to select the side with more dots) should not be expected to perform successfully on the task (Negen & Sarnecka, 2015). This parallels previous research on Piaget's conservation of number task, which ultimately found that preschoolers did not always understand the task instructions asking them which quantity was *more* (Elbers, 1986; Sinha & Carabine, 1981). More specifically, young children who score highly on the mathematical language assessment are signaling an understanding of the meaning of the mathematical concepts conveyed by the numerical and spatial relations queried. Children's mathematical language is key to their initial learning of these numerical relations and may allow them to broaden their understanding to abstract over larger set sizes (Purpura et al., 2019). When considering early childhood instruction, this finding suggests that opportunities to support preschoolers in developing their mathematical language skills may provide benefits. For example, brief storybook interventions have been found to improve children's mathematical language (Purpura et al., 2021).

Indeed, recent experimental studies have shown that reading interventions focusing on spatial and numerical words (e.g., *more*, *less*, *near*, *far*) can lead to improvements in both children's mathematical language and mathematics achievement (Purpura et al., 2021; Hassinger-Das et al., 2015; Purpura et al., 2016). This contrasts with the mixed findings of studies to promote children's mathematics achievement by improving their ANS (Honoré & Noël, 2016; Park et al., 2016; Wang et al., 2020). Future research should investigate whether interventions focused on improving the ANS benefit from being combined with interventions that support mathematical language. Moreover, future research should consider measures that assess ANS without the use of verbal instructions (e.g., Lindskog & Simms, 2021), such as a longitudinal study of infants' ANS with nonverbal paradigms mapping onto their mathematical language and achievement as preschoolers.

Limitations

The present study has several limitations. First, the present study cannot disentangle whether the overlap in variability between children's performance on the ANS task and the mathematical language measure could be due to their understanding of the word *more* (and in turn their interpretation of the task instructions). However, the mathematical language measure includes a number of terms in addition to the word *more*, and previous research on this measure has demonstrated that roughly 80% of children accurately respond to this item (Purpura &

Logan, 2015), resulting in a low difficulty score compared to other items on the measure (Purpura & Reid, 2016). Moreover, from a theoretical perspective, there is likely additional overlap between children's ANS, mathematical language, and mathematics achievement such that mathematical language helps children refine their approximate understanding of quality and bolster their understanding of the relations between symbolic numbers that is critical for success on broader mathematics achievement measures. An important next step for future research in this area will be to experimentally test the causal effects of a mathematical language intervention on children's ANS and mathematics achievement.

In a related point, one potential alternative hypothesis to our finding is that an understanding of mathematical language may rely on children's exact representations of cardinal numbers, rather than their representations of approximate quantities. However, this seems unlikely due to previous research showing young children aged 3.3 years can successfully verify sentences related to terms including *more* and *most* and rely on their approximate number system to complete these mathematical language tasks (Odic et al., 2013). Future work should examine these developmental processes in more depth to provide further evidence for how these skills relate over time.

A second limitation to the present study is that we took a narrow look at the association between children's ANS and mathematics achievement by focusing on preschoolers across a 5-month span of time. Therefore, our findings only inform our understanding of preschoolers although it is likely that the relations between executive functioning, numeral knowledge, and mathematical language with the ANS and mathematics achievement differ with older samples. In particular, the relation between EF and mathematics achievement when controlling for prior mathematics achievement would likely be stronger for older samples of children with more differentiation between EF and mathematical skills (Nguyen et al., 2019). Moreover, the children in our sample had parents with higher educational attainment compared to the general adult population in the U.S. (i.e., 64% of our sample had at least one parent who had obtained a bachelor's degree or higher, compared to 36% of adults in the U.S. aged 25 years or older; U.S. Census Bureau, 2020), which could potentially limit the generalizability of our results to other groups of children.

A third limitation is that in our attempt to discriminate the signal from the noise by testing multiple potential mediators in the same study, there were other relevant potential mediators that we did not assess. In particular, children's understanding of the cardinal values of number words and understanding of the relative magnitudes of number words and numerals have both been theorized to be crucial to developing mathematical understanding (Geary et al., 2017; Siegler, 2016). Both cardinality and symbolic magnitude understanding fall under the umbrella of children's broader understanding of symbolic number words and numerals, although we focused on numeral knowledge in the present study. Future research should consider the unique and potentially overlapping influence of children's cardinality, symbolic magnitude, and numeral knowledge as mediators of the relation between ANS and mathematics achievement.

Finally, the design of the present study does not allow for true causal inference - i.e., we have not shown evidence that children's higher ANS causes their higher mathematical language skills, which in turn causes higher mathematics achievement. Future research should use experimental designs to disentangle the causal nature of skill development.

Conclusion

In conclusion, the present study tested whether the relation between preschoolers' ANS and the change in their mathematics achievement was mediated by their EF, numeral knowledge, and mathematical language skills. We found evidence that mathematical language skills mediated the relation between ANS and mathematics achievement. These results suggest that early childhood educators should consider including opportunities for preschoolers to practice mathematical language, which may in turn support children's mathematics learning. Although children's ANS predicted variability in their mathematics achievement, it is clear that other mathematical skills can explain much of the same variability.

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Figure 1

Hypothesized model of mathematical development in early childhood

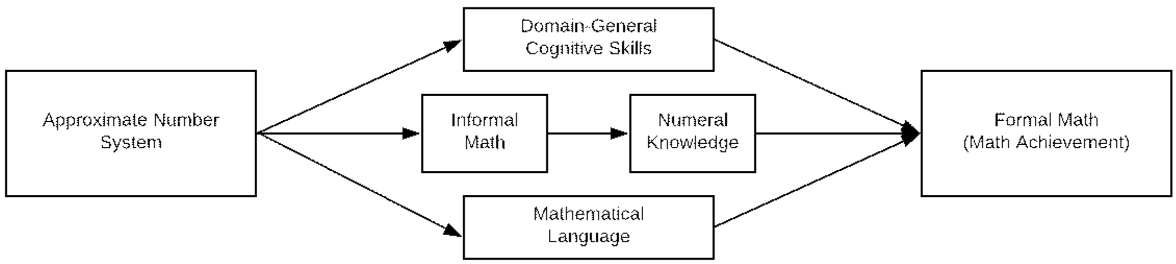
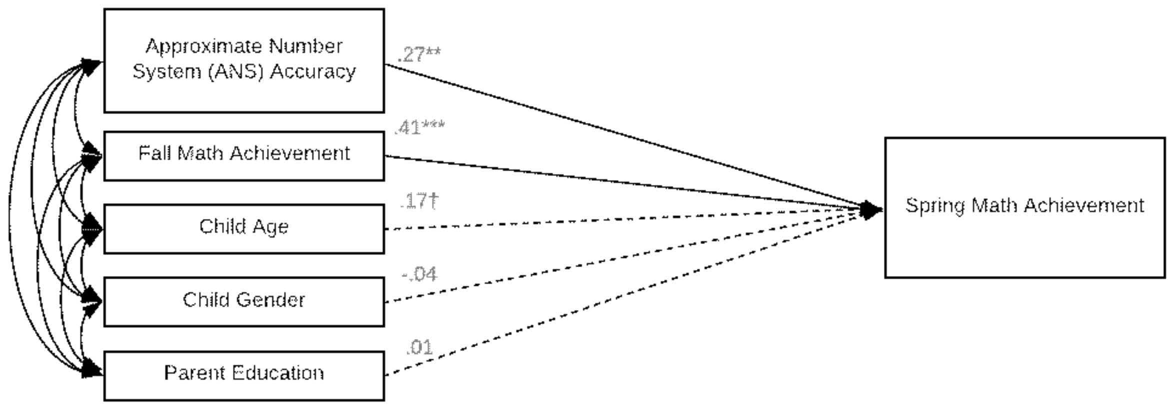


Figure 2

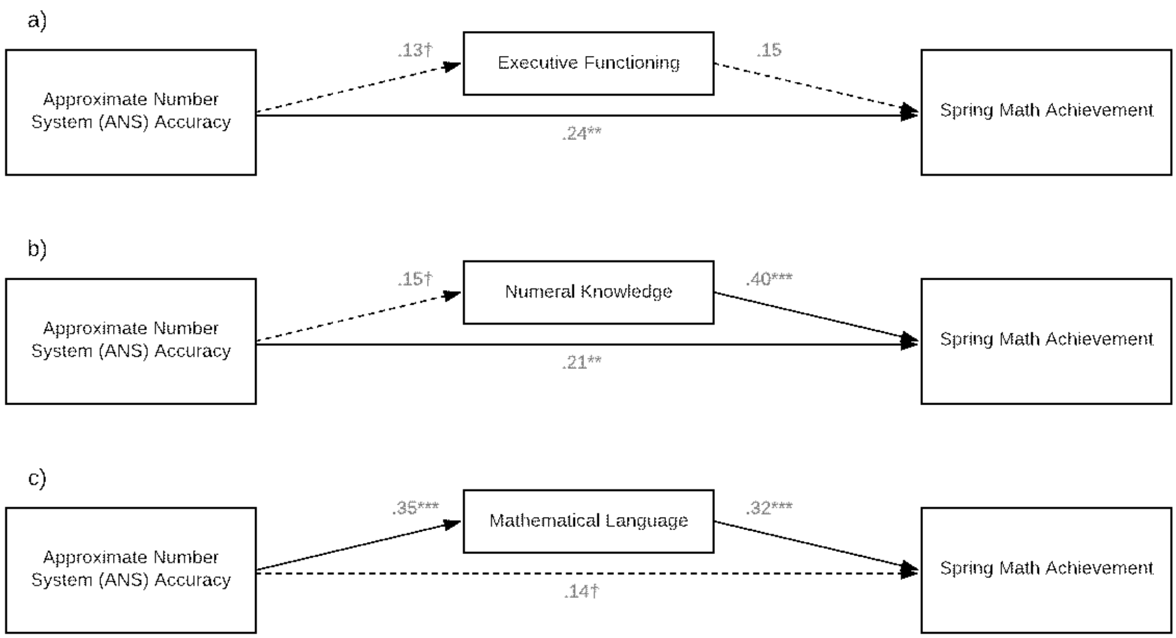
ANS accuracy predicting mathematics achievement, controlling for child fall mathematics achievement, age, gender, and parent education (Model 1)



Note. Solid lines represent paths statistically significant paths, dotted lines represent non-significant paths. *** $p < .001$, ** $p < .01$, † $p < .10$

Figure 3

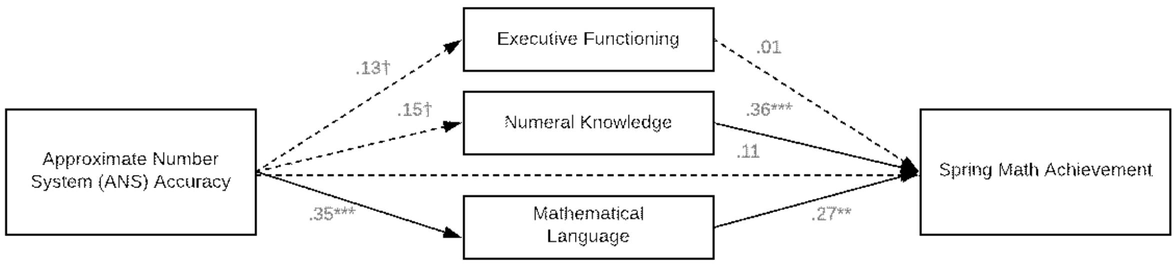
Mediators of ANS accuracy predicting mathematics achievement, controlling for child fall mathematics achievement, age, gender, and parent education (Models 2a – 2c)



Note. Solid lines represent paths statistically significant paths, dotted lines represent non-significant paths. Child fall mathematics achievement was the only control variable that was a significant predictor of spring mathematics achievement in models a – c. Child fall mathematics achievement predicted executive functioning, numeral knowledge, mathematical language; child age predicted executive functioning and mathematical language; no other control variables significantly predicted any of the mediating variables. *** $p < .001$, † $p < .10$.

Figure 4

Executive functioning, numeral knowledge, and mathematical language as mediators of ANS accuracy and mathematics achievement, controlling for child fall mathematics achievement, age, gender, and parent education (Model 3)



Note. Solid lines represent paths statistically significant paths, dotted lines represent non-significant paths. None of the control variables were a significant predictor of spring mathematics achievement. Child fall mathematics achievement predicted executive functioning, numeral knowledge, mathematical language; child age predicted executive functioning and mathematical language; no other control variables significantly predicted any of the mediating variables.

*** $p < .001$, ** $p < .01$, † $p < .10$.

Table 1. *Descriptive statistics and bivariate correlations*

	M	*SD*	*Range*	1	2	3	4	5	6	7	8	9	10	11	12
1. Child age	4.18	0.58	3.12 - 5.26	---											
2. Child gender (% female)	46.4	50.1	---	.21	---										
3. Parental education	6.94	1.67	3 - 9	$-.06^{ns}$	$.06^{ns}$	---									
4. Fall mathematics achievement	10.17	5.85	0 - 23	.60	$.00^{ns}$	.35	---								
5. ANS accuracy	66.94	16.12	27.5 - 100	.47	$.06^{ns}$	.33	.64	---							
6. Inhibitory control	19.88	8.59	0 - 36	.52	$-.06^{ns}$	$.06^{ns}$	.54	.37	---						
7. Cognitive flexibility	13.34	6.47	4 - 22	.43	$-.06^{ns}$	.29	.65	.46	.48	---					
8. Working memory	1.71	2.68	0 - 11	.34	$.00^{ns}$	$.17^{ns}$	.49	.47	.26	.25	---				
9. Numeral identification	5.09	2.81	0 - 9	.43	$.07^{ns}$	.30	.64	.54	.44	.50	.27	---			
10. Sets-to-numerals	2.83	1.75	0 - 5	.50	$.04^{ns}$	.25	.71	.52	.57	.52	.26	.66	---		
11. Mathematical language	11.25	3.84	2 - 16	.51	$-.03^{ns}$	.31	.68	.65	.61	.62	.38	.49	.64	---	

12. Spring mathematics achievement	13.55	5.95	0 - 24	.55	.02^{ns}	.19	.69	.62	.48	.51	.39	.60	.71	.68	---

Note. All correlations were significant at $p < .05$, unless noted as non-significant (^{ns}).