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Characteristics of π^-, $\bar{p}$, and Antinuclei from pp Collisions

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Characteristics of π^-, $\bar{p}$, and Antinuclei from pp Collisions

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An investigation of inclusive $pp \to \pi^- + ...$ in terms of the covariant Boltzmann factor (BF) including the chemical potential μ indicates a) that the temperature T increases less rapidly than expected from Stefan's law, b) that a scaling property holds for the fireball velocity of π^- secondaries, leading to a multiplicity law like $\sim E_{cm}^{1/2}$ at high energy, and c) that μ_π is related to the quark mass: $\mu_\pi = 2m_q - m_\pi$, the quark mass m_q determined by $T_{\pi-}$ at pp threshold being $m_q = 3T_{\pi-} \simeq 330$ MeV. Because of *threshold effects*, $T_{\bar{p}} < T_{\pi-}$, whereas $\mu_p/\mu_{\pi-} \simeq 3/2$ as expected from the quark contents of $\bar{p}$ and π. The antinuclei $\bar{d}$ and $\bar{t}$ $/\overline{He^3}$ observed in pp events are formed by coalescence of $\bar{p}$ and $\bar{n}$ formed in pp collisions. Semi-empirical formulae are proposed to estimate multiplicities of π^-, $\bar{p}$ and antinuclei.

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1. Introduction

Recently, an important refinement of the statistical approach to multiparticle production has been advanced by Chou, Yang and Yen [1,2]. They introduce the concept of the partition temperature and point out the ultimate difference between hadron production by $\bar{p}p/pp$ collisions and e^+e^- annihilation. Their prediction that there is only one temperature for $e^+e^- \to \bar{h}h$ [2] has been confirmed by various e^+e^- data [3]. For pp collisions, there remain questions regarding thermal equilibrium and the validity of Stefan's law for the well-known models formulated by Fermi [4], Landau [5], Hagedorn [6] and others.

An attempt is therefore made to investigate inclusive $pp \to \pi^-, \bar{p}, \bar{d}$ and $\overline{He^3}$ in terms of the following Boltzmann factor

$$f \sim e^{-[(E - bP_{\parallel}) - \mu]/T} \qquad (1)$$

where $E = \sqrt{P^2 + m^2}$ and $P_{\parallel}$ denote the energy and the longitudinal momentum of the hadron h under consideration with respect to the colliding axis of pp in the cms; the parameters are the temperature T, the velocity b of the fireball (FB) emitting h, and μ the chemical potential of h [7].

We use currently available data of $P_\perp$ distributions from $pp \to \pi^-$ to estimate the temperature T of π^- by limiting ourselves to low $P_\perp$ (Sec. 2). At $P_{lab} = 6.92$ GeV/c, corresponding to $\bar{p}p$ threshold, we find $T \simeq 110$ MeV leading to a mass $m = 3T \simeq 330$ MeV for the u/d quark constituents of $\bar{p}$ and p as has been noted in $e^+e^- \to \bar{p}p$ of a previous work [3b]. The increase of T with the energy is, in general, much less rapid than expected from Stefan's law. The FB velocity b of (1) has an important scaling property, leading to a multiplicity law for π^- : $n \sim E_{cm}^{1/2}$ similar to the well-known Fermi's law but without assuming Stefan's law.

An empirical formula for hadron multiplicity will be derived from (1) (Sec. 4) to estimate the chemical potential of π^- : $\mu_\pi \simeq 505$ MeV $\simeq 2\mathrm{m_q} - \mathrm{m}_\pi$ reflecting the property of π-production in the quark model $q + \bar{q} \rightarrow \pi$ behaving like a chemical process (Sec. 4).

The temperature of $\bar{p}$'s from pp collisions is found to be lower than for π^-'s (Sec. 5) because of threshold effects. It has been found that $\mu_{\bar{p}}/\mu_\pi \simeq \frac{3}{2}$ as expected from quark counting. The kinematic properties of $\bar{d}$'s indicate that they are not directly produced by pp collision and their multiplicity agrees with production by the final state interactions of $\bar{p}$'s and $\bar{n}$'s (Sec. 6), namely, the coalescence process. The same process accounts for $\bar{t}$ and $\overline{He^3}$ production (Sec. 7).

Some remarks will be made on temperature estimation using the $P_\perp$ distribution and on applications of our empirical formulae to estimate hadron multiplicities of the forthcoming Superconducting Super Collider (SSC).

2. Temperature behavior of inclusive $pp \rightarrow \pi^-$.

Consider π^- production by pp collisions at a given energy and assume its single-particle distribution to follow the Boltzmann factor (1). We estimate the temperature T using the transverse momentum distribution:

$$\frac{d\sigma}{dP_\perp^2} \sim \int_0^\infty e^{-E/T} dP_\parallel \simeq m_\perp K_1(\frac{m_\perp}{T}) \simeq \sqrt{m_\perp} e^{-m_\perp/T}. \tag{2}$$

where $m_\perp = \sqrt{P_\perp^2 + m^2}$ and $K_n(x)$ is the modified Bessel function of the second kind; $K_n(x) \sim \sqrt{\pi/2x} \cdot e^{-x}$ for large x.

Note a) that we may further simplify (2) by neglecting the π-mass, so that $m_\perp = P_\perp$ and $\frac{d\sigma}{dP_\perp} \sim P_\perp^{3/2} e^{-P_\perp/T}$, b) that the validity of (2) thus simplified has been tested [8a], and c) that T thus obtained is systematically higher than that with $m_\pi \neq 0$. The approximation $m_\pi = 0$ blows up the multiplicity formula derived from (1) (cf. infra, Eq. (12)); therefore, throughout this paper, unless otherwise stated, we keep $m_\pi \neq 0$.

We recall that the average $P_\perp$ according to (2) is

$$< P_\perp >= \sqrt{\pi m T/2} \cdot K_{5/2}(m/T)/K_2(m/T), \tag{3}$$

and that the energy of equipartition in the fireball system, i.e., $b = 0$ (cf. Sec. 3) is

$$\mathcal{E}^* = 3T + mK_1(m/T)/K_2(m/T), \tag{4}$$

Note that for $T << m$, $K_n(x) \simeq \sqrt{\frac{\pi}{2x}} e^{-x}[1 + \frac{4n^2-1}{8x^2} + ...]$, so that (4) reduces to

$$\mathcal{E}^* = m + 3T/2 \quad \text{for m} >> \text{T} . \tag{4-a}$$

as is well known in the kinetic theory of gases.

We use (2) to fit currently available data of inclusive $pp \to \pi^-$ [9], for $P_\perp \leq 1.5$ GeV, i.e., low $P_\perp$ describing the soft process of hadron production in pp collision. The result is shown in Fig. 1. The temperature T increases slowly with the energy, then tails off.

If we assume a power law behavior:

$$T = A \cdot (\sqrt{s} - 2m_p)^\alpha \tag{5}$$

and carry out a fit by dividing the data into two parts, we find (T in MeV and $\sqrt{s}$ in GeV)

$$\alpha = 0.154 \pm 0.014, \quad A = 100.2 \pm 8.0 \text{ for } \mathrm{P_{lab}} < 200 \text{ GeV/c}$$

and

$$\alpha = 0.079 \pm 0.005, \quad A = 116.0 \pm 8.3 \text{ for } \mathrm{P_{lab}} \geq 200 \text{ GeV/c}$$

The fits are shown by the dotted and the full curves in Fig. 1. Note that the exponent α departs significantly from the value 1/4 expected from Stefan's law.

As a check of our parameterization, consider the case of $\bar{p}p$ Collider, $\sqrt{s} = 540$ GeV being $\sim$ 2 orders of magnitude beyond the scale of Fig. 1. We estimate $T = 181 \pm 3$ MeV corresponding to $< P_\perp >= 443\pm4$ MeV/c according to (3), in agreement with 424 ± 1 MeV/c reported by the UA1 Collaboration [10].

Finally, it is interesting to consider the temperature $T = 110 \pm 3$ MeV of $pp \to \pi^- + ...$ at $P_{lab} = 6.92$ GeV/c corresponding to the *threshold* of $\bar{p}p$ production by pp collision. It reflects the effective mass of quark and antiquark constituents of the produced $\bar{p}p$ at rest in the cms of pp collision. Indeed, the intrinsic mass of the u or d quark constituents of $\bar{p}$ and p being zero, the effective mass at $T = 110$ MeV is

$$m_q = 3T \simeq 330 \text{ MeV}$$

according to (4). As expected from energy conservation for $\bar{p}p$ production at threshold, $2m_p/6 \cong 313$ MeV. This justifies a-posteriori our temperature estimation for inclusive $pp \to \pi^-$ using the distribution (2) for low $P_\perp$.

3. The fire-ball velocity b.

As is well known from cosmic rays, a characteristic feature of meson production by high energy pp collisions is the anisotropic angular distribution in the cms [4b]. We now consider the rest frame of mesons of the same hemisphere in the cms of $pp \to \pi^- + ...$, called the fire-ball (FB) system, moving along the axis of the pp collision with a velocity b (in units of c = 1) with respect to the cms. We transform the energy-momentum $(E, \vec{P})$ of a π meson to the FB system, $\mathcal{E}^* = \gamma_F(E - bP_\parallel)$, where $\gamma_F = 1/\sqrt{1-b^2}$ is the Lorentz factor of the FB. Assuming $T^* = \gamma_F T$ [8b], we get

$$\frac{E - bP_\parallel}{T} = \frac{E^*}{T^*}, \tag{6}$$

so that the Boltzmann factor becomes e^{-E^*/T^*} indicating that the angular distribution of mesons is isotropic in the FB system. Thus, we may use the angular distribution of $pp \to \pi^- + ...$ to estimate the FB velocity b of the Boltzmann factor (1).

That this property is actually related to scaling will become more apparent if, instead of b, we use the parameter λ to replace $P_{\|} \rightarrow \lambda P_{\|}$ in the expression of energy E in the cms such that

$$E - bP_{\|} = \sqrt{P_{\perp}^2 + \lambda^2 P_{\|}^2 + m^2}\ . \tag{7}$$

We get in the high energy approximation: $P_{\|} \simeq E$

$$b = 1 - \lambda \tag{8}$$

and

$$< P_{\perp} > / < P_{\|} >= \frac{\pi}{2} \cdot \lambda\ . \tag{9}$$

Noting that $< P_{\perp} >$ is limited whereas $< P_{\|} > \sim \sqrt{s}$, we may write

$$\lambda = C_p / \gamma_{cm} \tag{10}$$

where the subscript p refers to the incident proton of the pp collision. We find C_p = 2.0 for $pp \rightarrow \pi^- + ...$ for $P_{lab} \geq 18$GeV/c as reported previously [8b,c).

We have analyzed the $pp \rightarrow \pi^-$ data by comparing $< P_{\perp} >$ and $< P_{\|} >$, and then we deduced b using (9). The result is presented in Fig. 2; the curve being the prediction according to the scaling property (10). The dotted part represents the extrapolation of (10) below the validity of scaling as mentioned above. The agreement with the experimental data is, in general, very satisfactory.

Knowing the Lorentz factor of the FB, γ_F, we may estimate the FB mass using energy conservation [8b]

$$M^* = m_p \gamma_{cm} / \gamma_F \tag{11}$$

which, in turn, may be used to estimate the number of π^- produced in the pp collision without considering a statistical approach as used in the Fermi-Landau model [2,3]. We shall discuss this point in the next section.

Finally, we mention that the scaling property (10) can be extended to other reactions involving two asymmetric fireballs such as $a + p \to \pi^- + ...$, the projectile a being either a meson π^+, K^+, or a lepton, μ^+, e^+, ν. The coefficients C_a of (10) have the similarity property, as has been reported elsewhere [8d].

4. π^- Multiplicity

We now investigate the hadron multiplicity from pp collisions. We use the FB system described above, i.e., $b = 0$, so that there is only one parameter, the temperature $T^* = \gamma_F T = T$, the same as in the cms; this is obvious, in view of the invariance of the $P_\perp$ distribution. We get the hadron multiplicity by integrating (1):

$$n = \frac{N}{(2I+1)(2J+1)} \cdot T^3 (\frac{m}{T})^2 \; K_2(\frac{m}{T}) e^{\mu/T} \tag{12}$$

I and J being the isospin and the spin of the hadron produced by the pp collision, and N the coefficient. Note that $\bar{n}$ becomes infinity if we assume $m_\pi = 0$.

We use (12) to analyze π^- data of pp collisions [11] as shown in Fig. 3, assuming *a priori* values of T according to the fit to T as a function of $\sqrt{s}$, Eq. (5). The result of the fit to (12) is shown by the solid curve in Fig. 3. The parameters are

$$\mu_\pi = -505 \pm 63 \text{ MeV}$$

$$N_\pi = (4.68 \pm 0.62) \times 10^4 \text{ GeV}^{-3}$$

Remembering that the effective mass of a u/d quark is $\sim$ 330 MeV (Sec. 2), we note that

$$2m_q + \mu_\pi \simeq m_\pi \tag{13}$$

suggesting that the chemical potential μ_π may be regarded as the latent heat of π production by quark-antiquark annihilation: $q + \bar{q} \to \pi$.

We turn now to another method to estimate the π^- multiplicity using the kinematic properties of the FB as discussed in Sec. 3. Here, neglecting the π mass, we may assume $n(\pi)$ to be proportional to the available energy of the FB, i.e., $M^* - m_p$ and write, using (11),

$$n(\pi) = 2\alpha_p(\gamma_{cm}/\gamma_F - 1) \ , \tag{14}$$

where the subscript p refers to the incident proton as in (10), whereas the factor 2 is to account for two FB's moving in opposite directions along the axis of the pp collision. We find

$$2\alpha_p = 0.49 \pm 0.03 \ ,$$

the fit is shown by the dotted line in Fig. 3.

As a check of this simple formula (14), let us extrapolate it to the CERN $\bar{p}p$ Collider energy $\sqrt{s} = 540$ GeV. We get $\gamma_F = 8.49$ using (8) and (10) and find the charged multiplicity

$$n_{ch} = 2n_- = 32.9 \pm 2.0$$

compared to 29.1 ± 0.9 of UA5 Collaboration [12].

Finally, we note that $\gamma_F = 2\sqrt{\gamma_{cm} - 1}$ for pp collision. Consequently, at high energy, we expect

$$n \sim E_{cm}^{1/2} \ . \tag{15}$$

In spite of its resemblance to the well-known Fermi-Landau formula, our formula (15) is based entirely on general grounds of kinematic properties of the FB of pp collision without invoking Stefan's law.

5. $\bar{p}$ production

Next, we apply (12) to $\bar{p}$ production by pp collisions from $P_{lab} = 19$ to 1100 GeV/c as reported for ISR experiments by the Bologna Group [13]. The data are reproduced in Fig. 4, errors $\sim$ 10-15% being omitted as in their plot.

For this purpose, we have to determine the temperature $T_{\bar{p}}$, as there is no *a priori* reason to assume it to be the same as T_{π^-}. In this regard, consider $P_{lab} < 6.92$GeV/c, $n_{\bar{p}} = 0$, i.e., $T_{\bar{p}} = 0$ according to (12), whereas $T_{\pi^-} \simeq 110$ MeV. Therefore, generally speaking, there is a *threshold effect* for $\bar{p}$ so that at a given energy $T_{\bar{p}} < T_{\pi^-}$. However, because of large fluctuations inherent in the temperature estimate using the $P_\perp$ distribution, it is rather difficult to determine the small difference between these two temperatures of $\bar{p}$ and π^- at energies above the threshold of $\bar{p}$ production.

We therefore proceed as follows: to each average $P_\perp$ of $\bar{p}$ of Ref. 13a we attach a scale factor, i.e. the ratio of $< P_\perp >$ of π^- of the same experiment to that expected from T_{π^-} at the same energy according to our analysis discussed in Sec. 2 using (3). We then multiply the measured $< P_\perp >$ of $\bar{p}$ by this factor to estimate $T_{\bar{p}}$ by (3). The result for the Bologna experiment [13] is summarized in Fig. 5.

Knowing $T_{\bar{p}}$, we fit the $\bar{p}$ multiplicity with (12) and find

$$\mu_{\bar{p}} = -636 \pm 102 \text{ MeV}$$

$$N_{\bar{p}} = (2.02 \pm 0.54) \times 10^4 \text{ GeV}^{-3}$$

The fit is shown in Fig. 4.

Comparing with the fit to π^- multiplicity of the previous section, we note

$$\mu_{\bar{p}}/\mu_{\pi-} \simeq 1.27 \pm 0.26 \tag{16}$$

consistent with 3/2 as expected from the quark content of $\bar{p}$ and π^-. We find

$$N_{\bar{p}} \simeq N_{\pi^-}/2 \ ,$$

due to particle-antiparticle production of $\bar{p}$ in contrast to the single particle production of π of a given charge, +, -, or neutral. The formula (12) counts all πś , whereas for $\bar{p}$, it includes

$\bar{n}$ but does not account for p and n. Therefore, we find actually the same coefficient N for π and $\bar{p}$. This justifies *a posteriori* formula (12) as used in our analysis.

6. Formation of antideuteron

Lack of information on the $P_\perp$ distribution of $\bar{d}$'s observed in pp collisions makes it impossible to estimate the $\bar{d}$ temperature. Nonetheless, we may get some insight into the mechanism of their production by comparing their $P_\parallel$ distribution with that of $\bar{p}$'s observed in the same experiment.

Consider, for instance, an SPS experiment by the Bologna-Saclay-LAPP (BSL) Collaboration [14] at $P_{lab} = 200 - 240$ GeV/c, using Be target. Neglecting the Fermi motion of the target nucleon, we estimate the FB velocity of π^- : $b = 0.82$ by (10) so that $\gamma_F = 1.73$. If, for simplicity, we assume the same temperature $T = 147$ MeV for $\bar{p}$ and $\bar{d}$ as for π^- (see Fig. 1), the kinetic energy of $\bar{p}$ and $\bar{d}$ in the FB system is, according to Eq. (4-a)

$$K^* = \frac{3}{2}T \simeq 0.221 \text{ GeV} .$$

If we transform the momentum of $\bar{p}$ from the FB to the cms, we find the minimum $P_\parallel = \gamma_F(-P_\parallel^* + bE^*)$ with $E^* = K^* + M_{\bar{p}}$; this gives $x_{\bar{p}} \geq 0.047$ in agreement with the BSL data (Fig. 4 of Ref. 14). Likewise, if $\bar{d}$'s are produced directly by p-nucleus collisions as are $\bar{p}$'s, we expect $x_{\bar{d}} \geq 0.16$, in contrast with the data; note that ~95% of the $\bar{d}$'s observed in the BSL experiment [14] lie below $x = 0.10$, indicating that the antideuterion results from the final-state interaction of $\bar{p}$ and $\bar{n}$, just as the deuterons observed in p-nucleus reactions.

If we assume a Poisson distribution for $n_{\bar{p}}$, we may express the multiplicity of $\bar{d}$ as follows:

$$n_{\bar{d}} = C_{\bar{d}} \frac{(n_{\bar{p}})^2}{2!} \, e^{-\Delta\mu/T_{\bar{p}}} \tag{17}$$

where $C_{\bar{d}}$ is a coefficient and

$$\Delta\mu = \mu_{\bar{d}} - 2\mu_{\bar{p}} . \tag{18}$$

Note that we have approximated $e^{-n_{\bar{p}}} \simeq 1$ since we are dealing with $n_{\bar{p}}$ (see Fig.4).

We now proceed to analyze the data of the Bologna-Padova-Saclay Experiment [15], reproduced in Fig. 6. We compute n_{π^-} and $n_{\bar{p}}$ with (12), the temperatures of π^- and $\bar{p}$ being taken from Fig. 5, and carry out the fit to their data. We find

$$\Delta\mu = 0.047 \pm 0.140 \text{ GeV}$$

$$C_{\bar{d}} = (3.3 \pm 0.28) \cdot 10^{-3} ,$$

and the fit is shown by the solid curve in Fig. 6.

It is interesting to note that $\Delta\mu$ is very small compared to the chemical potential of the $\bar{p}$ suggesting that the binding energy of the antideuteron is rather small.

7. Production of antinuclei $\bar{t}/\overline{He_3}$

Antinuclei of mass number $A = 3$, $\bar{t}$ and $\overline{He_3}$ of $I = J = 1/2$, have been observed in high energy p-nucleus reactions; their production compared to $\bar{d}$ and $\bar{p}$ has been investigated by the Serpukhov Group [16a] using $p - A\ell$ at $P_{lab} = 70$ GeV/c. Their measurements at secondary $P = 20$ GeV/c in the forward direction, expressed in terms of cross-sections using their earlier experiment [16b], are replotted in Fig. 7.

The characteristic feature of these antinuclei is that their cross-sections decrease exponentially with A, as is noticed by the Serpukhov Group [16a]. This property has also been reported by an SPS experiment at $P_{lab} = 200$ GeV/c of Bologna-Padova-LAPP (BPL) Collaboration [17].

An attempt is made to account for this property using the coalescence model as in the case of $\bar{d}$ production. We shall derive a general formula for antinuclei production according to (12). Noting that here, $m >> T$ so that $K_2(x) \simeq \sqrt{\pi/2x} \cdot e^{-x}$, and keeping only the factor specific to the antinucleus of mass $m = AM$, M being the nucleon mass, we get

$$\sigma(A) \sim \frac{A^{3/2}}{(2I+1)(2J+1)} C \; \mathrm{p}^A e^{-A(M/T)} \tag{19}$$

where p denotes the probability for antinucleons to coalesce together to form a nucleus of mass number A, and the constant C specifies the statistics : $C = 1$ for independent particles, whereas $C = 1/A!$ for distinguishable particles. Note that the chemical potential is implicitly contained in the parameter p.

As regards the temperature T, it refers to $\bar{p}$ as mentioned in Sec. 6. Here we shall use the temperature deduced from the negative multiplicity of the reaction $p - A\ell$ producing $\bar{p}$, $\bar{d}$, $\overline{He_3}$ of the experiment. We estimate n_- to be ~2.2, about 12% higher than pp at 70 GeV/c [18], leading to $T \simeq 140$ MeV.

We use this temperature to fit the Serpukhov data with the empirical formula (18) and find

$$\text{for} \quad C = 1, \qquad \mathrm{p} = 0.27 \pm 0.13,$$

$$\text{for} \quad C = 1/A! \,, \mathrm{p} = 0.57 \pm 0.15 \; .$$

The fits are shown in Fig. 7 by the full curve and the dotted line, respectively.

A comparison with the data indicates that the fit favors the case C = 1, i.e., the antinucleons behave like independent particles to coalesce into antinuclei. The same result is obtained from the BPL data of reference [17].

Finally, it is worth noting that here we get $\mathrm{p} \simeq 1/3$ which may be interpreted as the average of the square of the cosine of the angle in the cms between the randomly oriented momenta of $\bar{p}$ and $\bar{n}$ giving rise to the antideuteron, and that the cross-section of antinuclei formation decreases approximately by a factor $\mathrm{p} \cdot e^{-M/T} \simeq 4 \times 10^{-4}$ for the mass number A to increase by one unit.

8. Remarks

We have investigated π^- and $\bar{p}$ production by pp collisions with the covariant Boltzmann factor (BF) (1) including the chemical potential μ of the secondary hadron. The values of μ for π^- and $\bar{p}$ suggest the quark contents of π's and $\bar{p}$'s, Eqs. (13) and (16), respectively. The quark mass deduced from the temperature of $pp \to \pi^- + ...$ at the threshold of $\bar{p}p$ production, i.e., $P_{lab} = 6.92$ GeV/c (Fig. 1) is ~330 MeV.

The parameter b of (1), the fireball velocity, is found to be related to scaling. This property leads to a simple formula (14) for π^- multiplicity, accurate to ~3%. The multiplicity of $\bar{p}$ may be computed using (12), the temperature $T_{\bar{p}}$ is shown in Fig. 5; whereas the antinuclei $\bar{d}$, $\bar{t}/\overline{He_3}$ and others may be estimated according to the coalescence formula (19).

As an application of the scaling property (10) we may extend (14) to the SSC energy using the $\bar{p}p$ Collider data. We find for the π^- multiplicity

$$n_- = 141 \pm 14 \ for \ \sqrt{s} = 40 \ TeV,$$

compared to $\simeq 85.3$ according to $n \sim E^{1/4}$ (15). This corresponds to

$$T \simeq 255 \ MeV$$

leading to

$$N_{\bar{p}} \simeq 17.2.$$

As for the temperature estimation, we bear in mind that it depends essentially on the method used to analyse the $P_\perp$ data and that besides (2), other formulae have been used in the literature to estimate the temperature. In this respect, we note especially the use of $P_\perp$ data in the central region, $x \simeq 0$. In this case, we have to take into account the possible transverse motion of the FB, namely the exponent of the BF (1) should include a term similar to $bP_\parallel$, i.e., $E - (aP_\perp + bP_\parallel)$ as discussed in Ref. 8b, so that, leaving aside the chemical potential, the appropriate form is

$$\begin{aligned} f_{x\simeq 0} &\sim e^{-(m_\perp - aP_\perp)/T} \\ &\simeq e^{-m_\perp/T'} \end{aligned} \tag{20}$$

with

$$T' = T/(1-a). \tag{21}$$

Clearly, for low $P_{\perp}$, a is negligible, and this form $e^{-m_{\perp}/T}$ is often used by other authors. It should be mentioned that T thus estimated is less than that using the $P_{\perp}$ distribution integrated over $P_{\parallel}$, i.e., Eq. (2), because of the well known seagull effect [19].

For large $P_{\perp}$, it has been found in a previous analysis of the ISR data that in general, $a < b$, whereas T obtained using (20) is, to some extent, higher than that of low $P_{\perp}$. However, the temperature of large $P_{\perp}$ may be overestimated by using (20), if no account is taken of the transverse velocity a of the FB [20]. But with appropriate parameterization, one may attempt a two-temperature model to describe the entire distribution of both low $P_{\perp}$ and high $P_{\perp}$, for the soft as well as the hard process of hadron production.

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References and Footnotes

1. T.T. Chou, Chen-Ning Yang and E. Yen, Phys. Rev. Lett. **54**, 510 (1985).
2. T.T. Chou and Chen-Ning Yang, (a) Phys. Rev. **D32**, 1692, (1985) and (b) Phys. Rev. Lett. **55**, 1359 (1985).
3. T.F. Hoang and Bruce Cork, Z. Phys. **C38**, 608 (1988) and T.F. Hoang, Phys. Rev. D, in press.
4. E. Fermi, (a) Prog. Theor. Phys. **5**, 570 (1950) and (b) Phys. Rev. **81**, 683 (1951).
5. L.D. Landau, Akad. Nauk. SSSR, sr. fiz. **17**, 51 (1953).
6. R. Hagedorn, Nouvo Cimento, Suppl. **3**, 167 (1965) and Nuovo Cimento, **56A**, 1027 (1968), and Rev. Lett. Nouvo Cimento, Vol. 6, N10 (1982).
7. See e.g., S. Weinberg, Gravitation and Cosmology, John Wiley, New York (1972), p. 530.
8. T.F. Hoang, (a) Nucl. Phys. **B38**, 333 (1972), (b) Phys. Rev. **D12**, 296 (1975), (c) ibid., **17**, 927 (1978), and (d) ibid., **24**, 1406 (1981).
9. Except one *np* experiment at 3 GeV/c, W.B. Fowler, et al., Phys. Rev. **95**, 1026 (1954), we use the following $pp \to \pi^- + \ldots$ data: (a) 6.92 GeV/c, S. Danieli, et al., Nucl. Phys. **B27**, 157 (1971); (b) 10 GeV/c, P. Almeida, et al., Phys. Rev. **174**, 1638 (1968); (c) 12 GeV/c, J.L. Day, et al., Phys. Rev. Lett. **18**, 1218 (1969), (d) 10, 88, 18, 21.1, and 24.1 GeV/c, D.B. Smith, et al., Phys. Rev. Lett. **23**, 1064 (1968); (e) 19.2 GeV/c, H. Bøgglid, et al., Nucl. Phys. **B27**, 1 (1971); (f) 28 GeV/c, R.S. Panvani, et al., Phys. Lett., **38B**, 55 (1972); (g) 69 GeV/c, V.V. Ammosov, et al., Phys. Lett. **42B**, 519 (1972), and F. Binon, et al., ibid., **30B**, 506 (1969); (h) 100 GeV/c, J.W. Chapman, et al., Phys. Lett. **47B**, 465 (1973); (i) 205 GeV/c, S. Barish, et al., Phys. Rev. **D9**, 2689 (1974), and G. Charlton, et al., Phys. Rev. Lett. **30**, 594 (1973); (j) 303 GeV/c, P.T. Dao, et al., Phys. Rev. Lett. **29**, 1627 (1972); and (k) $\nu s = 43, 60$ GeV, B. Alpan, et al., Nucl. Phys. **B100**, 237 (1975) and A. Breakstone, et al., Phys. Rev. **D30**, 528

(1984).

10. UA1 Collaboration, G. Arnisson, et al., Phys. Lett. **118B**, 167 (1982).

11. We use the data compiled by the Bologna Group, C. Albini, et al., Nuovo Cimento **101A**, (1976) and assume for π^- multoplicity $n_- = (n_{ch} - 2)/2$.

12. UA5 Collaboration, G.I. Alner, et al., Phys. Lett. **160B**, 193 and 199 (1985).

13. Bologna Group, (a) M. Antimucci, et al., Nuovo Cimento, **6**, 121 (1973) and (b) A.M. Rossi, et al., Nucl. Phys. **84B**, 269 (1975).

14. Bologna-Saclay-LAPP Collaboration, A. Bussieie, et al., Nucl. Phys. **174**, 1 (1980).

15. Bologna-Padova-Saclay Collaboration, N. Bozzoli, et al., Nucl. Phys. **B144**, 317 (1978).

16. Serpukhov-Yu.M. Antipov, et al., (a) Nucl. Phys. **31B**, 235 (1971) and (b) Phys. Lett. **34B**, 164 (1971).

17. Bologna-Padova-LAPP Collaboration, W. Bozzoli, et al., Nucl. Phys. **B144**, 317 (1978). The parameters of fit with (18) are: $fuc = 1$, $p = 0.31 \pm 0.02$ and $fuc = 1/A!$, $p = 0.77 \pm 0.09$, assuming $T = 150$ MeV.

18. We use the A-dependence of n_- of p-nucleus data of JINR experiment: N.K. Bish, et al., Sov. Journ. Nucl. Phys. **30**, 824 (1979).

19. See e.g. M. Deritchmann, et al., Nucl. Phys. **B70**, 189 (1974).

20. See, e.g., the two-temperature model of C.K. Chou, et al., (Phys. Rev. **D29**, 2127 (1984). They assume an exponential law for each $T : A \cdot e^{-P_\perp/T}$. According to their values for the experiment of $\bar{p}p$ Collider [10], namely $T_1 = 2774$ MeV, $T_2 = 664$ MeV and $A_1/A_2 = 500$, the average $P_\perp$ should be $\sim T_2$, compared to 424 ± 1 MeV/c, indicating a non-negligible transverse velocity $a \simeq 0.36$ of the fireball.

Figure Captions

1. Temperatures of inclusives $pp \to \pi^- + ...$ using the $P_\perp$ distribution (2). The curves are fits with $T \sim (\sqrt{s} - 2m_p)^\alpha$ for $P_\perp <$ and ≥ 100 GeV/c : $\alpha = 0.154 \pm 0.014$ and 0.097 ± 0.005, respectively, compared to 1/4 of the Stefan's law. The u/d quark mass deduced from T at $\bar{p}p$ threshold (arrow) is $m_q = 3T = 330$ MeV, see text.

2. Plot of the velocity b of the fireball of π^- from pp collisions vs. P_{lab}. The curve is the prediction according to the scaling property of b, Eq. (10), see text.

3. Multiplicity of π^- produced by pp collisions. The full line represents the empirical formula, Eq. (12), derived from the Boltzmann factor (1). The dotted line is the prediction of the fire-ball model, Eq. (14), see text.

4. Multiplicity of $\bar{p}$ from pp collisions. The curve represents the computation using Eq. (12) and temperature $T_{\bar{p}}$ of Fig. 5, the chemical potential being $\mu_{\bar{p}} \simeq \frac{3}{2}\mu_{\pi^-}$ as expected from quark contents of $\bar{p}$ and π, see text.

5. Comparison of temperatures of π^- and $\bar{p}$ from pp collisions. Note $T_{\bar{p}} = 0$ below threshold according to Eq. (12).

6. Antideuteron production by pp collisions, data compiled by the Bologna-Saclay-LAPP Group [Ref. 14]. The curve represents $\bar{d}$ production by the pick-up process.

7. $\bar{p}$, $\bar{d}$, and $\overline{He_3}$ production by 70 GeV/c proton on Be, Superkhov data [Ref. 16b]. The curves represent the coalescence model of antinuclei production Eq. (19), assuming either Bose statistics (full line) or Boltzmann statistics (dotted line).

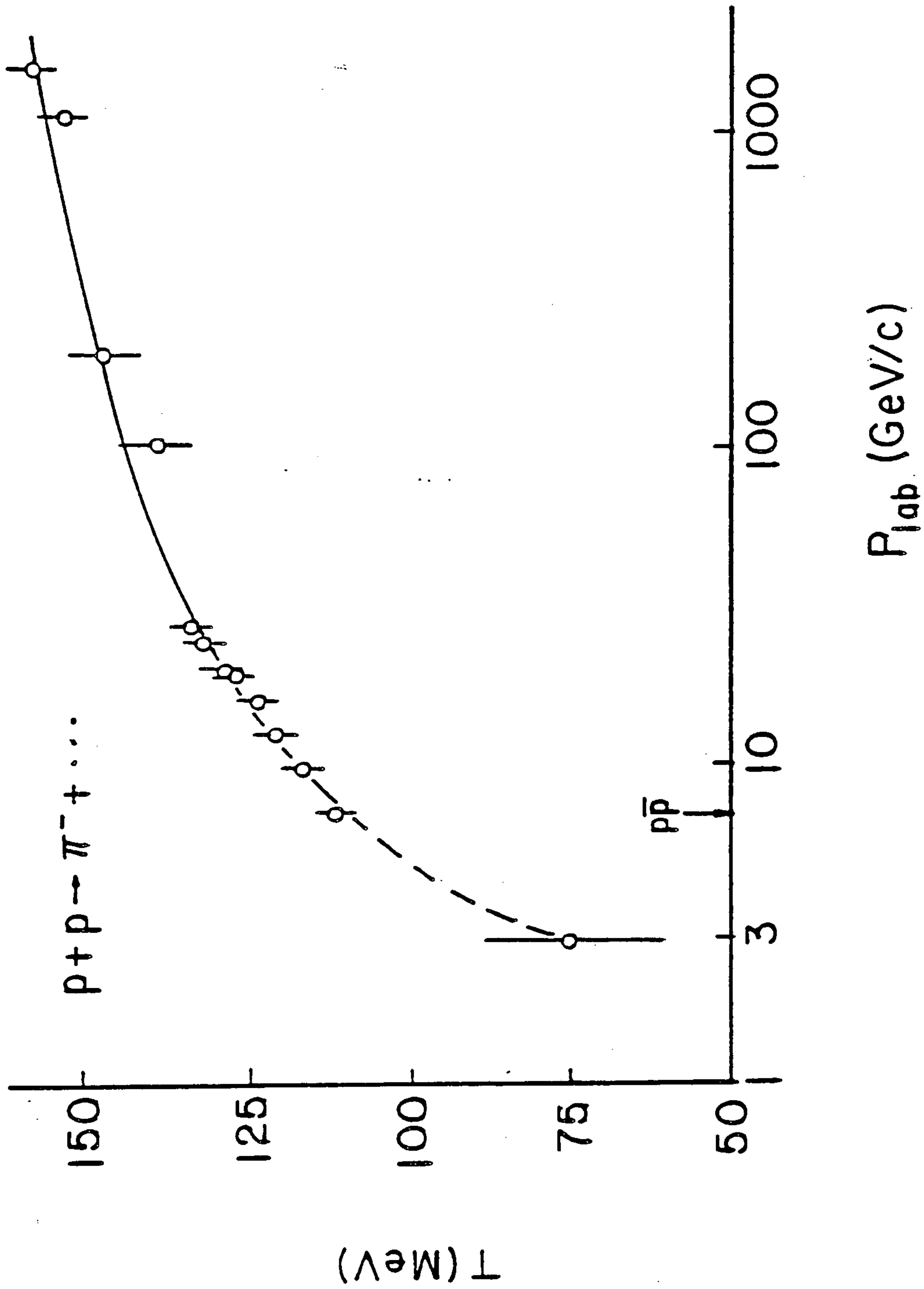
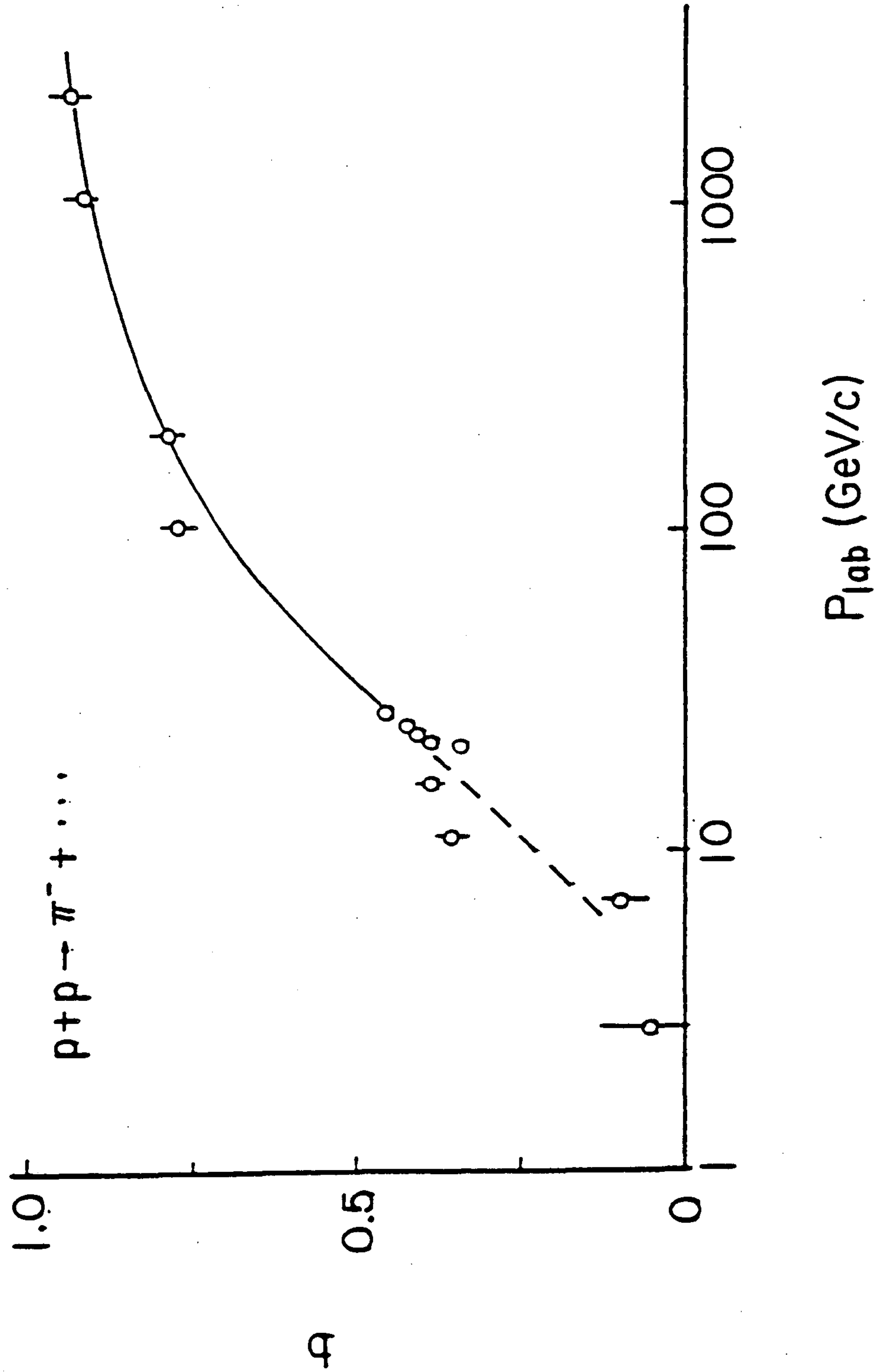
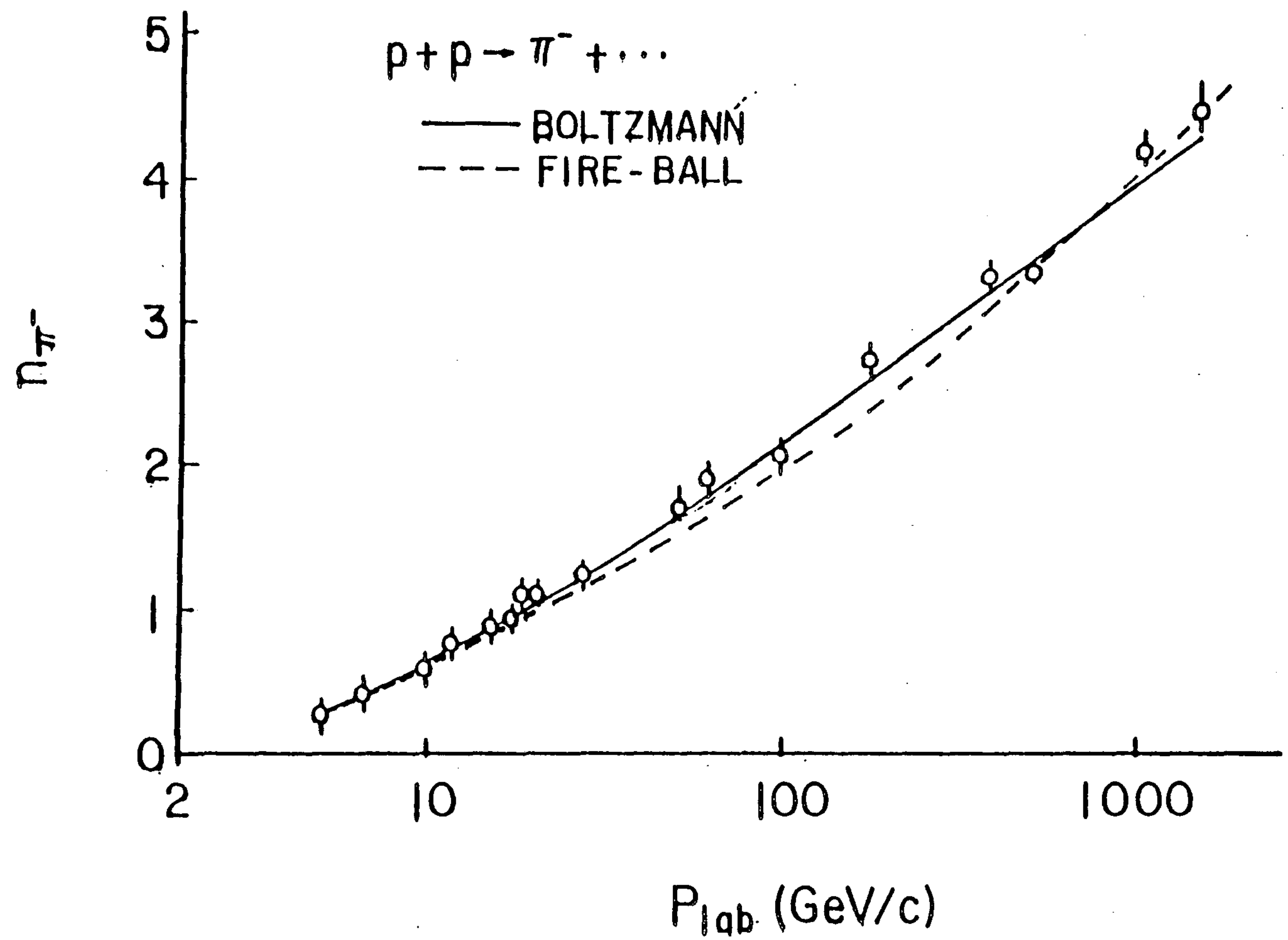


Fig. 1



Fig, 2



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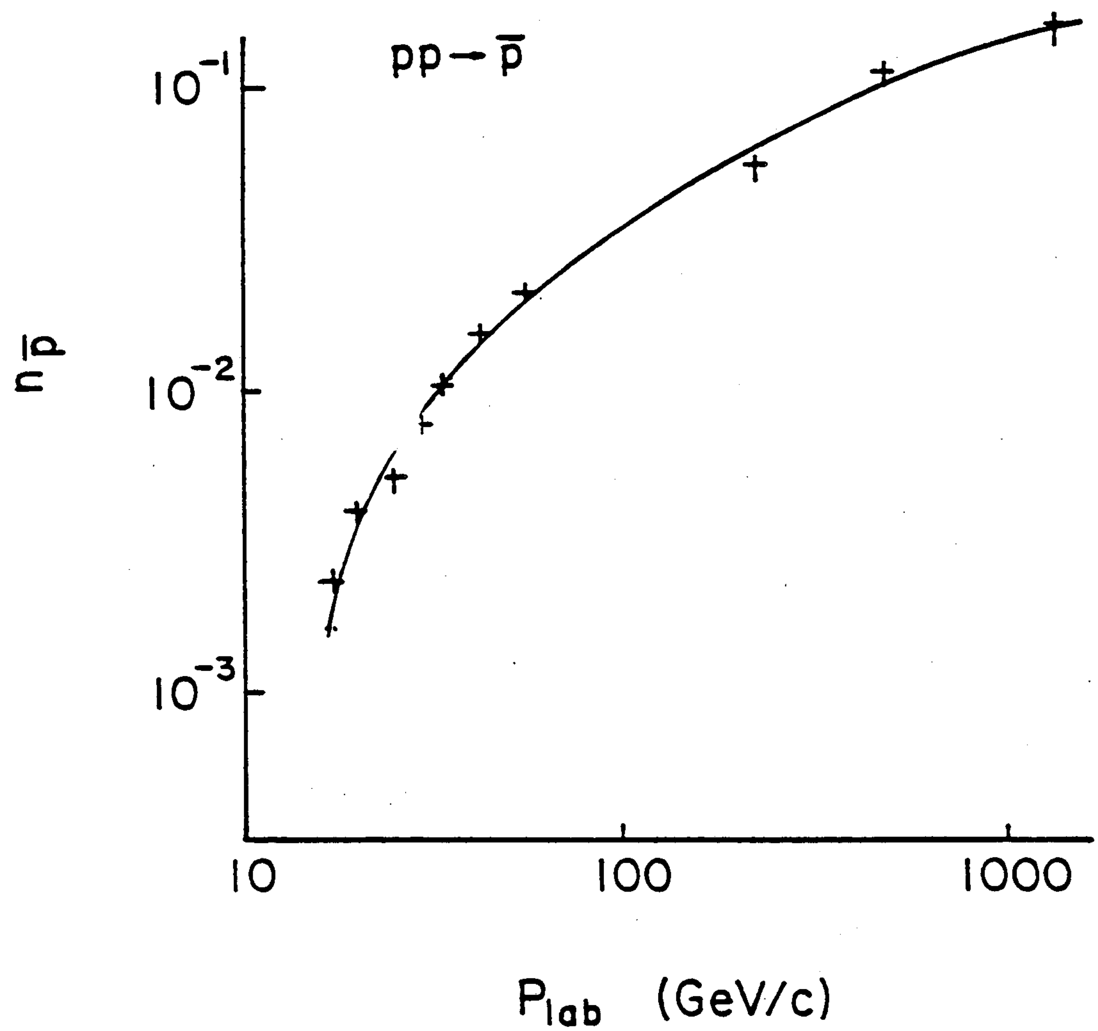


Fig. 4

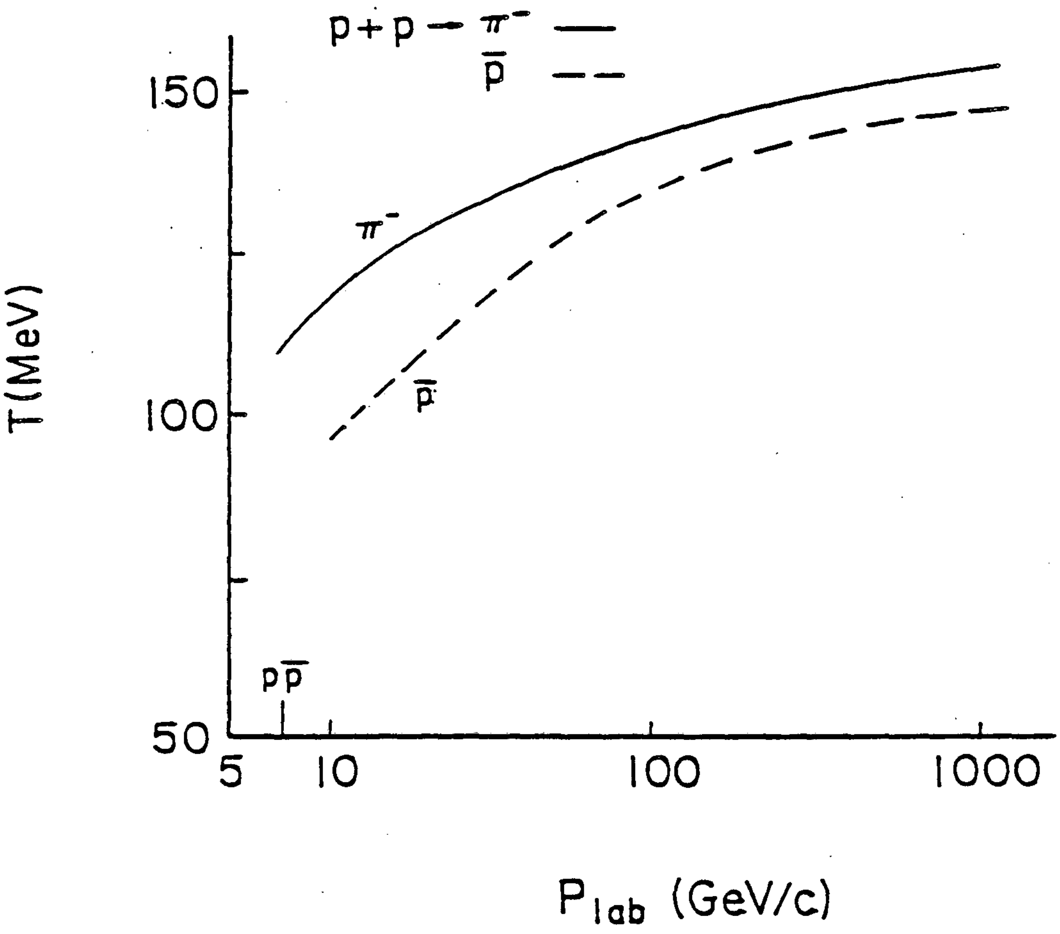


Fig. 5

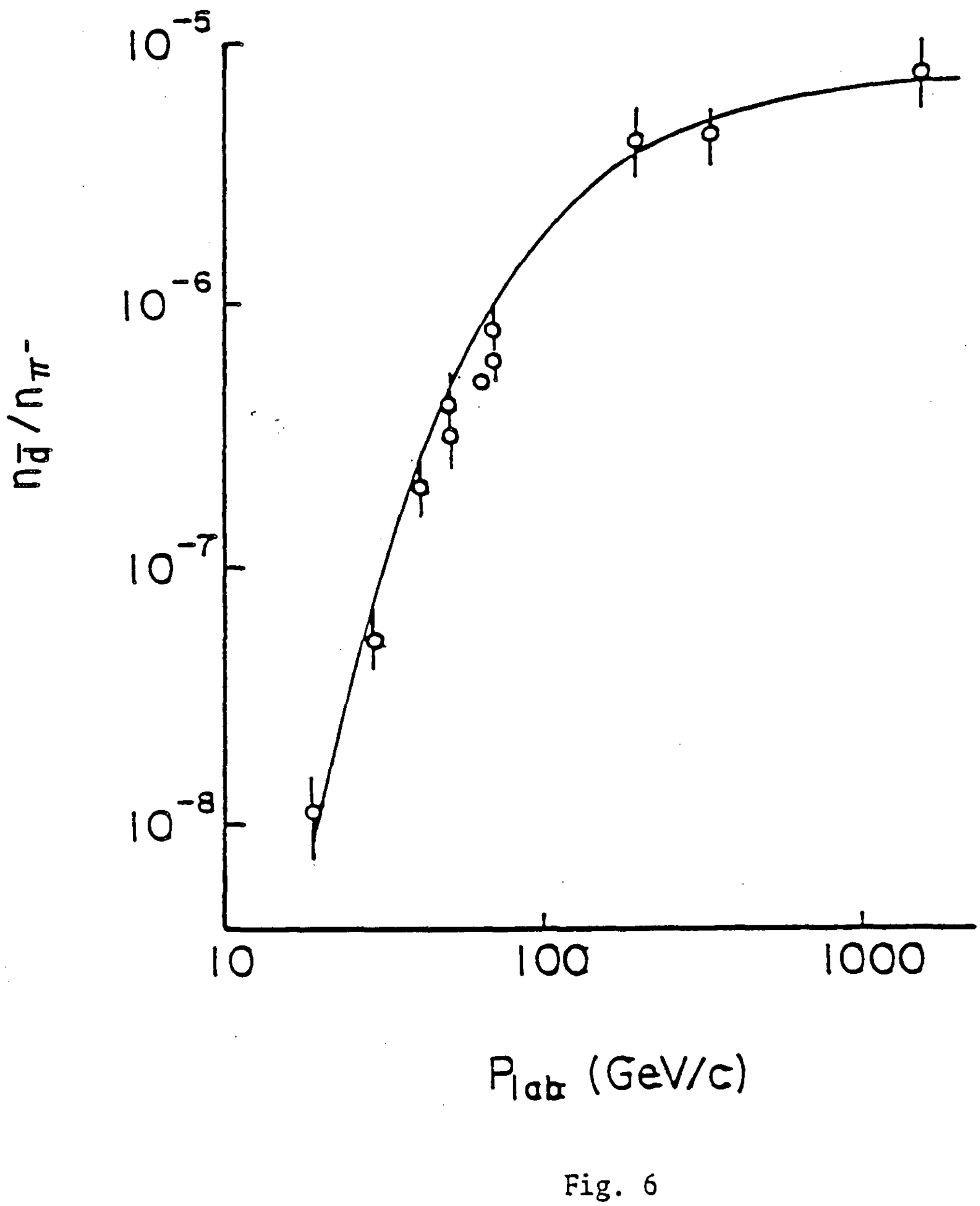
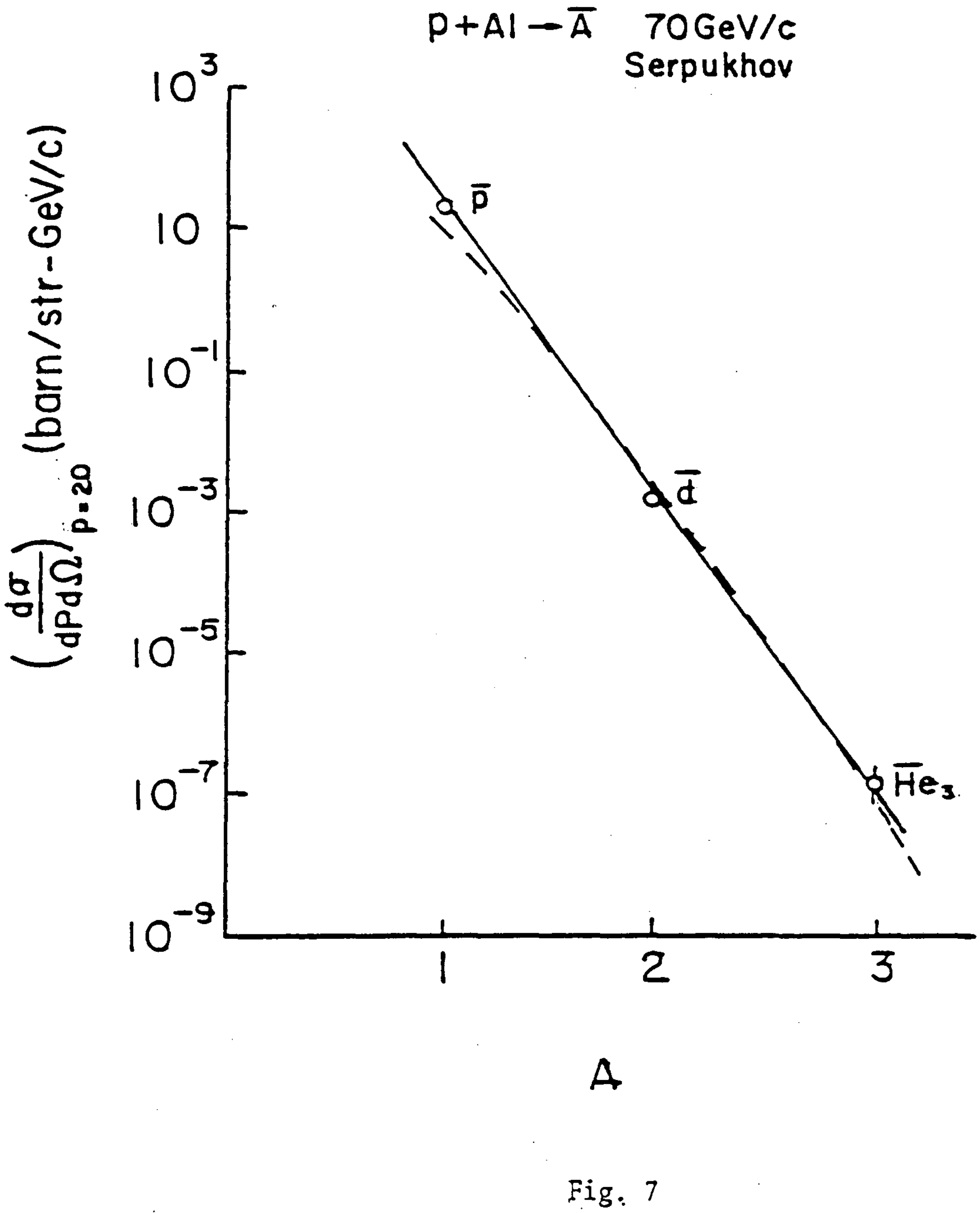


Fig. 6



p+Al → $\overline{A}$ 70GeV/c
Serpukhov
$\left(\frac{d\sigma}{dPd\Omega}\right)_{p=20}$ (barn/str-GeV/c)
10^3
10
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
$\overline{p}$
$\overline{d}$
$\overline{He}_3$
1
2
3
A

Fig. 7

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