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Essays on Macroeconomics

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by

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Abstract of the Dissertation

Essays on Macroeconomics

by

Luis Gonzalo Llosa

Doctor of Philosophy in Economics

University of California, Los Angeles, 2012

Professor Lee Ohanian, Chair

In these essays, I examine (i) the role of terms of trade in emerging countries and (ii) economic efficiency under endogenous information. The first chapter documents a negative relationship between the terms of trade - defined as the ratio of imports to the price of exports - and various macroeconomic variables such as output, consumption, investment and total factor productivity (TFP) in emerging economies. The second part of this chapter presents a small open economy business cycles model featuring intermediate imported inputs, monopolistic competition and input-output linkages. A calibrated version of the model reproduces the empirical facts documented in the first part. In particular, the model replicates the negative link between the terms of trade and TFP, providing an explanation to a long-standing puzzle in the business cycle literature. In addition, I show how terms of trade shocks help to increase the volatility of consumption above the volatility of output. The second and third chapters, which are part of an ongoing work with Venky Venkateswaran, examine economic efficiency under costly private information. In the second chapter, my co-author and I study the efficiency of equilibrium outcomes in models where monopolistically competitive firms acquire costly information about aggregate fundamentals before making pricing or quantity decisions. Using this framework we show that market power reduces the private value of information relative to its social value, causing too little investment in learning and inefficient cyclical fluctuations. Importantly, this is true even in an environment where the ex-post response to information is socially optimal, which is the case when firms choose labor input under uncertainty about aggregate productivity. When firms set nominal prices, however, their actions exhibit a inefficiently high sensitivity to their private signals. The

combination of this inefficiency in information use and market power makes the overall direction of the inefficiency in information acquisition ambiguous. In terms of policy, we show that the standard full information response to market power-related distortions can reduce welfare under endogenous uncertainty. We also show how our analysis applies to coordination games in general, using the beauty contest framework. In the third chapter my co-author and I show that the same inefficiencies characterized in the previous chapter take place in standard business cycle models. In a RBC framework with dispersed information about technology shocks, distortions due to market power have no effect on incentives to respond to information, but distort the private value of information, leading to an inefficiently low level of information acquired in equilibrium. In a monetary model with nominal price-setting by heterogeneously informed firms, inefficiencies arise in both the use and the acquisition of information.

The dissertation of Luis Gonzalo Llosa is approved.

Mark J. Garmaise

Ariel Thomas Burstein

Andrew Granger Atkeson

Lee Ohanian, Committee Chair

University of California, Los Angeles

2012

To my wife, Silvana, whose immense love kept me afloat in turbulent times.

To my parents, Luis and Eleana, who have supported me each step of the way.

To my Grandparents, Sabino and Judith, whose example has guided me throughout life.

There are no words to express my gratitude to all of them.

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CHAPTER 1

Terms of trade In Emerging Markets

1.1 Introduction

Do terms of trade have a real effect on small open economies? If they do, what is the mechanism behind? Several studies have noticed that the terms of trade - defined as the ratio of import prices to export prices - have a negative correlation with output and TFP and that larger terms of trade shocks play an important role in explaining larger business cycles, e.g. Mendoza [Men95], Kose [Kos02], Easterly et. al [EL93], Becker and Mauro [BM05], Izquierdo et. al. [IRT08], Kehoe and Ruhl [KR08]. However, recent research has challenged some of these empirical findings by showing that standard macro models predict that changes in the terms of trade have no first order effect on TFP if output is measured using chain-weighted methods, e.g. Kehoe and Ruhl [KR08].

This paper revisits the aforementioned questions using a small open economy (SOE) model in which imports are inputs in production, output markets are imperfectly competitive and firms are connected in an input-output network. Otherwise, the model nests the standard SOE model commonly used in quantitative macro, e.g. Mendoza [Men91]. Using this framework, this paper delivers the following results:

1. terms of trade shocks affect TFP in the same way as in the data.
2. terms of trade shocks increase the volatility of consumption relative to that of output.
3. Input-output linkages amplify the influence of terms of trade shocks on the real economy.

The main finding is that terms of trade affect negatively the aggregate output in this economy through TFP. To understand this result, first note that in the presence of intermediate inputs, output is really the total quantity of goods produced by firms net of the real opportunity cost of intermediate inputs. This notion of output corresponds to real value added and it essentially requires subtracting real cost of imports - i.e. cost of imports at constant terms of trade - from the gross output. Now, suppose that there is an (infinitesimal) increase in the terms of trade that causes a decrease in the use of imported inputs. As a result, for each unit of less imports, gross output falls by an amount equal to the marginal product of imported inputs (increasing output) and the real cost of imports falls by an amount equal to the constant terms of trade (decreasing output). The net effect depends on whether or not the marginal product is equal to terms of trade in equilibrium. Monopolistic behavior distorts this equalization, i.e. the marginal product of imports is higher than the terms of trade. It follows that after an increase in terms of trade, TFP and output falls.

In the model the influence of terms of trade on TFP is summarized by an elasticity. The absolute magnitude of the elasticity of TFP to terms of trade is proportional to the excess price over marginal cost or markup. This occurs because in monopolistic competitive equilibria markups create a constant positive wedge between the marginal product of imports and the terms of trade. Hence, the greater is the markup, or the lower the elasticity of substitution among competing products, the greater is the influence of the terms of trade on TFP. The intuition works as follows. With markups firms under-produce and thus use a sub-optimal low level of imports. A terms of trade improvement partially undoes this inefficiency yielding additional quantity of output. When the monopolistic distortion disappears, i.e. zero markups, the elasticity of TFP to the terms of trade collapses to zero. This knife-edge case corresponds to perfect competition which is a common assumption in standard business cycles models. Under perfect competition, profit maximization also optimizes output, which kills the first order effects of terms of trade. In other words, as a direct consequence of the envelope theorem, in a perfectly competitive environment the terms of trade effects on TFP are confined to be second or higher order.

Other elements of the model determine the magnitude of the elasticity of TFP to the terms of trade. One element is the degree of exposure to imported inputs as measured by the share of imports on output. The intuition here is that markups act like a tax on imports and the share of imports to output is the tax base. Hence, the share of imports on output re-scales the distortion (which affects all production) into output units. The interplay between the markup and the share of imports on output determines the magnitude of the direct effect of the terms of trade on TFP. If firms engage in inputs transactions to one another, then these interconnections will enlarge the magnitude the elasticity of TFP to the terms of trade. Specifically, each firm is indirectly affected by the terms of trade via those firms it supplies inputs to and those firms it buys inputs from. These indirect effects set in motion a sequence of feedback loops among firms that amplify aggregate shocks.

The last qualitative result is that terms of trade increase the volatility of consumption relative to that of output. The general idea is that consumption is really driven by permanent real income and not by output. Now, while output measures the quantity of final goods produced by domestic factors of production, real income measures the purchasing power of households' income generated by those factors of production. This difference implies that income is more elastic to the terms of trade than output. Consequently, terms of trade volatility increases income volatility (relative to the volatility of output), and through it, the volatility of consumption (relative to the volatility of output). Yet, access to international capital markets bound the volatility of consumption below the volatility of income.

I quantify to what extent the aforementioned results matter in the data. To that end, I calibrate the model to a sample of emerging economies and perform a series of numerical simulations. These simulations show that terms of trade shocks alone account for about a quarter of the volatility of the actual TFP volatility. The model also performs quite well for other moments of the data. Terms of trade shocks alone account for about 75 percent of the actual volatility of output. Under the baseline calibration the model is unable to generate excess volatility of consumption. However, under highly persistent, though stationary, terms of trade shocks the model is able to generate excess volatility of consumption. Putting restrictions to international capital mobility also generates the same result.

1.1.1 Related literature

This paper is connected with a strand of quantitative macro literature studying the role of terms of trade within SOE RBC framework, e.g. Mendoza [Men95] and Kose [Kos02]. In this models, technology shocks and terms of trade shocks are assumed to be correlated. I assume that terms of trade shocks are independent from technology shocks. Other papers have studied terms of trade fluctuations in models where imports are intermediate inputs in production, e.g. Kohli [Koh04], Kehoe and Ruhl [KR08], and Feenstra et al. [FMR09]. The main message of these papers is that terms of trade do not have a direct effect on output if output is measured using a chain-weighted method. The economic insight is that there is a envelope condition that guarantees that terms of trade have no first-order effects on TFP. A common theme in these analyses is the assumption of perfectly competitive markets. In this paper I relax this assumption given the empirical evidence in favor non-competitive markets, e.g. Broda and Weinstein [BW06] and Hendel and Nevo [HN06]. I show that, under monopolistic (and hence inefficient) competitive equilibria, terms of trade have first-order effects on TFP. This occurs because the monopolistic behavior of firms introduces a wedge between the marginal product of imports and the terms of trade, thus, breaking the envelope condition Kehoe and Ruhl refer to.

There are other papers analyzing the consequences of imperfect competition for TFP, e.g. Hall [Hal90] and Basu and Fernald [BF02] among others. In particular, there exist some noteworthy coincidences between Basu and Fernald and my work. Specifically these authors show that imperfect competition introduces several non-technological factors into industry TFP. One of these factors arises from aggregate industrial output (real value added), which is computed as the difference between gross output and intermediate inputs valued at their purchase price, not their marginal product. Importantly, for this result to hold, a fraction of intermediate inputs must be produced outside the industry. That is exactly the insight I emphasized in this paper, except that intermediates inputs must be produced by another country.

A recent group of papers analyze the role nonconvexities for the effects of terms of trade on aggregate outcomes, e.g. Alessandria et al. [AKM10] and Gopinath and Neiman [GN12]. This research shows that nonconvexities in imports have important consequences not only for the level of trade, but also for the response of industry aggregates after terms of trade shocks. In an parallel and independent work, Gopinath and Neiman build a monopolistic competitive model similar to the one considered here except that firms must pay a fixed cost for each input variety they decide to import.[1] There are important similarities and differences between their work and mine. Both papers are similar in the sense the impact of the terms of trade on TFP arises due to firms' monopolistic behavior. Furthermore, Gopinath and Neiman's model features a richer trade adjustment after terms of trade shock which create an additional mechanisms through which terms of trade affect TFP and at the same time allows them to match a set of new micro-facts.[2] In contrast, I consider a much simpler trade adjustment pattern while I expand the analysis to other macro consequences of terms of trade shocks such as the response of investment, consumption, labor and aggregate output.

This paper also contributes to the study of terms of trade effects on other macro variables. In the context of perfect competition, Kohli [Koh04] shows that output tends to underestimate the increase (decrease) in real income and welfare when the terms of trade improve (deteriorate). For example, from a balance-trade position, an improvement in the terms of trade implies that the same amount of exports can produce more imports. As a consequence, real income and welfare rise directly from that effect. In contrast,output, which focuses on production, subtracts this direct price effect. I show that this also true for the case of imperfect competition, i.e. real income responds more forcefully than output. Moreover, I connect this result with the so-called *excess volatility of consumption puzzle*, which as a salient feature of SOE business cycles, specially in emerging economies, e.g. Neumeyer and Perri [NP05]. Aguiar and Gopinath [AG07] provide an explanation to this phenomenon that is based permanent shocks to the growth rate of TFP. The key idea of their argument lies on the permanent income hypothesis, namely, consumption responds more to the permanent component of income than to the transitory one.[3] To

[1] Gopinath and Neiman [GN12] show that the size of the fixed cost and the ToT shock determine the adjustment in several margins: (i) the number of imported varieties, (ii) the number of importing firms and (iii) who import and who not (selection effect). These margins are important part of the trade adjustment pattern observed in the aftermath of 2001 Argentine crisis.

[2] For instance, in their model firm level import shares affect the level of productivity. Moreover, because a ToT shocks change the number of imported varieties, statistical import price index may differ from the ideal price index. This mismeasurement introduces an artificial ToT effect on TFP, see Feenstra et al [FMR09].

[3] Other common explanations to the excess volatility of consumption involve essentially shocks to

some extent, terms of trade shocks induce a similar result with the difference that the excess volatility of consumption arises from a difference between income and output.

My work is connected with a literature studying the role of intermediate inputs in macroeconomics, e.g. Basu [Bas95], Jones [Jon11], Acemoglu et. al. [ACO11] among others. Two lessons are derived from this literature. First, input-output linkages amplify disturbances in the economy, e.g. Jones [Jon11]. Second, the architecture of the input-output matrix matters for aggregate volatility, e.g. Acemoglu et. al. [ACO11]. Following Basu, in my model all domestically produced goods can serve either as final outputs or as inputs for the production of other goods. I modify this structure by adding imported intermediate inputs, which are supplied elastically (at a given price) by an external sector. Moreover, I have assumed that the domestic technology of production depends equally on imported inputs. This implies that the external sector plays the role of a general-purpose technology. terms of trade shocks can be interpreted as shocks to this general-purpose technology and the amplification occurs downstream as all firms using imports are interconnected to each other.

The rest of the paper is organized as follows. Section 1.2 presents some empirical regularities of emerging markets. Section 3.3 outlines the model. Section 3.3.5 discusses the inefficiency in TFP. Section 1.5 presents calibrates the model and presents the quantitative results. Section 2.8 concludes.

1.2 Data

This section documents some empirical regularities about business cycles and the terms of trade in emerging economies. First, terms of trade are highly countercyclical. Second, higher terms of trade volatility is associated with larger business cycles fluctuations. These empirical regularities are contrasted with those obtained for developed economies. This comparison shows that the aforementioned stylized facts are distinctive characteristics of emerging economies.

the real interest rates that not only affect the stochastic discount factor but also affect output through financial constraints, e.g. Neumeyer and Perri [NP05], Mendoza and Yue [MY11].

1.2.1 Sample and definitions

I consider a sample of emerging countries and developed countries. The sample of emerging countries consists of a list of non-oil exporters, no transition, middle-income developing countries.[4] The sample is restricted to those countries integrated to international capital markets as defined in Calvo et. al. [CIT06]. The sample of developed countries consists of OECD members prior to 1980.[5] The annual time series studied are real GDP, consumption, investment, real net exports, terms of trade and TFP. Detailed information about these time series is given in the data appendix. The analysis is restricted to the period between 1980 and 2008. The analysis is further restricted to those countries with complete data between 1980 and 2008.[6] The final sample is composed by 12 emerging economies and 22 developed economies.[7]

The variables of interest are real output, real consumption, real investment, real net exports, TFP and the terms of trade. Detailed information about these time series is given in the data appendix. Terms of trade (hereafter P_m) is defined as ratio of the import price deflator to the export price deflator.[8] Following the literature on real business cycles, e.g. Bergoeing et. al. [BKK02], *Total Factor Productivity* (TFP) is calculated as:

$$TFP_t = \frac{Y_t}{K_{t-1}^{\alpha} L_t^{1-\alpha}}$$

[4]The classification of middle-income developing country is taken from World Bank's World Development Indicator. See IMF [IMF00] for a list of transition economies. See Chapter 2 of IMF's World Economic Outlook 2006 for a list of oil exports. These are countries with an average share of fuel exports in total exports that exceeds 40 percent and the average value of fuel exports that exceeds 500 millions of dollars.This classification excludes large oil producers for which oil is not a key export, such as Canada, Ecuador, Mexico, and the United Kingdom.

[5]As in the case of emerging economies, oil exporters were left out of the analysis. The only country within this category is Norway.

[6]By focusing on this period, the analysis reduces the influence of possible structural breaks on the empirical facts. Aguiar and Gopinath [AG07] points out that some important stylized facts about emerging-market economies in the last decades are not present before 1980. This structural break can reflect the fact that many emerging-market economies were essentially closed economies or had tight controls on private capital flows before 1980.

[7]The group of emerging economies is conformed by Argentina, Brazil, Chile, Colombia, Indonesia, Korea, Mexico, Malaysia, Peru, Philippines, Thailand and Uruguay. The group of developed economies is conformed by Australia, Austria, Belgium, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, United Kingdom, Greece, Ireland, Iceland, Italy, Japan, Luxembourg, Netherlands, New Zealand, Portugal, Sweden and United States.

[8]Terms of trade are similarly defined by Backus et. al. [DK92]. The implicit price deflators from national accounts correspond to Paasche price indexes, i.e. individual import (resp. export) prices weighted by current imports (resp. export) quantities. The price deflators are constructed using exports and imports of goods and services.

where Y_t is the period t *real GDP*, K_{t-1} is the end of period $t-1$ *stock of capital* (beginning of period t stock of capital) and L_t is the labor input. The stock of capital is constructed using the perpetual inventory method under constant depreciation rate δ. The parameters α and δ are fixed at 0.36 and 0.06, respectively. These are the standard values used in the literature, see for example Bergoeing et. al. [BKK02]. These parameters are also used in the calibration.

All series are logged, except real net exports which are presented as a ratio to real output. I focus on the business cycle component by removing the stochastic trend from these series. The trends are calculated using the Hodrick-Prescott filter with smoothing parameter 100, which is the usual value for annual data.

1.2.2 Empirical regularities

This section documents some empirical regularities about business cycles and the terms of trade in emerging economies. These empirical regularities are contrasted with those obtained for developed economies. This comparison highlights some distinctive characteristics of emerging economies.

Part I of table 1.1 reports the standard deviation of the variables in percent units. As expected, the volatility of all variables is higher for emerging economies than for developed economies. This is also true for the terms of trade, which are on average twice as much volatile in emerging markets than in developed countries.

Data on individual countries suggest that emerging economies with larger terms of trade fluctuations also face larger business cycles fluctuations. To illustrate this, Figure 1.1 plots terms of trade volatility (horizontal-axis) against output volatility (left scatter plot) and TFP volatility (right scatter plot). To show the degree of association between business cycles volatility and terms of trade volatility within each group, the figure also plots the fitted values obtained from OLS regressions for each group. There seems to be a positive association between terms of trade volatility and business cycle volatility in emerging economies. In contrast, among developed economies terms of trade volatility and business cycle volatility seem to be disconnected.

Part II of table 1.1 reports the volatility of each variable relative to the volatility of

Table 1.1: Business cycles moments: Volatility

I. Volatility - standard deviation (in percent)	Y		C		I		NX		TFP		P_M	
Argentina	7.04	(0.97)	7.34	(1.11)	20.22	(3.32)	2.29	(0.34)	6.41	(0.77)	6.68	(1.00)
Brazil	3.61	(0.52)	4.13	(0.39)	9.86	(1.08)	1.56	(0.22)	3.61	(0.47)	8.09	(0.95)
Chile	4.60	(0.86)	6.48	(1.33)	15.10	(2.58)	2.96	(0.55)	4.57	(1.04)	8.70	(0.95)
Colombia	2.89	(0.28)	3.25	(0.32)	16.33	(2.66)	2.27	(0.27)	2.74	(0.19)	5.53	(0.95)
Indonesia	4.80	(0.93)	6.82	(1.05)	12.93	(2.47)	4.90	(1.02)	5.36	(0.86)	9.26	(1.77)
Korea	3.00	(0.50)	4.03	(0.89)	8.49	(1.27)	2.07	(0.43)	2.61	(0.54)	3.66	(0.36)
Mexico	3.77	(0.71)	5.38	(0.90)	12.98	(2.19)	2.04	(0.31)	3.14	(0.57)	6.52	(1.42)
Malaysia	4.35	(0.68)	6.98	(1.00)	19.33	(2.96)	6.20	(0.82)	3.64	(0.47)	3.72	(0.84)
Peru	6.55	(0.87)	6.27	(1.05)	17.37	(1.16)	1.98	(0.17)	7.70	(1.16)	8.90	(0.93)
Philippines	4.13	(0.84)	2.18	(0.42)	15.20	(3.16)	0.68	(0.07)	4.66	(0.97)	5.93	(0.71)
Thailand	5.17	(0.86)	5.25	(0.82)	18.87	(2.99)	4.32	(0.73)	4.70	(0.64)	4.11	(0.66)
Uruguay	6.70	(0.87)	8.63	(1.01)	26.11	(3.68)	2.29	(0.20)	5.99	(0.91)	5.86	(0.91)
Emerging[1]	4.72	(1.35)	5.56	(1.80)	16.07	(4.61)	2.80	(1.50)	4.60	(1.50)	6.41	(1.91)
Developed[1]	2.19	(0.63)	2.33	(0.98)	7.03	(2.34)	1.24	(0.70)	1.59	(0.60)	2.80	(1.45)

II. Volatility - Relative standard deviation	Y		C		I		NX		TFP		P_M	
Argentina	1.00	(0.00)	1.04	(0.05)	2.87	(0.15)	0.33	(0.02)	0.91	(0.08)	0.95	(0.21)
Brazil	1.00	(0.00)	1.14	(0.17)	2.73	(0.20)	0.43	(0.10)	1.00	(0.08)	2.24	(0.31)
Chile	1.00	(0.00)	1.41	(0.11)	3.28	(0.11)	0.64	(0.07)	0.99	(0.09)	1.89	(0.47)
Colombia	1.00	(0.00)	1.13	(0.09)	5.66	(0.85)	0.79	(0.11)	0.95	(0.08)	1.92	(0.38)
Indonesia	1.00	(0.00)	1.42	(0.35)	2.69	(0.15)	1.02	(0.32)	1.12	(0.07)	1.93	(0.59)
Korea	1.00	(0.00)	1.34	(0.13)	2.83	(0.22)	0.69	(0.08)	0.87	(0.17)	1.22	(0.23)
Mexico	1.00	(0.00)	1.43	(0.22)	3.45	(0.20)	0.54	(0.08)	0.83	(0.04)	1.73	(0.33)
Malaysia	1.00	(0.00)	1.61	(0.17)	4.45	(0.28)	1.43	(0.15)	0.84	(0.08)	0.86	(0.21)
Peru	1.00	(0.00)	0.96	(0.05)	2.65	(0.40)	0.30	(0.06)	1.17	(0.07)	1.36	(0.25)
Philippines	1.00	(0.00)	0.53	(0.04)	3.68	(0.17)	0.16	(0.03)	1.13	(0.07)	1.43	(0.28)
Thailand	1.00	(0.00)	1.01	(0.04)	3.65	(0.28)	0.83	(0.09)	0.91	(0.06)	0.79	(0.17)
Uruguay	1.00	(0.00)	1.29	(0.07)	3.89	(0.27)	0.34	(0.03)	0.89	(0.07)	0.87	(0.17)
Emerging[1]	1.00	(0.00)	1.19	(0.28)	3.49	(0.85)	0.63	(0.34)	0.97	(0.11)	1.43	(0.48)
Developed[1]	1.00	(0.00)	1.05	(0.26)	3.22	(0.64)	0.55	(0.20)	0.73	(0.19)	1.37	(0.84)

Note: Data are logged, except for net exports which are presented as the ratio of real net exports to real GDP. Data are detrended using the Hodrick-Prescott with a smoothing parameter of 100. Moments were estimated by GMM. Numbers in parenthesis are the standard errors.

[1] Simple cross-country average. The standard errors for the averages were computed assuming independence across countries.

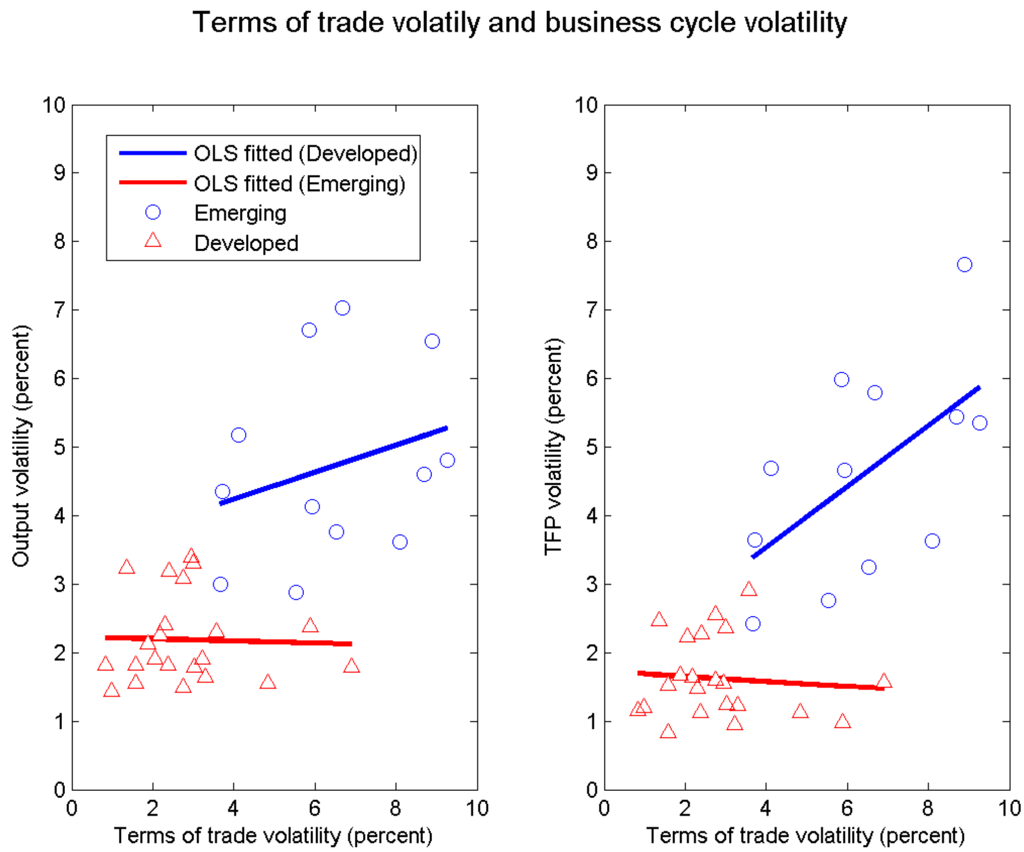

Figure 1.1: Terms of trade volatility and business cycle volatility

output. For the same level of output volatility, emerging economies have more volatility than developed economies. Importantly, emerging economies have a level of consumption volatility that exceeds the volatility of output while developed economies have a level of consumption volatility that is in almost line with the level of output volatility. This is a well documented distinctive feature of business cycles in emerging economies, see Neumeyer and Perri [NP05], Aguiar and Gopinath [AG07], Mendoza and Yue [MY11].

Part I of table 1.2 reports the correlation with output. The level of cyclicality is similar in both groups except for next exports and the terms of trade. In the case of net exports, they are strongly counter-cyclical in emerging economies, almost twice as much than developed economies. This is a well documented feature of business cycles in emerging economies, see Neumeyer and Perri [NP05], Aguiar and Gopinath [AG07], Mendoza and Yue [MY11]. With respect to the terms of trade, the evidence suggest that they are countercyclical in emerging economies whereas they are acyclical in developed

Table 1.2: Business cycles moments: Correlations

I. Correlation with output	Y		C		I		NX		TFP		P_M	
Argentina	1.00	(0.00)	0.95	(0.03)	0.97	(0.01)	-0.95	(0.02)	0.96	(0.01)	-0.45	(0.14)
Brazil	1.00	(0.00)	0.25	(0.26)	0.93	(0.02)	-0.50	(0.10)	0.94	(0.02)	-0.57	(0.14)
Chile	1.00	(0.00)	0.94	(0.02)	0.95	(0.03)	-0.84	(0.05)	0.95	(0.02)	-0.34	(0.11)
Colombia	1.00	(0.00)	0.86	(0.08)	0.65	(0.12)	-0.53	(0.19)	0.73	(0.11)	-0.32	(0.19)
Indonesia	1.00	(0.00)	0.47	(0.14)	0.93	(0.04)	-0.35	(0.14)	0.95	(0.01)	-0.15	(0.16)
Korea	1.00	(0.00)	0.92	(0.05)	0.86	(0.08)	-0.67	(0.19)	0.80	(0.05)	-0.59	(0.16)
Mexico	1.00	(0.00)	0.88	(0.05)	0.91	(0.05)	-0.65	(0.12)	0.98	(0.01)	-0.79	(0.07)
Malaysia	1.00	(0.00)	0.91	(0.03)	0.95	(0.02)	-0.88	(0.04)	0.82	(0.05)	-0.61	(0.07)
Peru	1.00	(0.00)	0.94	(0.03)	0.78	(0.07)	-0.64	(0.07)	0.92	(0.03)	-0.27	(0.13)
Philippines	1.00	(0.00)	0.93	(0.03)	0.93	(0.04)	-0.44	(0.18)	0.93	(0.03)	0.44	(0.09)
Thailand	1.00	(0.00)	0.97	(0.02)	0.96	(0.02)	-0.84	(0.08)	0.93	(0.02)	-0.33	(0.15)
Uruguay	1.00	(0.00)	0.97	(0.01)	0.91	(0.03)	-0.84	(0.04)	0.88	(0.07)	-0.10	(0.25)
Emerging	1.00	(0.00)	0.83	(0.22)	0.89	(0.09)	-0.68	(0.19)	0.90	(0.07)	-0.34	(0.30)
Developed	1.00	(0.00)	0.81	(0.12)	0.85	(0.09)	-0.35	(0.25)	0.78	(0.14)	-0.04	(0.31)
II. Correlation with terms of trade	Y		C		I		NX		TFP		P_M	
Argentina	-0.45	(0.14)	-0.41	(0.14)	-0.42	(0.14)	0.55	(0.13)	-0.41	(0.15)	1.00	(0.00)
Brazil	-0.57	(0.14)	-0.28	(0.17)	-0.55	(0.13)	0.42	(0.16)	-0.57	(0.18)	1.00	(0.00)
Chile	-0.34	(0.11)	-0.35	(0.12)	-0.40	(0.12)	0.38	(0.13)	-0.33	(0.13)	1.00	(0.00)
Colombia	-0.32	(0.19)	-0.30	(0.21)	-0.23	(0.15)	0.14	(0.20)	-0.40	(0.14)	1.00	(0.00)
Indonesia	-0.15	(0.16)	-0.57	(0.16)	-0.18	(0.15)	0.62	(0.13)	-0.18	(0.18)	1.00	(0.00)
Korea	-0.59	(0.16)	-0.42	(0.16)	-0.53	(0.13)	0.26	(0.14)	-0.56	(0.15)	1.00	(0.00)
Mexico	-0.79	(0.07)	-0.64	(0.13)	-0.77	(0.07)	0.63	(0.07)	-0.83	(0.04)	1.00	(0.00)
Malaysia	-0.61	(0.07)	-0.68	(0.07)	-0.54	(0.09)	0.50	(0.10)	-0.54	(0.10)	1.00	(0.00)
Peru	-0.27	(0.13)	-0.15	(0.15)	-0.44	(0.12)	0.33	(0.13)	-0.25	(0.14)	1.00	(0.00)
Philippines	0.44	(0.09)	0.41	(0.09)	0.54	(0.08)	0.07	(0.14)	0.25	(0.13)	1.00	(0.00)
Thailand	-0.33	(0.15)	-0.32	(0.14)	-0.33	(0.13)	0.41	(0.14)	-0.27	(0.13)	1.00	(0.00)
Uruguay	-0.10	(0.25)	-0.22	(0.25)	-0.07	(0.25)	0.27	(0.20)	-0.13	(0.25)	1.00	(0.00)
Emerging[1]	-0.34	(0.30)	-0.33	(0.27)	-0.33	(0.32)	0.38	(0.17)	-0.35	(0.26)	1.00	(0.00)
Developed[1]	-0.04	(0.31)	-0.21	(0.32)	-0.17	(0.30)	0.38	(0.25)	-0.00	(0.32)	1.00	(0.00)

Note: Data are logged, except for net exports which are presented as the ratio of real net exports to real GDP. Data are detrended using the Hodrick-Prescott with a smoothing parameter of 100. Moments were estimated by GMM. Numbers in parenthesis are the standard errors.
[1] Simple cross-country average. The standard errors for the averages were computed assuming independence across countries.

economies.[9] That is, in emerging economies: when terms of trade deteriorate (resp. improve) relative to their trend, the output level tends to be below (resp. above) its trend.[10]

Part II of table 1.2 reports the correlation with terms of trade. The correlations just confirm that terms of trade move counter-cyclically in emerging economies. Also, note that the terms of trade correlate negatively with TFP.

[9]There is a large degree of heterogeneity among developed countries with respect to the cyclical properties of terms of trade. Some countries have pro-cyclical terms of trade while others counter-cyclical terms of trade. See data appendix.

[10]Other studies have also document this fact in a broader set of developing economies and periods, e.g. Mendoza [Men95], Kose [Kos02], Easterly et. al [EL93], Becker and Mauro [BM05], Izquierdo et. al. [IRT08], Spatafora and Tytell [TS09]. See also Williamsom [Wil11] for an historical account.

1.2.3 Exogeneity of terms of trade

This part of the analysis argues that terms of trade for emerging economies are most likely to be exogenous to domestic economic conditions. Yet, to some extent, terms of trade in emerging economies may reflect domestic economic conditions.

One factor determining whether or not domestic conditions influence the terms of trade is the size of the country. Table 1.4 reports the size of emerging countries, measured by the share of world's trade. It is clear that emerging countries are small using this metric. For instance, only Korea and Mexico account no more than 2 percent of world's trade. Hence, with respect to size, it is sensible to assume that terms of trade are exogenous in emerging countries.

Table 1.3: Participation in world's trade

	Exports (% of world's total exports)	Imports (% of world's total imports)	Trade (Exports plus Imports) (% of world's total trade)
Argentina	0.35	0.31	0.33
Brazil	0.95	0.85	0.90
Chile	0.29	0.27	0.28
Colombia	0.20	0.22	0.21
Indonesia	0.81	0.71	0.76
Korea, Rep.	1.98	1.92	1.95
Malaysia	1.01	0.92	0.97
Mexico	1.47	1.48	1.47
Peru	0.13	0.13	0.13
Philippines	0.38	0.42	0.40
Thailand	0.79	0.80	0.79
Uruguay	0.05	0.05	0.05
Emerging[1]	0.70	0.67	0.69
Developed[1]	2.90	2.96	2.93

Note: Exports and imports are measured in current dollars (Source: World Bank's World Development Indicators database). Reported numbers are the average for the period 1980-2008.
[1] Simple cross-country average.

In addition to the size of the country, the terms of trade would be endogenous if exporters are not price takers. Particularly, exporters could exert some monopoly power if the elasticity of substitution of the goods they sell is relatively low. In relation to this, table 1.4 report the share of non-primary goods (manufacturing goods and services) on total exports. The rest is classified as primary goods and it includes categories such as raw food, fuels and minerals. This distinction is important because non-primary goods tend to be less substitutable than primary goods.[11] The table shows that, in contrast to

[11]It is expected that non-primary goods are more prone to differentiation than primary goods. Broda and Weinstein [BW06] estimate the elasticity of substitution among a wide range of U.S. imports. Their

developed economies, exports from emerging economies are clearly less concentrated on non-primary goods. This suggest that terms of trade fluctuations in emerging economies could reflect domestic economic conditions, but less so than in developed economies. Note that not all emerging countries follow this pattern. For instance, the shares of non-primary exports of Korea and Philippines look very similar to that of a typical developed economy.[12] Thus, one should expect that terms of trade in these countries are more prone to respond to domestic conditions than in a typical emerging economy.

Table 1.4: Trade composition

	Non-primary exports (% of total exports)	Non-primary imports (% of total imports)	Non-primary trade (% of total trade)
Argentina	32.68	84.50	56.58
Brazil	55.51	66.67	61.02
Chile	29.99	75.39	51.95
Colombia	45.55	81.04	63.67
Indonesia	42.02	74.41	57.35
Korea	92.69	64.66	78.79
Malaysia	62.68	82.08	72.18
Mexico	63.15	79.54	70.54
Peru	28.50	76.53	53.06
Philippines	71.89	67.02	69.43
Thailand	68.20	73.23	70.92
Uruguay	56.98	73.70	65.25
Emerging[1]	54.15	74.90	64.23
Developed[1]	76.34	76.06	76.26

Note: Exports and imports are measured in current dollars (Source: World Bank's World Development Indicators database). Reported numbers are the average for the period 1980-2008.
[1] Simple cross-country average.

In addition of being primary exporters, emerging economies export few primary commodity groups, see Chapter 4 of IMF [IMF12]. Hence, it is possible that a country plays a dominant role as supplier of a specific commodity. With respect to this, Broda [Bro04] shows that, among developing countries, only small number of them accounts for more than 15 percent of the total world exports of specific commodities. Moreover, those commodities in which the country has more than 15 percent of the world market account for a small share of the country's total exports.[13] Hence, to the extent that monopoly power is reflected in market shares, export prices in emerging economies are less likely to be

estimates shows that the elasticity of substitution is larger for primary imports (e.g. fuel) than for non-primary imports (e.g. footwear).

[12]The same heterogeneity is observed among developed economies. Trade composition in Australia, Iceland and New Zealand are more similar to a typical emerging economies. See appendix for details.

[13]Specifically, Broda [Bro04] shows that only 22 goods from 9 developing countries, out of 75, account for more than 15 percent of world trade between 1996-1997. On average, these goods account for 6 percent of a country's total exports.

influenced by domestic supply conditions.

The aforementioned structural characteristics of emerging economies affect the statistical properties of terms of trade in emerging economies. For instance, as shown in table 1.1, terms of trade volatility is larger in emerging economies than in developed economies. This excess volatility is certainly a function of the exposure to primary commodity exports. Bidarkota and Crucini [BC00] shows that real commodity prices are highly volatile and have a common stochastic component. The authors also shock that almost 50 percent of the volatility of terms of trade can be traced back to single commodity price. Importantly, such volatility is not statistically explained by domestic factors. Mendoza [Men95] and Broda [Bro04] test whether or not domestic economic conditions (captured by output, exports or imports) cause, in the Granger sense, the terms of trade. These studies find no evidence against the null hypothesis that domestic conditions do not Granger-cause the terms of trade of developing economies.

1.3 Model

The economy is populated by 3 types of agents- a representative household, a final goods producer and a continuum of intermediate producers (henceforth firms). The representative household is standard; she consumes, invest on physical capital, supplies labor and holds a risk-less non-contingent bond. The final good producer is also standard; it assembles a tradable good using the intermediate inputs produced by the continuum of intermediate producers. The final tradable good is consumed, used in the formation of new physical capital or exported. Firms produce one intermediate input using a technology that requires labor, capital, other domestic intermediate inputs and imported inputs. Firms sell their output in monopolistic competitive markets as in Dixit and Stiglitz [DS77]. Inputs transactions happen in perfectly competitive markets. The model is subject to two shocks: a shock to aggregate technology and a shock to the price of imported intermediate inputs. Despite all these features, the model is isomorphic to standard one-sector small open economy real business cycle model, e.g. Mendoza [Men91].

Household problem The problem of the household is standard. The household maximizes expected discounted utility

$$\max_{\{C_t, L_t, K_t, B_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t),$$

where parameter $\beta \in (0, 1)$ is the discount factor and $U(\cdot)$ is the period utility function which depends on consumption C_t and labor L_t. Maximization is subject to a budget constraint and capital accumulation equation:

$$C_t + I_t + B_t + \frac{\kappa}{2}(B_t - B)^2 = \frac{w_t}{P_t} L_t + \frac{r_t}{P_t} K_{t-1} + \frac{\Pi_t}{P_t} - R_{t-1} B_{t-1}, \tag{1.1}$$

$$\begin{aligned} I_t &= K_t - (1 - \delta) K_{t-1} + \left(\frac{\phi}{2}\right)\left(\frac{K_t}{K_{t-1}} - 1\right)^2 K_{t-1}, \qquad (1.2) \\ & K_{-1}, B_{-1} \qquad \text{given} \end{aligned}$$

The household buys one unit of consumption C_t at a price P_t. At the same price, she invest in new physical capital I_t, or saves in a one-period non-contingent riskless bond which B_t which pays a exogenous real (gross) interest rate R_t next period.[14] In addition, she pays a portfolio quadratic adjustment cost for holding a stock of bonds different than the steady bond holdings B.[15] The household supplies a fraction L_t of her time at wage rate w_t and gets a rental rent r_t for every unit of physical capital supplied to the market. She also receives a lump sum transfer of aggregate profits across firms, i.e. $\Pi_t \equiv \int_0^1 \Pi_t(i)\, di$. The capital accumulation includes a capital adjustment cost which is used in the numerical simulations to modulate investment volatility.[16] Parameter $\delta \in (0, 1)$ is the depreciation rate, κ and ϕ are positive real numbers. I assume that the real interest rate on bond holdings is constant and equal to the inverse of the discount factor.

[14] Note that the bond is denominated in units of the consumption good. If the financial contracts are denominated in units of the imported input, then a change in this price can change the real value of debt and hence bring different types of wealth effects. See Kose [Kos02].

[15] This guarantees that the stock of bonds is stationary. See Uribe and Schmidtt-Grohe [SU03].

[16] As long as households have access to foreign markets, they can separate their savings decisions from their investment decisions by financing any gap between the two with external resources. As a result, investment is too volatile in the standard frictionless model. See Mendoza [Men91] for a discussion.

The first order conditions of the problem can be written as:[17]

$$-\frac{U_{L,t}}{U_{C,t}} = \frac{w_t}{P_t} \tag{1.3}$$

$$1+\phi\left(\frac{K_t}{K_{t-1}}-1\right) = \beta\mathbb{E}_t\left\{\frac{U_{C,t+1}}{U_{C,t}}\left(\frac{r_{t+1}}{P_{t+1}}+(1-\delta)-\frac{\phi}{2}\left(\left(\frac{K_{t+1}}{K_t}\right)^2-1\right)\right)\right\} \tag{1.4}$$

$$1+\kappa\left(B_t-B\right) = \beta\mathbb{E}_t\left\{\frac{U_{C,t+1}}{U_{C,t}}R_t\right\} \tag{1.5}$$

which together with the budget constraint and physical accumulation equation characterize the optimality conditions of the household problem.

Final good producer problem The tradable final good is produced using a continuum $[0,1]$ of intermediate inputs $g_t(i)$. The production technology is a Dixit-Stiglizt aggregator with constant returns to scale,

$$G_t = \left(\int_0^1 g_t(i)^\theta \, di\right)^{\frac{1}{\theta}} \quad \text{with } 0<\theta\leq 1 \tag{1.6}$$

where θ controls the elasticity of substitution among intermediate goods, i.e. $\frac{1}{1-\theta}$. As $\theta \to 1$, intermediate inputs are perfect substitutes. The production of the final good satisfies its demand:

$$G_t = C_t + I_t + X_t$$

where X_t represents foreign consumption of the domestic good.

The final good producer takes its price (normalized to one) and the price of all the inputs as given. The final good producer problem is:

$$\max P_t G_t - \int_0^1 p_t(i)\, g_t(i)\, di$$

subject to (1.6) and where:

$$P_t = \left(\int_0^1 p_t(i)^{\frac{\theta}{\theta-1}} \, di\right)^{\frac{\theta-1}{\theta}}.$$

[17] U_C denotes the marginal utility of consumption at period t and U_L denotes the marginal dis-utility of labor at period t.

The first order condition for $g_t(i)$:

$$g_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{\frac{1}{\theta-1}} G_t. \tag{1.7}$$

Intermediate producer problem The economy is populated by a continuum of intermediate good producers indexed $i \in [0,1]$. These firms produce differentiated goods using primary inputs (capital and labor) and intermediate inputs. Each differentiated good can be used as in the production of the final good or in the production of other differentiated goods. I assume these producers have monopolistic power in output markets.

The production function of firm is,

$$q_t(i) = A_t(i)\left(k_t(i)^{\alpha} l_t(i)^{1-\alpha}\right)^{1-\mu}\left(d_t(i)^{\gamma} m_t(i)^{1-\gamma}\right)^{\mu} \tag{1.8}$$

where A_i is firm's i technology, q is gross output, k is capital, l is labor, d is a composite of domestic inputs and m_i are imported inputs. Hereafter I refer to $q_t(i)$ as *gross* output.

Domestic intermediate input is an Dixit-Stiglizt aggregator of all differentiated goods produced within the economy

$$d_t(i) = \left(\int_0^1 d_t(i,j)^{\theta} dj\right)^{\frac{1}{\theta}} \qquad \text{with } 0 < \theta < 1, \tag{1.9}$$

where $d(i,j)$ is the domestic intermediate used by firm i and produced by firm $j \in [0,1]$. Parameter θ controls the elasticity of substitution among intermediate goods, i.e. $\frac{1}{1-\theta}$. This is the same parameter that governs the elasticity of substitution competing products in the final good market. This assumption simplifies the analysis as firms face a demand with a unique elasticity of substitution.

Firm i production equalizes its demand:

$$q_t(i) = g_t(i) + \int_0^1 d_t(j,i)\, dj, \tag{1.10}$$

where $g(i)$ is the demand of firm i output used in the production of the final good and $d(j,i)$ is the aggregate demand of firm i output used in the production of good j.

Firm i problem is:

$$\begin{aligned}\Pi_t(i) &= \max_{\varkappa_t(i)} p_t(i)\, q_t(i) - r_t k_t(i) - w_t l_t(i) - \int_0^1 p_t(j)\, d_t(i,j)\, dj - P_{Mt} m_t(i) \qquad (1.11)\\ \text{with} &: \varkappa_t(i) \equiv \{p_t(i), q_t(i), k_t(i), l_t(i), d_t(i), d_t(i,j), m_t(i)\}\end{aligned}$$

subject to $(1.8), (1.9)$ and (1.10). Intermediate producer takes the price of imported inputs P_{Mt} as given. It also takes the rental rate r_t, the wage rate w_t and the price of all domestic intermediate inputs $p_t(j)$ as given. The problem can be solved in two stages. One stage minimizes costs given factor prices. This provides the optimal mix of factors. The second stage is the standard price decision under monopolistic competition. Appendix A provides the details. The f.o.c. of cost minimization problem are:

$$(1-\mu)\,\alpha \frac{q_t(i)}{k_t(i)} = \frac{r_t}{mc_t(i)} \qquad (1.12)$$

$$(1-\mu)(1-\alpha)\frac{q_t(i)}{l_t(i)} = \frac{w_t}{mc_t(i)} \qquad (1.13)$$

$$(1-\mu)(1-\alpha)\frac{q_t(i)}{d_t(i)} = \frac{\widetilde{P}_t}{mc_t(i)} \qquad (1.14)$$

$$(1-\mu)(1-\alpha)\frac{q_t(i)}{m_t(i)} = \frac{P_{Mt}}{mc_t(i)} \qquad (1.15)$$

$$d_t(i,j) = \left(\frac{p_t(j)}{\widetilde{P}_t}\right)^{\frac{1}{\theta-1}} d_t(i) \qquad (1.16)$$

where $\widetilde{P}_t$ is the shadow price of one unit of the domestic intermediate input aggregate $d_t(i)$. Given that θ controls the elasticity of substitution of competing final goods and intermediate domestic goods, the shadow price $\widetilde{P}_t$ equals the price of the final good P_t. Variable $mc_t(i)$ denotes the minimum unitary cost and is given by,

$$\begin{aligned}mc_t(i) &= \frac{\left(r_t^{\alpha} w_t^{1-\alpha}\right)^{1-\mu}\left(P_t^{\gamma} P_{Mt}^{1-\gamma}\right)^{\mu}}{\varsigma A_t(i)}\\ \varsigma &\equiv \left(\alpha^{\alpha}(1-\alpha)^{1-\alpha}\right)^{1-\mu}\left(\gamma^{\gamma}(1-\gamma)^{1-\gamma}\right)^{\mu}\end{aligned}$$

Given the optimal mix of factors of production, the pricing decision is given by the standard rule of markup over unitary cost :

$$p_t(i) = \frac{mc_t(i)}{\theta} \qquad (1.17)$$

where the markup is given by θ^{-1}.

Shocks Firm i production function is affected by $A_t(i)$, firm i technological level. I assume that $A_t(i)$ has a time variant aggregate component and firm idiosyncratic static component:

$$A_t(i) = \bar{A}(i) A_t$$

where the aggregate component follows an AR(1), in logs:

$$A_{t+1} = A_t^{\rho_a} \exp(\epsilon_{a,t+1}) \qquad \text{with } \epsilon_{a,t+1} \sim N\left(0, \sigma_a^2\right) \text{ and } 0 < \rho_a < 1 \tag{1.18}$$

In addition, this economy is affected by shocks to the price of imports, which follow an AR(1), in logs,

$$P_{Mt+1} = (P_{Mt})^{\rho_m} \exp(\epsilon_{m,t+1}) \qquad \text{with } \epsilon_{m,t+1} \sim N\left(0, \sigma_m^2\right) \text{ and } 0 < \rho_m < 1 \tag{1.19}$$

It is worthwhile to emphasize that the terms of trade shocks and technology shocks, ϵ_a and ϵ_m, are assumed to be independent.[18]

Definition of terms of trade The final good price P_t determines the price of exports. Hence, the terms of trade are defined by the real price of imports or the ratio of the price of imports to the price of the final good, i.e. P_{Mt}/P_t.

Definition of equilibrium Given the sequences of aggregate technology shocks and import price shocks, the equilibrium is defined by:
(i) a sequence of allocations C_t, K_t, B_t, L_t for the household,
(ii) a sequence of allocations G_t, $g_t(i)$ for the final good producers,
(iii) a sequence of allocations $k_t(i)$, $l_t(i)$, $d_t(i,j)$, $m_t(i)$, $q_t(i)$ and prices $p_t(i)$ for all i,
(iv) a sequence of final good prices P_t, real wages w_t and capital rental rates r_t,
such that:
(a) given (iv), (i) solves the problem of the household,

[18] Mendoza [Men95] and Kose [Kos02] consider stochastic processes in which the terms of trade shocks and technology shocks are correlated. I depart from this practice by assuming the both shocks are independent.

(b) given $p_t(i)$ and P_t, (ii) solves the problem of the final good producer,

(c) given (iv), (iii) solves the problem of the intermediate good producer i,

(d) markets clear:

$$\begin{aligned} L_t &= \int_0^1 l_t(i)\, di, && (1.20) \\ K_{t-1} &= \int_0^1 k_t(i)\, di, && (1.21) \\ q_t(i) &= g_t(i) + \int_0^1 d_t(j,i)\, dj \qquad \forall i, && (1.22) \\ G_t &= C_t + I_t + X_t && (1.23) \end{aligned}$$

where X_t are exports.

1.3.1 Symmetric equilibrium

The model departs from the standard one-sector small open economy real business cycle model, e.g. Mendoza [Men91], in four aspects. First, all import activity in the model is concentrated in intermediate inputs. Households cannot import final goods alone. This captures the idea that even finished imports (e.g. footwear) have to go through a number of processes in the importing country (e.g. unloading, transporting, insuring, repackaging, wholesaling and retailing) before they meet their final demand.[19] Second, trade in intermediate inputs create input-output linkages as in Basu [Bas95], Jones [Jon11], Acemoglu et. al. [ACO11]. Third, in addition to technology shocks, the model is affected by terms of trade shocks. Fourth, firms have monopolistic power over the goods they sell. As it will shown next, the model is isomorphic to the standard model.

Next, I characterize a symmetric equilibrium where $\bar{A}(i) = \bar{A}\ \forall\ i$.[20] In this symmetric equilibrium, $P_t = p_t(i)\,,\ Q_t = q_t(i)\,,\ K_{t-1} = k_t(i)\,,\ L_t = l_t(i)\,,\ D_t = d_t(i)\,,\ M_t = m_t(i)\,,$ $G_t = g_t(i)\ \forall\ i$ and $d_t(i) = d_t(i,j)\ \forall\ j$.

[19]These processes involve domestic factors services (e.g. capital, labor and other intermediate goods) and account for a significant proportion of the final price. It is also in line with the empirical evidence showing that about two thirds of world trade involves raw materials or intermediates, see Chapter 4 IMF's World Economic Outlook.

[20]Note that the marginal revenue product for each factor is equalized across firms. This allocative efficiency implies that, at the aggregate level, a symmetric equilibrium is isomorphic to an equilibrium with dispersed idiosyncratic productivity. A formal proof can be obtained from the author upon request.

The market clearing condition (1.22) at the symmetric equilibrium can be written as,

$$Q_t = G_t + D_t \tag{1.24}$$

where, from (1.8) :

$$Q_t = A_t \bar{A} \left(K_{t-1}^{\alpha} L_t^{1-\alpha}\right)^{1-\mu} \left(D_t^{\gamma} M_t^{1-\gamma}\right)^{\mu}$$

At the symmetric equilibrium, (1.12) through (1.15),

$$\begin{aligned}
(1-\mu)\,\alpha \frac{Q_t}{K_{t-1}} &= \frac{1}{\theta}\frac{r_t}{P_t} &(1.25)\\
(1-\mu)\,(1-\alpha)\,\frac{Q_t}{L_t} &= \frac{1}{\theta}\frac{w_t}{P_t} &(1.26)\\
\mu\gamma\frac{Q_t}{D_t} &= \frac{1}{\theta} &(1.27)\\
\mu\,(1-\gamma)\,\frac{Q_t}{M_t} &= \frac{1}{\theta}\frac{P_{Mt}}{P_t} &(1.28)
\end{aligned}$$

Note that the right hand side is the marginal product of each factor. The presence of a markup θ^{-1} drives wedge between the marginal product and the real factor prices. In particular, in equilibrium the marginal product of each factor is higher than its real factor price. This property plays an important role later in the analysis.

Combining equations (1.27) and (1.28), aggregate gross output can be rewritten as,

$$Q_t = \Omega \left(\frac{P_t}{P_{Mt}}\right)^{\frac{(1-\gamma)\mu}{1-\mu}} A_t^{\frac{1}{1-\mu}} K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{1.29}$$

where:

$$\Omega \equiv \left((1-\gamma)^{1-\gamma}\gamma^{\gamma}\right)^{\frac{\mu}{1-\mu}} (\mu\theta)^{\frac{\mu}{1-\mu}}\, \bar{A}^{\frac{1}{1-\mu}} \tag{1.30}$$

Continuing with the characterization of the symmetric equilibrium, substitute (1.27) and (1.29) into (1.24) yields,

$$G_t = (1-\mu\gamma\theta)\,\Omega \left(\frac{P_t}{P_{Mt}}\right)^{\frac{(1-\gamma)\mu}{1-\mu}} A_t^{\frac{1}{1-\mu}} K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{1.31}$$

Plugging this expression into (1.25) – (1.26),

$$\begin{aligned}
\left((1-\mu)\,\theta\Omega\left(\frac{P_t}{P_{Mt}}\right)^{\frac{(1-\gamma)\mu}{1-\mu}}\right)\frac{\alpha A_t^{\frac{1}{1-\mu}}K_{t-1}^{\alpha}L_t^{1-\alpha}}{K_{t-1}} &= \frac{r_t}{P_t},\\
\left((1-\mu)\,\theta\Omega\left(\frac{P_t}{P_{Mt}}\right)^{\frac{(1-\gamma)\mu}{1-\mu}}\right)\frac{(1-\alpha)\,A_t^{\frac{1}{1-\mu}}K_{t-1}^{\alpha}L_t^{1-\alpha}}{L_t} &= \frac{w_t}{P_t},\\
\left(\mu\theta\Omega\left(\frac{P_t}{P_{Mt}}\right)^{\frac{(1-\gamma)\mu}{1-\mu}}\right)\frac{\gamma A_t^{\frac{1}{1-\mu}}K_{t-1}^{\alpha}L_t^{1-\alpha}}{D_t} &= 1\\
\left(\mu\theta\Omega\left(\frac{P_t}{P_{Mt}}\right)^{\frac{(1-\gamma)\mu}{1-\mu}}\right)\frac{(1-\gamma)\,A_t^{\frac{1}{1-\mu}}K_{t-1}^{\alpha}L_t^{1-\alpha}}{M_t} &= \frac{P_{Mt}}{P_t}
\end{aligned}$$

These equations imply that exogenous increases in the real price of imports would reduce the incentives of the household to supply labor and capital to the market. Intuitively, times of high terms of trade, are also times where the reward of supplying capital and labor to the market falls.

The budget constraint (1.1) depends on household's real gross domestic income (hereafter income), defined as:

$$Z_t \equiv \frac{\Pi_t + r_t K_{t-1} + w_t L_t}{P_t},$$

This measures the purchasing power of the income generated by domestic factor of production. Note that $\Pi_t + r_t K_{t-1} + w_t L_t = P_t Q_t - P_t D_t - P_{Mt} M_t$. Plugging (1.27) , (1.28) , (1.29) into the above expression yields,

$$Z_t = (1-\mu\theta)\,\Omega\left(\frac{P_t}{P_{Mt}}\right)^{\frac{(1-\gamma)\mu}{1-\mu}} A_t^{\frac{1}{1-\mu}}K_{t-1}^{\alpha}L_t^{1-\alpha}. \tag{1.32}$$

Thus, household real domestic income responds negatively to the terms of trade, see equation (1.30). In other words, a deterioration of the terms of trade reduces the purchasing power of household's income. This effect is common in different small open economy models with terms of trade shocks, e.g. Mendoza [Men95], Kose [Kos02]. Yet, in this model the effect is amplified by the input multiplier $1/(1-\mu)$.

Plugging (1.32) into (1.1) delivers,

$$C_t + I_t + B_t + \frac{\kappa}{2}(B_t - \bar{B})^2 = (1-\mu\theta)\,\Omega\left(\frac{P_t}{P_{Mt}}\right)^{\frac{(1-\gamma)\mu}{1-\mu}} A_t^{\frac{1}{1-\mu}}K_{t-1}^{\alpha}L_t^{1-\alpha} - R_{t-1}B_{t-1}, \tag{1.33}$$

Combining the equations for the real wage and the real rental rate with the household's first order conditions,

$$-\frac{U_{L,t}}{U_{C,t}} = \left((1-\mu)\,\theta\Omega\left(\frac{P_t}{P_{Mt}}\right)^{\frac{(1-\gamma)\mu}{1-\mu}}\right)\frac{(1-\alpha)\,A_t^{\frac{1}{1-\mu}}K_{t-1}^{\alpha}L_t^{1-\alpha}}{L_t}, \tag{1.34}$$

$$1+\phi\left(\frac{K_t}{K_{t-1}}-1\right) = \beta\mathbb{E}_t\left\{\frac{U_{C,t+1}}{U_{C,t}}\left(\begin{array}{c}\left((1-\mu)\,\theta\Omega\left(\frac{P_{t+1}}{P_{Mt+1}}\right)^{\frac{(1-\gamma)\mu}{1-\mu}}\right)\frac{\alpha A_{t+1}^{\frac{1}{1-\mu}}K_t^{\alpha}L_{t+1}^{1-\alpha}}{K_t}\\ +(1-\delta)-\frac{\phi}{2}\left(\left(\frac{K_{t+1}}{K_t}\right)^2-1\right)\end{array}\right)\right\} \tag{1.35}$$

Equations (1.33)−(1.35) along with the capital accumulation equation (1.2) and the Euler equation for bond-holdings, (1.5) determine the equilibrium sequence $\{C_t, L_t, K_t, B_t\}_{t=0}^{\infty}$. These equations are isomorphic to those of standard one-sector small open economy real business cycles models, e.g. Mendoza (1991).

The differences between the standard model and this model are threefold. First, the markup θ^{-1} enters in the determination of the equilibrium. Second, aggregate technology shocks are scaled up by $1/(1-\mu)$. This is input multiplier arising from the input-output linkages in the model. Specifically, this multiplier works as follows: higher technology leads to more production for all firms, with increases the demand of intermediate goods, which increases production for all firms, and so on. The elasticity of output to aggregate intermediate inputs is μ. Hence, the overall multiplier is $1+\mu+\mu^2+... = 1/(1-\mu)$.[21] Third, the equilibrium responds to shocks to the terms of trade. The economic intuition for this effect is the following. When the economy is hit by an increase of P_{Mt}, firms utilize less imports. To the extent that imports cannot be perfectly substituted with other factors, lower imported intermediates reduce production for all firms. In the first round, the reduction in production is given by the elasticity of total production to imported intermediates, i.e. $\mu(1-\gamma)$. This effect is then amplified in further rounds by input-output linkages as explained before. Hence, overall effect is scaled up by the input multiplier $1/(1-\mu)$.[22]

[21] This formula reflects the simple architecture of the input-output matrix. In the model, each firm has the same number of downstream interconnections, i.e. it supplies inputs to every other firm. In addition, given that the production function is symmetric across firms, each firm relies equally on other firms' inputs. Hence, no firm plays a dominant role as supplier in this input-output network.

[22] Note in the model, all firms' production possibilities depend equally on imported inputs. This implies that the external sector plays the role of a general-purpose technology in the language of Acemoglu et. al. [ACO11]. In this sense, terms of trade shocks can be interpreted as shocks to this general-purpose

Note also that a terms of terms of trade shock enters into the system in the same way as a technology shock. For instance, a terms of trade deterioration reduces the incentives to invest and supply labor. Likewise, it also reduces the purchasing power of household's income. Note that the financial structure of the model assumes that households do not have access to complete set of contingent financial contracts. This structure limits the ability of the representative household to insure himself against shocks. As a consequence of this, household's consumption would respond to shocks, but less so than income.[23]

Next I focus the analysis on aggregate output. Without loss of generality, I normalized the price of final goods in the equilibrium, i.e. $P_t = 1 \; \forall \; t$. This implies that P_{Mt} is the real price of imports. Aggregate output is measured by the real gross domestic product (GDP). From the expenditure approach, nominal GDP equals the value of *gross* production of final goods, i.e. $G_t = C_t + I_t + X_t$, minus the cost of imported inputs, i.e. $P_{Mt}M_t$.[24] In practice, most emerging economies compute real GDP at constant prices, i.e. after fixing prices at a base year using Laspeyres indexes. I fix these prices at their steady state levels. In other words, the real price of imports P_{Mt}/P_t is fixed at its steady state level P_M. Given all this, output is given by,

$$Y_t = G_t - P_M \; M_t \tag{1.36}$$

A first-order approximation around its steady state yields,[25]

$$\widehat{y}_t \approx \left(\frac{G}{Y}\right)\widehat{g}_t - \left(\frac{P_M \; M}{Y}\right)\widehat{m}_t$$

technology and the amplification occurs downstream as all firms using imports are interconnected to each other.

[23]Only in the limiting case where the bond-holding cost is infinitely convex, i.e. $\kappa \to \infty$, consumption responds to both shocks as much as real domestic income does.

[24]Standard practices to compute real GDP are detailed in "System of National Accounts 2008" published jointly by European Commission, IMF, OECD, United Nations and World Bank and "Concepts and Methods of the U.S. National Income and Product Accounts" published by the Bureau of Economic Analysis. The inclusion of imports M_t in GDP as a offsetting entry deserves further explanation. From the point of the model, all imports are intermediate goods. This implies that imports are included in consumption, investment and exports. Therefore, to accurately reflect *domestic* final good production, imports of intermediates are subtracted from GDP to offset the contribution of foreign production in the final expenditures components.

[25]By using the steady state price of imports, it is guarantee that final good production and imports are correctly weighted in output. If another price were used, then output would be affected by the discrepancy between this price and the steady state price.

where $\widehat{y}_t \equiv \log(Y_t/Y)$ is the log-deviation of variable Y_t from its equilibrium steady state level Y.

From equations (1.28) and (1.31), evaluated at the steady state, delivers: $G = (1-\mu\gamma\theta)\ Q,\ P_M\ M = (1-\gamma)\ \mu\theta\ Q,\ Y = (1-\mu\theta)\ Q$. It follows that log-deviation of output from its steady state in the equilibrium can be rewritten as,

$$\widehat{y}_t = \frac{1-\mu\gamma\theta}{1-\mu\theta}\widehat{g}_t - \frac{(1-\gamma)\,\mu\theta}{1-\mu\theta}\widehat{m}_t, \tag{1.37}$$

Log-linearizing equations (1.28)and (1.31) yields,[26]

$$\begin{aligned} \widehat{g}_t &= -\frac{(1-\gamma)\,\mu}{1-\mu}\widehat{p}_{Mt} + \frac{1}{1-\mu}\widehat{a}_t + \alpha\widehat{k}_{t-1} + (1-\alpha)\,\widehat{l}_t, && (1.38)\\ \widehat{m}_t &= -\widehat{p_{Mt}} + \widehat{g}_t. && (1.39) \end{aligned}$$

Plugging these equations into (1.37),

$$\widehat{y}_t = -\frac{(1-\gamma)\,\mu}{(1-\mu)}\frac{1-\theta}{1-\mu\theta}\widehat{p}_{Mt} + \frac{1}{1-\mu}\widehat{a}_t + \alpha\widehat{k}_{t-1} + (1-\alpha)\,\widehat{l}_t \tag{1.40}$$

This response disappears in the limiting case of $\theta \to 1$.

Given the expression of output, the TFP in the equilibrium, as a deviation from its steady state, is:

$$\widehat{tfp}_t \equiv \widehat{y}_t - \left(\alpha\widehat{k}_{t-1} + (1-\alpha)\,\widehat{l}_t\right) = -\frac{(1-\gamma)\,\mu}{(1-\mu)}\frac{1-\theta}{1-\mu\theta}\widehat{p_{Mt}} + \frac{1}{1-\mu}\widehat{a}_t. \tag{1.41}$$

Hence, TFP reflects changes in both aggregate technology and the terms of trade.[27] The model implies that, up to a first order approximation, terms of trade shocks and aggregate TFP are negatively correlated. Section 3.3.5 focuses in understanding this effect.

Finally, it is useful to compare output with household's income. Up to a first order

[26]Recall that the price of the final good is normalized. Thus, $\widehat{p_{Mt}} = \widehat{\left(\frac{P_{Mt}}{P_t}\right)}$.

[27]Note that measured TFP is obtained by weighting capital and labor by their output elasticities, not their factor shares on aggregate output. Because monopoly profits exists in equilibrium, capital and labor income shares are lower than α and $1-\alpha$, respectively. Hence, computing the aggregate TFP using factor shares instead of output elasticities would introduce a bias in the measurement of productivity that changes with capital and labor utilization. See Hall [Hal90] for a discussion of this effect.

approximation, the latter is given by,

$$\widehat{z}_t = -\frac{(1-\gamma)\,\mu}{(1-\mu)}\widehat{p}_{Mt} + \frac{1}{1-\mu}\widehat{a}_t + \alpha\widehat{k}_{t-1} + (1-\alpha)\,\widehat{l}_t$$

Both output and income are alike. Note however that income responds more forcefully to the terms of trade than output.[28] Obviously, when the economy is driven by technology shocks, income and output would behave in the same way. This distinction would be useful later when interpreting the *excess volatility of consumption* found in the data.

1.4 The effect of terms of trade on TFP

The response of TFP to terms of trade shocks in equilibrium warrants further discussion.

To fix ideas, it is useful to compare the equilibrium the efficient allocation. The efficient allocation is obtained from solving the problem of a social planner who takes the price of imports and the interest as given and chooses quantities to maximize lifetime utility subject to resource constraints. The appendix provides the details. The comparison with the efficient allocation shows that there is a suboptimally low utilization of resources in equilibrium.[29] Moreover, around the efficient allocation TFP does not respond to terms of trade.

The response of TFP to terms of trade in equilibrium contrasts with Kehoe and Ruhl [KR08] who show that in standard models featuring perfect competition (i.e. price taking) the terms of trade have no first order effects on TFP.[30] It also contrasts with results showing that under constant markups TFP only changes with technology shocks, e.g. Jaimovich and Floetotto [JF08]. One exception in the literature is Basu and Fernald [BF02] who argue that, in the context of imperfect competition, industry TFP can be affected by intermediate input utilization.

[28] This result is also present in Kohli [Koh04]. He analyzes is performed in a perfectly competitive framework. In such a case, output does not respond *directly* to terms of trade shocks.

[29] As shown by Bilbiie et. al. [BGM08], in models with constant markups and labor as the sole input the steady state equilibrium is inefficient if only if the supply of labor is endogenous. This inefficiency is explained intuitively as follows. In the absence of a markup for the leisure good, households are less willing to substitute from leisure into consumption. That is, a suboptimally high amount of leisure is purchased since this is relatively cheaper good. As a consequence of this inefficient allocation, not enough resources (labor) are devoted to the production of the good with higher markup (consumption).

[30] A version of the model with perfect competition renders a fully efficient equilibrium and, hence, the effect of terms of trade on TFP is absent. A formal proof can be obtained upon request from the author.

1.4.1 Simple intuition

Basu and Fernald [BF02] explain their result with the following argument. First, industry real value added requires subtracting *real cost of intermediate inputs* - i.e. intermediate inputs at constant real factor prices - from the *gross output*. Now, suppose that there is an (infinitesimal) increase in the real prices of intermediates that causes a decrease in the use of intermediate inputs. As a result, for each unit of less intermediate inputs, gross output falls by an amount equal to the marginal product of intermediate good (decreasing real value added) and the real cost of intermediate inputs falls by an amount equal to the constant real factor prices (increasing value added). The net effect depends on whether or not the marginal product is equal to real factor prices in equilibrium. Monopolistic behavior distorts this equalization, i.e. the marginal product of intermediate inputs is higher than its real factor price. It follows that, even for small changes in the real factor price, real value added (output) falls.

In the model aggregate output is measured by real GDP, which is also the sum of real value added across production units (properly weighted). Given that output transactions among firms cancel each other out in the aggregate, the only intermediate input that matters is imported. This suggests that the logic put forth by Basu and Fernald [BF02] applies. To see this, let me reduce the model to a simpler setup. Assume that production requires labor and imported intermediate inputs. Moreover, to simplify the analysis further assume that the supply of labor is inelastic. In this context, it can be shown that output reduces to,

$$Y_t = Q_t - P_M M_t.$$

Now, consider the response of output to a infinitesimal change in the terms of trade starting from $P_{Mt} = P_M$, i.e. P_M is also the current terms of trade. Before computing this derivative, recall that in the above formula P_M has to remain constant, hence:

$$\frac{\partial Y_t}{\partial P_{M_t}} = \left(\frac{\partial Q_t}{\partial M_t} - P_M \right) \frac{\partial M_t}{\partial P_{M_t}}$$

This derivative is zero if and only if the marginal product of imports is equal to the terms of trade used to compute output. Otherwise, the terms of trade would have an effect on output. Recall that in equilibrium, the marginal product $\partial Q / \partial M$ and the terms of trade

P_M are linked by, $\partial Q/\partial M = \theta^{-1} P_M > P_M$. To determine the sign of $\partial Y/\partial P_M$ note the implicit function theorem implies,

$$\frac{\partial M_t}{\partial P_{M_t}} = \frac{1}{\theta} \frac{1}{\frac{\partial^2 Q_t}{(\partial M_t)^2}} < 0$$

It follows that $\partial Y/\partial P_M < 0$. It is straightforward to show that in this simple example output per worker is TFP and that $\partial TFP/\partial P_M < 0$.[31]

A positive gap between marginal productivity and the terms of trade, $\partial Q/\partial M > P_M$, represents the extra *quantity* of final goods that can be obtained after increasing the amount of imports. These opportunities are left unexploited in equilibrium since each firm chooses its level of imports to maximize profits, not welfare. A reduction of the terms of trade alleviates this inefficiency, bringing the economy closer to its efficient level. How this occurs depends on how imported inputs respond to the terms of trade. This is measured by the derivative of imports to the terms of trade $\partial M/\partial P_M$. This derivative depends on the rate at which diminishing returns set in, i.e. $\partial^2 Q/\left(\partial M\right)^2$. The economic interpretation is that any additional production brought by imports also increases the demand of imports. This cycle of positive feedbacks between production and imports eventually dies out as diminishing returns set in. Hence, the increase in production is greater than the elasticity of production to imports. When diminishing returns set in immediately, i.e. $\partial^2 Q/\left(\partial M\right)^2 \to -\infty$, a reduction in terms of trade does not deliver any extra production.

1.4.2 Policy

As shown by Bilbiie et. al. [BGM08], efficiency can be restored by properly designed subsidies. It is straightforward to show that setting a constant revenue subsidy $\tau = \theta^{-1}$ restores full efficiency. This section pays special attention those policies correcting inefficiency in TFP. The above analysis suggest that the inefficiency arises because the marginal product of imports is greater than terms of trade. Given this, I start with a

[31] Deriving the effect this way or using the log-linear approximation around the equilibrium renders exactly the same result, i.e. the same elasticity of TFP to terms of trade

constant subsidy on imports τ_m. The problem of the firm can be rewritten as,

$$\Pi_t(i) = \max_{\varkappa_t(i)} p_t(i) q_t(i) - r_t k_t(i) - w_t l_t(i) - \int_0^1 p_t(j) d_t(i,j) dj - \tau_m P_{Mt} m_t(i)$$

This subsidy affect the f.o.c's for $m(i)$, evaluated at symmetric allocation,

$$\frac{\mu(1-\gamma) Q_t}{M_t} = \frac{\tau_m}{\theta} \frac{P_{Mt}}{P_t}$$

To eliminate the effect of terms of trade on TFP in equilibrium, it is required that,

$$\tau_m = \frac{\theta(1-\gamma\mu)}{(1-\theta\gamma\mu)} < 1$$

Note that $\tau_m < \theta^{-1}$. In other words, to eliminate the inefficient response of TFP to the terms of trade, the subsidy has to be such that imports are suboptimally overused in equilibrium.[32]

Another possibility is to subsidize to domestic intermediate inputs or a subsidy all intermediates inputs. In the case of the latter, the problem of the firm is,

$$\Pi_t(i) = \max_{\varkappa_t(i)} p_t(i) q_t(i) - r_t k_t(i) - w_t l_t(i) - \tau_v \left(\int_0^1 p_t(j) d_t(i,j) dj + \tau_m P_{Mt} m_t(i) \right)$$

This subsidy affect the f.o.c's for $d(i)$ and $m(i)$, which at the symmetric equilibrium can be written as,

$$\begin{aligned} \frac{\mu(1-\gamma) Q_t}{D_t} &= \frac{\tau_v}{\theta} \\ \frac{\mu(1-\gamma) Q_t}{M_t} &= \frac{\tau_v}{\theta} \frac{P_{Mt}}{P_t} \end{aligned}$$

It is straightforward to show that subsidizing the intermediates with $\tau_v = \theta < 1$, eliminates the effect of terms of trade on TFP.

[32]This policy could potentially reduce welfare.

1.4.3 Measurement issues

It is important to consider how the results change when output is measured differently. The previous results are obtained using constant prices. Importantly, the prices used to compute output coincide with (or are arbitrarily close to) the current equilibrium prices, otherwise, output would be biased. This bias also affects the measurement of TFP. In particular, part of the change of TFP due to terms of trade fluctuations would be artificial. Suppose that output is measured using a price deflator P_M^0 rather that the current market price P_M. In principle, P_M^0 can be greater or lower to $\partial G/\partial M$. If P_M moves towards P_M^0, TFP would artificially rise. Conversely, if P_M moves away from P_M^0, TFP would artificially decline. Statisticians try to reduce this bias by updating price deflators to current market condition, e.g. chain-weighted Fisher index. The appendix discusses all these cases in more detail using first-order approximations. The main finding is that, as $P_M \to P_M^0$, the bias disappears and output decreases unambiguously with the terms of trade, i.e. $\partial Y/\partial P_M < 0$. Finally, the results were obtained under the expenditure approach. The same results hold under the producer or value added approach. Under this approach, output is measured by value added at constant prices $Y_t = Q_t - D_t - P_M M_t$. Log-linearizing around the steady state yields (1.40).

1.5 Quantitative results

The objective of this subsection is to evaluate the role of the terms of trade in driving the business cycles of small open economies. The analysis proceeds as follows. First, the model is log-linearized around the steady state. Second, the model is parameterized. Some of the parameters are chosen to capture some specific characteristics of the sample of emerging countries studied in section 1.2. Third, dynamics of the model after technology and terms of trade shocks are illustrated via impulse response functions. Fourth, the implied business cycles moments of the model are compared with the moments in the sample. Finally, using the policy functions, I construct a set counterfactuals showing what would have been the path of the emerging countries in the sample if only terms of trade shocks had hit them.

1.5.1 Preferences

The quantitative implications of the model are studied under two specifications for preferences. The first specification assumes period utility that is quasi-linear in consumption as in Greenwood et. al. [GHH88], hereafter GHH. Specifically,

$$U(C,L) = \frac{(C_t - \psi L_t^{\upsilon})^{1-\sigma}}{1-\sigma}$$

where $\sigma > 0$ is the coefficient of risk aversion and $\upsilon \in (0,1)$ controls the Firsch elasticity of labor supply, ψ is a scaling parameter. This is the benchmark case because its use is standard in the open economy business cycle literature, e.g. Mendoza [Men91].

The second specification for preferences assumes that consumption and leisure, i.e. i.e. $1 - L_t$, enter in the Cobb-Douglas fashion,

$$U(C,L) = \frac{\left(C_t^{\eta}(1-L_t)^{1-\eta}\right)^{1-\sigma}}{1-\sigma}$$

where $\sigma > 0$ is the coefficient of risk aversion and $0 < \eta < 1$ is a share parameter.

These two preferences differ in one particular aspect, see Neumeyer and Perri [NP05]. Under GHH preferences, the labor supply is independent from consumption. It follows that if the economy is hit by a shock, labor supply does not respond to the income effect. In contrast, for the Cobb-Douglas preferences specification, the labor supply depends negatively on consumption. Hence, a negative shock that causes a drop in consumption, it also induces an outward shift in the labor supply curve. Throughout this section I use GHH as benchmark. The second specification is studied for robustness check.

1.5.2 Log-linearization

The equilibrium sequence $\{C_t, L_t, K_t, B_t\}_{t=0}^{\infty}$ is solved by log-linearizing the following questions for endogenous variables (1.5), $(1.32) - (1.35)$ and the shocks $(1.18) - (1.19)$.

The log-linearized system is under GHH preferences is summarized next:

$$\upsilon \widehat{l}_t = \widehat{z}_t \tag{1.42}$$

$$\phi\left(\widehat{k}_t - \widehat{k}_{t-1}\right) = \mathbb{E}_t\widehat{u}_C(t+1) - \widehat{u}_C(t) + (1-\beta(1-\delta))\left(\mathbb{E}_t\widehat{z}_{t+1} - \hat{k}_t\right) \tag{1.43}$$

$$-\beta\phi\mathbb{E}_t\left(\widehat{k}_{t+1} - \widehat{k}_t\right) \tag{1.44}$$

$$\kappa B\hat{b}_t = \mathbb{E}_t\widehat{u}_C(t+1) - \widehat{u}_C(t) \tag{1.45}$$

$$\widehat{z}_t = -\frac{(1-\gamma)\mu}{(1-\mu)}\widehat{p}_{Mt} + \frac{1}{1-\mu}\widehat{a}_t + \alpha\widehat{k}_{t-1} + (1-\alpha)\widehat{l}_t \tag{1.46}$$

$$C\widehat{c}_t + K\widehat{k}_t + B\widehat{b}_t = Z\widehat{z}_t + RB\widehat{b}_{t-1} + (1-\delta)K\widehat{k}_{t-1} \tag{1.47}$$

$$\widehat{a}_t = \rho_a\widehat{a}_{t-1} + \epsilon_{a,t} \tag{1.48}$$

$$\widehat{p}_{Mt} = \rho_m\widehat{p}_{Mt-1} + \epsilon_{m,t} \tag{1.49}$$

with marginal utility of consumption $\widehat{u}_{Ct}$ given by:

$$\widehat{u}_C(t) = -\frac{\sigma C}{C-\psi L^{\upsilon}}\widehat{c}_t + \frac{\psi\upsilon L^{\upsilon}}{C-\psi L^{\upsilon}}\widehat{l}_t \tag{1.50}$$

Under Cobb-Douglas, equation (1.42) becomes,

$$\frac{1}{1-L}\widehat{l}_t + \widehat{c}_t = \widehat{z}_t \tag{1.51}$$

and the marginal utility of consumption becomes (1.50) :

$$\widehat{u}_C(t) = (\eta(1-\sigma)-1)\widehat{c}_t - (1-\eta)(1-\sigma)\frac{L}{1-L}\widehat{l}_t \tag{1.52}$$

Finally, note that the linearized system is affected by the markup only through the steady state.

1.5.3 Calibration

This section describes the calibration process. Table 1.5 summarizes the parameters under GHH preferences.

Some parameters and steady state conditions are set beforehand. The coefficient of relative risk aversion, σ, is set to 2, which is the typical value in the literature, e.g. Aguiar and Gopinath [AG07]. The preference parameters η and ψ are chosen to generate a share of time spent working of 1/3 in steady state. As it is standard in the RBC literature, I fix the depreciation rate, δ, at 0.06 and the elasticity of output with respect to capital, α, at 0.36. Following Neumeyer and Perri [NP05], the curvature of labor in the GHH preference specification υ is set to 1.6. The parameter κ in the bond holding quadratic cost function is set to the minimum value that guarantees that the equilibrium solution is stationary. This guarantees that the model replicated the dynamics of the frictionless economy. The steady state value of P_M is set to 1. I choose $\bar{A}$ so that Ω is one in the steady state.

A key parameter is the elasticity of substitution across competing products, $1/\left(1-\theta\right)$, which determines the markup, $1/\theta$. As a benchmark I take the estimates of the elasticity of substitution obtained for the U.S. economy, which the trade and industrial organization literatures locate between 3 to 10, e.g. Broda and Weinstein [BW06] and Hendel and Nevo [HN06]. Given this, I choose an elasticity of substitution of 5, $\theta = 0.8$, which implies that the markup is 1.25.

A subset of parameters is calibrated to match cross-country averages in the sample of emerging countries. The discount factor β is chosen to match an average real interest rate of 10 percent, which is close to the average real interest rates on sovereign debt in emerging economies, see Neumeyer and Perri [NP05]. The steady state bond holdings B is calibrated to match an average net exports to GDP ratio of 0.013 percent. The elasticity of gross output to intermediate goods, μ, is chosen such that $\mu\theta$, i.e. the ratio of the cost of intermediate to revenue in the model, is 0.50. This value to the actual average ratio of intermediate purchases to gross output in the sample, see United Nations Database. Parameter γ is chosen to generate an import to GDP ratio of 30 percent.

The remaining block of parameters, ϕ, ρ_a, σ_a, ρ_m and σ_m, target other moments from the sample. The parameter controlling the capital stock adjustment costs, ϕ, is calibrated to match the average ratio of investment volatility to output volatility in the sample. Parameters ρ_a and σ_a^2 are calibrated in the following way. The autoregressive coefficient is set to 0.4, close to the value used by Mendoza [Men91]. Parameter σ_a is calibrated to match cross-country sample average of the standard deviation of output. To calibrate ρ_m and σ_m I first fit an AR(1) to each emerging country's terms of trade

(HP filtered). The cross-country sample average OLS estimate of the AR(1) coefficient is 0.4099. Given this, I fix ρ_m to 0.40. Parameter σ_m is chosen to match cross-country average volatility of the terms of trade in the sample.[33]

Table 1.5: Baseline Calibration

Shocks	Name	Value
ρ_a	Persistence of technology shock	0.4000
σ_a	Volatility of technology shock	0.0063
ρ_m	Persistence of terms of trade shock	0.4000
σ_m	Volatility of terms of trade shock	0.0588
Utility parameters	Name	Value
β	Discount factor	0.9091
σ	Relative risk aversion	2.0000
υ	Labor curvature	1.6000
ψ	Scale parameter	0.1860
Technology parameters	Name	Value
$1/(1-\theta)$	Elasticity of substitution among goods	5.0000
μ	Exponent of production function	0.6250
α	Exponent of production function	0.3600
γ	Exponent of production function	0.7000
δ	Depreciation rate	0.0600
κ	Bond holding cost	0.0000
ϕ	Capital adjustment cost	0.2364
Steady state exogenous variables	Name	Value
B	Bond holdings	-0.0174
P_M	Terms of trade	1.0000
$\bar{A}$	Idiosyncratic technology	2.2592
A	Aggregate technology	1.0000

1.5.4 Impulse responses

This section presents the impulse response functions of the model to a terms of trade deterioration shock and a negative technology shock. To facilitate the comparison, I normalize the response of output to -1 percent upon shock. To generate this, the terms of trade shock has to 34 percent the calibrated value for σ_m while the technology shock has to be -34 percent of σ_a.

Figure 1.2 depicts the responses of the endogenous variables. The left (resp. right) panel plots impulse response functions of the terms of trade (resp. technology) shock. Note that the response of the economy to a terms of trade deterioration is similar to

[33] Specifically, $\sigma_m = \left(\frac{1}{12}\sum_{k=1}^{12}\sigma_k\left(P_M\right)\right)\left(1-\rho_m^2\right)$. Where $\sigma_k\left(P_M\right)$ is the GMM estimate of the standard deviation of the terms of trade in country k.

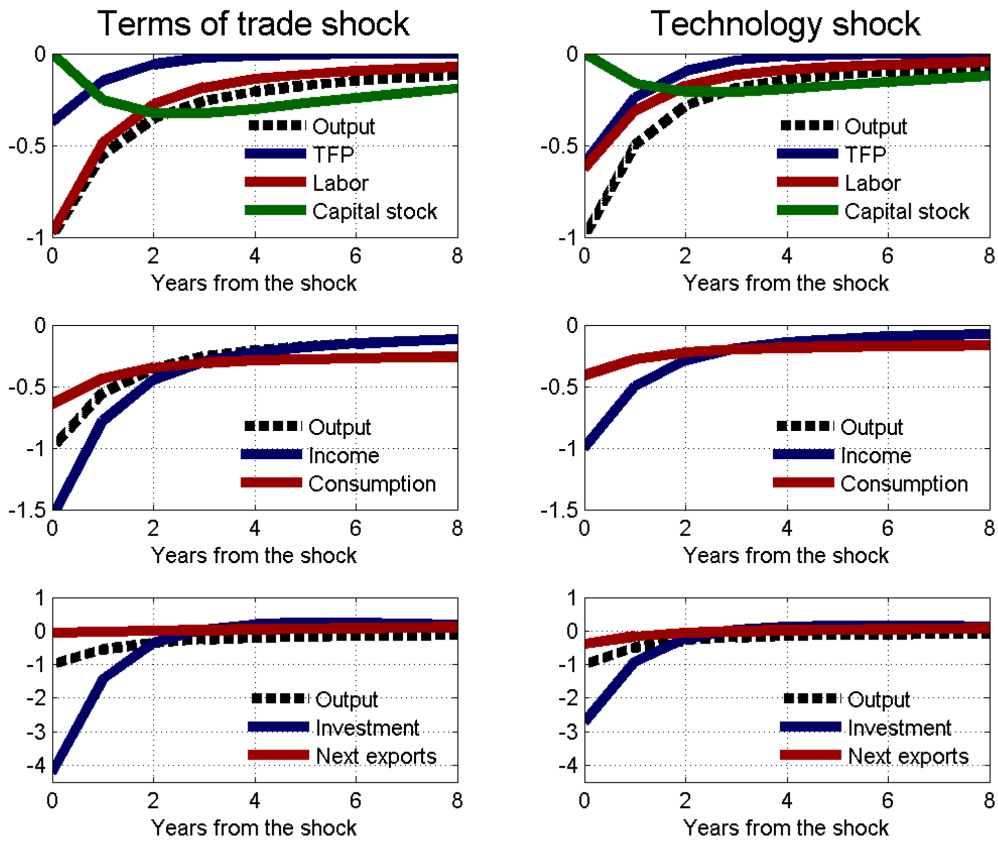

Figure 1.2: Impulse response functions

the response of the economy to negative technology shock. On one hand, a terms of trade shock reduces the households' incentives to supply labor, invest and consume fall. In addition, a deterioration of the terms of trade moves the economy farther away from efficiency and hence TFP falls. On the other hand, a technology shock works in exactly the same way, except that the fall in TFP is not caused by a reduction of efficiency.

The impulse responses also show some qualitative differences. Note that for the same level of output, consumption fall disproportionately more under the terms of trade shock than under the technology shock. The main explanation to this lies in difference between income and output. Income measures purchasing power, which is affected by changes in quantities and relative prices, while output only measures changes in quantities. As a consequence, under terms of trade shocks, income moves disproportionately more than output. As the household smooths out his consumption path relative to income, not output, it follows that consumption becomes closer to output. This is not possible under technology shocks because income moves proportionally with output. Still, in the benchmark calibration, in both cases consumption responds less than output.

1.5.5 Business Cycle Moments

This section reports the second moments implied by the model and compares them with the actual data. Table 1.6 reports the standard deviations, relative volatilities and cross correlations computed from artificial data generated by the model and from the sample of emerging countries, see section 1.2. For the model, the table reports three cases. Each case differs in the kind of shocks hitting the system.

Table 1.6: Simulated and actual moments

	Y	C	I	NX	TFP	P_M
I. Volatility - standard deviation (in percent)						
Data Emerging	4.72	5.56	16.07	2.80	4.61	6.41
Model P_M shocks only	3.61	2.53	14.07	1.22	1.25	6.24
Model A shocks only	3.08	1.42	8.07	1.41	1.79	0.00
Model Both shocks	4.72	2.92	16.30	1.92	2.18	6.24
II. Volatility - Relative standard deviation						
Data Emerging	1.00	1.19	3.49	0.63	0.97	1.43
Model P_M shocks only	1.00	0.70	3.94	0.33	0.35	1.74
Model A shocks only	1.00	0.46	2.63	0.46	0.58	0.00
Model Both shocks	1.00	0.62	3.49	0.41	0.47	1.35
III. Correlation with output						
Data Emerging	1.00	0.83	0.89	-0.68	0.89	-0.34
Model P_M shocks only	1.00	0.87	0.89	0.44	0.95	-0.95
Model A shocks only	1.00	0.87	0.94	0.93	0.98	n.a.
Model Both shocks	1.00	0.85	0.90	0.67	0.93	-0.72
IV. Correlation with terms of trade						
Data Emerging	-0.34	-0.33	-0.33	0.38	-0.35	1.00
Model P_M shocks only	-0.95	-0.75	-0.99	-0.43	-1.00	1.00
Model A shocks only	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Model Both shocks	-0.72	-0.64	-0.85	-0.26	-0.56	1.00

Note: Moments are the average across 100 simulations of 79 periods each. The moments are computed over the last 29 periods (same length as in the data) of each simulation.

Part I of table 1.6 reports the standard deviation. To quantify the contribution of different shocks to business cycle volatility, take the ratio of standard deviation implied by the model to the standard deviation observed in the data. Start with the case where terms of trade shocks are the only source of volatility in the model. The volatility ratio indicates that terms of trade shocks alone can account for about 70 percent of the actual output volatility in emerging economies.[34] The volatility ratio varies for other variables.

[34] Similar numbers have been reported in the literature. Mendoza [Men95] concludes that terms-of-trade shocks account for 45 to 60 percent of the observed variability of output. Kose [Kos02] concludes

For instance, terms of trade shocks generate a volatility that is 25 percent TFP volatility, 40 percent of consumption volatility and 85 percent of investment volatility. The model under technology shocks generates a similar volatility ratio for output. However, it generates a lower volatility ratio for consumption, 27 percent. Adding both shocks simultaneously increases the volatility ratios for all variables.

Part II reports the standard deviations of the variables relative to that of output. Note that in the data, household's consumption is 20 percent more volatile than output. The model is unable to replicate this excess volatility of consumption. Comparing the different cases shows that terms of trade shocks help to generate higher consumption volatility and hence brings the theory closer to the data. Yet, household's ability to smooth out his consumption is too strong in the model. In other words, capital flows based on the non contingent bond offered to households at a constant interest rate offers them enough insurance to shocks. This only occurs because moving capital in and out the country is costless. Next section explores the role of frictions to international capital flows in generating excess consumption volatility.

Part III reports the correlation of the variables with output. Note that the model only under terms of trade shocks predicts the right sign of the correlation except for net exports, which are highly countercyclical in the data. The same occurs when the model is simulated only under technology shocks. However, quantitatively, with terms of trade shocks the pro-cyclicality of net exports is weaker. Part IV reports the correlation with terms of trade. The model over-predicts the negative correlation of most variables. However, it does a decent job in the correlation between terms of trade and TFP.

1.5.5.1 Sensitivity analysis

This section presents a sensitivity analysis over the previous results. To simplify the analysis, this section focuses only on the following moments: the correlation between terms of trade and output $\rho(P_M, Y)$, the correlation between terms of trade and TFP $\rho(P_M, TFP)$, the correlation of net exports with output $\rho(NX, Y)$ and the relative volatility of consumption $\sigma(C)/\sigma(Y)$. Table 1.7 summarizes the results. The first

that about 88 percent of output volatility is accounted for by terms of trade shocks. Importantly, both studies consider technology and terms of trade shocks that are correlated.

row reports the moments from the data and the second row reports the moments implied by the model under the benchmark parameterization (GHH markups).

As a first pass, table 1.7 studies the role of preferences by computing the moments implied by Cobb-Douglas preferences. Relative to the baseline, Cobb-Douglas preferences generate less volatility in consumption and more procyclicality of net exports. This is because under Cobb-Douglas households are less willing to subtitute consumption for leisure. Part II reports the results under perfect competition (no markups). Without markups the correlation between TFP and the terms of trade is zero. Still, output and the terms of trade are negative correlated because labor and capital respond to terms of trade shocks. Under GHH preferences, removing markups reduces the correlation between net exports and output and rises the relative volatility of consumption slightly.

Part III of table 1.7 reports the result when terms of trade shocks are highly persistent, ρ_m is 0.90, keeping σ_m at its baseline value.[35] Under GHH preferences, with or without markups, consumption is more volatile than output, net exports are countercylical and terms of trade that are too pro-cyclical. The key insight lies on consumption behavior. In the model consumption is determined by the discounted present value of income (permanent income), not current income. Hence, when terms of trade shocks are persistent, both permanent income and consumption respond to them. The second part of the explanation is the fact that income responds more to terms of trade shocks than output. Thus, for high persistent terms of trade shocks it is possible that the volatility of consumption becomes higher than the volatility of output. Note that the results holds for quite different values of markups. In fact, as TFP does not respond to terms of trade shocks in the case perfect competition (without markups), the relative volatility of consumption can rise even more. This indicates that the effect does not rely on markups, and instead, it is a general consequence of the production structure of the model. Moreover, under Cobb-Douglas preferences (not reported), consumption volatility is still below output volatility. Hence, in addition to high persistent terms of trade shocks, it is required that preferences display high enough substitutability between consumption and leisure. It is worthwhile to mention that the model offers an explanation to the excess volatility of consumption that does not rely on non-stationary shocks processes as in Aguiar and

[35] There exists evidence that terms of trade and commodity prices are highly persistent. Bidarkota and Crucini [BC00] cannot reject the hypothesis that terms of trade follow a random walk.

Table 1.7: Sensitivity Analysis

	$\rho(Y,P_M)$	$\rho(TFP,P_M)$	$\rho(NX,Y)$	$\sigma(C)/\sigma(Y)$
Data emerging	-0.34	-0.35	-0.68	1.19
Baseline (GHH markups)	-0.72	-0.56	0.67	0.62
I. Preferences				
Cobb-Douglas markups	-0.70	-0.55	0.91	0.28
II. Perfect competition				
GHH no markups	-0.27	0.00	0.13	0.68
Cobb-Douglas no markups	-0.25	0.00	0.80	0.28
III. Higher persistence ($\rho_m = 0.90$)				
GHH markups	-0.81	-1.00	-0.21	1.06
GHH No markups	-0.56	0.00	-0.12	1.12
IV. Higher volatility ($\sigma_m = 0.10$)				
GHH markups	-0.96	-1.00	0.42	0.71
GHH no markups	-0.45	0.00	-0.08	0.85
V. Capital mobility				
GHH markups $\kappa = 10$	-0.61	-0.54	-0.32	0.97
GHH markups $\kappa = 100$	-0.63	-0.54	-0.62	1.09
GHH markups $\kappa = 1000$	-0.64	-0.54	-0.65	1.12
VI. Measurement				
GHH markups value added approach[1]	-0.72	-0.56		
GHH no markups value added approach[1]	-0.23	0.00		
GHH markups base year overvalued[2]	-0.51	0.32		
GHH no markups base year overvalued[2]	0.03	0.57		
GHH markups base year undervalued[3]	-0.80	-0.79		
GHH no markups base year undervalued[3]	-0.45	-0.39		
GHH markups Fisher Index[5]	-0.71	-0.56		
GHH no markups Fisher Index[5]	-0.23	0.00		
GHH markups TFP based on income shares[5]		-0.66		
GHH markups TFP based on income shares[6]		-0.66		

Note: Moments are the average across 100 simulations of 79 periods each. The moments are computed over the last 29 periods (same length as in the data) of each simulation. All models are calibrated to match the same targets.

[1] Value added approach: $Y_t = Q_t - D_t - P_M M_t$

[2] Output is computed as: $Y_t = G_t - 1.5 P_M M_t$

[3] Output is computed as: $Y_t = G_t - 0.5 P_M M_t$

[4] Chain-weighted Fisher index. See appendix for exact formula.

[5] $\widehat{tfp}_t = \widehat{y}_t - \alpha \frac{(1-\mu)\theta}{1-\mu\theta} \widehat{k}_{t-1} + (1-\alpha) \frac{(1-\mu)\theta}{1-\mu\theta} \widehat{l}_t$

[6] $\widehat{tfp}_t = \widehat{y}_t - \left(1 - (1-\alpha) \frac{(1-\mu)\theta}{1-\mu\theta}\right) \widehat{k}_{t-1} + (1-\alpha) \frac{(1-\mu)\theta}{1-\mu\theta} \widehat{l}_t$

Gopinath [AG07] or shocks to the interest rate, as in Mendoza [Men91], Neumeyer and Perri [NP05], Mendoza and Yue [MY11].

Part IV reports the moments when terms of trade shocks are more volatile, σ_m is 0.10, keeping ρ_m at its baseline value.[36] This increases the short run volatility of terms of trade. The ratio of consumption volatility to output volatility rises but it is less than one. This confirms that the key for excess volatility in consumption is the increase in the volatility of permanent income, not current income.

Part IV of table 1.7 explores the role of capital mobility. In the model, the mobility of international capital flows is determined by the parameter κ of the quadratic bond holding cost function, equation (1.5). A positive κ implies that household cannot transform today's' consumption into tomorrow's consumption costlessly.[37] As expected, decreasing capital mobility increases the volatility of consumption. For high enough values of κ, consumption is more volatile than output. This comes from the interplay between terms of trade shocks and tight restrictions on capital mobility. On one hand, terms of trade shocks affect income disproportionately more than output. One the other hand, imposing tight restrictions on capital mobility brings consumption closer to current income. As income responds proportionally more to terms of trade shocks than output, consumption volatility may rise above output volatility. It is worthwhile to mention that in a standard small open economy model, i.e. technology shocks, it is not be possible to generate such excess volatility of consumption even under very tight restrictions on capital mobility. Finally, note that imposing restrictions on capital mobility has obvious implications for the volatility of the current account. For example, under $\kappa = 1000$, the volatility of the current account (as a ratio of nominal output) is 0.02 times the volatility of output.[38] This is counterfactual given the evidence of large volatility of capital flows in emerging economies, see Calvo et. al. [CIT06].

Finally, part VI reports the sensitivity of the results to some measurement issues.

[36] This value is twice the value of the baseline σ_m. Compared to the data, σ_m of 0.1 is still a moderate value, see Bidarkota and Crucini [BC00].

[37] The calibration considered a value of κ small enough $\left(10^{-5}\right)$ to guarantee the stationarity of the model. In that sense, the calibration approximates a situation where, in response to shocks, households can save or borrow B_t to smooth consumption as much as they can.

[38] Note that the correlation of the real net exports (as a ratio of output) and output is aligned with the data. For large values of κ, $X_t \approx P_{Mt}M_t$, i.e. nominal trade balance. Hence, real net exports becomes $(P_{Mt} - P_M)\, M_t$.

The first robustness check shows what occurs if output is measured using the value added approach instead of the expenditure approach. Nothing really changes since both methods are equivalent. The second robustness check shows what occurs if output is measured using base year prices that are biased upwards. Specifically, base year terms of trade are 50 percent higher than steady state terms of trade. In this case, a terms of trade deterioration brings the current marginal productivity of imports closer to the base year terms of trade, implying an artificial gain in TFP. In the case of markups, the bias of base year prices is strong enough to overturn the correlation between terms of trade and TFP from negative to positive and to reduce the correlation between terms of trade and output. Without markups, the bias on base year prices makes the terms of trade acyclical with respect to output and increases the correlation between TFP and terms of trade from zero to positive. Next I report the correlations when base year prices are biased downwards. In this case, terms of trade become artificially more counter-cyclical. The fourth robustness check shows the correlations when output is measured using a chain-weighted Fisher index. This index updates price deflators with current and past price and quantity data. By doing so, it reduces the biases from base year prices. The results show that measuring output using the Fisher index replicates the results obtained with the first order approximation. The last robustness check considers the case in which output elasticities of labor and capital are mismeasured. Two cases are analyzed. In the first case, both output elasticities are replaced by their income shares.[39] In the second case, the elasticity of output with respect to labor is measured as the labor income share while the elasticity of capital is measured by one minus the labor share. With markups, the correlations are biased but still negative. Without markups (not reported), income shares coincide with output elasticities.

1.6 Conclusion

This paper revisited the connection between the terms of trade (terms of trade) and the real economy. I built a small open economy model in which imports are inputs in

[39]Hall [Hal90], Basu and Fernald [BF02] consider this case. The broad idea is that when measuring TFP one should weight capital and labor by their income shares, as they reflect the contribution of labor and capital to welfare.

production, output markets are imperfectly competitive and firms are connected in an input-output network. The existing input-output linkages amplify both technology and terms of trade shocks. In the model, TFP is a function of terms of trade. In particular, just like in the data, in the model a terms of trade deterioration (improvement) leads to a drop (increase) in TFP. In addition, the model predicts that terms of trade fluctuations increase the volatility of real consumption. Under some parameters, the volatility of consumption can be higher than the volatility of output. Hence, the model offers an alternative explanation for the observed excess consumption volatility puzzle found in the data.

1.7 Appendix

1.7.1 Intermediate good producer problem

Given the input-output linkages in the model, it is useful to think about firm i problem in three stages.

Cost minimization First, firm i finds the factor mix, i.e. $k_t(i), l_t(i), d_t(i), m_t(i)$ that minimizes the total cost of production taking as given factor prices. That is,

$$\begin{aligned} mc_t(i)\, q_t(i) \quad \equiv \quad & \min_{\{k_t(i), l_t(i), d_t(i), m_t(i)\}} r_t k_t(i) - w_t l_t(i) - \widetilde{P}_t d_t(i) - P_{Mt} m_t(i), \quad (1.53) \\ & \text{s.t.} : q_t(i) = A_t(i) \left(k_t(i)^{\alpha} l_t(i)^{1-\alpha}\right)^{1-\mu} \left(d_t(i)^{\gamma} m_t(i)^{1-\gamma}\right)^{\mu} \end{aligned}$$

where $\widetilde{P}_t$ is the shadow price of the domestic intermediate input aggregator $d_t(i)$ which would be defined below. First order conditions and the definition of total cost of production deliver equations $(1.12) - (1.15)$ and the formula for the unitary cost of production $mc_t(i)$.

Domestic intermediate input mix Given $d_t(i)$, firm i chooses the optimal mix of intermediate goods $d_t(i,j)$ taking their given prices. Firm i minimizes the cost of the domestic intermediate good composite:

$$\widetilde{P}_t d_t(i) \equiv \min_{\{d_t(i,j)\}_{j\in[0,1]}} \int_0^1 p_t(j)\, d_t(i,j)\, dj,$$
$$\text{s.t.: } d_t(i) = \left(\int_0^1 d_t(i,j)^\theta\, dj\right)^{\frac{1}{\theta}}$$

The first order condition for $d_t(i,j)$:

$$d_t(i,j) = \left(\frac{p_t(j)}{\widetilde{P}_t}\right)^{\frac{1}{\theta-1}} d_t(i)\,. \tag{1.54}$$

Substituting (1.54) in $d_t(i)$ delivers the following equation:

$$\widetilde{P}_t = P_t = \left(\int_0^1 p_t(i)^{\frac{\theta}{\theta-1}}\, di\right)^{\frac{\theta-1}{\theta}}. \tag{1.55}$$

Pricing decision Finally, firm i sells its output to the market at a price $p_t(i)$ in a monopolistic competitive fashion. The firm maximizes profits:

$$\Pi_t(i) = \max_{p_t(i)} p_t(i)\, q_t(i) - mc_t(i)\, q_t(i)\,,$$

subject to (1.8), (1.9) and (1.10). Combining these equations into single demand,

$$q_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{\frac{1}{\theta-1}} \left(G_t + \left(\frac{p_t(i)}{P_t}\right)^{\frac{1}{\theta-1}} \int_0^1 d_t(j)\, dj\right),$$

The first order condition of the above problem yields (1.17).

1.7.2 Social planning problem

The social planner takes the price of imports and the interest as given and chooses quantities to maximize lifetime utility subject to (1.2), (1.6), (1.8), (1.10), (1.9), (1.21), (1.20), (1.23) and a constraint on international capital flows:

$$B_t + \kappa (B_t - B)^2 - R_{t-1} B_{t-1} = X_t - P_{Mt} \int_0^1 m_t(i)\, di$$

The problem can be stated as follows,

$$
\begin{aligned}
& \max_{\chi_t \mathbb{E}0} \sum_{t=0}^{\infty} \beta^t U\left(C_t, L_t\right) \\
\chi_t \quad \equiv \quad & \left\{C_t, K_t, B_t, g_t\left(i\right), m_t\left(i\right), k_t\left(i\right), l_t\left(i\right), d_t\left(i\right), m_t\left(i\right), d_t\left(i,j\right)\right\}
\end{aligned}
$$

subject to:

$$
\begin{aligned}
\left(\int_0^1 g_t\left(i\right)^{\theta} di\right)^{\frac{1}{\theta}} &= C_t + K_t - \left(1-\delta\right) K_{t-1} - \left(\frac{\phi}{2}\right)\left(\frac{K_t}{K_{t-1}} - 1\right)^2 K_{t-1} \\
& \quad + B_t + \kappa\left(B_t - B\right)^2 - R_{t-1}B_{t-1} + P_{Mt}\int_0^1 m_t\left(i\right) di, \\
g_t\left(i\right) + \int_0^1 d_t\left(j,i\right) dj, &= A_t\left(i\right)\left(k_t\left(i\right)^{\alpha} l_t\left(i\right)^{1-\alpha}\right)^{1-\mu}\left(d_t\left(i\right)^{\gamma} m_t\left(i\right)^{1-\gamma}\right)^{\mu} \\
d_t\left(i\right) &= \left(\int_0^1 d_t\left(i,j\right)^{\theta} dj\right)^{\frac{1}{\theta}}, \\
K_{t-1} &= \int_0^1 k_t\left(i\right) di, \\
L_t &= \int_0^1 l_t\left(i\right) di,
\end{aligned}
$$

For any given distribution of idiosyncratic productivities $A_t\left(i\right)$, it can be shown that the social planner solution delivers the same allocative distribution of resources as the equilibrium. This I focus in a symmetric allocation $Q_t = q_t\left(i\right), K_{t-1} = k_t\left(i\right), L_t = l_t\left(i\right)$, $D_t = d_t\left(i\right), M_t = m_t\left(i\right), G_t = g_t\left(i\right) \; \forall \; i$ and $d_t\left(i\right) = d_t\left(i,j\right) \; \forall \; j$. Hence the social planner is reduced to:

$$
\max_{\{C_t, K_t, B_t, M_t, D_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, L_t\right)
$$

$$
A_t\left(K_{t-1}^{\alpha} L_t^{1-\alpha}\right)^{1-\mu}\left(D_t^{\gamma} M_t^{1-\gamma}\right)^{\mu} - D_t - P_{Mt}M_t \quad = \quad C_t + B_t + \kappa\left(B_t - B\right)^2 - R_{t-1}B_{t-1}
$$

$$
I_t \quad = \quad K_t - \left(1-\delta\right) K_{t-1} + \left(\frac{\phi}{2}\right)\left(\frac{K_t}{K_{t-1}} - 1\right)^2 K_{t-1}
$$

It can be shown that the efficient allocation is characterized by the following equations:

$$
G_t = \left(1 - \mu\gamma\right)\Omega\left(\frac{1}{P_{Mt}}\right)^{\frac{(1-\gamma)\mu}{1-\mu}} A_t^{\frac{1}{1-\mu}} K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{1.56}
$$

$$\left(\mu\Omega_t^E\left(\frac{1}{P_{Mt}}\right)^{\frac{(1-\gamma)\mu}{1-\mu}}\right)\frac{\gamma A_t^{\frac{1}{1-\mu}}K_{t-1}^{\alpha}L_t^{1-\alpha}}{D_t} = 1, \tag{1.57}$$

$$\left(\mu\Omega_t^E\left(\frac{1}{P_{Mt}}\right)^{\frac{(1-\gamma)\mu}{1-\mu}}\right)\frac{(1-\gamma)A_t^{\frac{1}{1-\mu}}K_{t-1}^{\alpha}L_t^{1-\alpha}}{M_t} = P_{Mt}, \tag{1.58}$$

$$-\frac{U_{L,t}}{U_{C,t}} = \left((1-\mu)\Omega\left(\frac{1}{P_{Mt}}\right)^{\frac{(1-\gamma)\mu}{1-\mu}}\right)\frac{(1-\alpha)A_t^{\frac{1}{1-\mu}}K_{t-1}^{\alpha}L_t^{1-\alpha}}{L_t}, \tag{1.59}$$

$$1+\phi\left(\frac{K_t}{K_{t-1}}-1\right) = \beta\mathbb{E}_t\left\{\frac{U_{C,t+1}}{U_{C,t}}\left(\begin{array}{c}\left((1-\mu)\Omega\left(\frac{1}{P_{Mt+1}}\right)^{\frac{(1-\gamma)\mu}{1-\mu}}\right)\frac{\alpha A_{t+1}^{\frac{1}{1-\mu}}K_t^{\alpha}L_{t+1}^{1-\alpha}}{K_t}\\ +(1-\delta)-\frac{\phi}{2}\left(\left(\frac{K_{t+1}}{K_t}\right)^2-1\right)\end{array}\right)\right\} \tag{1.60}$$

$$C_t+I_t+B_t+\frac{\kappa}{2}(B_t-\bar{B})^2 = (1-\mu)\Omega\left(\frac{1}{P_{Mt}}\right)^{\frac{(1-\gamma)\mu}{1-\mu}}A_t^{\frac{1}{1-\mu}}K_{t-1}^{\alpha}L_t^{1-\alpha}-R_{t-1}B_{t-1}, \tag{1.61}$$

where:

$$\Omega \equiv \left((1-\gamma)^{1-\gamma}\gamma^{\gamma}\right)^{\frac{\mu}{1-\mu}}\mu^{\frac{\mu}{1-\mu}}\bar{A}^{\frac{1}{1-\mu}} \tag{1.62}$$

Equations (1.33)−(1.34) along with the capital accumulation equation (1.2) and the Euler equation for bond-holdings, (1.5) determine the equilibrium sequence $\{C_t, L_t, K_t, B_t\}_{t=0}^{\infty}$. These equations are isomorphic to those of standard one-sector small open economy real business cycles models, e.g. Mendoza (1991). Comparing the social planning solution with the equilibrium one it is easy to show that θ affects the equilibrium allocation.

Inefficiencies in steady state First, I focus in the steady state, assuming that $A_t = P_t = 1\ \forall\ t$. Relative to the efficient allocation in the steady state, the equilibrium steady state has a suboptimally low level of resources, i.e.

$$L^{EQ} = \theta^{\frac{1}{1-\mu}\frac{1}{1-\alpha}\frac{1}{\upsilon-1}}L^E < L^E, \qquad K^{EQ} = \theta^{\frac{1}{1-\mu}\frac{1}{1-\alpha}\frac{\upsilon}{\upsilon-1}}K^E < K^E,$$

$$D^{EQ} = \theta^{\frac{1}{1-\mu}\frac{1}{1-\alpha}\frac{\upsilon}{\upsilon-1}}D^E < D^E, \qquad M^{EQ} = \theta^{\frac{1}{1-\mu}\frac{1}{1-\alpha}\frac{\upsilon}{\upsilon-1}}M^E < M^E.$$

where E stands for efficient (solution to social planning problem) and EQ stands for equilibrium. The key element behind this suboptimality is the markup over marginal cost, i.e. θ^{-1}. Intuitively, as a consequence of this markup, the marginal productivities of all factors of production are greater than their corresponding real factor prices. In other words, the markup acts like a tax on factors' demands. Note also that labor is relatively more distorted by this inefficiency than any other input.[40] Naturally, these suboptimal levels of inputs imply a suboptimally low levels of production in equilibrium,

$$
\begin{aligned}
G^{EQ} &= \left(\frac{1-\mu\theta\gamma}{1-\mu\gamma}\right)\theta^{\frac{\mu}{1-\mu}\frac{1}{1-\alpha}\frac{\upsilon}{\upsilon-1}}G^{E}, \\
GDP^{EQ} &= \left(\frac{1-\mu\theta}{1-\mu}\right)\theta^{\frac{\mu}{1-\mu}\frac{1}{1-\alpha}\frac{\upsilon}{\upsilon-1}}GDP^{E}.
\end{aligned}
$$

Inefficiency in TFP It is expected that the inefficiencies in the steady state will spill over onto the first-order approximation of the stochastic model. In what follows I pay particularly attention to the inefficient response of TFP to terms of trade shocks.

$$
\begin{aligned}
\widehat{g}_t &= -\frac{(1-\gamma)\,\mu}{1-\mu}\widehat{p_{Mt}} + \frac{1}{1-\mu}\widehat{a}_t + \alpha\widehat{k}_{t-1} + (1-\alpha)\,\widehat{l}_t, && (1.63) \\
\widehat{m}_t &= -\widehat{p_{Mt}} + \widehat{g}_t. && (1.64)
\end{aligned}
$$

Turning to the characterization of TFP under the socially optimum allocation, it can be shown that around the efficient allocation output, measured as in (1.36) , is given by:

$$
\widehat{y}_t = \frac{(1-\mu\gamma)}{1-\mu}\widehat{g}_t - \frac{(1-\gamma)\,\mu}{1-\mu}\widehat{m}_t \qquad (1.65)
$$

where $\widehat{z}_t \equiv \log(Z_t/Z)$ is the log-deviation of variable Z_t from its efficient steady state level Z. Note that the coefficients multiplying $\widehat{g}$ and $\widehat{m}$ do not coincide with those in equilibrium, see equation (1.65).

The equation characterizing $\widehat{g}$ and $\widehat{m}$ are isomorphic to those in the equilibrium. In fact, up to a first order approximation, they look exactly the same as (1.38) and (1.39) , except that they respond to the efficient levels of capital and labor. Replacing these

[40] As shown by Bilbiie et. al. [BGM08], in models with constant markups and labor as the sole input the equilibrium is inefficient if only if the supply of labor is endogenous. This inefficiency is explained intuitively as follows. In the absence of a markup for the leisure good, households are less willing to substitute from leisure into consumption. That is, a suboptimally high amount of leisure is purchased since this is relatively cheaper good. As a consequence of this inefficient allocation, not enough resources (labor) are devoted to the production of the good with higher markup (consumption). In the context of the present model, even in the case of inelastic labor supply, $\upsilon \to \infty$, the steady state equilibrium would still be inefficient.

equation into (1.65), delivers,

$$\widehat{y}_t = \frac{1}{1-\mu}\widehat{a}_t + \alpha\widehat{k}_{t-1} + (1-\alpha)\,\widehat{l}_t$$

The above implies that up to a first order approximation, TFP around the efficient allocation is given by,

$$\widehat{tfp}_t = \widehat{y}_t - \left(\alpha\widehat{k}_{t-1} + (1-\alpha)\,\widehat{l}_t\right) = \frac{1}{1-\mu}\widehat{a}_t,$$

only reflects technological shock.

1.7.3 Measurement of output

Assume that production requires labor and imported intermediate inputs. Moreover, to simplify the analysis further assume that the supply of labor is inelastic. First consider *output at constant base prices*:

$$Y_t = Q_t - P_{M,0}M_t$$

where $P_{M,0}$ are the terms of trade in the base year.

Now consider the effect of a deterioration of the terms of trade - i.e. an increase in P_{Mt}- on output at constant prices:

$$\frac{\partial Y_t}{\partial P_{M,t}} = \left(\frac{\partial Q_t}{\partial M_t} - P_M\right)\frac{\partial M_t}{\partial P_{M_t}} - P_{M,0}\frac{\partial M_t}{\partial P_{M,t}} = \left(\frac{P_{M,t}}{\theta} - P_{M,0}\right)\frac{\partial M_t}{\partial P_{M,t}}, \tag{1.66}$$

which can be rewritten as:

$$\frac{\partial Y_t}{\partial P_{M,t}} = (P_{M,t} - P_{M,0})\,\frac{\partial M_t}{\partial P_{M,t}} + \frac{1-\theta}{\theta}P_{M,t}\frac{\partial M_t}{\partial P_{Mt}}. \tag{1.67}$$

The elasticity of output to terms of trade:

$$\frac{\partial y_t}{\partial p_{M,t}} = \frac{P_{M,t} - P_{M,0}}{P_{M,t}}\left(\frac{P_{M,t}M_t}{Y_t}\right)\left(\frac{\partial m_t}{\partial p_{M,t}}\right) + \frac{1-\theta}{\theta}\left(\frac{P_{M,t}M_t}{Y_t}\right)\left(\frac{\partial m_t}{\partial p_{M,t}}\right), \tag{1.68}$$

where low-cases denote variables in logs, i.e. $y = \log Y$. The first term is the bias created by base year prices.

Now consider the *chain weighted Fisher index* of output. Under Fisher chain-weighted output, I have:

$$Y_{t+1} = \frac{Q_{t+1} - P_{M,t+1}M_{t+1}}{P_{t+1}^{Fisher}},$$

where the Fisher chain-weighted price index is the geometric average of the Paasche and Laspeyres indices between the current period and the previous period:

$$P_{t+1}^{Fisher} = \left(\frac{Q_{t+1} - P_{M,t+1}M_{t+1}}{Q_{t+1} - P_{M,t}M_{t+1}}\right)^{\frac{1}{2}} \left(\frac{Q_t - P_{M,t+1}M_t}{Q_t - P_{M,t}M_t}\right)^{\frac{1}{2}} P_t^{Fisher}.$$

Kehoe and Ruhl [KR08] show that this yields the Fisher chain-weighted quantity index:

$$Y_{t+1} = \left(\frac{Q_{t+1} - P_{M,t+1}M_{t+1}}{Q_t - P_{M,t+1}M_t}\right)^{\frac{1}{2}} \left(\frac{Q_{t+1} - P_{M,t}M_{t+1}}{Q_t - P_{M,t}M_t}\right)^{\frac{1}{2}} Y_t.$$

The first order change of the logarithm of chain-weighted output is approximated as:

$$y_{t+1} - y_t \approx \frac{\partial Y_t}{\partial P_{M,t+1}} \left(P_{M,t+1} - P_{M,t}\right).$$

Differentiating the natural logarithm of chain-weighted real GDP:

$$\begin{aligned}
\frac{\partial y_{t+1}}{dP_{M,t+1}} &= \frac{\frac{\partial Q_{t+1}}{\partial M_{t+1}}\frac{\partial M_t}{\partial P_{M,t}} - P_{M,t+1}\frac{\partial M_{t+1}}{\partial P_{M,t+1}} - M_{t+1}}{2\left(Q_{t+1} - P_{M,t+1}M_{t+1}\right)} \\
&\quad + \frac{M_t}{2\left(Q_t - P_{M,t+1}M_t\right)} + \frac{\frac{\partial Q_{t+1}}{\partial M_{t+1}}\frac{\partial M_t}{\partial P_{M,t}} - P_{M,t}\frac{\partial M_{t+1}}{\partial P_{M,t+1}}}{2\left(Q_{t+1} - P_{M,t}M_{t+1}\right)}.
\end{aligned}$$

Since $\partial Q_t/\partial M_t = \theta^{-1}P_{M,t}$, the above simplifies to:

$$\begin{aligned}
\frac{\partial y_{t+1}}{\partial P_{M,t+1}} &= \frac{1-\theta}{\theta}\frac{P_{M,t+1}\frac{\partial M_{t+1}}{\partial P_{M,t+1}}}{\left(Q_{t+1} - P_{M,t+1}M_{t+1}\right)} - \frac{M_{t+1}}{2\left(Q_{t+1} - P_{M,t+1}M_{t+1}\right)} \\
&\quad + \frac{M_t}{2\left(Q_t - P_{M,t+1}M_t\right)} + \frac{\left(P_{M,t+1} - P_{M,t}\right)\frac{\partial M_t}{\partial P_{M,t}}}{2\left(Q_{t+1} - P_{M,t}M_{t+1}\right)}.
\end{aligned} \tag{1.69}$$

The first term of the right hand side of (1.69) captures the effect imperfect competition. Evaluating the above expression at $P_{M,t+1} = P_{M,t}$, this terms remains,

$$\frac{\partial y_t}{\partial P_{M,t}} = \frac{1-\theta}{\theta}\frac{P_{M,t}\frac{\partial M_t}{\partial P_{M,t}}}{\left(Q_t - P_{M,t}M_t\right)}.$$

I can rewrite this as:

$$\frac{\partial y_t}{\partial p_{M,t}} = \frac{1-\theta}{\theta} \frac{P_{M,t} M_t}{P_t^{Fisher} Y_t} \left(\frac{\partial m_t}{\partial p_{M,t}} \right) < 0. \tag{1.70}$$

1.7.4 Data

Most of the data comes from World Bank's World Development Indicators (WDI) and Penn World Tables 7.0 (PWT). These databases contain annual data from 1960 to 2008. The following describes the construction of the variables used in the paper. All series are constructed using all the available data, including data prior to the sample analyzed (1980-2008).

Data on real GDP, consumption, investment, imports and exports comes from WDI. The correspondence between the variables analyzed in the paper and WDI data is the following (WDI's mnemonics are given in parenthesis). *Real GDP*: GDP is constant local currency (NY.GDP.MKTP.KN). *Consumption*: household final consumption expenditure in constant local currency (NE.CON.PRVT.KN). *Investment*: gross fixed capital formation in constant local currency (NE.GDI.FTOT.KN). *Imports*: imports of goods and services in constant local currency (NE.IMP.GNFS.KN). *Exports*: exports of goods and services in constant local currency (NE.EXP.GNFS.KN). *Nominal imports*: imports of goods and services in current local currency (NE.IMP.GNFS.CN). *Nominal exports*: exports of goods and services in current local currency (NE.EXP.GNFS.CN).

Terms of trade are constructed as implicit deflators from the national accounts. The price deflator of imports (exports) is computed as the ratio of nominal imports (nominal exports) to imports at constant prices (exports at constant prices).

Total Factor Productivity is computed as follows:

$$TFP_t = \frac{GDP_t}{K_{t-1}^{\alpha} L_t^{1-\alpha}}$$

where GDP_t is the period t *real GDP*, K_{t-1} is the end of period $t-1$ *stock of capital* (beginning of period t stock of capital) and L_t is a measure of labor input utilized in production. The parameter α is set at 0.36, which is the standard value used in the RBC literature.

The *stock of capital* is constructed using the perpetual inventory method. This consists in constructing recursively a time series for K_t using:

$$K_t = (1 - \delta) K_{t-1} + I_t$$

where I_t is investment in period t and parameter δ is the depreciation rate. The series of investment is the one described earlier. The depreciation rate is fixed at 0.06, which is the standard value used in the RBC literature. The recursive constructed of capital is initialized using the steady state condition of capital under balanced growth path.[41]

The *labor* input is proxied by total hours from PWT. The data are not reported directly by PWT and instead recovered as follows (PWT's mnemonics are given in parenthesis). Total hours is recovered as the product of population (POP) with PPP Converted GDP Per Capita (RGDPL), divided by PPP Converted GDP Laspeyres per hour worked by employees at 2005 constant prices (rgdpl2th). The information is available for most countries in the sample, except for Indonesia, Malaysia, The Philippines, Thailand and Uruguay. For these countries, the labor input is approximated by total employment. The data are not reported directly by PWT and instead recovered as follows. Employment is recovered as the product of population (POP) with PPP Converted GDP Per Capita (RGDPL), divided by PPP Converted GDP Laspeyres per person counted in total employment at 2005 constant prices (rgdpl2te).

[41] That is $K_0 = I_0 / (\delta + g)$, where g is growth rate of investment in balanced growth path. The latter is estimated by the average growth rate of investment in the sample.

CHAPTER 2

Efficiency with Endogenous Information Choice

2.1 Introduction

A large and growing literature in modern macroeconomics focuses on the role of dispersed information in understanding fluctuations in economic activity. The main contribution of this paper is to demonstrate a new source of inefficiency in this class of models - one that arises only when learning is costly. We show that, in a standard business cycle model augmented to allow for endogenously acquired private information, incentives of monopolistically competitive firms to learn about aggregate shocks are typically not aligned with the social value of doing so. This leads to a suboptimal level of information acquired in equilibrium, distorting both the level of economic activity as well as its sensitivity to shocks, and generating to fluctuations that are inefficient relative to a natural constrained-efficient benchmark. Importantly, this inefficiency can be present even in environments where the incentives to respond to information are in line with social objectives, illustrating the importance of explicitly modeling the information choice decision. Our findings hold for various types of shocks (real or nominal) as well as for different decision variables (quantities versus prices) and are obtained under a general specification for the information acquisition technology which nests various commonly used specifications.

This inefficiency arises from 2 distinct sources. The first is that a firm with market power (i.e. the ability to affect prices through its actions) does not internalize all the benefits of better aligning its decisions with fundamentals. In other words, under imperfect competition, the *private* value of information, the change in expected profits, is less than the social value, which reflects changes in expected social surplus. As a result, in a *laissez faire* equilibrium, monopolistically competitive firms tend to make suboptimally low levels of investment in information. Importantly, this holds even if the sensitivity of actions to such information is the socially optimal one. A real business cycle version of our model - where firms make labor input decisions under uncertainty about aggregate productivity shocks - exhibits this combination of *ex-ante* inefficiency and *ex-post* efficiency.

The second source of inefficiency emerges when information is *used* suboptimally. In our general equilibrium environment, this occurs when firms set nominal prices under uncertainty (and let quantities be determined by realized demand conditions). We find that firms set prices that are 'too sensitive' to their private signals, because they do not internalize their contribution to overall uncertainty in the economy. More precise information exacerbates this inefficiency, partly (and in some cases, completely) offsetting the direct benefits of taking actions under better information. Private decisions do not reflect this trade-off and therefore, tend to overvalue information.

When firms make quantity choices under uncertainty, only the first of these two sources is operational and the equilibrium features under-acquisition of information relative to the welfare maximizing benchmark. When firms set nominal prices, however, both channels are present, making the overall direction of the inefficiency ambiguous. We divide the parameter space into various regions depending on whether we see over- or under-acquisition of information in equilibrium. When the elasticity of substitution between the firms' products is low, market power is high and the first effect tends to dominate. The opposite happens when goods are highly substitutable or when the quality of information is low.

Our results have a number of implications. First, they show that conclusions about the efficiency of aggregate fluctuations derived under complete or exogenous information do not extend to an environment with costly learning. Second, policies aimed at correcting market power-related distortions have additional effects when information is endogenously chosen in equilibrium. When information is used in a socially optimal fashion, policies which align changes in social surplus with private payoffs always improve welfare. In fact, in our CES environment, the standard complete information policy response to non-competitive behavior -a constant revenue subsidy - is also the optimal policy with endogenous information and achieves the constrained-efficient solution. However, this is no longer true with inefficiency in information use, as in the price-setting environment. Then, policies which correct market power alone can even reduce welfare. This counter-intuitive result emerges because such policies can worsen the inefficiency in information choice. When the equilibrium features an suboptimally high level of information acquisition, giving firms a greater share of the total surplus exacerbates the inefficiency. This can, under some circumstances, more than offset the beneficial effects of removing the noncompetitive distortion. It is important to note that this is an effect which arises only when information choice is modeled explicitly. We show the optimal policy in such an environment must be state-contingent and takes the form of a countercyclical revenue subsidy. Finally, our results also point to situations where public information can lead to welfare losses because it crowds out private information production. Intuitively, this occurs when the equilibrium features an inefficiently low of private learning.

Though our focus is the fully articulated business cycle environment, we also show how our analysis applies to coordination games in general, using the beauty contest framework of Morris and Shin [MS98] and others. A general insight emerges - what matters for information use is only the *relative* importance of the various components of the payoff function, but the value of information is also influenced by the *absolute* level of the payoff. We show that externalities in payoff functions (e.g. as in Morris and Shin [MS02] or Angeletos and Pavan [AP07]) can distort the level of social versus private payoffs and cause private and social values of information to diverge, even if they leave incentives to use such information undistorted[1].

It is worth emphasizing that our results are derived with very little structure on the learning technology beyond those necessary to guarantee an interior solution. Our specification encompasses several commonly used formulations (e.g. rational inattention, costly signals). Also, while we focus on private signals for most of our analysis, the sources of inefficiency highlighted are relevant to the acquisition of public information as well[2].

This paper bears a direct connection to the body of work embedding heterogeneous

[1] In an independent paper, Colombo, Femminis and Pavan [CFP12] arrive at the same result, using a general quadratic specification for payoffs.

[2] In Section 2.7.5, we illustrate this using the beauty contest model.

information in business cycle models. One branch of this literature [3] takes the information structure as exogenous and derives implications for equilibrium responses. A second strand [4] endows agents with a learning technology and allows them to endogenously determine the extent of information, as in this paper. The main difference between this paper and this latter group is that we are concerned primarily with efficiency, while most of the other papers focus on other properties of equilibrium outcomes.

Two independent recent papers are important exceptions[5]. Colombo, Femminis and Pavan [CFP12] and Mackowiak and Wiederholt [MW11b] investigate the efficiency of information choice in a quadratic utility framework[6]. Colombo, Femminis and Pavan [CFP12] also find that efficiency in information use does not imply optimal information choice and characterize the link between payoff externalities and efficiency. In a rational inattention framework with quadratic utility, Mackowiak and Wiederholt [MW11b] study the optimality of attention allocated to rare events. The insights from these papers, while related, are not directly applicable to the fully articulated microfounded environments that are the focus of this paper. Our analytical framework allows us to derive closed-form expressions for the objects of interest and thus allows us to capture all effects of information acquisition, without resorting to approximations[7]. Thus, our analysis provides a comprehensive picture of the incentives to acquire information in a standard macroeconomic environment, leading to more robust conclusions about welfare and setting the stage for a quantitative evaluation. Moreover, our results - on the sources and magnitude of the inefficiency - are directly interpretable in terms of model primitives, viz. preference and technology parameters.

[3]For example, Woodford [Woo03], Moscarini [Mos04], Angeletos and Pavan [AP07], Angeletos and La'O [AL08, AL09], Nimark [Nim08], Hellwig [Hel08b, Hel08a], Lorenzoni [Lor09, Lor10], and Hellwig and Venkateswaran [HV09b]. The large literature on noisy rational expectations models in asset pricing, including seminal work by Hellwig [Hel80] and Diamond and Verrecchia [DV81], mostly falls under this category, as does the recent work on global games, following Morris and Shin [MS98, MS02].

[4]For example, Mackowiak and Wiederholt [MW09, MW11a] consider a setting where agents face a constraint on their ability to process information, while Hellwig and Veldkamp [HV09a], Gorodnichenko [Gor08] and Reis [Rei06] introduce explicit costs of planning or acquiring information. In the asset pricing context, Grossman and Stiglitz [GS80], Ganguli and Yang [GY09], Barlevy and Veronesi [BV00] and Veldkamp [Vel06b, Vel06a] all consider environments where information is chosen endogenously. Myatt and Wallace [MW10] study a beauty contest setting where agents choose what signals to pay attention to.

[5]Chahrour [Cha12] also looks at the welfare implications of costly *public* signals.

[6]In an unpublished working-paper version of Hellwig and Veldkamp [HV09b], information acquisition is shown to be efficient in a beauty-contest model without externalities.

[7]For example, 'non-strategic' effects of uncertainty on payoffs are ruled out in Colombo, Femminis and Pavan [CFP12].

Our work complements earlier work on the efficiency of information use under exogenous information. Angeletos and Pavan [AP07] show that information is used inefficiently in equilibrium when private and social incentives to coordinate are different (Hellwig [Hel05] and Roca [Roc10] analyze these incentives in a general equilibrium monetary model). Our paper, on the other hand, compares social and private incentives to learn. Amador and Weill [AW10] also study the efficiency when the extent of information is endogenously determined in equilibrium. However, this occurs through learning from endogenous objects and not, as in this paper, through the acquisition of costly information. Finally, our findings on the effects of market power on information choice also contribute to a broader agenda studying the efficiency implications of imperfect competition - see, for example, Bilbie et. al [BGM08] and the references therein.

The rest of the paper is organized as follows. In section 2.2, we use a simple model to show how imperfect competition distorts incentives to learn. Section 2.3 lays out the full model, which embeds information acquisition in a general equilibrium real business cycle model with aggregate shocks. The next 3 sections consider three commonly used versions of this environment. Section 3.2 is a real business cycle environment, where firms make labor input choices under imperfect information about aggregate productivity shocks. Sections 2.5 and 2.6 repeat the analysis under price-setting and nominal shocks respectively. Section 2.7 studies information choice in a reduced-form coordination game with quadratic payoffs. Section 2.8 contains a brief conclusion. Proofs are collected in the Appendix.

2.2 A Simple Example

The purpose of this section is to build intuition about the connection between market power and value of information. We study a simple environment where a single monopolist makes production choices under uncertainty. She is endowed with a technology that transforms the numeraire, denoted N, into final goods, denoted Q, according to

$$Q = AN^{\frac{1}{\delta}}, \qquad \delta > 1,$$

where A is a log-normally distributed technology shock, i.e. $a \equiv \ln A \sim N(0, \sigma_a^2)$. The profit of the monopolist is given by

$$\Pi = PQ - N,$$

where P is the price of the final good in terms of the numeraire. The monopolist faces a representative consumer with a utility function

$$C = \left(\frac{\theta}{\theta - 1}\right) Q^{\frac{\theta-1}{\theta}} - PQ, \qquad \theta > 1.$$

Optimization by the consumer implies

$$P = Q^{\frac{-1}{\theta}}.$$

The total social surplus is

$$U = \left(\frac{\theta}{\theta - 1}\right) Q^{\frac{\theta-1}{\theta}} - N.$$

Using the consumer's optimality condition, we can rewrite this as

$$U = \left(\frac{\theta}{\theta - 1}\right) PQ - N.$$

Thus, in this constant demand elasticity environment, there is a simple relationship between the consumer's utility and revenue. As $\theta \to \infty$, the difference vanishes, i.e. profits equal the social surplus.

When making her production decision, the monopolist faces uncertainty about the realization of the technology shock A. In particular, she only sees a noisy signal

$$s = a + e, \qquad e \sim N(0, \sigma^2),$$

and chooses input[8], N. Formally, her problem is

$$\begin{aligned}\Pi &= \max_N \quad \mathbb{E}\left[PQ \mid s\right] - N \\ &= \max_N \quad \mathbb{E}\left[\left(AN^{\frac{1}{\delta}}\right)^{\frac{\theta-1}{\theta}} \mid s\right] - N,\end{aligned}$$

where the operator $\mathbb{E}\left[\cdot|s\right]$ represents the expectation conditional on the signal s. The first order condition is

$$\frac{\theta-1}{\theta\delta}\mathbb{E}\left[A^{\frac{\theta-1}{\theta}} \mid s\right] N^{\frac{\theta-1}{\theta\delta}-1} = 1.$$

Standard results for conditional expectations of log-normal random variables imply

$$\log\ N = \kappa + \alpha \cdot s,$$

where

$$\begin{aligned}\alpha &= \frac{\delta(\theta-1)}{1-\theta+\theta\delta}\left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_\varepsilon^2}\right), \\ \kappa &= \left(\frac{\theta\delta}{1-\theta+\theta\delta}\right)\log\frac{\theta-1}{\theta\delta} + \frac{1}{2}\left(\frac{\theta\delta}{1-\theta+\theta\delta}\right)\left(\frac{\theta-1}{\theta}\right)^2\left(\frac{\sigma_a^2\sigma^2}{\sigma_a^2+\sigma^2}\right).\end{aligned}$$

Before analyzing the value of information, it is instructive to examine the efficiency properties of this policy more closely. Consider the surplus-maximizing response function, i.e. the solution to

$$U = \max_N \quad \mathbb{E}\left[\left(\frac{\theta}{\theta-1}\right)PQ \mid s\right] - N.$$

It is easy to show that the solution takes the same form as the monopolist's policy, with

$$\begin{aligned}\alpha^* &= \alpha, \\ \kappa^* &= \kappa + \left(\frac{\theta\delta}{1-\theta+\theta\delta}\right)\ln\left(\frac{\theta}{\theta-1}\right) \quad > \quad \kappa.\end{aligned}$$

In other words, the elasticity of labor input with respect to the signal (and therefore, to the fundamental) is the socially optimal one but the monopolist chooses a suboptimally

[8] The results go through even if the monopolist had to choose prices instead of input.

low average level of labor input. Thus, the monopolist uses information *efficiently* even though she finds it optimal to restrict production.

The private value of information to the monopolist is the sensitivity of the (ex-ante) expected profit to the variance of the noise in the signal. A straightforward application of the envelope theorem yields

$$\frac{\partial \mathbb{E}\Pi}{\partial \sigma^2} = -\frac{\alpha^2}{2}\left(\frac{1-\theta+\theta\delta}{\theta\delta}\right)\mathbb{E}N < 0.$$

where $\mathbb{E}$ takes expectations over the realizations of the aggregate shocks and the signals. $\mathbb{E}N$ is the unconditional expectation of input. The derivative is negative, i.e. profits decline with poorer information. Analogously, the social value is the change in expected total surplus i.e. $\frac{\partial \mathbb{E}U}{\partial \sigma^2}$. We can show that social surplus is proportional to profits, i.e.

$$\begin{aligned}
\mathbb{E}U &= \underbrace{\left[\frac{\left(\frac{\theta}{\theta-1}\right)\frac{\theta\delta}{\theta-1}-1}{\frac{\theta\delta}{\theta-1}-1}\right]}_{>1}\mathbb{E}\Pi \\
\Rightarrow \frac{\partial \mathbb{E}U}{\partial \sigma^2} &= \left[\frac{\left(\frac{\theta}{\theta-1}\right)\frac{\theta\delta}{\theta-1}-1}{\frac{\theta\delta}{\theta-1}-1}\right]\frac{\partial \mathbb{E}\Pi}{\partial \sigma^2} \quad < \quad \frac{\partial \mathbb{E}\Pi}{\partial \sigma^2}.
\end{aligned}$$

Thus, noisier signals lead to a greater loss of utility compared to profits. The source of the difference, the $\frac{\theta}{\theta-1}$ in the numerator, is simply the constant of proportionality between consumer surplus and revenue. Intuitively, the effect on profits from better information underestimates the change in social surplus because revenues do not capture all the utility gained by the consumer. Only in the limiting case of infinite demand elasticity does the private value coincide with the social value.

Figure 3.3 provides a graphical illustration for this intuition. The profit maximizing choice for a given level of the technology shock under perfect information, N, leads to a full information profit of Π. The corresponding social surplus is denoted U. Under imperfect information, the firm chooses a scale of production that is lower on average than under full information. The expected profit drops to Π^e while the social surplus drops to U^e. Since the utility curve is steeper than the profit curve at N^*, the private loss from less information ($\Pi - \Pi^e$) underestimates the social loss ($U - U^e$).

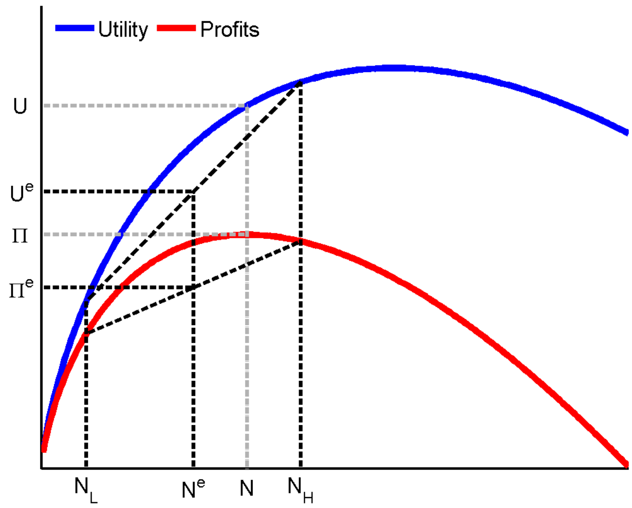

Figure 2.1: Profits and Utility

In the general equilibrium environments studied in the rest of the paper, the underlying demand structure will be more complicated, but this basic intuition carries over almost exactly. Firms internalize the effects of better information on prices and therefore, attach a lower value to their own learning and therefore, tend to acquire less than the socially optimal amount of information. However, other equilibrium linkages can cause information to be used inefficiently, which can overwhelm this channel for underinvestment in learning.

2.3 A Microfounded Business Cycle Model

In this section, we lay out a microfounded business cycle model with dispersed information. The fully-articulated and flexible specification will allow us to examine the efficiency of equilibrium information choice under various assumptions about the nature of shocks (real vs. nominal) and decisions (prices vs. quantity). These cases will be examined in detail in the next 3 sections.

Time is discrete, $t = 0, 1, 2....$ The economy is populated by a continuum of entrepreneurs and a final good producer. The entrepreneurs or firms as we will sometimes refer to them in our exposition, each have access to a technology, which transforms labor into a differentiated intermediate good. These technologies, are located on a continuum of informationally-separate islands, with one firm per island. Firms make two decisions - an *ex-ante* information choice, modeled as the precision of a private signal about an aggregate shock and an *ex-post* production/pricing choice.

Preferences and Technology: Entrepreneur i enjoys a per-period utility according to[9]

$$C_{it} - N_{it} - \upsilon(\sigma_{ei}^2),$$

where C_{it} is consumption of final goods and N_{it} the labor input[10]. The last term reflects the cost of acquiring private information. The agent is subject to a budget constraint

$$P_t C_{it} = P_{it} Y_{it} \ .$$

Production of intermediate goods is described by a decreasing returns to scale production function:

$$Y_{it} = A_t N_{it}^{\frac{1}{\delta}} \ ,$$

where $\delta > 1$ and A_t is aggregate productivity.

The final good is a CES composite of the intermediate goods

$$Y_t = \left(\int_0^1 Y_{it}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \ ,$$

where the parameter θ is the elasticity of substitution between intermediate goods. Throughout the paper, we will assume that $\theta > 1$.

Finally, aggregate variables are linked by the following quantity equation:

$$P_t Y_t = M_t \ ,$$

where M_t is the (exogenous) level of nominal demand.

In the following three sections, we study in detail 3 versions of this general framework:

[9]The assumption of linearity is not crucial for any of our results, but simplifies the expressions considerably.

[10]We model the entrepreneur as choosing how much of his own effort to commit to production. This backyard production specification is only for simplicity. In earlier versions of this paper, we worked with explicit labor markets on each island and our results go through almost exactly. .

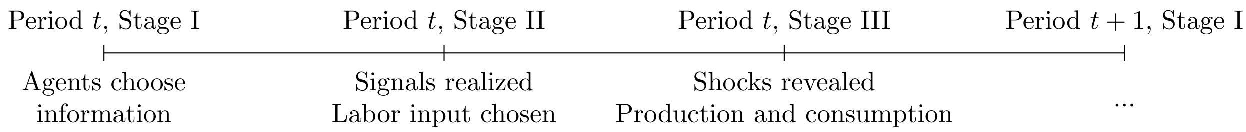

Figure 2.2: Timeline of Events

- Quantity (labor input) choice with aggregate productivity shocks
- Price choice with aggregate productivity shocks
- Price choice with aggregate nominal shocks

2.4 Model I: Quantity choice with productivity shocks

In this version, the only source of aggregate uncertainty is the level of aggregate technology A_t. Nominal demand is constant, i.e. $M_t = M \quad \forall t$. Note that under complete information, this is the canonical real business cycle model, with monopolistic competition replacing the standard representative firm assumption[11].

Firms observe a private signal about the aggregate productivity shock and choose labor input. Then, production takes place, the firms sell their output and buy the final good for consumption. Figure 3.1 shows the timing of events in each period.

We will show that information about the aggregate shock is used efficiently in this environment, but the incentives to learn are suboptimally low. As a result, the *laissez-faire* equilibrium with endogenous information exhibits inefficient fluctuations, even though the same economy under the assumption of exogenous information does not. The intuition is similar to the simple example in the previous section - imperfect substitutability leads to a wedge between the private value of information and its the social value. As a result, agents in equilibrium expend a suboptimally low level of effort in information acquisition. Only in a limiting case, as goods becomes perfect substitutes, does the equilibrium achieve efficiency.

Aggregate productivity is log-normally distributed, i.e. $\log A_t \equiv a_t \sim N\left(0, \sigma_a^2\right)$. For

[11] Angeletos and La'O [AL09] study a similar environment with dispersed but exogenous information. The main modeling difference is that they have many firms on each island, a feature that is easy to incorporate into our setup.

simplicity, we focus on the case where this is an i.i.d shock, but our results go through for more general stochastic processes as well[12].

Information structure: Before choosing labor input, each agent observes a private signal s_{it} about the current productivity shock:

$$s_{it} = a_t + e_{it} \ ,$$

where $e_{it} \sim N\left(0, \sigma_{ei}^2\right)$. The variance of the noise term, σ_{ei}^2 is the variance chosen *ex-ante* by the firm.

Optimality: The competitive firm producing the final good solves :

$$\max \quad P_t Y_t - \int_0^1 P_{it} Y_{it} di \ ,$$

$$Y_t = \left(\int_0^1 Y_{it}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \ ,$$

where P_{it} is the price of intermediate good i. Optimality yields the usual demand function for good i

$$Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\theta} Y_t \ . \tag{2.1}$$

Substituting from the budget constraint, we can write the intermediate producer's objective in Stage II as follows:

$$\Pi_{it} = \max_{N_{it}} \quad \mathbb{E}_{it} \left(\frac{P_{it}}{P_t} A_t N_{it}^{\frac{1}{\delta}} - N_{it} \right) \ ,$$

where the operator $\mathbb{E}_{it}\left(\cdot\right)$ represents the expectation conditional on firm i's information $\mathcal{I}_{it}$, i.e. $\mathbb{E}_{it}\left(\cdot\right) \equiv \mathbb{E}_t\left(\cdot \ |\mathcal{I}_{it}\right)$.

Substituting from the demand function (2.1),

$$\Pi_{it} = \max_{N_{it}} \quad \mathbb{E}_{it} \left[\left(\frac{Y_{it}}{Y_t} \right)^{\frac{-1}{\theta}} A_t N_{it}^{\frac{1}{\delta}} - N_{it} \right] , \tag{2.2}$$

The solution is to choose an input level that equates expected marginal revenue to

[12]For example, if a_t is an $AR(1)$ process, our results go through exactly with the aggregate shock now interpreted as the current innovation to the aggregate productivity level.

marginal cost

$$\mathbb{E}_{it}\left[\left(\frac{\theta-1}{\delta\theta}\right)Y_t^{\frac{1}{\theta}}A_t^{\frac{\theta-1}{\theta}}N_{it}^{\frac{\theta-1-\theta\delta}{\delta\theta}}\right]=1. \tag{2.3}$$

Rearranging,

$$N_{it}^{\frac{1+\theta\delta-\theta}{\delta\theta}}=\frac{\theta-1}{\delta\theta}\mathbb{E}_{it}\left[Y_t^{\frac{1}{\theta}-\gamma}A_t^{\frac{\theta-1}{\theta}}\right]. \tag{2.4}$$

Information acquisition: In the first stage of each period, before signals are realized, each agent chooses the extent of information to acquire, taking as given choices of other firms in the economy. The unconditional expectation of profits is defined as:

$$\hat{\Pi}_{it}\left(\sigma_{ei}^2,\sigma_e^2\right)\equiv\mathbb{E}\Pi_{it}\ , \tag{2.5}$$

where $\mathbb{E}$ takes expectations over the realizations of the aggregate shocks and the signals

The problem of the agent in the first stage can then be written as:

$$\max_{\sigma_{ei}^2}\quad\hat{\Pi}_{it}\left(\sigma_{ei}^2,\sigma_e^2\right)-\upsilon\left(\sigma_{ei}^2\right)\ ,$$

where $\upsilon(\cdot)$ is the cost of information[13] as a function of the noise in the signal with $\upsilon'(\cdot)<0,\ \upsilon''(\cdot)>0$. Our focus in this paper is on differences in the *value* of information to private agents and to the planner, so we wish to impose as little structure as possible on the *cost* of information. The only additional assumption we make is that the solution to the above information choice problem (and later, that of the planner) lies in the interior[14], i.e. is characterized by :

$$\frac{\partial\hat{\Pi}}{\partial\sigma_{ei}^2}-\upsilon'(\sigma_{ei}^2)=0\ . \tag{2.6}$$

[13]For example, under the rational inattention paradigm, as in Sims [Sim03], this would be determined by the cost of information processing capacity, which is defined as the extent of reduction in entropy about the fundamental shock. Alternatively, if information choice takes the form of deciding how many signals to acquire, the function $\upsilon(\cdot)$ is interpreted as the total cost of acquiring a basket of signals with the same informational content as a single signal with precision σ_e^2.

[14]A necessary condition is that the cost function is sufficiently convex, i.e.

$$\frac{\partial^2\hat{\Pi}}{\partial\hat{\sigma}_e^2\partial\hat{\sigma}_e^2}-\frac{\partial^2\upsilon}{\partial\hat{\sigma}_e^2\partial\hat{\sigma}_e^2}<0\ .$$

2.4.1 Equilibrium

A equilibrium is (i) a set of information choices for each firm (ii) island-specific labor inputs as functions of the signal on the island (iii) aggregate consumption and output as functions of the aggregate state such that: (a) the labor input is optimal, given island-specific information and wages and the functions in (iii) above, (b) taking the behavior of aggregates in (iii) as given, the information choice in (i) solves the Stage I problem, (c) markets clear and (d) the functions in (iii) are correct.

We focus on symmetric equilibria, where all agents acquire the same amount of information in stage I and follow the same strategies in stage II. The characterization of the equilibrium in stage II essentially follows the same procedure as in Angeletos and La'O [AL09]. We begin with a conjecture that, in equilibrium, firms follow a symmetric labor input policy of the form[15]

$$n_{it} = k_2 + \alpha s_{it} \ , \tag{2.7}$$

where k_2 and α are coefficients to be determined in equilibrium. The former determines the (unconditional) average level of (the log of) employment, while the latter is the elasticity. The details of this guess-and-verify approach are in the Appendix. The expressions for the response coefficients are given in the following result.

Proposition 1 *In a symmetric equilibrium, labor input is given by (2.7), with*

$$\alpha = \left(\frac{\delta}{\delta-1}\right)\left[\frac{\sigma_a^2}{\sigma_a^2 + \left(\frac{1+\delta\theta-\theta}{\delta\theta-\theta}\right)\sigma_e^2}\right] \ , \tag{2.8}$$

$$k_2 = \left(\frac{\theta\delta}{1+\theta\delta-\theta}\right)\log\left(\frac{\theta-1}{\delta\theta}\right) + \left[\frac{1-\theta+\theta\delta}{\theta(\delta-1)}\right]\frac{\alpha\sigma_e^2}{2} + \left[\frac{1}{\theta(\delta-1)}\right]\frac{\alpha^2\sigma_e^2}{2}. \tag{2.9}$$

where σ_e^2 is the variance of the error in agents' signals.

The expression for α has an intuitive interpretation. The first part $\frac{\delta}{\delta-1}$ is simply the full information elasticity of employment to a productivity shock. Under incomplete information, this is downweighted by the second part, an adjusted signal-to-noise ratio. The adjustment essentially increases the weight of the noise (by a factor $\frac{1+\delta\theta-\theta}{\delta\theta-\theta} > 1$), reflecting the well-known effect of strategic complementarities. In other words, firms in

[15] Hereafter, variables in small cases denote variables in logs, i.e. $x \equiv \log(X)$

this economy have an incentive to coordinate their actions (due to the imperfect substitutability of the goods they produce). Since the informational friction dampens the overall response of the economy to the fundamental, agents find it optimal to respond less than one-for-one to their expectations of fundamental.

Finally, we characterize the information acquisition decision in stage I. We begin by noting that the maximized stage II profit function, equation (3.12), depends on both the information choices of the agent herself as well as everybody else in the economy. The latter enter payoffs through the aggregate response coefficients, α and k_2. Ex-ante expected profits, conditional on a choice of individual error variance σ_{ei}^2, are obtained by taking expectations over the realization of the random variable $\mathbb{E}_{it}(a_t)$.

A symmetric stationary equilibrium can thus be represented as a fixed point problem in σ_e^2:

$$\sigma_e^2 = \text{argmax}_{\sigma_{ei}^2} \ \hat{\Pi}\left[\sigma_{ei}^2, \alpha(\sigma_e^2), k_2(\sigma_e^2)\right] - \upsilon\left(\sigma_{ei}^2\right),$$

where we make explicit the dependence of α and k_2 on σ_e^2 according to the equilibrium relationships (3.14)-(3.15).

In the Appendix we show that, under the assumption of an interior solution, the optimality condition associated with this problem is

$$-\frac{1}{2}\left(\frac{\theta-1}{\delta\theta}\right)\hat{\Pi}\alpha^2 = \upsilon'(\sigma_e^2), \tag{2.10}$$

where $\hat{\Pi}$ is the unconditional expected profit and α is the equilibrium response coefficient. Since both these objects are themselves functions of σ_e^2, this is a fixed point relation in σ_e^2 and completes the characterization of equilibrium.

2.4.2 Efficiency in Information Use

We now turn to its efficiency properties. We begin by showing that information use is optimal[16]. To achieve this, we compare the equilibrium coefficients α and k_2 to those chosen by a planner, who is interested in maximizing household utility. Importantly, the planner is assumed to be information-constrained, i.e. cannot pool information across

[16]This subsection is essentially a replication of the welfare results in Angeletos and La'O [AL09].

islands but is free to choose how agents respond to the signals. We exploit log-normality and restrict attention to symmetric log-linear policy rules of the form:

$$n_{it} = \tilde{k}_2 + \tilde{\alpha}\ s_{it}\ . \tag{2.11}$$

Then, it is straightforward to derive the aggregate labor input, consumption and welfare are:

$$\begin{aligned} N_t &= \exp\left[\tilde{k}_2 + \tilde{\alpha} a_t + \frac{1}{2}\tilde{\alpha}^2\sigma_e^2\right], \\ C_t &= Y_t = \exp\left[\left(1 + \frac{\tilde{\alpha}}{\delta}\right) a_t + \frac{\tilde{k}_2}{\delta} + \frac{1}{2}\left(\frac{\theta - 1}{\theta}\right)\frac{\tilde{\alpha}^2}{\delta^2}\sigma_e^2\right], \end{aligned}$$

and the corresponding ex-ante expectations

$$\begin{aligned} \hat{N}\left(\tilde{k}_2, \tilde{\alpha}\right) &= E(N_t) = \exp\left[\tilde{k}_2 + \frac{1}{2}\tilde{\alpha}^2(\sigma_a^2 + \sigma_e^2)\right], \\ \hat{C}\left(\tilde{k}_2, \tilde{\alpha}\right) &= E(C_t) = \exp\left[\frac{1}{2}\left(1 + \frac{\tilde{\alpha}}{\delta}\right)^2 \sigma_a^2 + \frac{\tilde{k}_2}{\delta} + \frac{1}{2}\left(\frac{\theta - 1}{\theta}\right)\frac{\tilde{\alpha}^2}{\delta^2}\sigma_e^2\right], \\ \mathbb{U} &= \hat{C} - \hat{N}. \end{aligned} \tag{2.12}$$

The efficient use of information is characterized by coefficients α^* and k_2^* that maximize utility, i.e.

$$(\alpha^*, k_2^*) = \text{argmax}_{\tilde{k}_2, \tilde{\alpha}} \quad \hat{C}\left(\tilde{k}_2, \tilde{\alpha}\right) - \hat{N}\left(\tilde{k}_2, \tilde{\alpha}\right).$$

The optimality conditions of this problem are

$$C^* = \delta N^*, \tag{2.13}$$

$$C^*\left[\frac{1}{\delta}\left(1 + \frac{\alpha^*}{\delta}\right)\sigma_a^2 + \left(\frac{\theta - 1}{\theta\delta}\right)\frac{\alpha^*}{\delta}\sigma_e^2\right] = N^*\alpha^*\left(\sigma_a^2 + \sigma_e^2\right).$$

Using the first equation, the second condition can be rewritten as

$$\left[\frac{1}{\delta}\left(1 + \frac{\alpha^*}{\delta}\right) - \frac{\alpha^*}{\delta}\right]\sigma_a^2 = \frac{\alpha^*}{\delta}\sigma_e^2\left[1 - \frac{\theta - 1}{\theta\delta}\right]. \tag{2.14}$$

The two sides of this equation reflect the trade-off faced by the planner. A stronger response to the signal makes actions better aligned with the fundamental, but also increases the inefficient variation in them. The planner sets α to equate the marginal benefit from the former channel to the marginal cost from the latter.

Agents in equilibrium face a similar trade-off. Consider the ex-ante profit of firm which takes as given the responses of other firms (α, k_2) and chooses the coefficients of its own best response $(\hat{\alpha}, \hat{k}_2)$

$$
\begin{aligned}
&\exp\left\{\left(\frac{\theta-1}{\theta\delta}\right)\hat{k}_2 + \frac{1}{\theta\delta}k_2 + \frac{1}{2}(\theta-1)\frac{\alpha^2}{\theta^2\delta^2}\sigma^2\right\} \\
&\exp\left\{\frac{1}{2}\left[\left(\frac{\theta-1}{\theta}\right)\left(1+\frac{\hat{\alpha}}{\delta}\right)+\frac{1}{\theta}\left(1+\frac{\alpha}{\delta}\right)\right]^2\sigma_a^2 + \frac{1}{2}\hat{\alpha}^2\left(\frac{\theta-1}{\theta\delta}\right)^2\sigma_e^2\right\} \\
&-\exp\left\{\hat{k}_2 + \frac{1}{2}\hat{\alpha}^2\left(\sigma_a^2+\sigma_e^2\right)\right\}.
\end{aligned}
$$

The associated optimality conditions are

$$
\begin{aligned}
C_i &= \left(\frac{\theta}{\theta-1}\right)\delta N_i\,, \\
\left\{\frac{1}{\delta}\left[\frac{\theta-1}{\theta}\left(1+\frac{\hat{\alpha}}{\delta}\right)+\frac{1}{\theta}\left(1+\frac{\alpha}{\delta}\right)\right]\sigma_a^2\left(\frac{\theta-1}{\theta}\right)+\hat{\alpha}\left(\frac{\theta-1}{\theta\delta}\right)^2\sigma_e^2\right\}C_i &= N_i\hat{\alpha}\left(\sigma_a^2+\sigma_e^2\right).
\end{aligned}
$$

Using the first equation and invoking symmetry, the second condition becomes

$$
\left(\frac{\theta-1}{\theta}\right)\frac{1}{\delta}\left(1+\frac{\alpha}{\delta}\right)\sigma_a^2 + \frac{\alpha}{\delta}\left(\frac{\theta-1}{\theta}\right)\left(\frac{\theta-1}{\theta\delta}\right)\sigma_e^2 = \left(\frac{\theta-1}{\theta}\right)\frac{\alpha}{\delta}\left(\sigma_a^2+\sigma_e^2\right).
$$

Re-arranging,

$$
\left(\frac{\theta-1}{\theta}\right)\left[\frac{1}{\delta}\left(1+\frac{\alpha}{\delta}\right)-\frac{\alpha}{\delta}\right]\sigma_a^2 = \left(\frac{\theta-1}{\theta}\right)\frac{\alpha}{\delta}\sigma_e^2\left[1-\frac{\theta-1}{\theta\delta}\right]. \tag{2.15}
$$

Comparing (2.14) and (2.15), we see that the private benefits and costs of a stronger response are proportional to those faced by the planner, with a scaling factor $\frac{\theta-1}{\theta}$. In other words, the trade-off faced by agents in equilibrium is the same as the social tradeoff. Note that this only applies to the choice of the response coefficient α. The usual monopoly inefficiency of restricted production applies here as well, but only as a distortion to the average level of activity k_2. Formally, the following result shows that the equilibrium α

coincides with the corresponding socially optimal coefficient, but k_2 is inefficiently low. Moreover, the difference between k_2 and k_2^* is invariant to the information structure and vanishes in the competitive limit as $\theta \to \infty$.

Proposition 2 *For a given σ_e^2, the planner's optimal response coefficients are:*

$$\alpha^* = \alpha, \tag{2.16}$$

$$k_2^* = k_2 + \frac{\delta}{\delta - 1} \log\left(\frac{\theta}{\theta - 1}\right), \tag{2.17}$$

where α and k_2 are as defined in Proposition 19.

Thus, the distortion caused by imperfect competition takes the form of a constant scaling down of labor input, but does not distort the elasticity of aggregate employment with respect to the shock. This result has an important implication - when information is exogenous, the average level of activity in this economy is inefficiently low, but fluctuations are constrained efficient.

2.4.3 Efficiency of Information Choice

Next, we show that, despite the optimal response to signals *ex-post*, the ex-ante information acquisition decision is inefficient. Our benchmark is the level of information that maximizes *ex-ante* utility in a symmetric equilibrium, i.e.

$$\max_{\sigma_e^2} \quad \mathbb{U}\left(\sigma_e^2\right) - \upsilon\left(\sigma_e^2\right) \ ,$$

where $\mathbb{U}$ is the expected utility characterized in (3.20).

We restrict attention to the case where the solution to the above problem is interior, i.e. characterized by the first-order condition[17]:

$$\frac{\partial \mathbb{U}}{\partial \sigma_e^2} = \frac{\partial \upsilon}{\partial \sigma_e^2}. \tag{2.18}$$

[17] As with the equilibrium information choice, we also need to assume that the cost function is sufficiently convex, i.e.

$$\frac{\partial^2 \mathbb{U}}{\partial \sigma_e^2 \partial \sigma_e^2} - \frac{\partial^2 \upsilon}{\partial \sigma_e^2 \partial \sigma_e^2} < 0\ .$$

Comparing (3.24) to (2.6), it is easy to see that information choice is efficient if, and only if, the marginal value to the planner, $\partial\mathbb{U}/\partial\sigma_e^2$ coincides with the private value to the firm, $\partial\hat{\Pi}/\partial\sigma_{ei}^2$.

The next proposition presents the main result of this section. It shows that the intuition from the simple example in Section 2 goes through in this richer general equilibrium environment as well. In any symmetric equilibrium, there is a constant wedge between the private value of information by firms and its social value.

Proposition 3 *In a symmetric equilibrium, the private value of information is always less than its social value, i.e.*

$$\frac{\partial\mathbb{U}}{\partial\sigma_e^2} = \left(1 + \frac{\delta}{(\theta-1)(\delta-1)}\right)\left(\frac{\partial\hat{\Pi}}{\partial\sigma_{ei}^2}\right)_{\sigma_e^2=\sigma_{ei}^2} < 0 \qquad \forall\ \sigma_e^2 \in \mathbb{R}^+. \tag{2.19}$$

Therefore, the level of information acquired in equilibrium is inefficiently low.

From (2.19), it is easy to see that the inefficiency is related to the elasticity of substitution, θ. The intuition is similar to the simple example - with imperfect substitutability, marginal revenue is strictly less than the marginal surplus. Therefore, more information, or equivalently better alignment of actions with fundamentals, causes a smaller improvement in profits relative to total utility. As a result, the monopolist attaches a lower value to learning than the social planner and therefore, acquires less than the socially optimal level of information. The extent of this underacquisition is decreasing in θ. As goods become more and more substitutable, the difference between marginal revenue and marginal surplus shrinks. In the perfectly competitive limit, as $\theta \to \infty$, the gap between the social value and the private value of information vanishes[18].

The implications for efficiency of equilibrium outcomes are immediate. When information is exogenous, the average level of activity is inefficiently low but cyclical fluctuations are constrained efficient. However, this is no longer true when information is endogenous.

[18]Note that market power and imperfect substitutability are controlled by the same parameter θ. One can easily extend this framework to parameterize these two forces separately. For example, in Angeletos and La'O [AL09], a continuum of firms on each island produce differentiated inputs, which are bundled together to produce a final good. Imperfect substitutability of these island-specific final goods leads to aggregate demand linkages, while the differentiated nature of inputs gives firms on each island market power. Our efficiency results extend to this environment as well, with this latter parameter playing the role of θ.

Too little information is acquired in equilibrium and through its effects on k_2 and α, this suboptimality influences both the average level of activity in equilibrium as well as the elasticity with respect to the shock. The sign of the effect on the former is in general ambiguous, but the response coefficient α is lower, i.e. the sensitivity of employment (and therefore, of output) to the technology shock is inefficiently muted. This is a novel source of inefficiency in this class of models - one that is absent both under the canonical full information assumption as well as under exogenous information (e.g. Angeletos and LaO [AL09]).

A less obvious implication relates to the social value of public information. Suppose entrepreneurs also had access to a free public signal about aggregate productivity in this environment. It is straightforward to show that this reduces the value of private signals information, leading to lower investments in information. In other words, public information crowds out private information. Since the latter was already being produced at an inefficiently low level, this is detrimental to welfare and in some cases, can overwhelm the direct benefit of more information[19]. Note that this effect is not present when information is exogenous. In that case, public information can reduce welfare only if it is *used* inefficiently. With endogenous information, however, the social value of public information can be negative, even in the absence of *ex-post* inefficiencies.

Finally, we turn to the policy implications of our inefficiency result, which are discussed in the next subsection.

2.4.4 Optimal Policy

We show that constrained efficiency is restored in this environment if policy is used to correct the average level distortion in production. In particular, a constant revenue subsidy, equal to the markup not only removes the firm's incentives to underproduce, but also leads it to invest the socially optimal amount in information production.

Given an arbitrary revenue subsidy Λ, the problem of the firm becomes:[20]

$$\Pi_{it} = \max_{N_{it}} \quad \mathbb{E}_{it}\left[\Lambda P_{it} Y_{it} - N_{it}\right] \quad ,$$

[19] Colombo, Femminis and Pavan [CFP12] find a similar result with in a quadratic utility framework.
[20] In addition, a lump sum transfer $\tau_R \int P_{it} Y_{it} di$ is subtracted from the budget constraints.

It is easy to show that the level distortion in activity is removed, i.e. k_2 equals k_2^*, if the subsidy satisfies

$$\Lambda = \frac{\theta}{\theta - 1} \ .$$

More interestingly, this subsidy also aligns marginal revenue with the change in total surplus and therefore, equates the private marginal value of information to the social value, leading to both *ex-post* and *ex-ante* efficiency.

Proposition 4 *A symmetric equilibrium with a constant revenue subsidy* $\Lambda = \frac{\theta}{\theta-1}$ *is constrained efficient, i.e. it attains the optimal allocation of the information constrained planner.*

Thus, in a real business cycle environment with endogenous information, policies aimed at correcting market-power related inefficiencies have an additional benefit - they also remove the wedge between private and social value of information, eliminating inefficiencies (in both the average level and the fluctuations) arising from suboptimal information choice. This conclusion, however, depends crucially on the fact that information is efficiently used - as the price-setting model in the following section will highlight.

2.5 Model II: Price-setting with productivity shocks

In this section, we modify the environment in the previous section and assume that entrepreneurs set nominal prices (as opposed to choosing labor input) after observing private signals of aggregate productivity[21]. Formally, the intermediate goods producers now choose nominal prices for their products and commit to producing any amount demanded at that price[22]. The entrepreneur's problem is now:

$$\max_{P_{it}} \ E_{it} \left(\frac{P_{it}}{P_t}\right)^{1-\theta} Y_t - \left[\left(\frac{P_{it}}{P_t}\right)^{-\theta} \frac{Y_t}{A_t}\right]^{\delta} . \tag{2.20}$$

[21] Lorenzoni [Lor09] studies a similar environment with exogenous dispersed information.

[22] Whether firms compete by choosing prices or quantities is a matter of some debate. See Aiginger[Aig99] for a survey. One of the studies cited in that paper describes a survey of Austrian manufacturing on their main strategic variable. About 38% of the 930 firms surveyed said they produce a specific quantity, thereafter permitting demand to decide price conditions while the remaining said they set prices leaving competitors and the market to determine quantity sold.

As with quantity choice, optimality equates expected marginal revenue to expected marginal cost:

$$(\theta-1)\,P_{it}^{-\theta}\;E_{it}\left[P_t^{\theta-1}Y_t\right]=\theta\delta P_{it}^{-\theta\delta-1}E_{it}\left[P_t^{\theta\delta}\frac{Y_t^{\delta}}{A_t^{\delta}}\right]. \tag{2.21}$$

A comparison (2.21) and the optimality condition under labor input choice, equation (2.3) in the previous section, reveals an important difference between the two environments. When a firm chooses its labor input under uncertainty, its marginal cost is known, or more generally, unaffected by the actions of other agents. This is not the case under price setting - the marginal cost to a firm from changing its own price depends on the aggregate price level P_t. Therefore, the sensitivity of average prices to the shock affects each firm's uncertainty about its own marginal cost. As we will see, this additional interaction leads to an externality and will lead to both *ex-post* and *ex-ante* inefficiencies.

The solution strategy follows the same guess-and-verify procedure as in the previous section. We begin with a conjecture that individual prices are set according to:

$$p_{it}=k_2+\alpha s_{it}\;. \tag{2.22}$$

The expressions for the response coefficients in a symmetric equilibrium are derived in the Appendix and collected in the following result.

Proposition 5 *In a symmetric equilibrium, firms follow a pricing rule of the form (2.22), with*

$$\begin{aligned}\alpha &= \left(\frac{-\delta}{\delta-1}\right)\left[\frac{\sigma_a^2}{\sigma_a^2+\left(\frac{1+\delta\theta-\theta}{\delta-1}\right)\sigma_e^2}\right],\\ (\delta-1)k_2 &= \ln\left(\frac{\theta\delta}{\theta-1}\right)+(\delta-1)\,m+\frac{1}{2}\alpha^2\sigma_e^2\left[\theta^2\delta^2-\delta(\theta-1)^2+1-\theta\right]+\frac{1}{2}\sigma_a^2\left[\delta^2(1+\alpha)^2-\alpha^2\right],\end{aligned}$$

where σ_e^2 is the variance of the error in agents' signals.

As with quantity choice, the adjustment to the signal-to-noise ratio in the expression for α reveals a coordination motive - strategic complementarities further dampen the response of the aggregate price level to the shock.

2.5.1 Efficiency in information use

We define the socially optimal response as the utility-maximizing choice of an information-constrained planner, who is free to set the response coefficients α and k_2 but is subject to all the other equilibrium constraints. In particular, given a cross-sectional distribution of prices $\{P_{it}\}$, the aggregate price level P_t and output Y_t are determined by the zero-profit condition of the final goods producer and the quantity equation respectively.

Proposition 6 *For a given σ_e^2, the planner's optimal response coefficients are:*

$$\begin{aligned} \alpha^* &= \left(\frac{-\delta}{\delta-1}\right)\left[\frac{\sigma_a^2}{\sigma_a^2+\theta\left(\frac{1+\delta\theta-\theta}{\delta-1}\right)\sigma_e^2}\right], \qquad (2.23) \\ (\delta-1)k_2^* &= -\ln\left(\frac{\theta}{\theta-1}\right)+(\delta-1)\,k_2(\alpha^*), \end{aligned}$$

where $k_2(\alpha^*)$ denotes is the equilibrium level coefficient in Proposition 5 with α^* replacing α.

Thus, the equilibrium features prices that are too responsive[23] to signals, i.e. $\alpha^* > \alpha$. In other words, information used suboptimally when monopolistically competitive firms set prices and optimally when the choice variable is labor input instead. The intuition is related to the marginal cost uncertainty mentioned earlier. Firms do not take into account their contribution to the uncertainty faced by other firms in the economy and as a result, set prices that are too responsive to private signals[24].

The level coefficient also is suboptimal - but now it includes not only the usual markup distortion but also the effects of the inefficient sensitivity to information. Importantly, the latter persist even as the former disappears, e.g as θ tends to infinity.

[23] Hellwig [Hel05] shows a similar result in an environment with monetary shocks.

[24] What is crucial for the efficiency results is that the marginal cost is uncertain but that it depends on the actions of other firms. If, for example, firms committed to an output level, i.e. Y_{it}, they would still be uncertain about marginal cost (because A_t is not known), but this is unaffected by the behavior of other agents. As a result, information is still used efficiently. Similarly, in the single firm environment of Section 2, responses to signals are efficient even with price choice.

2.5.2 Efficiency of information choice

Not surprisingly, the *laissez-faire* choice of signal precision in this environment is suboptimal. Recall that information choice was suboptimal in the quantity choice model, even without *ex-post* inefficiencies in its effect on actions, so the results in Proposition 6 make *ex-ante* efficiency even less likely. The characterization of the socially optimal information choice follows the same procedure as section 3.2.6. Information choice is efficient if, and only if, the marginal social value of learning, $\partial \mathbb{U}/\partial \sigma_e^2$ coincides with the private value, $\partial \hat{\Pi}/\partial \sigma_{ei}^2$.

There is one additional complication. Unlike the quantity choice environment, the social value of information is not always positive (even though information is always privately valuable). This is because changing information now has two effects on welfare. The first, or direct effect, is simply the value of better alignment with fundamentals. The second, or indirect, effect arises because the (inefficient) response coefficient changes with the level of information. Formally, as we show in the Appendix, the social value can be decomposed as follows,

$$\begin{aligned}
\frac{\partial \mathbb{U}}{\partial \sigma_e^2} &= -\mathbb{U}\left(\frac{\theta\delta\left(1-\theta+\theta\delta\right)}{2\left(\delta-1\right)}\alpha^2+\frac{\delta\left(\theta-1\right)\left(1-\theta+\theta\delta\right)\sigma^2}{\left(\delta-1\right)}\alpha\frac{d\alpha}{d\sigma^2}\right) \\
&= -\mathbb{U}\frac{\theta\delta\left(1-\theta+\theta\delta\right)}{\left(\delta-1\right)}\left[\frac{\alpha^2}{2}+\left(\frac{\theta-1}{\theta}\right)\sigma^2\alpha\frac{d\alpha}{d\sigma^2}\right].
\end{aligned}$$

The two terms inside the square brackets represent the two effects. The first, the direct one, implies that information is socially valuable, while the second term, the indirect effect, goes in the opposite direction since $\alpha \cdot d\alpha/d\sigma^2 < 0$. The social value of information can be negative when the latter outweighs the former. Holding other parameters fixed, this is more likely when θ is high. Intuitively, higher θ makes realized production levels more sensitive to price differences, making marginal cost uncertainty particularly damaging. In the other direction, as we approach unit elasticity, $\theta \to 1$, the indirect effect becomes arbitrarily small[25] and the equilibrium responses coincide with the planner's.

Obviously, if the social value of information is negative at the equilibrium choice of σ_e^2, information choice is trivially inefficient (increasing the noise in the signals will raise

[25] As $\theta \to 1$, expenditure shares are close to constant, so the strategic linkage becomes very weak.

utility *and* save on information costs). The more interesting case is when both the profit-maximizing and utility-maximizing information choices are in the interior, i.e. we are in the region where the social value is positive,i.e. $\partial\mathbb{U}/\partial\sigma_e^2 < 0$. This will always be the case if information is sufficiently cheap. Formally, we assume that the cost function $v(\cdot)$ is such that both the equilibrium and the socially optimal level of information satisfy the following condition:

Assumption 1 $(\theta-2)(1-\theta+\theta\delta)\,\sigma_e^2 \leq (\delta-1)\,\theta\sigma_a^2$.

Conditional on being in this region[26], the utility maximizing choice is characterized by equating $\partial\mathbb{U}/\partial\sigma_e^2$ to the marginal cost. The next proposition shows that information acquisition is typically inefficient, though the direction is ambiguous[27]. The result essentially divides the parameter space into two regions, depending on whether the equilibrium exhibits too much or too little information production. Only for a non-generic combination of parameters does the equilibrium choice of σ_e^2 coincide with the utility maximizing level.

Proposition 7 *Suppose $\theta > 2$ and the conditions of Assumption 1 are met. Then, there is over-acquisition of information in equilibrium if the following condition holds at the equilibrium σ_e^2:*

$$\sigma_e^2 > \frac{\theta(\delta-1)}{(1-\theta+\theta\delta)}\left[\frac{(1-\theta+\theta\delta)+\delta}{\theta(\theta-1)(\delta-1)+2(1-\theta+\theta\delta)(\theta-2)}\right]\sigma_a^2 .$$

If inequality is reversed, there is underacquisition.

The underlying intuition is not hard to see. There are two sources of inefficiency in information choice, working in opposite directions. As in the quantity choice environment, imperfect substitutability gives firms pricing power, which implies that do not appropriate all the benefits of better information, pushing them towards underacquisition. However, the excess sensitivity of equilibrium responses to information makes increasing it less valuable from a social point of view. The combination of these two forces leaves us with

[26]Note that this always holds for $\theta < 2$. If $\theta > 2$, then we need σ_e^2 to be sufficiently low.

[27]For brevity, we only present results for the case where $\theta > 2$ (the empirically relevant case for most macroeconomic models).

the ambiguous finding in the proposition. Higher θ or σ_e^2 exacerbates the inefficiency in information use, strengthening the second channel and making over-acquisition more likely.

A full quantitative investigation is beyond the scope of this paper, but it is easy to verify that reasonable calibrations of incomplete information monetary models easily satisfy this condition. In other words, the empirically relevant region of the parameter space seems to be one where the equilibrium information acquisition is more than the social optimal level. As an illustrative case, set $\theta = 4, \delta = 1.5$. Then, the noise in private signals only needs to be one-sixth as volatile as aggregate productivity for the above condition to hold. In other words, we need only a very modest departure from full information to see over-acquisition of information in equilibrium.

The implications of this finding for the constrained efficiency of fluctuations as well as the social value of public information are similar to that under the labor input choice model of the previous section. However, the presence of both sources of inefficiency has important implications for policy. We turn to this issue in the following subsection.

2.5.3 Optimal Policy

Consider the constant subsidy aimed at correcting the monopoly distortion studied in section 2.4.4 Recall that, with labor input choice, this policy restored constrained efficiency. However, when firms set prices instead, it can have unintended consequences and even reduce welfare. In other words, the desirability of subsidies to correct underproduction might hinge on the nature of firms' decision variable firms - prices or quantities.

To see why the revenue subsidy can be detrimental to welfare, note that aligning marginal revenue with the marginal social surplus increases the private value of information and leads to more learning. However, this additional investment in information acquisition can be socially suboptimal. To put it differently, without the policy, the two sources of inefficiency partially offset each other. If only one of them is removed, the economy bears the full brunt of the other, which could more than overcome the direct benefits of the policy. In other words, incomplete policy responses can do more harm than good.

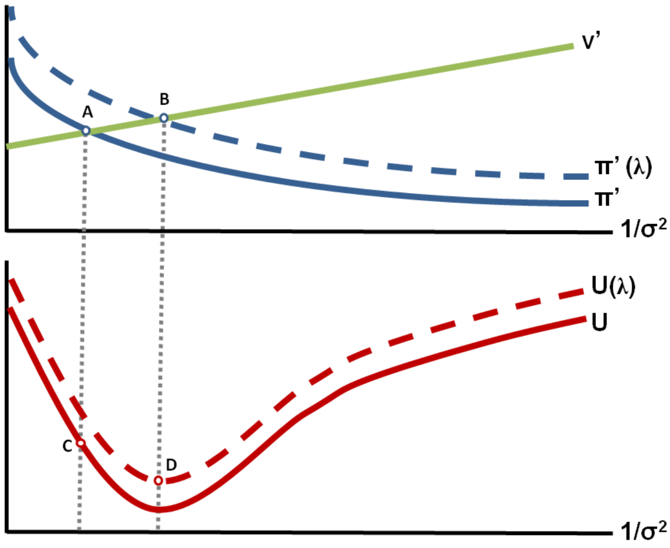

Figure 2.3: Effect of subsidy on welfare

Figure 2.3 illustrates such a case. The top panel depicts information choice in equilibrium, where the marginal cost of information (υ') intersects the marginal benefit (π'). The corresponding level of welfare is shown in the bottom panel. Without the subsidy (the solid lines), the equilibrium features over-acquisition of information (note that the variable on the x-axis is precision, the inverse of σ_e^2). In fact, at the equilibrium choice, the social value is negative ! The subsidy raises the private value of information and therefore, leads to more learning. The removal of the monopoly distortion to average production raises utility for all levels of information (the direct effect of removing the markup distortion), but the new equilibrium is associated with a lower level of welfare than without the subsidy (the point D in the bottom panel compared to C).

Finally, we characterize the optimal policy in this environment. For ease of comparison with the quantity choice model, we consider a revenue subsidy of the form

$$\Lambda \ A_t^{\delta\tau} ,$$

where the policy parameter τ is the sensitivity of the subsidy to fundamentals[28]. We begin by deriving the values of Λ and τ that lead to *ex-post* efficiency, for a given level of noise in signals σ_e^2.

[28]In the quantity choice model, the optimal policy set $\tau = 0$, i.e. the subsidy was invariant to the realization of the fundamental.

Proposition 8 *Given σ_e^2, equilibrium allocations coincide with the choices of the planner, i.e.* $(\alpha, k_2) = (\alpha^*, k_2^*)$ *if the subsidy coefficients satisfy*

$$\begin{aligned} \tau &= \frac{\alpha^*}{\alpha^{eq}} - 1 \qquad < \ 0 \,, \\ \Lambda &= \left(\frac{\theta}{\theta - 1}\right) \exp\left\{\frac{\sigma_a^2 \delta \tau^* \left(2\alpha^* - \delta \tau^*\right)}{2}\right\} . \end{aligned} \tag{2.24}$$

The optimal policy is a state-contingent revenue subsidy - decreasing in the technology shock A_t. This countercyclicality dampens the effect of the shock on firm's profits and therefore, reduces the firms' incentives to adjust prices in response to an expected shock, fixing the excess sensitivity problem in equilibrium responses. The level coefficient Λ has the usual markup correction, with an adjustment for level effects arising from the state-contingent part.

More importantly, this policy also implements the socially optimal level of information acquisition, as the following result shows. Formally,

Proposition 9 *A symmetric equilibrium under the policy described in Proposition 8 (evaluated at the socially optimal σ_e^2), is constrained efficient, i.e. it attains the optimal allocation of the information constrained planner.*

In other words, the general insight from the quantity choice model goes through here as well - fixing *ex-post* inefficiencies in equilibrium responses also aligns private and social benefits from learning, leading ex-ante efficiency. Unlike the quantity choice model however, this requires policy to be state-contingent.

2.6 Model III: Price choice with nominal shocks

In this section, we show that the results from the previous section apply to learning about aggregate nominal shocks as well. In particular, firms adjust their prices by too much in response to expected changes in money supply. The combination of this inefficiency and market power once again leads to an ambiguous sign on the inefficiency in information choice.

The environment is identical to that of the previous section except that productivity is now constant (i.e. $A_t = A$) but aggregate nominal demand is stochastic. In particular, M_t is an iid[29], log-normally distributed random variable, i.e. $\log M_t \equiv m_t \sim N(0, \sigma_m^2)$. Intermediate goods producers choose nominal prices for their products and commit to producing any amount demanded at that price. Before setting prices, each firm observes a private signal s_{it} about the current monetary shock:

$$s_{it} = m_t + e_{it} \ ,$$

where $e_{it} \sim N(0, \sigma_{ei}^2)$. The variance of the noise term, σ_{ei}^2 is the variance chosen in stage I by the firm.

As before, the competitive firm producing the final good operates after the monetary shock is realized. Therefore, the problem of this firm remains the same, i.e. demand for intermediate goods is given by 2.1. Intermediate goods producer's choose prices prior to the realization of the monetary shock.

The intermediate producer's problem is:

$$\max_{P_{it}} \ E_{it} \left(\frac{P_{it}}{P_t}\right)^{1-\theta} Y_t - \left[\left(\frac{P_{it}}{P_t}\right)^{-\theta} \frac{Y_t}{A}\right]^{\delta} .$$

As before, we guess (and verify) that equilibrium prices follow:

$$p_{it} = k_2 + \alpha s_{it} \ . \tag{2.25}$$

The response coefficients in a symmetric equilibrium are collected in the following result.

Proposition 10 *In a symmetric equilibrium, firms follow a pricing rule of the form (2.25), with*

$$\alpha = \left[\frac{\sigma_m^2}{\sigma_m^2 + \left(\frac{1-\theta+\theta\delta}{\delta-1}\right)\sigma_e^2}\right] , \tag{2.26}$$

$$k_2 = \frac{1}{(\delta-1)} \ln \frac{\theta\delta}{\theta-1} - \delta a + \frac{(\delta^2-1)(1-\alpha)^2}{2(\delta-1)}\sigma_m^2 + \frac{\delta\theta(1-\theta+\theta\delta)}{2(\delta-1)}\alpha^2\sigma_e^2 + \frac{(\delta-1)(\theta-1)}{2(\delta-1)}\alpha^2\sigma_e^2 \tag{2.27}$$

[29] Again, for simplicity, we assume that nominal demand is iid, though it is straightforward to extend the analysis to richer stochastic processes.

where σ_e^2 is the variance of the error in agents' signals.

A symmetric stationary equilibrium can thus be represented as a fixed point problem in σ_e^2:

$$\sigma_e^2 = \text{argmax}_{\sigma_{ei}^2} \; \hat{\Pi}\left[\sigma_{ei}^2, \alpha(\sigma_e^2), k_2(\sigma_e^2)\right] - \upsilon\left(\sigma_{ei}^2\right) \; ,$$

where we make explicit the dependence of α and k_2 on σ_e^2 according to the equilibrium relationships (2.26)-(2.27).

2.6.1 Efficiency in Information Use

The socially efficient response function takes the same form as (2.25) with the coefficients given in the following result.

Proposition 11 *For a given σ_e^2, the planner's optimal response coefficients are:*

$$\begin{aligned} \alpha^* &= \left[\frac{\sigma_m^2}{\sigma_m^2 + \theta\left(\frac{1-\theta+\theta\delta}{\delta-1}\right)\sigma^2}\right] \\ (\delta-1)k_2^* &= \ln\left(\frac{\theta}{\theta-1}\right) + (\delta-1)\,k_2(\alpha^*). \end{aligned}$$

where the dependence of $k_2(\alpha^*)$ denotes the equilibrium level coefficient with α^* replacing α.

Again, the equilibrium features prices that are too responsive to signals, i.e. $\alpha^* < \alpha$. The intuition is very similar to the productivity shocks case - firms do not fully internalize the effect of their pricing decisions on the marginal cost uncertainty faced by other firms. As a result, they react too much to private signals, relative to the planner's solution.

2.6.2 Efficiency in Information Choice

Next, we compare the amount of information acquired in equilibrium to the utility-maximizing level. As with productivity shocks case discussed in section 2.5, the presence of direct and indirect effects means that the marginal social value of information is not always positive. We restrict attention to the region where this value is indeed positive. A sufficient condition is

Assumption 2 $(\theta - 2)(1 - \theta + \theta\delta)\,\sigma_e^2 \leq (\delta - 1)\,\theta\sigma_m^2$.

Conditional on being in this region, whether the social planner acquires more or less information than the equilibrium depends only on the marginal value to the planner, $\partial\mathbb{U}/\partial\sigma_e^2$, versus the private value to the firm, $\partial\hat{\Pi}/\partial\hat{\sigma}_e^2$.

The following result mirrors Proposition 7 and shows that the equilibrium can feature both under- and over-acquisition. Again, as before, we only present results for the case where $\theta > 2$

Proposition 12 *Suppose $\theta > 2$ and the conditions of Assumption 2 are met. Then, there is over-acquisition of information in equilibrium if the following condition holds:*

$$\sigma_e^2 \geq \left[\frac{\delta\theta\,(\delta - 1)}{(1 - \theta + \theta\delta)\,[2\,(\theta - 1) + (\theta - 2)\,\theta\delta]}\right]\sigma_m^2 \ .$$

In the spirit of the simple numerical illustration following Proposition 7, suppose $\theta = 4, \delta = 1.5$. Then, the condition in the above result amounts to requiring that the variance of noise in signals be at least five percent of the variance of money supply, a small deviation from full monetary neutrality.

2.6.3 Optimal Policy

We conclude our discussion of this version of the model by characterizing optimal policy in this environment. In line with section 2.5, we consider revenue subsidies of the form,

$$\Lambda\ M_t^{(1-\delta)\tau}\ .$$

The following proposition characterizes the policy coefficients that correct both the sources of inefficiency in the equilibrium response functions.

Proposition 13 *Given σ_e^2, equilibrium allocations coincide with the choices of the plan-*

ner, i.e. $(\alpha, k_2) = (\alpha^*, k_2^*)$ *if the subsidy coefficients satisfy*

$$\begin{aligned} \tau &= \frac{\alpha^*}{\alpha^{eq}} - 1 \qquad < \quad 0, \qquad\qquad (2.28)\\ \Lambda &= \left(\frac{\theta}{\theta - 1}\right)^{\frac{1}{\delta-1}} \exp\left\{\frac{\sigma_m^2 \tau^* \left(2\left(1-\alpha^*\right) + \left(1-\delta\right)\tau^*\right)}{2}\right\}. \end{aligned}$$

As we would expect, this policy also removes the wedge between private and social value of information, ensuring that signal precisions in equilibrium are socially optimal. Formally,

Proposition 14 *A symmetric equilibrium with the policy described in Proposition 2.28 is constrained efficient, i.e. it attains the optimal allocation of the information constrained planner.*

2.7 A Beauty Contest Model

In this section, we study information choice in a beauty contest model, in the spirit of the global games literature, see Morris and Shin [MS98, MS02][30]. Though more abstract than the micro-founded environments of the previous sections, this setup will allow us to both demonstrate the applicability of our main results to coordination games more generally as well as to draw connections to earlier work on the efficiency properties of economies with dispersed information. We show that social and private value of information are, in general, different and so information acquisition in equilibrium is typically inefficient relative to a socially optimal benchmark. This inefficiency can arise be due to the suboptimal *use* of information, but it can be present even when information is used efficiently.

2.7.1 Payoffs and Information

There is a continuum of agents, indexed by $i \in [0, 1]$. The game in played in two stages. In stage I, agents choose how much private information (measured by the precision of a private signal about an aggregate fundamental) to acquire subject to a cost function. In

[30]It is also possible to demonstrate our main results in the more general quadratic payoff structure, as in Angeletos and Pavan [AP07].

stage II, signals are realized and agent i chooses an action $x_i \in \mathbb{R}$ to maximize expected the following *private* payoff function:

$$\Pi_i = \max_{x_i} \ - \mathbb{E}_i \left[\phi \left(x_i - a\right)^2 + \psi \left(x_i - \bar{x}\right)^2\right],$$

where $\bar{x} \equiv \int_0^1 x_i di$ is the average action of all agents, A is the underlying aggregate state and $\mathbb{E}_i\left(\cdot\right) \equiv \mathbb{E}\left(\cdot|\mathcal{I}_i\right)$ is the expectation operator conditional on agent i's information set $\mathcal{I}_i$. The random variable a represents an aggregate state, which is normally distributed with mean zero and variance σ_a^2.

The payoff function for agent i has two components. The first component is linked to the (squared) deviation between the underlying state θ and agent i's action x_i. The second part is the squared distance between i's action and the average action of all the other agents in the economy, denoted $\bar{x}$. The two components capture the idea that an agent's payoff depends not only on fundamentals but also on actions of other agents (a feature that was present in the business cycle environments studied earlier). The parameters ϕ and ψ index the relative importance of these two components in private payoffs. For ease of exposition, we focus on the case where these two weights are positive, though this is not essential for our results.

Before choosing x_i, each agent has access to a private signal s_i about the fundamental:

$$s_i = a + e_i \ ,$$

where $e_i \sim N\left(0, \sigma_{ei}^2\right)$. This variance $\hat{\sigma}_e^2$ is the result of choices made in stage I by the agent. The noise term e_i is independent of a and independent across the population, i.e. $\mathbb{E}\left(e_i \, e_j\right) = 0$ for $i \neq j$. The agent's information set consists only of the common prior and this private signal.

Let $\hat{\Pi}(\cdot)$ denote the expected payoff in stage II (prior to the realization of the signals s_i):

$$\hat{\Pi}_i\left(\sigma_{ei}^2, \sigma_e^2\right) \ \equiv \ \mathbb{E}\left(\Pi_i\right),$$

where $\mathbb{E}\left(\cdot\right)$ is the expectation operator prior to the realization of signals, σ_{ei}^2 is the variance of the agent's own private signal and σ_e^2 is the variance of the signals of all the other agents in the economy[31]. The problem of the agent in the first stage can then be written as:

$$\max_{\hat{\sigma}_e^2} \;\; \hat{\Pi}_i \left(\sigma_{ei}^2, \sigma_e^2\right) - \upsilon\left(\sigma_{ei}^2\right) \;\; ,$$

where $\upsilon(\cdot)$ is the cost of information as a function of the noise in the signal. For now, we impose only that $\upsilon'(\cdot) < 0, \upsilon''(\cdot) > 0$.

2.7.2 Equilibrium

We start with the equilibrium in stage II. The agent's maximization problem directly yields the following first order condition:

$$x_i = \frac{\phi}{\phi+\psi}\mathbb{E}_i(a) + \frac{\psi}{\phi+\psi}\mathbb{E}_i(\bar{x}) \quad .$$

We conjecture (and verify) that, in a symmetric equilibrium, the average action is linked to the realization of the fundamental θ according to this linear relationship:

$$\bar{x} = \alpha a \quad .$$

Given this conjecture,

$$\begin{aligned} x_i &= \frac{\phi+\psi\alpha}{\phi+\psi}\mathbb{E}_i(a) \\ x_i &= \left(\frac{\phi+\psi\alpha}{\phi+\psi}\right)\left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_{ei}^2}\right)(a+e_i) = \hat{\alpha}(a+e_i) \quad , \\ \Longrightarrow \hat{\alpha} &= \left(\frac{\phi+\psi\alpha}{\phi+\psi}\right)\left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_{ei}^2}\right). \end{aligned} \tag{2.29}$$

The conjecture is verified when

$$\hat{\alpha} = \alpha \; ,$$

which leads to the following result.

[31] We restrict attention to symmetric equilibria, where all agents make the same information acquisition choices.

Proposition 15 *The unique symmetric equilibrium is given by $x_i = \alpha^{eq}\, s_i$, where*

$$\alpha^{eq} = \frac{\phi\sigma_a^2}{\phi\sigma_a^2 + (\phi+\psi)\sigma_e^2}\ . \tag{2.30}$$

Information Acquisition: Next, we turn to the ex-ante information acquisition decision in stage I. Recall that each agent chooses the precision of her private signals, subject to a cost function $\upsilon(\cdot)$. At the optimum, each agent equates the marginal value of more information to its cost. In a symmetric equilibrium, the envelope theorem implies that this *private* marginal value of information is the same for all agents and is given by:

$$\frac{\partial\hat{\Pi}_i}{\partial\hat{\sigma}_\epsilon^2} = -\left(\phi+\psi\right)\left(\alpha^{eq}\right)^2\ . \tag{2.31}$$

Under the assumption of an interior optimum[32], the optimality condition in stage I becomes:

$$-\left(\phi+\psi\right)\left(\alpha^{eq}\right)^2 = \upsilon'(\sigma_e^2)\ .$$

Noting that λ^{eq} is in turn a function of the (symmetric) information choice, the above condition is a fixed point in σ_e^2 and completes the characterization of the equilibrium with endogenous information acquisition.

2.7.3 Welfare

Next, we study the efficiency properties of the equilibrium characterized in the previous subsection. The first step is to define a social welfare criterion, which is assumed to take the same form as private payoffs:

$$\begin{aligned} W &= U - \int \upsilon\left(\sigma_{ei}^2\right) di \\ &= -\mathbb{E}\left[\phi^*\int_0^1 \left(x_i - a\right)^2 di + \psi^*\int_0^1 \left(x_i - \bar{x}\right)^2 di\right] - \int \upsilon\left(\sigma_{ei}^2\right) di. \end{aligned}$$

where ϕ^* and ψ^* are both positive. Thus, welfare is declining in the average deviations from the fundamental θ and the cross-sectional dispersion in actions, but with weights that are potentially different from the ones in private payoffs. These differences arise due

[32]We assume that $\upsilon(\cdot)$ is such that the optimum is reached at an interior point.

to externalities, e.g. as in Morris and Shin [MS02].

At the symmetric equilibrium characterized in the previous subsection, welfare (before information acquisition costs) is

$$U = -\phi^*(\alpha^{eq} - 1)^2\sigma_a^2 - (\phi^* + \psi^*)\,(\alpha^{eq})^2\,\sigma_e^2\ .$$

The social value of information is given by

$$\frac{dU}{d\sigma_e^2} = -(\phi^* + \psi^*)\alpha^2 + \frac{dU}{d\alpha}\,\frac{d\alpha}{d\sigma_e^2}\ . \tag{2.32}$$

The information choice (assumed to be in the interior) that maximizes social welfare is one at which this marginal benefit is equal to the marginal cost of information $\upsilon'(\sigma_e^2)$. Recall that the equilibrium information choice was characterized by equating the private marginal value (2.31) to the marginal cost. Thus, the inefficiency in information choice is determined by the difference between social and private marginal values, i.e. $\frac{dU}{d\sigma_e^2}$ and $\frac{\partial\hat{\Pi}_i}{\partial\sigma_{\epsilon i}^2}$.

It is useful to first characterize the efficient use of information. This is modeled as the choice of an information-constrained planner who directly chooses agents' actions to maximize the above objective. We restrict attention to linear response functions of the form:

$$x_i = \alpha s_i\ .$$

The efficient use of information is then the solution to

$$\begin{aligned}\mathbb{U} &= \max_{\alpha}\ \ -\phi^*(\alpha-1)^2\mathbb{E}a^2 + (\phi^* + \psi^*)\alpha^2\sigma_e^2 \\ &= \max_{\alpha}\ \ -(\phi^*(\alpha-1)^2\sigma_a^2 + (\phi^* + \psi^*)\alpha^2\sigma_e^2)\ .\end{aligned}$$

The first order condition[33] of the above problem is:

$$\frac{\partial\mathbb{U}}{\partial\alpha} : \phi^*(\alpha-1)\sigma_a^2 + (\phi^* + \psi^*)\alpha\sigma_e^2 = 0\ . \tag{2.33}$$

[33]The second-order condition requires that $\phi^*\sigma_\theta^2 + (\phi^* + \psi^*)(\sigma_\theta^2 + \sigma_e^2) \geq 0$.

Re-arranging, we derive the following result:

Proposition 16 *The socially efficient linear response coefficient, denoted α^* is*

$$\alpha^* = \frac{\phi^* \sigma_a^2}{\phi^* \sigma_a^2 + (\phi^* + \psi^*)\sigma_e^2}. \tag{2.34}$$

Comparing the two response coefficients, α^{eq} and α^*, we see that equilibrium responses are efficient if, and only if, the agents attach the same relative weight to the two types of deviations in their private payoffs as the planner does. Formally,

Proposition 17 *For a given σ_e^2, equilibrium response is efficient if, and only if, the relative weights of the two components are equal in the private and social payoff function, i.e.*

$$\alpha^{eq} = \alpha^* \qquad \Leftrightarrow \qquad \frac{\phi}{\psi} = \frac{\phi^*}{\psi^*} \ .$$

This finding is an instance of a well-known feature of these models (see, for example, Angeletos and Pavan [AP07]) - differences between the social and private costs of dispersion and volatility can lead to information being used in a socially sub-optimal manner.

With some algebra, we can rewrite the social value of information in (2.32) as

$$\frac{dU}{d\sigma_e^2} = \frac{d\Pi}{d\sigma_{ei}^2}\left[\left(\frac{\phi^* + \psi^*}{\phi + \psi}\right) - 2\frac{\phi^*}{\phi}\left(\frac{\alpha^{eq}}{\alpha^*} - 1\right)\right]. \tag{2.35}$$

This equation is key for understanding our inefficiency result. It shows that social and private marginal values of information can diverge for two reasons. First suppose $\alpha^{eq} = \alpha^*$, i.e. information use is socially optimal. Then, the second term inside the brackets is zero. Even so, information can contribute more (less) to social welfare than to private profits if $\phi^* + \psi^*$ is greater (smaller) than $\phi + \psi$. In other words, externalities can lead to inefficiencies through differences in the overall level of social welfare and private payoffs, even if they do not distort the relative importance of the two components of payoffs.

What happens when information use is inefficient ? For concreteness, consider the case where $\phi^* + \psi^* = \phi + \psi$ but $\alpha^{eq} > \alpha^*$, i.e. there are no level differences but agents respond too much to their signals, relative to the planner's solution. Then, the term inside the square bracket is less than 1, i.e. the social value of information is lower than the private value. Intuitively, more information has an additional effect - it makes the ex-post inefficiency more severe. The opposite happens when $\alpha^{eq} < \alpha^*$.

The information-constrained optimum in this economy, i.e. the outcome when the planner chooses both the amount of information and its ex-post use, is also easy to characterize. By the envelope theorem, the social marginal value of information in this allocation is given by

$$\frac{dU^*}{d\sigma_e^2} = -\left(\phi^* + \psi^*\right)\left(\alpha^*\right)^2 \ , \tag{2.36}$$

Equating this to the marginal cost yields fixed point relationship that defines the information-constrained optimum.

$$-\left(\phi^* + \psi^*\right)\left(\alpha^*\right)^2 = v'(\sigma_e^2) \ .$$

This level of information differs from the equilibrium one for the 2 reasons discussed earlier. The first is linked to the suboptimality in information use referred to earlier, i.e. to the fact that α^{eq} may not be equal to α^*. However, even if the equilibrium information use is efficient, i.e. $\alpha^{eq} = \alpha^*$, the private marginal value of information can still diverge from the socially optimal level because of a level effect, i.e. the difference between $\phi^* + \psi^*$ and $\phi + \psi$. To see this more clearly, note that we can rewrite this social value as follows:

$$\frac{dU^*}{d\sigma_e^2} = \underbrace{\frac{\partial \hat{\Pi}_i}{\partial \hat{\sigma}_e^2}}_{\text{Private value}} - \frac{1}{(\phi + \psi)} \underbrace{\left\{ \left[\left(\frac{\phi^* + \psi^*}{\phi + \psi} \right) - 1 \right] \left(\alpha^*\right)^2 + \left[\left(\alpha^*\right)^2 - \left(\alpha^{eq}\right)^2 \right] \right\}}_{\text{'Externalities'}} \ .$$

2.7.4 Implementing the Information-Constrained Optimum

In this subsection, we consider the nature of interventions that are necessary to correct the information-related inefficiencies in the equilibrium characterized above. Given a precision of signals, σ_e^2, we will show that efficiency in information use can be restored through a 'tax', which aligns the social and private weights attached to the two payoff components. However, in line with the general intuition behind the findings in the previous subsection, we will show that this, by itself, is not sufficient to align private incentives to acquire information with the social ones.

We start with the sub-optimal nature of information use. Formally, we consider a tax, τ, of the following form[34]

$$\Pi_i = \max_{x_i} \; - \mathbb{E}_i(\phi\tau\,(x_i - a)^2 + \psi\,(x_i - \bar{x})^2)\ .$$

For a given tax τ, the equilibrium response coefficient is:

$$\alpha^\tau = \frac{\phi\tau\sigma_a^2}{\phi\tau\sigma_a^2 + (\phi\tau + \psi)\sigma_e^2}\ .$$

Any response coefficient α can be implemented by setting the tax appropriately, i.e. by solving the following equation for τ,

$$\alpha = \frac{\phi\tau\sigma_a^2}{\phi\tau\sigma_a^2 + (\phi\tau + \psi)\sigma_e^2}\ .$$

In particular, to implement α^*, the socially optimal response, the tax is simply

$$\tau^* = \frac{\phi^*}{\psi^*}\,\frac{\psi}{\phi}\ .$$

The expression for the optimal tax rate is intuitive - it corrects the inefficiency in information use by aligning the relative weights of the two components in the private and social payoff functions. However, this correction by itself is not enough to align the private incentives to acquire information with those of the planner. The marginal private value of information under τ^*, is given by:

$$\frac{\partial\hat{\Pi}_i}{\partial\sigma_{ei}^2} = -\left(\phi\tau^* + \psi\right)\left(\alpha^*\right)^2 = -\frac{\psi}{\psi^*}(\phi^* + \psi^*)(\alpha^*)^2\ .$$

Thus, the private marginal value of information is equal to the social marginal value if, and only if[35], $\psi = \psi^*$. In other words, even if payoffs are distorted by policy to achieve efficiency in the use of information, information choice still remains inefficient. In

[34]This formulation is not the only way to restore efficiency in use of information. The key point, however, is that correcting the inefficiency in information use is not sufficient to get the economy to the information-constrained optimum.

[35]Note that this condition depends on the nature of tax that was introduced. If, for example, the distortion was a tax to the second component of the payoff function, then we need $\phi^* = \phi$ for the optimal tax to ensure efficiency in information acquisition as well, we need $\phi = \phi^*$.

general, in order to restore efficiency along both these margins, we need 2 distinct forms of intervention - one which aligns the relative weights in private and social payoffs and another which corrects the level distortions. Here, we propose one such implementation. In addition to the τ policy discussed earlier, we employ another 'tax', denoted κ, which affects total payoffs. Then, the private payoff is :

$$\Pi_i = \max_{x_i} \; - \kappa \;\; \mathbb{E}_i(\phi\tau \left(x_i - a\right)^2 + \psi \left(x_i - \bar{x}\right)^2) \; .$$

We can then show that the following policy achieves the constrained-efficient allocation.

$$\begin{aligned} \tau &= \tau^* = \frac{\phi^*}{\psi^*}\,\frac{\psi}{\phi}, \\ \kappa &= \frac{\psi^*}{\psi}\; . \end{aligned}$$

2.7.5 Public Signals

While the analysis in this paper has focused on the acquisition of private information, the economic forces leading to inefficiency also affect incentives to learn through public signals. Here, we demonstrate this by extending the beauty contest model to include both public and private signals. The payoff structure is the same as before but agents' information set now also has the following additional signal:

$$S_i = a + \rho_i \, \epsilon \;\; ,$$

where $\epsilon \sim N\left(0, \hat{\sigma}_\epsilon^2\right)$ is a common noise term, while ρ_i reflects the extent to which agent i's signal is affected by that noise term. As $\rho_i \to \infty$, this signal becomes worthless from the perspective of forecasting a. In the other direction, as $\rho_i \to 0$, this becomes an arbitrarily precise signal of the fundamental.

The information cost is now a function of both the public and private information choices, i.e $\upsilon(\sigma_{ei}^2, \rho^2)$. One interpretation is that agent chooses both the precision and the degree of commonality in her information and could face potentially non-separable costs. As in the baseline model, we impose very little structure on this cost function, beyond monotonicity and curvature assumptions needed to ensure interior solutions to the optimization problems of the agent and the planner. The following proposition characterizes the equilibrium and socially optimal response functions. It confirms that efficiency in information use is obtained if the relative weights are the same in private and social payoffs.

Proposition 18 1. *There exists a pair of constants, α_1^{eq} and α_2^{eq} such that actions in a symmetric equilibrium are given by*

$$x_i = \alpha_1^{eq} s_i + \alpha_2^{eq} S_i \ .$$

2. *There exist constants α_1^* and α_2^* such that the symmetric socially optimal response function is*

$$x_i = \alpha_1^* s_i + \alpha_2^* S_i \ .$$

3. *Given a symmetric information structure, i.e. with the same (σ_e^2, ρ^2) for all agents in the economy, the two sets of response coefficients are equal if, and only if, $\frac{\phi}{\psi} = \frac{\phi^*}{\psi^*}$.*

We then turn to the choice of commonality, ρ^2. In general, inefficiencies in information use also drive a wedge between the social and private value of commonality. More interestingly, however, they can differ even when information is used optimally, i.e. $\frac{\phi}{\psi} = \frac{\phi^*}{\psi^*}$. To see this, note that the private marginal value of observing the public signal more precisely is

$$\frac{\partial \Pi}{\partial \rho^2} = -\phi \left(\alpha_2^{eq}\right)^2 \sigma_\epsilon^2 \ .$$

The social value is

$$\begin{aligned} \frac{\partial U}{\partial \rho^2} &= -\phi^* \left(\alpha_2^*\right)^2 \sigma_\epsilon^2 = -\phi^* \left(\alpha_2^{eq}\right)^2 \sigma_\epsilon^2 \ . \\ &= \left(\frac{\phi^*}{\phi}\right) \frac{\partial \Pi}{\partial \rho^2}. \end{aligned}$$

Again, as with private information choice, the *level* of social-versus-private payoffs matter for incentives to invest in information.

2.8 Conclusion

The preceding sections highlight a novel source of inefficiency in a class of business cycle models used widely in modern macroeconomics. *Ex-post* inefficiencies feed back into *ex-ante* incentives to invest in information, even when these inefficiencies leave responses to the information undistorted. This in turn leads to suboptimal levels of learning and through that, *ex-post* equilibrium outcomes that are constrained inefficient, both in terms of average levels and elasticity to fundamental shocks.

There are several directions for future work. With a view to maintaining analytical tractability, we have made several simplifying assumptions. For example, we focus exclusively on static decisions, but the channels we highlight also have implications for intertemporal decisions (e.g. through capital accumulation, pricing with nominal frictions etc.). Similarly, for expositional simplicity, we rule out additional shocks (aggregate or idiosyncratic) and other sources of information. Relaxing some of these assumptions will require the use of numerical methods, but will allow a quantitative evaluation of the inefficiency and the policy interventions necessary to correct it. On the theoretical side, exploring the connections between the payoff-linked inefficiencies in this paper with others identified by the literature (e.g. the inefficiency in Amador and Weill [AW10]) is another interesting direction for future work.

2.9 Proofs of Results

2.9.1 Model I: Quantity choice with productivity shocks

2.9.1.1 Equilibrium

We solve for equilibrium by studying the problem of an individual entrepreneur i, who takes as given the information choices of all other entrepreneurs $j \neq i$ in the economy. In a symmetric equilibrium, she conjectures (correctly) that all other firms follow a log-linear policy rule:

$$n_{jt} = k_2 + \alpha s_{jt} \ .$$

This implies

$$
\begin{aligned}
y_{jt} &= a_t + \frac{n_{jt}}{\delta} = a_t + \frac{k_2}{\delta} + \frac{\alpha}{\delta} s_{jt} = \left(1 + \frac{\alpha}{\delta}\right) a_t + \frac{k_2}{\delta} + \frac{\alpha}{\delta} e_{jt} \ , \\
y_t &= \int y_{jt} \, dj = \left(1 + \frac{\alpha}{\delta}\right) a_t + \frac{k_2}{\delta} + \frac{1}{2}\left(\frac{\theta - 1}{\theta}\right) \frac{\alpha^2}{\delta^2} \sigma_e^2 \ , \\
\left(\frac{\theta - 1}{\theta}\right) a_t + \frac{1}{\theta} y_t &= \left(\frac{\theta - 1}{\theta} + \frac{1}{\theta} + \frac{\alpha}{\theta\delta}\right) a_t + \frac{k_2}{\theta\delta} + \frac{1}{2}(\theta - 1) \frac{\alpha^2}{\theta^2\delta^2} \sigma_e^2 \ .
\end{aligned}
$$

We then guess (and verify) that i 's best response takes the form

$$
n_{it} = \hat{k}_2 + \hat{\alpha} s_{it} \ .
$$

Using this, we write i's objective function as

$$
\begin{aligned}
&\hat{K}_2^{\frac{\theta-1}{\theta\delta}} \exp\left\{\frac{1}{2}(\theta - 1) \frac{\alpha^2}{\theta^2\delta^2} \sigma_e^2\right\} E_{it}\left(A_t^{\frac{\theta-1}{\theta}\left(1+\frac{\hat{\alpha}}{\delta}\right) + \frac{1}{\theta}\left(1+\frac{\alpha}{\delta}\right)} e_{it}^{\hat{\alpha}\frac{\theta-1}{\theta\delta}} K_2^{\frac{1}{\theta\delta}}\right) - \hat{K}_2 E_{it} A_t^{\hat{\alpha}} e_{it}^{\hat{\alpha}} \\
= \ &\exp\left\{\frac{\theta - 1}{\theta\delta} \hat{k}_2 + \frac{1}{\theta\delta} k_2 + \frac{1}{2}(\theta - 1) \frac{\alpha^2}{\theta^2\delta^2} \sigma_e^2\right\} \\
&\exp\left\{\left[\frac{\theta - 1}{\theta}\left(1 + \frac{\hat{\alpha}}{\delta}\right) + \frac{1}{\theta}\left(1 + \frac{\alpha}{\delta}\right)\right] a_t + \hat{\alpha} \frac{\theta - 1}{\theta\delta} e_{it}\right\} \\
&- \exp\left\{\hat{k}_2\right\} \exp\left\{\hat{\alpha} a_t + \hat{\alpha} e_{it}\right\} \ .
\end{aligned}
$$

The unconditional expectation is[36]

$$
\underbrace{\begin{array}{c} \exp\left\{\frac{\theta-1}{\theta\delta}\hat{k}_2 + \frac{1}{\theta\delta}k_2 + \frac{1}{2}(\theta-1)\frac{\alpha^2}{\theta^2\delta^2}\sigma_e^2\right\} \\ \exp\left\{\frac{1}{2}\left[\frac{\theta-1}{\theta}\left(1+\frac{\hat{\alpha}}{\delta}\right) + \frac{1}{\theta}\left(1+\frac{\alpha}{\delta}\right)\right]^2 \sigma_a^2 + \frac{1}{2}\hat{\alpha}^2\left(\frac{\theta-1}{\theta\delta}\right)^2 \sigma_{ei}^2\right\} \end{array}}_{C} \underbrace{- \exp\left\{\hat{k}_2\right\} \exp\left\{\frac{1}{2}\hat{\alpha}^2\left(\sigma_a^2 + \sigma_{ei}^2\right)\right\}}_{N} \tag{2.37}
$$

FOC:

$\hat{k}_2$

$$
\left(\frac{\theta - 1}{\theta\delta}\right) C = N \ . \tag{2.38}
$$

[36]We need to verify that firms have no incentive to change their response coefficients after seeing the signal, i.e. we should check that the response also maximizes conditional expected profits. This is easy to show under log-normality.

$\hat{\alpha}$

$$\begin{aligned}
\left\{\left[\frac{\theta-1}{\theta}\left(1+\frac{\hat{\alpha}}{\delta}\right)+\frac{1}{\theta}\left(1+\frac{\alpha}{\delta}\right)\right]\sigma_a^2\left(\frac{\theta-1}{\theta\delta}\right)+\hat{\alpha}\left(\frac{\theta-1}{\theta\delta}\right)^2\sigma_{ei}^2\right\}C &= \hat{\alpha}\left(\sigma_a^2+\sigma_{ei}^2\right)N \\
\left[\frac{\theta-1}{\theta}\left(1+\frac{\hat{\alpha}}{\delta}\right)+\frac{1}{\theta}\left(1+\frac{\alpha}{\delta}\right)\right]\sigma_a^2+\hat{\alpha}\left(\frac{\theta-1}{\theta\delta}\right)\sigma_{ei}^2 &= \hat{\alpha}\left(\sigma_a^2+\sigma_{ei}^2\right) .
\end{aligned}$$

In a symmetric equilibrium, $\sigma_{ei}^2 = \sigma_e^2$, $\hat{k}_2 = k_2$ and $\hat{\alpha} = \alpha$. These conditions then become

$$\begin{aligned}
\left(\frac{\theta-1}{\theta\delta}\right)\exp\left\{\frac{1}{\delta}k_2+\frac{1}{2}\left(1+\frac{\alpha}{\delta}\right)^2\sigma_a^2+\frac{1}{2}\alpha^2\left(\frac{\theta-1}{\theta\delta}\right)^2\sigma_e^2\right\} &= \exp\left\{k_2+\frac{1}{2}\hat{\alpha}^2\left(\sigma_a^2+\sigma_e^2\right)\right\} , \\
\left(1+\frac{\alpha}{\delta}-\alpha\right)\sigma_a^2 &= \alpha\left(1-\frac{\theta-1}{\theta\delta}\right)\sigma_e^2 .
\end{aligned}$$

Rearranging, we get the expressions in Proposition 19.

The expression for the private value of information on the left hand side of (2.10) is obtained by a direct application of the envelope theorem to (2.37) along with (2.38).

$$\begin{aligned}
\frac{\partial\hat{\Pi}}{\partial\sigma_{ei}^2} &= C\frac{1}{2}\hat{\alpha}^2\left(\frac{\theta-1}{\theta\delta}\right)^2-N\frac{1}{2}\hat{\alpha}^2 \\
&= \frac{1}{2}\hat{\alpha}^2\left[\left(\frac{\theta\delta}{\theta-1}\right)\left(\frac{\theta-1}{\theta\delta}\right)^2-1\right]N \\
&= -\frac{1}{2}\hat{\alpha}^2\left(\frac{1-\theta+\theta\delta}{\theta\delta}\right)N \\
&= -\frac{1}{2}\hat{\alpha}^2\left(\frac{\theta-1}{\theta\delta}\right)\hat{\Pi} , \qquad (2.39)
\end{aligned}$$

where the last step makes use of the fact that, in equilibrium, $\hat{\Pi} = C - N = \left(\frac{1-\theta+\theta\delta}{\theta-1}\right)N$.

2.9.1.2 Efficiency in information choice

Expected utility is given by

$$\begin{aligned}
\mathbb{U} &= \exp\left[\frac{1}{2}\left(1+\frac{\alpha}{\delta}\right)^2\sigma_a^2+\frac{k_2}{\delta}+\frac{1}{2}\left(\frac{\theta-1}{\theta}\right)\frac{\alpha^2}{\delta^2}\sigma_e^2\right]-\exp\left[k_2+\frac{1}{2}\alpha^2\sigma_a^2+\frac{1}{2}\alpha^2\sigma_e^2\right] \\
&= C^*\exp\left(\frac{k_2-k_2^*}{\delta}\right)-N^*\exp\left(k_2-k_2^*\right),
\end{aligned}$$

where k_2^* is the optimal response coefficient and (C^*, N^*) the corresponding unconditional expectations of consumption and labor input. Using (2.13), we get

$$\begin{aligned}\mathbb{U} &= \left[\delta \exp\left(\frac{k_2 - k_2^*}{\delta}\right) - \exp\left(k_2 - k_2^*\right)\right] N^* \\ &= \left[\delta \exp\left(\frac{k_2 - k_2^*}{\delta}\right) - \exp\left(k_2 - k_2^*\right)\right] \frac{\mathbb{U}^*}{\delta - 1},\end{aligned}$$

where $\mathbb{U}^* = C^* - N^*$.Then,

$$\frac{\partial \mathbb{U}}{\partial \sigma_e^2} = \left[\delta \exp\left(\frac{k_2 - k_2^*}{\delta}\right) - \exp\left(k_2 - k_2^*\right)\right] \frac{1}{\delta - 1} \frac{\partial \mathbb{U}^*}{\partial \sigma_e^2}.$$

The term in the square bracket is independent of σ_e^2. The envelope theorem implies,

$$\begin{aligned}\frac{\partial \mathbb{U}^*}{\partial \sigma_e^2} &= C^* \frac{1}{2}\left(\frac{\theta - 1}{\theta}\right)\frac{\alpha^2}{\delta^2} - N^* \frac{1}{2}\alpha^2 \\ &= -\frac{1}{2}\alpha^2\left(1 - \frac{\theta - 1}{\theta\delta}\right) N^*,\end{aligned}$$

Substituting,

$$\begin{aligned}\frac{\partial \mathbb{U}}{\partial \sigma_e^2} &= -\frac{1}{2}\alpha^2\left(\frac{1 - \theta + \theta\delta}{\theta\delta}\right)\left[\delta \exp\left(\frac{k_2 - k_2^*}{\delta}\right) - \exp\left(k_2 - k_2^*\right)\right]\frac{N^*}{\delta - 1} \\ &= -\frac{1}{2}\alpha^2\left(\frac{1 - \theta + \theta\delta}{\theta\delta}\right)\frac{1}{\delta - 1}\mathbb{U} \\ &= -\frac{1}{2}\alpha^2\left[\frac{1 - \theta + \theta\delta}{(\theta - 1)(\delta - 1)}\right]\mathbb{U}\left(\frac{\theta - 1}{\theta\delta}\right) \\ &= -\frac{1}{2}\alpha^2\left[1 + \frac{\delta}{(\theta - 1)(\delta - 1)}\right]\mathbb{U}\left(\frac{\theta - 1}{\theta\delta}\right). \qquad (2.40)\end{aligned}$$

Since $\mathbb{U} = \hat{\Pi}$, we have the result in Proposition 21.

2.9.1.3 Policy

To see that the constant revenue subsidy also aligns private and social values of information, note that the only change in the derivation of the private value of information above is in (2.39). Since the level distortion to output is not present under this subsidy, $\hat{\Pi} = (\delta - 1) N$ instead of $\hat{\Pi} = \left(\frac{1-\theta+\theta\delta}{\theta-1}\right) N$. Then, it is easy to see that the resulting expression for private value is identical to the social value in (2.40).

2.9.2 Model II: Price setting with productivity shocks

2.9.2.1 Equilibrium

As with the quantity choice, we begin with the problem of entrepreneur i, who believes (correctly) that everybody else is acting according to

$$p_{jt} = k_2 + \alpha s_{jt} = k_2 + \alpha a_t + \alpha e_{jt} \ .$$

The corresponding aggregate relationships are

$$\begin{aligned}
p_t &= k_2 + \alpha a_t + \frac{1}{2}(1-\theta)\alpha^2\sigma_e^2 \ , \\
y_t &= m - k_2 - \alpha a_t - \frac{1}{2}(1-\theta)\alpha^2\sigma_e^2 \ , \\
y_{jt} &= -\theta\left(p_{jt} - p_t\right) + y_t = -\alpha\theta e_{jt} + \frac{\theta}{2}(1-\theta)\alpha^2\sigma^2 + m - k_2 - \alpha a_t - \frac{1}{2}(1-\theta)\alpha^2\sigma_e^2 \\
&= -\alpha\theta\varepsilon_{jt} + m - k_2 - \alpha a_t - \frac{1}{2}\alpha^2\sigma_e^2(\theta-1)^2 \ , \\
n_{jt} &= \left(y_{jt} - a_t\right)\delta = -\alpha\theta\delta\varepsilon\backslash e_{jt} + \delta m - \delta k_2 - \delta(\alpha+1)a_t - \frac{1}{2}\delta\alpha^2\sigma_e^2(\theta-1)^2 \ , \\
n_t &= \delta m - \delta k_2 - \delta(\alpha+1)a_t - \frac{1}{2}\delta\alpha^2\sigma_e^2(\theta-1)^2 + \frac{1}{2}\alpha^2\theta^2\delta^2\sigma_e^2 \\
&= \delta m - \delta k_2 - \delta(\alpha+1)a_t + \frac{1}{2}\alpha^2\sigma_e^2\delta\left[\theta^2\delta - (\theta-1)^2\right] \ .
\end{aligned}$$

We then solve for i's optimal policy

$$p_{it} = \hat{k}_2 + \hat{\alpha}s_{it} \ .$$

The objective function can be written as

$$
\begin{aligned}
& \hat{K}_2^{1-\theta} K_2^{\theta-2} \exp\left[\frac{1}{2}(\theta-2)(1-\theta)\alpha^2\sigma_e^2\right] M E_{it}\left[A_t^{\alpha(\theta-2)+\hat{\alpha}(1-\theta)} e_{it}^{\hat{\alpha}(1-\theta)}\right] - \\
& \hat{K}_2^{-\theta\delta} K_2^{\delta(\theta-1)} \exp\left[-\frac{1}{2}\delta(1-\theta)^2\alpha^2\sigma_e^2\right] M^\delta E_{it} A_t^{\alpha\delta(\theta-1)-\delta-\theta\delta\hat{\alpha}} e_{it}^{-\theta\delta\hat{\alpha}} \\
= \; & M \exp\left\{(1-\theta)\hat{k}_2 + (\theta-2)k_2\right\} \exp\left[\frac{1}{2}(\theta-2)(1-\theta)\alpha^2\sigma_e^2\right] \\
& \exp\left\{\left[\alpha(\theta-2) + \hat{\alpha}(1-\theta)\right] a_t + \hat{\alpha}(1-\theta) e_{it}\right\} \\
& -M^\delta \exp\left\{-\theta\delta\hat{k}_2 + \delta(\theta-1)k_2\right\} \exp\left[-\frac{1}{2}\delta(1-\theta)^2\alpha^2\sigma_e^2\right] \\
& \exp\left\{\left[\alpha\delta(\theta-1) - \delta - \theta\delta\hat{\alpha}\right] a_t - \theta\delta\hat{\alpha} e_{it}\right\} \; .
\end{aligned}
$$

The unconditional expectation is

$$
\underbrace{\begin{array}{c} M \exp\left[\frac{1}{2}(\theta-2)(1-\theta)\alpha^2\sigma_e^2\right] \exp\left\{(1-\theta)\hat{k}_2 + (\theta-2)k_2\right\} \\ \exp\left\{\frac{1}{2}\left[\alpha(\theta-2)+\hat{\alpha}(1-\theta)\right]^2 \sigma_a^2 + \frac{1}{2}\hat{\alpha}^2(1-\theta)^2\sigma_{ei}^2\right\} \end{array}}_{C}
$$
$$
- \underbrace{\begin{array}{c} M^\delta \exp\left\{-\theta\delta\hat{k}_2 + \delta(\theta-1)k_2\right\} \exp\left[-\frac{1}{2}\delta(1-\theta)^2\alpha^2\sigma_e^2\right] \\ \exp\left\{\frac{1}{2}\left[\alpha\delta(\theta-1) - \delta - \theta\delta\hat{\alpha}\right]^2 \sigma_a^2 + \frac{1}{2}\theta^2\delta^2\hat{\alpha}^2\sigma_{ei}^2\right\} \end{array}}_{N}
$$

FOC

$\hat{k}_2$:

$$
(\theta-1)C = \theta\delta N \; ,
$$

$\hat{\alpha}$:

$$
C\left\{\left[\alpha(\theta-2)+\hat{\alpha}(1-\theta)\right](1-\theta)\sigma_a^2 + \hat{\alpha}(1-\theta)^2\sigma_{ei}^2\right\} = -N\left[\alpha\delta(\theta-1) - \delta - \theta\delta\hat{\alpha}\right]\theta\delta\sigma_a^2 + \theta^2\delta^2\hat{\alpha}\sigma_{ei}^2 \, .
$$

In a symmetric equilibrium, the FOC for k_2 becomes

$$\begin{aligned}
& \frac{1}{2}(\theta-2)(1-\theta)\alpha^2\sigma_e^2 + m - k_2 + \frac{1}{2}\left[-\alpha\right]^2\sigma_a^2 + \frac{1}{2}\alpha^2(1-\theta)^2\sigma_e^2 \\
= \; & \ln\left(\frac{\theta\delta}{\theta-1}\right) + \delta m - \delta k_2 - \frac{1}{2}\delta(1-\theta)^2\alpha^2\sigma_e^2 + \frac{1}{2}\left[\delta(1+\alpha)\right]^2\sigma_a^2 + \frac{1}{2}\theta^2\delta^2\alpha^2\sigma_e^2 \\
\Longrightarrow (\delta-1)\,k_2 \; = \; & \ln\left(\frac{\theta\delta}{\theta-1}\right) + (\delta-1)\,m + \frac{1}{2}\sigma_a^2\left[\delta^2(1+\alpha)^2 - \alpha^2\right] \\
& + \frac{1}{2}\alpha^2\sigma_e^2\left[\theta^2\delta^2 - \delta(1-\theta)^2 - (\theta-2)(1-\theta) - (1-\theta)^2\right] \\
\Longrightarrow (\delta-1)\,k_2 \; = \; & \ln\left(\frac{\theta\delta}{\theta-1}\right) + (\delta-1)\,m + \frac{1}{2}\alpha^2\sigma_e^2\left[\theta^2\delta^2 - \delta(1-\theta)^2 + 1 - \theta\right] + \frac{1}{2}\sigma_a^2\left[\delta^2(1+\alpha)^2 - \alpha^2\right]
\end{aligned}$$

The FOC for $\hat{\alpha}$ simplifies to,

$$\begin{aligned}
\left(\frac{\theta\delta}{\theta-1}\right)\left\{\left[\alpha(\theta-2) + \alpha(1-\theta)\right](1-\theta)\sigma_a^2 + \alpha(1-\theta)^2\sigma_e^2\right\} &= \left[-\alpha\delta(\theta-1) + \delta + \theta\delta\alpha\right]\theta\delta\sigma_a^2 + \theta^2\delta^2\alpha\sigma_e^2 \\
\left(\frac{\theta\delta}{\theta-1}\right)\left\{\alpha(\theta-1)\sigma_a^2 + \alpha(\theta-1)^2\sigma_e^2\right\} &= \left[\delta\left(\alpha+1\right)\right]\theta\delta\sigma_a^2 + \theta^2\delta^2\alpha\sigma_e^2 \\
\left(\frac{\theta\delta}{\theta-1}\right)\left\{\alpha\sigma_a^2 + \alpha(\theta-1)\sigma_e^2\right\} &= \left(\frac{\theta\delta}{\theta-1}\right)\left\{\left[\delta\left(\alpha+1\right)\right]\sigma_a^2 + \alpha\sigma_e^2\theta\delta\right\} \\
\alpha\sigma_a^2 + \alpha(\theta-1)\sigma_e^2 &= \left[\alpha\delta + \delta\right]\sigma_a^2 + \theta\delta\alpha\sigma_e^2 \\
-\delta\sigma_a^2 &= \alpha\left[(\delta-1)\,\sigma_a^2 + (1-\theta+\theta\delta)\sigma_e^2\right] \; ,
\end{aligned}$$

Rearranging yields the expressions in Proposition 5.

The private value of information is a direct application of the envelope theorem:

$$\begin{aligned}
\frac{\partial\hat{\Pi}}{\partial\sigma_{ei}^2} &= \frac{1}{2}\hat{\alpha}^2(1-\theta)^2 C - \frac{1}{2}\theta^2\delta^2\hat{\alpha}^2 N \\
&= -\frac{1}{2}\hat{\alpha}^2\left\{-(1-\theta)^2\left(\frac{\theta\delta}{\theta-1}\right) + \theta^2\delta^2\right\} N \\
&= -\frac{1}{2}\hat{\alpha}^2\theta\delta(1-\theta+\theta\delta)N \\
&= -\frac{1}{2}\hat{\alpha}^2\theta\delta\,(\theta-1)\,\hat{\Pi} \; .
\end{aligned}$$

2.9.2.2 Efficiency in information use

The planner's problem to pick the optimal response coefficients:

$$\max_{\alpha,k_2} \quad \exp\left[m - k_2 + \frac{1}{2}\alpha^2\sigma_a^2 - \frac{1}{2}(1-\theta)\alpha^2\sigma_e^2\right] - \exp\left[\delta m - \delta k_2 + \delta^2(\alpha+1)^2\frac{1}{2}\sigma_a^2 + \frac{1}{2}\alpha^2\sigma_e^2\delta\left[\theta^2\delta - (\theta-1\right.\right.$$

The optimality conditions:

$$\begin{aligned}
\underbrace{\exp\left[m - k_2 + \frac{1}{2}\alpha^2\sigma_a^2 - \frac{1}{2}(1-\theta)\alpha^2\sigma_e^2\right]}_{C^*} &= \delta \quad \underbrace{\exp\left[\delta m - \delta k_2 + \delta^2(\alpha+1)^2\frac{1}{2}\sigma_a^2 + \frac{1}{2}\alpha^2\sigma_e^2\delta\left[\theta^2\delta - (\theta - \right.\right.}_{N^*} \\
\alpha\sigma_a^2 + \alpha(\theta-1)\sigma_e^2 &= \delta(\alpha+1)\sigma_a^2 + \alpha\sigma_e^2\left[\theta^2\delta - (\theta-1)^2\right] \\
&= \delta(\alpha+1)\sigma_a^2 + \alpha\sigma_e^2\left[\theta(2-\theta+\theta\delta) - 1\right] \\
&= \delta\alpha\sigma_a^2 + \alpha\sigma_e^2\left[\theta(1-\theta+\theta\delta) + \theta - 1\right] + \delta\sigma_a^2 \\
\alpha\sigma_a^2 &= \delta\alpha\sigma_a^2 + \alpha\sigma_e^2\left[\theta(1-\theta+\theta\delta)\right] + \delta\sigma_a^2 \;.
\end{aligned}$$

Solving, we get the coefficients in Proposition 6.

2.9.2.3 Efficiency in information choice

Expected utility is given by

$$\mathbb{U} = \exp\left[m - k_2 + \frac{1}{2}\alpha^2\sigma_a^2 - \frac{1}{2}(1-\theta)\alpha^2\sigma_e^2\right] - \exp\left[\delta m - \delta k_2 + \delta^2(\alpha+1)^2\frac{1}{2}\sigma_a^2 + \frac{1}{2}\alpha^2\sigma_e^2\delta\left[\theta^2\delta - (\theta-1)^2\right]\right.$$

Then, after some algebra,

$$\frac{\partial\mathbb{U}}{\partial\sigma^2} = -\mathbb{U}\frac{\theta\delta\left(1-\theta+\theta\delta\right)}{(\delta-1)}\left[\frac{\alpha^2}{2} + \left(\frac{\theta-1}{\theta}\right)\sigma^2\alpha\frac{d\alpha}{d\sigma_e^2}\right] \;.$$

Replacing $d\alpha/d\sigma_e^2$ and rearranging,

$$\frac{\partial\mathbb{U}}{\partial\sigma_e^2} = -\frac{\delta\left(1-\theta+\theta\delta\right)}{\delta-1}\left[\frac{\theta\left(\delta-1\right)\sigma_a^2 - \left(\theta-2\right)\left(1-\theta+\theta\delta\right)\sigma_e^2}{\left(\delta-1\right)\sigma_a^2 + \left(1-\theta+\theta\delta\right)\sigma_e^2}\right]\alpha^2\mathbb{U} \;.$$

Given condition (1), it follows that

$$\frac{\partial \mathbb{U}}{\partial \sigma_e^2} \leq 0 \ .$$

Suppose $\theta > 2$ and the conditions of Assumption 1 are met. Then, there is over-acquisition of information if and only if:

$$\sigma_e^2 > \frac{\theta\left(\delta-1\right)}{\left(1-\theta+\theta\delta\right)}\left[\frac{\left(1-\theta+\theta\delta\right)+\delta}{\theta\left(\theta-1\right)\left(\delta-1\right)+2\left(1-\theta+\theta\delta\right)\left(\theta-2\right)}\right]\sigma_a^2$$

We have the result in Proposition 7.

2.9.2.4 Policy

Given a revenue subsidy of the form

$$\Lambda \ A_t^{\delta\tau},$$

the unconditional expectation now becomes

$$\underbrace{\begin{matrix} M\Lambda \exp\left[\frac{1}{2}(\theta-2)(1-\theta)\alpha^2\sigma_e^2\right]\exp\left\{(1-\theta)\hat{k}_2+(\theta-2)k_2\right\} \\ \exp\left\{\frac{1}{2}\left[\alpha(\theta-2)+\hat{\alpha}(1-\theta)\right]^2\sigma_a^2+\frac{1}{2}\hat{\alpha}^2(1-\theta)^2\sigma_{ei}^2\right\}\end{matrix}}_{C} - \underbrace{\begin{matrix} M^\delta \exp\left\{-\theta\delta\hat{k}_2+\delta(\theta-1)k_2\right\}\exp\left[-\frac{1}{2}\delta(1-\theta)^2\alpha^2\sigma_e^2\right] \\ \exp\left\{\frac{1}{2}\left[\alpha\delta(\theta-1)-\delta-\theta\delta\hat{\alpha}\right]^2\sigma_a^2+\frac{1}{2}\theta^2\delta^2\hat{\alpha}^2\sigma_{ei}^2\right\}\end{matrix}}_{N} \ .$$

The FOC for $\hat{k}_2$:

$$\left(\theta-1\right)C = \theta\delta N \ ,$$

and for α :

$$\left\{\left[\alpha(\theta-2)+\hat{\alpha}(1-\theta)+\delta\tau\right](1-\theta)\sigma_a^2+\hat{\alpha}(1-\theta)^2\sigma_{ei}^2\right\}C = \left\{-\left[\alpha\delta(\theta-1)-\delta-\theta\delta\hat{\alpha}\right]\theta\delta\sigma_a^2+\theta^2\delta^2\hat{\alpha}\sigma_{ei}^2\right\} N$$

Using the FOC for $\hat{k}_2$ and invoking symmetry

$$\begin{aligned}
\left(\frac{\theta\delta}{\theta-1}\right)\left\{\left[-\alpha+\delta\tau\right](1-\theta)\sigma_a^2+\alpha(1-\theta)^2\sigma_e^2\right\} &= \left\{\delta(1+\alpha)\theta\delta\sigma_a^2+\theta^2\delta^2\hat{\alpha}\sigma_e^2\right\} \\
\left[\alpha-\delta\tau\right]\sigma_a^2+\alpha(1-\theta)\sigma_e^2 &= \delta(1+\alpha)\sigma_a^2+\theta\delta\hat{\alpha}\sigma_e^2 \\
-\delta\sigma_a^2(1+\tau) &= \alpha\left[(\delta-1)\,\sigma_a^2+(1-\theta+\theta\delta)\sigma_e^2\right] \\
\alpha &= \frac{-\delta\sigma_a^2(1+\tau)}{\left[(\delta-1)\,\sigma_a^2+(1-\theta+\theta\delta)\sigma_e^2\right]} \\
\alpha &= (1+\tau)\,\alpha^{eq}\ . \qquad (2.41)
\end{aligned}$$

Invoking symmetry in the FOC for k_2

$$\begin{aligned}
& M\Lambda\exp\left[\frac{1}{2}(\theta-2)(1-\theta)\alpha^2\sigma_e^2\right]\exp\left\{-k_2\right\}\exp\left\{\frac{1}{2}\left[-\alpha+\delta\tau\right]^2\sigma_a^2+\frac{1}{2}\alpha^2(1-\theta)^2\sigma_e^2\right\} \\
= & \left(\frac{\theta\delta}{\theta-1}\right)M^{\delta}\exp\left\{-\delta k_2\right\}\exp\left[-\frac{1}{2}\delta(1-\theta)^2\alpha^2\sigma_e^2\right]\exp\left\{\frac{1}{2}\left[-\delta(1+\alpha)\right]^2\sigma_a^2+\frac{1}{2}\theta^2\delta^2\alpha^2\sigma_e^2\right\}
\end{aligned}$$

In logs,

$$\begin{aligned}
& m+\lambda+\frac{1}{2}(\theta-2)(1-\theta)\alpha^2\sigma_e^2-k_2+\frac{1}{2}\left[-\alpha+\delta\tau\right]^2\sigma_a^2+\frac{1}{2}\alpha^2(1-\theta)^2\sigma_e^2 \\
&= \ln\left(\frac{\theta\delta}{\theta-1}\right)+\delta m-\delta k_2-\frac{1}{2}\delta(1-\theta)^2\alpha^2\sigma_e^2+\frac{1}{2}\left[\delta(1+\alpha)\right]^2\sigma_a^2+\frac{1}{2}\theta^2\delta^2\alpha^2\sigma_e^2 \\
(\delta-1)\,k_2 &= \ln\left(\frac{\theta\delta}{\theta-1}\right)+(\delta-1)\,m+\frac{1}{2}\alpha^2\sigma_e^2\left[\theta^2\delta^2-\delta(1-\theta)^2-(\theta-2)(1-\theta)-(1-\theta)^2\right] \\
&\quad +\frac{1}{2}\sigma_a^2\left[\delta^2(1+\alpha)^2-(-\alpha+\delta\tau)^2\right]-\lambda \\
(\delta-1)\,k_2 &= \ln\left(\frac{\theta\delta}{\theta-1}\right)+(\delta-1)\,m+\frac{1}{2}\alpha^2\sigma_e^2\left[\theta^2\delta^2-\delta(1-\theta)^2+1-\theta\right] \\
&\quad +\frac{1}{2}\sigma_a^2\left[\delta^2(1+\alpha)^2-(-\alpha+\delta\tau)^2\right]-\lambda\ . \qquad (2.42)
\end{aligned}$$

Any response function (α, k_2) can be implemented by setting the policy parameters (τ, λ) to satisfy (2.41) and (2.42). In particular, to implement the socially optimal (α^*, k_2^*), we need

$$
\begin{aligned}
\tau^* &= \frac{\alpha^*}{\alpha^{eq}} - 1 \ , \\
\lambda^* &= \ln\left(\frac{\theta\delta}{\theta - 1}\right) + (\delta - 1)\, m + \frac{1}{2}\alpha^{*2}\sigma^2 \left[\theta^2\delta^2 - \delta(1-\theta)^2 + 1 - \theta\right] + \\
&\quad \frac{1}{2}\sigma_a^2 \left[\delta^2(1+\alpha^*)^2 - (-\alpha^* + \delta\tau^*)^2\right] - (\delta - 1)\, k_2^* \\
&= \ln\left(\frac{\theta}{\theta - 1}\right) + \frac{\sigma_a^2 \delta\tau^* \left(2\alpha^* - \delta\tau^*\right)}{2} \ .
\end{aligned}
$$

which proves Proposition 8.

To see that this also aligns private and social values, first note that the policy implements the socially optimal response by construction, so we can directly apply the envelope theorem to get

$$\frac{d\mathbb{U}}{d\sigma_e^2} = -\frac{1}{2}\alpha^2\theta\delta(1-\theta+\theta\delta)EN^* \ .$$

Now, recall that, in equilibrium, private value is

$$\frac{d\hat{\Pi}}{d\sigma_{ei}^2} = -\frac{1}{2}\hat{\alpha}^2\theta\delta(1-\theta+\theta\delta)EN \ .$$

Since $EN = EN^*$with efficient response functions,

$$\frac{d\hat{\Pi}}{d\sigma_{ei}^2} = \frac{d\mathbb{U}}{d\sigma_e^2}$$

establishing the result in Proposition 9.

2.9.3 Model III: Price setting with nominal shocks

The proofs for the results in this section are almost identical to those of section 2.5, so in the interest of brevity, we omit them.

2.9.4 A Beauty Contest Model

Proof of Proposition 15 Follows directly by setting $\hat{\alpha} = \alpha$ in (2.29) and solving.

Proof of Proposition 16 We solve (2.33) for α.

Proof of Proposition 17 Follows from the comparison of the expressions for α and α^*.

Proof of Proposition 18 We start with a conjecture about the average action,

$$\bar{x} = \alpha_1 a + \alpha_2 S \, .$$

Then, the optimality condition of the agent implies

$$x_i = \left(\frac{\phi}{\phi + \psi} + \frac{\psi}{\phi + \psi} \alpha_1 \right) \mathbb{E}_i(a) + \frac{\psi}{\phi + \psi} \alpha_2 S \ .$$

Integrating over i,

$$x_i = \left(\frac{\phi}{\phi + \psi} + \frac{\psi}{\phi + \psi} \alpha_1 \right) \bar{\mathbb{E}}(a) + \frac{\psi}{\phi + \psi} \alpha_2 S \ .$$

Next, note that we can write

$$\mathbb{E}_i(a) = \delta_1 s_i + \delta_2 S \ ,$$

where $\delta_1 = \frac{\frac{1}{\sigma_e^2}}{\frac{1}{\sigma_e^2} + \frac{1}{\rho^2 \sigma_\epsilon^2} + \frac{1}{\sigma_\theta^2}}$ and $\delta_2 = \frac{\frac{1}{\rho^2 \sigma_\epsilon^2}}{\frac{1}{\sigma_e^2} + \frac{1}{\rho^2 \sigma_\epsilon^2} + \frac{1}{\sigma_\theta^2}}$. This implies that the cross-sectional average expectation $\bar{\mathbb{E}}(a) = \delta_1 a + \delta_2 S$. Substituting in the expression for $\bar{a}$ yields a system of linear equations. The solution is

$$\alpha_1^{eq} = \frac{\phi \delta_1}{\phi + \psi(1 - \delta_1)} \qquad \alpha_2^{eq} = \frac{\phi + \psi}{\phi} \frac{\delta_2}{\delta_1} \alpha_1^{eq} \ .$$

The planner's optimality conditions for α_1^* and α_2^* are

$$\phi^*(\alpha_1^* + \alpha_2^* - 1)\sigma_a^2 + (\phi^* + \psi^*)\alpha_1 \sigma_e^2 = 0 \ ,$$
$$\phi^*(\alpha_1^* + \alpha_2^* - 1)\sigma_a^2 + \phi_2^{*\alpha*} \rho^2 \sigma_\epsilon^2 = 0 \ .$$

Solving yields

$$\alpha_1^* = \frac{\phi^* \delta_1}{\phi^* + \psi^*(1 - \delta_1)} \ , \qquad \alpha_2^* = \frac{\phi^* + \psi^*}{\phi^*} \frac{\delta_2}{\delta_1} \alpha_1^* \ .$$

Comparing the two sets of coefficients yields the last part of the proposition.

CHAPTER 3

Inefficiencies in Business Cycle Models under Endogenous Information

3.1 Introduction

In this paper we demonstrate the conditions under which information acquisition is inefficient in business cycle models. In particular, we consider 2 benchmark environments, studied extensively by the literature on dispersed information referred to earlier in the second chapter. The first environment is a micro-founded general equilibrium real business cycle RBC model. The second environment is a general equilibrium model of a monetary economy under price setting. A feature that these two environments have in common is that agents care not only about fundamentals but also about the actions taken by other agents. These payoff linkages are a source of strategic complementarity/substitutability in agents choices.

We start with a micro-founded general equilibrium RBC model, where agents on informationally-separate 'islands' choose to acquire information about aggregate technology shocks before participating in local labor markets. The environment features a rich set of payoff linkages, arising through general equilibrium interactions. The information structure here not only affects the response of the economy to shocks, but also has implications for the average *level* of activity. Our analytical characterization of equilibrium allows us to characterize these implications quite sharply. We show that agents respond to information in a socially optimal fashion[1], but the equilibrium features a wedge between the social and private value of additional information, causing agents to invest an inefficiently low amount in information acquisition. This wedge can be traced to market power, arising from imperfect substitutability of goods produced in the economy. Effi-

[1]See Angeletos and La'O [AL09] for the same finding in a very similar environment.

ciency is obtained only in a limiting case where all goods are perfect substitutes or the market power distortion is corrected by appropriate policy. In other words, policies which offset the effect of monopoly pricing have the additional benefit of aligning private and social incentives to acquire information, a novel effect which arises only with endogenous information choice.

Our second environment studies the information acquisition problem in a general equilibrium model of a monetary economy, where firms post nominal prices under imperfect information about shocks to aggregate nominal demand. In such an environment, but with exogenous information, Hellwig [Hel05] demonstrates the inefficiency of equilibrium responses to private signals. As in the beauty contest model, this inefficiency also has an effect on ex-ante incentives to acquire information, but the payoff linkages also have an independent effect on the value of information, as evidenced by the fact that information choice is inefficient even when firms set prices according to the socially optimal response function. The net effect of these forces on information choice is ambiguous in sign, but we characterize the region of the parameter space where the equilibrium features overinvestment in information.

Our analytical framework also allows us to explore other interesting questions related to information choice. We use the price-setting application to demonstrate two such extensions. First, we examine the optimal information choice under the assumption that firms are able to coordinate their ex-ante investments in information. In our environment, this leads to a striking result - the collusive optimum features no learning ! This occurs because, in equilibrium, information acquisition is subject to a negative externality - an individual firm's expected profits decline when all other firms in the economy become more informed. This exactly offsets the benefits to those firms and so, when this effect is internalized, information has no value and therefore, will not be acquired at all. Next, we explore the role of strategic considerations in the information acquisition decision. In particular, we characterize how an individual firm's incentives to acquire its own information are affected by the amount of information acquired by other firms. We find that, in the empirically plausible regions of the parameter space, information acquisition is a strategic complement i.e. the better information the overall economy, the greater is the incentive for a firm to become better informed.

The rest of the paper is organized as follows. Section 3.2 embeds information acquisition in a general equilibrium real business cycle model with productivity shocks. Section 3.3 presents the second application - a nominal price-setting model with monetary shocks. Proofs are collected in the Appendix.

3.2 A Real Business Cycle Model

In this section, we lay out our first application - a micro-founded business cycle model with dispersed information about aggregate productivity shocks. The setup closely follows that of Angeletos and La'O [AL09]. On informationally-separate islands, firms and households trade labor services, the only input in a decreasing returns to scale technology. Importantly, the labor market operates under imperfect information about the productivity shock. As in the previous section, the information structure is endogenous and in equilibrium, reflects private incentives to learn. We assume that each firm is specialized in the production of an intermediate input which is imperfectly substitutable with other inputs in the production of the final good. Firms act in a monopolistically competitive fashion. Our assumptions on preferences and technologies are fairly standard, but we make a few simplifying assumptions (e.g. no capital) in the interest of analytical tractability.

The nature of general equilibrium linkages between firms and households implies that the extent of information available has two kinds of effects on economic activity. The first exerts its influence on the average level of economic activity. The second acts through the sensitivity of economic activity to the realization of the aggregate productivity shock. As we will see, in this economy, only the first channel is a source of inefficiency[2].

Our main result is that the equilibrium in this economy does not attain the information-constrained optimum. In particular, market power arising from imperfect substitutability leads to a wedge between the private value of information and its the social value. As a result, firms in equilibrium will acquire a suboptimally low level of information. Importantly, this efficiency arises despite the fact that information is used efficiently ex-post, i.e. the response of firm choices to the realizations of the signals is the socially optimal one.

[2]The price-setting economy with monetary shocks in Section 3.3, on the other hand, features inefficiencies in both.

Only in a limiting case, as goods become perfect substitutes and market power disappears, does the equilibrium achieve efficiency in information acquisition. Efficiency is also restored when market power is offset by appropriate policies, which in this environment takes the form a constant revenue subsidy.

3.2.1 Preferences, Technology and Information

Time is discrete, $t = 0, 1, 2....$ The economy has a single representative household, which consists of 4 types of agents - a single consumer, a continuum of entrepreneurs, a continuum of workers and a final good producer. The entrepreneurs each have access to a technology, which transforms labor into a differentiated intermediate good according to an identical decreasing returns to scale production function. These technologies, or firms as we will refer to them in our exposition, are located on a continuum of islands, with one firm per island. Every period, the household sends one of its workers to each island. The firm and the worker on an island trade labor services after observing all the available information on that island. Then, production takes place and the firms sell their output in a monopolistic competitive fashion to the final good producer, pays its workers and pays dividends. The only source of uncertainty in the model is an aggregate technology shock, which affects the productivity of all the firms in the economy.

At the beginning of each period, every entrepreneur decides how much information (about the realization of the aggregate shock) to acquire. This information takes the form of the precision of a signal, that is made available on her island. Importantly, we assume that it also becomes available to the worker on that island before the labor market opens[3]. In other words, wages and labor input on each island are determined under imperfect information about aggregate conditions. After the labor market shuts down, the aggregate shock becomes commonly known, production takes place. The worker and entrepreneur return to the household with their respective shares of output one each and deliver them to the consumer. Figure 3.1 shows the timing of events in each period.

[3] We adopt this assumption for simplicity. It is not difficult to extend our results to an environment where the local labor market plays an informational role, e.g. where workers get an independent signal about aggregate conditions. An alternative interpretation of the current setup is that of a continuum of worker-entrepreneurs operating on informationally-separate islands. Under this interpretation, we can eliminate the need for local labor markets altogether - each entrepreneur maximizes her contribution to expected utility.

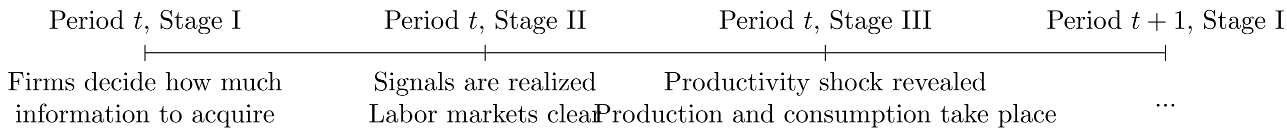

Figure 3.1: Timeline of Events

We now make explicit assumptions about preferences and technologies in this economy.

The Household: The lifetime utility of the household is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \int_0^1 N_{it} di - \int_0^1 \upsilon(\sigma_i^2) di \right) \qquad 0 < \gamma < \infty,$$

where C_t is denoted consumption, N_{it} is the labor input on island i, σ_i^2 is the variance of the noise term in the island-specific signal (to be described later) and the last term is the entrepreneur's cost of information. Parameter β is the discount factor and γ represents the degree of risk aversion of households.

Households maximize utility subject to a budget constraint[4]:

$$C_t \leq \int_0^1 W_{it} N_{it} di + \int_0^1 \Pi_{it} di \ ,$$

where W_{it} denotes island-specific wages. In addition to labor income, the household receives the sum of all profits from intermediate producers, denoted by $\int \Pi_{it} di$,.

Final good producer: The single final good is produced using a continuum $[0, 1]$ of intermediate inputs Y_{it}. The production function is a Dixit-Stiglitz aggregator with constant returns to scale. The competitive firm producing the final good solves the following static problem:

$$\max \ Y_t - \int_0^1 P_{it} Y_{it} di \ ,$$

$$Y_t = \left(\int_0^1 Y_{it}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \ ,$$

where P_{it} is the price of intermediate good i.

[4]The households also have access to markets in Arrow-Debreu securities. Crucially, these markets are assumed to operate only in the last stage, i.e. are unavailable to firms and workers on the islands. Since we work with a representative household, we keep the exposition simple by omitting the relevant terms from the budget constraint.

Note that intermediate inputs are imperfectly substitutable in production. Imperfect substitutability disappears as θ goes to infinity. Parameter θ also indexes the strength of aggregate demand externalities (or the sensitivity of optimal firm profits to aggregate output), see Angeletos and Pavan [AP07]. Throughout the analysis we assume that $\theta > 1$.

Intermediate producers: There is a continuum of intermediate good producers indexed $i \in [0, 1]$. The production function is a standard decreasing returns to scale with labor as the sole input.

$$Y_{it} = A_t N_{it}^{\frac{1}{\delta}} \ ,$$

where $\delta > 1$ and A_t is the aggregate productivity which is assumed to be log-normal, i.e. $\log A_t \equiv a_t \sim N(0, \sigma_a^2)$. For expositional simplicity, we focus on the case where this is an i.i.d shock, but our results go through for more general stochastic processes as well[5]. This is the only source of fundamental uncertainty in the model.

Information structure: Before labor markets open, the firm and worker on each island see a signal s_{it} about the current productivity shock:

$$s_{it} = a_t + e_{it} \ ,$$

where $e_{it} \sim N(0, \hat{\sigma}^2)$ and $\hat{\sigma}_e^2$ is the variance chosen in stage I by the firm.

Labor markets: Firms and workers on an island take the island-specific wage as given and choose labor demand and supply to maximize expected profits and utility respectively. Formally, a firm chooses labor to maximize the expected value of profits:

$$\Pi_{it} = \max_{N_{it}} \ \mathbb{E}_{it} Q_t \left(P_{it} Y_{it} - W_{it} N_{it} \right) \ ,$$

where Q_t is the household's stochastic discount factor (defined below). The operator $\mathbb{E}_{it}(\cdot)$ represent the expectation conditional on firm i's information $\mathcal{I}_{it}$, i.e. $\mathbb{E}_{it}(\cdot) \equiv \mathbb{E}_t(\cdot \,|\mathcal{I}_{it})$. Under monopolistic competition, firms take into account the effect of their labor choice on their price P_{it} (defined below).

[5]For example, if a_t is an $AR(1)$ process, our results go through exactly with the aggregate shock now interpreted as the current innovation to the aggregate productivity level.

Similarly, the worker on island i solves

$$\max_{N_{it}} \qquad \mathbb{E}_{it} Q_t W_{it} N_{it} - N_{it} \ .$$

Information acquisition: In the first stage of each period, firms choose the amount of information, taking as given information choices of other firms. Expected profits prior to the realization of the signal and the aggregate state is defined by:

$$\hat{\Pi}_{it}\left(\hat{\sigma}_e^2\right) \equiv \mathbb{E}_{t-1}\Pi_{it} \ ,$$

where $\mathbb{E}_{t-1}$ is the expectation conditional on information available at the time of the first stage decision i.e. the (commonly known) history until $t-1$.

The problem of the firm in the first stage can then be written as:

$$\max_{\hat{\sigma}_e^2} \quad \hat{\Pi}_{it}\left(\hat{\sigma}_e^2\right) - \upsilon\left(\hat{\sigma}_e^2\right) \ ,$$

where $\upsilon(\cdot)$ is the cost of information as a function of the noise in the signal. We will assume that $\upsilon'(\cdot) < 0, \ \upsilon''(\cdot) > 0$. As before, we will assume that $\upsilon(\cdot)$ is such that an interior solution is obtained. We discuss these assumptions in the equilibrium characterization.

3.2.2 Optimality

We solve the model backwards starting from the last stage.

Stage III: Complete information: In the last stage of each period, there is perfect information of the aggregate state. Optimization by households and the representative final good producer, combined with market clearing, implies the following set of equilibrium conditions:

$$Y_t = \left(\int_0^1 Y_{it}^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}} , \tag{3.1}$$

$$Y_{it} = A_t N_{it}^{\frac{1}{\delta}} , \tag{3.2}$$

$$P_{it} = Y_t^{\frac{1}{\theta}} Y_{it}^{-\frac{1}{\theta}} , \tag{3.3}$$

$$Q_t = C_t^{-\gamma} , \tag{3.4}$$

$$C_t = Y_t . \tag{3.5}$$

Stage II: Labor markets: The first order condition of the firm and worker take the form,

$$\mathbb{E}_{it} Q_t \left[\left(\frac{\theta-1}{\delta\theta}\right) Y_t^{\frac{1}{\theta}} A_t^{\frac{\theta-1}{\theta}} N_{it}^{\frac{\theta-1-\theta\delta}{\delta\theta}} - W_{it}\right] = 0 \tag{3.6}$$

$$\mathbb{E}_{it} Q_t W_{it} = 1 \tag{3.7}$$

Substituting for W_{it} from the worker's optimality condition and for Q_t from stage III, we derive the following expression for labor input on island i.[6]

$$N_{it}^{\frac{1+\theta\delta-\theta}{\delta\theta}} = \frac{\theta-1}{\delta\theta}\left(\mathbb{E}_{it} Y_t^{\frac{1}{\theta}-\gamma} A_t^{\frac{\theta-1}{\theta}}\right) \tag{3.8}$$

Stage I: Information acquisition Under the assumption of an interior solution, the firm's optimal information choice is characterized by the following optimality condition:

$$\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2} - \frac{\partial v}{\partial \hat{\sigma}_e^2} = 0 ,$$

[6] In our framework, market power and strategic linkages due to imperfect substitutability are controlled by the same parameter θ. One can easily extend this framework to parameterize these two forces separately. For example, Angeletos and La'O [AL09] work with an environment where different islands produce differentiated goods from a continuum from differentiated inputs. Imperfect substitutability in the former (i.e. the final goods produced on the islands) is a source of aggregate demand linkages, while the latter gives the intermediate producers on each island market power. Importantly, our efficiency results extend naturally to that environment as well.

3.2.3 Equilibrium

A stationary equilibrium is (i) a set of information choices for each firm (ii) island-specific wages and labor input as functions of the signal on the island (iii) aggregate consumption and output as functions of the aggregate state such that: (a) the labor input is optimal for the worker and the firm, given island-specific information and wages and the functions in (iii) above, (b) taking the behavior of aggregates in (iii) as given, the information choice in (i) solves the Stage I problem, (c) markets clear and (d) the functions in (iii) are correct, i.e. consistent with choices of firms and workers.

We focus on symmetric stationary equilibria, where all firms acquire the same amount of information in stage I and follow the same labor hiring strategies in stage II. The characterization of the equilibrium in stage II essentially follows the same procedure as in Angeletos and La'O [AL09]. We begin with a conjecture about the aggregate labor input:

$$N_t \equiv \int_0^1 N_{it} di = A_t^{\alpha} K_2 \ ,$$

or, in logs[7]

$$n_t = k_2 + \alpha a_t \ , \tag{3.9}$$

where α and k_2 are constants to be determined in equilibrium. The former determines the sensitivity of aggregate labor to productivity shocks whereas the latter affects the level of aggregate labor input (and therefore, of economic activity). Both these coefficients will play an important role in our analysis. In a symmetric equilibrium, we can show that (3.9) implies the following about aggregate output:

$$y_t = \frac{1}{\delta} k_2 - \frac{1}{2} \left(\frac{1 + \theta\delta - \theta}{\theta\delta} \right) \frac{\alpha^2}{\delta} \sigma_e^2 + \left(\frac{\delta + \alpha}{\delta} \right) a_t \ . \tag{3.10}$$

Recall, from (3.8), that the labor input on island i is characterized by:

$$n_{it} = \frac{\theta\delta}{1 + \theta\delta - \theta} \log \left(\frac{\theta - 1}{\delta\theta} \right) + \frac{\theta\delta}{1 + \theta\delta - \theta} \log \left[\mathbb{E}_{it} \left(Y_t^{\frac{1}{\theta} - \gamma} A_t^{\frac{\theta - 1}{\theta}} \right) \right] \ .$$

We substitute for y_t using (3.10) and, under the assumption that aggregate variables are

[7] Hereafter, variables in small cases denote variables in logs, i.e. $x \equiv \log(X)$

conditional log-normally distributed[8], derive

$$n_{it} = \frac{\theta\delta}{1+\theta\delta-\theta}\log\left(\frac{\theta-1}{\delta\theta}\right) + \frac{\theta\delta}{1+\theta\delta-\theta}\left(\frac{1}{\theta}-\gamma\right)\left(\frac{1}{\delta}k_2 - \frac{1}{2}\left(\frac{1+\theta\delta-\theta}{\theta\delta}\right)\frac{\alpha^2}{\delta}\sigma_e^2\right) \tag{3.11}$$

$$+ \phi_1\mathbb{E}_{it}(a_t) + \phi_2\mathbb{V}_{it}\ ,$$

where:

$$\begin{aligned}
\phi_1 &\equiv \frac{\theta\delta}{1+\theta\delta-\theta}\left[\left(\frac{\delta+\alpha}{\delta}\right)\left(\frac{1}{\theta}-\gamma\right)+\frac{\theta-1}{\theta}\right]\ ,\\
\phi_2 &\equiv \frac{\theta\delta}{1+\theta\delta-\theta}\frac{\left[\left(\frac{\delta+\alpha}{\delta}\right)\left(\frac{1}{\theta}-\gamma\right)+\frac{\theta-1}{\theta}\right]^2}{2} > 0\ ,
\end{aligned}$$

and $\mathbb{E}_{it}$ and $\mathbb{V}_{it}$ are the mean and variance of the distribution of a_t, conditional on the information in island i. Using standard results for Bayesian updating, these are given by:

$$\begin{aligned}
\mathbb{E}_{it}\left(a_t\right) &= \frac{\sigma_a^2}{\sigma_a^2+\hat{\sigma}_e^2}s_{it}\ ,\\
\mathbb{V}_{it} &= \frac{\sigma_a^2\hat{\sigma}_e^2}{\sigma_a^2+\hat{\sigma}_e^2}\ ,
\end{aligned}$$

where $\hat{\sigma}_e^2$ is the variance of the error term in the firm's signal.

Plugging the optimal labor into the firm's profit function, we get the following expression for maximized profit

$$\Pi_{it} = K_1 K_2^{\frac{1-\theta\gamma}{1+\theta\delta-\theta}}\exp\left(\phi_1\mathbb{E}_{it}\left(a_t\right)+\phi_2\mathbb{V}_{it}\left(a_t\right)-\left(\frac{1}{\theta}-\gamma\right)\frac{1}{2}\frac{\alpha^2}{\delta}\sigma_e^2\right)\ , \tag{3.12}$$

where

$$K_1 \equiv \frac{1+\theta\delta-\theta}{\theta-1}\left(\frac{\theta-1}{\delta\theta}\right)^{\frac{\delta\theta}{1+\theta\delta-\theta}} > 0\ .$$

Notice that the conjectured behavior of the aggregate labor (3.9) affects the firm's payoff in two ways. First, the level coefficient, k_2, affects positively the level of profits in the second stage. Second, the labor elasticity to the aggregate shock, i.e. α, enters into the coefficients ϕ_1 and ϕ_2 and has an additional level effect through the last term in the exponent.

[8] This will be shown to be true in equilibrium.

In a symmetric equilibrium, where all firms choose the same amount of information and follow the same hiring rule, the cross-sectional distribution of labor is log-normal. Then, by definition,

$$n_t = \bar{\mathbb{E}}\left(n_{it}\right) + \frac{1}{2}\mathbb{D} \ ,$$

where $\bar{\mathbb{E}}\left(\cdot\right)$ and $\mathbb{D}$ denote the cross-sectional mean and variance of labor inputs on the islands:

$$\begin{aligned} \bar{\mathbb{E}}\left(n_{it}\right) &= \int_0^1 n_{it} di \ , \\ \mathbb{D} &= \int_0^1 \left(n_{it} - \bar{\mathbb{E}}\left(n_{it}\right)\right)^2 \ di \ . \end{aligned}$$

Next, we derive these cross-sectional moments. First, using the expression for n_{it},

$$\bar{\mathbb{E}}\left(n_{it}\right) = \frac{\theta\delta}{1+\theta\delta-\theta}\log\left(\frac{\theta-1}{\delta\theta}\right) + \frac{\theta\delta}{1+\theta\delta-\theta}\left(\frac{1}{\theta}-\gamma\right)\left(\frac{1}{\delta}k_2 - \frac{1}{2}\frac{1+\theta\delta-\theta}{\theta\delta}\frac{\alpha^2}{\delta}\sigma_e^2\right) \tag{3.13}$$

$$+\,\phi_1 \int \mathbb{E}_{it}(a_t)di + \phi_2\mathbb{V} \ .$$

Substituting the Bayesian updating formulae into (3.13), we get

$$\begin{aligned} \bar{\mathbb{E}}\left(n_{it}\right) = &\frac{\theta\delta}{1+\theta\delta-\theta}\log\left(\frac{\theta-1}{\delta\theta}\right) + \frac{\theta\delta}{1+\theta\delta-\theta}\left(\frac{1}{\theta}-\gamma\right)\left(\frac{1}{\delta}k_2 - \frac{1}{2}\frac{1+\theta\delta-\theta}{\theta\delta}\frac{\alpha^2}{\delta}\sigma_e^2\right) \\ &+\phi_1\frac{\sigma_a^2}{\sigma_a^2+\sigma_e^2}a_t + \phi_2\mathbb{V} \ , \end{aligned}$$

where we invoke the law of large numbers to show that

$$\int_0^1 s_{it} di = \int_0^1 \left(a_t + e_{it}\right) di = a_t + \int_0^1 e_{it} \ di = a_t \ .$$

Similarly,

$$\begin{aligned} n_{it} - \bar{\mathbb{E}}(n_{it}) &= \frac{\sigma_a^2}{\sigma_a^2+\sigma_e^2}\phi_1 e_{it} \ , \\ \Rightarrow \quad \mathbb{D} &= \left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_e^2}\right)^2 \phi_1^2\sigma_e^2 \ . \end{aligned}$$

The next result completes the guess-and-verify procedure and characterizes the response coefficients.

Proposition 19 *In a symmetric equilibrium, aggregate labor input is given by (3.9), with*

$$\alpha = \frac{\delta\theta\left(1-\gamma\right)\sigma_a^2}{\left[\left(\delta-1\right)\theta+\gamma\theta\right]\sigma_a^2+\left(1+\delta\theta-\theta\right)\sigma_e^2}, \tag{3.14}$$

$$k_2 = \frac{\theta\delta}{1+\theta\delta-\theta}\log\left(\frac{\theta-1}{\delta\theta}\right)+\frac{\left(1+\theta\delta-\theta\right)\left(1-\gamma\right)}{\theta\left(\delta-1+\gamma\right)}\frac{\alpha\sigma_e^2}{2}+\frac{1+\theta\delta-\theta}{\theta\left(\delta-1+\gamma\right)}\frac{\alpha^2\sigma_e^2}{2} \tag{3.15}$$

where σ_e^2 is the variance of the error in the signals.

3.2.4 Information acquisition

Next, we examine the information acquisition decision in stage I. Consider the maximized stage II profit function, equation (3.12), which we reproduce here

$$\Pi_{it} = K_1 K_2^{\frac{1-\theta\gamma}{1+\theta\delta-\theta}}\exp\left(\phi_1\mathbb{E}_{it}\left(a_t\right)+\phi_2\mathbb{V}_{it}\left(a_t\right)-\left(\frac{1}{\theta}-\gamma\right)\frac{1}{2}\frac{\alpha^2}{\delta}\sigma_e^2\right).$$

In stage I, the firm takes as given the information choices of other firms, or equivalently, the aggregate coefficients, α and k_2. Expected profits, conditional on a choice of individual error variance $\hat{\sigma}_e^2$, are given by taking expectations over the realization of the random variable $\mathbb{E}_{it}(a_t)$. Exploiting log-normality (and dropping the time subscript), this ex-ante expected profit is:

$$\hat{\Pi}(\hat{\sigma}_e^2,\alpha,k_2) = K_1 K_2^{\frac{1-\theta\gamma}{1+\theta\delta-\theta}}\exp\left(\frac{\phi_1^2}{2}\frac{\left(\sigma_a^2\right)^2}{\sigma_a^2+\hat{\sigma}_e^2}+\phi_2\frac{\sigma_a^2\hat{\sigma}_e^2}{\sigma_a^2+\hat{\sigma}_e^2}-\left(\frac{1}{\theta}-\gamma\right)\frac{1}{2}\frac{\alpha^2}{\delta}\sigma_e^2\right). \tag{3.16}$$

Note that expected profits is a function of the firm's own variance, $\hat{\sigma}_e^2$ as well as the aggregate coefficients, which in turn are determined by the information choices of all firms in the economy. It is straightforward to show that expected profits are decreasing (and convex) in the variance of the error in firm's own signal i.e.[9]

$$\frac{\partial\hat{\Pi}}{\partial\hat{\sigma}_e^2} = -\frac{\theta-1}{\delta\theta}\hat{\Pi}\frac{\alpha^2}{2} < 0 \qquad \forall\,\hat{\sigma}_e^2\in\mathbb{R}^+, \tag{3.17}$$

[9] Recall that the information acquired in equilibrium affects individual profits through k_2 and α., which are taken as given by the firm when choosing its own investment in information. Thus, the effect of overall information σ_e^2, on these coefficients is the basic source of the externality in information choice. We study this in the Appendix 3.4.1.

$$\frac{\partial^2 \hat{\Pi}}{\partial \hat{\sigma}_e^2 \partial \hat{\sigma}_e^2} = -\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2} \left\{ \frac{2}{(\sigma_u^2 + \hat{\sigma}_e^2)} - \frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2} \right\} > 0 \ .$$

The problem of the firm in stage I is:

$$\max_{\hat{\sigma}_e^2} \ \hat{\Pi}\left(\hat{\sigma}_e^2\right) - \upsilon\left(\hat{\sigma}_e^2\right) \ , \tag{3.18}$$

where $\upsilon\left(\hat{\sigma}_e^2\right)$ is the cost function.

As discussed earlier, the main focus of this paper is on private versus social value of information, so we wish to impose as little structure as possible on the cost of information. Therefore, instead of specifying a functional form for $\upsilon(\cdot)$, we directly assume that the cost function is such that the solution to the firm's (and later in the analysis, the social planner's) problem will always lead to an interior solution. This requires assuming that the cost function is sufficiently convex, i.e.

$$\frac{\partial^2 \hat{\Pi}}{\partial \hat{\sigma}_e^2 \partial \hat{\sigma}_e^2} - \frac{\partial^2 \upsilon}{\partial \hat{\sigma}_e^2 \partial \hat{\sigma}_e^2} < 0 \ .$$

A symmetric stationary equilibrium can thus be represented as a fixed point problem in σ_e^2:

$$\sigma_e^2 = \mathrm{argmax}_{\hat{\sigma}_e^2} \ \hat{\Pi}\left(\hat{\sigma}_e^2, \alpha, k_2\right) - \upsilon\left(\hat{\sigma}_e^2\right) \ ,$$

where α and k_2 are functions of σ_e^2 as given by (3.14)-(3.15).

3.2.5 Efficiency in Information Use

We now turn to the efficiency properties of the equilibrium characterized above. First, we show, as in Angeletos and La'O [AL09], that information use is optimal. In particular, we compare the equilibrium coefficients α and k_2 to those chosen by a social planner, who is interested in maximizing household utility. Importantly, the planner is information-constrained, i.e. cannot pool information across islands. We show that, given σ_e^2, the equilibrium response differs from the optimal one only through a constant distortion of the average level of employment (captured by k_2) but the sensitivity to the signal (the coefficient α) coincides with the choice of the planner.

To characterize the planner's optimum, we assume that, in stage II, all firms follow a linear labor-hiring rule of the form:

$$n_{it} = \tilde{k}_2 - \frac{1}{2}\tilde{\alpha}^2\sigma_e^2 + \tilde{\alpha}\ s_{it}\ . \tag{3.19}$$

It is straightforward to show that the aggregate employment and consumption are then given by,

$$\begin{aligned} N_t &= \int_0^1 N_{it} di = \widetilde{K}_2 A_t^{\tilde{\alpha}}\ , \\ C_t &= \widetilde{K}_2^{\frac{1}{\delta}} A_t^{\frac{\delta+\tilde{\alpha}}{\delta}} \exp\left(-\frac{1}{2}\left(\frac{1+\theta\delta-\theta}{\theta\delta}\right)\frac{\widetilde{\alpha}^2}{\delta}\sigma_e^2\right)\ . \end{aligned}$$

The next step is to express the utility of the household in equilibrium as a function of the amount of information. Using the relationships derived above, the period utility of the household is

$$\begin{aligned} \mathbb{U} &= \frac{1}{1-\gamma}\exp\left(\frac{1-\gamma}{\delta}\tilde{k}_2 + \left((1-\gamma)\left(\frac{\delta+\tilde{\alpha}}{\delta}\right)\right)^2\frac{\sigma_a^2}{2} - \frac{1-\gamma}{2}\left(\frac{1+\theta\delta-\theta}{\theta\delta}\right)\frac{\tilde{\alpha}^2}{\delta}\sigma_e^2\right) \\ &\quad - \exp\left(\tilde{k}_2 + \frac{\tilde{\alpha}^2}{2}\sigma_a^2\right)\ . \end{aligned} \tag{3.20}$$

The efficient use of information is then defined by coefficients α^* and k_2^* that maximize utility, i.e.

$$(\alpha^*, k_2^*) = \text{argmax}_{\tilde{k}_2,\tilde{\alpha}} \quad \mathbb{U}(\tilde{k}_2, \tilde{\alpha})\ .$$

The next result shows that the socially optimal sensitivity to the signal α^* coincides with the corresponding equilibrium coefficient, but the level coefficient k_2^* does not.

Proposition 20 *For a given σ_e^2, the planner's optimal response coefficients are:*

$$\alpha^* = \alpha\ , \tag{3.21}$$

$$k_2^* = k_2 + \frac{\delta}{\delta-1+\gamma}\log\left(\frac{\theta}{\theta-1}\right)\ . \tag{3.22}$$

where α and k_2 are as defined in Proposition 19.

The above result reiterates the efficiency results of Angeletos and La'O [AL09] - firms internalize their market power and restrict their output, but this distortion is invariant to the state of the world and therefore, to the information structure. To see the intuition, note that the difference between the efficient quantity choice and the equilibrium arises from the fact that the planner equates marginal cost to expected price, while the firm equates it to expected marginal revenue. The CES specification implies that the ratio of price to marginal revenues is the same in every state of the world, leading to the above result.

3.2.6 Efficiency of Information Choice

In this subsection, we compare the level of information acquired in equilibrium to a socially optimum level. Our main result is that, despite the efficiency result in the previous subsection, the ex-ante acquisition decision is suboptimal.

We restrict attention to the case where utility, net of information acquisition costs, is maximized at an interior level of information choice. In other words, the solution to the following problem

$$\max_{\sigma_e^2} \mathbb{U}\left(\sigma_e^2\right) - \upsilon\left(\sigma_e^2\right) \ , \tag{3.23}$$

is characterized by the usual first-order condition:

$$\frac{\partial \mathbb{U}}{\partial \sigma_e^2} = \frac{\partial \upsilon}{\partial \sigma_e^2} \ , \tag{3.24}$$

where $\mathbb{U}$ is given by (3.20).

As with the equilibrium information choice, we also need to assume that the cost function is sufficiently convex, i.e.

$$\frac{\partial^2 \mathbb{U}}{\partial \sigma_e^2 \partial \sigma_e^2} - \frac{\partial^2 \upsilon}{\partial \sigma_e^2 \partial \sigma_e^2} < 0 \ .$$

Conditional on being in this region, whether the social planner acquires more or less information than the equilibrium depends only on the marginal value to the planner, $\partial \mathbb{U}/\partial \sigma_e^2$, versus the private value to the firm, $\partial \hat{\Pi}/\partial \hat{\sigma}_e^2$.

The next proposition presents the main result of this section. It shows that information acquisition is inefficient. In particular, in any symmetric equilibrium, there is a (constant) wedge between the private value of information by firms and its value to the planner.

Proposition 21 *In a symmetric equilibrium, the private value of information is always less than its social value, i.e.*

$$\frac{\partial \mathbb{U}}{\partial \sigma_e^2} = \left(1 + \frac{\delta}{(\theta - 1)(\delta - 1 + \gamma)}\right) \left(\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2}\right)_{\sigma_e^2 = \hat{\sigma}_e^2} < 0 \qquad \forall \ \sigma_e^2 \in \mathbb{R}^+$$

Therefore, the level of information acquired in equilibrium is inefficiently low.

It is important to note that this inefficiency result obtains despite the fact that firms respond to information in a socially optimal fashion (by Proposition 20). In other words, when information is exogenous, the cyclical response of firms in equilibrium are the same as those induced by an information-constrained planner. However, this is no longer true when information is endogenous. In particular, the equilibrium will feature less investment in information acquisition and therefore, exhibit fluctuations that are in general different from those under the planner's information.

Moreover, since informational choices also have an indirect effect on the level coefficient k_2, the average level of activity in equilibrium will also be inefficient[10]. This is a new source of inefficiency in this class models - one that is absent in models with exogenous information (e.g. Angeletos and LaO [AL09]).

Finally, note that the extent of under-acquisition depends on the degree of market power, parameterized by θ. When this disappears, e.g. in the perfectly competitive limit as $\theta \to \infty$, the gap between the social value and the private value to the firm also vanishes. In the following subsection, we will show that efficiency can be restored by a natural policy intervention.

3.2.7 Optimal Policy

Next, we characterize the optimal policy in this environment. We show that a constant revenue subsidy, which corrects the monopoly power distortion in employment, also aligns

[10] The sign of this effect is, in general, ambiguous.

the private and social values of information, leading to ex-ante efficiency in information acquisition.

Under this policy, the problem of the firm becomes:[11]

$$\Pi_{it} = \max_{N_{it}} \mathbb{E}_i Q_t \left[(1+\tau_R) P_{it} Y_{it} - W_{it} N_{it} \right] \quad ,$$

The first order condition for labor is:

$$N_{it} = \left(\frac{1}{\delta} \frac{\theta - 1}{\theta} (1+\tau_R) \left(\mathbb{E}_{it} Q_t Y_t^{\frac{1}{\theta}} A_t^{\frac{\theta-1}{\theta}} \right) \right)^{\frac{\delta\theta}{1+\theta\delta-\theta}} \quad . \tag{3.25}$$

It is straightforward to show that the level distortion in the response function is removed, i.e. k_2 equals k_2^*, if the subsidy satisfies

$$1 + \tau_R = \frac{\theta}{\theta - 1} > 1 \ .$$

What about information acquisition ? Recall from Proposition 21 that the private value of information was less than the social value. By subsidizing firms' revenues, the policy described above also raises the private marginal value of information. Remarkably, it brings the private value exactly in line with the planner's valuation, as the following result shows.

Proposition 22 *A symmetric equilibrium with a constant revenue subsidy $\tau_R = 1/\theta$ is constrained efficient, i.e. it attains the optimal allocation of the information constrained planner.*

In other words, policies aimed at correcting market power distortions in activity have an additional effect when information is endogenous - they also serve to remove the wedge between private and social value of information. As a result, they eliminate the inefficiencies (in both the average level and the fluctuations) arising from the suboptimally low levels of information production in equilibrium.

[11] In addition, a lump sum transfer $\tau_R \int P_{it} Y_{it} di$ is subtracted from the income side of the household's budget constraint.

3.3 A Nominal Price-Setting Model

In this section, we study information acquisition in a standard micro-founded model of nominal price-setting under dispersed information about monetary shocks. The model environment closely follows Hellwig [Hel05]. In line with the results in that paper, we find that payoff externalities lead to information being used inefficiently in equilibrium. In particular, firms pay too much attention to private signals about innovations to money supply. This inefficiency also causes the incentives of firms to acquire information to diverge from social incentives, but, as we show, these incentives remain distorted even when the inefficiency in use is not present.

3.3.1 Preferences, Technology and Information

Time is discrete, $t = 0, 1, 2....$ The economy is populated by 3 types of agents - a representative household, a continuum of intermediate producers and a final goods producer.

Household: The household solves the following problem:

$$\max_{\{C_t, N_t, M_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - N_t\right) \ ,$$

subject to a cash in advance constraint

$$P_t C_t \leq M_{t-1} + T_t \ ,$$

and a budget constraint

$$P_t C_t + M_t \leq W_t N_t + \Pi_t + \left(M_{t-1} - P_{t-1} C_{t-1}\right) + T_t \ .$$

Government: The government's budget constraint is given by

$$M_t = M_{t-1} + T_t \ ,$$

where M_t is the stock of money supply. The (exogenous) law of motion for M_t is

$$M_t = M_{t-1} U_t \ .$$

In other words, money supply is assumed to follow a random walk in logs[12]

$$m_t = m_{t-1} + u_t \ ,$$

where $u_t \sim N\left(0, \sigma_u^2\right)$. This shock to the stock of money is the only source of aggregate uncertainty in the model.

Final good producer: The single final good is produced using a continuum $[0, 1]$ of intermediate inputs Y_{it}. The production function is a Dixit-Stiglitz aggregator with constant returns to scale. The final good producing firm solves the following static problem:

$$\max \ \ P_t Y_t - \int_0^1 P_{it} Y_{it} di \ ,$$

subject to

$$Y_t = \left(\int_0^1 Y_{it}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \ ,$$

where θ is the elasticity of substitution, $\theta > 1$.

Intermediate producers: There is a continuum of intermediate good producers indexed $i \in [0, 1]$. These firms make decisions in different stages in every period. In the first stage of the period, each firm chooses the variance of the error term in its private signal about the aggregate state subject to a cost function, $\upsilon(\hat{\sigma}_e^2)$. The properties of this function will be specified later. In the second stage, the firm observes its signal and sets prices to maximize expected profits, conditional on its information set. After this, in stage III, markets for goods and labor open, wages are determined and production takes place. Figure 3.2 shows the timing of events in each period.

The production function is a standard decreasing returns to scale technology with labor as the sole input.

$$Y_{it} = \left(\delta N_{it}\right)^{\frac{1}{\delta}} \ ,$$

[12]Hereafter, lower case variables are in logs, e.g. $x_t \equiv \ln X_t$.

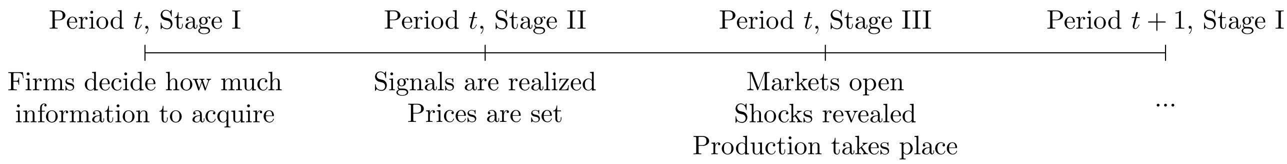

Figure 3.2: Timeline of Events

where $\delta > 1$.

In stage II, an intermediate producer sets a nominal price to maximize the expected value of profits (weighted by the household's stochastic discount factor):

$$\Pi_{it} = \max_{P_{it}} \quad \mathbb{E}_{it} Q_t \left[P_{it} Y_{it} - W_t N_{it} \right] \ ,$$

where $\mathbb{E}_{it}(\cdot)$ represent the expectation conditional on firm's i information set $\mathcal{I}_{it}$, i.e. $\mathbb{E}_{it}(\cdot) \equiv \mathbb{E}_t(\cdot \, |\mathcal{I}_{it})$. Note that, by setting a price, the firm commits to delivering any quantity at that price when markets open in stage III.

Information and signal structure: Before setting prices in stage II, each firm has access to a private signal s_{it} about the current innovation to money supply:

$$s_{it} = u_t + e_{it} \ ,$$

where $e_{it} \sim N(0, \hat{\sigma}_e^2)$ and $\hat{\sigma}_e^2$ is the variance chosen in stage I by the firm. In stage III, i.e. after prices are set, markets open and the aggregate state becomes commonly known. Therefore, at the time of setting prices in period t, the firm's information set consists of the aggregate state (money supply) at the end of the previous period and its private signal about the current innovation i.e. $\mathcal{I}_{it}$ consists of $\{M_{t-1}, s_{it}\}$.

Information acquisition problem: In stage I of the period, intermediate firms choose the amount of information of their private signal, taking as given information choices of other firms. Expected profits prior to the realization of the signal and the aggregate state is defined by:

$$\hat{\Pi}_{it}\left(\hat{\sigma}_e^2\right) \equiv \mathbb{E}_{t-1} \Pi_{it} \ ,$$

where $\mathbb{E}_{t-1}$ is the expectation conditional on information available at the time of the first stage decision i.e. the (commonly known) history until $t-1$.

The problem of the firm in the first stage can then be written as:

$$\max_{\hat{\sigma}_e^2} \quad \hat{\Pi}_{it}\left(\hat{\sigma}_e^2\right) - \upsilon\left(\hat{\sigma}_e^2\right) \ ,$$

where $\upsilon(\cdot)$ is the cost of information as a function of the noise in the signal. We assume that $\upsilon'(\cdot) < 0, \upsilon''(\cdot) > 0$.

3.3.2 Optimality

As before, we solve the model backwards starting from the last stage.

Stage III: Complete information - In the last stage of each period, both household and firms have perfect information of the aggregate state. Optimization by households and the final goods producer, combined with market clearing, implies the following set of equilibrium conditions:

$$P_t C_t = M_t \ , \tag{3.26}$$

$$W_t = \gamma M_t \quad \text{where} \quad \gamma = \beta e^{\frac{\sigma_m^2}{2}} \ , \tag{3.27}$$

$$Q_t = \frac{1}{W_t} \ , \tag{3.28}$$

$$P_t = \left(\int_0^1 P_{it}^{1-\theta} di\right)^{\frac{1}{1-\theta}} \ , \tag{3.29}$$

$$N_t = \int_0^1 N_{it} di \ , \tag{3.30}$$

$$Y_t = C_t \ . \tag{3.31}$$

The production decisions of firm i are pinned down by the demand of the final goods producer (given the prices set in stage II):

$$Y_{it} = Y_t \left(\frac{P_{it}}{P_t}\right)^{-\theta} \ .$$

Stage II: Price-setting - Firms set prices to maximize expected profits, taking into account the nature of equilibrium allocations in stage III:

$$\max_{P_{it}} \quad \mathbb{E}_{it} Q_t \left[P_{it} Y_{it} - W_t N_{it} \right] \ ,$$

subject to:

$$\begin{aligned} Y_{it} &= Y_t \left(\frac{P_{it}}{P_t} \right)^{-\theta} \ , \\ Y_{it} &= (\delta N_{it})^{\frac{1}{\delta}} \ . \end{aligned}$$

Plugging these constraints and equilibrium conditions from Stage III, this profit maximization problem can be written as:

$$\max_{P_{it}} P_{it}^{1-\theta} \mathbb{E}_{it} \left(P_t^{\theta-1} \right) - \frac{\gamma}{\delta} P_{it}^{-\theta\delta} \mathbb{E}_{it} \left(M_t^{\delta} P_t^{\delta(\theta-1)} \right) \ .$$

The first order condition is:

$$P_{it} = \left(\frac{\gamma\theta}{\theta - 1} \right)^{\frac{1}{1+\theta\delta-\theta}} \left[\frac{\mathbb{E}_{it} \left(M_t^{\delta} P_t^{\delta(\theta-1)} \right)}{\mathbb{E}_{it} \left(P_t^{\theta-1} \right)} \right]^{\frac{1}{1+\theta\delta-\theta}} \ . \tag{3.32}$$

Stage I: Information acquisition Under the assumption of an interior solution, the firm's optimal information choice is characterized by the following optimality condition:

$$\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2} - \frac{\partial \upsilon}{\partial \hat{\sigma}_e^2} = 0 \ .$$

3.3.3 Equilibrium

A stationary equilibrium is (i) a set of information choices in Stage I for each firm (ii) a set of pricing rules (iii) aggregate variables C_t, N_t, W_t, Y_t and P_t as functions of the aggregate history (iv) intermediate production Y_{it} and labor input N_{it} such that (a) taking W_t and P_t as given, the household choices C_t and N_t solve the household's maximization problem (b) taking P_t and P_{it} as given, the choices of Y_t and Y_{it} solve the final goods producer's problem (d) taking the functions in (iii) as given, the pricing rules in (ii) maximize expected profits of the intermediate goods producer, conditional on its information (e) taking the behavior of aggregates in (iii) as given, the information choice in (i) solves the Stage I problem (f) Markets clear i.e. $N_t = \int N_{it}\, di$, $Y_{it} = \delta N_{it}^{\frac{1}{\delta}}$ and $Y_t = C_t$.

We focus on symmetric stationary equilibria, where all intermediate producers acquire the same amount of information in Stage I and follow the same pricing strategies in Stage II. We start the characterization of such an equilibrium with a conjecture about the aggregate price level:

$$P_t = M_{t-1} U_t^{\alpha} K_2 \ ,$$

or, in logs

$$p_t = m_{t-1} + \alpha u_t + k_2 \ , \tag{3.33}$$

where $\alpha \in [0,1]$ and k_2 are constants to be determined in equilibrium. The former determines the sensitivity of aggregate prices to monetary shocks whereas the latter affects the level of aggregate prices. Both these coefficients will play an important role in our analysis.

Intermediate producers We substitute the equilibrium conjecture (3.33) for aggregate prices into the first order condition (3.32) and assuming[13] conditional log-normality, take logs :

$$\begin{aligned} p_{it} =& m_{t-1} + \frac{(1-r)}{\delta} \ln\left(\frac{\gamma\theta}{\theta-1}\right) + r k_2 \\ &+ (1-r+\alpha r)\, \mathbb{E}_{it}(u_t) \\ &+ \frac{1}{2}(1-r+\alpha r)\left(\delta + (\delta+1)\,\alpha\,(\theta-1)\right) \mathbb{V}_{it}(u_t) \ , \end{aligned}$$

where

$$r \equiv \frac{(\delta-1)(\theta-1)}{1+\theta\delta-\theta} \ \in [0,1] \ ,$$

and $\mathbb{E}_{it}(u_t)$ and $\mathbb{V}_{it}(u_t)$ are the posterior mean and variance, respectively, conditional on the firm's information set $\mathcal{I}_{it}$.

The firm's optimal price thus has 3 components. The first is a constant term, consisting of the (commonly known) level of last period's money supply and the level coefficients in aggregate prices. The second represents the firm's optimal response to the expected

[13]This will be verified later.

innovation in money supply. The parameter r controls the nature of strategic interactions. The greater the value of r, the more the firm's optimal reaction depends on α, the sensitivity of aggregate prices to the current shock. The third term is an adjustment to the price to account for the fact that the firm is uncertain about the realization of the shock. This 'precautionary' term emerges from the asymmetric nature of the firm's profit function. If the firm's relative price is higher than the optimum, it loses market share. A low relative price leads to higher quantities sold but due to diminishing returns, these additional units are produced at an increased marginal cost. Given the specific forms of demand and production functions, the latter is a much more costly phenomenon i.e. profits decline much more sharply with a low relative price than a high one. As a result, when the firm is uncertain about the position of its demand curve, the optimal price is a little higher than the expected value of the target price. Figure 3.3 illustrates this feature with a simple example where the aggregate shock is assumed to take only 1 of 2 possible values - low U_- and high U_+, $(U_+ > U_-)$. The left panel depicts the profit function as a function of P_i under the two realizations of the monetary shock. Notice that for a particular realization of the shock, the profit function is steeper for prices that are below the profit maximizing price. That is, charging prices that are too low is a costlier mistake than charging prices that are too high. In the right panel of Figure 3.3, we show the *expected* profit function as a function of price P_i and varying degrees of uncertainty about the aggregate shock. The two lines keep the expected value of the shock constant, but vary the levels of variance (low σ_L^2 and high σ_H^2). Expected-profits are strictly decreasing in uncertainty. Also, as uncertainty increases, the optimal price (i.e. the one which maximizes expected profit) increases, reflecting the asymmetric nature of the penalty for charging sub-optimal prices.

Using standard results for Bayesian updating, the firm's posterior expectation and variance about the state of the economy are:

$$\mathbb{E}_{it}\left(u_t\right) = \frac{\sigma_u^2}{\sigma_u^2 + \hat{\sigma}_e^2} s_{it} \,, \tag{3.34}$$

$$\mathbb{V}\left(u_t\right) = \frac{\sigma_u^2 \hat{\sigma}_e^2}{\sigma_u^2 + \hat{\sigma}_e^2} \,, \tag{3.35}$$

where $\frac{\sigma_u^2}{\sigma_u^2 + \hat{\sigma}_e^2}$ is the signal to noise ratio.

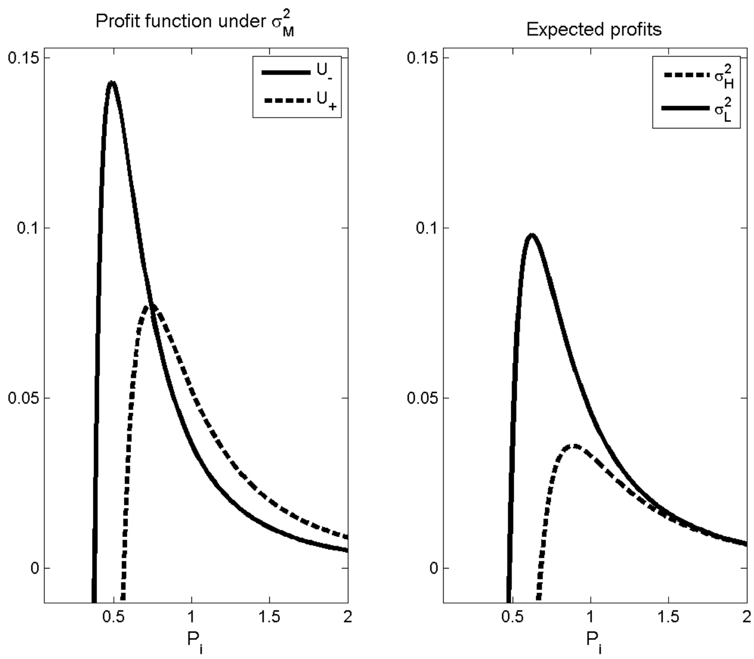

Figure 3.3: Effect of Uncertainty

Plugging the optimal price into the firm's profit function, we get the following expression for maximized profit

$$\Pi_{it} = e^{\phi_1 \mathbb{E}_{it}(u_t) + \phi_2 \mathbb{V}_{it}(u_t)} \; K_2^{(\theta-1)(1-r)} \; K_1 \; , \tag{3.36}$$

where

$$\begin{aligned}
\phi_1 &\equiv (1-\theta)(1-r)(1-\alpha) < 0 \; , \\
\phi_2 &\equiv \frac{1}{2}(1-\theta)(1-r)\left(\delta\left(1+\alpha(\theta-1)\right)^2 - \alpha^2\theta(\theta-1)\right) < 0 \; , \\
K_1 &\equiv \left(\frac{\theta-1}{\gamma\theta}\right)^{\frac{\theta\delta}{1+\theta\delta-\theta}} \left[\frac{1}{(\theta-1)(1-r)}\right] > 0 \; .
\end{aligned}$$

Notice that the conjectured behavior of the aggregate price (3.33) affects the firm's payoff in two ways. First, the level effect, k_2, affects positively the level of profits in the second stage. Second, the price elasticity to the aggregate shock, i.e. α, enters into the coefficients ϕ_1 and ϕ_2. It is easy to show that

$$\frac{\partial \phi_1}{\partial \alpha} > 0 \; , \qquad \frac{\partial \phi_2}{\partial \alpha} < 0 \; .$$

In other words, the more responsive aggregate prices are to the nominal shock, the lower is the sensitivity of firm's profits to the expected nominal shock, but the greater is the cost of uncertainty.

Equilibrium in Stage II In a symmetric equilibrium, where all firms choose the same amount of information and follow the same pricing rule, the cross-sectional distribution of prices is log-normal. Therefore, taking logs on both sides of the aggregate price expression (3.29) yields:

$$p_t = \bar{\mathbb{E}}\left(p_{it}\right) + \frac{(1-\theta)}{2}\mathbb{D} \ ,$$

where $\bar{\mathbb{E}}\left(\cdot\right)$ and $\mathbb{D}$ denote the cross-sectional mean and dispersion in prices:

$$\begin{aligned} \bar{\mathbb{E}}\left(p_{it}\right) &= \int_0^1 p_{it} di \ , \\ \mathbb{D} &= \int_0^1 \left(p_{it} - \bar{\mathbb{E}}\left(p_{it}\right)\right)^2 \ di \ . \end{aligned}$$

Next, we derive these cross-sectional moments. First, using the expression for p_{it}, the cross-sectional mean is given by

$$\bar{\mathbb{E}}\left(p_{it}\right) = m_{t-1} + \frac{1-r}{\delta}\ln\left(\frac{\gamma\theta}{\theta-1}\right) + rk_2 + (1-r+\alpha r)\int \mathbb{E}_{it}(u_t)\ di + g(\alpha)\mathbb{V} \ , \quad (3.37)$$

where

$$g(\alpha) \equiv \frac{1}{2}\left(1-r+\alpha r\right)\left(\delta + (\delta+1)\,\alpha\,(\theta-1)\right) \ .$$

Substituting the Bayesian updating formulae into (3.37), we get

$$\bar{\mathbb{E}}\left(p_{it}\right) = m_{t-1} + \frac{1-r}{\delta}\ln\left(\frac{\gamma\theta}{\theta-1}\right) + rk_2 + (1-r+\alpha r)\,\frac{\sigma_u^2}{\sigma_u^2+\sigma_e^2}u_t + g(\alpha)\mathbb{V} \ ,$$

where we invoke the law of large numbers to note that

$$\int_0^1 s_{it} di = \int_0^1 (u_t + e_{it})\ di = u_t + \int_0^1 e_{it}\ di = u_t \ .$$

Similarly,

$$p_{it} - \bar{E}(p_{it}) = (1 - r + r\alpha)\frac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2} e_{it} \ ,$$

$$\Rightarrow \quad \mathbb{D} = (1 - r + r\alpha)^2 \left(\frac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2}\right)^2 \sigma_e^2 \ .$$

The next result completes the guess-and-verify procedure and characterizes the response coefficients.

Proposition 23 *In a symmetric equilibrium, the aggregate price level is given by (3.33), with*

$$\alpha = \frac{(1-r)\,\sigma_u^2}{(1-r)\,\sigma_u^2 + \sigma_e^2} \quad \in [0\ 1] \ , \tag{3.38}$$

$$k_2 = \frac{1}{\delta} \ln\left(\frac{\gamma\theta}{\theta - 1}\right) + \frac{g(\alpha)}{1-r}\mathbb{V} + \frac{(1-\theta)}{2\,(1-r)}\mathbb{D} \quad > 0 \ , \tag{3.39}$$

where, as defined earlier,

$$\begin{aligned} g &\equiv \frac{(1 - r + \alpha r)\,(\delta + (\delta + 1)\,\alpha\,(\theta - 1))}{2} \ , \\ \mathbb{V} &\equiv \frac{\sigma_u^2 \sigma_e^2}{\sigma_u^2 + \sigma_e^2} \ , \\ \mathbb{D} &\equiv (1 - r + r\alpha)^2 \left(\frac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2}\right)^2 \sigma_e^2 = \alpha^2 \sigma_e^2 \ . \end{aligned}$$

The expression for α has an intuitive interpretation. It takes the form of a signal-to-noise ratio, except that the variance of the fundamental (in this case, the nominal shock) is adjusted to account for the degree of complementarity. Greater the complementarity, i.e. higher the r, the lower the weight on private signals and higher the reliance on commonly known information, which in this case is just the money stock in the previous period.

3.3.4 Information acquisition

Next, we examine the information acquisition decision in Stage I. Consider the maximized stage II profit function, equation (3.36),

$$\Pi_{it} = e^{\phi_1 \mathbb{E}_{it}(u_t) + \phi_2 \mathbb{V}_{it}(u_t)} \; K_2^{(\theta-1)(1-r)} \; K_1 \; . \tag{3.40}$$

In Stage I, the firm takes as given the information choices of other firms, or equivalently, the aggregate coefficients, ϕ_1, ϕ_2 and k_2. Expected profits, conditional on a choice of individual error variance $\hat{\sigma}_e^2$, are given by taking expectations over the realization of the random variable $\mathbb{E}_{it}(u_t)$. Exploiting log-normality (and dropping the time subscript), this ex-ante expected profit is:

$$\hat{\Pi}(\hat{\sigma}_e^2, \alpha, k_2) = e^{\left(\frac{\phi_1^2}{2} \frac{\left(\sigma_u^2\right)^2}{\sigma_u^2 + \hat{\sigma}_e^2} + \phi_2 \frac{\sigma_u^2 \hat{\sigma}_e^2}{\sigma_u^2 + \hat{\sigma}_e^2} \right)} K_2^{(\theta-1)(1-r)} K_1 \; . \tag{3.41}$$

Note that expected profits is a function of the firm's own variance, $\hat{\sigma}_e^2$ as well as the aggregate coefficients, which in turn are determined by the information choices of all firms in the economy.

It is straightforward to show that expected profits are decreasing (and convex) in the variance of the error in firm's own signal i.e.

$$\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2} = \hat{\Pi} \left(-\frac{\phi_1^2}{2} + \phi_2 \right) \left(\frac{\sigma_u^2}{\sigma_u^2 + \hat{\sigma}_e^2} \right)^2 < 0 \qquad \forall \; \hat{\sigma}_e^2 \in \mathbb{R}^+ \; , \tag{3.42}$$

$$\frac{\partial^2 \hat{\Pi}}{\partial \hat{\sigma}_e^2 \partial \hat{\sigma}_e^2} = - \frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2} \left\{ \frac{2}{(\sigma_u^2 + \hat{\sigma}_e^2)} - \frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2} \right\} > 0 \; .$$

The problem of the firm in Stage I is:

$$\max_{\hat{\sigma}_e^2} \;\; \hat{\Pi}\left(\hat{\sigma}_e^2, \alpha, k_2\right) - \upsilon\left(\hat{\sigma}_e^2\right) \; , \tag{3.43}$$

where $\upsilon\left(\hat{\sigma}_e^2\right)$ is the cost function. As before, we assume that $\upsilon\left(\cdot\right)$ is sufficiently convex so that the solution to this problem is an interior one and is characterized by the first-order condition.

A symmetric stationary equilibrium can thus be represented as a fixed point problem in σ_e^2

$$\sigma_e^2 = \text{argmax}_{\hat{\sigma}_e^2} \ \hat{\Pi}\left(\hat{\sigma}_e^2, \alpha, k_2\right) - \upsilon\left(\hat{\sigma}_e^2\right) \ ,$$

where α and k_2 are functions of σ_e^2 as given by (3.38)-(3.39).

3.3.5 Efficiency in Information Use

As we did for the RBC model, we begin by examining the efficiency properties of information use in equilibrium. In line with the findings in Hellwig [Hel05], we show that firms place too much reliance on private signals, relative to an information-constrained social planner, who is interested in maximizing household utility.

To characterize the socially optimal use, we assume that, in Stage II, all firms follow a linear pricing rule of the form:

$$p_{it} = m_{t-1} + \tilde{k}_2 - \frac{1-\theta}{2}\tilde{\alpha}^2\sigma_e^2 + \tilde{\alpha}\ s_{it} \ . \tag{3.44}$$

It is straight forward to see that the aggregate price is then given by

$$p_t = m_{t-1} + \tilde{k}_2 + \tilde{\alpha}\ u_t \ ,$$

and life-time utility of the household is

$$\mathbb{U} = -\tilde{k}_2 - \frac{1}{\delta}\exp\left\{-\delta\tilde{k}_2 + \frac{\delta^2}{2}\left[\frac{\theta}{(1-r)}\tilde{\alpha}^2\sigma_\epsilon^2 + (1-\tilde{\alpha})^2\sigma_u^2\right]\right\} \ .$$

The efficient use of information is then defined by coefficients α^* and k_2^* that maximize utility, i.e.

$$(\alpha^*, k_2^*) = \text{argmax}_{\tilde{k}_2,\tilde{\alpha}} \quad \mathbb{U}(\tilde{k}_2, \tilde{\alpha}) \ .$$

The next proposition lays out the optimal response coefficients and shows equilibrium prices that are suboptimally higher on average and too sensitive to nominal shocks.

Proposition 24 *1. The coefficients that maximize the life-time utility of the house-*

hold are

$$\alpha^* = \frac{(1-r)\,\sigma_u^2}{(1-r)\,\sigma_u^2 + \theta\sigma_e^2}\,, \tag{3.45}$$

$$k_2^* = \frac{\delta\theta}{2\,(1-r)}\alpha^*\sigma_e^2\,. \tag{3.46}$$

2. For a given σ_e^2, these coefficients are lower than the equilibrium ones

$$\alpha^* < \alpha \qquad \textit{and} \qquad k_2^* < k_2\,.$$

3.3.6 Efficiency of Information Acquisition

In this subsection, we compare the level of information acquired in equilibrium to a socially optimum level. In order to disentangle the effect of the inefficiency in information use identified earlier, we start by comparing the equilibrium information acquisition to the choice of a planner who is also subject to the same inefficiency, i.e. who takes the equilibrium in Stage II as given. In other words, we study the problem of a planner who gets to choose only the amount of information acquired ex-ante, but cannot affect the equilibrium responses. Not surprisingly, the equilibrium features a suboptimal level of information acquisition. We then revisit the optimality of information choice under the assumption of efficient use and find that private incentives to acquire information are still not aligned to social ones.

The first step is to express the utility of the household in equilibrium as a function of the amount of information. Using the equilibrium relationships derived in the previous section, we have:

$$C_t = \frac{U_t^{(1-\alpha)}}{K_2}\,,$$

$$N_t = \frac{1}{\delta}\left(C_t\right)^{\delta}\int_0^1\left(\frac{P_{it}}{P_t}\right)^{-\theta\delta} di\,.$$

More algebra yields the following expression for utility:

$$\mathbb{U}\left(\sigma_e^2\right) = \frac{1}{\delta}\left[\ln\left(\frac{\theta-1}{\gamma\theta}\right) - \frac{\theta-1}{\gamma\theta}\right] - \frac{\delta}{2}\frac{\sigma_u^2\sigma_e^2\left((1-r)\,\theta\sigma_u^2 + \sigma_e^2\right)}{\left((1-r)\,\sigma_u^2+\sigma_e^2\right)^2}\,. \tag{3.47}$$

The next result replicates the findings in Hellwig [Hel05] that equilibrium welfare is not monotonically increasing in the precision of the private signal.

Proposition 25 *1. Suppose $\theta \leq 2$. Then, welfare decreases with the error in firms' signals i.e.* $\frac{d\mathbb{U}}{d\sigma_e^2} < 0 \ \ \forall \sigma_e^2$.

2. If $\theta > 2$, welfare is decreasing in σ_e^2 only if the σ_e^2 is sufficiently small, i.e.

$$\frac{\partial \mathbb{U}}{\partial \sigma_e^2} < 0 \qquad \textit{if} \qquad \sigma_e^2 < \frac{\theta}{(\theta - 2)} \sigma_u^2 (1 - r) \ .$$

The above result shows that the inefficiency in equilibrium information use can be so extreme that more information actually reduces welfare. To see the intuition behind the dependence on θ, recall that the difference between the equilibrium response coefficient and the socially optimal one was increasing in θ. As a result, when θ is high, we need to impose additional restrictions to ensure that information is still socially desirable. For the rest of our analysis, we will assume that these conditions hold at the equilibrium information choice. Formally, we assume

Assumption 3 $(\theta - 2)\sigma_e^2 < \theta \sigma_u^2 (1 - r)$

As before, we also assume that the cost function is such that utility, net of information acquisition costs, is maximized at an interior point. In other words, the solution to the following problem

$$\max_{\sigma_e^2} \mathbb{U}\left(\sigma_e^2\right) - \upsilon\left(\sigma_e^2\right) \ , \tag{3.48}$$

is characterized by the usual first-order condition:

$$\frac{\partial \mathbb{U}}{\partial \sigma_e^2} - \frac{\partial \upsilon}{\partial \sigma_e^2} = 0 \ .$$

Conditional on being in this region, whether the social planner acquires more or less information than the equilibrium depends only on the marginal value to the planner, $\partial \mathbb{U} / \partial \sigma_e^2$, versus the private value to the firm, $\partial \hat{\Pi} / \partial \hat{\sigma}_e^2$.

The next proposition, our main efficiency result for this model, shows that information acquisition is typically inefficient, though the direction is ambiguous. The proposition characterizes the regions of the parameter space where there is over-acquisition of information in equilibrium. For brevity, we only present results for the case where $\theta > 2$ (which is the case for most calibrations of macroeconomic models) but similar results can be obtained for the other case as well.

Proposition 26 *Suppose $\theta > 2$ and the conditions of Assumption 3 are met, so the marginal value of information to the planner is positive. Then, there is over-acquisition of information in equilibrium if the following condition holds:*

$$\sigma_e^2 \geq \left[\frac{\theta\gamma - (\theta - 1)}{(\theta - 1) + \gamma(\theta - 2)}\right](1 - r)\,\sigma_u^2 \,.$$

Note that if $\gamma < \frac{\theta-1}{\theta}$, then the condition in the proposition is always met, i.e. firms invest a sub-optimally high amount in information.

A full-fledged numerical investigation is beyond the scope of this paper, but we note that common calibrations of incomplete information monetary models satisfy the condition in the first statement. In other words, the empirically relevant region of the parameter space seems to be one where the equilibrium information acquisition is more than the social optimal level. For an illustrative case, set $\theta = 4, \delta = 1.5$. Also, given that σ_m^2 is usually very small in most calibrations, we have $\gamma \approx \beta > \frac{\theta-1}{\theta} = \frac{3}{4}$. The condition on σ_e^2 is equivalent to the condition that prices are not too responsive to contemporaneous nominal shocks, i.e. $\alpha < 0.84$. In other words, so long as heterogeneity in information generates even modest real effects from nominal shocks, there will be over-acquisition of information in equilibrium.

Recall that firms place too much reliance on private signals relative to the social optimum. Yet, Proposition 26 shows that the direction of the inefficiency in information choice is ambiguous. This divergence between the signs of the ex-ante and ex-post inefficiencies is reinforced by our next result, which shows that, even when the use of information is efficient, firms do not fully internalize all the effects of their information choice. To show this, we again compare information acquired in equilibrium to the social planner's choice, under the assumption that prices are set using the socially efficient response coefficients α^* and k_2^*. The following proposition shows that the inefficiency in information acquisition persists even in this case, though the exact conditions governing over/under-acquisition are different.

Proposition 27 *Suppose firms follow the pricing rule (3.44), with* $\tilde{\alpha} = \alpha^*$ *and* $\tilde{k}_2 = k_2^*$. *Then, if* $\gamma > 1$*, there is over-acquisition of information in equilibrium. Otherwise, there is under-acquisition of information in equilibrium.*

3.3.7 The Collusive Optimum

In this subsection and the next, we take a closer at the interaction between firms' information choices. In particular, we characterize how an individual firm's profits and its incentives to acquire information are affected by the information choice of other firms. We start with the effects on profits. We show the existence of a negative externality - a firm's expected profits decline when other firms in the economy are better informed. It turns out that this negative effect exactly offsets the positive effects of information on individual firms' profits. When firms internalize this externality, e.g. by maximizing total profits, the optimal choice is to acquire no information at all !

Recall that the average amount of information in the economy enters the firm's profits through the aggregate price level, specifically through the (endogenous) coefficients α and k_2. The expression for the ex-ante profit (3.41) leads to the following observation

Lemma 1 *The firm's ex-ante profit is decreasing in the elasticity of aggregate prices to the nominal shock and increasing in the level of aggregate prices i.e.* $\frac{\partial \hat{\Pi}}{\partial \alpha} < 0$ *and* $\frac{\partial \hat{\Pi}}{\partial k_2} > 0$.

Thus, an individual firm's profits are decreasing in the elasticity of prices to nominal shocks, but increasing in the overall level of prices. Next, from (3.38), it is easy to see that α depends negatively on σ_e^2, i.e.

$$\frac{\partial \alpha}{\partial \sigma_e^2} = -\frac{\alpha}{\left((1-r)\,\sigma_u^2 + \sigma_e^2\right)} \quad < 0 \ .$$

In other words, the more accurate are the signals of other firms, the more responsive is the aggregate price level to nominal shocks. In the limit, as $\sigma_e^2 = 0$, aggregate prices will fully adjust to shocks, i.e. $\alpha = 1$.

The relationship of the level coefficient k_2 with information is less clear. The expression for k_2, equation (3.39) comprises a term which is linear in price dispersion as well as a term where the posterior variance, $\mathbb{V}$, is multiplied by a function of α. Price dispersion is non-monotonic in the precision of firms' private information. As σ_e^2 increases, the dispersion of the firms' signals increases, but firms place less weight on them. If the former effect dominates[14], dispersion increases with σ_e^2, otherwise it decreases. Posterior variance, on the other, always increases with the variance of the error term. However, the sensitivity of the price level to the posterior variance, $g(\alpha)$, is an decreasing function of σ_e^2. The intuition for this stems from our earlier discussion on why firms set higher prices in response to greater uncertainty. A higher σ_e^2 implies a lower α. This implies that the aggregate price level comoves less positively with the nominal shock, effectively reducing the uncertainty about its target. The combination of these two forces makes this term also non-monotonic with respect to information.

As a result of these distinct forces, the overall effect of σ_e^2 on k_2 is in general ambiguous. The next result provides a complete characterization:

Lemma 2 *1. Suppose $\theta \leq 2$. Then, the aggregate price is, on average, increasing in the variance of the firms' signals i.e. $\frac{dk_2}{d\sigma_e^2} > 0 \quad \forall \; \sigma_e^2$.*

2. Suppose $\theta > 2$. Then, the aggregate price is, on average, increasing in the variance of the firms' signals only if the variance is sufficiently small i.e. $\frac{dk_2}{d\sigma_e^2} > 0$ if $\sigma_e^2 < \frac{\theta(1-r)}{(\theta-2)}\sigma_u^2$.

Thus, the overall amount of information in the economy affects the aggregate price level (and through it, profits) in complicated ways. However, it turns out that, in a symmetric equilibrium, we can characterize the net effect of information on firm profits quite sharply. In particular, there is a negative externality in any symmetric equilibrium - i.e. others' information choices have negative effects on the firm's profits. Formally,

Proposition 28 *In a symmetric equilibrium, a firm's expected profits increase with σ_e^2, the variance of the error term in the signals of other firms, i.e.*

$$\frac{\partial \hat{\Pi}}{\partial \sigma_e^2} > 0 \; .$$

[14]This happens as long as $(1-r)\sigma_u^2 > \sigma_e^2$.

This negative externality makes equilibrium information acquisition suboptimal from the perspective of maximizing total profit. To make this point more formally, we compare the equilibrium information choice to a natural benchmark. The *team profit*, denoted $\hat{\Pi}^T$ is the combined expected profit earned by all firms in the Stage II equilibrium. The next proposition shows that the externality is quite powerful and when internalized, undoes all the beneficial effects of greater information (recall that an individual firm's profit is always increasing in its own information).

Proposition 29 *In a symmetric equilibrium, i.e. where the error in all firms' signals has the same variance σ_e^2, the expected team profit is independent of that variance i.e.*

$$\frac{d\hat{\Pi}^T}{d\sigma_e^2} = 0 \ .$$

Therefore, the symmetric information choice that maximizes the collective profit is no information i.e. $\sigma_e^2 = \infty$.

In other words, the increase in an individual firm's profits by improving the quality of its own information is exactly offset by the negative effect it has on others' profits. A direct implication of this striking result is that if firms were able to collude on their information acquisition decision and information was costly, the unique symmetric outcome would be to acquire no information at all.

This implication continues to be valid even if information is used in a socially optimal fashion. The next result shows this formally by imposing the planner's optimal response coefficients α^* and k_2^*.

Proposition 30 *When the response function is the socially optimal one, i.e. characterized by α^* and k_2^*, a firm's expected profits increase with σ_e^2, the variance of the error term in the signals of other firms, i.e.*

$$\frac{\partial\hat{\Pi}}{\partial\sigma_e^2} > 0 \ .$$

Further, the expected team profit is independent of that variance of the error in the firms' *signals, i.e.*

$$\frac{d\hat{\Pi}^T}{d\sigma_e^2} = 0 \ .$$

Therefore, the symmetric information choice that maximizes the collective profit is no information i.e. $\sigma_e^2 = \infty$.

3.3.8 Strategic Motives in Information Acquisition

Next, we take a closer look at the role of strategic considerations in the incentives to acquire information. Hellwig and Veldkamp [HV09b] show that, with a quadratic objective function, the information choice decision inherits the strategic nature of agents actions - if actions are strategic complements, information choices become subject to complementarity as well. This subsection investigates the applicability of their finding to the environment of this section. We show that the basic intuition in Hellwig and Veldkamp [HV09b] is at work here as well, but there are other forces at work - in particular, those acting through the *level* of aggregate price. Once these additional effects are taken into account, the nature of strategic interaction in information choice is in general ambiguous, even though actions are unambiguously strategic complements.

Recall that, in any interior solution to the firm's Stage I problem (3.43), the firm equalizes the marginal value of information to the marginal cost i.e.

$$\frac{\partial\hat{\Pi}}{\partial\hat{\sigma}_e^2} = \upsilon'(\hat{\sigma}_e^2) \ .$$

Our focus is the effect of other firms' information on the term on the left hand side. We say information acquisition decisions are *strategic complements* if

$$\frac{\partial^2\hat{\Pi}}{\partial\sigma_e^2\partial\hat{\sigma}_e^2} > 0$$

and *strategic substitutes* otherwise. In other words, if the firm's marginal value of information is increasing in others information, information choices are complements. As before, we start by examining the effect of the slope and level coefficients α and k_2.

Lemma 3 *The marginal value of information to a firm is increasing in the elasticity* *of aggregate prices to nominal shocks and the overall level of prices i.e.* $\frac{\partial^2 \hat{\Pi}}{\partial \alpha \partial \hat{\sigma}_e^2} < 0$ *and* $\frac{\partial^2 \hat{\Pi}}{\partial k_2 \partial \hat{\sigma}_e^2} < 0$.

Thus, more responsive aggregate prices not only affect the level of the firm's profits (Lemma 1), but also increase the sensitivity of the firm's profit to its own information. This is intuitive - when prices comove more with the nominal shocks, the firm's target price becomes more volatile and therefore, acquiring information becomes more attractive.

Combining the first part of the lemma with the definition of α in (3.38) leads to our next result:

Proposition 31 *Suppose k_2 is fixed. Then, an increase in the overall amount of information in the economy increases the marginal value of information for a firm.*

The above result is essentially the insight in Hellwig and Veldkamp [HV09b]. Without any level effects, the complementarity in firm's pricing decisions (as parameterized by r) enters the information choice as well.

This finding does not generally hold once level effects are explicitly taken into account. The non-monotonicity of k_2 with overall information (as demonstrated in Lemma 2) spills over into the effect of overall information on a firm's marginal value of information. The next proposition divides the parameter space into regions according to the sign of the overall effect:

Proposition 32

1. *Suppose $\sigma_u^2 < \frac{4(\delta-1)}{\delta^2 \theta}$. Then, information acquisition decisions are strategic complements i.e.* $\frac{\partial^2 \hat{\Pi}}{\partial \sigma_e^2 \partial \hat{\sigma}_e^2} > 0$.
2. *Suppose $\frac{4(\delta-1)}{(1-r)\delta^2 \theta} < \sigma_u^2$. Then, decisions are strategic substitutes i.e.* $\frac{\partial^2 \hat{\Pi}}{\partial \sigma_e^2 \partial \hat{\sigma}_e^2} < 0$.
3. *If $\frac{4(\delta-1)}{\delta^2 \theta} < \sigma_u^2 < \frac{4(\delta-1)}{(1-r)\delta^2 \theta}$, then this relationship is ambiguous and depends on the value of $\hat{\sigma}_e^2$.*

Parameterizations commonly used in macro models are in the region where information choices are strategic complements. For example, with $\theta = 4, \delta = 1.5$, there is strategic complementarity in information acquisition so long as the innovations to money supply have a variance less than 0.22, which is consistent with standard calibrations (e.g. see Golosov and Lucas [GL07], who use $(0.0062)^2$ in an annual model). Even with $\theta = 20, \delta = 1.1$, the cutoff level of the variance is 0.02.

3.4 Proofs of Results

3.4.1 A Real Business Cycle Model

Proofs for 3.2.3 This outlines the main steps in the derivation of the equilibrium. We start with a guess about the law of motion for n_t. This guess is verified through the following steps.

Start from a conjecture for firm i labor (in logs):

$$n_{it} = \hat{k}_2 + \hat{\alpha} s_{it} \tag{3.49}$$

Define aggregate employment:

$$N_t = \int_0^1 N_{it} di$$

By the Central Limit Theorem,

$$n_t = \bar{\mathbb{E}}\left(n_{it}\right) + \frac{1}{2}\mathbb{D}$$

where $\bar{\mathbb{E}}\left(\cdot\right)$ and $\mathbb{D}$ denote the cross-sectional mean and variance of labor inputs on the islands:

$$\begin{aligned} \bar{\mathbb{E}}\left(n_{it}\right) &= \int_0^1 n_{it} di \ , \\ \mathbb{D} &= \int_0^1 \left(n_{it} - \bar{\mathbb{E}}\left(n_{it}\right)\right)^2 \ di \ . \end{aligned}$$

From (3.49):

$$n_t = \hat{k}_2 + \hat{\alpha} a_t + \frac{1}{2}\hat{\alpha}^2 \sigma_e^2$$

Thus, given (3.9) we get the following equivalencies:

$$\begin{aligned}\alpha &= \hat{\alpha},\\ k_2 &= \hat{k}_2 + \frac{1}{2}\hat{\alpha}^2\sigma_e^2,\end{aligned}$$

where α and k_2 are unknown parameters. Plugging (3.2)and (3.49) into (3.1), get (3.10). Combine equations (3.6) and (3.7) to get:

$$N_{it} = \left(\frac{\theta-1}{\delta\theta}\left(\mathbb{E}_{it}Q_tY_t^{\frac{1}{\theta}}A_t^{\frac{\theta-1}{\theta}}\right)\right)^{\frac{\delta\theta}{1+\theta\delta-\theta}}$$

Substituting the equilibrium conditions (3.1)-(3.5) and (3.10) into the last equation and using log-normality yields (3.11). The rest consist on computing $\bar{\mathbb{E}}(n_{it})$ and $\mathbb{D}$ from above. Using the definition of n_t, delivers two fixed points for the unknown coefficients α and k_2,

$$\begin{aligned}\alpha &= \frac{\theta\delta}{1+\theta\delta-\theta}\left(\left(\frac{\delta+\alpha}{\delta}\right)\left(\frac{1}{\theta}-\gamma\right)+\frac{\theta-1}{\theta}\right)\frac{\sigma_a^2}{\sigma_a^2+\sigma_e^2}\\ k_2 &= \frac{\theta\delta}{1+\theta\delta-\theta}\log\left(\frac{\theta-1}{\delta\theta}\right)+\frac{\theta\delta}{1+\theta\delta-\theta}\left(\frac{1}{\theta}-\gamma\right)\left(\frac{1}{\delta}k_2-\frac{1}{2}\frac{1+\theta\delta-\theta}{\theta\delta}\frac{\alpha^2}{\delta}\sigma_e^2\right)\\ &\quad+\left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_e^2}\right)^2\phi_1^2\sigma_e^2+\phi_2\frac{\sigma_a^2\sigma_e^2}{\sigma_a^2+\sigma_e^2}\end{aligned}$$

where ϕ_1 and ϕ_2 are given in the main text. After some algebra, these two fixed points can be rewritten as (3.14) and (3.15).

Proofs for 3.2.4 Substituting (3.11) back into profits yields (3.12). Expectations at Stage I, i.e. $\mathbb{E}_{t-1}$, of (3.12) delivers (3.16). Compute the first and second derivatives of (3.12) with respect to the firm's own noise, $\hat{\sigma}_e^2$. After some steps, we get (3.17). It is straightforward to show that profits are decreasing and convex with respect to $\hat{\sigma}_e^2$.

Proofs for 3.2.5 Start with a conjecture for individual employment like (3.19), where $\widetilde{\alpha}$ and $\widetilde{k}_2$ are unknown coefficients to be solved next. Define the household's period utility as:

$$\mathbb{U} \equiv (1-\beta)\,\mathbb{E}_{t-1}\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\gamma}}{1-\gamma}-\int_0^1 N_{it}di\right) \tag{3.50}$$

Using the same steps as in section (3.2.3), it is possible to show that (3.19) implies

$$\begin{aligned} N_t &= \int_0^1 N_{it} di = \widetilde{K}_2 A_t^{\widetilde{\alpha}} , \\ C_t &= \widetilde{K}_2^{\frac{1}{\delta}} A_t^{\frac{\delta+\widetilde{\alpha}}{\delta}} \exp\left(-\frac{1}{2}\left(\frac{1+\theta\delta-\theta}{\theta\delta}\right)\frac{\widetilde{\alpha}^2}{\delta}\sigma_e^2\right) . \end{aligned}$$

Using the relationships derived above, the period utility of the household can be written as (3.20). Define the coefficients α^* and k_2^* as $(\alpha^*, k_2^*) = \arg\max_{\widetilde{k}_2,\tilde{\alpha}} \quad \mathbb{U}(\widetilde{k}_2, \tilde{\alpha})$. Then, from (3.20) we get following set of first order conditions,[15]

$$\log\frac{1}{\delta} + \frac{1-\gamma}{\delta}k_2 + \left((1-\gamma)\left(\frac{\delta+\alpha}{\delta}\right)\right)^2 \frac{\sigma_a^2}{2} - \frac{1-\gamma}{2}\frac{1+\theta\delta-\theta}{\theta\delta}\frac{\alpha^2}{\delta}\sigma_e^2 = k_2 + \frac{\alpha^2}{2}\sigma_a ,$$

$$(1-\gamma)\left(\frac{\delta+\alpha}{\delta}\right)\sigma_a^2 - \frac{1+\theta\delta-\theta}{\theta\delta}\alpha\sigma_e^2 = \alpha\sigma_a^2 .$$

After simplifying these two first order conditions equations we get (3.21) and (3.22).

Proofs for 3.2.6 The following are useful results. First note that the first order condition for $\widetilde{k}_2$ implies:

$$\mathbb{E}_{t-1}\left(C_t\left(k_2^*, \alpha^*\right)\right)^{1-\gamma} = \delta\mathbb{E}_{t-1}N_t\left(k_2^*, \alpha^*\right) \tag{3.51}$$

where $C_t\left(k_2^*, \alpha^*\right)$ and $N_t\left(k_2^*, \alpha^*\right)$ are the levels of consumption and employment under (k_2^*, α^*). Second, given the formula in Proposition 20 it follows that

$$N_t\left(k_2, \alpha\right) = \left(\frac{\theta-1}{\theta}\right)^{\frac{\delta}{(\delta-1+\gamma)}} N_t\left(k_2^*, \alpha^*\right) , \tag{3.52}$$

$$\left(C_t\left(k_2, \alpha\right)\right)^{1-\gamma} = \left(\frac{\theta-1}{\theta}\right)^{\frac{1-\gamma}{(\delta-1+\gamma)}} \left(C_t\left(k_2^*, \alpha^*\right)\right)^{1-\gamma} , \tag{3.53}$$

which hold state by state.Third, combining (3.51) with (3.52)-(3.53) we have:

$$\mathbb{E}_{t-1}\left(C_t\left(k_2, \alpha\right)\right)^{1-\gamma} = \delta\left(\frac{\theta}{\theta-1}\right)\mathbb{E}_{t-1}N_t\left(k_2, \alpha\right) \tag{3.54}$$

[15]It is straightforward to prove that the second order conditions holds.

Note that ex-ante profits are proportional to aggregate employment. Plugging equation (3.8) back into the profits, we get:

$$\Pi_{it} = \frac{1 + \theta\delta - 1}{\theta - 1} N_{it}$$

Expected value at Stage I, i.e. $\mathbb{E}_{t-1}$ delivers the formula for ex-ante profits,

$$\hat{\Pi}\left(k_2, \alpha\right) \equiv \mathbb{E}_{t-1}\Pi_{it} = \frac{1 + \theta\delta - 1}{\theta - 1} N_t\left(k_2, \alpha\right)$$

where $N_t\left(k_2, \alpha\right)$ is given by (3.10). Computing the derivative with respect to noise, we get:

$$\frac{\partial \hat{\Pi}\left(k_2, \alpha\right)}{\partial \sigma_e^2} = -\frac{\left(1 + \theta\delta - \theta\right)}{\theta\delta} \frac{\left(1 - \gamma\right)}{\delta - 1 + \gamma} \hat{\Pi}\left(k_2, \alpha\right) \frac{\alpha^2}{2}$$

The latter is the overall effect of information of profits. Note that, in symmetric equilibrium $\sigma_e^2 = \hat{\sigma}_e^2$, The effect of the firm own noise on profits has the functional form of (3.17). Thus, the overall effect does not coincide with the value of private information. Plugging (3.54) in the definition of $\mathbb{U}$ given in (3.50), we have:

$$\mathbb{U}(k_2, \alpha) = \frac{\delta\theta - \left(1 - \gamma\right)\left(\theta - 1\right)}{\left(1 - \gamma\right)\left(\theta - 1\right)} \mathbb{E}_{t-1} N_t\left(k_2, \alpha\right),$$

which using the previous result about ex-ante profits can be written as:

$$\mathbb{U}(k_2, \alpha) = \frac{\left(\delta - 1\right)\theta + \gamma\left(\theta - 1\right) + 1}{\left(1 - \gamma\right)\left(1 + \theta\delta - \theta\right)} \hat{\Pi}\left(k_2, \alpha\right)$$

It follows that, after using (3.17) in a symmetric equilibrium,

$$\frac{\partial \mathbb{U}(k_2, \alpha)}{\partial \sigma_e^2} = \left(1 + \frac{\delta}{\left(\theta - 1\right)\left(\delta - 1 + \gamma\right)}\right)\left(\frac{\partial \hat{\Pi}(k_2, \alpha)}{\partial \hat{\sigma}_e^2}\right)_{\sigma_e^2 = \hat{\sigma}_e^2} < 0 \ \forall \ \sigma_e^2 \in \mathbb{R}^+$$

The following lemma helps to determine the direction of inefficiency.

Lemma 4 *If $\partial \mathbb{U}/\partial \sigma_e^2$ is greater (smaller) than $\partial \hat{\Pi}/\partial \hat{\sigma}_e^2$, then there is over-acquisition (under-acquisition) of information. If $\partial \mathbb{U}/\partial \sigma_e^2$ equals $\partial \hat{\Pi}/\partial \hat{\sigma}_e^2$, then information acquired in equilibrium is socially optimal.*

Proof. This proof focuses on the region where both the equilibrium and social planner information acquisition problems have an interior optimum. Define σ_{eq}^2 as the information acquired in a symmetric equilibrium

$$\sigma_{eq}^2 \equiv \arg\max_{\hat{\sigma}^2} \hat{\Pi}\left(\sigma^2, \hat{\sigma}^2\right) - \upsilon\left(\hat{\sigma}^2\right) \ .$$

where the subscript "e" in the variance term has been erased for exposition. Recall the first and second order conditions:

$$\begin{aligned} \frac{\partial \hat{\Pi}}{\partial \hat{\sigma}^2} - \frac{\partial \upsilon}{\partial \hat{\sigma}^2} &= 0 \ , \\ \frac{\partial^2 \hat{\Pi}}{\partial \hat{\sigma}^2 \partial \hat{\sigma}^2} - \frac{\partial^2 \upsilon}{\partial \hat{\sigma}^2 \partial \hat{\sigma}^2} &< 0 \ . \end{aligned}$$

Notice that the second order condition indicates that $\partial \upsilon / \partial \hat{\sigma}^2$ crosses $\partial \hat{\Pi} / \partial \hat{\sigma}^2$ from below. In other words, for $\sigma^2 < \sigma_{eq}^2$, $\partial \upsilon / \partial \hat{\sigma}^2 < \partial \hat{\Pi} / \partial \hat{\sigma}^2$ and for $\sigma^2 > \sigma_{eq}^2$, $\partial \upsilon / \partial \hat{\sigma}^2 > \partial \hat{\Pi} / \partial \hat{\sigma}^2$. Define σ_{sp}^2 as the information acquired by the social planner:

$$\sigma_{sp}^2 \equiv \arg\max_{\sigma_e^2} \mathbb{U}\left(\sigma^2\right) - \upsilon\left(\sigma_e^2\right) \ .$$

Recall the first and second order conditions:

$$\begin{aligned} \frac{\partial \mathbb{U}}{\partial \hat{\sigma}^2} - \frac{\partial \upsilon}{\partial \hat{\sigma}^2} &= 0 \ , \\ \frac{\partial^2 \mathbb{U}}{\partial \hat{\sigma}^2 \partial \hat{\sigma}^2} - \frac{\partial^2 \upsilon}{\partial \hat{\sigma}^2 \partial \hat{\sigma}^2} &< 0 \ . \end{aligned}$$

Note that, because $\partial \upsilon / \partial \hat{\sigma}_e^2 < 0$, the first condition for an interior solution requires $\partial \mathbb{U} / \partial \sigma^2 < 0$. Also notice that the second order condition indicates that $\partial \upsilon / \partial \sigma^2$ crosses $\partial \mathbb{U} / \partial \hat{\sigma}^2$ from below. In other words, for $\sigma^2 < \sigma_{sp}^2$, $\partial \upsilon / \partial \sigma^2 < \partial \mathbb{U} / \partial \sigma^2$ and for $\sigma^2 > \sigma_{sp}^2$,

$\partial v/\partial\sigma^2 > \partial\mathbb{U}/\partial\sigma^2$. Given all these properties, the following is true

$$\frac{\partial\hat{\Pi}\left(\sigma^2,\hat{\sigma}^2\right)}{\partial\hat{\sigma}^2}|_{\hat{\sigma}^2=\sigma^2=\sigma^2_{eq}} < \frac{\partial\mathbb{U}\left(\sigma^2\right)}{\partial\sigma^2}|_{\sigma^2=\sigma^2_{eq}} \Rightarrow \sigma^2_{eq} < \sigma^2_{sp} \ .$$

To prove this, suppose to the contrary that $\sigma^2_{eq} > \sigma^2_{sp}$. Then, by definition of σ^2_{eq}

$$\frac{\partial\hat{\Pi}\left(\sigma^2,\hat{\sigma}^2\right)}{\partial\hat{\sigma}^2}|_{\hat{\sigma}^2=\sigma^2=\sigma^2_{eq}} = \frac{\partial v\left(\hat{\sigma}^2\right)}{\partial\hat{\sigma}^2}|_{\hat{\sigma}^2=\sigma^2=\sigma^2_{eq}} \ .$$

From the social planner problem, if $\sigma^2_{eq} > \sigma^2_{sp}$

$$\frac{\partial v\left(\hat{\sigma}^2\right)}{\partial\hat{\sigma}^2}|_{\hat{\sigma}^2=\sigma^2=\sigma^2_{eq}} > \frac{\partial\mathbb{U}\left(\sigma^2\right)}{\partial\sigma^2}|_{\sigma^2=\sigma^2_{eq}} \ .$$

Contradiction. ■

Proofs for 3.2.7 Following the same steps as in 3.2.3, the law of motion of total employment under τ_R is,

$$n_t = \frac{\theta\delta}{1+\theta\delta-\theta}\log\left(1+\tau_R\right) + k_2 + \alpha a_t.$$

To implement $n_t\left(k_2^*,\alpha^*\right)$, it follows that $\tau_R = 1/\theta$.
Next we compute the social value of information under τ_R. Since τ_R is such that the response coefficients are $\left(k_2^*,\alpha^*\right)$ the social value information can be computed directly from (3.20) evaluated at the optimum, i.e. $\widetilde{k}_2 = k_2^*$ and $\widetilde{\alpha} = \alpha^*$. The envelope condition implies, after using (3.51)

$$\frac{\partial\mathbb{U}\left(k_2^*,\alpha^*\right)}{\partial\sigma_e^2} = -\frac{1+\theta\delta-\theta}{\theta\delta}\frac{\alpha^{*2}}{2}\mathbb{E}_{t-1}N_t\left(k_2^*,\alpha^*\right)$$

Plugging equations (3.25) back into profits and use $\tau_R = 1/\theta$, we get:

$$\Pi_{it} = \frac{1+\theta\delta-1}{\theta-1}N_{it}$$

Expected value at Stage I, i.e. $\mathbb{E}_{t-1}$ delivers the formula for ex-ante profits,

$$\hat{\Pi}\left(k_2^*,\alpha^*\right) \equiv \mathbb{E}_{t-1}\Pi_{it} = \frac{1+\theta\delta-1}{\theta-1} N_t\left(k_2^*,\alpha^*\right) \ ,$$

which holds state by state.

These results imply that:

$$\frac{\partial \mathbb{U}\left(k_2^*,\alpha^*\right)}{\partial \sigma_e^2} = -\frac{\theta-1}{\theta\delta}\frac{\alpha^{*2}}{2}\hat{\Pi}\left(k_2^*,\alpha^*\right),$$

which exactly the functional form of the private value of information given in (3.17), evaluated at $\left(k_2^*,\alpha^*\right)$.

Externalities in the quantity choice model Note that a firm's expected profits are decreasing in its own noise $\hat{\sigma}_e^2$,

$$\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2} < 0 \ .$$

The *team profit*, denoted by $\hat{\Pi}^T$, is the combined expected profit earned by all firms in the stage II equilibrium. As a team, firms can collude in their investment of information. The difference is that under collusion, firms are concerned about the overall effect of information on the team profit. This value can be represented by the total derivative of the team profits with respect to the noise of the signal σ_e^2. In a symmetric outcome this derivative is the sum of two factors

$$\frac{d\hat{\Pi}^T}{d\sigma_e^2} = \left(\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2}\right)_{\hat{\sigma}_e^2=\sigma_e^2} + \frac{\partial \hat{\Pi}}{\partial \sigma_e^2} \ . \tag{3.55}$$

The first factor corresponds to the derivative of a firm's expected profits evaluated at the symmetric outcome, i.e. $\hat{\sigma}_e^2 = \sigma_e^2$. The second factor corresponds to the externality.

We have

$$\frac{d\hat{\Pi}^T}{d\sigma_e^2} = \frac{(1+\theta\delta-\theta)}{\theta-1}\frac{(1-\gamma)}{\delta-1+\gamma}\left(\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2}\right)_{\hat{\sigma}_e^2=\sigma_e^2}$$

Thus, if $\gamma < 1$, then $d\hat{\Pi}^T/d\sigma_e^2 < 0$. Moreover,

$$\text{if } \frac{1+\theta\delta-\theta}{\theta-1}\frac{1-\gamma}{\delta-1+\gamma} > 1 \Longrightarrow \frac{d\hat{\Pi}^T}{d\sigma_e^2} < \left(\frac{\partial\hat{\Pi}}{\partial\hat{\sigma}_e^2}\right)_{\hat{\sigma}_e^2=\sigma_e^2} ,$$

the team's value of information at σ_e^2 is higher than individual firm's value of information. And,

$$\text{if } \frac{1+\theta\delta-\theta}{\theta-1}\frac{1-\gamma}{\delta-1+\gamma} < 1 \Longrightarrow \left(\frac{\partial\hat{\Pi}}{\partial\hat{\sigma}_e^2}\right)_{\hat{\sigma}_e^2=\sigma_e^2} < \frac{d\hat{\Pi}^T}{d\sigma_e^2} < 0 ,$$

the team's value of information at σ_e^2 is lower than individual firm's value of information. On the other hand, if $\gamma > 1$, then $d\hat{\Pi}^T/d\sigma_e^2 > 0$ and the team's value of information at σ_e^2 is zero.

With respect to the externality, from equation (3.55) we have that,

$$\text{if } \frac{d\hat{\Pi}^T}{d\sigma_e^2} < \left(\frac{\partial\hat{\Pi}}{\partial\hat{\sigma}_e^2}\right)_{\hat{\sigma}_e^2=\sigma_e^2} \Longrightarrow \frac{\partial\hat{\Pi}}{\partial\sigma_e^2} < 0,$$

and the externality is positive, i.e. more information acquired by others, the higher is payoff to the firm.

On the other hand,

$$\text{if } \frac{d\hat{\Pi}^T}{d\sigma_e^2} > \left(\frac{\partial\hat{\Pi}}{\partial\hat{\sigma}_e^2}\right)_{\hat{\sigma}_e^2=\sigma_e^2} \Longrightarrow \frac{\partial\hat{\Pi}}{\partial\sigma_e^2} > 0,$$

and the externality is positive, i.e. more information acquired by others, the higher is payoff to the firm. One special case is when $d\hat{\Pi}^T/d\sigma_e^2 > 0$. In such a case, the externality is strong enough to discourage investment in information completely in the collusive outcome.

3.4.2 A Price Setting Model

Proof of Lemma 1 From the ex-ante expected profit (3.41) and given that $\phi_1 < 0$, $d\phi_1/d\alpha > 0$ and $d\phi_2/d\alpha < 0$,

$$\frac{\partial\hat{\Pi}}{\partial\alpha} = \hat{\Pi}\left(\phi_1\frac{d\phi_1}{d\alpha}\sigma_u^2 + \frac{d\phi_2}{d\alpha}\hat{\sigma}_e^2\right)\frac{\sigma_u^2}{\sigma_u^2+\hat{\sigma}_e^2} < 0 .$$

From the ex-ante expected profit (3.41),

$$\frac{\partial \hat{\Pi}}{\partial k_2} = (\theta - 1)(1 - r)\frac{\hat{\Pi}}{K_2} > 0 \, .$$

Proof of Lemma 2 After plugging α, equation (3.38), into k_2, equation (3.39),

$$k_2 = \frac{1}{\delta}\ln\left(\frac{\gamma\theta}{\theta - 1}\right) + \frac{\delta\sigma_u^2}{2(1 - r)}\left(\frac{\sigma_e^2\left((1 - r)\theta\sigma_u^2 + \sigma_e^2\right)}{\left((1 - r)\sigma_u^2 + \sigma_e^2\right)^2}\right) \, .$$

Hence:

$$\frac{dk_2}{d\sigma_e^2} = \frac{\delta\sigma_u^2}{2}\left(\frac{(1 - r)\theta\sigma_u^2 + (2 - \theta)\sigma_e^2}{\left((1 - r)\sigma_u^2 + \sigma_e^2\right)^3}\right) \, .$$

If $\theta \leq 2$,

$$\frac{dk_2}{d\sigma_e^2} > 0 \ \forall \sigma_e^2 \, .$$

If $\theta > 2$, the effect can be non-monotonic, i.e.

$$\frac{dk_2}{d\sigma_e^2} > 0 \qquad \text{if} \qquad \sigma_e^2 < \frac{(1 - r)\theta}{(\theta - 2)}\sigma_u^2 \, .$$

Proof of Lemma 3 From equation (3.42) and using Lemma 1 and $\phi_1 < 0$, $\phi_2 < 0$, $d\phi_1/d\alpha > 0$ and $d\phi_2/d\alpha < 0$,

$$\begin{aligned}
\frac{\partial^2 \hat{\Pi}}{\partial\alpha\partial\hat{\sigma}_e^2} &= \hat{\Pi}\left(-\phi_1\frac{d\phi_1}{d\alpha} + \frac{d\phi_2}{d\alpha}\right)\left(\frac{\sigma_u^2}{\sigma_u^2 + \hat{\sigma}_e^2}\right)^2 + \frac{\partial \hat{\Pi}}{\partial\alpha} < 0 \, , \\
\frac{\partial^2 \hat{\Pi}}{\partial k_2\partial\hat{\sigma}_e^2} &= \frac{\partial \hat{\Pi}}{\partial k_2}\left(-\frac{\phi_1^2}{2} + \phi_2\right)\left(\frac{\sigma_u^2}{\sigma_u^2 + \hat{\sigma}_e^2}\right)^2 < 0 \, .
\end{aligned}$$

Proof of Proposition 31

Follows directly from Lemma 3 along with $d\alpha/d\sigma_e^2 < 0$.

Proof of Proposition 24 Suppose firms commit to follow this rule

$$p_{it} = m_{t-1} + \tilde{k_2} - \frac{1 - \theta}{2}\tilde{\alpha}^2\sigma_e^2 + \tilde{\alpha}s_{it} \, .$$

Average prices are given by

$$p_t = m_{t-1} + \tilde{k_2} + \tilde{\alpha}u_t \, .$$

The utility of the household can be expressed as

$$\mathbb{U} = -\tilde{k_2} - \frac{1}{\delta}\exp\left\{-\delta\tilde{k_2} + \frac{\delta^2}{2}\left[\frac{\theta}{(1-r)}\tilde{\alpha}^2\sigma_\epsilon^2 + (1-\tilde{\alpha})^2\sigma_u^2\right]\right\} .$$

Then, the optimal use of information is defined by

$$(\alpha^*, k_2^*) = \text{argmax}_{\tilde{k_2},\tilde{\alpha}} \quad \mathbb{U}(\tilde{k_2}, \tilde{\alpha}) .$$

The focs

$$\begin{aligned} \frac{2\theta}{(1-r)}\tilde{\alpha}\sigma_\epsilon^2 - 2(1-\tilde{\alpha})^2\sigma_u^2 &= 0 , \\ -1 - \exp\left\{-\delta\tilde{k_2} + \frac{\delta^2}{2}\left[\frac{\theta}{(1-r)}\tilde{\alpha}^2\sigma_\epsilon^2 + (1-\tilde{\alpha})^2\sigma_u^2\right]\right\} &= 0 . \end{aligned}$$

It is straightforward to show that the solution to this problem is characterized by

$$\begin{aligned} \alpha^* &= \frac{(1-r)\sigma_u^2}{(1-r)\sigma_u^2 + \theta\sigma_e^2} , \\ k_2^* &= \frac{\delta\theta}{2(1-r)}\alpha^*\sigma_e^2 . \end{aligned}$$

For a fixed σ_e^2, it follows that α^* is lower than the equilibrium α. Also, for a fixed σ_e^2, the price level under the socially efficient information use of information is lower than the price level under the equilibrium use of information. To prove this, suppose

$$k_2 < k_2^* .$$

or, after replacing the values for k_2 and k_2^*

$$\frac{1}{\delta}\ln\left(\frac{\gamma\theta}{\theta-1}\right) + (\delta-1)(1-r)^2\theta\left(\sigma_u^2\right)^2 + (\delta-1)\theta\left(\sigma_e^2\right)^2 + (1-r)\left(\delta\left(\theta^2+1\right)-2\right)\sigma_u^2\sigma_e^2 < 0 .$$

which cannot be true. By contradiction, for a fixed σ_e^2, $k_2 > k_2^*$.

Proof of Proposition 25 From equation (3.47),

$$\frac{\partial \mathbb{U}}{\partial \sigma_e^2} = -\frac{\delta (1-r) (\sigma_u^2)^2}{2} \left[\frac{(1-r)\, \theta \sigma_u^2 + (2-\theta)\, \sigma_e^2}{((1-r)\, \sigma_u^2 + \sigma_e^2)^3} \right] .$$

Note that if $\theta \leq 2$, then $\partial \mathbb{U}/\partial \sigma_e^2 < 0\ \forall \sigma_e^2$. If $\theta > 2$ the sign of $\partial \mathbb{U}/\partial \sigma_e^2$ depends on σ_e^2. If $\sigma_e^2 < (1-r)\theta\sigma_u^2/\left(\theta-2\right)$, then $\partial \mathbb{U}/\partial \sigma_e^2 < 0$. Otherwise, $\partial \mathbb{U}/\partial \sigma_e^2 > 0$.

Proof of Proposition 26 There is under acquisition of information in equilibrium if

$$\frac{\partial \mathbb{U}}{\partial \sigma_e^2} < \frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2} ,$$

or

$$((\theta-1) + \gamma (\theta-2))\, \sigma_e^2 < (1-r)\, (\theta\gamma - (\theta-1))\, \sigma_u^2 . \tag{3.56}$$

The proof focuses on the case of socially valuable information, i.e. $\frac{d\mathbb{U}}{d\sigma_e^2} < 0$.

- If $\theta > 2$ and $\sigma_e^2 < (1-r)\, \theta\sigma_u^2/\left(\theta-2\right)$, there are two cases.

  Case 1A: Suppose, $\theta\gamma - (\theta-1) > 0$, then condition (3.56) requires

$$\sigma_e^2 < (1-r) \left(\frac{\theta\gamma - (\theta-1)}{(\theta-1) + \gamma(\theta-2)} \right) \sigma_u^2 .$$

  Suppose that

$$\begin{aligned} \frac{(1-r)\,\theta}{(\theta-2)} \sigma_u^2 &< (1-r) \left(\frac{\theta\gamma - (\theta-1)}{(\theta-1) + \gamma(\theta-2)} \right) \sigma_u^2 \\ &\Rightarrow \theta(\theta-1) < -(\theta-2)(\theta-1) \end{aligned}$$

  which cannot be true.

  There is under acquisition of information if

$$\sigma_e^2 < (1-r) \left(\frac{\theta\gamma - (\theta-1)}{(\theta-1) + \gamma(\theta-2)} \right) \sigma_u^2 ,$$

  and there over acquisition of information if

$$\sigma_e^2 \geq (1-r) \left(\frac{\theta\gamma - (\theta-1)}{(\theta-1) + \gamma(\theta-2)} \right) \sigma_u^2 .$$

Case 1B: Suppose, $\theta\gamma - (\theta - 1) < 0$, then condition (3.56) information is always over acquired in equilibrium.

- If $\theta < 2$, there are three cases.

  Case 2A: Suppose $\theta\gamma - (\theta - 1) < 0$ and $(\theta - 1) + \gamma (\theta - 2) > 0$ or $\gamma \in (0\ ,\ \ (\theta - 1)/\theta)$, then there is always over acquisition in equilibrium.

  Case 2B: Suppose $\theta\gamma - (\theta - 1) > 0$ and $(\theta - 1) + \gamma (\theta - 2) > 0$ or $\gamma \in ((\theta - 1)/\theta\ ,\ \ (\theta - 1)/(2 - \theta))$ There is under acquisition of information if

$$\sigma_e^2 < (1 - r)\left(\frac{\theta\gamma - (\theta - 1)}{(\theta - 1) + \gamma(\theta - 2)}\right)\sigma_u^2 \ ,$$

  and there over acquisition of information if

$$\sigma_e^2 \geq (1 - r)\left(\frac{\theta\gamma - (\theta - 1)}{(\theta - 1) + \gamma(\theta - 2)}\right)\sigma_u^2 \ .$$

  Case 2C: Suppose $\theta\gamma - (\theta - 1) > 0$ and $(\theta - 1) + \gamma (\theta - 2) < 0$ or $\gamma \in ((\theta - 1)/(2 - \theta), \infty)$, then there is always under acquisition in equilibrium.

Proof of Proposition 27 Suppose firms use information efficiently at Stage II. Then, a firm profit at Stage I can be written as

$$\hat{\Pi}\left(\hat{\sigma}_e^2, \alpha^*, k_2^*\right) = 1 - \frac{\gamma}{\delta}\exp\left\{-\delta k_2^* + \frac{\delta^2}{2}\left[\frac{\theta}{(1 - r)}\alpha^{*2}\hat{\sigma}_e^2 + (1 - \alpha^*)^2\sigma_u^2\right]\right\} \ .$$

At the symmetric outcome, the marginal value of information for an individual firm, after replacing k_2^*, is:

$$\frac{\partial\hat{\Pi}}{\partial\hat{\sigma}_e^2} = -\frac{\gamma}{2}\frac{\delta\theta}{(1 - r)}\alpha^{*2} < 0 \ .$$

Notice that when information is used efficiently, the firm's marginal value of information to its own information choice is constant, i.e. $\partial^2\hat{\Pi}/\partial\hat{\sigma}_e^2\partial\hat{\sigma}_e^2 = 0$.

The household's lifetime utility when information is used efficiently (after replacing k_2^* and α^*) is

$$\mathbb{U} = -\frac{\delta\theta}{2(1 - r)}\alpha^*\sigma_e^2 - \frac{1}{\delta} \ .$$

The marginal value of information for the social planner is given by

$$\frac{\partial \mathbb{U}}{\partial \sigma_e^2} = -\frac{\delta}{2}\frac{\theta}{(1-r)}\alpha^{*2} < 0$$

Also, note that

$$\frac{\partial \mathbb{U}}{\partial \hat{\sigma}_e^2} = \frac{1}{(1-r)}\frac{\delta\theta^2\alpha^{*2}}{\left((1-r)\,\sigma_u^2 + \theta\sigma_e^2\right)^2} > 0$$

Using Lemma 4, if the marginal value of information to the social planner is smaller (greater) than the marginal value of information to the firm, then there is over-acquisition (under-acquisition) of information in equilibrium. Over-acquisition of information in equilibrium happens when $\partial \mathbb{U}/\partial \sigma^2$ greater than $\partial \hat{\Pi}/\partial \sigma^2$. It is easy to see that this is true so long as $\gamma > 1$. If this condition does not hold, there is under-acquisition of information.

Proof of Proposition 28 Note that a firm's expected profits are decreasing in its own noise $\hat{\sigma}_e^2$,

$$\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2} < 0\ .$$

Compute the derivative of the team profits with respect to the noise of the signal σ_e^2. In a symmetric outcome this derivative is the sum of two factors

$$\frac{d\hat{\Pi}^T}{d\sigma_e^2} = \frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2}|_{\hat{\sigma}_e^2 = \sigma_e^2} + \frac{\partial \hat{\Pi}}{\partial \sigma_e^2}\ . \tag{3.57}$$

The first factor corresponds to the derivative of a firm's expected profits evaluated at the symmetric outcome, i.e. $\hat{\sigma}_e^2 = \sigma_e^2$. The second factor corresponds to the externality.

Combine Proposition 31 with the expression from equation (3.57) so that in a symmetric equilibrium:

$$\begin{aligned} \frac{d\hat{\Pi}^T}{d\sigma_e^2} &= \frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2}|_{\hat{\sigma}_e^2 = \sigma_e^2} + \frac{\partial \hat{\Pi}}{\partial \sigma_e^2} = 0 \quad \forall \sigma_e^2 \\ &\Rightarrow \frac{\partial \hat{\Pi}}{\partial \sigma_e^2} = -\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2}|_{\hat{\sigma}_e^2 = \sigma_e^2} > 0\ . \end{aligned}$$

Hence, there is a negative externality in any symmetric equilibrium.

Proof of Proposition 29 Plug α from equation (3.38) into the ex-ante expected profit (3.41). The team profits are given by

$$\Pi^T = e^{(\theta-1)(1-r)\frac{1}{\delta}\log\left(\frac{\gamma\theta}{\theta-1}\right)} K_1 = \frac{1}{\gamma\theta\,(1-r)}\ ,$$

which does not depend on σ^2, i.e.

$$\frac{d\hat{\Pi}^T}{d\sigma_\epsilon^2} = 0\ .$$

Proof of Proposition 30 The proof is analogous to the proof for Proposition 31. Team profits are obtained after replacing (α^*, k_2^*) into $\hat{\Pi}\left(\hat{\sigma}_e^2, \alpha^*, k_2^*\right)$,

$$\Pi^T = 1 - \frac{\gamma}{\delta}\ ,$$

which requires $\delta > \gamma$ to guarantee positive profits. Hence the marginal value of information is:

$$\frac{d\Pi^T}{d\sigma_e^2} = 0\ .$$

To prove that sign of the externality, we proceed as in the proof for Proposition 28.

Proof of Proposition 32 From equation (3.42):

$$\frac{\partial^2\hat{\Pi}}{\partial\sigma_e^2\partial\hat{\sigma}_e^2} = \left(\frac{\sigma_u^2}{\sigma_u^2+\hat{\sigma}_e^2}\right)^2 \left\{\hat{\Pi}\left(-\phi_1\frac{d\phi_1}{d\sigma_e^2} + \frac{d\phi_2}{d\sigma_e^2}\right) + \frac{\partial\hat{\Pi}}{\partial\hat{\sigma}_e^2}\left(-\frac{\phi_1^2}{2}+\phi_2\right)\right\}\ .$$

Using Proposition 28, in any symmetric outcome

$$\frac{\partial\hat{\Pi}}{\partial\hat{\sigma}_e^2} = -\hat{\Pi}\left(-\frac{\phi_1^2}{2}+\phi_2\right)\left(\frac{\sigma_u^2}{\sigma_u^2+\sigma_e^2}\right)^2\ .$$

Hence

$$\frac{\partial^2\hat{\Pi}}{\partial\sigma_e^2\partial\hat{\sigma}_e^2} = \left(\frac{\sigma_u^2}{\sigma_u^2+\hat{\sigma}_e^2}\right)^2 \hat{\Pi}\left\{\left(-\phi_1\frac{d\phi_1}{d\sigma_e^2} + \frac{d\phi_2}{d\sigma_e^2}\right) - \left(-\frac{\phi_1^2}{2}+\phi_2\right)^2\left(\frac{\sigma_u^2}{\sigma_u^2+\sigma_e^2}\right)^2\right\}\ ,$$

where

$$-\phi_1\frac{d\phi_1}{d\sigma_e^2} + \frac{d\phi_2}{d\sigma_e^2} = \frac{(\theta-1)^2(1-r)^3\,\theta\,(\delta-1)\,\sigma_u^2\left(\sigma_e^2+\sigma_u^2\right)}{\left((1-r)\,\sigma_u^2+\sigma_e^2\right)^3} > 0\ .$$

Plug the previous expression into $d^2\hat{\Pi}/d\sigma_e^2 d\hat{\sigma}_e^2$

$$\frac{\partial^2\hat{\Pi}}{\partial\sigma_e^2\partial\hat{\sigma}_e^2} = \frac{1}{4}\left(-\phi_1\frac{d\phi_1}{d\sigma_e^2}+\frac{d\phi_2}{d\sigma_e^2}\right)\left\{\frac{(1-r)\left(4(\delta-1)-\delta^2\theta\sigma_u^2\right)\sigma_u^2+\left(4(\delta-1)-(1-r)\delta^2\theta\sigma_u^2\right)\sigma_e^2}{(\delta-1)\left((1-r)\sigma_u^2+\sigma_e^2\right)}\right\}.$$

Note that if $\sigma_u^2 < 4(\delta-1)/\delta^2\theta$, then decisions are strategic complements, i.e. $\partial^2\hat{\Pi}/\partial\sigma_e^2\partial\hat{\sigma}_e^2 > 0$. On the other hand, if $4(\delta-1)/(1-r)\delta^2\theta < \sigma_u^2$ then decisions are strategic substitutes, i.e. $\partial^2\hat{\Pi}/\partial\sigma_e^2\partial\hat{\sigma}_e^2 > 0$. Finally, if $4(\delta-1)/\delta^2\theta < \sigma_u^2 < 4(\delta-1)/(1-r)\delta^2\theta$, then the sign of $\partial^2\hat{\Pi}/\partial\sigma_e^2\partial\hat{\sigma}_e^2$ depends on the value of σ_e^2.

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