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Fundamental Concepts

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This chapter provides a brief review of several topics that help in understanding air pollution in urban areas. We first describe methods used to quantify concentrations of different species in air. We then introduce the variables that are used to characterize the vertical structure of the atmospheric layer in which pollutants are transported. The last section of this chapter describes meteorological concepts relevant to air pollution.



EXPRESSING CONCENTRATIONS

The most obvious way of expressing concentrations in air is in terms of **mass per unit volume**, which is simply the mass of the species in a given volume divided by the volume. Concentrations are usually expressed in terms of milligrams/cubic meter (mg/m^3 , $\text{mg} = 10^{-3} \text{g}$) or micrograms/cubic meter ($\mu\text{g}/\text{m}^3$, $\mu\text{g} = 10^{-6} \text{g}$). Concentrations of particulate matter in the atmosphere are customarily expressed in mass units.

The concentration of a gas in a mixture of gases is most often expressed in terms of the **mixing ratio** (q) defined by

$$q = \frac{\text{concentration of species in a volume}}{\text{concentration of mixture in the same volume}}. \quad (2.1)$$

The concentration is usually expressed in terms of moles of the gas per unit volume. The mixing ratio is simply the ratio of the number of moles of the species to the number of moles of the mixture in the volume used to calculate concentrations. This means that the mixing ratio does not change if the volume changes. We can always convert mixing ratio to mass per unit volume concentration units at any temperature and pressure by calculating the density of the mixture for these conditions.

To see how this is done, let us review the perfect gas law, which states that:

$$pV = nR_u T, \quad (2.2)$$

where p is the pressure in Newtons/m² (a unit referred to as the Pascal denoted by Pa), V is the volume of the gas in m³, n is the number of moles of the gas, R_u is the universal gas constant and is equal to 8.314 J/(mol K), and T is the temperature in Kelvin.

We can restate Eq. (2.2) in terms of the density of the gas ρ , by expressing the number of moles, n by

$$n = m_g \times 1000/M_w, \quad (2.3)$$

where m_g is the mass of the gas in kg and M_w is the molecular weight of the gas—mass in grams of 1 mole or 6.02×10^{23} molecules of the gas.

Substituting Eq. (2.3) in Eq. (2.2) yields:

$$p = \left(\frac{m_g}{V}\right) \left(\frac{1000R_u}{M_w}\right) T. \quad (2.4)$$

Now, ρ , the density of the gas in mass units is

$$\rho = \frac{m_g}{V}. \quad (2.5)$$

If we define the gas constant specific to the gas as

$$R_g = \frac{1000R_u}{M_w} \quad (2.6)$$

an alternate form of the gas law becomes:

$$p = \rho R_g T. \quad (2.7)$$

If we assume that nitrogen (78% by volume) and oxygen (21% by volume) are the primary components of air, the average molecular weight of air is about $(28 \times 0.78 + 32 \times 0.21)/0.99 = 29$. Let us calculate the gas constant specific to air, R_g by taking $M_w = 29$ g/mole,

$$R_g = \frac{1000 \text{ g/kg} \times 8.314 \text{ J/(mole K)}}{29 \text{ g/mole}} = 287 \text{ J/kg K} \quad (2.8)$$

It is now easy to estimate the density of air at any temperature and pressure. For example, let us estimate the density near the ground where the temperature is 300 K, and the pressure is approximately 10^5 N/m². Using Eq. (2.7), we get

$$\rho_a = p/(R_g T) = \frac{10^5 \text{ N/m}^2}{287 \text{ J/(kg K)} \times 300 \text{ K}} \approx 1.2 \text{ kg/m}^3 \quad (2.9)$$

(notice that Joule = Newton · Meter).

The mixing ratio defined in Eq. (2.1) is essentially a mass ratio. In air pollution work, it is customary to work with **volumetric mixing ratios**. Let us define the volumetric mixing ratio by considering two species, A and B , enclosed in a container with volume V . The gases in the container are subject to a constant atmospheric pressure through a moveable piston exposed to the atmosphere. Now assume that gas B is absorbed using some reagent so that only gas A remains in the container. Because the external pressure remains the same, the piston moves down to enclose a smaller volume occupied by A , denoted by V_A . The volumetric mixing ratio of A is then V_A/V and that of B is $(1 - V_A/V)$. This procedure to estimate volumetric mixing ratios can be readily related to molar ratios using the perfect gas law.

Denote the atmospheric pressure by p . Let N be the total number of moles initially in the container, and the number of moles of A be N_A . Then the gas law before and after absorption of B reads:

$$pV = NR_u T, \quad (2.10a)$$

and

$$pV_A = N_A R_u T. \quad (2.10b)$$

We see immediately that the volumetric mixing ratio is simply the mole fraction of the species,

$$\frac{V_A}{V} = \frac{N_A}{N}. \quad (2.11)$$

The mole fraction can be converted in mass per unit volume units at a specified temperature, p_o , and temperature, T_o , using the molar concentration of the mixture of gases given by

$$\frac{N}{V} = \frac{p_o}{R_u T_o}. \quad (2.12)$$

For $p_o = 1.013 \times 10^5 \text{ N/m}^2$ (1 atm) and $T_o = 298 \text{ K}$, the molar concentration works out to be

$$n_a = \frac{N}{V} = \frac{1.013 \times 10^5 \text{ N/m}^2}{8.314 \text{ J/(mol K)} \times 298 \text{ K}} \approx 41 \text{ mol/m}^3. \quad (2.13)$$

This concentration is independent of the species. Let us convert 100 ppb of ozone to mass per unit volume units measured at 1 atmosphere and 25°C , which corresponds to the above molar concentration. Now $100 \text{ ppb} = 100 \times 10^{-9}$ expressed as a mole fraction, which translates into $10^{-7} \times 41 \text{ mol/m}^3$. To convert this into mass units, we multiply this concentration by the molecular mass of ozone which is 48 g/mol , and obtain $1.96 \times 10^{-4} \text{ g/m}^3$ or $197 \text{ }\mu\text{g/m}^3$.

This concentration can be referred to some other temperature and pressure, say 0.8 atmospheres and 273 K by noticing that the total molar concentration is directly proportional to pressure, and inversely proportional to temperature. Then the concentration works out to be $197 \times (0.8/1) \times (298/273) = 172 \text{ }\mu\text{g/m}^3$.

It is just as easy to go from mass per unit volume units to molar mixing ratio by:

- Converting the concentration of the species into mol/m^3 units using the molecular weight of the relevant species;
- Dividing the concentration by the molar concentration of the mixture of gases at the pressure and temperature that the concentration was measured.

To illustrate this conversion, assume that the concentration of sulfur dioxide (SO_2) is measured to be $1000 \text{ }\mu\text{g/m}^3$ at 10°C and a pressure of 900 mb. Let us express this concentration in mixing ratio units. The molecular mass of SO_2 is 64 g/mol , so that the molar concentration is $(1000/64) \times 10^{-6} \text{ mol/m}^3$. We need to calculate the density of air (including the gas) at the given pressure and temperature. Note that the

pressure is expressed in terms of millibar = 10^{-3} bar, where bar is an atmospheric pressure unit equal to 10^5 N/m² (1 N/m² = Pa). Then Eq. (2.13) yields $900 \times 10^{-3} \times 10^5$ N/m² / (8.314 J/(mol K) \times ($273 + 20$) K) = 37 mol/m³ for the molar density of air. The mixing ratio becomes $(1000/64) \times 10^{-6} / 37 \times 10^6 = 0.42$ ppm or 420 ppb.

It is sometimes tempting to dismiss atmospheric species concentrations as small simply because they are expressed as parts per billion—1 molecule out of a billion air molecules. This is simply an artifact of the choice of units as can be seen by expressing the concentration in molecules per cubic meter. For example, 1 ppb is approximately $10^{-9} \times 41$ (mol/m³) \times 6×10^{23} molecules/mol = 2.5×10^{16} molecules/m³, which translates into a large exposure if we consider the fact that we breathe in about 1 m³/h. This illustrates the fact that terms such as large or small have little meaning; concentration levels assume meaning only when converted into effects of concern, such as that related to human health.

With these preliminaries out of the way, we can discuss the composition of air.



THE COMPOSITION OF AIR

Air primarily consists of nitrogen and oxygen, with nitrogen constituting 78% by volume, and oxygen making up about 21%. This means that 78% of the molecules in any given volume of air consist of nitrogen, and 21% consists of oxygen. The remaining 1% consists of minor constituents such as Argon, Neon, Xenon, and Krypton, whose concentrations are essentially steady with time. There are other minor constituents that are more variable, the most important of which are water vapor and carbon dioxide. Water vapor concentrations are highly variable, ranging from 1% to 10% by volume. As we will see later, the phase changes undergone by these relatively small concentrations of water vapor have profound effects on the energetics of the atmosphere.

The background carbon dioxide concentration is about 350 ppm by volume; which means that 350 molecules of a total of 1 million molecules of air is carbon dioxide. This value is believed to be increasing at a rate of 1.5 ppm/year because of human activity, such as combustion of fossil fuels and deforestation. Both water vapor and carbon dioxide play important

roles in the Greenhouse effect, which refers to the warming caused by the absorption of thermal radiation by these gases.

Chemical species, referred to as air pollutants, contribute to the variable composition of the atmosphere. The species of importance and their typical concentrations are given in the following table:

Species	Concentration (ppb) Polluted	Concentration (ppb) Clean
Sulfur dioxide	100	10
Carbon monoxide	30 (ppm)	3 (ppm)
Nitrogen oxide	50	5
Hydrocarbons	500	50
Ozone	100	10

The composition of pollutants in the atmosphere is governed by the transport and dispersion of anthropogenic emissions into the atmospheric boundary layer, which refers to the lower part of the atmosphere, in contact with the Earth's surface, that is affected by heat and mass transfer between the ground and the atmosphere. The next chapter describes how the boundary layer is characterized in dispersion applications. Here we provide some preliminaries required to understand this chapter.



HYDROSTATIC EQUILIBRIUM

Measurements indicate that the pressure at the Earth's surface is about 10^5 N/m^2 , a quantity that is referred to as a **bar**. Standard atmospheric pressure is defined as 1013.25 millibar, where 1 millibar (mb) = 10^{-3} bar. Standard atmospheric pressure corresponds to the pressure at the bottom of column of mercury 76 cm high. Because pressure changes are usually much smaller than a bar, atmospheric pressure is usually expressed in millibars.

We live at the bottom of a large pool of air. The pressure felt at the Earth's surface is caused by the weight of the air above our heads. This means that we should expect the air pressure to go down as we go higher into the atmosphere. Let us use these ideas to get an idea of the variation of pressure in the atmosphere. We know that the density of air is about 1 kg/m^3 near the ground. If we assume that the density does not vary

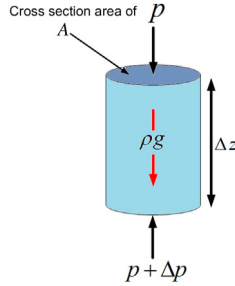


Figure 2.1 Cylinder of air with cross-sectional area A in hydrostatic equilibrium.

substantially in the first few kilometers near the ground, we expect the pressure to decrease by the weight of 1 km, which is about $1000 \times 1 \times 9.8 \approx 10^4 \text{ N/m}^2$ (9.8 is the acceleration due to gravity) for every kilometer we go up into the atmosphere. Thus we expect a pressure drop of about 100 mb for every kilometer we rise into the atmosphere. This cannot be true at all heights because we have not accounted for the decrease in density of air with height. Before deriving the equations to allow us to do this, let us calculate the effective height of the atmosphere assuming that the density goes to zero at the top. Then the average density of the air above the surface is about 0.5 kg/m^3 . The height of the column of air that gives rise to the surface pressure of 10^5 N/m^2 is $10^5 / (9.8 \times 0.5) \approx 2 \times 10^4 \text{ m} = 20 \text{ km}$. This suggests that we expect a small fraction of the atmosphere to exist above this height.

To derive a quantitative relationship between pressure and height, we need to assume that there is a balance between gravitational and pressure forces. This assumption is generally valid at horizontal scales of tens of kilometers. However, rapid vertical motion associated with convective motion or orographic forcing can invalidate this assumption.

To analyze hydrostatic equilibrium, consider the cylinder of air shown in [Fig. 2.1](#).

Downward force due to gravity = density of air \times Volume of cylinder $\times g = \rho A \Delta z g$

Upward force due to pressure = $pA - (p + \Delta p)A = -\Delta p A$

Equating the two forces, and taking the limit $\Delta z \rightarrow 0$, we obtain

$$\frac{dp}{dz} = -\rho g. \quad (2.14)$$

This is the hydrostatic equation that allows us to calculate the vertical pressure variation in the atmosphere. We can eliminate the density in the

equation using the gas law, $p = \rho R_a T$, where R_a is the gas constant of air, to obtain:

$$\frac{dp}{dz} = -\frac{gp}{R_a T}, \quad (2.15)$$

which can be integrated if we know the temperature variation with height, as follows:

$$\frac{p}{p_o} = \exp \left[-\frac{g}{R_a} \int_0^z \frac{dz}{T} \right]. \quad (2.16)$$

It is easy to calculate the pressure variation in an atmosphere, in which the temperature decreases at a constant lapse rate, γ , which is the rate at which the temperature decreases with height

$$T = T_o - \gamma z, \quad (2.17)$$

where γ can be taken to be about 6.5 K/km and T_o is the surface temperature. Substituting Eq. (2.17) into Eq. (2.16) and integrating yields

$$\frac{p}{p_o} = \left[\frac{T_o - \gamma z}{T_o} \right]^{g/R_a \gamma}. \quad (2.18)$$

Let us use Eq. (2.18) to estimate the pressure at 10 km, where the temperature is approximately $(300 - 6.5 \times 10) = 235$ K. Taking $R_a = 287$ J/(kg K), and $\gamma = 6.5$ K/km, we find

$$\frac{p}{p_o} = \left(\frac{235}{300} \right)^{g/R_a \gamma} = \left(\frac{235}{300} \right)^{5.26} = 0.277 \quad (2.19)$$

and

$$\frac{\rho}{\rho_o} = \left(\frac{235}{300} \right)^{(g/R_a \gamma) - 1} = \left(\frac{235}{300} \right)^{4.26} = 0.35. \quad (2.20)$$

Thus $p \approx 277$ mb, which is consistent with measurements. The air density at this height is about $1.2 \times 0.35 = 0.42$ kg/m³. Notice that the density falls off less rapidly than the pressure, which explains why we can use the surface density to approximate the pressure gradient below 5 km. At 20 km, the pressure works out to be about 50 mb, which means that 95% of the mass of the atmosphere is below this height.

We next use our knowledge of pressure variation in the atmosphere to define the concept of potential temperature.



THE POTENTIAL TEMPERATURE

We can define the potential temperature, θ , by considering a unit mass of dry air undergoing a change of state by moving vertically. If this parcel does not exchange heat with its surroundings, the first law of thermodynamics states

$$du + pdv = 0, \quad (2.21)$$

where u is the specific internal energy (internal energy per unit mass) and v is the specific volume. Using the perfect gas law, Eq. (2.21) can be restated as

$$C_p dT - v dp = 0$$

or

$$(2.22)$$

$$\frac{dT}{T} - \frac{R_a}{C_p} \frac{dp}{p} = 0,$$

where C_p is the specific heat of air at constant pressure. This equation can be integrated to obtain

$$\theta = T \left(\frac{p_0}{p} \right)^{R_a/C_p}, \quad (2.23)$$

where θ is the temperature at p_0 , a reference pressure taken to be 10^5 N/m^2 , a quantity that is referred to as a **bar** in atmospheric literature. Then the potential temperature, θ , of a parcel of air at a pressure, p , and temperature, T , is defined as that achieved by the parcel when it is moved adiabatically to the reference pressure p_0 . If this parcel is moved adiabatically to another pressure, its temperature changes, but its potential temperature does not change; it is a conserved quantity.

The potential temperature is a useful variable that is used to characterize energy changes in the atmosphere. Notice that by dealing with the potential temperature rather than the absolute temperature, we do not have to worry about the expansion work associated with the pressure

changes in the atmosphere. Our analysis has not included the effects of heat released during condensation of the water vapor in an air parcel.

ATMOSPHERIC STABILITY

The concept of stability can be illustrated using the motion of a ball on different surfaces, as illustrated in Fig. 2.2.

The ball in the bottom right figure is in unstable equilibrium because any small motion of the ball results in the ball moving away from its initial position in the direction of the initial push. The ball in the top middle figure is in neutral equilibrium because the motion of the ball is proportional to the push; the ball moves to its new position showing no tendency to move away. The ball in the bottom left figure has a tendency to move back to its original position when pushed in any direction; it is in stable equilibrium.

The preceding concepts on equilibrium can be extended to the motion of a parcel in an atmosphere with the three gradients of potential temperature as shown in Fig. 2.3.

The solid lines in the three figures represent profiles of potential temperature, and the shaded circles represent air parcels whose stability is being examined. Consider the middle figure, which shows the potential temperature decreasing with height. Let us move an air parcel vertically in this atmosphere, and assume that this motion is adiabatic. This means that during this motion, the potential temperature of the parcel remains constant. The parcel's potential temperature is shown as a vertical line.

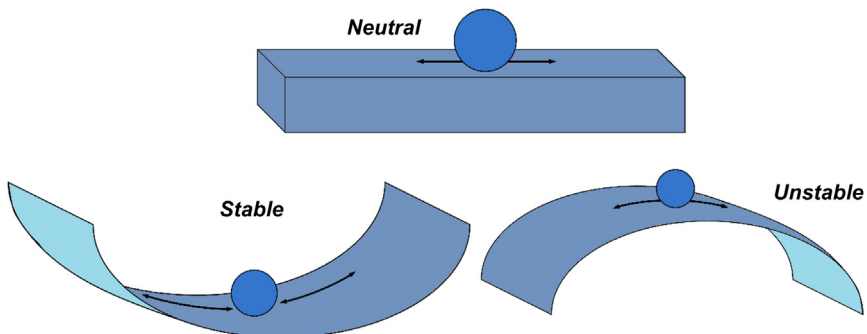


Figure 2.2 Stability of the ball's position relative to its surroundings.

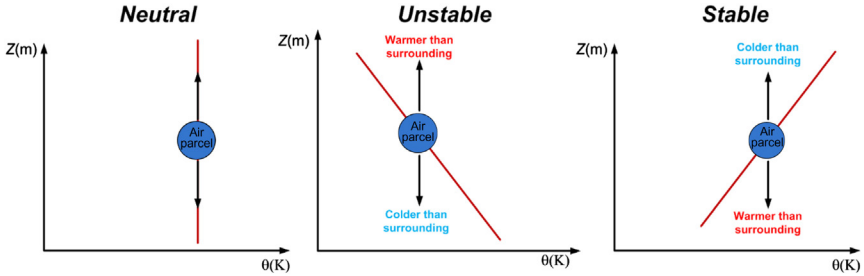


Figure 2.3 Stability of motion of air parcel relative to its surroundings.

When the air parcel is moved upwards to its new position, it is warmer and hence less dense than its surroundings. It is thus subject to an upward buoyancy force which continues to push it away from its initial position. The motion of the air parcel is also unstable when it is pushed downwards. The air parcel is denser than its surroundings, and thus continues to sink away from its initial position. We see that when the potential temperature decreases with height, the atmosphere is unstable to air motion. Any motion is amplified in such an atmosphere.

It is easy to see why an atmosphere with a constant potential temperature is indifferent to air motion. Such an atmosphere, shown in the left figure, is referred to as neutral. The right figure shows a stable atmosphere, which resists vertical motion. If a parcel is moved upwards, it is denser than its surroundings and falls back to its original position. When moved downwards, the parcel is lighter than the surrounding atmosphere, and thus rises back to its original position.

The atmosphere will tend to neutral conditions when it undergoes vertical mixing. It is only when the potential temperature is constant that further mixing does not lead to change in the temperature.

Note that when the potential temperature is constant, the actual temperature decreases with height at the adiabatic lapse rate. It is easy to calculate the temperature decrease in a layer with constant potential temperature as follows. We can use Eq. (2.23), and the hydrostatic balance to derive the relationship between the vertical gradients of the potential temperature and the absolute. Differentiating Eq. (2.23) with respect to z yields

$$\frac{1}{\theta} \frac{d\theta}{dz} = \frac{1}{T} \frac{dT}{dz} - \frac{R_a}{C_p p} \frac{dp}{dz}. \quad (2.24)$$

Using Eq. (2.15) to eliminate the vertical pressure gradient, we find

$$\frac{d\theta}{dz} = \frac{\theta}{T} \left[\frac{dT}{dz} + \frac{g}{C_p} \right]. \quad (2.25)$$

Because $\theta/T \cong 1$, we can calculate the potential temperature gradient from

$$\frac{d\theta}{dz} \cong \frac{dT}{dz} + \frac{g}{C_p}. \quad (2.26)$$

If the potential temperature is constant, Eq. (2.26) implies that

$$\frac{dT}{dz} = -\frac{g}{C_p}. \quad (2.27)$$

This temperature decrease with height is called the adiabatic lapse rate, whose value can be readily worked out to be

$$\frac{dT}{dz} = -\frac{g}{C_p} = -\frac{9.81 \text{ m/s}^2}{1000 \text{ J/(kg K)}} \cong -\frac{10}{1000} \text{ K/m}. \quad (2.28)$$

So if the temperature in the atmosphere decreases faster than the adiabatic lapse rate, the atmosphere is unstable. If the temperature decrease is less than the adiabatic lapse rate, the atmosphere is stable.

When the potential temperature is constant through a layer, the layer is well mixed in the sense that in the absence of external heat input there can be no changes in the potential temperature. To see this, imagine moving an air parcel from one height to another. Initially because its temperature is the same as its surroundings, there is no heat transfer into the air parcel. Any motion does not change this situation because one can imagine this motion to consist of steps in which the parcel is moved adiabatically, and then allowed to come to equilibrium with its surroundings. In its new location, the parcel has exactly the same temperature as its surroundings because adiabatic motion of the parcel does not change the potential temperature, which is identical to the potential temperature of its surroundings. If the potential temperatures are equal, so are the actual temperatures, and there is no heat transfer, and hence no change in the potential temperature of the parcel or its surroundings.

Notice that when we lift a parcel adiabatically, or if a layer is well mixed, the temperature decreases with height. It is this decrease in temperature that leads to the condensation of vapor in moist air parcels, and hence the formation of clouds. However, this condensation results in

the heating of the parcel, and the temperature decrease is smaller than the dry adiabatic lapse rate. The rate of decrease in temperature of the saturated parcel, referred to as the wet adiabatic lapse rate, γ_w , is always smaller than the dry adiabatic lapse rate, γ_d . This makes physical sense because the heat released during condensation of the water vapor has to decrease the rate at which the temperature decreases during dry conditions.



THE ORIGIN OF LARGE-SCALE WINDS

The winds that carry the pollution away from their sources are initiated by the differential heating of the Earth's surface by solar radiation. The intensity of solar radiation is highest near the equator, and becomes small near the poles. This difference in radiation translates into temperature differences at the ground, which in turn give rise to large-scale convective motion. Once initiated, this motion is governed by several forces that determine the subsequent evolution of the wind. These forces of most importance to large-scale motion are (1) the pressure gradient force, (2) the Coriolis force, and (3) the frictional force. Let us consider each of them in turn.

The Pressure Gradient Force

The vertical pressure gradient is proportional to the local air density:

$$\frac{dp}{dz} = -\rho g. \quad (2.29)$$

Eq. (2.29) tells us that the rate at which the pressure decreases with height increases with density. This means that the pressure falls off faster in a cold (dense) atmosphere than in a warm atmosphere. Let us use this observation to determine the role of differential heating in generating large-scale winds.

Assume that the Earth as represented in Fig. 2.4:

In the figure, the air is warm over the equator and cold over the North Pole. We assume that the surface pressures at equator and North Pole are the same. As discussed earlier, the pressure decreases with height more rapidly in the cold (dense) air than in the warm air. This means that

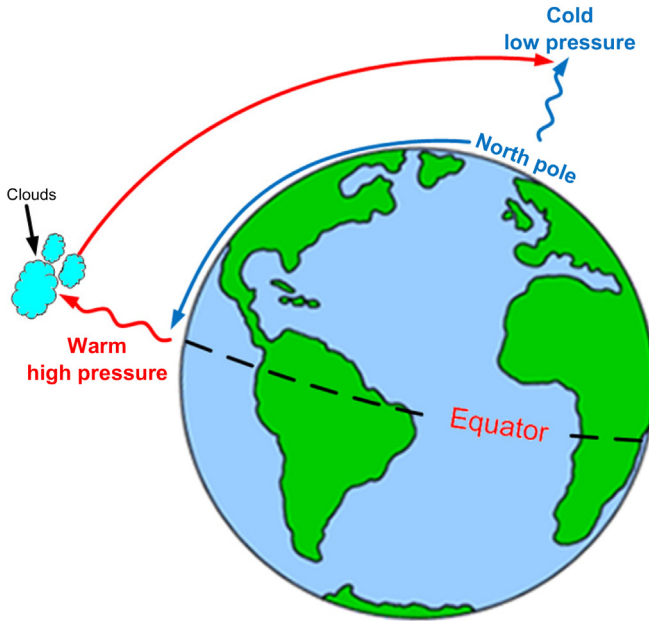


Figure 2.4 Representation of Earth's surface to relate temperature gradients to pressure gradients.

at all heights above the surface; the pressure over the equator is higher than that over the pole at the same height over the Earth's surface. The pressure at High pressure is greater than that at Low pressure, which results in a horizontal pressure gradient that drives the air toward the north. This flow is compensated by flow from the north to the south at the surface. There is rising motion over the equator, and descending motion over the North Pole to complete the convective circulation pattern. In the absence of the Earth's rotation, we would expect to see such a pattern. However, the Earth does rotate, and the circulation pattern is considerably more complicated than this picture.

As the air flows toward the north, the Earth's rotation causes it to turn toward the west. This turning is caused by the Coriolis force, which is a pseudo force that we need to invoke in a rotating frame of reference. The Coriolis force also deflects the southward moving flow toward the east. This general pattern of circulation is seen in cells near the equator, and near the poles. However, in mid-latitudes between 40° and 60° , the flow is much less organized in the vertical. The large-scale flow is westerly

(from the west), and superimposed on this flow are structures referred to as cyclones and anticyclones. These structures are responsible for the weather we see in the mid-latitudes. In this chapter, we will not discuss weather patterns in any great detail. We will confine our attention to the underlying forces that determine air motion. We have already discussed the pressure gradient force. The next section discusses the Coriolis force.

The Coriolis Force

To understand the origin of the Coriolis force, imagine conducting the following experiment with a turntable. Joe is on the rotating turntable shooting beads along the X -axis, which is rotating with him. The situation is shown in Fig. 2.5.

If the beads are shot out from the center of the turntable, they would have no tangential velocity and thus move along their original X -direction as shown in the figure. This is what, Moe, the stationary observer expects; the successive positions of one of these beads are shown as **A** and **B**. On the other hand, Joe, on the turntable, thinks that the bead ought to be at positions **A'** and **B'**. Because the bead is observed to be at **A** and **B**, Joe has to invent a force, called the Coriolis force, that forces the beads to their right as they move along Joe's X -axis.

We can derive an expression for the Coriolis acceleration by assuming that the beads are forced to move along the X -axis in Joe's rotating coordinate system; assume that the beads move along a wire stretched out along the radius, as shown in Fig. 2.6. Then in order for us to keep the beads from deflecting to the right, we need to apply a Coriolis force to the left.

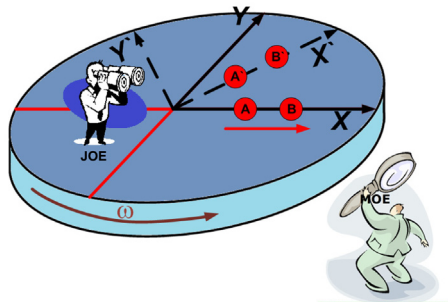


Figure 2.5 Motion of a ball on a turntable as viewed by two different observers.

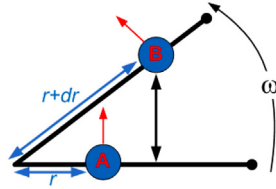


Figure 2.6 Motion of a ball along a wire on a rotating turntable.

Let us calculate the magnitude of the associated Coriolis acceleration by examining the distance traveled by the bead over a small time interval.

Assume that the turntable rotates anticlockwise over a small time interval Δt . If the angular velocity of rotation is ω , the corresponding angle through which the X -axis rotates is $\omega\Delta t$. Now the tangential velocity at position A is ωr . To calculate the Coriolis acceleration, let us use the expression for the distance traveled in the tangential direction:

$$s = ut + \frac{1}{2}at^2, \quad (2.30)$$

where u is the initial velocity and a is the acceleration. Now

$$s = \omega(r + \Delta r)\Delta t \quad (2.31)$$

where

$$\Delta r = v_r\Delta t$$

and v_r is the radial velocity. Substituting Eq. (2.31) into Eq. (2.30), and using $u = \omega r$, we find that the Coriolis acceleration is

$$a_c = 2v_r\omega. \quad (2.32)$$

It turns out that the Coriolis acceleration does not depend on the particle (or bead) traveling radially. The velocity, v_r , can be in any direction on the plane that is normal to the vector of rotation, $\vec{\omega}$. The acceleration is normal to the velocity vector, $v_r\vec{r}$, and when $a_c\vec{r}$ is rotated toward $v_r\vec{r}$, a right handed screw will point toward $\vec{\omega}$.

On the Earth, the rotation rate is a maximum at the pole, and decreases with the latitude as shown in Fig. 2.7

If the rotation rate is ω at the pole, the rotation rate of the plane parallel to the Earth's surface at a latitude, λ , is $\omega\sin\lambda$, and the Coriolis acceleration of a particle moving with a velocity v_r on the plane is

$$a_c = 2v_r\omega\sin\lambda. \quad (2.33)$$

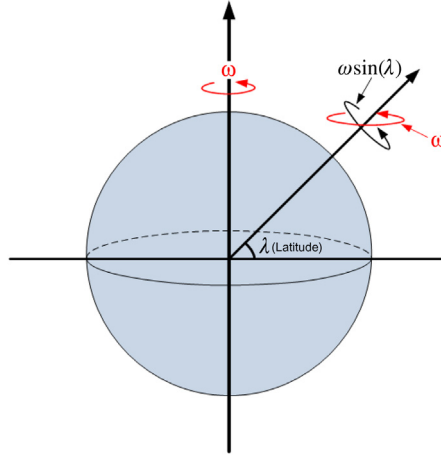


Figure 2.7 Rotation of the Earth as a function of latitude.

In the atmospheric literature, we define the Coriolis parameter, f , as follows

$$f = 2\omega \sin\lambda. \quad (2.34)$$

Notice that f has the units of inverse time. Its inverse $1/f$ is an important time scale that governs several atmospheric phenomena, including the dynamics of the atmospheric boundary layer. We will discuss this briefly in a later section. The magnitude of f at latitude of 45° is given by

$$f = 2 \times \frac{2\pi}{24 \text{ h} \times 3600 \text{ s/h}} \sin(45) \cong 1 \times 10^{-4} \text{ s}^{-1}. \quad (2.35)$$

The next section discusses the winds created by the balance between the pressure gradient force and the Coriolis force.



THE GEOSTROPHIC WIND

We saw earlier that the pressure gradient force generates air motion from a region of high pressure to that of low pressure. As the air accelerates toward the region of low pressure, it is subject to the Coriolis force, which in the northern hemisphere, deflects the wind to the right. This is depicted in [Fig. 2.8](#).

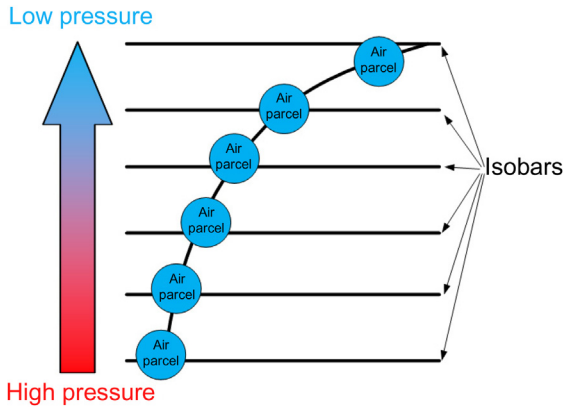


Figure 2.8 Deflection of moving air parcel by Coriolis force.

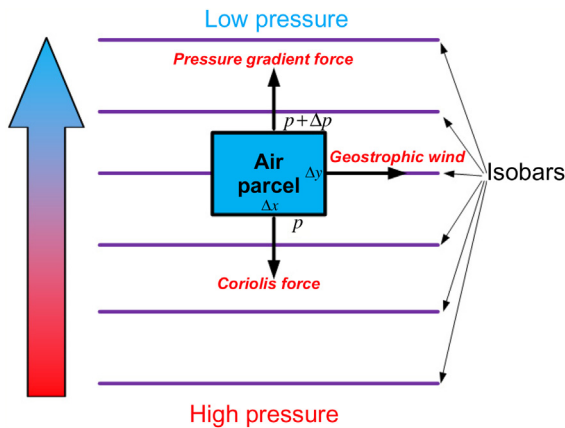


Figure 2.9 Balance between pressure gradient force and Coriolis force to produce the geostrophic wind.

The horizontal lines represent regions of equal pressure at some height above the ground; these lines are called isobars. As the air parcel moves from high pressure to low pressure, it is deflected to the right by the Coriolis force. Over long-time scales, the pressure gradient and the Coriolis forces reach a state of balance, and the wind corresponding to this balance is referred to as the geostrophic wind.

We can derive an expression for the geostrophic wind, V_g , using Fig. 2.9.

Consider an air parcel with dimensions Δx and Δy , and unit depth. Take the density of the air parcel to be ρ . Then the Coriolis force acting downwards is given by

$$\text{Coriolis Force} = f\rho(\Delta x\Delta y)V_g. \quad (2.36)$$

The pressure gradient force acting upwards is given by:

$$\begin{aligned} \text{Pressure gradient force} &= [p - (p + \Delta p)]\Delta x \\ &= -\Delta p\Delta x \end{aligned} \quad (2.37)$$

Equating the two forces gives us the required expression for the geostrophic wind:

$$V_g = -\frac{1}{\rho f} \frac{\Delta p}{\Delta y}. \quad (2.38)$$

Eq. (2.38) provides useful estimates of the winds at several kilometers above the ground. The equation tells us that horizontal pressure gradients are associated with winds. We can estimate the magnitude of these pressure gradients by taking

$$\begin{aligned} V_g &= 10 \text{ m/s} \\ \rho &= 1 \text{ kg/m}^3 \\ f &= 10^{-4} \text{ s}^{-1} \\ \Delta y &= 100 \text{ km} \end{aligned}$$

Then Eq. (2.38) yields

$$\begin{aligned} \Delta p &= -10 \text{ m/s} \times 1 \text{ kg/m}^3 \times 10^{-4} \text{ s}^{-1} \times 100 \text{ km} \times 1000 \text{ m/km} \\ &= 100 \text{ N/m}^2 = 1 \text{ mb} \end{aligned} \quad (2.39)$$

We see that horizontal pressure gradients are of the order of a few mb over 100 km. In contrast, vertical pressure gradients are of the order of 100 mb over 1 km.

Because horizontal winds are associated with pressure gradients, it is often possible to infer the magnitude of winds from maps of pressures. The fact that upper air winds are approximately Geostrophic suggests Guy Ballot's Law: if the wind is at your back, the high pressure is on your right in the northern hemisphere.

Most weather maps usually show surface isobars, which appear as patterns of high and low pressures. These patterns, which are examined in the next section, are important indicators of weather.



HIGH- AND LOW-PRESSURE CENTERS

Large-scale disturbances in the upper atmosphere lead to the formation of localized high- and low-pressure regions, which are commonly referred to as highs and lows. High-pressure regions are associated with large-scale descending motion or subsidence, which leads to compressive heating of the air and the formation of regions in which the temperature increases with height. As we saw earlier, these temperature inversions resist vertical motion, and can thus act as lids to vertical dispersion of pollution. Because descending motion prevents the formation of clouds, highs are associated with clear skies with warm temperatures. Such conditions are conducive to the formation of smog.

Low-pressure regions are associated with large-scale ascending motion of air. This leads to the formation of clouds, which in turn can lead to rain. The formation of clouds is accompanied by condensation and hence heating, which, in turn, can lead to further buoyancy-induced vertical motion. Thus air motion in lows is unstable because of this positive feedback between vertical motion and cloud formation. Weather in lows is generally inclement. It is easy to see that one does not expect air pollution buildup in low-pressure regions.

Low-pressure regions are referred to as cyclones because the air motion around them is **cyclonic**, or in the same direction as the Earth's rotation; this rotation is anticlockwise in the northern hemisphere. High-pressure regions are referred to as anticyclones because the flow around them is clockwise. To understand the direction of flows around highs and lows, we need to examine the forces affecting these flows. Let us first consider the flow around a high-pressure region, as shown in Fig. 2.10.

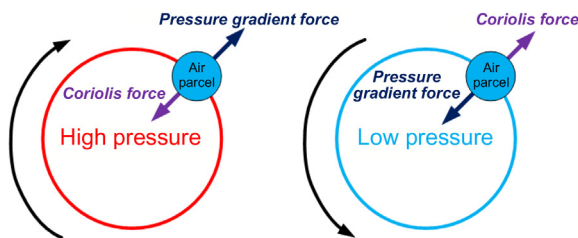


Figure 2.10 Flow around high- and low-pressure centers.

The figure shows an air parcel in the flow around a high-pressure center. The pressure gradient force is outward, while the Coriolis force is directed toward the center of the high. The Coriolis force has to be greater than the pressure gradient force to provide the net inward force to keep the air parcel circulating around the center of the high. If the flow was anticlockwise, both the Coriolis and the pressure gradient forces would be directed outward, and flow around the center would not be possible.

Now consider the flow around a low or cyclone. Here the flow is counterclockwise, with the difference between the inward pressure gradient force and the outward Coriolis force providing the force to keep the air parcel moving around the low. In principle, the flow around the low can be clockwise because both the Coriolis and the pressure gradient forces can add to provide the required inward force. However, such a flow is unusual because the flow around the low originates from winds that are initially in geostrophic balance: the Coriolis force opposes the pressure gradient force. The anticlockwise flow around a low maintains this relationship between the two forces.

The majority of the cyclones and anticyclones are migratory in the sense that they are formed in different parts of the Earth and then move with the large-scale winds until they are dissipated. However, some of these systems are semipermanent because they are associated with the global circulation pattern setup by differential heating of the Earth's surface. The weather in Los Angeles is governed by semipermanent anticyclone associated with the descending region of the **Hadley** cell extending from the equator to about 30°N and S latitudes. This high-pressure center causes the generally good weather marked by the lack of clouds and rain during most of the year. The winter rains occur when the high-pressure region move southwards during winter. The undesirable effect of warm temperatures is the smog that forms when pollutants react under sunny, light wind conditions. Therefore we can blame part of our air pollution problems on weather.

The flows around highs and lows are not parallel to the isobars because of the effects of friction. The wind is directed into the low, and away from the high; there is low-level convergence into a low-pressure region, and divergence at low levels of a high. These modifications by friction are treated in the next section.

EFFECTS OF FRICTION

Friction refers to processes that decrease the wind that would exist under geostrophic balance. The slowing down is caused by the transfer of vertical momentum from upper layers of the atmosphere to lower layers by turbulence. Because the horizontal velocity at the ground has to be zero, any transfer of momentum will lead to reduction of the geostrophic velocity (Fig. 2.11).

We can examine the effects of friction by first assuming that the flow is initially geostrophic, so that the pressure gradient force is balanced by the Coriolis force. When friction slows the velocity, the Coriolis force, F , decreases because it decreases with the velocity. The pressure gradient force, P , is now greater than the Coriolis force, and the wind is deflected toward lower pressure until a new state of force balance is reached. This is illustrated in Fig. 2.11.

We can determine the cross-isobaric angle, θ , by performing a force balance on the air parcel. We find

$$\begin{aligned} F \cos \theta &= C \sin \theta \\ \text{and} & \\ F \sin \theta + C \cos \theta &= P \end{aligned} \quad (2.40)$$

Eq. (2.40) allows us to solve for the two unknowns, θ and the horizontal velocity, v_H , if we know the relationship between the frictional force, F , and v_H .

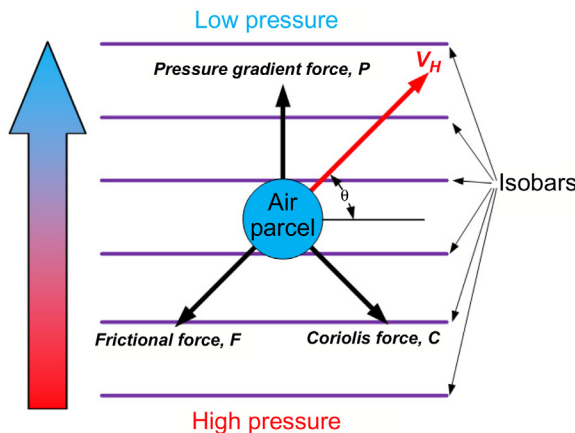


Figure 2.11 Turning of the wind by frictional forces.

Notice that the cross-isobaric angle increases with the frictional force, which means that it has its largest value close to ground, and decreases as we go up in height. Another way of saying this is that the wind rotates clockwise as the height increases. The clockwise rotation and the accompanying increase of velocity is commonly referred to as the Ekman spiral in honor of the German meteorologist, Ekman.

The frictionally induced flow across isobars also occurs in high- and low-pressure centers. The wind flows into a low and away from a high. This is consistent with the low-level convergence and divergence required to maintain lows and highs.



FRONTS

This section provides a very brief description of the phenomena of fronts, which play a major role in determining our weather. Fronts represent the boundaries between air masses with markedly different characteristics, especially temperature. These air masses acquire their properties by moving slowly or stagnating over different regions of the Earth's surface. For example, a large mass of cold air can form by southward moving air that spends a long period of time over the continents that are cold during winter. Similarly, a warm air mass can form in air originating from equatorial regions, and then moving slowly over the warm oceans.

The boundary between the cold and warm air masses is referred to as a front, a name that is associated with the time, the Second World War, during which the phenomena was first discovered and studied. The role of these fronts in determining weather in the mid-latitudes was first suggested by a group of Norwegian meteorologists, who likened the activity at the boundaries between warm and cold air masses to the fronts of advancing and retreating armies.

The weather associated with a front is caused by lateral motion along the boundary between the cold and warm air masses. The usual pattern of motion resembles a wave, which is illustrated in [Fig. 2.12](#).

On the left side of the wave, the cold air mass pushes southward along the cold front, while the warm air pursues the retreating cold air along the warm front. A low-pressure center is located at the crest of the wave,

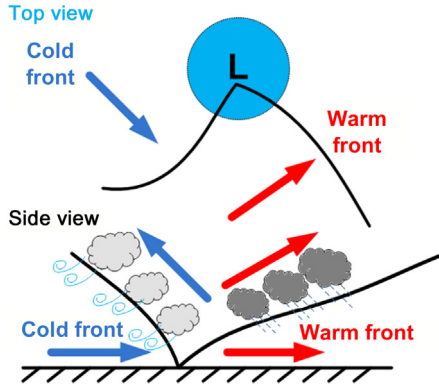


Figure 2.12 Schematic representation of warm and cold fronts.

and as described earlier, the flow around it is cyclonic. At the fronts, the warm air rides over the denser cold air mass. The vertical motion of the warm, usually moist, air at the fronts, results in the formation of clouds and hence rain. The advancing cold front forces the warm air vertically, which results in the formation of clouds and rain at the front. The retreating warm front is shallower than the cold front is, and the vertical motion of the overriding warm is less intense than at the cold front. The weather is generally more unstable at the cold front than at the warm front. Thunderstorms usually occur at the cold front; at the warm front, the more gradual upward motion results in precipitation that is usually light and continuous, but it occurs over a larger area than at the cold front.

The slopes of the fronts are of the order 1 in 100: the vertical scale is of the order of kilometers, while the horizontal scale is of the order of hundreds of kilometers. The weather pattern associated with the passage of the frontal system can be understood by placing yourself on the right of the warm front. As the pattern moves from left to right, you will first notice clouds that are layered in appearance, and whose bases are about 3 km; these are associated with the gentle vertical motion along the retreating warm front. Steady rain will start falling when the warm front is several hundred kilometers from you. This will be accompanied by falling pressure. As the front approaches you, the clouds will start to thicken, and the rain will become heavier and more intermittent, and the cloud bases will become lower signaling the approach of the warm air.

You can see from [Fig. 2.12](#), that as the warm front passes you, the wind direction at the surface will shift from the south, southeast, to

southwest. The temperature will rise, and the precipitation will disappear. Within 24 h, the cold front will approach, and it will signal its presence through rapidly falling pressures and deepening clouds. The passage of the leading edge of the cold front will be accompanied by a rapid change in the wind direction to the northwest, rising pressure, and showers and thunderstorms associated with deep clouds. These rains will last for a few hours until you are in the cold air mass behind the front. The weather now is cold and crisp, and the air is usually clear.

This chapter provides the preliminaries required for a detailed discussion of the processes within the atmospheric boundary layer, which are described in Chapter 3, Fundamentals of Micrometeorology and Dispersion.