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Abstract

It is shown that all the phase transitions in and out of freely flowing traffic reported earlier for a German site could be caused by bottlenecks, as are all the transitions observed at two other sites examined here. Furthermore, all the evidence indicates that bottlenecks cause these transitions in a predictable way, and no evidence is found that stoppages (jams) appear spontaneously in free flow traffic for no apparent reason. The most salient phenomena observed at all locations are explained in terms of a simple theory specific to traffic.

1. CAUSES

This report provides additional empirical observations and an alternative explanation of some traffic data from a German highway [1, 2, 3]. A few studies of North American freeways have also identified related traffic flow properties and the origin of traffic disturbances [4, 5]. Reassuringly, the observations on both continents are consistent with each other. However, it will also be shown that neither the German nor the North American data support the conclusion in [1]: that free flowing traffic will spontaneously break down randomly, without obvious reasons, and then remain in that state due to traffic's tendency to self-maintain congestion. Rather, all the evidence indicates that traffic "breaks down" (queues form) at locations of freeway inhomogeneities (bottlenecks) due to reproducible exogenous reasons, and that, following breakdown, the bottleneck flow behaves in a predictable way. Furthermore, there is no evidence that queues self-maintain away from a bottleneck [6]. To the contrary, all indications are that highways behave like crowds of people going through a series of queues at well defined locations; e.g., fans who wait in line to buy football tickets and then queue through one or more gates to enter the stadium. Some definitions derived from this analogy are introduced below; see e.g., [8] for additional explanations.

Free flow (or *unqueued*) conditions refer to that state of traffic where small disturbances can only flow forward in space (e.g., as in supersonic gas flow). People walking away from the ticket window, for example, are in free flow because if one of them momentarily slows down just a little, this does not affect the queue upstream of the server. The free flow status of highway traffic can be determined by correlating the curves of cumulative flow at successive detectors with a time lag, as described in

[4] and shown below. Conditions that are not free flow will be called *queued*. In queued traffic, disturbances can and do flow upstream, and this can also be determined from the set of cumulative curves. *Bottlenecks* are those inhomogeneous locations such as a ticket window where queues can form and persist with free flow downstream. A bottleneck in this state will be said to be *active*. Note that a bottleneck may be deactivated by a queue from a downstream bottleneck that blocks it and also when its own queue dissipates. If the flow through an active bottleneck is nearly constant and reproducible this flow will be called the bottleneck *capacity*.

Queues may exhibit instabilities. For example, a football fan who is queued some distance from the ticket window may occasionally allow large gaps to form in front of her before advancing (causing the queue to advance in fits and starts) because she knows that this sporadic motion will not delay her ticket purchase. The ticket agent may thus serve the queue at a regular rate. Note too that football fans do not choose to wait in line due to some innate tendency to self-maintain their queued condition, but rather because the bottleneck does not have the "capacity" to serve them rapidly enough. It will be shown that the evidence available in German and North American traffic data contains nothing at all to contradict this analogy; i.e., that queues grow when bottlenecks become active and that instabilities are the result and not the cause of queues.

Merges: Included here is evidence that traffic instabilities upstream of an active merge bottleneck appear not to influence the bottleneck flow significantly, i.e., one can define a "merge capacity", that queues grow shortly after the merging flows exceed this capacity, and that queues dissipate predictably. This explains the breakdown described in Figs. 1, 2 and 3 of [1], as well as those observed on North American sites.

Figure 2(b) of [1] shows that at 7:16 AM a drop in the time series of measured speeds for the passing lanes was recorded at the first detector (D3) downstream of the merge, while the speeds upstream and downstream of it remained higher. This change in speeds was later detected upstream and downstream, suggesting that the bottleneck became active and a queue began to grow near detector D3 at around 7:16 AM. The sharper changes in speed observed later at upstream locations are what one would expect if the back end of such a queue was growing in the upstream direction; it passed over detector D2 at 7:22 AM and detector D1 later. It is further stated in [1] that the "phase change" (i.e., the onset of queueing) was brought about by an increase in the on-ramp flow and that

the phase change was self-maintained for about two hours.

Interestingly, this is the prediction of the merge bottleneck theory in [9]. This theory holds that when a queue is generated on the freeway, the bottleneck flow stabilizes at a predictable level and is maintained until the freeway queue dissipates. The data presented in [1] suggests that the fluctuations in ramp flow are absorbed by the mainline flow, as predicted by the theory. Incomplete proof of this is contained in Fig. 2(e) of [1], which presents the rather stable time-series of flow in the left lane at the two detectors immediately downstream of the merge. The figure shows that these average flows did not change appreciably after the onset of queueing, suggesting that the cumulative number of vehicles entering the merge after this event did not change significantly either.

Far from exceptional, the behavior of the German bottleneck was reproduced qualitatively at a North American site analyzed in [5], and the main results are summarized below. The analysis is based on cumulative flow curves because they are more informative than the time series data of counts and speed used in [1]. The results are also presented in a form suitable for comparison with [1].

The analyzed site is a segment of the Queen Elizabeth Way in Toronto, Canada, with the geometry of Fig. 1(a). This freeway is instrumented with detectors that record vehicle counts, occupancies (i.e., the "dimensionless" measure of density obtained by loop detectors [8]) and time averaged speeds over 30-second intervals. (The labels shown by the detector stations are those used by the Ontario Ministry of Transport.) Typical of several other days that were analyzed at this location, the data shown here were gathered on the morning of May 3, 1995.

Figure 1(b) presents transformed N-curves of cumulative flow (in all lanes) versus time for the 4 detector locations of our site during the onset of queueing. Note that an untransformed N-curve gives the cumulative number of vehicles to have passed detector station i by time t, starting the counts (N = 0) with the passage of a reference vehicle. Thus, horizontal separations between N-curves are trip times and vertical separations the accumulation between detectors [8, 10]. In our figure, the curves (along with their respective time axes) have been shifted to the right by the average free flow trip time between the respective detector and detector 25, so that the vertical separation between curves now represents the excess vehicular accumulation between detectors due to vehicular delays. Such a shift is advantageous because two superimposed curves indicate that traffic in the intervening segment is freely flowing—since every feature of the upstream N-curve is passed to its downstream neighbor

later. In addition, the figure only shows the difference between each curve and the line $N = q_o t$ because this background flow reduction magnifies details without changing the excess accumulations [4]. The superimposed curve portions in this figure indicate that traffic was initially in free flow and remained in free flow between detectors 24 and 25. The marked separation of curves 24/25 from curve 23 from 6:29 AM onward (as shown by the arrow) indicates that a bottleneck was activated a little earlier between detectors 24 and 23. The subsequent separation of curve 23 from curve 22 indicates when the queue arrived to detector 23.

Figure 1(c) presents the transformed N-curves for stations 22 and 25 for the entire rush. (Additional tests explained in [5] confirmed that the bottleneck was active during this time.) Note the persistent displacement between these curves (i.e., the queue) while the bottleneck was active. As shown by the dashed line, the queue discharge rate measured at detector 25 varied slowly in time, but N never deviated by more than about 50 vehicles from this trend line with average rate 6,470 vph. Also evident in the figure is the maximum flow of 6,970 vph that persisted for 12.5 minutes and appears to have triggered the queue's formation. No surges or large fluctuations in the on-ramp flows appear to have accompanied this event. This is evident in Fig. 1(d); the transformed N-curve from Cawthra Road never deviated by more than about 10 vehicles from a trend line with average flow 770 vph.

Repeated observations at this and at another freeway merge in Toronto indicated that queue formations were always accompanied by brief periods of excessively high arrival rates and that the average discharge flows were reproducible from day to day [5]. Having demonstrated that the Toronto queue formed due to a bottleneck, we next show that the German data are qualitatively similar to Toronto's and thus, the former likely describes the activation of a merge bottleneck as well.

Figure 2 presents time-series plots of the average vehicle speeds at the Toronto site measured over 1-minute intervals in each travel lane and at each of the four detectors, as was done in [1]. As on the German site, speeds upstream of the bottleneck at detectors 23 and 22 (Figs. 2(c) and 2(d) respectively) drop markedly during the "phase change" to the queued state. Also reported in [1], speeds dropped downstream of the merge when the queue formed (as occurred here at station 24 by 6:29 AM) with smaller reductions exhibited a little later further downstream (as occurred here at station 25). This is logical; it indicates that vehicles gradually accelerated after discharging from the

bottleneck queue at an average rate equal to its capacity, as shown in Fig. 1(c) of this report and in Fig. 3(a) of [1].

Figure 3 presents scatter plots of 1-minute occupancies (densities) versus flows in the left-most travel lane of the Toronto site. Unshaded, smaller circles denote observations subsequent to the arrival of the queue formation waves at each location. Reference [1] demonstrates that the 1-minute flow-density data fluctuate markedly upstream of the merge, and this can also be seen here at station 22 (Fig. 3(a)). The reference also contains plots similar to Figs. 3(b) and 3(c), again indicating that vehicles gradually accelerated downstream of the bottleneck. In fact, Figs. 2 and 3 are so similar to Figs. 2(a-d) and 3(c) of [1] there is little doubt that the German data describe a bottleneck formed by merging vehicles. Thus, one cannot claim based on the supplied evidence that a spontaneous local breakdown occurred.

Diverges: It is further shown in [1] (Fig. 4) that a phase transition into the queued state also occurred at a different location and time. This disturbance was characterized as spontaneous and due to a random occurrence because it formed away from a merge. No efforts appear to have been made to identify its source even though the disturbance started next to an off-ramp with much lower speeds in the shoulder lanes at locations near the exit (see detector D9 in Fig. 4 of [1]) which is a "signature" of a brief interruption caused by an oversaturated (i.e. queued) diverge. The disturbance then focused and propagated as predicted with the queue instability theory in [11].

A diverge on a narrow freeway is not qualitatively different from a single line of cars at an uncontrolled T-junction. Here, a left- turning car may force everyone behind to wait (e.g. because of opposing traffic) even if it is safe for other vehicles to proceed. Likewise, if a freeway off-ramp cannot accept the traffic wishing to exit, a queue entrapping some through vehicles will form on the freeway and reduce the flow, perhaps even blocking all the lanes further upstream. These effects are more pronounced if flows are close to saturation and the freeway is narrow as in [1]. A mathematical theory of the diverge can be found in Secs. 3.3 and 4 of [12], and a preliminary theory for wide freeways where traffic may sort itself by lane depending on destination can be found in [13].

The onset of queueing next to the off-ramp can be triggered in at least two ways: 1) a queue from the off-ramp spills-over and blocks the freeway traffic and 2) the off-ramp is unqueued but an increase in the exit flow greater than the off-ramp capacity creates a freeway queue. According to [12] a

recovery wave like that of Fig. 4 of [1] should be issued from the off-ramp in the latter case if the flow of exiting vehicles approaching the off-ramp drops below the saturation level of the ramp. Note that the effects due to 2) would occur *even if the rate at which vehicles arrived to the queue and the exit ramp flow remained nearly constant*; i.e., they could be interpreted as occurring for no apparent reason. The information in Fig. 4 of [1] does not include the ramp flows, but from the node conservation law it appears that both the freeway flows (at station D9) and the ramp flows (the difference between the flows of D9 and D10) were close to saturation prior to the genesis of the disturbance. Its short duration suggests a disturbance of type 2, although it could also be due to exiting traffic (perhaps involving trucks) that required some lane changing. The development of this disturbance into a stoppage that propagates upstream is consistent with the theory proposed in [11] as explained in Sec. 2.

The situation observed in [1] is not exceptional. Evidence that off-ramp queues can block adjacent freeway lanes can be found even when freeways are wide and upstream flows well below saturation; i.e., when a disturbance would seem even less likely to affect adjacent lanes. As an illustration of this, Figs. 4(a)-(b) describe the evolution of an off-ramp queue on a segment of Interstate 880 in Hayward, California (U.S.A.). At its upstream end, the segment has four regular lanes and a median lane for car pools and buses, numbered as shown. (Also shown in the figure are the detector numbers used by the California Department of Transportation.) The shadings drawn here correspond to the 2-minute occupancies (densities) measured by the detectors at two different times during the afternoon rush on March 8, 1993. An occupancy of about 25% or more corresponds to queued traffic on this freeway [14] so that unshaded portions denote free flow. Figure 4(a) shows that the disturbance originates at the off-ramp while (b) shows that as the queue propagated upstream, it moved in the transverse direction and blocked all regular lanes. The queue eventually passed detector 20; see part (c).

Figure 5 presents the same data in the form used in [1]; data from the car pool and bus lane are not shown. Figure 5(a) shows that the flows at the upstream end of the section were always well below saturation in most of the regular lanes. This explains why the queue did not propagate upstream of detector 20 or reduce appreciably the output flow measured at detector 8 (Fig. 5(b)). Figure 5 also reveals the speed reductions brought by the queue. Three similarities with [1] stand out:

(1) larger speed reductions in the shoulder lanes at locations near the exit (part d), (2) upstream propagation of the speed reductions with a delay and (3) equally low speeds in all lanes at locations sufficiently far upstream of the exit (part h).

In summary, the attributes of the California speeds are similar to those from [1], although the drop in the former persists for over 1 hour. Furthermore, although the flow drop on the California site was not severe, the occupancy shading on the off-ramp leaves no doubt that this ramp spilled over onto the freeway (reason 1) for more than 1 hour. Thus, the flow and speed data of Fig. 5, which might otherwise be even more puzzling than those of the German site, can be explained in terms of a clear cause. In our view, one cannot claim that traffic queues form spontaneously on homogeneous roadways and then self-maintain themselves based on the available evidence.

2. EFFECTS

It was argued in the previous section that traffic can flow smoothly through a bottleneck, and yet be unstable upstream of it. There is no contradiction in this. It turns out that both phenomena are contained in a simple model [11] which was motivated by Edie and Foote's classic observation of "stop-and-go" traffic upstream of bottlenecks [16, 17]. It was noted in [17] that a decelerating (or just decelerated) platoon of traffic adopted a set of states on the flow-density (q, k) plane that were consistently different from those observed for accelerating platoons. It was speculated in [11] that these states could be described by two curves such as the D- and A-curves of Fig. 6a, accelerated traffic always being on the A-curve, decelerated traffic on the D-curve and transitioning traffic in between. Although we know that this theory is not precise, and may not be entirely correct, it predicts enough things right (see below) to suggest that a correct theory is probably a modification of it. This theory is also appealing because it does not borrow unwarranted elements from the theory of materials flow, such as waves that overtake vehicles. As explained in [18], this is an unreasonable feature of the "high order" fuid dynamic theories such as that used in [19]. The theory in [11] is simplified below by assuming that the transitions between the A-states and the D-states are quick. It will be shown that, when combined with existing models of bottleneck behavior, this model can explain qualitatively the observed phenomena in both German and North American highways.

Let us now consider the result of perturbations to a long platoon (queue) of vehicles in a given

stationary state, represented by point o' on the q-k plane; see Fig. 6(a). Assume that one of the vehicles in this platoon allows a gap to grow in front of it and later resumes its original speed in a retarded position. This is called a "deceleration disturbance". Then, given the A- and D-curves, the response of the following vehicles can be predicted. One would expect the following platoon to decelerate, adopting a state such as D on the D-curve, and later return to its original speed, moving to state o on the A-curve.[1] A following vehicle would then experience state sequence o',D,o, as indicated by the arrows. The result for all vehicles can be conveniently displayed on the time-space (t, x) plane using a scale that includes many vehicles; see Fig. 6(b). The dotted lines of this figure are sample vehicle trajectories and the solid lines are waves that separate regions of homogeneity. Note that states D and o expand as they propagate through the line of vehicles. Observers stationed at fixed locations would see state sequence o',D,o upstream of the disturbance's origin and state sequence o',o downstream.

Consideration shows that an acceleration disturbance, introduced when a vehicle decides to close the gap in front of it, could cause a state sequence such as o',a,O to appear and expand into the solution--also shown by arrows on Fig. 6(a).

If the initial state is on the D-curve (o' → O) we find that acceleration disturbances do not introduce expanding acceleration states into the solution because the two waves enclosing any such state would travel with the same speed. However, deceleration disturbances do expand. Thus, state "D" of Fig. 6(b) is unstable and will decay toward higher density states as additional perturbations occur; e.g., through the sequence D,E,d shown in Fig. 6(b). Eventually a growing "jam" state, "J", would appear in the solution. This decay process is in agreement with the spreading "jam" shown in Fig. 3 of [2].

The reverse occurs if the initial state is on the A-curve (o' → o). Then, state "o" would decay toward lower densities after an acceleration disturbance, e.g. through a sequence such as o,a,O, but the same state would not decay as a result of a deceleration disturbance.

For the curves of our example one would expect to find sufficiently far upstream of the "lead vehicle" a fluctuating sequence between states "J" and "a", where "a" is the state corresponding to

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[1] Lower case letters are used for states on the A-curve and capital letters for the D-curve; the same letter is used to denote states with the same vehicular velocity.

the largest speed that all the vehicles in the traffic line can sustain; see Fig. 6(a).[2] This sequence is no longer disturbed by additional perturbations and would be propagated up the line of cars without change with a velocity equal to the slope of line a-J of Fig. 6(a). This prediction is consistent with the unchanging propagating stoppages described in Figs. 2 and 3 of [3].

This simple theory also implies that (k, q) detector data would tend to form clockwise hysteresis loops on the (k, q) plane, as is frequently observed, with deviations from this pattern due to statistical fluctuations caused by driver differences. This can be understood from the figure by considering the sequence of states seen by an observer of unstable traffic at a fixed location.

Insofar as traffic is not always unstable, one would expect deceleration disturbances of a size significant to trigger the effects of Fig. 6(b) to be relatively rare, e.g. when a truck moves into the platoon or a vehicle slows down to change lanes. However, if vehicles show a tendency to "close the gap" (e.g., to prevent other vehicles from cutting into their line) so that they react to minor acceleration disturbances, then the platoon would behave as if these disturbances were continually present. Thus, A-states would be short-lived and would decay quickly toward "a". On a macroscopic scale where this process is not shown, each deceleration disturbance could then be modeled as a deceleration-acceleration pair of disturbances as in Fig. 6(c).

A plot on the flow-density plane of the stationary states that appear at a fixed location on the (t, x) plane would then trace the D-curve. Indirect evidence of this is given in Fig. 11 of [14], which shows few non-stationary points falling outside the stationary curve.[3]

This simple model can also shed light on a central question of this paper: whether queue instabilities can cause the flow through an active bottleneck to drop significantly below the maximum possible and then sustain itself in this state for an extended period. Part (d) of the figure shows what would happen in this simple theory if at location x_b there was a bottleneck that would preclude flows greater than q_b from crossing it; see Fig. 6(a). As the surge of traffic "f" reaches the bottleneck this theory would predict that vehicles would slow down and adopt a (decelerated) state D, consistent

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[2] With A- and D-curves of different shapes the system may settle into a fluctuation between high and low density states that are not at the extremes of the curves.

[3] Some points fall outside the envelope for very low speeds, and this cannot be explained with the simplified model presented here.

with the bottleneck service rate. If acceleration states are short-lived, as in Fig. 6(c), and insofar as $q_b < q_a$, we see from the geometry that no waves with positive speed could ever reach the bottleneck; i.e., in this theory, with quick decaying accelerated states, disturbances could not affect the flow through the bottleneck. Disturbance pairs would look as the one shown on Fig. 6(d); they would grow through the queue in the upstream direction and would dissipate on reaching the back of the queue, as would occur in Fig. 6(d) if the drawing had been extended further in time. Not shown, the vehicle trajectories in the part of this figure that is enclosed in a rectangle would be qualitatively similar to the often-quoted data set presented by Treiterer and Myers [11]. The low speed of the lead vehicles in these data, combined with the high density in which they are embedded and the presence of backward-moving disturbances indicate that the disturbance in [11] was created within a queue as in Fig. 6(d). Thus, one cannot conclude based on [11] that such disturbances reduce the flow through a bottleneck, and much less that of a freely flowing traffic stream.

Finally, the reader can also verify that the introduction of a temporary bottleneck in a traffic stream that is close to saturation will generate a disturbance with the general characteristics of the one shown in Fig. 4 of [1], and that a time-dependent bottleneck (e.g., the freeway merge) will generate a series of upstream, 1-minute observations much as the light small circles of Fig. 3(a) and the corresponding figure of [1].

Note that disturbances within a queue could affect the flow through a bottleneck if acceleration states with flow less than "a" were not short-lived. In this case states such as "e", "d" and "o" of Fig. 6(a) could propagate forward in space, as occurs for state o in Fig. 6(b), reach the bottleneck before having decayed and starve it for flow. Most researchers and practicing engineers do not believe that this is a significant effect, however, as evidenced by the standard "capacity manuals" that are currently in use. This is one of the reasons why the assertions made in [1] would be extremely interesting if they could be observed and reproduced. In fact, they would not just be interesting, but also important for control purposes. If disturbances do not affect significantly the rate of bottleneck flow, then the time that a vehicle spends in a system of given length, including the trip time to reach the back of the queue plus the time within it, is independent of where the intermediate changes in speed take place; see e.g. [20] for an explanation. Control schemes, should then aim to keep the bottleneck flowing at capacity, which can be achieved relatively simply by preventing downstream queues from backing

up into the bottleneck [21, 9]. However, if queue disturbances could starve the bottleneck for flow, then control schemes would also have to be devised to prevent acceleration disturbances from persisting *upstream* of the bottleneck. This would be much more difficult to achieve.

To be sure, the simple macroscopic theory presented in this section does not explain everything. For example, it does not explain the oscillating transient that is recurrently observed in the data of [5] as the merge bottleneck became saturated, which the theory idealizes by a singularity (point "S" in Fig. 6(d)), and it does not explain some of the high density measurements in [14]. But the theory gets enough things right to suggest that an improved model should probably be a modification of it.

The theory is also incomplete because it does not explain the genesis of the disturbances or the rate at which they grow. An extension of the theory would be needed to explain these phenomena because observations of different bottlenecks indicate that the periods of oscillation and growth are quite site-specific. For example, the Holland Tunnel bottleneck of [16] exhibited regular oscillation periods on the order of 2 minutes, but observations upstream of other bottlenecks (such as those in [1, 2, 3]) are more sporadic. This should not be surprising, however, given that the vehicular interactions that take place at merge, diverge, weave, lane drop or sag bottlenecks are very different; they should be studied as such.

3. CONCLUSION

A model of freeway traffic flow as a network of (possibly) unstable queues, with bottlenecks and particles that have specific destinations, explains much observed phenomena including the most puzzling in [1, 2, 3]. In this theory, much of what is observed depends on the status of the bottlenecks. Insofar as the behavior of these critical points in their many possible forms is only partially understood today, additional experimental work seems warranted. These authors believe that this is the research area where careful experimental observation will yield the greatest payoffs in the near future.

Acknowledgement

Endnotes

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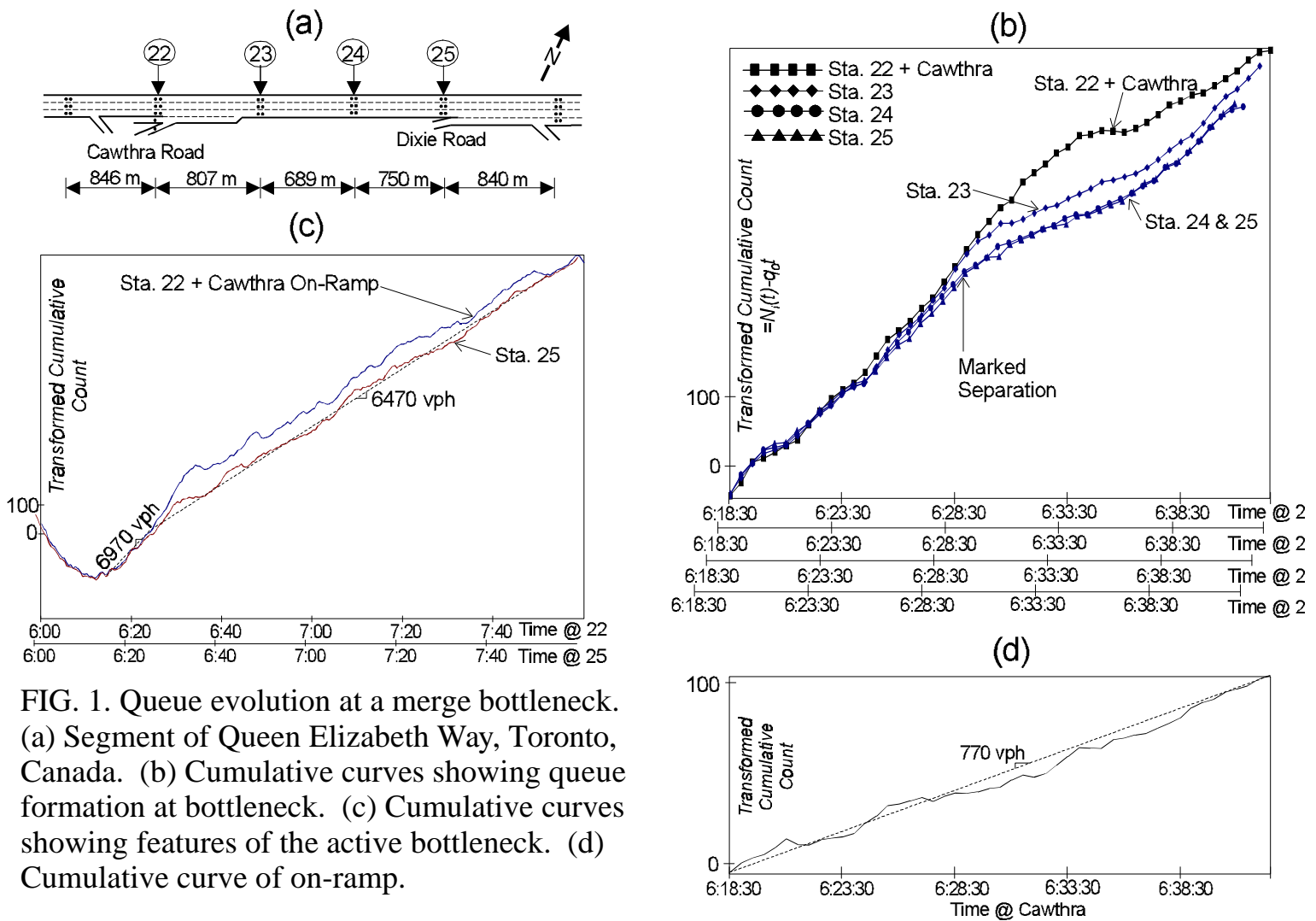[21] Newman, L., A. Dunnet and J. Meirs, Traff. Eng. (June) pp. 14-25 (1969).

FIG. 1. Queue evolution at a merge bottleneck. (a) Segment of Queen Elizabeth Way, Toronto, Canada. (b) Cumulative curves showing queue formation at bottleneck. (c) Cumulative curves showing features of the active bottleneck. (d) Cumulative curve of on-ramp.
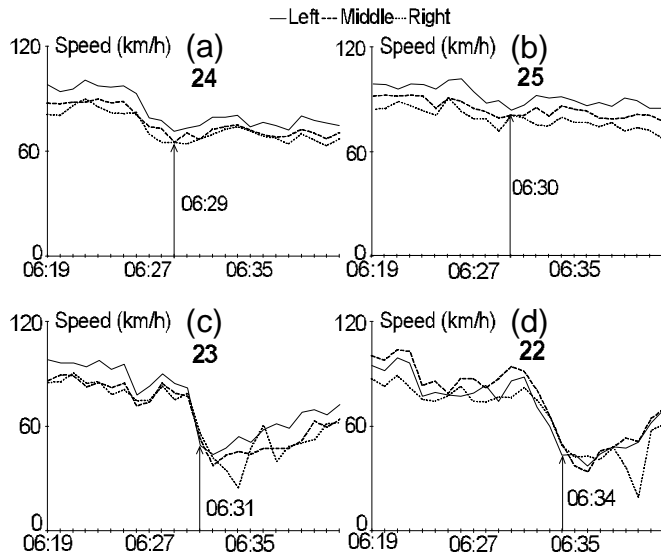
FIG. 2. Time series of vehicle speeds at merge bottleneck. (a) Station 24. (b) Station 25. (c) Station 23. (d) Station 22.
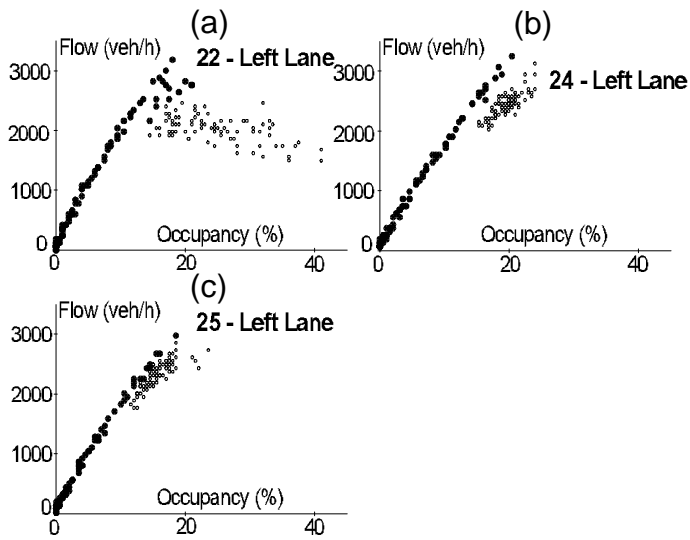
FIG. 3. Occupancy versus flow at merge bottleneck, 6:00 a.m. to 7:54 a.m. (a) Station 22. (b) Station 24. (c) Station 25.
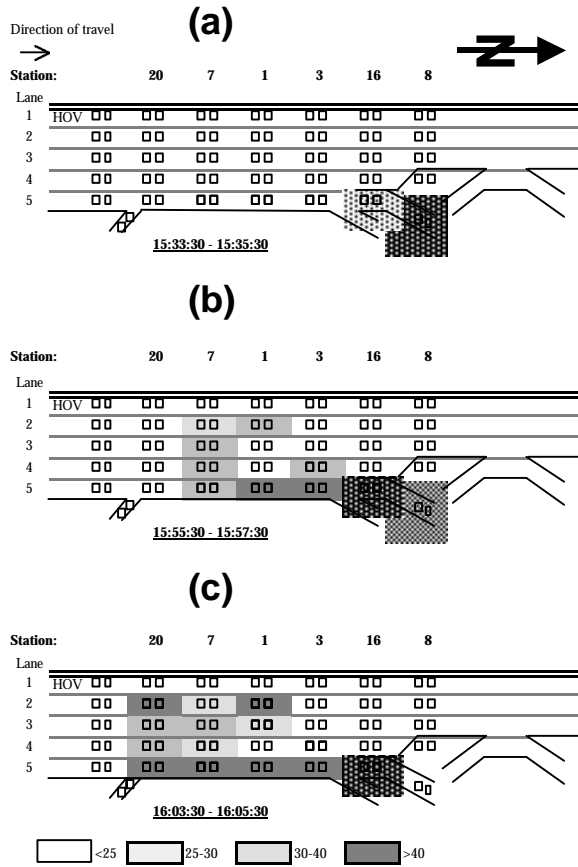
FIG. 4. Queue Evolution at a Diverge Bottleneck:
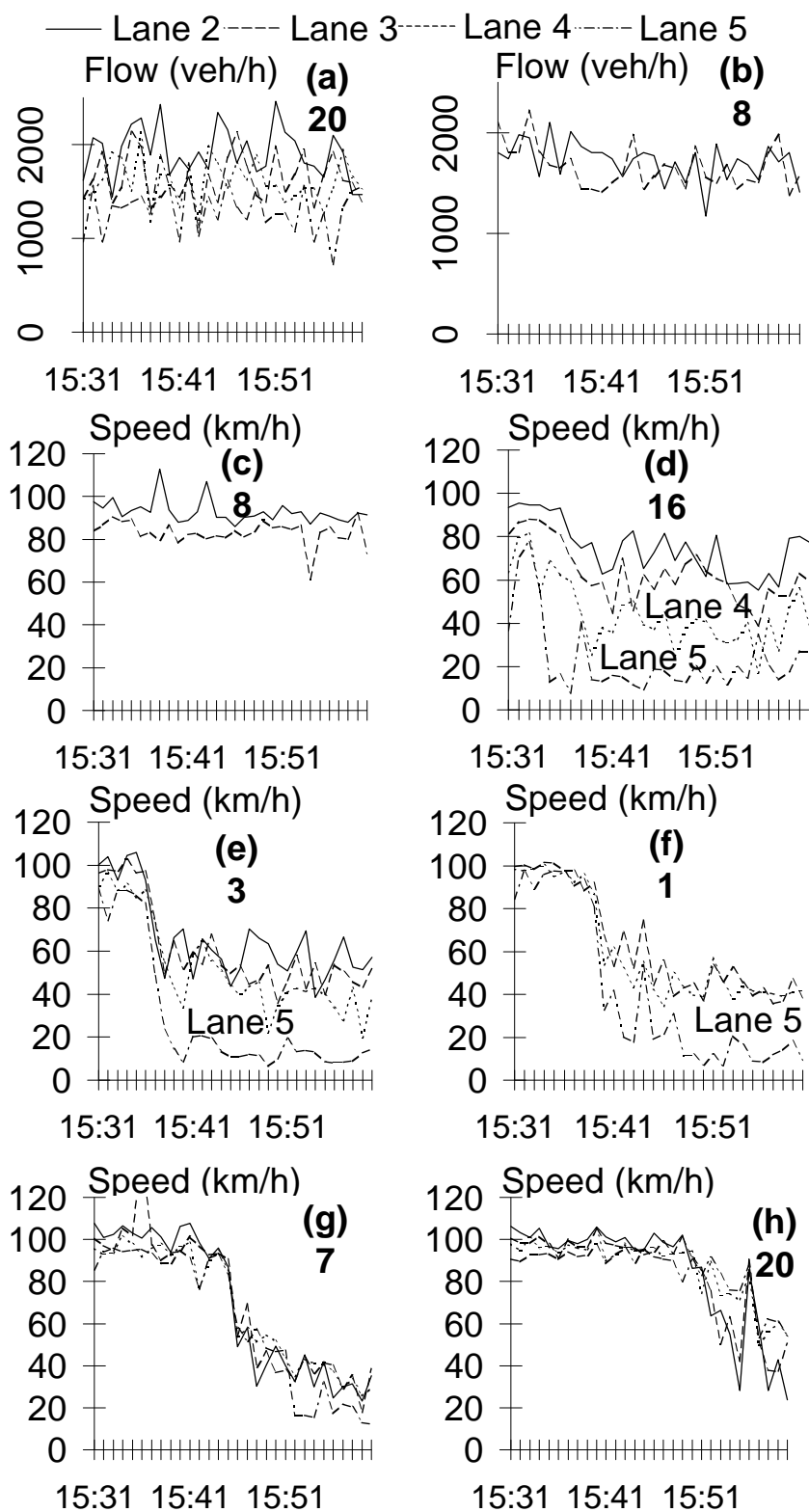   Shading intensities indicate occupancy rates

FIG. 5. Time series of flows and vehicle speeds at diverge bottleneck. (a) Flow at Station 20. (b) Flow at Station 8. (c) Speed at Station 8. (d) Speed at Station 16. (e) Speed at Station 3. (f) Speed at Station 1. (g) Speed at Station 7. (h) Speed at Station 20.
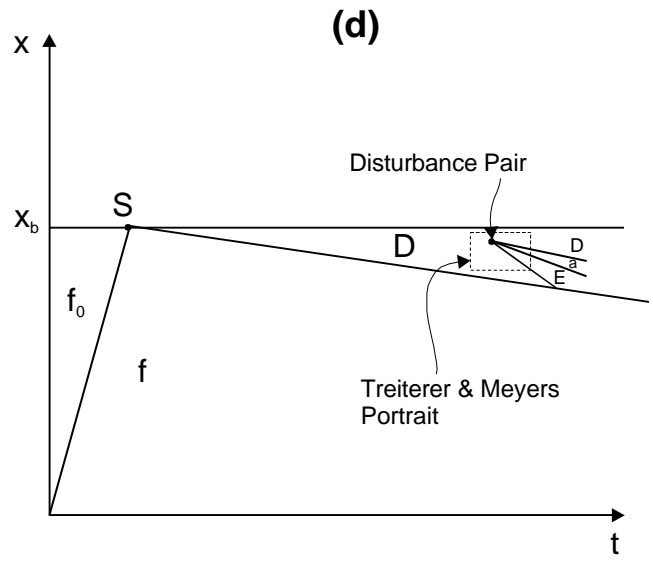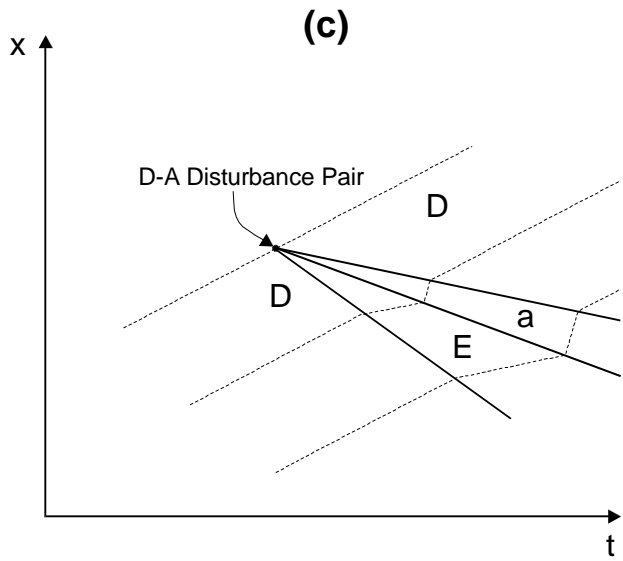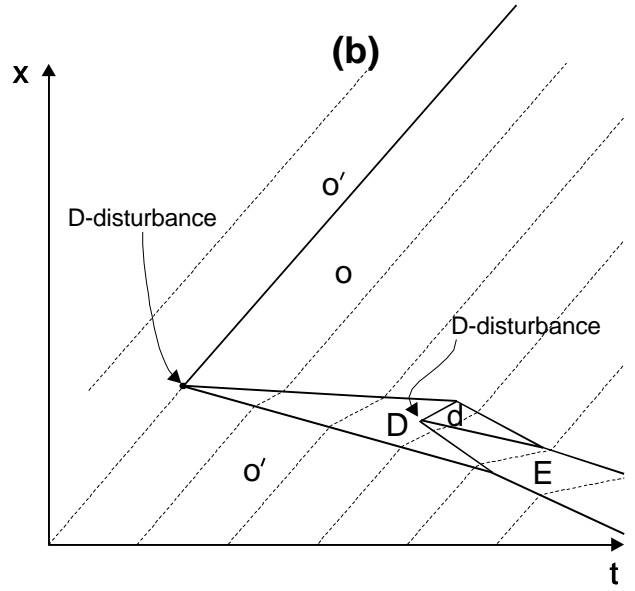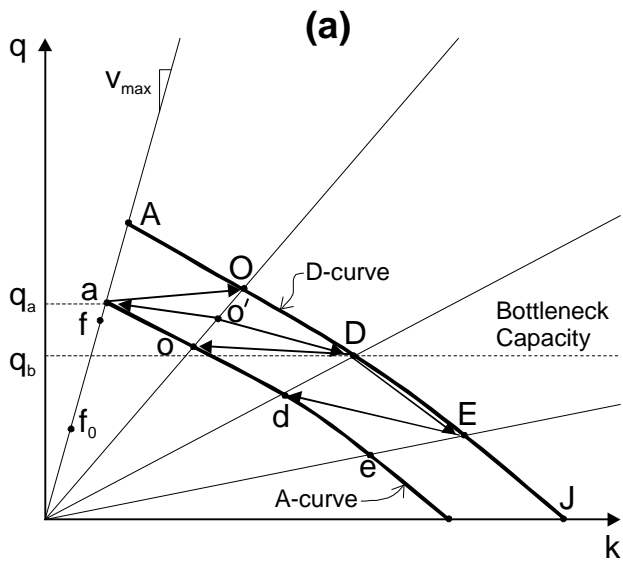
(a)

q

v_{max}

A

O D-curve

q_a a
f o′
q_b o D Bottleneck
Capacity

f_0
d
E
e
A-curve J

k

(b)

x

D-disturbance

o′
o

D-disturbance

D d

o′
E

t

(c)

x

D-A Disturbance Pair

D

D
E a

t

(d)

x

x_b S

Disturbance Pair

D D
a
E

f_0

f

Treiterer & Meyers
Portrait

t

FIGURE 6