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Short Horizon Return Reversals and the Bid-Ask Spread

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Abstract

This paper shows that the pattern of short term negative serial covariances for stock returns over different return measurement intervals is consistent with the implications of inventory-based market microstructure models. The results of the tests developed in this paper indicate that the short horizon return reversals can to a large extent be explained by transitory components in prices related to imbalances in specialists' inventory positions. The evidence also suggests that the stock market may not be as liquid or resilient as suggested by some earlier studies.

The evidence that short horizon stock returns exhibit negative serial correlation has recently attracted a lot of attention. Although this empirical regularity had been documented more than two decades ago by Cootner (1964) and Fama (1965), until recently, the short term return reversals were not considered economically important. Much of the recent interest can be attributed to evidence suggesting that contrarian trading strategies designed to exploit these return reversals provide investors with opportunities to achieve significant abnormal returns. For instance, Jegadeesh (1990) documents that a contrarian trading strategy designed to exploit the negative serial correlation in monthly stock returns yields abnormal returns of about 2% per month. Subsequently, Lehmann (1990) examined a similar trading rule applied to weekly data and found that over twenty-six consecutive six month periods, the trading rule never lost money.[1]

One explanation that Jegadeesh and Lehmann consider is that the negative serial correlation in returns is caused by the bid-ask spread. Appealing to Roll's (1984) model, where the bid-ask spread induces negative serial correlation only for returns measured over adjacent intervals, Jegadeesh and Lehmann examine a modified strategy in which one day is skipped between the dates when the returns used to form the portfolios are measured and the dates when the contrarian strategies are initiated. The authors argue that since the profits to these modified strategies are also significantly positive, the observed return reversals are not due to the bid-ask spread and are suggestive of an inefficient market where stock prices over-react to information.

This paper presents evidence that indicates that higher order serial correlations can in fact be generated by the bid-ask spread. In contrast to Roll's model, which considers only the component of the spread that is due to the specialists' (or market

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[1]Lo and MacKinlay (1990a) decompose the contrarian strategy profits into components related to the negative serial correlation in stock returns and to the positive serial correlation in market returns. They find that both these components are individually significant.

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makers') order processing and other fixed costs, we focus on the component of the spread that is due to the specialists' inventory costs.[2] This component, analyzed in papers by Amihud and Mendelson (1980), Ho and Stoll (1981) and Stoll (1989), will arise when the specialists are risk-averse and thus incur costs when the shares they buy or sell lead them to hold undiversified portfolios.

Consider for example the case where a number of investors require liquidity and sell shares; requiring the specialist to take an unusually large inventory. To induce the specialist to hold these shares, the bid price must be low enough to enable him to earn a positive excess return on his inventory to compensate for the risk of holding an undiversified portfolio. Hence, liquidity induced price declines are likely to be followed by price increases, and similarly, liquidity induced price increases are likely to be followed by price declines. If the liquidity demands have a very large effect on inventory positions (or if the factors that led to an excess demand for liquidity persist for a number of days) then it is likely that these excess inventory positions will persist beyond a single day. As a result, the specialist's reservation price is likely to be relatively low for a number of days, which in turn implies that stock returns will exhibit serial correlation over non-adjacent days.[3]

As Grossman and Miller (1988) emphasize, the above described relation between inventory costs and return reversals does not require a monopolistic specialist. In

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[2] An additional component of the spread that has received a great deal of attention arises because of adverse selection. This component was analyzed in models by Glosten and Milgrom (1986) and Copeland and Galai (1983). The bid-ask spread in these models does not, however, have an effect on the serial correlation of returns and is not explicitly considered in this paper. In the Glosten and Milgrom model, market makers buy and sell shares at their expected values conditional on either buy or sell order arriving. Hence, the bid price equals the expected value of the shares conditioned on a sell order arriving and the ask price is the expected value conditional on a buy order arriving. Since transaction prices occur at the expected values of the shares given all public information, price changes from period to period are serially uncorrelated in their setting.

[3] The discussion above is in the context of the Ho and Stoll model. The behavior of stock returns in Amihud and Mendelson is also consistent with the discussion here but in their model the monopolistic market maker is not risk averse but is subject to inventory constraints. They show that when the market maker sets his prices optimally the quoted prices are mean reverting.

reality, there are a number of investors that provide liquidity to the market. The empirical question that is of interest is whether or not the inventory positions of these providers of liquidity will generate the pattern of negative serial correlation that we observe.

To explore this issue in more detail we develop a simple statistical model that allows us to decompose the serial covariance of stock returns into four components: a component that relates to the decay of the temporary component of stock prices caused by the specialist's (or other liquidity providers') inventory imbalances, a component that accounts for the effect on serial covariances of the bid-ask bounce considered by Roll, and two additional components that relate to interactions between the order flow and temporary components of stock prices caused by inventory imbalances. Using some results of Ho and Stoll, we show that when the specialist's inventory levels are relevant for setting the bid and ask prices, the magnitude of the serial covariances of returns are typically smaller for returns measured over short intervals than for returns measured over long intervals. This indicates that Roll's measure of the implicit spread may underestimate the actual bid-ask spread when the serial covariance of returns are measured over short (e.g. daily) intervals and that it overestimates the spread when serial covariances of returns are measured over sufficiently long (e.g. weekly) intervals. Additionally, we show that the slope coefficient in a regression of the serial covariance of returns on the squared spread is larger in magnitude for long measurement intervals than for short intervals.

The results of our empirical tests are consistent with these predictions. Specifically, the quoted spreads are on average larger than the estimates of Roll's implicit measure based on one-day returns and they are smaller than the implicit spread based on ten-day returns. In addition, the magnitude of the slope coefficient of a cross-sectional regression of the estimated serial covariance of returns on the squared

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spread increases in magnitude monotonically as the return measurement interval is increased from one to ten days. Our results also indicate that the squared spread explains a large fraction of the magnitude of observed serial covariances and that these serial covariances are significantly higher during periods of higher than average volume.

The rest of the paper is organized as follows: In the next section we relate the serial covariances at different return measurement intervals to the bid-ask spread. The empirical tests and the results are presented in section 2. Section 3 briefly discusses further tests that illustrate the importance of inventory imbalances and section 4 contains our conclusions.

I The Relation Between Serial Covariance and the Bid-Ask Spread

Let p'_t and p_t be the natural logarithm of the 'true' price (a hypothetical price that would occur in the absence of transaction costs) and the transaction price at time t respectively. Let,

$$p_t = p'_t + z_t,$$

where z_t, the temporary component of the transaction price is determined by the bid-ask spread and is assumed to be independent of the true price.

In the context of the inventory model of Ho and Stoll, z_t can be decomposed into the sum of two components, $\frac{S}{2}\theta_t$ and δ_t, i.e.,

$$z_t = \frac{S}{2}\theta_t + \delta_t.$$

In the above expression,

$$\theta_t = 1 \quad \text{if the last transaction is a buy,}$$
$$= -1 \quad \text{if the last transaction is a sell,}$$

S, the total spread, is assumed to be constant over time, consistent with the results of Ho and Stoll and

δ_t is the adjustment to the quoted prices made by the dealer in response to inventory imbalances. When the dealer has positive (negative) excess inventory δ_t is set to be negative (positive) in order to encourage purchases (sales) and discourage sales (purchases). In Ho and Stoll, the magnitude of δ_t depends upon a number of unobservable parameters such as the wealth, the risk aversion parameters and the planning horizon of the specialist. However, in their model, these parameters affect δ_t in the same way as they affect the size of the spread so that δ_t can be specified as a simple linear function of the inventory level and the size of the spread,

$$\delta_t = -\alpha' I_{t-1}(S - C),$$

where α' is a positive constant that is related to the average order size, I_t is the level of inventory imbalance at time t, and C represents those components of the spread that are not directly related to inventory costs, (such as monopoly profits and fixed costs). In what follows, we will use a stylized version of the above expression that assumes that C is proportional to S cross-sectionally, allowing us to specify δ_t as

$$\delta_t = -\alpha I_{t-1} S. \tag{1}$$

This specification of δ_t is similar to that in Stoll (1989) where the specialist temporarily increases the bid and ask prices by an amount proportional to the spread after each transaction.

Under the hypothesis that the 'true' prices follow a random walk, the serial covariance of the changes in observed prices across a measurement interval of k periods is given by the sum of four terms,

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$$\text{cov}(p_{t+k} - p_t, p_t - p_{t-k}) = \text{cov}(z_{t+k} - z_t, z_t - z_{t-k})$$

$$= \text{cov}(\delta_{t+k} - \delta_t, \delta_t - \delta_{t-k}) + \frac{S^2}{4}\text{cov}(\theta_{t+k} - \theta_t, \theta_t - \theta_{t-k})$$

$$+ \frac{S}{2}\text{cov}(\delta_{t+k} - \delta_t, \theta_t - \theta_{t-k}) + \frac{S}{2}\text{cov}(\theta_{t+k} - \theta_t, \delta_t - \delta_{t-k}),$$

$$(2)$$

each of which can be considered separately.

The first term is related to the decay of the inventory component of the bid and ask prices. Using equation (1), this term can be expressed as,

$$\text{cov}(\delta_{t+k} - \delta_t, \delta_t - \delta_{t-k}) = -\alpha^2 S^2(\sigma_I^2 + \gamma_{2k} - 2\gamma_k),$$

where σ_I^2 is the variance of I_t and γ_k is its kth-order serial covariance. Unlike the serial covariance in returns due to the bounce between the bid and ask prices, the serial covariance due to the decay of δ will persist beyond a single transaction. In Ho and Stoll, $\gamma_k \to 0$ as k becomes large since the inventory imbalance of the specialist is stationary. Therefore,

$$\lim_{k \to \infty} \text{cov}(\delta_{t+k} - \delta_t, \delta_t - \delta_{t-k}) = -\alpha^2 S^2 \sigma_I^2.$$

For small k, however, $\gamma_k > \gamma_{2k}$ and hence $\text{cov}(\delta_{t+k} - \delta_t, \delta_t - \delta_{t-k}) > -\alpha^2 S^2 \sigma_I^2$. In other words, the magnitude of the serial covariance in returns due to the decay of δ will be larger for large k than for small k.

The second term in (2) can be expressed as,

$$\frac{S^2}{4}\text{cov}(\theta_{t+k} - \theta_t, \theta_t - \theta_{t-k}) = -S^2(1 - \pi_k)^2,$$

where π_k is the unconditional probability that a buy (sell) order at time t will be followed by a buy (sell) order at time $t+k$. The derivation of this expression is similar to that in the generalization of Roll's model considered by Choi, Salandro and Shastri (1988). In Ho and Stoll the specialist changes the δ_t between transactions

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in order to encourage offsetting transactions so that π_k is less than .5 for low k making this component of the serial covariance larger in magnitude than $-.25S^2$ for small k.[4] However, this component tends to $-.25S^2$ for large k as in Roll.

The third component of serial covariance in (2) is related to the average adjustment to δ made by the specialist in response to a buy or a sell order. Specifically,

$$\frac{S}{2}\text{cov}(\delta_{t+k} - \delta_t, \theta_t - \theta_{t-k}) = -\alpha\frac{S^2}{2}\text{cov}(I_{t+k-1} - I_{t-1}, \theta_t - \theta_{t-k})$$
$$= -\alpha\frac{S^2}{2}\{\text{cov}(I_{t+k-1} - I_{t-1}, \theta_t) - \text{cov}(I_{t+k-1} - I_{t-1}, \theta_{t-k})\}.$$

Note that θ_t is positive when the transaction at time t is a buy and that a buy order leads to a reduction in the specialist's inventory level. Therefore, the first term within brackets in the above expression is negative for small k. As k increases this term becomes smaller since any inventory imbalance is offset in the long run. The second term in the above expression is also negative since the transaction at time $t - k$ has a bigger effect on the inventory position at time $t - 1$ than at time $t + k - 1$. The above discussion implies that the third component can be written as,

$$\text{cov}(\delta_{t+k} - \delta_t, \theta_t - \theta_{t-k}) = S^2 f_{1k}, \tag{3}$$

where f_{1k} is positive and becomes small as k becomes large.[5]

The last term in (2) is,

$$\frac{S}{2}\text{cov}(\theta_{t+k} - \theta_t, \delta_t - \delta_{t-k}) = -\alpha\frac{S^2}{2}\text{cov}(\theta_{t+k} - \theta_t, I_{t-1} - I_{t-k-1}),$$
$$= -\alpha\frac{S^2}{2}\{\text{cov}(\theta_{t+k}, I_{t-1} - I_{t-k-1}) - \text{cov}(\theta_t, I_{t-1} - I_{t-k-1})\}.$$

Since positive changes in inventory leads to a decrease in δ which in turn increases the probability that the transaction at time t is a buy order, both the covariances

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[4]However, if the order flow over short intervals is positively serially correlated over short intervals for reasons outside the model, as found by Choi, Salandro and Shastri in the options market, then this component of serial covariance will be smaller in magnitude than $-.25S^2$.

[5]Even for large k this term is not zero due to $cov(I_{t-1}, \theta_t)$. This covariance is positive since positive inventory imbalance leads to negative δ which in turn encourages buy orders (negative θ). However, this effect is likely to be of second order importance in our analysis.

within the brackets are negative. The effect of the second term within the brackets dominates that of the first term because the changes in the specialist's inventory from time $t - k - 1$ to $t - 1$ is likely to have a bigger effect on the order flow at time t than that at time $t + k$. Therefore, the last component can be expressed as,

$$\frac{S}{2}\text{cov}(\theta_{t+k} - \theta_t, \delta_t - \delta_{t-k}) = S^2 f_{2k},$$

where f_{2k} is positive but probably small in magnitude.

The preceding analysis indicates that expression (2) can be written as,

$$\text{cov}(p_{t+k} - p_t, p_t - p_{t-k}) = -\alpha^2 S^2 (\sigma_I^2 + \gamma_{2k} - 2\gamma_k) - S^2 (1 - \pi_k)^2$$
$$+ S^2 f_{1k} + S^2 f_{1k}. \tag{4}$$

This representation of the serial covariances of stock returns can be compared with Roll's (1984) predictions for both large and small $k's$. In the case considered by Roll only the second term in the above expression is relevant and π_k is assumed to equal .5. Thus, in Roll's model,

$$\text{cov}(p_{t+k} - p_t, p_t - p_{t-k}) = -.25 S^2. \tag{5}$$

For large k, the sum of the first two terms on the right-hand-side of the expression (4) is less than $-.25 S^2$ and the last two terms are close to zero. This implies that for returns measured over sufficiently long intervals the magnitude of the negative serial covariance should be greater than that predicted by Roll, and that the coefficient from a cross-sectional regression of serial covariance on S^2 should be less than $-.25$. For small k, the decay of the temporary component is likely to be small and hence the first term will be close to zero. In addition, the third component is positive and likely to be of importance for small k. As a result, for small k the serial covariance in observed returns is likely to be greater than $-.25 S^2$.[6]

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[6] For instance, Stoll (1989) presents a simplified version of Ho and Stoll where the effect of the

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II Empirical Tests

In this section we empirically test the implications of the inventory model discussed in the last section. Our data set consists of quoted year-end closing spreads for securities traded on the New York Stock Exchange over the period 1963 to 1979 and daily stock returns from CRSP. The bid-ask spread data were originally obtained from the Fitch Quotations for a study by Stoll and Whaley (1983).[7] As in Stoll and Whaley (1983) the averages of the quoted spreads at the end of years t and $t-1$ are used as the spreads for year t. All securities that had spread data available and had at least 100 observations to estimate the means and the serial covariances were included in the sample. On average, there were 1214 securities per year in the sample.

II.1 Estimates of Serial Covariances

It is well known that the standard estimate of serial covariance is downward biased in small samples (see Kandell (1954) and Harris (1990)). To circumvent this small sample bias we estimate the serial covariances each year using an estimate of the expected returns from the previous year. Specifically, our serial covariance estimate for year t is,

$$\widehat{\mathrm{Scov}}_{ik}^{t} = \frac{1}{T} \sum_{n=1}^{T} (\sum_{m=1}^{k} R_{in+m-1} - k\overline{R}_{it-1})(\sum_{m=1}^{k} R_{in-m}), \tag{6}$$

where T is the number of overlapping sample points in year t, R_{in} is the continuously compounded return on day n and $\overline{R}_{it-1}$ is the sample mean daily return estimated from the data in year $t-1$. As we show in the appendix the serial covariance estimate given by (6) is unbiased.

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order flow on the changes in the temporary component are sufficiently important that in the short run the serial covariance in returns is smaller than $-.25S^2$ in magnitude.

[7] We would like to thank Hans Stoll for providing us this data.

The serial covariance from equation (6) is separately estimated for each security for each calendar year. These estimates for the individual securities are averaged across the cross-section every year. The estimates of the average serial covariances and the standard errors are computed from the time-series of the cross-sectional estimates. These estimates measured over intervals of 1, 3, 5, 7 and 10 days are presented in Table 1. Consistent with our analysis, the estimate of average serial covariance (t-statistic) for the entire sample monotonically decreases from .002 × 10^{-4} (.05) for $k = 1$ to -2.12×10^{-4} (−3.02) for $k = 10$.[8],[9]

The serial covariance estimates within the size quintiles are also presented in Table 1.[10] Virtually the same pattern of serial covariances is observed for all size-based groups. The small firms generally exhibit more negative serial covariance than the large firms at almost all lags. For instance, the average 10-day return serial covariance for the quintile of smallest firms is -4.9×10^{-4} while the corresponding estimate for the quintile of largest firms is -1.2×10^{-4}.

II.2 Quoted Spread Vs. Roll's Implicit Measure

As we showed in section 1, if inventory considerations affect bid and ask prices, Roll's implicit measure of the spread will overstate the quoted spread when the serial covariance is measured over a sufficiently long interval. This section empirically examines whether or not this is the case.

Roll's implicit measure of the spread is derived by solving equation (5) for S, an

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[8]The estimates of average serial covariance using the standard serial covariance estimator ranged from $-.008 \times 10^{-4}$ for $k = 1$ to -3.74×10^{-4} for $k = 10$. Consistent with the analytic results in Kandell and Stuart (1976) and Harris (1990), all these estimates are smaller than the unbiased estimates obtained here and also, the extent of measured bias is increasing in the measurement interval k.

[9]However, the evidence of postive serial covariances for the two large-firms quintiles at $k = 1$ is not consistent with our predictions. This result could possibly be explained by a more general inventory-based market microstructure model that allows for positively correlated order flows.

[10]Securities are assigned to different size-quintiles at the beginning of each year. The size-quintile cutoffs are based on the market values of all securities traded on the NYSE at the beginning of each year.

substituting an estimate of the serial covariance. For example, if the serial covariance is estimated with k-period returns, Roll's implicit measure is given by,

$$S^k_{imp} = 2\sqrt{-\widehat{Scov_k}} \quad \text{if} \quad -\widehat{Scov_k} > 0$$
$$= -2\sqrt{\widehat{Scov_k}} \quad \text{if} \quad -\widehat{Scov_k} < 0.$$

As Harris (1990) shows, in small samples, Roll's implicit measure is severely biased downwards when the variance of the serial covariance estimate is large.[11] Harris also reports that the analytic estimates of this bias based on low order Taylor series expressions are extremely inaccurate. To minimize this bias we estimate Roll's implicit spread using serial covariance estimates that are averaged across size-based groups of securities. Based on (5) the relation between the estimate of average serial covariance within a given group, denoted by $\overline{Scov_k}$, and the quoted spread is given by,

$$\mathrm{E}(\overline{Scov_k}) = -.25(\overline{S}^2 + \sigma^2_s),$$

where $\overline{S}$ is the average within group spread and σ^2_s is the within group variance of quoted spread.

Based on this expression the modified version of Roll's implicit measure of the average spread within a group is given by,

$$\overline{S}^k_{imp} = 2\sqrt{-\overline{Scov_k} + .25\sigma^2_s} \quad \text{if} \quad (-\overline{Scov_k} + .25\sigma^2_s) > 0$$
$$= -2\sqrt{\overline{Scov_k} - .25\sigma^2_s} \quad \text{if} \quad (-\overline{Scov_k} + .25\sigma^2_s) < 0.$$

The estimates of average implicit spreads within size-based groups over the sample period 1963-79 along with the average quoted spreads are given in Table 2.

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[11]By Jensen's inequality the expected value of the square root of a random variable is less than the square root of its expected value. Since the estimate of serial covariance is unbiased Roll's implicit measure of spread is biased downwards.

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The estimates of the implicit spreads are negative when calculated with a return measurement interval of one day and they are smaller than the average quoted spreads for $k = 3$. However, as our model predicts, these estimates are larger than the average quoted spreads for longer return measurement intervals. For example, the average quoted spread for all the securities in our sample is 1.5% of the price. In comparison, the implicit spread from serial covariances measured over three and ten day intervals are 1.1% and 2.7% respectively. The results are qualitatively similar within each of the size-based subsamples.

II.3 Regression Tests

To test the relation between serial covariances and bid-ask spreads predicted from our analysis in the last section we substitute estimates of the serial covariances for the actual serial covariances, and fit the following testable version of equation (4):

$$\widehat{\text{Scov}}_{ik}^{t} = a_k^t + b_k^t S_{it}^2 + e_{ik}^t, \tag{7}$$

where $\widehat{\text{Scov}}_{ik}^{t}$ is the estimated serial covariance of k-period returns on security i in year t. The analysis in the last section suggests that b_k should be smaller in magnitude than $-.25$ for small k and larger in magnitude than $-.25$ for large k.

Regression (7) is fitted separately for each of the seventeen years in the sample period. The average of the yearly coefficient estimates and the corresponding t-statistics obtained from the time-series of the yearly coefficient estimates are presented in Table 3. The slope coefficient estimates (t-statistics) reported in this table decrease monotonically with an increase in the measurement interval from $-.096(-8.3)$ for $k = 1$ to $-.251(-3.04)$ for $k = 10$ days and are all significant at the one percent level. The t-statistics at the bottom of this table indicate that the slope coefficient in the regression with $k = 1$ is significantly less negative than the corresponding estimate for $k = 3$ which is in turn less negative than the corre-

sponding estimate for $k = 5$. Although the slope coefficients continue to increase in magnitude for larger k the increases are not statistically significant. These findings are consistent with our prediction and support the hypothesis that at least part of the observed higher order negative serial correlation in stock returns is attributable to inventory costs.

II.4 Size-Effect?

Although the preceding results are consistent with our analysis we carry out additional tests to ensure that the observed nature of the cross-sectional relation between the squared spread and the estimate of serial covariance is not spurious. The regression results in Roll (1984) indicate that the serial covariances of small firm returns are generally larger in magnitude than that of the large firms and the results in Stoll and Whaley (1983) and our results in Table 2 indicate that the quoted spreads are bigger for small firms. Arguably, squared spread could have served as a proxy for firm size in regression (7), in which case we need to consider possible 'data snooping' biases formally discussed by Lo and MacKinlay (1990b). To address this concern we estimate the following multivariate regression where the variable LOGSIZE, the natural logarithm of the market value of equity at the beginning of the year, is included as an additional explanatory variable:

$$\widehat{\text{Scov}}_{ik}^{t} = a_{k}^{t} + b_{1k}^{t} S_{it}^{2} + b_{2k}^{t} \text{LOGSIZE}_{it} + u_{ik}^{t}. \tag{8}$$

Table 4 presents the average of the yearly coefficient estimates from regression (8) and the corresponding t-statistics. The estimates of the coefficient of S^2, b_{1k}, remains roughly the same as the corresponding slope coefficients in the univariate regression (7). These estimates are again significant at the one percent level and they decrease monotonically as the measurement interval k is increased. The coefficient of LOGSIZE is insignificant and negative for k equal to 1 and is positive and generally

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significant for k greater than 1. However, as can be seen from the R^2s, LOGSIZE adds very little explanatory power to the regressions. This finding indicates that our earlier results were not driven by the relation between spread and firm size and indicates that the squared spread is more important in explaining the cross-sectional differences in serial covariances than firm size.[12]

II.5   Instrumental Variable Regressions

The next question that is of interest is the extent to which the magnitude of the serial covariance of returns can be explained by the spread. This can be evaluated by examining the intercept in regression (7). These intercepts, reported in Table 3, are about half the average serial covariance of returns measured over intervals longer than a day. For instance, the estimate of average serial covariance with a five-day measurement interval is -1.182×10^{-4} while the intercept in regression (7) with $k = 5$ is $-.507 \times 10^{-4}$. These point estimates suggest that at least half the average serial covariance can be explained by the spread.

However, there are two explanations for why the intercepts in the set of regressions in Table 3 are different from zero that are consistent with the serial covariances being determined entirely by market microstructure considerations. One possibility is that the specific inventory model we use is misspecified. In reality, the assumptions of the Ho and Stoll model are likely to be violated in ways that imply that the spread only partly captures the inventory related component of prices. For example, the order flow process in reality may be different from the Poisson arrival process assumed by them. Secondly, any error in the measured spread will bias the estimates of the slope coefficient in (7) towards zero which in turn would bias the intercept downwards. Measurement errors in the bid-ask spread can arise either

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[12]To check whether our results are sensitive to the choice of the size variable we also fitted regression (8) with size-decile rank in the place of LOGSIZE and obtained similar results.

from the restriction that the bid and ask prices are quoted in eighths or because the spread data are point sampled while the serial covariance is estimated using returns over the entire year.

To overcome the errors-in-variables problem we estimate an instrumenatal variable regression. The instrumental variables that we use for the squared spread in this regression are LOGSIZE, the average price during the year (PRC) and the variance of daily returns (VAR).[13] Stoll (1978) has previously documented that these variables are related to the spread.

The following estimates are obtained from the first stage regression fitted with the pooled cross-sectional and time-series data:

$$S_{it}^2 = .00051 - .00000166 \text{PRC}_{it} - .000138 \text{LOGSIZE}_{it} + .5354 \text{VAR}_{it} + u_{it} \quad (9)$$

$$R_{adj}^2 = .275$$

In the second stage, the following cross-sectional regression is estimated with the fitted values of the squared spread, $\hat{S}^2$, from regression (9) as the explanatory variable,

$$\widehat{\text{Scov}}_{ik}^t = \alpha_k^t + \beta_k^t \hat{S}_{it}^2 + e_{ik}^t. \quad (10)$$

Table 5 presents the mean and the t-statistics of the coefficient estimates from the above regressions obtained from the yearly time-series of the cross-sectional regression estimates. Consistent with the presence of errors-in-variables, the estimated slope coefficients from these instrumental variable regressions, except for the case where $k = 1$, are bigger in magnitude than the estimates obtained in the corresponding OLS regressions. For $k > 3$ these coefficients are less than $-.25$,

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[13] The variance for year t is estimated with the data in the year $t - 1$ in order to avoid picking up in-sample correlation between the estimated variance and serial covariance. Further, we use the estimate of 20-day return variance divided by twenty rather than the estimate of one-day return variance since the latter estimate and serial covariance will be correlated if the 'true' serial covariance of returns is persistant.

although the difference is not statistically significant. As was the case in the OLS regressions, the slope coefficients in the instrumental variables regression vary with the measurement interval, again supporting the inventory model.

The estimates of the intercepts are generally closer to zero in the instrumental variable regressions than in the corresponding OLS regressions. For example, in the regression with $k = 5$ the intercept is $-.227 \times 10^{-4}$ while the corresponding estimate of average serial covariance is -1.182×10^{-4}. The point estimate of the intercept is, therefore, less than twenty percent of the estimate of the average serial covariance suggesting that a large part of the magnitude of observed serial covariances can be explained by the bid-ask spread.

As discussed earlier, a possible explanation of the negative intercept that is still observed is that the spread does not fully capture the effect of inventory imbalances on the bid and ask prices. Although it is difficult to unambiguously test this explanation we conduct some exploratory investigation by using the variance of returns (VAR) as an additional proxy for inventory risk. The motivation for the choice of this proxy is that the considerations that lead the risk averse specialist to respond to inventory imbalances by altering his bid and ask prices are likely to be more pronounced for high variance stocks than for low variance stocks. This implies that if the price effects of inventory imbalances are not fully captured by $\hat{S}^2$ then VAR should be negatively related to the serial covariances cross-sectionally. To test this hypothesis we estimate the following regression:

$$\widehat{\text{Scov}}_{ik}^t = \alpha_k^t + \beta_{1k}^t \hat{S}_{it}^2 + \beta_{2k}^t \text{VAR}_{it} + e_{ik}^t. \tag{11}$$

The average of the yearly estimates of the regression coefficients and the corresponding t-statistics are presented in Table 5 (Panel B). Except for the case where $k = 1$, the coefficients of VAR in (11) are negative. The positive slope coefficient on VAR for $k = 1$ could possibly be explained by the order flow process being different from

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the assumptions of Ho and Stoll (see fn. 9). However, the negative slope coefficients in the regressions with $k > 1$ support the hypothesis that the short horizon return reversals are related to the inventory components of the bid and ask prices.

III Trading Volume and Return Reversals

This section examines how trading volume relates to the serial covariance of returns. Since it is more likely that inventory imbalances will arise when trading volume is relatively high, the temporary component of stock prices is likely to be greater during times of high volume. A number of articles have presented evidence that is consistent with this relation between volume of transactions and return reversals. These articles examine price movements following days of unusual trading volume. The basic idea is that the magnitude of the price pressure is likely to be related to the volume of trades. An early example of such a study is Kraus and Stoll (1972) which examined price behavior following large block sells and found significant positive returns.[14] A more recent article by Blume, MacKinlay and Terker (1990) examined the relation between price changes following the October 1987 stock market crash, and volume on the day of the crash. They found that the stocks that had the heaviest volume on the day of the crash, recovered the most on the following day, suggesting that part of decline on the day of the crash was due to price pressure.

If trading volume is related to inventory imbalances in general, then serial covariances over periods of high trading volume should be of higher magnitude than serial covariances over periods of low trading volume. To classify each 1, 3, 5, 7, and 10 day period as a high volume period or a low volume period we first measure the volume of each stock in each period relative to the stock's volume over the previous

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[14]See also the more recent paper by Holthausen, Leftwich and Mayers (1990) who argue that prices recover almost immediately following block trades.

6 months.[15] If the relative volume for a stock is higher than the cross-sectional median in a given period t then that period is classified as a high volume period. We estimate the serial covariance separately in the high volume period as

$$\text{Cov}_{high\ volume} = \text{E}(r_{it} \times r_{it+1}|t = high) - (\text{E}(r))^2,$$

and similarly in the low volume period. The serial covariances are estimated within two year subperiods from 1971 to 1990.

Table 6 presents the average differences in these serial covariances $\text{Cov}_{low\ volume} - \text{Cov}_{high\ volume}$ for the entire sample and for five size-based subsamples. Under the hypothesis that the observed return reversals are generated by inventory imbalances, these differences should be positive and of larger magnitude for the returns of the smaller stocks. In contrast to this prediction, the serial covariances of 1 day returns are lower in magnitude on the high volume days than on the low volume days. This is consistent with our earlier finding that 1 day serial covariances for the larger stocks are positively serially correlated, probably reflecting the fact that the order flow is positively serially correlated. These results are consistent with the positive serial correlation of the order flow being of greater magnitude during times of high volume than during times of low volume. For 3, 5, and 7 day returns the serial covariances are larger in magnitude for the high volume days supporting the inventory cost hypothesis. This finding holds for each size-based quintile and, as we hypothesized, the differences in serial covariances is greater for smaller firms than it is for the larger firms. For returns measured over 10 day intervals the high and low volume serial covariances are not significantly different reflecting the low precision of these estimates.

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[15]The volume data were obtained from the Cornell University Stock Price file for the period 1970-1985 and from the CRSP file from 1986 onwards.

IV Conclusion

The evidence presented in this paper suggests that most of the short horizon return reversals can be explained by the way specialists set bid and ask prices, taking into account their inventory imbalances. The results support the implications of market microstructure models by Ho and Stoll (1981) and others that examine the dynamics of the bid and ask prices quoted by risk averse specialists.

Although these findings are consistent with an efficient market they do indicate that the stock market is not as liquid or resilient as some previous research has suggested. For instance, a previous study by Dann, Mayers and Raab (1977) concluded that block trades are absorbed very quickly by the market since they find that the stock price within 15 minutes after the execution of a large block trade is an unbiased estimate of the closing price. However, our results suggest that price pressure effects last well beyond one day and hence one might have to look at price changes over longer periods in order to assess the price resilience following block trades. Our results are consistent with Harris and Gurel (1986), who find that it takes about three weeks for the price pressure effects related to concentrated trading in stocks that are newly included in the S&P 500 index to dissipate.

Our results have implications relevant to the current debate about transaction taxes and initial margin requirements. For example, Summers and Summers (1989) have argued that a transaction tax is likely to reduce speculation and thereby reduce volatility. We would suggest that policymakers proceed along this path with caution. The evidence in this paper indicates that the negative serial correlation in stock returns, and hence the excess volatility, is related to the lack of liquidity in the market. Therefore, any impediment to trade that results in reduced market liquidity is likely to result in an increase in stock price volatility.

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Table 1. Estimates of serial covariances of returns measured over different observation intervals.

Unbiased estimates of k-day return serial covariance are obtained using the following estimator:

$$\widehat{Scov}_{ik}^{t} = \frac{1}{T} \sum_{n=1}^{T} (\sum_{m=1}^{K} R_{in+m-1} - k\overline{R}_{it-1})(\sum_{m=1}^{k} R_{in-m}),$$

where T is the number of overlapping in year t, R_{in} is the continuously compounded return on day n and $\overline{R}_{it-1}$ is the sample mean daily return estimated from the data in year $t-1$. The estimates are obtained for each security i within the calender year t and these estimates are averaged across the cross-section. The mean of the the time-series of the cross-sectional averages and the associated t-statistics are presented below. S1 through S5 are quintiles of the smallest through the largest firms in the sample.* The t-statistics are presented in parentheses. The sample period is 1963-1979.

$k =$	1	3	5	7	10
S1	-0.563[†]	-1.687	-2.568	-3.746	-4.885
	(-2.86)	(-5.48)	(-4.98)	(-4.26)	(-3.40)
S2	-0.069	-0.794	-1.472	-2.209	-2.725
	(-0.78)	(-5.62)	(-4.13)	(-3.64)	(-2.85)
S3	0.081	-0.450	-0.947	-1.445	-1.651
	(1.60)	(-4.56)	(-3.13)	(-2.92)	(-2.37)
S4	0.170	-0.157	-0.601	-1.024	-1.013
	(4.15)	(-3.39)	(-2.64)	(-2.13)	(-1.28)
S5	0.197	-0.040	-0.587	-1.154	-1.234
	(4.57)	(-0.93)	(-3.29)	(-3.24)	(-2.73)
All	0.002	-0.571	-1.182	-1.830	-2.117
	(0.05)	(-7.91)	(-4.40)	(-3.72)	(-3.02)

* - The cutoffs for the size-quintiles are based on all the firms traded on the NYSE at the beginning of a given year out of which some are excluded from our sample. Therefore the number of firms in the groups S1 through S5 are different and hence the estimates of the serial covariance of returns for the entire sample need not equal the average of the estimates within the five groups.

† - The estimates of average serial covariances are multiplied by 10^4 and reported in this table.

Table 2. Roll's implicit measures of spread and the quoted spreads

The within size-group pooled cross-sectional time-series average estimates of serial covariances of k-day returns are inverted to obtain Roll's implicit measure of bid-ask spread. Specifically the implicit spread is obtained using the expression below,

$$\overline{S}_{imp}^{k} = 2\sqrt{-\overline{Scov}_k + .25\sigma_s^2} \quad \text{if } \left(-\overline{Scov}_k + .25\sigma_s^2\right) > 0$$
$$= -2\sqrt{\overline{Scov}_k - .25\sigma_s^2} \quad \text{if } \left(-\overline{Scov}_k + .25\sigma_s^2\right) < 0,$$

where $\overline{S}_{imp}^{k}$ is the implicit estimate of the average spread with a given size-group, $\overline{Scov}_k$ is the within size-group average of the estimates of serial covariance of k-day returns and σ_s^2 is the within size-group variance of quoted spread. S1 through S5 are quintiles of the smallest through the largest firms in the sample. The sample period is 1963-79.

$k =$	Implicit Spread					Quoted Spread
	1	3	5	7	10	
S1	-0.006	0.020	0.027	0.034	0.039	0.025
S2	-0.009	0.015	0.022	0.028	0.032	0.018
S3	-0.009	0.012	0.019	0.024	0.025	0.014
S4	-0.010	0.006	0.015	0.020	0.019	0.011
S5	-0.010	0.001	0.016	0.023	0.022	0.008
Column Average	-0.009	0.011	0.020	0.026	0.027	0.015

Table 3. Relation between the square of the spread and the serial co-variance of returns measured over different intervals.

Regression Model $\widehat{Scov}_{ik} = a_k + b_k S_i^2 + e_{ik}$,

where $\widehat{Scov}_{ik}$ is the estimate of k-day return serial covariance of security i and S_i^2 is the squared spread. The above cross-sectional regression is fitted each year and the average estimates of the regression parameters are obtained from the time-series of these estimates. The t-statistics are presented in parentheses. $\overline{R}_{adj}^2$ is the average of the adjusted R^2 in the individual cross-sectional regressions. The sample period is 1963-79.

$k =$	1	3	5	7	10
$\hat{a}_k \times 10^4$	.311	.004	-.507	-1.035	-1.144
	(7.36)	(.06)	(-2.40)	(-2.65)	(-2.00)
$\hat{b}_k$	-.096	-.189	-.225	-.243	-.251
	(-8.30)	(-11.52)	(-8.95)	(-5.40)	(-3.04)
$\overline{R}_{adj}^2$	.298	.194	.103	.073	.052
$\hat{b}_k - \hat{b}_1$	–	-.093	-.129	-.147	-.155
	–	(-6.49)	(-5.48)	(-3.53)	(-1.99)

Table 4. Relation between the serial covariance of returns measured across different intervals, squared spread and firm size.

Regression Model $\widehat{\text{Scov}}_{ik} = a_k + b_{1k}S_i^2 + b_{2k}\text{LOGSIZE} + e_{ik}$,

where $\widehat{\text{Scov}}_{ik}$ is the estimate of k-day return serial covarince of security i, S^2 is the squared spread and LOGSIZE is the natural logarithm of the market value of equity at the beginning of a given year. The cross-sectional regression is fitted each year and the average estimates of the regression parameters are obtained from the time-series of these estimates. The t-statistics are presented in parentheses. $\overline{R}_{adj}^2$ is the average of the adjusted R^2 in the individual cross-sectional regressions. The sample period is 1963-79.

$k =$	1	3	5	7	10
$\hat{\alpha}_k \times 10^4$	0.530	-0.739	-1.604	-2.686	-3.929
	(2.85)	(-2.06)	(-2.08)	(-2.66)	(-2.47)
$\hat{b}_{1k}$	-0.101	-0.184	-0.212	-0.220	-0.229
	(-8.37)	(-10.40)	(-9.08)	(-5.58)	(-3.13)
$\hat{b}_{2k} \times 10^4$	-0.018	0.062	0.092	0.137	0.231
	(-1.15)	(2.33)	(1.80)	(2.14)	(2.31)
$\overline{R}_{adj}^2$	0.302	0.200	0.109	0.079	0.056
$\hat{b}_k - \hat{b}_1$	—	-0.083	-0.111	-0.119	-0.128
	—	(-5.16)	(-5.08)	(-3.29)	(-1.86)

Table 5. Instrumental variable regression estimates.
The first stage regression is fitted using pooled cross-section time-series data and the following estimates arae obtained:

$$S_{it}^2 = .00177 - .00000166 \text{PRC}_{it} - .000138 \text{LOGSIZE}_{it} + .5354 \text{VAR}_{it} + u_{it}$$

$$R_{adj}^2 = .275$$

where S^2 is the squared spread and the instrumental variable PRC is the average price, LOGSIZE is the natural logarithm of the market value of equity at the beginning of the year and VAR is the daily return variance estimated with returns data from year $t - 1$ (see fn. 13). The second stage regression specifications using the fitted values of the squared spread $\hat{S}^2$ from the above regression are:

$$\text{Panel A:} \quad \widehat{\text{Scov}}_{ik}^t = \alpha_k^t + \beta_k^t \hat{S}_{it}^2 + e_{ik}^t \quad \text{and}$$

$$\text{Panel B:} \quad \widehat{\text{Scov}}_{ik}^t = \alpha_k^t + \beta_{1k}^t \hat{S}_{it}^2 + \beta_{2k}^t \text{VAR}_{it} + e_{ik}^t,$$

where $\widehat{\text{Scov}}_{ik}^t$ is the estimate of k-day return serial covariance of security i in year t. The cross-sectional regression is fitted each year and the average estimates of the regression parameters are obtained from the time-series of these estimates. The t-statistics are presented in parentheses. $\overline{R}_{adj}^2$ is the average of the adjusted R^2 in the individual cross-sectional regressions. The sample period is 1963-1979.

Panel A:

$k =$	1	3	5	7	10
$\hat{\alpha}_k \times 10^4$	0.252	0.123	-0.227	-0.548	-0.531
	(4.86)	(1.63)	(-2.45)	(-2.21)	(-1.12)
$\hat{\beta}_k$	-0.065	-0.200	-0.260	-0.336	-0.378
	(-4.36)	(-10.34)	(-7.33)	(-5.16)	(-3.33)
$\overline{R}_{adj}^2$	0.067	0.108	0.076	0.070	0.050
$\hat{\beta}_k - \hat{\beta}_1$	—	-0.135	-0.195	-0.271	-0.313
	—	(-14.36)	(-6.99)	(-4.69)	(-2.99)

Panel B:

$k =$	1	3	5	7	10
$\hat{\alpha}_k \times 10^4$	0.2028	0.2034	-0.0499	-0.2397	-0.3064
	(4.36)	(3.27)	(-0.55)	(-1.07)	(-0.78)
$\hat{\beta}_{1k}$	-0.0928	-0.1703	-0.1757	-0.1844	-0.2872
	(-4.85)	(-7.00)	(-5.29)	(-3.80)	(-2.65)
$\hat{\beta}_{2k}$	0.0292	-0.0569	-0.1254	-0.2212	-0.1625
	(2.34)	(-2.76)	(-3.56)	(-3.39)	(-1.43)
$\overline{R}^2_{adj}$	0.0831	0.1145	0.0848	0.0862	0.0619
$\hat{\beta}_{1k} - \hat{\beta}_{11}$	—	-0.0775	-0.0829	-0.0916	-0.1944
	—	(-5.65)	(-2.48)	(-1.95)	(-2.08)

Table 6. Differences in serial covariances of returns over low volume and high volume periods.

This table presents the average differences between serial coavariances of k-day returns in the low volume periods and in the high volume periods. S1 through S5 are quintiles of the smallest through the largest firms in the sample. The t-statistics are presented in parentheses. The sample period is 1971-1990.

$k =$	1	3	5	7	10
S1	-0.5192[†]	1.3997	2.9577	2.8570	3.1494
	(-1.51)	(3.28)	(2.63)	(2.07)	(1.00)
S2	-0.1723	0.9240	1.3653	1.4136	0.4436
	(-1.93)	(3.13)	(2.23)	(1.41)	(0.27)
S3	-0.1555	0.8334	1.1562	1.1269	0.7310
	(-2.84)	(2.97)	(2.61)	(1.52)	(0.68)
S4	-0.1386	0.3819	0.6119	0.3598	0.1900
	(-4.30)	(2.95)	(1.94)	(0.69)	(0.24)
S5	-0.1440	0.1989	0.1778	0.0083	-0.3333
	(-4.47)	(2.19)	(1.05)	(0.05)	(-1.43)
All	-0.2259	0.7476	1.2538	1.1531	0.8361
	(-2.42)	(3.44)	(2.77)	(1.69)	(0.70)

† - The differences in the estimates of serial covariances are multiplied by 10^4 and reported in this table.

Appendix A

The standard estimator of serial covariance of returns using T observations is

$$\widehat{SC}(R_t, R_{t-1}) = \frac{1}{T-1} \sum_{i=2}^{T} R_i R_{i-1} - \left(\frac{1}{T} \sum_{i=1}^{T} R_i\right)^2 \qquad (A1)$$

Taking the expectations of the terms in (A1) we get,

$$E(\widehat{SC}(R_t, R_{t-1})) = E(R_t R_{t-1}) - \mu^2 - \frac{\sigma^2}{T} - \frac{2}{T^2} \sum_{i=1}^{T} (T-i)\gamma_i \qquad (A2)$$

where μ, σ^2 and γ_i are the mean, the variance and the ith order serial covariance of returns respectively. Under the hypothesis that the returns are serially uncorrelated the last term in (A2) is zero and the small sample bias in the serial covariance estimator (A1) is $-\frac{\sigma^2}{T}$. This result is contained in Marriot and Pope (1954) and Harris (1990).

Note that when returns are serially correlated, as suggested by our results in Section 1, the last term in (A2) is non-zero and hence the small sample bias contains terms involving the return serial covariances.

The estimator of serial covariance that we propose can be written as,

$$\widehat{Scov}(R_t, R_{t-1}) = \frac{1}{T-1} \sum_{i=2}^{T} R_i R_{i-1} - \left(\frac{1}{T} \sum_{i=1}^{T} R_i\right)\left(\frac{1}{T} \sum_{i=-T+1}^{0} R_i\right) \qquad (A3)$$

Taking the expectations of the terms in (A3) we get,

$$E(\widehat{Scov}(R_t, R_{t-1})) = E(R_t R_{t-1}) - \mu^2 - \frac{1}{T^2}\left(\sum_{i=1}^{T} i\gamma_i + \sum_{i=T+1}^{2T} (2T-i)\gamma_i\right) \qquad (A4)$$

Note that the term σ^2/T which is the major source of bias in (A2) is not contained in (A4). The last term in (A4) is zero when returns are serially uncorrelated. Even when the daily returns are serially correlated as observed, the last term on the right-hand side is trivial for actual values of serial covariance detected in the data.