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One-Loop Corrections to the S and T Parameters in a Three Site Higgsless Model

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Abstract: In this paper we compute the one-loop chiral logarithmic corrections to the S and T parameters in a highly deconstructed Higgsless model with only three sites. In addition to the electroweak gauge bosons, this model contains a single extra triplet of vector states (which we denote $\rho^{\pm}$ and ρ^0), rather than an infinite tower of "KK" modes. We compute the corrections to S and T in 't Hooft-Feynman gauge, including the ghost, unphysical Goldstone-boson, and appropriate "pinch" contributions required to obtain gauge-invariant results for the one-loop self-energy functions. We demonstrate that the chiral-logarithmic corrections naturally separate into two parts, a model-independent part arising from scaling below the ρ mass, which has the same form as the large Higgs-mass dependence of the S or T parameter in the standard model, and a second model-dependent contribution arising from scaling between the ρ mass and the cutoff of the model. The form of the universal part of the one-loop result allows us to correctly interpret the phenomenologically derived limits on the S and T parameters (which depend on a "reference" Higgs-boson mass) in this three-site Higgsless model. Higgsless models may be viewed as dual to models of dynamical symmetry breaking akin to "walking technicolor", and in these terms our calculation is the first to compute the subleading $1/N$ corrections to the S and T parameters. We also discuss the reduction of the model to the "two-site" model, which is the usual electroweak chiral lagrangian, noting the "non-decoupling" contributions present in the limit $M_\rho \to \infty$.

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Keywords: Dimensional Deconstruction, Electroweak Symmetry Breaking, Higgsless Theories, Fermion Delocalization, Precision Electroweak Tests, Chiral Lagrangian.

Contents

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1. Introduction

Higgsless models [1] accommodate electroweak symmetry breaking without the introduction of a fundamental scalar Higgs boson [2]. In these models, the unitarity of longitudinally-polarized electroweak gauge-boson scattering is achieved through the exchange of extra vector bosons [3, 4, 5, 6], rather than scalars. Based on TeV-scale [7] compactified five-dimensional gauge theories with appropriate boundary conditions [8, 9, 10, 11], these models provide effectively unitary descriptions of the electroweak sector beyond the TeV energy scale. They are not, however, renormalizable, and must be viewed as effective theories valid below a cutoff energy scale inversely proportional to the five-dimensional gauge-coupling squared. Above this energy scale, some new "high-energy" completion, which is valid to higher energies, must be present.

Deconstruction [12, 13] is a technique to build four-dimensional gauge theories, with appropriate gauge symmetry breaking patterns. which approximate – at least over some energy range – the properties of a five-dimensional theory. Deconstructed Higgsless models [14, 15, 16, 17, 18, 19, 20] have been used as tools to compute the general properties of Higgsless theories, and to illustrate the phenomenological properties of this class of models.

In the simplest realization of Higgsless models, the ordinary fermions are localized (on "branes") in the extra dimension. Such models necessarily [20] give rise to large tree-level corrections to the electroweak S parameter, and are not phenomenologically viable. It has been shown, however, that by relaxing the fermion locality constraint [21, 22, 23, 24, 25] – more correctly, by allowing fermions to propagate in the compactified fifth dimension and identifying the ordinary fermions with the lowest KK fermion states – it is always [26] possible to choose the fermion wavefunction in the fifth dimension so that all four-fermion electroweak quantities at tree-level have their standard model forms.[1]

Recently, a detailed investigation of a highly deconstructed three site Higgsless model [27] – in which the only vector states are the ordinary electroweak gauge bosons and a single triplet of $\rho^{\pm}$ and ρ^0 vector states – has been completed.[2] Although relatively simple in form, the model was shown to be sufficiently rich to incorporate the interesting physics issues related to fermion masses and electroweak observables. Calculations were presented addressing the size of corrections[3] to αT, $b \to s\gamma$, and $Z \to b\bar{b}$.

In this paper we compute the one-loop chiral logarithmic corrections to the S and T parameters [28, 29, 30] in the three site Higgsless model, in the limit $M_W \ll M_\rho \ll \Lambda$, where Λ is the cutoff of the effective theory. We compute these corrections in 't Hooft-Feynman gauge, including the ghost, unphysical Goldstone-boson, and appropriate "pinch" contributions [31, 32] required to obtain gauge-invariant results for the one-loop self-energy functions.

For the S-paramter, we find the result

$$\begin{aligned}
\alpha S_{3-site} = & \left[\frac{4s^2 M_W^2}{M_\rho^2}\left(1-\frac{x_1 M_\rho^2}{2M_W^2}\right)\right]_{\mu=\Lambda} + \frac{\alpha}{12\pi}\log\frac{M_\rho^2}{M_{Href}^2} \\
& - \frac{41\alpha}{24\pi}\log\frac{\Lambda^2}{M_\rho^2} + \frac{3\alpha}{8\pi}\left(\frac{x_1 M_\rho^2}{2M_W^2}\right)\log\frac{\Lambda^2}{M_\rho^2} \\
& -8\pi\alpha(c_1(\Lambda)+c_2(\Lambda)) \ .
\end{aligned} \tag{1.1}$$

where the parameter x_1 measures the amount of fermion delocalization, M_{Href} is the reference Higgs boson mass used in the definition of the S-parameter, and $c_{1,2}$ are higher order counter-terms [18]. The parameters M_ρ^2, M_W^2, and x_1 are renormalized at one-loop[4] and, to this order, in the first term of Eqn. (1.1) they should be understood to be evaluated at the scale

[1] It should be emphasized, however, that there is no explanation in any of these models (which are only low-energy effective theories) for the amount of delocalization. In particular, there is no dynamical reason why the fermion delocalization present *must* be such as to make the value of αS small.

[2] Note that the $\rho^{\pm}$ and ρ^0 here correspond to the $W'^{\pm}$ and Z' in that paper.

[3] In the original version of [27], we used the notation $\Delta\rho$ rather than αT. To the order we are working, they are identical: $Y \propto (\Delta\rho - \alpha T)$ vanishes in an ideally delocalized model [26].

[4] In a forthcoming paper [33] we will report the results of a full renormalization-group analysis of the $\mathcal{O}(p^4)$ terms in the three-site Higgsless model effective theory, allowing us to independently confirm the results in Eqns. (1.1) and (1.2), and to express the values of αS and αT in terms of low-energy parameters.

Λ. Note that the chiral-logarithmic corrections naturally separate into two parts, a model-independent part arising from scaling below the ρ mass, which has the same form as the large Higgs-mass dependence of the S-parameter in the standard model, and a second model-dependent contribution arising from scaling between the ρ mass and the cutoff of the model. The form of the model-independent part of the one-loop result allows us to correctly interpret the phenomenologically derived limits on the S parameter (which depend on a "reference" Higgs-boson mass [28]) in this three-site Higgsless model.

Similarly, we obtain for T

$$\alpha T_{3-site} = -\frac{3\alpha}{16\pi c^2}\log\frac{M_\rho^2}{M_{Href}^2} - \frac{3\alpha}{32\pi c^2}\log\frac{\Lambda^2}{M_\rho^2} + \frac{4\pi\alpha\, c_0(\Lambda)}{c^2}\,, \tag{1.2}$$

where M_{Href} is the reference Higgs-boson mass, c is approximately cosine of the standard weak mixing angle (see Eqn. (2.6)) , and $c_0(\Lambda)$ is the relevant $\mathcal{O}(p^4)$ custodial isospin-violating counterterm renormalized at scale Λ. Again, note the separation into model-independent and model-dependent pieces and the standard-model-like dependence on the "reference" Higgs-boson mass.

The next few sections of the paper introduce the model and the form of the Lagrangian in terms of the gauge eigenstates and mass eigenstates. We then present the results of our computations of the one-loop corrections to the self-energy functions of the W and Z bosons. Subsequently, we compute the one-loop corrections to the S and T parameters arising from the gauge sector and arrive at the results summarized above. We then turn to the relationship between the $M_\rho \to \infty$ limit of the three-site model and the usual electroweak chiral lagrangian [34, 35], discussing the importance of the "non-decoupling" contributions [36] which arise in this limit.

We conclude the paper by discussing the relationship of our results to the general expectations for the form of these corrections in models with a strongly-interacting symmetry breaking sector. Higgsless models may be viewed as dual [37, 38, 39, 40] to models of dynamical symmetry breaking [41, 42] akin to "walking technicolor" [43, 44, 45, 46, 47, 48], and in these terms our calculation is the first to compute the subleading $1/N$ corrections to the S and T parameters. The model we discuss is in the same class as models of extended electroweak gauge symmetries [49, 50] motivated by hidden local symmetry models [51, 52, 53, 54, 55] of chiral dynamics in QCD. We specifically compare our findings to the corresponding results in the "vector limit" [56] of hidden local symmetry models.

2. The Three-Site Model

The three-site Higgsless model analyzed in this paper is illustrated in Fig. 1 using "moose notation" [57]. The model incorporates an $SU(2)_L \times SU(2)_V \times U(1)_B$ gauge group with couplings g_0, g_1, and g_2 respectively, and 2 nonlinear $(SU(2) \times SU(2))/SU(2)$ sigma models in which the global symmetry groups in adjacent sigma models are identified with the corresponding factors of the gauge group. The symmetry breaking between the middle $SU(2)$

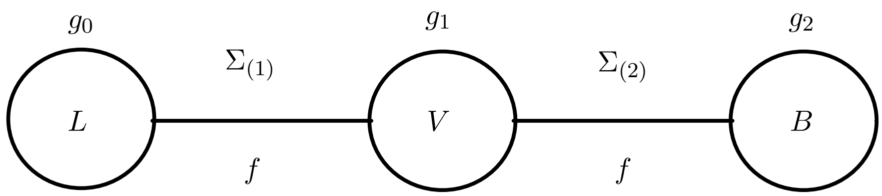

Figure 1: The three-site Higgsless model analyzed in this paper is illustrated using "moose notation" [57]. The model incorporates an $SU(2)_L \times SU(2)_V \times U(1)_B$ gauge group with couplings g_0, g_1, and g_2 respectively, and two nonlinear $(SU(2) \times SU(2))/SU(2)$ sigma models in which the global symmetry groups in adjacent sigma models are identified with the corresponding factors of the gauge group.

and the $U(1)$ follows an $SU(2)_L \times SU(2)_R/SU(2)_V$ symmetry breaking pattern with the $U(1)$ embedded as the T_3-generator of $SU(2)_R$. The leading order lagrangian in the model is given by

$$\begin{aligned}\mathcal{L}_{(2)} &= \frac{f^2}{4}\sum_{i=1}^{2} \mathrm{tr}[D_\mu \Sigma_{(i)}^\dagger D^\mu \Sigma_{(i)}] \\ &\quad -\frac{1}{2g_0^2}\mathrm{tr}[L_{\mu\nu}]^2 - \frac{1}{2g_1^2}\mathrm{tr}[V_{\mu\nu}]^2 - \frac{1}{2g_2^2}\mathrm{tr}[R_{\mu\nu}]^2 \,, \end{aligned} \tag{2.1}$$

where $L_{\mu\nu}$, $V_{\mu\nu}$, and $R_{\mu\nu}$ are the matrix field-strengths of the three gauge groups, $R_\mu = B_\mu \frac{\sigma_3}{2}$, and the covariant derivatives acting on $\Sigma_{(i)}$ are defined as

$$D_\mu \Sigma_{(1)} = \partial_\mu \Sigma_{(1)} - i L_\mu \Sigma_{(1)} + i \Sigma_{(1)} V_\mu \,, \tag{2.2}$$

$$D_\mu \Sigma_{(2)} = \partial_\mu \Sigma_{(2)} - i V_\mu \Sigma_{(2)} + i \Sigma_{(2)} R_\mu \,. \tag{2.3}$$

The 2×2 unitary matrix fields $\Sigma_{(1)}$ and $\Sigma_{(2)}$ may be parametrized by the Nambu-Goldstone (GB) boson fields $\pi_{(1)}$ and $\pi_{(2)}$:

$$\Sigma_{(i)} = e^{2i\pi_{(i)}/f} \,, \qquad \text{for} \quad i = 1, 2 \,, \tag{2.4}$$

with the decay constant[5] f.

This model (see [27] for details) approximates the standard model in the limit

$$x = g_0/g_1 \ll 1 \;, \qquad y = g_2/g_1 \ll 1 \;, \tag{2.5}$$

in which case we expect a massless photon, light W and Z bosons, and a heavy set of bosons $\rho^\pm$ and ρ^0 with $M_W \ll M_\rho$. Numerically, then, $g_{0,2}$ are approximately equal to the standard

[5]For simplicity, here we take the same decay constant f for both links.

model $SU(2)_W$ and $U(1)_Y$ couplings, and we therefore define an angle θ such that $s = \sin\theta$, $c = \cos\theta$, and

$$g_0^2 \approx \frac{4\pi\alpha}{s^2} = \frac{e^2}{s^2}\ , \quad g_2^2 \approx \frac{4\pi\alpha}{c^2} = \frac{e^2}{c^2}\ , \quad \frac{s}{c} = \frac{g_2}{g_0} \tag{2.6}$$

where α is the fine-structure constant and e the charge of the electron.

2.1 Fermion Couplings and αS at Tree-Level

In general, the standard model fermions may be delocalized along the three-site moose in the sense that their weak couplings arise from both sites 0 and 1 [58, 25]

$$\mathcal{L}_f = \vec{J}_L^\mu \cdot ((1 - x_1)L_\mu + x_1 V_\mu) + J_Y^\mu B_\mu\ , \tag{2.7}$$

where $\vec{J}_L^\mu$ and J_Y^μ are the fermionic weak and hypercharge currents, respectively, and $0 \leq x_1 \ll 1$ is a measure of the amount of fermion delocalization. This expression is not separately gauge invariant under $SU(2)_0$ and $SU(2)_1$. Rather, the fermions should be viewed as being charged under $SU(2)_0$, and the terms proportional to x_1 should be interpreted as arising from the operator of the form

$$\mathcal{L}'_f = -x_1 \cdot \bar{\psi}_L (i \not{D} \Sigma_{(1)} \Sigma_{(1)}^\dagger) \psi_L\ , \tag{2.8}$$

in unitary gauge. We will be interested only in the light fermions (*i.e.* all standard model fermions except for the top-quark), and will therefore ignore the couplings giving rise to fermon masses (these are discussed in detail in [27]).

Some degree of fermion delocalization is desirable for phenomenological reasons. Diagonalizing the gauge-boson mass matrix and computing the relevant tree-level four-fermion processes, one may compute the value of the S-parameter at tree-level, with the result [58, 25]

$$\alpha S^{tree} = \frac{4s^2 M_W^2}{M_\rho^2}\left(1 - \frac{x_1 M_\rho^2}{2M_W^2}\right)\ . \tag{2.9}$$

Current experimental bounds on αS are $\mathcal{O}(10^{-3})$ [59]. Since exchange of the ρ meson is necessary to maintain the unitarity of longitudinally polarized W-boson scattering, we must require that $M_\rho \leq \mathcal{O}(1\,\text{TeV})$ – leading, for localized fermions with $x_1 = 0$, to a value of αS^{tree} which is too large. For the three-site model to be viable, therefore, the value of the fermion delocalization parameter must be chosen to reduce the value of αS^{tree} [21, 22, 23, 24, 25, 26].[6]

2.2 Duality and The Size of Radiative Electroweak Corrections

By duality [37], tree-level computations in the 5-dimensional theory represent the leading terms in a large-N expansion [60] of the strongly-coupled dual gauge theory akin to "walking technicolor" [43, 44, 45, 46, 47, 48]. The mass of the W boson, M_W, is proportional to a weak gauge-coupling, g_{ew}, (which is fixed in the large-N approximation) times the f-constant

[6] See footnote 1.

for the electroweak chiral symmetry breaking of the strongly-coupled theory. Therefore, we expect M_W^2/M_ρ^2 to scale as [60, 61]

$$\frac{M_W^2}{M_\rho^2} = \mathcal{O}\left(\frac{g_{ew}^2 N}{(4\pi)^2}\right) , \tag{2.10}$$

which is the expected behavior of the S-parameter in the large-N limit [28, 10].

We now specify the limit in which we will perform our analysis. As shown below (see Eqns. (4.1, 4.2)), in the small x limit

$$\frac{M_W^2}{M_\rho^2} \approx \frac{x^2}{4} , \tag{2.11}$$

so that the tree-level value of αS vanishes if $x_1 = x^2/2$. In what follows, therefore, we will assume that $x_1 = \mathcal{O}(x^2)$. Overall, then, we work in the limit

$$1 \gg |\alpha S^{tree}| = \mathcal{O}\left(\frac{g_{ew}^2 N}{(4\pi)^2}\right) \gg |\alpha S^{one-loop}| = \mathcal{O}\left(\frac{g_{ew}^2}{(4\pi)^2}\right) > 0 , \tag{2.12}$$

which is manifestly consistent with the large-N approximation. Once we choose the value of x_1 to make the size of αS^{tree} consistent with the phenomenological bound of $\mathcal{O}(10^{-3})$, one-loop electroweak corrections $\alpha S^{one-loop}$ become potentially relevant.

Note also that $\alpha T^{tree} \approx 0$ in these models, independent of the degree of fermion delocalization [20, 25]. The one-loop corrections to αT are therefore of interest. Those arising from the extended fermion sector have been shown[2] to place strong lower bounds on the masses of the KK fermions [27]. Those arising from the gauge sector are considered below.

In practice, in calculating corrections to the gauge-boson self-energy functions, we will work in the leading-log approximation, and to order α; we will neglect corrections $\mathcal{O}(\alpha x^2 M_W^2)$ or $\mathcal{O}(\alpha x^2 p^2)$, but keep those of order $\mathcal{O}(\alpha x^2 M_\rho^2)$.

3. Gauge Sector Lagrangian

In order to obtain the relevant interaction terms to compute the one-loop electroweak corrections, we expand the link variables Σ_1 and Σ_2 as follows

$$\Sigma_{(i)} = 1 + 2i\frac{\pi_{(i)}}{f} - \frac{2\pi_{(i)}^2}{f^2} + \mathcal{O}(\pi^3) , \qquad \text{for } i = 1, 2 . \tag{3.1}$$

Furthermore, it is convenient to change the normalization of the gauge-boson fields so that the gauge-boson kinetic energy terms in Eqn. (2.1) are canonically normalized, and to introduce the following vectors in "link" and "site" space, respectively

$$\vec{\pi^a} = \begin{pmatrix} \pi_{(1)}^a \\ \pi_{(2)}^a \end{pmatrix} , \qquad \vec{A_\mu^a} = \begin{pmatrix} L_\mu^a \\ V_\mu^a \\ R_\mu^a \end{pmatrix} , \tag{3.2}$$

with $R^1_\mu = R^2_\mu = 0$, and $R^3_\mu = B_\mu$. In terms of these quantities, the lagrangian (2.1) decomposes into the following pieces:

$$\begin{aligned}\mathcal{L}_{(2)} &= \mathcal{L}^{AA}_{\pi\pi} + \mathcal{L}_{\pi AA} + \mathcal{L}_{\pi\pi A} + \mathcal{L}_{\pi\pi AA} \\ &\quad + \mathcal{L}_{AAA} + \mathcal{L}_{AAAA} + \cdots ,\end{aligned} \tag{3.3}$$

where we have ignored the interaction terms including more than three GB fields since these terms do not generate vertices relevant to the one-loop processes of interest.

3.1 $\mathcal{L}^{AA}_{\pi\pi}$: Kinetic Energy and Gauge-Fixing Terms

Terms in the lagrangian $\mathcal{L}^{AA}_{\pi\pi}$ are quadratic in the GB fields $\vec{\pi}^a$ or gauge fields $\vec{A}^a_\mu$

$$\mathcal{L}^{AA}_{\pi\pi} = \mathcal{L}^{\rm kin}_{\rm gauge} + \frac{1}{2}\left(\partial_\mu \vec{\pi}^a - f\left(D\cdot G\cdot \vec{A}^a_\mu\right)\right)^T \cdot \left(\partial^\mu \vec{\pi}^a - f\left(D\cdot G\cdot \vec{A}^{\mu a}\right)\right) , \tag{3.4}$$

where the kinetic terms of the gauge fields $\vec{A}^a_\mu$ are included in $\mathcal{L}^{\rm kin}_{\rm gauge}$, D is a 2×3 difference matrix in the link/site space defined as

$$D = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix} , \tag{3.5}$$

and G is the gauge coupling-constant matrix with the diagonal elements (g_0, g_1, g_2).

It is convenient to introduce charge eigenstate fields

$$\vec{A}^\pm_\mu = (L^\pm_\mu, V^\pm_\mu)^T , \qquad \vec{A}^0_\mu = (L^3_\mu, V^3_\mu, B_\mu)^T , \tag{3.6}$$

$$\vec{\pi}^\pm = (\pi^\pm_{(1)}, \pi^\pm_{(2)})^T , \qquad \vec{\pi}^0 = (\pi^3_{(1)}, \pi^3_{(2)})^T , \tag{3.7}$$

where

$$L^\pm_\mu = \frac{L^1_\mu \mp i L^2_\mu}{\sqrt{2}} , \qquad V^\pm_\mu = \frac{V^1_\mu \mp i V^2_\mu}{\sqrt{2}} , \tag{3.8}$$

and with $\pi^\pm_{(i)}$ and $\pi^0_{(i)}$ $(i = 1,2)$ defined analogously. Using the charge eigenstate fields, the mass terms of the gauge fields are expressed as

$$\vec{A}^{+T}_\mu M^2_{\rm CC} \vec{A}^{\mu -} + \frac{1}{2}\vec{A}^{0T}_\mu M^2_{\rm NC} \vec{A}^{\mu 0} , \tag{3.9}$$

where M^2_{CC} and M^2_{NC} are the mass matrices for the charged and neutral gauge bosons

$$M^2_{\rm CC} = \frac{f^2}{4}\begin{pmatrix} g_0^2 & -g_0 g_1 \\ -g_0 g_1 & 2g_1^2 \end{pmatrix} , \tag{3.10}$$

$$M^2_{\rm NC} = \frac{f^2}{4}\begin{pmatrix} g_0^2 & -g_0 g_1 & 0 \\ -g_0 g_1 & 2g_1^2 & -g_1 g_2 \\ 0 & -g_1 g_2 & g_2^2 \end{pmatrix} . \tag{3.11}$$

The lagrangian $\mathcal{L}_{\pi\pi}^{AA}$ includes quadratic mixing terms between the GB fields $\vec{\pi}^a$ and the gauge fields $\vec{A}_\mu^a$. These terms are eliminated by adding the following R_ξ gauge fixing term[7] [20]:

$$\mathcal{L}_{\mathrm{GF}} = -\frac{1}{2\xi}\left(\vec{\mathcal{G}}^T\right)^a \cdot \left(\vec{\mathcal{G}}\right)^a , \tag{3.12}$$

where

$$\vec{\mathcal{G}}^a = \left[\left(\partial_\mu \vec{A}^{\mu a}\right) + \frac{\xi f}{2}\left(G \cdot D^T \cdot \vec{\pi}^a\right)\right] . \tag{3.13}$$

After fixing the gauge, the unphysical Goldstone boson fields acquire the gauge-dependent masses $M_{\pi^\pm}$ and M_{π^0}

$$M_{\pi^\pm}^2 = \frac{\xi f^2}{4}\begin{pmatrix} g_0^2 + g_1^2 & -g_1^2 \\ -g_1^2 & g_1^2 \end{pmatrix} , \tag{3.14}$$

$$M_{\pi^0}^2 = \frac{\xi f^2}{4}\begin{pmatrix} g_0^2 + g_1^2 & -g_1^2 \\ -g_1^2 & g_1^2 + g_2^2 \end{pmatrix} . \tag{3.15}$$

The lagrangian $\mathcal{L}_{\pi\pi}^{AA}$ combined with the gauge-fixing term in $\mathcal{L}_{\mathrm{GF}}$ then become

$$\begin{aligned}\mathcal{L}_{\pi\pi}^{AA} + \mathcal{L}_{\mathrm{GF}} = & -\vec{A}_\mu^{+T}[D_{A^\pm}^{\mu\nu}]\vec{A}_\nu^- - \frac{1}{2}\vec{A}_\mu^{0T}[D_{A^0}^{\mu\nu}]\vec{A}_\nu^0 \\ & -\vec{\pi}^{+T}[D_{\pi^\pm}]\vec{\pi}^- - \frac{1}{2}\vec{\pi}^{0T}[D_{\pi^0}]\vec{\pi}^0 ,\end{aligned} \tag{3.16}$$

where

$$\left[D_{A^\pm,A^0}^{\mu\nu}\right] = \left(-\mathbf{1}\,\partial^2 - M_{CC,NC}^2\right) g^{\mu\nu} + \left(1 - \frac{1}{\xi}\right)\mathbf{1}\,\partial^\mu\partial^\nu , \tag{3.17}$$

$$[D_{\pi^{\pm,0}}] = \left(-\mathbf{1}\,\partial^2 - M_{\pi^{\pm,0}}^2\right) . \tag{3.18}$$

3.2 The Fadeev-Popov Ghost Lagrangian $\mathcal{L}_{\mathrm{FP}}$

Next we introduce the ghost terms corresponding to the gauge fixing terms in Eqn. (3.12)

$$\mathcal{L}_{\mathrm{FP}} = -\bar{C}_I^a \cdot (\Gamma^{ab})_{IJ} \cdot C_J^b , \tag{3.19}$$

where C_I^a and $\bar{C}_J^a$ $(I, J = 0, 1, 2)$ are respectively the Fadeev-Popov (FP) ghost and the anti-ghost fields corresponding to the gauge groups on the Ith-and Jth-site, and

$$(\Gamma^{ab})_{IJ} = g_J \cdot \frac{\delta \mathcal{G}_I^a}{\delta \Theta_J^b} , \tag{3.20}$$

[7] We take the same gauge parameter for all the gauge groups.

with Θ_I^a being the infinitesmal generator of the gauge transformations. The infinitesimal transformation laws for the gauge fixing functions $\mathcal{G}_I^a$ are immediately derived from those for the gauge fields $\vec{A}_\mu^a$ and the GB fields $\vec{\pi}^a$ [8]

$$\delta L_\mu^a = D_\mu \Theta_0^a = \left(\partial_\mu \delta^{ac} + g_0 \epsilon^{abc} L_\mu^b\right) \Theta_0^c \,, \tag{3.21}$$

$$\delta V_\mu^a = D_\mu \Theta_1^a = \left(\partial_\mu \delta^{ac} + g_1 \epsilon^{abc} V_\mu^b\right) \Theta_1^c \,, \tag{3.22}$$

$$\delta B_\mu = \partial_\mu \Theta_2 \,, \tag{3.23}$$

$$\delta \pi_{(1)}^a = \frac{f}{2}(g_0 \Theta_0^a - g_1 \Theta_1^a) + \mathcal{O}(\pi^2) \,, \tag{3.24}$$

$$\delta \pi_{(2)}^a = \frac{f}{2}(g_1 \Theta_1^a - g_2 \Theta_2^a) + \mathcal{O}(\pi^2) \,. \tag{3.25}$$

Defining the charge eigenstates for the FP ghost fields, we find

$$\mathcal{L}_{\rm FP} = \mathcal{L}_{\rm FP}^{\rm kin.} + \mathcal{L}_{\rm FP}^{\rm int.} \,, \tag{3.26}$$

where

$$\begin{aligned} \mathcal{L}_{\rm FP}^{\rm kin.} &= -\bar{C}_i^+ \left(\partial^2 \delta_{ij} - \xi [M_{CC}^2]_{ij}\right) C_j^- + \text{h.c.} \\ &\quad -\bar{C}_I^0 \left(\partial^2 \delta_{IJ} - \xi [M_{NC}^2]_{IJ}\right) C_J^0 \,, \end{aligned} \tag{3.27}$$

$$\mathcal{L}_{\rm FP}^{\rm int.} = i g_i \left(A_{\mu i}^+ \left\{ \partial^\mu \bar{C}_i^- C_i^3 - \partial^\mu \bar{C}_i^3 C_i^- \right\} + A_{\mu i}^3 \partial^\mu \bar{C}_i^+ C_i^- \right) + \text{h.c.} \,, \tag{3.28}$$

with $A_{\mu i}^+ = (L_\mu^+, V_\mu^+)^T$ and $A_{\mu i}^3 = (L_\mu^3, V_\mu^3)^T$, and where we sum over the repeated indices ($i, j = 0, 1$ and $I, J = 0, 1, 2$).

3.3 The Non-Abelian Interactions $\mathcal{L}_{AAA}$ and $\mathcal{L}_{AAAA}$

The Non-Abelian interaction terms among the gauge fields L_μ^a and V_μ^a are

$$\begin{aligned} \mathcal{L}_{AAA} &= i g_0 \left[\left(\partial_\mu L_\nu^+ - \partial_\nu L_\mu^+\right) L^{\mu -} L^{\nu 3} + \partial_\mu L_\nu^3 L^{\mu +} L^{\nu -} \right] \\ &\quad + i g_1 \left[\left(\partial_\mu V_\nu^+ - \partial_\nu V_\mu^+\right) V^{\mu -} V^{\nu 3} + \partial_\mu V_\nu^3 V^{\mu +} V^{\nu -} \right] + \text{h.c.} \,, \end{aligned} \tag{3.29}$$

$$\begin{aligned} \mathcal{L}_{AAAA} &= \frac{g_0^2}{2} \left[L_\mu^+ L_\nu^- \left(L^{\mu +} L^{\nu -} + L^{\mu -} L^{\nu +} \right) - 2 L_\mu^+ L_\nu^+ L^{\mu -} L^{\nu -} \right] \\ &\quad + g_0^2 \left[L_\mu^+ L_\nu^- L^{\mu 3} L^{\nu 3} - L_\mu^+ L^{\mu -} L_\nu^3 L^{\nu 3} \right] \\ &\quad + \frac{g_1^2}{2} \left[V_\mu^+ V_\nu^- \left(V^{\mu +} V^{\nu -} + V^{\mu -} V^{\nu +} \right) - 2 V_\mu^+ V_\nu^+ V^{\mu -} V^{\nu -} \right] \\ &\quad + g_1^2 \left[V_\mu^+ V_\nu^- V^{\mu 3} V^{\nu 3} - V_\mu^+ V^{\mu -} V_\nu^3 V^{\nu 3} \right] . \end{aligned} \tag{3.30}$$

[8] Here we omit terms including more than two GB fields, since these interactions are irrelevant to the processes we are concerned with.

3.4 The Goldstone Boson Interactions $\mathcal{L}_{\pi AA}, \mathcal{L}_{\pi\pi A}$ and $\mathcal{L}_{\pi\pi AA}$

The remaining Goldstone Boson interactions necessary for our computations are expressed as follows:

$$\mathcal{L}_{\pi AA} = -\frac{g_1 f}{2}\epsilon^{abc}\left(g_0 L^a_\mu V^{\mu b}\pi^c_{(1)} + g_2 V^a_\mu R^{\mu b}\pi^c_{(2)}\right) , \tag{3.31}$$

$$\mathcal{L}_{\pi\pi A} = -\frac{1}{2}\epsilon^{abc}\partial_\mu\pi^a_{(1)}\pi^b_{(1)}\left(g_0 L^{\mu c} + g_1 V^{\mu c}\right) - \frac{1}{2}\epsilon^{abc}\partial_\mu\pi^a_{(2)}\pi^b_{(2)}\left(g_1 V^{\mu c} + g_2 R^{\mu c}\right) , \tag{3.32}$$

$$\mathcal{L}_{\pi\pi AA} = \frac{1}{2}\epsilon^{eab}\epsilon^{ecd}\left(g_0 g_1 L^a_\mu \pi^b_{(1)} V^{\mu c}\pi^d_{(1)} + g_1 g_2 V^a_\mu \pi^b_{(2)} R^{\mu c}\pi^d_{(2)}\right) . \tag{3.33}$$

These interaction terms may be rewritten in terms of the charge eigenstate fields as[9]

$$\mathcal{L}_{\pi AA} = i\frac{g_1 f}{2}\left[g_0\left(L^3_\mu V^{\mu +} - V^3_\mu L^{\mu +}\right)\pi^-_{(1)} - g_2 B_\mu V^{\mu +}\pi^-_{(2)} + g_0 L^+_\mu V^{\mu -}\pi^3_{(1)}\right] + \text{h.c.} , \tag{3.34}$$

$$\begin{aligned}\mathcal{L}_{\pi\pi A} = \frac{i}{2}\Bigg[& g_0\partial_\mu\pi^+_{(1)}\pi^-_{(1)}L^{\mu 3} + g_1\left(\partial_\mu\pi^+_{(1)}\pi^-_{(1)} + \partial_\mu\pi^+_{(2)}\pi^-_{(2)}\right)V^{\mu 3} + g_2\partial_\mu\pi^+_{(2)}\pi^-_{(2)}B^\mu \\ & - \left(\pi^3_{(1)} \overleftrightarrow{\partial}_\mu \pi^-_{(1)}\right)\left(g_0 L^{\mu +} + g_1 V^{\mu +}\right) - g_1\left(\pi^3_{(2)} \overleftrightarrow{\partial}_\mu \pi^-_{(2)}\right)V^{\mu +}\Bigg] + \text{h.c.} ,\end{aligned} \tag{3.35}$$

$$\begin{aligned}\mathcal{L}_{\pi\pi AA} = \frac{g_1}{2}\Bigg[& g_0\pi^+_{(1)}\pi^-_{(1)}\left(L^+_\mu V^{\mu -} + L^3_\mu V^{\mu 3}\right) - g_0\pi^+_{(1)}\pi^3_{(1)}\left(L^-_\mu V^{\mu 3} + L^3_\mu V^{\mu -}\right) \\ & + g_0\pi^3_{(1)}\pi^3_{(1)}L^+_\mu V^{\mu -} - g_0\pi^+_{(1)}\pi^+_{(1)}L^-_\mu V^{\mu -} - g_2\pi^-_{(2)}\pi^3_{(2)}B_\mu V^{\mu +} + g_2\pi^+_{(2)}\pi^-_{(2)}B_\mu V^{\mu 3}\Bigg] \\ & + \text{h.c.} .\end{aligned} \tag{3.36}$$

4. Mass Eigenstate Fields

To facilitate our computation of the one-loop corrections to αS and αT, we express the interactions derived above in terms of mass eigenstate fields. As we are interested in the limit $x = g_0/g_1 \ll 1$, we will diagonalize the mass matrices perturbatively in x.

The charged gauge boson mass matrix $M^2_{\rm CC}$ has the eigenvalues

$$M^2_W = \frac{g_1^2 f^2}{4}\left[\frac{x^2}{2} - \frac{x^4}{8} + \mathcal{O}(x^6)\right] , \tag{4.1}$$

$$M^2_{\rho^\pm} = \frac{g_1^2 f^2}{4}\left[2 + \frac{x^2}{2} - \frac{x^4}{8} + \mathcal{O}(x^6)\right] . \tag{4.2}$$

Expanding the gauge-eigenstate fields in terms of the mass eigenstates, we find

$$L^\pm_\mu = v^L_{W^\pm} W^\pm_\mu + v^L_{\rho^\pm}\rho^\pm_\mu , \tag{4.3}$$

$$V^\pm_\mu = v^V_{W^\pm} W^\pm_\mu + v^V_{\rho^\pm}\rho^\pm_\mu , \tag{4.4}$$

[9]Here we define $(A \overleftrightarrow{\partial}_\mu B) \equiv (\partial_\mu A)B - A(\partial_\mu B)$.

where

$$
\begin{aligned}
v^L_{W^\pm} &= 1 - \tfrac{x^2}{8} + \cdots\,, \qquad v^L_{\rho^\pm} = -\tfrac{x}{2}\left(1 + \tfrac{x^2}{8} + \cdots\right)\,, \\
v^V_{W^\pm} &= \tfrac{x}{2}\left(1 + \tfrac{x^2}{8} + \cdots\right)\,, \qquad v^V_{\rho^\pm} = 1 - \tfrac{x^2}{8} + \cdots\,.
\end{aligned}
\tag{4.5}
$$

The neutral gauge boson mass matrix M^2_{NC} has one zero eigenvalue, corresponding to the photon, and the two non-zero eigenvalues

$$
M^2_Z = \frac{g_1^2 f^2}{4}\left[\frac{x^2}{2c^2} - \frac{(1-t^2)^2 x^4}{8c^4} + \mathcal{O}(x^6)\right]\,, \tag{4.6}
$$

$$
M^2_{\rho^0} = \frac{g_1^2 f^2}{4}\left[2 + \frac{x^2}{2c^2} + \frac{(1-t^2)^2 x^4}{8} + \mathcal{O}(x^6)\right]\,, \tag{4.7}
$$

where the angles $s = \sin\theta$ and $c = \cos\theta$ are defined in Eqn. (2.6), and $t = \tan\theta = s/c$. Expanding the neutral gauge-eigenstate fields in terms of mass eigenstates, we find

$$
L^3_\mu = v^L_A A_\mu + v^L_Z Z_\mu + v^L_{\rho^0}\rho^0_\mu\,, \tag{4.8}
$$

$$
V^3_\mu = v^V_A A_\mu + v^V_Z Z_\mu + v^V_{\rho^0}\rho^0_\mu\,, \tag{4.9}
$$

$$
B_\mu = v^B_A A_\mu + v^B_Z Z_\mu + v^B_{\rho^0}\rho^0_\mu\,, \tag{4.10}
$$

where

$$
\begin{aligned}
v^L_A &= s\left(1 - \tfrac{1}{2}s^2x^2 + \cdots\right)\,, \quad v^L_Z = c\left(1 - \tfrac{c^2x^2(1+2t^2-3t^4)}{8} + \cdots\right)\,, \quad v^L_{\rho^0} = -\tfrac{x}{2}\left(1 + \tfrac{x^2(1-3t^2)}{8}\right)\,, \\
v^V_A &= sx\left(1 - \tfrac{1}{2}s^2x^2 + \cdots\right)\,, \quad v^V_Z = \tfrac{cx(1-t^2)}{2}\left(1 + \tfrac{c^2x^2(1-t^2)^2}{8} + \cdots\right)\,, \quad v^V_{\rho^0} = 1 - \tfrac{x^2(1+t^2)}{8} + \cdots\,, \\
v^B_A &= c\left(1 - \tfrac{1}{2}s^2x^2 + \cdots\right)\,, \quad v^B_Z = -s\left(1 + \tfrac{c^2x^2(3-2t^2-t^4)}{8} + \cdots\right)\,, \quad v^B_{\rho^0} = -\tfrac{xt}{2}\left(1 - \tfrac{x^2(3-t^2)}{8} + \cdots\right)\,.
\end{aligned}
\tag{4.11}
$$

In obtaining the photon wavefunctions $v^{L,V,B}_A$, we have expanded the electromagnetic coupling e in powers of x as

$$
\begin{aligned}
\frac{1}{e^2} &= \frac{1}{g_0^2} + \frac{1}{g_1^2} + \frac{1}{g_2^2} \\
&= \frac{1}{g_0^2 s^2}\left(1 + s^2x^2 + \cdots\right)\,.
\end{aligned}
\tag{4.12}
$$

Since the mass matrices for the ghost fields are (see Eqn. (3.27)) equal to those of the vector bosons, up to an overall factor of ξ, the corresponding relationships between the gauge-eigenstate and mass-eigenstate ghost fields are

$$
C^\pm_{(0)} = v^L_{W^\pm} C_{W^\pm} + v^L_{\rho^\pm} C_{\rho^\pm}\,, \tag{4.13}
$$

$$
C^\pm_{(1)} = v^V_{W^\pm} C_{W^\pm} + v^V_{\rho^\pm} C_{\rho^\pm}\,, \tag{4.14}
$$

and

$$
C^3_{(0)} = v^L_A\ C_A + v^L_Z\ C_Z + v^L_{\rho^0} C_{\rho^0}\,, \tag{4.15}
$$

$$
C^3_{(1)} = v^V_A\ C_A + v^V_Z\ C_Z + v^V_{\rho^0}\ C_{\rho^0}\,, \tag{4.16}
$$

$$
C^3_{(2)} = v^B_A\ C_A + v^V_Z\ C_Z + v^B_{\rho^0} C_{\rho^0}\,. \tag{4.17}
$$

Similarly, the charged GB matrix $M^2_{\pi^\pm}$ has the eigenvalues ξM_W^2 and $\xi M^2_{\rho^\pm}$, and the neutral GB matrix $M^2_{\pi^0}$ has the eigenvalues ξM_Z^2 and $\xi M^2_{\rho^0}$. The mass matrices for the Goldstone bosons are given in Eqns. (3.15) and (3.14). Expanding the eigenvectors in powers of x we find that the GB fields are expressed in terms of the mass eigenstate fields $\pi_{W^\pm,Z}$ and $\pi_{\rho^\pm,\rho^0}$ as

$$\pi^{\pm,3}_{(1)} = v^{(1)}_{\pi_{W^\pm,Z}} \pi_{W^\pm,Z} + v^{(1)}_{\pi_{\rho^\pm,0}} \pi_{\rho^{\pm,0}} \,, \tag{4.18}$$

$$\pi^{\pm,3}_{(2)} = v^{(2)}_{\pi_{W^\pm,Z}} \pi_{W^\pm,Z} + v^{(2)}_{\pi_{\rho^\pm,0}} \pi_{\rho^{\pm,0}} \,, \tag{4.19}$$

where

$$\begin{aligned} v^{(1)}_{\pi_Z} &= \tfrac{1}{\sqrt{2}}\left(1 - \tfrac{(1-t^2)x^2}{4} + \cdots\right) \,, \; v^{(1)}_{\pi_{\rho^0}} = \tfrac{1}{\sqrt{2}}\left(-1 - \tfrac{(1-t^2)x^2}{4} + \cdots\right) \,, \\ v^{(2)}_{\pi_Z} &= \tfrac{1}{\sqrt{2}}\left(1 + \tfrac{(1-t^2)x^2}{4} + \cdots\right) \,, \;\; v^{(2)}_{\pi_{\rho^0}} = \tfrac{1}{\sqrt{2}}\left(1 - \tfrac{(1-t^2)x^2}{4} + \cdots\right) \,, \end{aligned} \tag{4.20}$$

with $t = 0$ for the wavefunctions of $\pi_{W^\pm,\rho^\pm}$.

5. One-Loop Corrections to the Gauge Boson Self-Energies

In order to compute the one-loop corrections to the S and T parameters, we must evaluate the relevant contributions to the gauge-boson self-energies [28, 29]. Using the results of the previous section, the gauge-sector interactions may be written in terms of the mass-eigenstate fields, yielding (order by order in x) the interactions necessary for our calculations. The gauge-sector interactions, written in the mass-eigenstate basis, are summarized in Appendix A, and the relevant diagrams are shown in figs. 2 and 3.

We define the self-energy amplitudes for the SM gauge bosons as

$$\int d^4x \, e^{-ipx} \langle 0 | \mathrm{T} \mathcal{A}^\mu_i(x) \mathcal{A}^\nu_j(0) | 0 \rangle = i g^{\mu\nu} \Pi_{\mathcal{A}_i \mathcal{A}_j}(p) + (p^\mu p^\nu \text{ term}) \,, \tag{5.1}$$

where i and j denote the species of the SM gauge bosons. In the present calculation, we choose the 't Hooft-Feynman gauge $\xi = 1$. The amplitudes are evaluated by using the Feynman integral formulae given in Appendix B; as described there, the formulae are derived using dimensional regularization and renormalized at the cutoff scale Λ of the effective theory.

5.1 Neutral Gauge Boson Self-Energies

The values of the individual diagrams in Fig. 2 are shown in Appendix C. Putting these contributions together, we obtain the photon self-energy

$$\Pi_{AA}(p^2) = \frac{e^2}{(4\pi)^2} \cdot p^2 \left[\left(3 \log \frac{\Lambda^2}{M_W^2} + 3 \log \frac{\Lambda^2}{M^2_{\rho^\pm}} \right) \right] , \tag{5.2}$$

the ZA mixing self-energy

$$\begin{aligned} \Pi_{ZA}(p^2) = \frac{e^2}{(4\pi)^2 sc} &\left[2M_W^2 \log \frac{\Lambda^2}{M_W^2} + (2c^2 - 1) M_Z^2 \log \frac{\Lambda^2}{M^2_{\rho^\pm}} \right. \\ &\left. + p^2 \left(\frac{18c^2 + 1}{6} \log \frac{\Lambda^2}{M_W^2} + \frac{3(2c^2 - 1)}{2} \log \frac{\Lambda^2}{M^2_{\rho^\pm}} \right) \right] , \end{aligned} \tag{5.3}$$

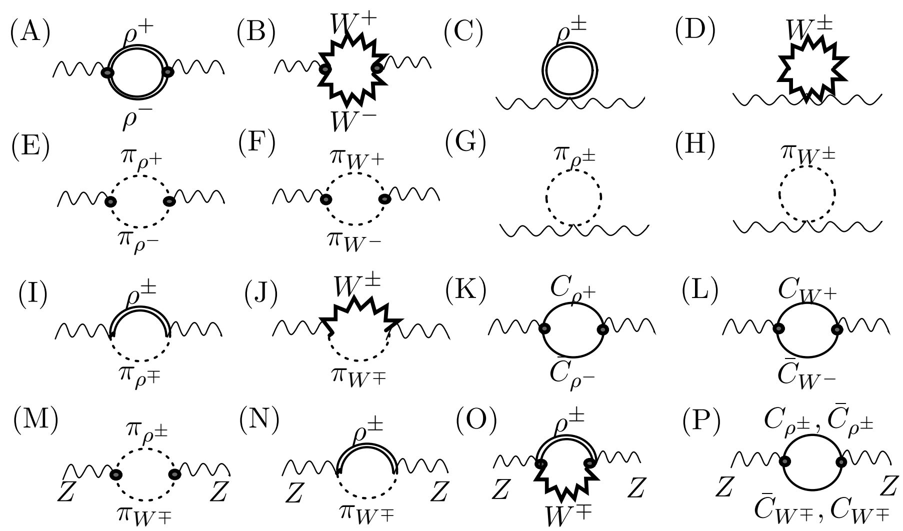

Figure 2: One-loop diagrams for the Z boson and photon self-energies $\Pi_{ZZ,ZA,AA}$ in the three-site model. Each external line in diagrams (A) – (L) can be either a photon or a Z boson; (M), (N), (O) and (P) apply only to the Z boson. Expressions for the relevant vertices are given in Appendix A. Dots on vertices denote derivative couplings. As described in the text, the calculation is done in 't Hooft-Feynman gauge. The $\pi_{W/Z,\rho}$ and $C_{W/Z,\rho}$ fields, respectively, denote the 't Hooft-Feynman gauge unphysical Goldstone bosons and the ghost fields corresponding to the electroweak and ρ bosons.

and the Z-boson self-energy

$$\begin{aligned}\Pi_{ZZ}(p^2) = \frac{e^2}{(4\pi)^2 s^2 c^2}\Bigg[&\left\{4c^2 - \frac{3}{2}\right\} M_W^2 \log\frac{\Lambda^2}{M_W^2} + \left\{\frac{56c^2-47}{8} M_W^2 + \frac{11}{8} M_Z^2 - \frac{3}{8} M_{\rho^\pm}^2\right\} \log\frac{\Lambda^2}{M_{\rho^\pm}^2} \\ &+ p^2 \left(\left\{3c^4 + \frac{1}{3}c^2 - \frac{1}{12}\right\} \log\frac{\Lambda^2}{M_W^2} + \left\{3c^4 - 3c^2 + \frac{17}{24}\right\} \log\frac{\Lambda^2}{M_{\rho^\pm}^2} \right) \Bigg] .\end{aligned} \tag{5.4}$$

These expressions are correct to leading-log approximation, and to order α; we neglect corrections $\mathcal{O}(\alpha x^2 M_W^2)$ or $\mathcal{O}(\alpha x^2 p^2)$, but keep those of order $\mathcal{O}(\alpha x^2 M_\rho^2)$ – and must therefore also account for the difference[10] between $M_{\rho^+}^2$ and $M_{\rho^0}^2$ in contributions proportional to αM_ρ^2.

We note that these results for Π_{ZA} and Π_{ZZ} are not transverse. While in the case of the Z-boson, one expects a scalar contribution renormalizing the Z-boson mass, the ZA mixing self-energy, properly defined, *must* be transverse by electromagnetic gauge-invariance. Therefore the calculation is not yet complete. As is well-known, a gauge-invariant result is obtained only after inclusion of the appropriate pieces (the so-called "pinch contributions") of one-loop vertex corrections and box diagrams [31, 32]. In 't Hooft-Feynman gauge the only

[10] $M_{\rho^0}^2 = M_{\rho^+}^2 + s^2 M_W^2/c^2 + \ldots$

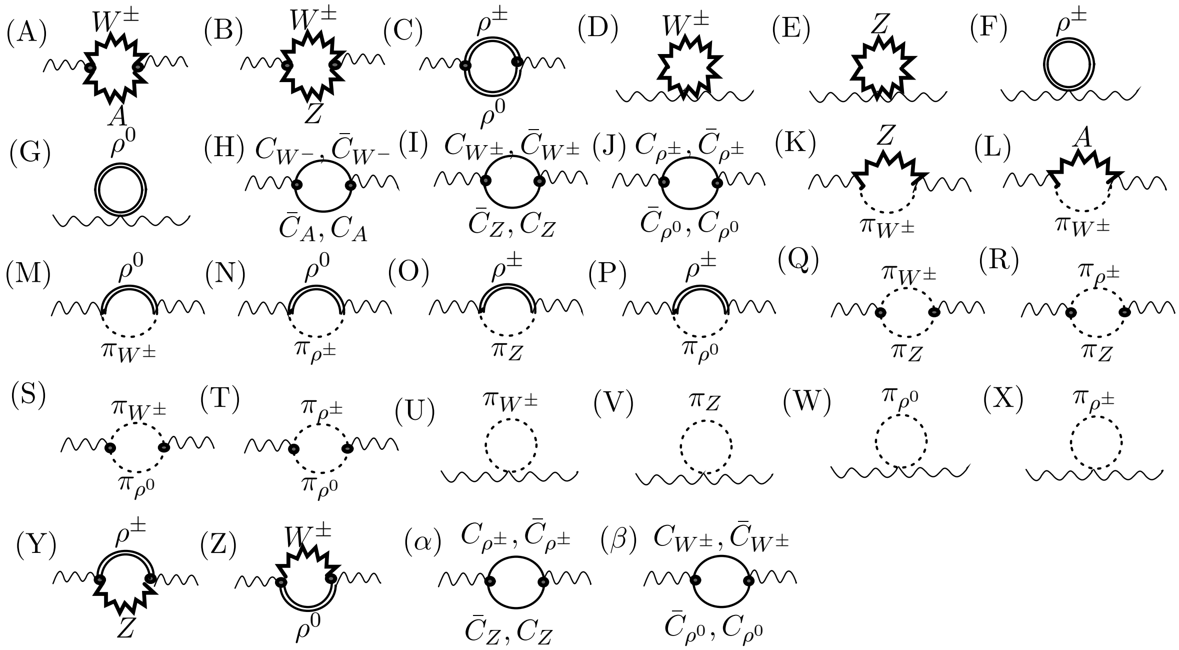

Figure 3: One-loop diagrams for the W boson self-energy Π_{WW} in the three-site model. The relevant vertices are listed given in Appendix A. Dots on vertices denote derivative couplings. As described in the text, the calculation is done in 't Hooft-Feynman gauge. The $\pi_{W/Z,\rho}$ and $C_{W/Z,\rho}$ fields denote the 't Hooft-Feynman gauge unphysical Goldstone bosons and ghost fields corresponding to the electroweak and ρ bosons.

such contributions arise from diagrams containing triple-vector-boson vertices, as illustrated for the electroweak and ρ gauge bosons in Fig. 4.

5.2 Charged Gauge Boson Self-Energies

The values of the individual diagrams in Fig. 3 are shown in Appendix D. Putting these contributions together, and using the relation $M^2_{\rho^0} \approx M^2_{\rho^\pm} + \frac{s^2}{c^2} M^2_W$, we obtain

$$\Pi_{WW}(p^2) = \frac{e^2}{(4\pi)^2 s^2}\left[\left\{\frac{13}{4}M_W^2 - \frac{3}{4}M_Z^2\right\}\log\frac{\Lambda^2}{M_W^2} + \left\{\frac{12c^2+5}{8}M_W^2 + \frac{3}{8}M_Z^2 - \frac{3}{8}M^2_{\rho^\pm}\right\}\log\frac{\Lambda^2}{M^2_{\rho^\pm}} \right.$$
$$\left. + p^2\left(\frac{13}{4}\log\frac{\Lambda^2}{M_W^2} + \frac{17}{24}\log\frac{\Lambda^2}{M^2_{\rho^\pm}}\right)\right]. \tag{5.5}$$

Again, the complete result will include pinch contributions.

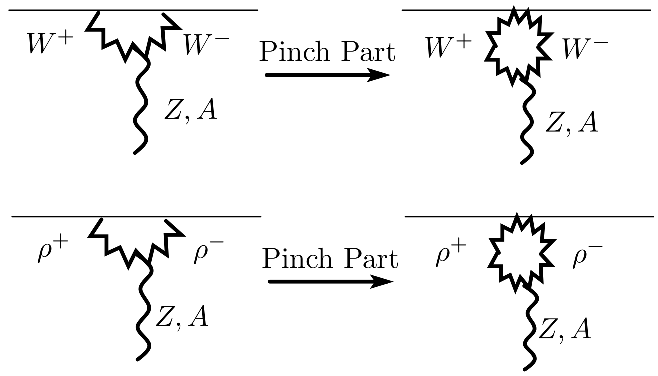

Figure 4: Vertex correction diagrams constructed from the SM and ρ gauge boson loops which contribute to the pinch part of the self-energies [31, 32] for the Z and A bosons. The external fermion lines are arbitrary, and may represent fermions charged under any combination of $SU(2)_0$ or $SU(2)_1$.

6. Pinch Contributions

6.1 Pinch Contributions in the Standard Model

We begin by reviewing the results of the pinch contributions to the Z and A self-energy functions in the standard model: see the first row of diagrams in Fig. 4. As discussed in detail in refs. [31, 32], the pinch contributions arise (in 't-Hooft-Feynman gauge) from the momentum dependence of the triple gauge-boson vertex, and yield effects proportional to the following commutator of generators times coupling constants

$$[\frac{g}{\sqrt{2}}T^+, \frac{g}{\sqrt{2}}T^-] = g^2 T_3 \ , \tag{6.1}$$

where the two terms in the commutator arise from the contraction of the momentum with the two charged-W vertices, and $g = e/s$ is the weak coupling constant. The factors $gT^\pm/\sqrt{2}$ arise from the $W^\pm$ couplings to the external fermion line currents $J^\mu_W = gJ^\mu_\pm/\sqrt{2}$.

The pinch parts of the vertex corrections are proportional to the value of $g^2 J^\mu_3$ on the relevant external fermion line. They are therefore universal, *i.e.* they depend only on the charges of the external fermion lines. As we will see, this property allows their effects to be incorporated into the gauge-boson self-energy functions. To do so, we will need to re-express this current in terms of the (tree-level) currents to which the photon and Z bosons couple. In the standard model, the relationship between the neutral mass-eigenstate and gauge-eigenstate fields is given by

$$\begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix} = \begin{pmatrix} c & -s \\ s & c \end{pmatrix} \begin{pmatrix} W^\mu_3 \\ B^\mu \end{pmatrix} \ , \tag{6.2}$$

and, therefore, the relationship between the currents to which the mass eigenstates couple to the symmetry currents of the theory is given by

$$\begin{pmatrix} J_Z^\mu \\ J_A^\mu \end{pmatrix} = \begin{pmatrix} c & -s \\ s & c \end{pmatrix} \begin{pmatrix} \frac{e}{s} J_3^\mu \\ \frac{e}{c} J_Y^\mu \end{pmatrix} . \tag{6.3}$$

Note that, in this notation, the J_A and J_Z currents *include* the relevant tree-level couplings. Inverting this relationship, we may solve for $g^2 J_3^\mu$ in terms of $J_{A,Z}$, and we find

$$\frac{e^2}{s^2} J_3^\mu = \left(e\frac{c}{s} J_Z^\mu + e J_A^\mu \right) . \tag{6.4}$$

Let us first consider the pinch contribution to Π_{AA}, where

$$\Pi_{AA}^{\mu\nu} = i\, g^{\mu\nu} \Pi_{AA} + \dots \tag{6.5}$$

and we neglect the terms proportional to $p^\mu p^\nu$, which will not contribute to the universal corrections. The pinch parts of photon vertex corrections to the four-fermion scattering amplitudes, then, are of the form

$$\mathcal{M}|^{\gamma}_{one-loop} \propto \mathcal{A} \cdot \frac{1}{p^2} \cdot e\, G(p^2, M_W^2) \cdot (e\frac{c}{s}\mathcal{Z}' + e\mathcal{A}') + (e\frac{c}{s}\mathcal{Z} + e\mathcal{A}) \cdot e\, G(p^2, M_W^2) \cdot \frac{1}{p^2} \cdot \mathcal{A}' . \tag{6.6}$$

The factors in the first term, read from left to right, represent the coupling of the photon to one external fermion line, the photon propagator, the $\gamma W^+ W^-$ coupling proportional to e, the relevant one-loop pinch vertex correction function $G(p^2, M_W^2)$ to the second photon-fermion vertex, and lastly the coupling of $g^2 J_3^\mu$ to the other external fermion line (with "primed" charges). The second term arises from applying the pinch vertex correction to the first fermion-vertex instead. A contribution to the self-energy function $\Delta\Pi_{AA}$, on the other hand, generally gives rise to a correction of the form

$$\mathcal{M}|^{AA}_{one-loop} \propto \mathcal{A} \cdot \frac{1}{p^2} \cdot \Delta\Pi_{AA}(p^2) \cdot \frac{1}{p^2} \cdot \mathcal{A}' . \tag{6.7}$$

Thus Eqn. (6.6) may be viewed as yielding a contribution to Π_{AA}

$$\Delta\Pi_{AA}(p^2) = 2e^2 p^2 G(p^2, M_W^2) . \tag{6.8}$$

Comparing this result for the standard model pinch contribution to that of [31, 32], we find

$$\Delta\Pi_{AA}^{SM}(p^2) = 4e^2 p^2 F_2(M_W, M_W; p^2) , \tag{6.9}$$

we see that $G(p^2, M_W^2) \equiv 2F_2(M_W, M_W; p^2)$, where F_2 is defined in Appendix B. Note that the function $G(p^2, M_W^2)$ will be the same in every standard model pinch contribution to the self-energies of the Z and A bosons. In addition, since the loop-functions depends only on the (universal) form of the triple gauge-boson vertex and the masses of the gauge-bosons, by substituting the appropriate gauge-bosons in the more general expression $F_2(M_A, M_B; p^2)$, we may use this result immediately to compute the relevant loop-function in any pinch contribution in either the standard model or the three-site model.

The one-loop contribution in Eqn. (6.6) also gives rise to contributions proportional to the product of the photon and Z charges of the external fermions, and hence also corrects $\Pi_{ZA}(p^2)$. In general, a correction to Π_{ZA} would give rise to a contribution to the four-fermion amplitude of the form

$$\mathcal{M}|^{ZA}_{one-loop} \propto \mathcal{A} \cdot \frac{1}{p^2} \cdot \Delta\Pi_{ZA}(p^2) \cdot \frac{1}{p^2 - M_Z^2} \cdot \mathcal{Z}' + \mathcal{Z} \cdot \frac{1}{p^2 - M_Z^2} \cdot \Delta\Pi_{ZA}(p^2) \cdot \frac{1}{p^2} \cdot \mathcal{A}' \ . \tag{6.10}$$

Hence, from Eqn. (6.6), we find a contribution

$$\Delta\Pi^{\gamma}_{AZ} = 2e^2 \frac{c}{s}(p^2 - M_Z^2) F_2(M_W, M_W; p^2) \ . \tag{6.11}$$

The pinch parts of the Z vertex corrections to the four-fermion scattering amplitudes are of the form

$$\mathcal{M}|^{Z}_{one-loop} \propto \mathcal{Z} \cdot \frac{1}{p^2 - M_Z^2} \cdot e\frac{c}{s} 2F_2 \cdot (e\frac{c}{s}\mathcal{Z}' + e\mathcal{A}') + (e\frac{c}{s}\mathcal{Z} + e\mathcal{A}) \cdot e\frac{c}{s} 2F_2 \cdot \frac{1}{p^2 - M_Z^2} \cdot \mathcal{Z}' \ , \tag{6.12}$$

where we have abbreviated $F_2 = F_2(M_W, M_W; p^2)$ and the $e\frac{c}{s}$ factor arises from the ZW^+W^- vertex. We see that this gives rise to a correction to Π_{ZA}

$$\Delta\Pi^{Z}_{ZA} = 2e^2 \frac{c}{s} p^2 F_2(M_W, M_W; p^2) \ , \tag{6.13}$$

and, hence, the total pinch contribution to $\Delta\Pi_{AZ}$ is

$$\Delta\Pi^{SM}_{AZ} = \Delta\Pi^{\gamma}_{ZA} + \Delta\Pi^{Z}_{ZA} = 2e^2 \frac{c}{s}(2p^2 - M_Z^2) F_2(M_W, M_W; p^2) \ , \tag{6.14}$$

in agreement with [31, 32].

Eqn. (6.12) also makes a contribution proportional to the product of the Z charges of the external fermions. If we write corrections to Π_{ZZ} in the general form,

$$\mathcal{M}|^{ZZ}_{one-loop} \propto \mathcal{Z} \cdot \frac{1}{p^2 - M_Z^2} \cdot \Delta\Pi_{ZZ}(p^2) \cdot \frac{1}{p^2 - M_Z^2} \cdot \mathcal{Z}' \ , \tag{6.15}$$

then, Eqn. (6.12) yields the pinch contribution

$$\Delta\Pi^{SM}_{ZZ} = 4e^2 \frac{c^2}{s^2}(p^2 - M_Z^2) F_2(M_W, M_W; p^2) \ , \tag{6.16}$$

in agreement with [31, 32].

An analogous calculation, arising from W-boson vertex corrections and corresponding to the commutator of one charged and one neutral current, yields the corresponding pinch correction for the W boson propagator [31, 32]

$$\Delta\Pi^{SM}_{WW} = \frac{4e^2}{s^2}(p^2 - M_W^2) \left[c^2 F_2(M_Z, M_W; p^2) + s^2 F_2(0, M_W; p^2)\right] \ , \tag{6.17}$$

in which the two terms represent the contributions from vertex corrections with internal ZW and γW states, respectively.

6.2 Additional Pinch Contributions in the Three Site Model

We next consider the pinch contributions in the three site model [27], as illustrated in the second row of diagrams in Fig. 4. In this model, up to corrections of order $\mathcal{O}(x^2)$, the mass-eigenstate charged gauge bosons are related to the gauge-eigenstates through (*cf.* Eqn. (4.5))

$$\begin{pmatrix} W^+_\mu \\ \rho^+_\mu \end{pmatrix} = \begin{pmatrix} 1 & \frac{x}{2} \\ -\frac{x}{2} & 1 \end{pmatrix} \begin{pmatrix} L^+_\mu \\ V^+_\mu \end{pmatrix} . \tag{6.18}$$

In order to understand the form of the pinch contributions, we need to understand the currents to which these gauge-bosons couple

$$\begin{pmatrix} J^\mu_W \\ J^\mu_{\rho\pm} \end{pmatrix} = \begin{pmatrix} 1 & \frac{x}{2} \\ -\frac{x}{2} & 1 \end{pmatrix} \begin{pmatrix} (1-x_1)\frac{e}{\sqrt{2}s} J^\mu_\pm \\ \frac{e}{\sqrt{2}sx}(J^{\mu\prime}_\pm + x_1 J^\mu_\pm) \end{pmatrix} , \tag{6.19}$$

where J^μ represents the current associated with fermions primarily charged under $SU(2)_0$ delocalized by an amount x_1 – the ordinary fermions – and $J^{\mu\prime}$ represents any matter charged primarily $SU(2)_1$ (see Eqn. (2.8)). Here we approximate $g_0 \approx e/s$ and $g_1 = e/sx$. From this, we find

$$J^\mu_{\rho\pm} = \frac{e}{\sqrt{2}sx} J^{\mu\prime}_\pm - \frac{xe}{2\sqrt{2}s}(1 - \frac{2x_1}{x^2}) J^\mu_\pm , \tag{6.20}$$

where we have neglected terms of order $\mathcal{O}(x_1 x^2) = \mathcal{O}(x^3)$. Note that, as required by ideal delocalization, the ordinary fermions do not couple to the charged-ρ bosons when $x_1 = x^2/2$.

Comparing to Eqn. (6.1), we see that the pinch contributions arising from ρ-boson vertex corrections in the three site model will be proportional to

$$\left[\frac{e}{\sqrt{2}sx}T^{+\prime} - \frac{xe}{2\sqrt{2}s}\left(1-\frac{2x_1}{x^2}\right)T^+, \frac{e}{\sqrt{2}sx}T^{-\prime} - \frac{xe}{2\sqrt{2}s}\left(1-\frac{2x_1}{x^2}\right)T^-\right] = \frac{e^2}{s^2}{T_3}' + \frac{x^2e^2}{4s^2}\left(1-\frac{2x_1}{x^2}\right)^2 T_3 . \tag{6.21}$$

The second term above is proportional to x^2, and is therefore irrelevant in what follows.

The pinch contributions are therefore proportional to the value of $e^2 J^{\mu\prime}_3/s^2x^2$ on the relevant external fermion line. As in Eqn. (6.4), the key to understanding the pinch contributions is to determine the relationship between $J^{\mu\prime}_3$ and the currents to which the neutral mass-eigenstates couple. Diagonalizing the mass-squared matrix, we find that the relationship between the neutral boson gauge- and mass-eigenstates is given by (*cf.* Eqn. (4.11))

$$\begin{pmatrix} Z^\mu \\ A^\mu \\ \rho^\mu \end{pmatrix} = \begin{pmatrix} c & -s & \frac{c^2-s^2}{2c}x \\ s & c & sx \\ -\frac{x}{2} & -\frac{sx}{2c} & 1 \end{pmatrix} \begin{pmatrix} L^\mu_3 \\ B^\mu \\ V^\mu_3 \end{pmatrix} , \tag{6.22}$$

where the rotation matrix is orthogonal up to corrections of $\mathcal{O}(x^2)$. The relationship between the currents to which the neutral mass-eigenstates couple and the symmetry currents is given, therefore, by

$$\begin{pmatrix} J^\mu_Z \\ J^\mu_A \\ J^\mu_\rho \end{pmatrix} = \begin{pmatrix} c & -s & \frac{c^2-s^2}{2c}x \\ s & c & sx \\ -\frac{x}{2} & -\frac{sx}{2c} & 1 \end{pmatrix} \begin{pmatrix} (1-x_1)\frac{e}{s}J^\mu_3 \\ \frac{e}{c}J^\mu_Y \\ \frac{e}{sx}(J^{\mu\prime}_3 + x_1 J^\mu_3) \end{pmatrix} , \tag{6.23}$$

allowing for fermion delocalization. Inverting the matrix, we find the relations

$$\frac{e}{sx}(J_3^{\mu\prime} + x_1 J_3^\mu) = J_\rho^\mu + sxJ_A^\mu + \frac{c^2-s^2}{2c} xJ_Z^\mu \ , \tag{6.24}$$

and

$$\frac{e}{s}(1-x_1)J_3^\mu = cJ_Z^\mu + sJ_A^\mu - \frac{x}{2}J_\rho^\mu \ . \tag{6.25}$$

Noting that $x_1 = \mathcal{O}(x^2)$, and therefore neglecting it on the left hand side of Eqn. (6.25), we may rearrange these equations to find

$$\frac{e^2}{s^2x^2}J_3^{\mu\prime} = \frac{e}{sx}\left(1-\frac{x_1}{2}\right)J_\rho^\mu + e\left(1-\frac{x_1}{x^2}\right)J_A^\mu + e\frac{c^2-s^2}{2cs}\left(1-\frac{2c^2}{c^2-s^2}\frac{x_1}{x^2}\right)J_Z^\mu \ . \tag{6.26}$$

This equation will allow us to extract the pinch contributions in the three site model – note that the operator $e^2 J_3^{\mu\prime}/s^2x^2$ has, counter-intuitively, relevant weak-size couplings to the ordinary fermion currents J_A^μ and J_Z^μ!

In analogy with our calculations in the standard model, we may immediately read off the form of the ρ-boson vertex correction contributions to photon exchange

$$\mathcal{M}|^{\gamma}_{one-loop} = \mathcal{A}\cdot\frac{1}{p^2}\cdot 2e\tilde{F}_2\cdot\left[e\left(1-\frac{x_1}{x^2}\right)\mathcal{A}' + e\frac{c^2-s^2}{2cs}\left(1-\frac{2c^2}{c^2-s^2}\frac{x_1}{x^2}\right)\mathcal{Z}'\right] + (\mathcal{A},\mathcal{Z}\leftrightarrow\mathcal{A}',\mathcal{Z}') \ , \tag{6.27}$$

where the $2e\tilde{F}_2$ includes both the $\gamma\rho^+\rho^-$ coupling e, and the loop-function $2\tilde{F}_2 = 2F_2(M_\rho, M_\rho; p^2)$. Similarly, the ρ-boson vertex corrections to Z exchange may be written

$$\begin{aligned}\mathcal{M}|^{Z}_{one-loop} =& \mathcal{Z}\cdot\frac{1}{p^2-M_Z^2}\cdot 2\frac{e(c^2-s^2)}{2cs}\tilde{F}_2\cdot\left[e\left(1-\frac{x_1}{x^2}\right)\mathcal{A}' + e\frac{c^2-s^2}{2cs}\left(1-\frac{2c^2}{c^2-s^2}\frac{x_1}{x^2}\right)\mathcal{Z}'\right] \\ &+ (\mathcal{A},\mathcal{Z}\leftrightarrow\mathcal{A}',\mathcal{Z}') \ ,\end{aligned} \tag{6.28}$$

where the $Z\rho^+\rho^-$ coupling is proportional to $e(c^2-s^2)/2cs$ to leading order (see Table 5 of Appendix A.4).

We may then compute the corresponding pinch contributions, by comparing Eqns. (6.27) and (6.28) to Eqns. (6.7), (6.10), and (6.15). From this we obtain

$$\Delta\Pi_{AA}^{3-site}(p^2) = 4e^2\left(1-\frac{x_1}{x^2}\right)p^2F_2(M_\rho, M_\rho; p^2) \ , \tag{6.29}$$

$$\Delta\Pi_{ZA}^{\gamma,3-site}(p^2) = \frac{e^2(c^2-s^2)}{cs}\left(1-\frac{2c^2}{c^2-s^2}\frac{x_1}{x^2}\right)(p^2-M_Z^2)F_2(M_\rho, M_\rho; p^2) \ , \tag{6.30}$$

$$\Delta\Pi_{ZA}^{Z,3-site}(p^2) = \frac{e^2(c^2-s^2)}{cs}\left(1-\frac{x_1}{x^2}\right)p^2F_2(M_\rho, M_\rho; p^2) \ , \tag{6.31}$$

$$\Delta\Pi_{ZZ}^{3-site}(p^2) = \frac{e^2(c^2-s^2)^2}{s^2c^2}\left(1-\frac{2c^2}{c^2-s^2}\frac{x_1}{x^2}\right)(p^2-M_Z^2)F_2(M_\rho, M_\rho; p^2) \ . \tag{6.32}$$

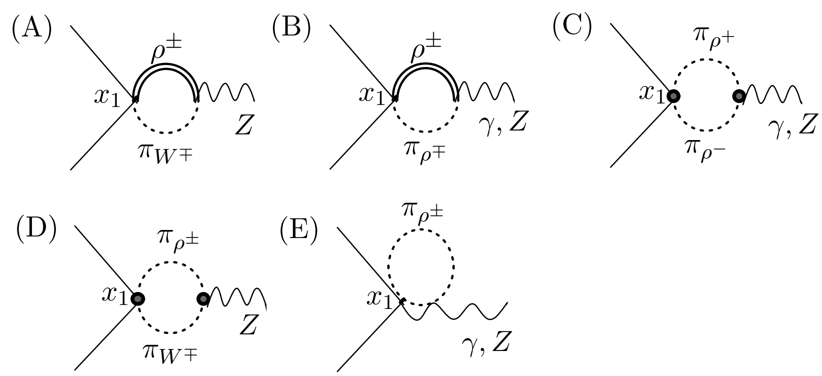

Figure 5: One-loop vertex contributions to neutral-current processes arising from the delocalization operator $\mathcal{L}'_f$. As described in the appendix, since they are universal – *i.e.* proportional to the charges of the external fermions – these contributions can be incorporated into the neutral gauge boson self-energy functions $\Pi_{ZA,ZZ}$. Eots on vertices denote derivative couplings. The delocalization operator is proportional to $x_1 = \mathcal{O}(x^2)$, and therefore only the contributions proportional to M_ρ^2, which are illustrated here, contribute to this order. There are analogous contributions to charged-current processes, which result in corrections to Π_{WW}.

Precisely analogous computations, arising from W-boson vertex corrections and corresponding to the commutator $[J^\mu_{\rho\pm}, J^\nu_\rho]$, yields the pinch correction for the W boson propagator

$$\Delta\Pi^{3-site}_{WW}(p^2) = \frac{e^2}{s^2}\left(1 - \frac{2x_1}{x^2}\right)(p^2 - M_W^2)F_2(M_\rho, M_\rho; p^2)\ . \tag{6.33}$$

Note that, consistent with isospin symmetry, $\Delta\Pi_{ZZ}$ in Eqn. (6.32) reduces to $\Delta\Pi_{WW}$ in Eqn. (6.33) in the limit $c \to 1$ with e/s held fixed.

Finally to the order in which we work, expanding $F_2(M_\rho, M_\rho; p^2)$ for $p^2 \simeq M^2_{W,Z} \ll M_\rho^2$, we see that we may approximate $F_2(M_\rho, M_\rho; p^2) \approx F_2(M_\rho, M_\rho; 0)$, and these pinch contributions only affect the values of the self-energies at zero momentum.

As described here, the "pinch" contributions are determined entirely by the gauge symmetry and fermion coupling structure(s) of the theory. Examining Eqn. (6.23) however, one notes that the couplings of the fermions (with current J^μ) to the ρ are suppressed by x. The vertex diagrams illustrated in the lower row of Fig. 4, therefore, do not contribute in the case of ordinary fermions on the external lines! Diagrammatically, as shown explicitly in Appendix E, for ordinary fermions the contributions in Eqns. (6.29) – (6.33) may be shown to arise instead from $\gamma - \rho$, $Z - \rho$, and $W - \rho$ mixing corrections to the four-fermion scattering amplitudes.

7. Fermion Delocalization Contributions

Consider the fermion delocalization operator of Eqn. (2.8)

$$\mathcal{L}'_f = -x_1 \cdot \bar\psi_L (i \not{D} \Sigma_{(1)} \Sigma^\dagger_{(1)}) \psi_L\ ,$$

where $D_\mu \Sigma_{(1)} = \partial_\mu \Sigma_{(1)} - i g_0 L_\mu \Sigma_{(1)} + i g_1 \Sigma_{(1)} V_\mu$. The link variable $\Sigma_{(1)}$ is expanded as

$$\Sigma_{(1)} = e^{i2\pi_{(1)}/f} = 1 + 2i\frac{\pi_{(1)}}{f} - \frac{2\pi_{(1)}^2}{f^2} + \cdots . \tag{7.1}$$

From this, we see that the delocalization operator of Eqn. (2.8) is expressed as

$$\begin{aligned}\mathcal{L}'_f = & -x_1 g_1 \frac{2}{f}\epsilon^{abc}\pi^a_{(1)} V^b_\mu J^{\mu c}_L - x_1 \frac{2}{f^2}\epsilon^{abc}\pi^a_{(1)}\partial_\mu \pi^b_{(1)} J^{\mu c}_L \\ & + x_1 g_1 \frac{2}{f^2}\epsilon^{eab}\epsilon^{ecd} V^a_\mu \pi^b_{(1)}\pi^c_{(1)} J^{\mu d}_L + \cdots ,\end{aligned} \tag{7.2}$$

where $J^{\mu a}_L = \bar{\psi}_L \gamma^\mu T^a \psi_L$. In terms of the mass- and charged-eigenstate fields, to leading order in x, we find

$$\begin{aligned}\mathcal{L}'_f|_{NC} = & -i\frac{e^2}{s^2 M_{\rho^\pm}}\left(\frac{x_1}{x^2}\right) J^{\mu 3}_L \Big[\rho^+_\mu \pi_{W^-} - \rho^+_\mu \pi_{\rho^-}\Big] + \text{h.c.} \\ & -i\frac{e^2}{2s^2 M^2_{\rho^\pm}}\left(\frac{x_1}{x^2}\right) J^{\mu 3}_L \Bigg[\left(\pi_{W^+} \overleftrightarrow{\partial}_\mu \pi_{W^-}\right) + \left(\pi_{\rho^+} \overleftrightarrow{\partial}_\mu \pi_{\rho^-}\right) \\ & \qquad - \left(\pi_{W^+} \overleftrightarrow{\partial}_\mu \pi_{\rho^-}\right) - \left(\pi_{\rho^-} \overleftrightarrow{\partial}_\mu \pi_{W^+}\right)\Bigg] \\ & -\frac{e^3}{s^2 M^2_{\rho^\pm}}\left(\frac{x_1}{x^2}\right) J^{\mu 3}_L \left(A_\mu + \frac{c^2 - s^2}{2sc} Z_\mu\right)\left(\pi_{W^+}\pi_{W^-} + \pi_{\rho^+}\pi_{\rho^-}\right) + \cdots ,\end{aligned} \tag{7.3}$$

where we have used $f \approx \sqrt{2} M_{\rho^\pm}/g_1$ and $g_0 \approx e/s$ which are valid only to leading order in x.

As illustrated in Fig. 5, these couplings in the delocalization operator give rise to vertex-corrections to neutral- and charged-current four-fermion processes.[11] These diagrams are evaluated in Appendix F, and their effects are summarized by the effective interactions in Eqn. (F.10). Using the relation (*cf.* Eqn. (6.3))

$$\frac{e}{s} J^\mu_{3\,L} = \left(c J^\mu_Z + s J^\mu_A\right), \tag{7.4}$$

we may rewrite Eqn. (F.10) in terms of the currents J^μ_A and J^μ_Z,

$$\mathcal{L}_{\text{eff}} = \left\{G_1 J^\mu_Z + \frac{s}{c} G_1 J^\mu_A\right\} Z_\mu + \left\{sc G_2 J^\mu_Z + s^2 G_2 J^\mu_A\right\} A_\mu , \tag{7.5}$$

where we have abbreviated $G_{1,2} = G_{1,2}(M^2_{\rho^\pm}; x_1)$, and these functions are defined in Eqns. (F.11) and (F.12). Note that these vertex corrections are *universal* – *i.e.* proportional to the charges of the external fermions. As in the case of the "pinch" contributions, their effects may be incorporated into corrections of the gauge-boson self-energy functions.

[11] As described in [33], these contributions correspond to renormalizations of the parameter x_1. As noted previously (see footnote 1), there is no explanation for the amount of delocalization in this low-energy effective theory.

The contribution of these corrections to four-fermion amplitudes mediated by the photon ($\mathcal{M}_\gamma^{one-loop}$) and Z boson ($\mathcal{M}_Z^{one-loop}$) exchange may then be written

$$\mathcal{M}_\gamma^{one-loop} = \left\{scG_2\mathcal{Z} + (1+s^2G_2)\mathcal{A}\right\}\frac{1}{p^2}\left\{scG_2\mathcal{Z}' + (1+s^2G_2)\mathcal{A}'\right\}, \tag{7.6}$$

$$\mathcal{M}_Z^{one-loop} = \left\{(1+G_1)\mathcal{Z} + \frac{s}{c}G_1\mathcal{A}\right\}\frac{1}{p^2-M_Z^2}\left\{(1+G_1)\mathcal{Z}' + \frac{s}{c}G_1\mathcal{A}'\right\}, \tag{7.7}$$

where, as before, $\mathcal{A}(')$ and $\mathcal{Z}(')$ are photon- and Z-charges carried by the external fermions. From the forms of Eqns.(7.6) and (7.7), we can read off the corrections to the self-energy functions,

$$\Delta\Pi_{AA}(p^2) = 2s^2G_2\cdot p^2\,, \tag{7.8}$$

$$\Delta\Pi_{ZA}^{\gamma}(p^2) = scG_2\cdot(p^2-M_Z^2)\,, \tag{7.9}$$

$$\Delta\Pi_{ZA}^{Z}(p^2) = \frac{s}{c}G_1\cdot p^2\,, \tag{7.10}$$

$$\Delta\Pi_{ZZ}(p^2) = 2G_1\cdot(p^2-M_Z^2)\,. \tag{7.11}$$

By an analogous computation, or by noting the isospin symmetry relation, $\Delta\Pi_{WW} = \Delta\Pi_{ZZ}|_{c\to 1}$ with (e/s) fixed, we can also easily read off the corresponding correction to the W boson self-energy function.

Substituting Eqns.(F.11) and (F.12) into Eqns.(7.8)-(7.11) we find

$$\Delta\Pi_{AA}^{delocal}(p^2) = \frac{4e^2}{(4\pi)^2}\left(\frac{x_1}{x^2}\right)p^2\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{7.12}$$

$$\Delta\Pi_{ZA}^{delocal}(p^2) = \frac{e^2}{(4\pi)^2 sc}\left(\frac{x_1}{x^2}\right)\left[\left(4c^2-\frac{1}{4}\right)p^2 - 2M_W^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{7.13}$$

$$\Delta\Pi_{ZZ}^{delocal}(p^2) = \frac{e^2}{(4\pi)^2 s^2}\left(4c^2-\frac{1}{2}\right)(p^2-M_Z^2)\left(\frac{x_1}{x^2}\right)\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{7.14}$$

$$\Delta\Pi_{WW}^{delocal}(p^2) = \frac{7e^2}{2(4\pi)^2 s^2}(p^2-M_W^2)\left(\frac{x_1}{x^2}\right)\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,. \tag{7.15}$$

8. Total Gauge Boson Self-Energies

8.1 Neutral Gauge Bosons

Including the standard model and three-site "pinch corrections", and the corrections arising from the fermion delocalization operator, we find the neutral gauge boson self-energies

$$\overline{\Pi}_{AA}(p^2) = \Pi_{AA}(p^2) + 4e^2p^2\left\{F_2(M_W, M_W; p^2) + F_2(M_{\rho^\pm}, M_{\rho^\pm}; p^2)\right\}\,, \tag{8.1}$$

$$\overline{\Pi}_{ZA}(p^2) = \Pi_{ZA}(p^2) + \frac{2e^2c}{s}(2p^2 - M_Z^2)F_2(M_W, M_W; p^2) + \frac{e^2(c^2-s^2)}{sc}\left(p^2\left\{2 + \frac{3}{4(c^2-s^2)}\left(\frac{x_1}{x^2}\right)\right\} - M_Z^2\right)F_2(M_{\rho^\pm}, M_{\rho^\pm}; p^2)\,, \tag{8.2}$$

$$\overline{\Pi}_{ZZ}(p) = \Pi_{ZZ}(p^2) + \frac{4e^2c^2}{s^2}(p^2 - M_Z^2)F_2(M_W, M_W; p^2) + \frac{e^2(c^2-s^2)^2}{s^2c^2}\left\{1 + \frac{3c^2}{2(c^2-s^2)^2}\left(\frac{x_1}{x^2}\right)\right\}(p^2 - M_Z^2)F_2(M_{\rho^\pm}, M_{\rho^\pm}; p^2)\,, \tag{8.3}$$

where the $\Pi_{AA,ZA,ZZ}$ were computed in Section 5.1, and the function F_2 is defined in Appendix B. Evaluating and simplifying, we have

$$\overline{\Pi}_{AA}(p^2) = \frac{e^2}{(4\pi)^2}p^2\left[7\log\frac{M_{\rho^\pm}^2}{M_W^2} + 14\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\right], \tag{8.4}$$

$$\overline{\Pi}_{ZA}(p^2) = \frac{e^2}{(4\pi)^2 sc}p^2\left[\left\{7c^2 + \frac{1}{6}\right\}\log\frac{M_{\rho^\pm}^2}{M_W^2} + \left\{14c^2 - \frac{10}{3} + \frac{3}{4}\left(\frac{x_1}{x^2}\right)\right\}\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\right], \tag{8.5}$$

$$\begin{aligned}\overline{\Pi}_{ZZ}(p^2) = \frac{e^2}{(4\pi)^2 s^2c^2}\Bigg[&-\frac{3}{2}M_W^2\log\frac{M_{\rho^\pm}^2}{M_W^2}\\ &+\left(\left\{3c^2 - \frac{27}{8} - \frac{3}{2}\left(\frac{x_1}{x^2}\right)\right\}M_W^2 + \frac{3}{8}M_Z^2 - \frac{3}{8}M_{\rho^\pm}^2\right)\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\\ &+p^2\Bigg(\left\{7c^4 + \frac{1}{3}c^2 - \frac{1}{12}\right\}\log\frac{M_{\rho^\pm}^2}{M_W^2}\\ &+\left\{14c^4 - \frac{20}{3}c^2 + \frac{39}{24} + \frac{3}{2}c^2\left(\frac{x_1}{x^2}\right)\right\}\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\Bigg)\Bigg]\,.\end{aligned} \tag{8.6}$$

up to $\mathcal{O}(\alpha x^2 p^2)$ or $\mathcal{O}(\alpha x^2 M_W^2)$. Note that the modified AA and ZA self-energies are purely transverse. The ZZ self-energy, $\overline{\Pi}_{ZZ}(p^2)$, represents, in part, a renormalization of the electroweak symmetry breaking scale[12] similar to the corresponding one-loop renormalization proportional to the Higgs boson mass-squared in the standard model [34].

8.2 Charged Gauge Boson

Including the standard model and three-site "pinch corrections", and the corrections arising

[12] That is, a renormalization of the electroweak F-constant, equal to the vacuum expectation value of the Higgs in the standard model

from the fermion delocalization operator, we find the charged gauge boson self-energy

$$\overline{\Pi}_{WW}(p^2) = \Pi_{WW}(p^2) + \frac{4e^2}{s^2}(p^2 - M_W^2)\left[c^2 F_2(M_Z, M_W; p^2) + s^2 F_2(0, M_W; p^2)\right] + \frac{e^2}{s^2}\left\{1 + \frac{3}{2}\left(\frac{x_1}{x^2}\right)\right\}(p^2 - M_W^2) F_2(M_{\rho^0}, M_{\rho^\pm}; p^2) ., \tag{8.7}$$

where Π_{WW} is computed in Section 5.2, and the function F_2 is defined in Appendix B. Evaluating and simplifying, we find

$$\begin{aligned}\overline{\Pi}_{WW}(p^2) = \frac{e^2}{(4\pi)^2 s^2}\Bigg[&-\frac{3}{4}(M_W^2 + M_Z^2)\log\frac{M_{\rho^\pm}^2}{M_W^2} \\ &+ \left\{\frac{12c^2 - 9}{8} M_W^2 - \frac{3}{8} M_Z^2 - \frac{3}{8} M_{\rho^\pm}^2 - \frac{3}{2}\left(\frac{x_1}{x^2}\right) M_W^2\right\}\log\frac{\Lambda^2}{M_{\rho^\pm}^2} \\ &+ p^2\left(\frac{29}{4}\log\frac{M_{\rho^\pm}^2}{M_W^2} + \left\{\frac{215}{24} + \frac{3}{2}\left(\frac{x_1}{x^2}\right)\right\}\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\right)\Bigg].\end{aligned} \tag{8.8}$$

Note that, in the custodial isospin symmetric limit $\sin^2\theta \to 0$ (*i.e.* $c^2 \to 1$ and $M_Z^2 \to M_W^2$), Eqn. (8.6) reduces to Eqn. (8.8).

9. Precision Electroweak Corrections

9.1 The S Parameter and Counterterms

The neutral gauge boson self-energies contribute to the S parameter as [28]

$$\frac{\alpha S_{1-\text{loop}}}{4s^2c^2} = \overline{\Pi}'_{ZZ}(0) - \overline{\Pi}'_{AA}(0) - \frac{c^2 - s^2}{sc}\overline{\Pi}'_{ZA}(0)\,, \tag{9.1}$$

By using Eqs.(8.4)-(8.6), the leading correction to the S parameter is evaluated in the limit $M_{\rho^\pm,0}^2 \gg M_W^2$ as

$$\alpha S_{1-\text{loop}} = \frac{\alpha}{12\pi}\log\frac{M_\rho^2}{M_W^2} - \frac{41\alpha}{24\pi}\log\frac{\Lambda^2}{M_\rho^2} + \frac{3\alpha}{4\pi}\left(\frac{x_1}{x^2}\right)\log\frac{\Lambda^2}{M_\rho^2}\,. \tag{9.2}$$

Note that the first term arises from "scaling" between M_W and M_ρ – and has a coefficient precisely equal to the leading-log contribution from a heavy Higgs boson[28]

$$\alpha S_{Higgs} = \frac{\alpha}{12\pi}\log\left(\frac{M_H^2}{M_W^2}\right)\ . \tag{9.3}$$

This allows us to match our calculation to the phenomenological extractions of S which depend on a reference standard model Higgs-boson mass.

The dependence on the renormalization scale (here taken to be the cutoff Λ of the effective theory) is cancelled by the scale-dependence of the appropriate counterms [18]. The $\mathcal{O}(p^4)$ counterterms relevant to $S_{1-\text{loop}}$ are given by

$$\mathcal{L}_{(4)} = c_1 g_1 g_2 \text{Tr}[V_{\mu\nu}\Sigma_{(2)}B^{\mu\nu}\frac{\sigma_3}{2}\Sigma_{(2)}^\dagger] + c_2 g_1 g_0 \text{Tr}[L_{\mu\nu}\Sigma_{(1)}V^{\mu\nu}\Sigma_{(1)}^\dagger]\,. \tag{9.4}$$

By using Eqn. (4.8)-(4.10), these may be written in terms of the mass eigenstate fields as

$$\mathcal{L}_{(4)}|^{\rm quad}_{Z,A} = \frac{i}{2}\delta_{ZZ}(Z_\mu D^{\mu\nu} Z_\nu) + i\delta_{ZA}(Z_\mu D^{\mu\nu} A_\nu) + \frac{i}{2}\delta_{AA}(A_\mu D^{\mu\nu} A_\nu)\,, \tag{9.5}$$

where $D^{\mu\nu} = -\partial^2 g^{\mu\nu} + \partial^\mu \partial^\nu$ and

$$\delta_{ZZ} = \frac{e^2(c^2-s^2)}{c^2 s^2}\left[-s^2 c_1 + c^2 c_2\right], \tag{9.6}$$

$$\delta_{ZA} = \frac{e^2}{2sc}\left[(c^2-3s^2)c_1 + (3c^2-s^2)c_2\right], \tag{9.7}$$

$$\delta_{AA} = 2e^2\left[c_1 + c_2\right]. \tag{9.8}$$

From this, applying Eqn. (9.1), we find the contribution to S

$$\delta S_{1-\rm loop} = -8\pi(c_1+c_2)\,. \tag{9.9}$$

Adjusting for the reference Higgs mass, using Eqn. (9.3) and adding the contribution from the counterterms in Eqn. (9.9), we arrive at our final result (Eqn. (1.1))

$$\begin{aligned}\alpha S_{3-site} = & \left[\frac{4s^2 M_W^2}{M_\rho^2}\left(1-\frac{x_1 M_\rho^2}{2M_W^2}\right)\right]_{\mu=\Lambda} + \frac{\alpha}{12\pi}\log\frac{M_\rho^2}{M_{Href}^2} \\ & - \frac{41\alpha}{24\pi}\log\frac{\Lambda^2}{M_\rho^2} + \frac{3\alpha}{8\pi}\left(\frac{x_1 M_\rho^2}{2M_W^2}\right)\log\frac{\Lambda^2}{M_\rho^2} \\ & -8\pi\alpha(c_1(\Lambda)+c_2(\Lambda))\ ,\end{aligned}$$

where the tree-level expression and the counterterms are now understood to be evaluated at scale Λ.

9.2 αT and a Counterterm

The T parameter [28] is expressed in terms of the W and Z boson self-energies as

$$\alpha T_{1-loop} = \frac{\overline{\Pi}_{WW}(0)}{M_W^2} - \frac{\overline{\Pi}_{ZZ}(0)}{M_Z^2}\,. \tag{9.10}$$

Noting

$$\frac{M_{\rho^\pm}^2}{M_W^2} - \frac{M_{\rho^\pm}^2}{c^2 M_Z^2} = \frac{s^2(4c^2-1)}{c^2} + \mathcal{O}(x^2)\,, \tag{9.11}$$

from Eqns.(8.8) and (8.6), we have

$$\alpha T_{1-loop} = -\frac{3\alpha}{16\pi c^2}\log\frac{M_\rho^2}{M_W^2} - \frac{3\alpha}{32\pi c^2}\log\frac{\Lambda^2}{M_\rho^2}\,. \tag{9.12}$$

Note that, as in the case of the S-parameter, the first term arises from "scaling" between M_W and M_ρ – and has precisely the same form as the leading-log contribution from a heavy Higgs boson [28]

$$\alpha T_{Higgs} = -\frac{3\alpha}{16\pi c^2}\log\frac{M_H^2}{M_W^2}\ . \tag{9.13}$$

This allows us to match our calculation to the phemenological extractions of T which depend on a reference standard model Higgs-boson mass.

The dependence on the renormalization scale (here taken to be the cutoff, Λ, of the effective theory) is cancelled by the scale-dependence of the appropriate counterterm. The $\mathcal{O}(p^4)$ counterterm[13] relevant to T_{1-loop} is

$$\delta\mathcal{L} = c_0 g_2^2 f^2\left(\mathrm{tr}[D_\mu\Sigma_{(2)}\frac{\tau_3}{2}\Sigma^\dagger_{(2)}]\right)^2 = \frac{4\pi\alpha\, c_0}{c^2}f^2\left(\mathrm{tr}[D_\mu\Sigma_{(2)}\frac{\tau_3}{2}\Sigma^\dagger_{(2)}]\right)^2\ . \tag{9.14}$$

Using Eqns. (4.6), (4.9) and (4.10), in unitary gauge we read off a correction to the Z boson mass (but *not* the W boson mass) from Eqn. (9.14):

$$\delta M_Z^2 = -\frac{4\pi\alpha\, c_0}{c^2}\, M_Z^2\,, \tag{9.15}$$

which leads to a contribution to αT:

$$[\delta\,(\alpha T)]_{1-\mathrm{loop}} = \frac{4\pi\alpha\, c_0(\Lambda)}{c^2}\ . \tag{9.16}$$

Adjusting for the reference Higgs mass, M_{Href}, using Eqn. (9.13) and adding the contribution from the counterterm in Eqn. (9.14), we then arrive at the final result quoted in Eqn. (1.2):

$$\alpha T_{3-site} = -\frac{3\alpha}{16\pi c^2}\log\frac{M_\rho^2}{M_{Href}^2} - \frac{3\alpha}{32\pi c^2}\log\frac{\Lambda^2}{M_\rho^2} + \frac{4\pi\alpha\, c_0(\Lambda)}{c^2}\ .$$

In addition to this contribution, there will typically be additional contributions to the T-parameter[2] arising from isospin-violation in the fermion sector [27].

10. Reduction to the Two-Site Model

In the limit $M_{Higgs}\to\infty$, which corresponds to taking the self-coupling of the Higgs to infinity, the standard model formally reduces to the electroweak chiral lagrangian [34, 35] which may be viewed as the "two-site" model illustrated in Fig. 6. Consider the limit $M_\rho\to\infty$ in the three-site model, obtained by taking the coupling g_1 in Fig. 1 to infinity. In either case one is taking a *dimensionless* coupling to infinity, and the ordinary decoupling theorem [36] does not apply.

[13] Note that the tree-level value of αT is $\mathcal{O}(M_W^4/M_\rho^4)$ in Higgsless models [20], and therefore this counterterm is formally of $\mathcal{O}(p^4)$. This is manifest in Eqn. (9.14) by the fact that the counterterm is proportional to g_2^2.

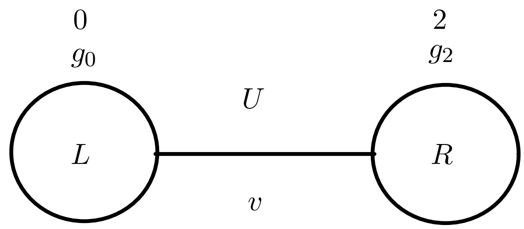

Figure 6: Two-site nonlinear model which is, formally, the limit of the standard model as $M_{Higgs} \to \infty$ [34, 35].

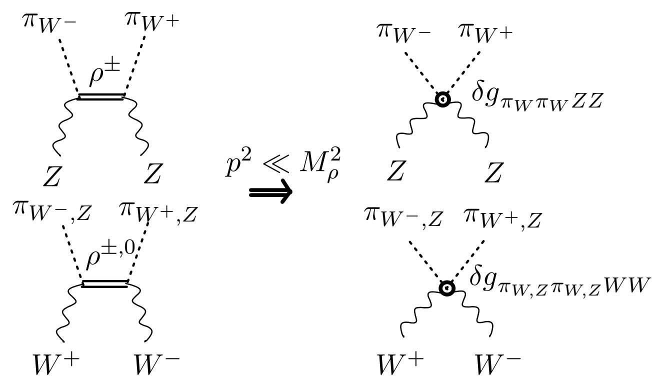

Figure 7: ρ-exchange diagrams contributing to the $\pi_{W^+}\pi_{W^-}ZZ$, $\pi_{W^+}\pi_{W^-}W^+W^-$ and $\pi_Z\pi_Z W^+W^-$ interaction terms at energy scales $p^2 \ll M_\rho^2$.

Indeed, as noted above, there are interaction vertices proportional to M_ρ (see Eqn. (A.1)) in the three-site model. These yield corrections to $\overline{\Pi}_{ZZ}$ and $\overline{\Pi}_{WW}$ that are proportional to $\ln M_\rho^2/M_W^2$ without any $1/M_\rho^2$ suppression factor. Ordinarily, one would expect such contributions to the low-energy behavior of the theory to arise only from diagrams without propagating ρ bosons. In this case, however, the non-decoupling is manifested by the presence of terms proportional to $\ln M_\rho^2/M_W^2$ in the amplitudes for diagram $(N)_{ZZ}$ of Fig. 2 and diagrams $(M)_{WW}$ and $(O)_{WW}$ of Fig. 4, all of which include propagating ρ fields. Examining the contributions of these diagrams in detail (see Eqns. (C.38, D.13, D.15)) reveals that these diagrams contribute only to $\overline{\Pi}_{ZZ,WW}(0)$, and therefore affect αT but not αS.

Nonetheless, we have seen above that the one-loop leading-log contribution to T arising from scaling between M_W and M_ρ has precisely the same form as the leading-log contribution from a heavy Higgs boson. In retrospect, this is an expected result. The chiral-logarithmic contributions of this kind depend only on the low-energy theory valid at energy scales between M_W and M_ρ. The leading order – $\mathcal{O}(p^2)$ – interactions in this energy regime are determined entirely by gauge-invariance and chiral low-energy theorems. Since the gauge- and chiral-symmetries of the three-site model at energies below M_ρ are precisely the same as those in

	$\pi_{W^+}\pi_{W^-}ZZ$	$\pi_{W^+}\pi_{W^-}W^+W^-$	$\pi_Z\pi_Z W^+W^-$	$\pi_Z\pi_Z ZZ$
2-site	$-e^2$	0	0	0
3-site	$\frac{e^2(c^2-s^2)^2}{4s^2c^2}$	$\frac{e^2}{4s^2}$	$\frac{e^2}{4s^2}$	0

Table 1: This table lists the coupling constants of the tree-level two-pion/two-gauge-boson interactions arising from the $\mathcal{O}(p^2)$ interactions in the 2-site and 3-site models. Adding the non-decoupling contributions arising from ρ-exchange illustrated in Fig. 7, Eqn. (10.1), we see that the 3-site interactions reduce to those of the 2-site model at energies less than M_ρ.

the standard model, the $\mathcal{O}(p^2)$ interactions must be the same in both theories — and therefore the chiral-logarithmic corrections arising from this energy regime must also be the same in both theories [51, 52, 53, 54].

Examining the pion and Goldstone boson interactions in the $\mathcal{O}(p^2)$ lagrangian, we find that the only differences between the three-site and two-site model relevant to the calculation of the gauge-boson self-energies occur in the two-pion/two-gauge-boson interactions summarized in table 1. We may see how the three-site to two-site reduction occurs explicitly.[14] Starting in the three-site model, we have the $\mathcal{L}_{\pi\pi AA}$ vertices listed in table 1. We also have $\pi\pi AA$ interactions mediated by ρ-exchange, as illustrated in the left panel of Fig. 7. At energies $E < M_\rho$, we may integrate out the ρ-mesons and, at tree-level, we obtain additional $\pi\pi AA$ interactions as illustrated in the right panel of Fig. 7. The correspondingly induced couplings are evaluated to be

$$\delta g_{\pi_W\pi_W WW} = -\frac{e^2}{4s^2}\,, \qquad \delta g_{\pi_Z\pi_Z WW} = -\frac{e^2}{4s^2}\,,$$
$$\delta g_{\pi_W\pi_W ZZ} = -\frac{e^2}{4s^2c^2}\ . \tag{10.1}$$

Combining these contributions with the three-site couplings given in table 1, we find that the three-site model interactions reduce to those of the two-site model at energies less than M_ρ.

11. Discussion

We have computed the one-loop corrections to the S and T parameters in a highly-deconstructed three site Higgless model. Higgsless models may be considered as dual [37, 38, 39, 40] to models of dynamical symmetry breaking [41, 42] akin to "walking technicolor" [43, 44, 45, 46, 47, 48], and in these terms our calculation is the first to compute the subleading $1/N$ corrections to the S and T parameters. We find that the chiral-logarithmic corrections naturally separate into a model-independent part arising from scaling below the ρ mass, which has the same form as the large Higgs-mass dependence of the S or T parameter in the standard model, and a second model-dependent contribution arising from scaling between the ρ mass and the cutoff

[14] See Figs. 2 and 3 of ref. [55].

of the model. The former allows us to correctly interpret the phenomenologically derived limits on the S and T parameters (in terms of a "reference" Higgs-boson mass) in this three-site Higgsless model. We also discussed the reduction of the model to the "two-site" model, which is the usual electroweak chiral lagrangian, noting the "non-decoupling" contributions present in the limit $M_\rho \to \infty$.

Our analysis has focused on contributions to the S and T parameters from the extended electroweak gauge sector. In principle, there would also be contributions from the extended fermion sector of the model. We calculated these contributions[2] to αT in the three-site model [27] and demonstrated that they are sizable enough to place strong lower bounds on the masses of the KK fermions. Specifically, the enlarged fermion sector results from adding fermions with Dirac masses (M) and the bound from αT is $M \geq 1.8$ TeV. The contributions of these Dirac fermions to the S parameter decouple in the large-M limit; the lower bound on M is large enough to render their $\mathcal{O}(M_W^2/16\pi^2 M^2)$ contributions to αS negligible.

In the limit in which the vector fields at sites 0 and 2 are treated as external gauge fields (*i.e.*, not as dynamical fields) the three-site model is equivalent to the "vector limit" [56] (with a=1) of "Hidden Local Symmetry" [51, 52, 53, 54, 55] models of chiral dynamics in QCD. The calculation of one-loop corrections to the S-parameter presented here – in the limit of "brane-localized" fermions, $x_1 = 0$ – is equivalent to those presented in [62, 55]. Note, however, the contributions to the T parameter arise from one-loop diagrams involving the vector-boson at site 2, and cannot be reproduced in the limit that one treats this gauge-boson as external.

In a forthcoming publication [33] we will report the results of a full renormalization-group analysis of the $\mathcal{O}(p^4)$ terms in the three-site Higgsless model effective theory, allowing us to independently confirm the results in Eqns. (1.1) and (1.2), and to express the values of αS and αT in terms of low-energy parameters.

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A. Interactions of Mass Eigenstate Fields

In this appendix, we rewrite the gauge-sector interactions from Section 3 in terms of mass eigenstate fields using Eqns. (4.3) – (4.4) and (4.8) – (4.14), expanding those in powers of x and keeping the terms which give rise to a non-trivial contribution to the gauge boson self-energy functions in the leading log approximation, up to corrections of order $\mathcal{O}(\alpha x^2 M_W^2)$.

A.1 Three-Point Vertices Proportional to Gauge Boson Masses: $\mathcal{L}_{\pi AA}$

Rewriting eqn. (3.34) in terms of mass eigenstate fields yields

$$\begin{aligned}\mathcal{L}_{\pi AA} = i\Bigg[\sum_{n=A,Z,\rho^0} \Bigg\{ &\left(g^{\pi_{W^-}}_{nW^+}\, n_\mu W^{\mu +} + g^{\pi_{W^-}}_{n\rho^+}\, n_\mu \rho^{\mu +}\right)\pi_{W^-} \\ &+ \left(g^{\pi_{\rho^-}}_{nW^+}\, n_\mu W^{\mu +} + g^{\pi_{\rho^-}}_{n\rho^+}\, n_\mu \rho^{\mu +}\right)\pi_{\rho^-}\Bigg\} \\ + \sum_{\alpha,\beta=W^\pm,\rho^\pm}^{\alpha\neq\beta} &\left\{ g^{\pi_Z}_{\alpha\beta}\,\alpha_\mu\beta^\mu\pi_Z + g^{\pi_{\rho^-}}_{\alpha\beta}\,\alpha_\mu\beta^\mu\pi_{\rho^-}\right)\Bigg] + \text{h.c.}\,.\end{aligned} \tag{A.1}$$

The couplings are expressed in terms of the gauge and Goldstone boson wavefunctions given in Eqns.(4.5), (4.11) and (4.20) as

$$g^{\pi_{W^-}}_{n\alpha} = \frac{eM_{\rho^\pm}}{\sqrt{2}s}\left(1+\frac{4s^2-1}{8}x^2\right)\left(\left[v^L_n v^V_\alpha - v^V_n v^L_\alpha\right] v^{(1)}_{\pi_{W^\pm}} - t\cdot v^B_n v^V_\alpha v^{(2)}_{\pi_{W^\pm}}\right)\,, \tag{A.2}$$

$$g^{\pi_{\rho^-}}_{n\alpha} = \frac{eM_{\rho^\pm}}{\sqrt{2}s}\left(1+\frac{4s^2-1}{8}x^2\right)\left(\left[v^L_n v^V_\alpha - v^V_n v^L_\alpha\right] v^{(1)}_{\pi_{\rho^\pm}} - t\cdot v^B_n v^V_\alpha v^{(2)}_{\pi_{\rho^\pm}}\right)\,, \tag{A.3}$$

$$g^{\pi_Z}_{\alpha\beta} = \frac{eM_{\rho^\pm}}{\sqrt{2}s}\left(1+\frac{4s^2-1}{8}x^2\right)\left(v^L_\alpha v^V_\beta - v^V_\alpha v^L_\beta\right) v^{(1)}_{\pi_Z} \qquad (\alpha\neq\beta)\,, \tag{A.4}$$

$$g^{\pi_{\rho^0}}_{\alpha\beta} = \frac{eM_{\rho^\pm}}{\sqrt{2}s}\left(1+\frac{4s^2-1}{8}x^2\right)\left(v^L_\alpha v^V_\beta - v^V_\alpha v^L_\beta\right) v^{(1)}_{\pi_{\rho^0}} \qquad (\alpha\neq\beta)\,, \tag{A.5}$$

for $\alpha,\beta = W^\pm,\rho^\pm$ and $n = A, Z, \rho^0$. In obtaining the expressions for these couplings, we have used Eqn. (4.12) and

$$g_1 f = \sqrt{2}M_{\rho^\pm}\left(1-\frac{x^2}{8}+\cdots\right)\,, \tag{A.6}$$

which follows from Eqn. (4.2). The explicit expressions for each of these couplings is shown in table 2. From table 2, in the isospin symmetric limit $s\to 0$ (or $c\to 1$) with (e/s) fixed, we find

$$\begin{aligned} &g^{\pi_{\rho^-}}_{ZW^+}|_{c\to1} = g^{\pi_{\rho^0}}_{W^+W^-} = 0\,, \qquad g^{\pi_{W^-}}_{\rho^0W^+}|_{c\to1} = g^{\pi_{W^-}}_{Z\rho^+}|_{c\to1} = g^{\pi_Z}_{W^-\rho^+}|_{c\to1}\,,\\ &g^{\pi_{\rho^-}}_{\rho^0W^+}|_{c\to1} = g^{\pi_{\rho^+}}_{Z\rho^-}|_{c\to1} = g^{\pi_{\rho^0}}_{W^-\rho^+}|_{c\to1}\,, \end{aligned} \tag{A.7}$$

as expected.

n	$g^{\pi_{W^-}}_{nW^+}$	$g^{\pi_{\rho^-}}_{nW^+}$
A	$-eM_W$	0
Z	$\frac{esM_W}{c}$	$\frac{eM_W x^2}{8sc}\left(1-\frac{1}{c^2}\right)$
ρ^0	$-\frac{eM_{\rho^\pm}}{2s}\left(1-\frac{x^2(4c^2-4+\frac{3}{c^2})}{8}\right)$	$\frac{eM_{\rho^\pm}}{2s}\left(1+\frac{x^2(-4c^2+4+\frac{1}{c^2})}{8}\right)$

n	$g^{\pi_{W^-}}_{n\rho^+}$	$g^{\pi_{\rho^-}}_{n\rho^+}$
A	0	$-eM_{\rho^\pm}$
Z	$\frac{eM_{\rho^\pm}}{2sc}\left(1-\frac{x^2(8c^2-6+\frac{1}{c^2})}{8}\right)$	$-\frac{e(c^2-s^2)M_{\rho^\pm}}{2sc}\left(1+\frac{x^2}{8(c^2-s^2)c^2}\right)$
ρ^0	$\frac{esM_W}{2c^2}$	$\frac{esM_W}{2c^2}$

$g^{\pi_Z}_{\alpha\beta}$	$\beta = W^+$	$\beta = \rho^+$
$\alpha = W^-$	0	$\frac{eM_{\rho^\pm}}{2s}\left(1-\frac{x^2(4c^2+1-\frac{2}{c^2})}{8}\right)$
$\alpha = \rho^-$	$-\frac{eM_{\rho^\pm}}{2s}\left(1-\frac{x^2(4c^2+1-\frac{2}{c^2})}{8}\right)$	0

$g^{\pi_{\rho^0}}_{\alpha\beta}$	$\beta = W^+$	$\beta = \rho^+$
$\alpha = W^-$	0	$-\frac{eM_{\rho^\pm}}{2s}\left(1+\frac{x^2(-4c^2+7-\frac{2}{c^2})}{8}\right)$
$\alpha = \rho^-$	$\frac{eM_{\rho^\pm}}{2s}\left(1+\frac{x^2(-4c^2+7-\frac{2}{c^2})}{8}\right)$	0

Table 2: The Goldstone boson couplings proportional to gauge boson masses.

The interaction vertices in eqn. (A.1) include terms that explicitly mix the standard model and new-physics sectors of the model. The second term in line one of Eqn. (A.1) includes an interaction ($Z\rho\pi_W$) contributing to diagram $(N)_{ZZ}$ of Fig. 2. The first term of line one includes an interaction ($W\rho\pi_W$) contributing to diagram $(M)_{WW}$ of Fig. 3; the first term of line three includes a $W\rho\pi_Z$ interaction contributing to diagram $(O)_{WW}$ of the same figure. All three of these terms contribute to diagrams whose amplitudes are explicitly found to have non-decoupling contributions proportional to $\ln(M_\rho^2/M_W^2)$, unsuppressed by factors of $1/M_\rho^2$.

A.2 Three-Point Vertices Dependent on Derivatives: $\mathcal{L}_{\pi\pi A}$

Rewriting Eqn. (3.35) in terms of mass eigenstate fields yields

$$
\begin{aligned}
\mathcal{L}_{\pi\pi A} = i\Bigg\{ & \sum_{n=A,Z,\rho^0} n_\mu \Bigg[g^n_{\pi_{W^+}\pi_{W^-}} \partial^\mu \pi_{W^+}\pi_{W^-} + g^n_{\pi_{\rho^+}\pi_{\rho^-}} \partial^\mu \pi_{\rho^+}\pi_{\rho^-} \\
& + g^n_{\pi_{W^+}\pi_{\rho^-}} \left(\partial^\mu \pi_{W^+}\pi_{\rho^-} + \partial^\mu \pi_{\rho^+}\pi_{W^-} \right) \Bigg] \\
& + \sum_{\alpha=W^\pm,\rho^\pm} \alpha^\mu \Bigg[g^\alpha_{\pi_Z\pi_{W^-}} \left(\pi_Z \overleftrightarrow{\partial}_\mu \pi_{W^-} \right) + g^\alpha_{\pi_{\rho^0}\pi_{\rho^-}} \left(\pi_{\rho^0} \overleftrightarrow{\partial}_\mu \pi_{\rho^-} \right) \\
& + g^\alpha_{\pi_Z\pi_{\rho^-}} \left(\pi_Z \overleftrightarrow{\partial}_\mu \pi_{\rho^-} \right) + g^\alpha_{\pi_{\rho^0}\pi_{W^-}} \left(\pi_{\rho^0} \overleftrightarrow{\partial}_\mu \pi_{W^-} \right) \Bigg] \Bigg\} \\
& + \text{h.c.}\,. \qquad \text{(A.8)}
\end{aligned}
$$

The couplings are expressed in terms of the gauge and Goldstone boson wavefunctions given in Eqns.(4.5), (4.11) and (4.20) as

$$g^n_{\pi_\alpha\pi_\beta} = \frac{e}{2s}\left(1+\frac{s^2}{2}x^2\right)\left[v^{(1)}_{\pi_\alpha} v^{(1)}_{\pi_\beta} (v^L_n + \frac{1}{x} v^V_n) + v^{(2)}_{\pi_\alpha} v^{(2)}_{\pi_\beta} (t \cdot v^B_n + \frac{1}{x} v^V_n) \right], \qquad \text{(A.9)}$$

$$g^\alpha_{\pi_Z\pi_\beta} = -\frac{e}{2s}\left(1+\frac{s^2}{2}x^2\right)\left[v^{(1)}_{\pi_Z} v^{(1)}_{\pi_\beta} (v^L_\alpha + \frac{1}{x} v^V_\alpha) + \frac{1}{x} v^{(2)}_{\pi_Z} v^{(2)}_{\pi_\beta} v^V_\alpha \right], \qquad \text{(A.10)}$$

$$g^\alpha_{\pi_{\rho^0}\pi_\beta} = -\frac{e}{2s}\left(1+\frac{s^2}{2}x^2\right)\left[v^{(1)}_{\pi_{\rho^0}} v^{(1)}_{\pi_\beta} (v^L_\alpha + \frac{1}{x} v^V_\alpha) + \frac{1}{x} v^{(2)}_{\pi_{\rho^0}} v^{(2)}_{\pi_\beta} v^V_\alpha \right], \qquad \text{(A.11)}$$

for $\alpha, \beta = W^\pm, \rho^\pm$ and $n = A, Z, \rho^0$. In obtaining the expressions for these couplings, we have used Eqn. (4.12). The explicit expression for each of these couplings is shown in table 3. From table 3, in the isospin symmetric limit $s \to 0$ (or $c \to 1$) with (e/s) fixed, we find

$$
\begin{aligned}
g^Z_{\pi_{W^+}\pi_{W^-}}|_{c\to 1} &= g^{W^+}_{\pi_Z\pi_{W^-}}|_{c\to 1}\,, \qquad g^{W^+}_{\pi_Z\pi_{\rho^-}}|_{c\to 1} = g^Z_{\pi_{W^+}\pi_{\rho^-}}|_{c\to 1} = g^{W^+}_{\pi_{\rho^0}\pi_{W^-}}|_{c\to 1}\,, \\
g^Z_{\pi_{\rho^+}\pi_{\rho^-}}|_{c\to 1} &= g^{W^+}_{\pi_{\rho^0}\pi_{\rho^-}}|_{c\to 1}\,, \qquad g^{\rho^0}_{\pi_{W^+}\pi_{\rho^-}}|_{c\to 1} = g^{\rho^+}_{\pi_Z\pi_{\rho^-}} = g^{\rho^+}_{\pi_{\rho^0}\pi_{W^-}}|_{c\to 1}\,, \qquad \text{(A.12)}
\end{aligned}
$$

up to an overall sign, as expected.

The interaction vertices in Eqn. (A.8) include terms that explicitly mix the standard model and new-physics sectors of the model. The terms on line two of (A.8) include interactions $(Z\pi_W\pi_\rho)$ contributing to diagram $(M)_{ZZ}$ of Fig. 2; the terms of line four include $W\pi_Z\pi_\rho$ and $W\pi_\rho\pi_W$ interactions contributing, respectively, to diagrams $(R)_{WW}$ and $(S)_{WW}$ of Fig. 3.

A.3 Four-Point Vertices: $\mathcal{L}_{\pi\pi AA}$

Rewriting Eqn. (3.36) in terms of mass eigenstate fields yields

n	$g^n_{\pi_{W^+}\pi_{W^-}}$	$g^n_{\pi_{W^+}\pi_{\rho^-}}$	$g^n_{\pi_{\rho^+}\pi_{\rho^-}}$
A	e	0	e
Z	$\frac{e(c^2-s^2)}{2sc}$	$-\frac{e}{4sc}\left(1+\frac{x^2(-8c^2+8-\frac{1}{c^2})}{8}\right)$	$\frac{e(c^2-s^2)}{2sc}\left(1+\frac{x^2}{4(c^2-s^2)}\right)$
ρ^0	$\frac{e}{2sx}$	$\frac{e(c^2-s^2)x}{8sc^2}$	$\frac{e}{2sx}$

$g^\alpha_{\pi_Z\pi_\beta}$	$\beta=W^-$	$\beta=\rho^-$
$\alpha=W^+$	$-\frac{e}{2s}$	$\frac{e}{4s}\left(1+\frac{x^2(-4c^2-1+\frac{4}{c^2})}{8}\right)$
$\alpha=\rho^+$	irrelevant	$-\frac{e(c^2-s^2)x}{8sc^2}$

$g^\alpha_{\pi_{\rho^0}\pi_\beta}$	$\beta=W^-$	$\beta=\rho^-$
$\alpha=W^+$	$\frac{e}{4s}\left(1+\frac{x^2(-4c^2+7-\frac{4}{c^2})}{8}\right)$	$-\frac{e}{2s}\left(1+\frac{x^2(-4c^2+7-\frac{1}{c^2})}{8}\right)$
$\alpha=\rho^+$	$-\frac{ex}{8sc^2}$	$-\frac{e}{2sx}$

Table 3: The Goldstone boson couplings dependent on derivatives. The expression of the $g^{\rho^+}_{\pi_Z\pi_W^-}$ coupling is not shown (denoted as "irrelevant") since the vertex constructed from this coupling does not contribute to the self-energy functions of the standard model gauge bosons W, Z and photon.

$$
\begin{aligned}
\mathcal{L}_{\pi\pi AA} = \; & \pi_{W^+}\pi_{W^-}\left(\sum_{n=A,Z,\rho^0} g^{nn}_{\pi_{W^+}\pi_{W^-}} n_\mu n^\mu + \sum_{n=A,Z,\rho^0}^{n\neq m} g^{nm}_{\pi_{W^+}\pi_{W^-}} n_\mu m^\mu\right) \\
& +\pi_{\rho^+}\pi_{\rho^-}\left(\sum_{n=A,Z,\rho^0} g^{nn}_{\pi_{\rho^+}\pi_{\rho^-}} n_\mu n^\mu + \sum_{n,m=A,Z,\rho^0}^{n\neq m} g^{nm}_{\pi_{\rho^+}\pi_{\rho^-}} n_\mu m^\mu\right) \\
& +\pi_{W^+}\pi_{W^-}\left(\sum_{\alpha=W^\pm,\rho^\pm} g^{\alpha\alpha}_{\pi_{W^+}\pi_{W^-}} \alpha_\mu\alpha^\mu + \sum_{\alpha,\beta=W^\pm,\rho^\pm}^{\alpha\neq\beta} g^{\alpha\beta}_{\pi_{W^+}\pi_{W^-}} \alpha_\mu\beta^\mu\right) \\
& +\pi_Z\pi_Z\left(\sum_{\alpha=W^\pm,\rho^\pm} g^{\alpha\alpha}_{\pi_Z\pi_Z} \alpha_\mu\alpha^\mu + \sum_{\alpha,\beta=W^\pm,\rho^\pm}^{\alpha\neq\beta} g^{\alpha\beta}_{\pi_Z\pi_Z} \alpha_\mu\beta^\mu\right) \\
& +\pi_{\rho^0}\pi_{\rho^0}\left(\sum_{\alpha=W^\pm,\rho^\pm} g^{\alpha\alpha}_{\pi_{\rho^0}\pi_{\rho^0}} \alpha_\mu\alpha^\mu + \sum_{\alpha,\beta=W^\pm,\rho^\pm}^{\alpha\neq\beta} g^{\alpha\beta}_{\pi_{\rho^0}\pi_{\rho^0}} \alpha_\mu\beta^\mu\right). \qquad \text{(A.13)}
\end{aligned}
$$

These couplings are expressed in terms of the gauge and Goldstone boson wavefunctions given in Eqns. (4.5), (4.11) and (4.20) as

$$g^{nn}_{\pi_\alpha\pi_\alpha} = \frac{e^2}{s^2x}\left(1+s^2x^2\right)\left[v^{(1)}_{\pi_\alpha}v^{(1)}_{\pi_\alpha}v^L_n v^V_n + t\cdot v^{(2)}_{\pi_\alpha}v^{(2)}_{\pi_\alpha}v^B_n v^V_n\right], \tag{A.14}$$

$$g^{nm}_{\pi_\alpha\pi_\alpha} = \frac{e^2}{s^2x}\left(1+s^2x^2\right)\left[v^{(1)}_{\pi_\alpha}v^{(1)}_{\pi_\alpha}(v^L_n v^V_m + v^V_n v^L_m) + t\cdot v^{(2)}_{\pi_{\rho^+}}v^{(2)}_{\pi_\alpha}(v^B_n v^V_m + v^V_n v^B_m)\right], \tag{A.15}$$

$$g^{\beta\beta}_{\pi_\alpha\pi_\alpha} = \frac{e^2}{s^2x}\left(1+s^2x^2\right)v^{(1)}_{\pi_\alpha}v^{(1)}_{\pi_\alpha}v^L_\beta v^V_\beta, \tag{A.16}$$

$$g^{\beta\gamma}_{\pi_\alpha\pi_\alpha} = \frac{e^2}{2s^2x}\left(1+s^2x^2\right)v^{(1)}_{\pi_\alpha}v^{(1)}_{\pi_\alpha}\left(v^L_\beta v^V_\gamma + v^L_\gamma v^V_\beta\right), \qquad \text{for } \beta\neq\gamma \tag{A.17}$$

$$g^{\beta\beta}_{\pi_Z\pi_Z} = \frac{e^2}{s^2x}\left(1+s^2x^2\right)v^{(1)}_{\pi_Z}v^{(1)}_{\pi_Z}v^L_\beta v^V_\beta, \tag{A.18}$$

$$g^{\beta\gamma}_{\pi_Z\pi_Z} = \frac{e^2}{2s^2x}\left(1+s^2x^2\right)v^{(1)}_{\pi_Z}v^{(1)}_{\pi_Z}\left(v^L_\beta v^V_\gamma + v^L_\gamma v^V_\beta\right), \qquad \text{for } \beta\neq\gamma \tag{A.19}$$

$$g^{\beta\beta}_{\pi_{\rho^0}\pi_{\rho^0}} = \frac{e^2}{s^2x}\left(1+s^2x^2\right)v^{(1)}_{\pi_{\rho^0}}v^{(1)}_{\pi_{\rho^0}}v^L_\beta v^V_\beta, \tag{A.20}$$

$$g^{\beta\gamma}_{\pi_{\rho^0}\pi_{\rho^0}} = \frac{e^2}{2s^2x}\left(1+s^2x^2\right)v^{(1)}_{\pi_{\rho^0}}v^{(1)}_{\pi_{\rho^0}}\left(v^L_\beta v^V_\gamma + v^L_\gamma v^V_\beta\right), \qquad \text{for } \beta\neq\gamma, \tag{A.21}$$

where subscripts α,β and γ denote $W^\pm$ and $\rho^\pm$ and n does A, Z and ρ^0. In obtaining the expressions for these couplings, we have used Eqn. (4.12). The explicit expression for each of these couplings is shown in table 4. From table 4, in the isospin symmetric limit $s\to 0$ (or $c\to 1$) with (e/s) fixed, we find

$$\begin{aligned} g^{ZZ}_{\pi_{\rho^+}\pi_{\rho^-}}|_{c\to1} &= g^{W^+W^-}_{\pi_{\rho^+}\pi_{\rho^-}}|_{c\to1} = g^{W^+W^-}_{\pi_{\rho^0}\pi_{\rho^0}}|_{c\to1}, \\ g^{\rho^0Z}_{\pi_{\rho^+}\pi_{\rho^-}}|_{c\to1} &= 2g^{W^+\rho^-}_{\pi_{\rho^+}\pi_{\rho^-}}|_{c\to1} = 2g^{W^+\rho^-}_{\pi_{\rho^0}\pi_{\rho^0}}|_{c\to1}, \end{aligned} \tag{A.22}$$

up to an overall sign, as expected.

A.4 Three-Point Vertices among the Gauge Bosons: $\mathcal{L}_{AAA}$

Rewriting Eqn. (3.29) in terms of mass eigenstate fields yields

$$\begin{aligned} \mathcal{L}_{AAA} = i\Bigg\{ &\sum_{n=A,Z,\rho^0} g^n_{W^+W^-}\left(W^+_{\mu\nu}W^{\mu-}n^\nu + \frac{1}{2}n_{\mu\nu}W^{+\mu}W^{-\nu}\right) \\ &+g^n_{W^+\rho^-}\left(\left(W^+_{\mu\nu}\rho^{\mu-} + \rho^+_{\mu\nu}W^{\mu-}\right)n^\nu + \frac{1}{2}n_{\mu\nu}\left(W^{\mu+}\rho^{\nu-} + \rho^{\mu+}W^{\nu-}\right)\right) \\ &+g^n_{\rho^+\rho^-}\left(\rho^+_{\mu\nu}\rho^{\mu-}n^\nu + \frac{1}{2}n_{\mu\nu}\rho^{+\mu}\rho^{-\nu}\right)\Bigg\} + \text{h.c.}, \end{aligned} \tag{A.23}$$

$g^{mn}_{\pi_{W^+}\pi_{W^-}}$	$n=A$	$n=Z$	$n=\rho^0$
$m=A$	e^2	$\frac{e^2(c^2-s^2)}{sc}$	$\frac{e^2(c^2-s^2)^2}{4s^2c^2}$
$m=Z$	$\frac{e^2(c^2-s^2)}{sc}$	$\frac{e^2(c^2-s^2)^2}{4s^2c^2}$	irrelevant
$m=\rho^0$	irrelevant	irrelevant	irrelevant

$g^{mn}_{\pi_{\rho^+}\pi_{\rho^-}}$	$n=A$	$n=Z$	$n=\rho^0$
$m=A$	e^2	$\frac{e^2(c^2-s^2)}{sc}\left(1+\frac{x^2}{4(c^2-s^2)}\right)$	$\frac{e^2(c^2-s^2)^2}{4s^2c^2}\left(1+\frac{x^2}{2(c^2-s^2)}\right)$
$m=Z$	$\frac{e^2(c^2-s^2)}{sc}\left(1+\frac{x^2}{4(c^2-s^2)}\right)$	$\frac{e^2(c^2-s^2)^2}{4s^2c^2}\left(1+\frac{x^2}{2(c^2-s^2)}\right)$	$\frac{e^2(c^2-s^2)}{2s^2cx}$
$m=\rho^0$	$\frac{e^2}{sx}$	$\frac{e^2(c^2-s^2)}{2s^2cx}$	irrelevant

$g^{\alpha\beta}_{\pi_{W^+}\pi_{W^-}}$	$\beta=W^-$	$\beta=\rho^-$
$\alpha=W^+$	$\frac{e^2}{4s^2}$	irrelevant
$\alpha=\rho^+$	irrelevant	irrelevant

$g^{\alpha\beta}_{\pi_{\rho^+}\pi_{\rho^-}}$	$\beta=W^-$	$\beta=\rho^-$
$\alpha=W^+$	$\frac{e^2}{4s^2}\left(1+\frac{x^2(-2c^2+3)}{2}\right)$	$\frac{e^2}{4s^2x}$
$\alpha=\rho^+$	$\frac{e^2}{4s^2x}$	irrelevant

$g^{\alpha\beta}_{\pi_Z\pi_Z}$	$\beta=W^-$	$\beta=\rho^-$
$\alpha=W^+$	$\frac{e^2}{4s^2}$	irrelevant
$\alpha=\rho^+$	irrelevant	irrelevant

$g^{\alpha\beta}_{\pi_{\rho^0}\pi_{\rho^0}}$	$\beta=W^-$	$\beta=\rho^-$
$\alpha=W^+$	$\frac{e^2}{4s^2}\left(1+\frac{x^2(-2c^2+4-\frac{1}{c^2})}{2}\right)$	$\frac{e^2}{4s^2x}$
$\alpha=\rho^+$	$\frac{e^2}{4s^2x}$	irrelevant

Table 4: The Goldstone boson couplings between two gauge bosons and two Goldstone bosons. The vertices denoted as "irrelevant" do not generate leading log corrections to the gauge boson self-energy functions we are concerned with.

where $\mathcal{A}_{\mu\nu} = \partial_\mu \mathcal{A}_\mu - \partial_\nu \mathcal{A}_\mu$ ($\mathcal{A} = A, Z, \rho^{\pm,0}, W^\pm$). These couplings are expressed by using the wavefunctions given in Eqns.(4.5) and (4.11) as

$$g^n_{W^+W^-} = \frac{e}{s}\left(1+\frac{1}{2}s^2x^2\right)\left[(v^L_W)^2 v^L_n + \frac{1}{x}(v^V_W)^2 v^V_n\right], \tag{A.24}$$

$$g^n_{W^+\rho^-} = \frac{e}{s}\left(1+\frac{1}{2}s^2x^2\right)\left[v^L_W v^L_{\rho^\pm} v^L_n + \frac{1}{x}v^V_W v^V_{\rho^\pm} v^V_n\right], \tag{A.25}$$

$$g^n_{\rho^+\rho^-} = \frac{e}{s}\left(1+\frac{1}{2}s^2x^2\right)\left[(v^L_{\rho^\pm})^2 v^L_n + \frac{1}{x}(v^V_{\rho^\pm})^2 v^V_n\right], \tag{A.26}$$

for $n = A, Z, \rho$. In obtaining the expressions for these couplings, we have used Eqn. (4.12). The explicit expression for each of these couplings is shown in table 5. From table 5, in the isospin symmetric limit $s \to 0$ (or $c \to 1$) with (e/s) fixed, we find

$$g^Z_{\rho^+\rho^-}|_{c\to 1} = g^{\rho^0}_{W^+\rho^-}|_{c\to 1}\,, \qquad g^{\rho^0}_{W^+W^-}|_{c\to 1} = g^Z_{W^+\rho^-}|_{c\to 1}\,, \tag{A.27}$$

as expected.

n	$g^n_{W^+W^-}$	$g^n_{W^+\rho^-}$	$g^n_{\rho^+\rho^-}$
A	e	0	e
Z	$\frac{ec}{s}$	$-\frac{ex}{4sc}$	$\frac{e(c^2-s^2)}{2sc}\left(1+\frac{x^2(4-\frac{1}{c^2})}{8(c^2-s^2)}\right)$
ρ^0	- $\frac{ex}{4s}$	$\frac{e}{2s}\left(1+\frac{x^2(-4c^2+8-\frac{1}{c^2})}{8}\right)$	$\frac{e}{sx}$

Table 5: The three-point couplings among the gauge fields.

A.5 Four-Point Vertices among the Gauge Bosons: $\mathcal{L}_{AAAA}$

Rewriting Eqn. (3.30) in terms of the mass eigenstate fields yields

$$\begin{aligned}
\mathcal{L}_{AAAA} = & \sum_{n=A,Z,\rho^0} \Big\{ g^{nn}_{W^+W^-}(W^+_\mu W^-_\nu n^\mu n^\nu - W^+_\mu W^-_\mu n^\nu m^\nu) \\
& \qquad + g^{nn}_{\rho^+\rho^-}(\rho^+_\mu \rho^-_\nu n^\mu n^\nu - \rho^+_\mu \rho^-_\mu n^\nu m^\nu) \Big\} \\
& + \sum_{n,m=A,Z,\rho^0}^{n\neq m} \Big\{ g^{nm}_{W^+W^-} \left(W^+_\mu W^-_\nu (n^\mu m^\nu + m^\mu n^\nu) - 2W^+_\mu W^-_\mu n^\nu m^\nu \right) \\
& \qquad + g^{nm}_{\rho^+\rho^-} \left(\rho^+_\mu \rho^-_\nu (n^\mu m^\nu + m^\mu n^\nu) - 2\rho^+_\mu \rho^-_\mu n^\nu m^\nu \right) \Big\} \\
& + g^{\rho^0\rho^0}_{W^+\rho^-} \left(\rho^0_\mu \rho^0_\nu W_{\mu+} \rho^{\nu-} - \rho^0_\mu \rho^{\mu 0} W^+_\nu \rho^{\nu-} + \text{h.c.} \right) \\
& + g^{\rho^+\rho^-}_{W^+\rho^-} \left(\rho^+_\mu \rho^-_\nu W^{\mu+} \rho^{\nu-} - \rho^+_\mu \rho^{\mu-} W^+_\nu \rho^{\nu-} + \text{h.c.} \right) \\
& + g^{\rho^+\rho^-}_{W^+W^-} \left(2W^+_\mu W^-_\nu \rho^{\mu+} \rho^{\nu-} - W^+_\mu W^-_\nu \rho^{\mu-} \rho^{\nu+} - W^+_\mu W^{\nu-} \rho^+_\nu \rho^{\nu-} \right) \\
& + g^{W^+W^-}_{W^+W^-} W^+_\mu W^-_\nu \left(W^{\mu+} W^{\nu-} - W^{\mu-} W^{\nu+} \right) , \qquad \text{(A.28)}
\end{aligned}$$

where we have neglected terms which do not affect the gauge boson self-energy functions at the one-loop level. These relevant couplings are expressed by using the wavefunctions given

$g^{mn}_{W^+W^-}$	$n=A$	$n=Z$	$n=\rho^0$
$m=A$	e^2	$\frac{e^2c}{s}$	irrelevant
$m=Z$	$\frac{e^2c}{s}$	$\frac{e^2c^2}{s^2}$	irrelavant
$m=\rho^0$	irrelevant	irrelevant	$\frac{e^2}{4s^2}\left(1+\frac{x^2(-4c^2+9-\frac{1}{c^2})}{4}\right)$

$g^{mn}_{\rho^+\rho^-}$	$n=A$	$n=Z$
$m=A$	e^2	$\frac{e^2(c^2-s^2)}{2sc}\left(1+\frac{x^2(4-\frac{1}{c^2})}{8(c^2-s^2)}\right)$
$m=Z$	$\frac{e^2(c^2-s^2)}{2sc}\left(1+\frac{x^2(4-\frac{1}{c^2})}{8(c^2-s^2)}\right)$	$\frac{e^2(c^2-s^2)^2}{4s^2c^2}\left(1+\frac{x^2(8c^2-5+\frac{1}{c^2})}{4(c^2-s^2)^2}\right)$
$m=\rho^0$	$\frac{e^2}{sx}$	$\frac{e^2(c^2-s^2)}{2s^2cx}$

$$g^{\rho^0\rho^0}_{W^+\rho^-}=\frac{e^2}{2s^2x}\,,\quad g^{\rho^+\rho^-}_{W^+\rho^-}=\frac{e^2}{2s^2x}\,,\quad g^{\rho^+\rho^-}_{W^+W^-}=\frac{e^2}{4s^2}\left(1+(2-c^2)x^2\right)\,,\quad g^{W^+W^-}_{W^+W^-}=\frac{e^2}{2s^2}\,.$$

Table 6: The four-point couplings among the gauge fields. The vertices denoted as "irrelevant" do not generate leading log corrections to the gauge boson self-energy functions.

in Eqns.(4.5) and (4.11) as

$$g^{nm}_{W^+W^-}=\frac{e^2}{s^2}\left(1+s^2x^2\right)\left[(v^L_{W^\pm})^2v^L_nv^L_m+\frac{1}{x^2}(v^V_{W^\pm})^2v^V_nv^V_m\right]\,, \tag{A.29}$$

$$g^{nm}_{\rho^+\rho^-}=\frac{e^2}{s^2}\left(1+s^2x^2\right)\left[(v^L_{\rho^\pm})^2v^L_nv^L_m+\frac{1}{x^2}(v^V_{\rho^\pm})^2v^V_nv^V_m\right]\,, \tag{A.30}$$

$$g^{\rho^0\rho^0}_{W^+\rho^-}=\frac{e^2}{s^2}\left(1+s^2x^2\right)\left[v^L_{W^\pm}v^L_{\rho^\pm}(v^L_{\rho^0})^2+\frac{1}{x^2}v^V_{W^\pm}v^V_{\rho^\pm}(v^V_{\rho^0})^2\right]\,, \tag{A.31}$$

$$g^{\rho^+\rho^-}_{W^+\rho^-}=\frac{e^2}{s^2}\left(1+s^2x^2\right)\left[v^L_{W^\pm}(v^L_{\rho^\pm})^3+\frac{1}{x^2}v^V_{W^\pm}(v^V_{\rho^\pm})^3\right]\,, \tag{A.32}$$

$$g^{\rho^+\rho^-}_{W^+W^-}=\frac{e^2}{s^2}\left(1+s^2x^2\right)\left[(v^L_{W^\pm})^2(v^L_{\rho^\pm})^2+\frac{1}{x^2}(v^V_{W^\pm})^2(v^V_{\rho^\pm})^2\right]\,, \tag{A.33}$$

$$g^{W^+W^-}_{W^+W^-}=\frac{e^2}{2s^2}\left(1+s^2x^2\right)\left[(v^L_{W^\pm})^4+\frac{1}{x^2}(v^V_{W^\pm})^4\right]\,, \tag{A.34}$$

for $m,n=A,Z,\rho$. In obtaining the expressions for these couplings, we have used Eqn. (4.12). The explicit expression for each of these couplings is shown in table 6. From table 6, in the isospin symmetric limit $s\to 0$ (or $c\to 1$) with (e/s) fixed, we find

$$\begin{aligned} g^{\rho^0\rho^0}_{W^+W^-}|_{c\to 1} &= g^{\rho^+\rho^-}_{W^+W^-}|_{c\to 1}=g^{\rho^+\rho^-}_{ZZ}|_{c\to 1}\,,\\ g^{\rho^0 Z}_{\rho^+\rho^-}|_{c\to 1} &= g^{\rho^0\rho^0}_{W^+\rho^-}=g^{\rho^+\rho^-}_{W^+\rho^-}\,, \end{aligned} \tag{A.35}$$

as expected.

A.6 FP Ghost Terms in the Mass Eigenstate Basis

Rewriting Eqn. (3.28) in terms of mass eigenstate fields yields

$$\mathcal{L}_{FP}^{\text{int}} = i \sum_{n=A,Z,\rho^0} \Bigg[g^n_{W^+W^-} n_\mu \partial^\mu \bar{C}_{W^+} C_{W^-} + g^n_{\rho^+\rho^-} n_\mu \partial^\mu \bar{C}_{\rho^+} C_{\rho^-} + g^n_{W^+\rho^-} n_\mu (\partial^\mu \bar{C}_{W^+} C_{\rho^-} + \partial^\mu \bar{C}_{\rho^+} C_{W^-}) \Bigg] + \text{h.c.} \,. \tag{A.36}$$

Note that these couplings are equal to the three-point couplings among the gauge bosons listed in table 5 due to gauge invariance.

B. Feynman Integral Formulae

We define the following Feynman integrals:

$$iF_1^{\mu\nu}(M_A, M_B; p^2) \equiv \int \frac{d^4k}{(2\pi)^4} \frac{(2k+p)^\mu (2k+p)^\nu}{\left[k^2 - M_A^2\right]\left[(k+p)^2 - M_B^2\right]} \,, \tag{B.1}$$

$$iF_2(M_A, M_B; p^2) \equiv \int \frac{d^4k}{(2\pi)^4} \frac{1}{\left[k^2 - M_A^2\right]\left[(k+p)^2 - M_B^2\right]} \,, \tag{B.2}$$

$$iF_3(M) \equiv \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - M^2)} \,. \tag{B.3}$$

By introducing Feynman parameters, and performing dimensional regularization, these integrals are evaluated as

$$(4\pi)^2 F_1^{\mu\nu}(M_A, M_B; p^2) = g^{\mu\nu} \Bigg[\left(M_A^2 + M_B^2 - \frac{1}{3}p^2 \right) \cdot \left(\frac{1}{\bar{\epsilon}} + 1 \right) - 2\int_0^1 dx \Delta_A^B(p^2) \log \Delta_A^B(p^2) \Bigg] + (p^\mu p^\nu \text{ term}) \,, \tag{B.4}$$

$$(4\pi)^2 F_2(M_A, M_B; p^2) = \frac{1}{\bar{\epsilon}} - \int_0^1 dx \log \Delta_A^B(p^2) \,, \tag{B.5}$$

$$(4\pi)^2 F_3(M) = M^2 \cdot \frac{1}{\bar{\epsilon}} - \left(M^2 \log M^2 + M^2 \right) \,, \tag{B.6}$$

where

$$\Delta_A^B(p^2) = x M_A^2 + (1-x) M_B^2 - x(1-x) p^2 \,, \tag{B.7}$$

$$\frac{1}{\bar{\epsilon}} = \frac{2}{\epsilon} - \gamma + \log 4\pi \,. \tag{B.8}$$

Interpreting the results in terms of a dimensional cutoff representing the cutoff of the effective theory, we make the replacement

$$\frac{1}{\bar{\epsilon}} \to \log \Lambda^2 \,. \tag{B.9}$$

Equivalently, the replacement above may be viewed as evaluating the counterterms, which cancel divergences, renormalized at the scale of the cutoff.

In applying these formulae to extract the leading-log corrections to the S and T parameters arising from loops of ρ-mesons, we encounter two separate cases. In one case, the loop-corrections involve only propagating ρ-mesons . We refer to this as the degenerate heavy mass case, in which $M_A^2 \simeq M_B^2 \gg |p^2| = \mathcal{O}(M_W^2)$. In the other case – which we refer to as the hierarchical mass case – loop-corrections involve one ρ-meson and one light gauge-boson, and $M_A^2 \gg M_B^2 \simeq |p^2| = \mathcal{O}(M_W^2)$.

We first examine the degenerate heavy mass case in which, from Eqns. (B.4)-(B.6), we find the approximate formulae,

$$
\begin{aligned}
(4\pi)^2 F_1^{\mu\nu}(M_A, M_B; p^2) &\simeq g^{\mu\nu}\left[\left(M_A^2 + M_B^2 - \frac{1}{3}p^2\right)\log\frac{\Lambda^2}{M_A^2}\right. \\
&\qquad \left. + M_A^2 + M_B^2 + \frac{1}{3}p^2 + \mathcal{O}(p^4/M_A^2)\right] \\
&\qquad + (p^\mu p^\nu \text{ term}) , \qquad \text{(B.10)} \\
(4\pi)^2 F_2(M_A, M_B; p^2) &\simeq \log\frac{\Lambda^2}{M_A^2} + \frac{1}{6}\frac{p^2}{M_A^2} + \mathcal{O}(p^4/M_A^4) , \qquad \text{(B.11)} \\
(4\pi)^2 F_3(M_A) &= M_A^2 \log\frac{\Lambda^2}{M_A^2} - M_A^2 . \qquad \text{(B.12)}
\end{aligned}
$$

Consider next the hierarchical mass case. In this case, the expression for the function F_3 remains the same as in Eqn. (B.12). In order to derive the approximate formulae for the functions $F_1^{\mu\nu}$ and F_2, we may rewrite the expressions (B.4) and (B.5) as follows:

$$
\begin{aligned}
(4\pi)^2 F_1^{\mu\nu}(M_A, M_B; p^2) &= g^{\mu\nu}\left[\left(M_A^2 + M_B^2 - \frac{1}{3}p^2\right)\log\frac{\Lambda^2}{M_A^2} + M_A^2 + M_B^2 - \frac{1}{3}p^2\right. \\
&\qquad -2\int_0^1 dx\,\left(xM_A^2 + (1-x)M_B^2 - x(1-x)p^2\right) \\
&\qquad\qquad \left.\times \log\left(x + (1-x)\frac{M_B^2}{M_A^2} - x(1-x)\frac{p^2}{M_A^2}\right)\right] \\
&\qquad + (p^\mu p^\nu \text{ term}) , \qquad \text{(B.13)} \\
(4\pi)^2 F_2(M_A, M_B; p^2) &= \log\frac{\Lambda^2}{M_A^2} - \int_0^1 dx \log\left(x + (1-x)\frac{M_B^2}{M_A^2} - x(1-x)\frac{p^2}{M_A^2}\right) . \\
&\qquad \text{(B.14)}
\end{aligned}
$$

Expanding the right hand sides in terms of $1/M_A^2$, we find

$$(4\pi)^2 F_1^{\mu\nu}(M_A, M_B; p^2) = g^{\mu\nu}\left[\left(M_A^2 + M_B^2 - \frac{1}{3}p^2\right)\log\frac{\Lambda^2}{M_A^2} + \frac{3}{2}(M_A^2 + M_B^2 - \frac{5}{18}p^2)\right] + \mathcal{O}(\frac{M_B^4}{M_A^2}) + (p^\mu p^\nu \,\text{term}) \,, \tag{B.15}$$

$$(4\pi)^2 F_2(M_A, M_B; p^2) = \log\frac{\Lambda^2}{M_A^2} + 1 + \frac{M_B^2}{M_A^2}\log\frac{M_B^2}{M_A^2} + \frac{1}{2}\frac{p^2}{M_A^2} + \mathcal{O}(\frac{M_B^4}{M_A^4}\log\frac{M_B^2}{M_A^2}) \,, \tag{B.16}$$

where we have used [63]

$$\int_0^1 dx \Delta_1^{\epsilon_1}(\epsilon_2) = -1 - \epsilon_1 \log\epsilon_1 - \frac{1}{2}\epsilon_2 + \cdots \,, \tag{B.17}$$

$$\int_0^1 dx x \Delta_1^{\epsilon_1}(\epsilon_2) = -\frac{1}{4} + \frac{1}{2}\epsilon_1 - \frac{1}{6}\epsilon_2 + \cdots \,, \tag{B.18}$$

$$\int_0^1 dx (1-x)\Delta_1^{\epsilon_1}(\epsilon_2) = -\frac{3}{4} - \epsilon_1\log\epsilon_1 - \frac{1}{2}\epsilon_1 - \frac{1}{3}\epsilon_2 + \cdots \,, \tag{B.19}$$

$$\int_0^1 dx x(1-x)\Delta_1^{\epsilon_1}(\epsilon_2) = -\frac{5}{36} + \frac{1}{3}\epsilon_1 - \frac{1}{12}\epsilon_2 + \cdots \,, \tag{B.20}$$

with $\Delta_1^{\epsilon_1}(\epsilon_2) = x \cdot 1 + (1-x)\epsilon_1 - x(1-x)\epsilon_2$. Note the large logarithm, $\log(M_B^2/M_A^2)$, in the expression for F_2.

As a sample application of Eqns.(B.15) and (B.16), consider evaluating the amplitudes $(M)_{ZZ}$ and $(N)_{ZZ}$, from diagrams (M) and (N) in Fig. 2 , in the limit $M_{\rho^\pm}^2 \gg M_W^2$:

$$\begin{aligned}
(M)_{ZZ} &\simeq \frac{ie^2}{8s^2c^2}\left\{1 + \frac{x^2\left(-8c^2 + 8 - \frac{1}{c^2}\right)}{4}\right\}F_1^{\mu\nu}(M_{\rho^\pm}, M_W; p^2) + \mathcal{O}(\alpha x^2 M_W^2) \\
&\simeq \frac{ie^2}{8s^2c^2}\left[F_1^{\mu\nu}(M_{\rho^\pm}, M_W; p^2) + \frac{x^2\left(-8c^2 + 8 - \frac{1}{c^2}\right)}{4}g^{\mu\nu}F_3(M_{\rho^\pm})\right] \\
&\quad + \mathcal{O}(\alpha x^2 M_W^2) \,,
\end{aligned} \tag{B.21}$$

$$\begin{aligned}
(N)_{ZZ} &\simeq \frac{-ie^2 g^{\mu\nu}}{2s^2c^2}M_{\rho^\pm}^2\left\{1 - \frac{x^2\left(8c^2 - 6 + \frac{1}{c^2}\right)}{4}\right\}F_2(M_{\rho^\pm}, M_W; p^2) + \mathcal{O}(\alpha x^2 M_W^2) \\
&\simeq \frac{-ie^2 g^{\mu\nu}}{2s^2c^2}\left[F_3(M_{\rho^\pm}) - F_3(M_W) - \frac{x^2\left(8c^2 - 7 + \frac{1}{c^2}\right)}{4}F_3(M_{\rho^\pm})\right] \\
&\quad + \mathcal{O}(\alpha x^2 M_W^2) \,,
\end{aligned} \tag{B.22}$$

where $x^2 \approx 4M_W^2/M_{\rho^\pm}^2$. It is easy to see that, in the leading log approximation, these expressions precisely equal Eqns. (C.37) and (C.38) in Appendix C.

C. Feynman Graph Results: Neutral Gauge Bosons

In this appendix, we present the results of each contribution to the neutral gauge-boson self-energy functions $\Pi_{AA,ZA,ZZ}$, as shown in Fig. 2.

C.1 Photon Self-Energy Amplitude Π_{AA}

$$(A)_{\gamma\gamma} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2}\left[9M_{\rho^\pm}^2 + \frac{19}{6}p^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{C.1}$$

$$(B)_{\gamma\gamma} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2}\left[9M_W^2 + \frac{19}{6}p^2\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{C.2}$$

$$(C)_{\gamma\gamma} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2}\left[-6M_{\rho^\pm}^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{C.3}$$

$$(D)_{\gamma\gamma} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2}\left[-6M_W^2\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{C.4}$$

$$(E)_{\gamma\gamma} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2}\left[2M_{\rho^\pm}^2 - \frac{1}{3}p^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{C.5}$$

$$(F)_{\gamma\gamma} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2}\left[2M_W^2 - \frac{1}{3}p^2\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{C.6}$$

$$(G)_{\gamma\gamma} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2}\left[-2M_{\rho^\pm}^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{C.7}$$

$$(H)_{\gamma\gamma} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2}\left[-2M_W^2\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{C.8}$$

$$(I)_{\gamma\gamma} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2}\left[-2M_{\rho^\pm}^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{C.9}$$

$$(J)_{\gamma\gamma} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2}\left[-2M_W^2\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{C.10}$$

$$(K)_{\gamma\gamma} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2}\left[-M_{\rho^\pm}^2 + \frac{1}{6}p^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{C.11}$$

$$(L)_{\gamma\gamma} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2}\left[-M_W^2 + \frac{1}{6}p^2\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{C.12}$$

where we have neglected terms of $\mathcal{O}(\alpha x^2 M_W^2)$.

C.2 Photon/Z boson Mixing Amplitude Π_{ZA}

$$(A)_{Z\gamma} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 sc}\left[\frac{c^2-s^2}{4}\left(18M_{\rho^\pm}^2+\frac{19}{3}p^2\right)+\frac{9}{4}\left(4c^2-1\right)M_Z^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{C.13}$$

$$(B)_{Z\gamma} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 sc}\left[c^2\left(9M_W^2+\frac{19}{6}p^2\right)\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{C.14}$$

$$(C)_{Z\gamma} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 sc}\left[\frac{c^2-s^2}{4}\left(-12M_{\rho^\pm}^2\right)-\frac{3}{2}\left(4c^2-1\right)M_Z^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{C.15}$$

$$(D)_{Z\gamma} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 sc}\left[-6c^2M_W^2\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{C.16}$$

$$(E)_{Z\gamma} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 sc}\left[\frac{c^2-s^2}{4}\left(4M_{\rho^\pm}^2-\frac{2}{3}p^2\right)+c^2M_Z^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{C.17}$$

$$(F)_{Z\gamma} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 sc}\left[\frac{c^2-s^2}{4}\left(4M_W^2-\frac{2}{3}p^2\right)\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{C.18}$$

$$(G)_{Z\gamma} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 sc}\left[\frac{c^2-s^2}{4}\left(-4M_{\rho^\pm}^2\right)-c^2M_Z^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{C.19}$$

$$(H)_{Z\gamma} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 sc}\left[\frac{c^2-s^2}{4}\left(-4M_W^2\right)\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{C.20}$$

$$(I)_{Z\gamma} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 sc}\left[\frac{c^2-s^2}{4}\left(-4M_{\rho^\pm}^2\right)-\frac{1}{2}M_Z^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{C.21}$$

$$(J)_{Z\gamma} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 sc}\left[2(1-c^2)M_W^2\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{C.22}$$

$$(K)_{Z\gamma} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 sc}\left[\frac{c^2-s^2}{4}\left(-2M_{\rho^\pm}^2+\frac{1}{3}p^2\right)-\frac{1}{4}(4c^2-1)M_Z^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{C.23}$$

$$(L)_{Z\gamma} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 sc}\left[c^2\left(-M_W^2+\frac{1}{6}p^2\right)\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{C.24}$$

where we have used $x^2M_{\rho^\pm}^2\approx 4M_W^2$ and neglected terms of $\mathcal{O}(\alpha x^2M_W^2)$.

C.3 Z Boson Self-Energy Amplitude Π_{ZZ}

$$(A)_{ZZ} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2 c^2}\left[\frac{(c^2-s^2)^2}{4}\left(9M_{\rho^\pm}^2+\frac{19}{6}p^2\right)+\left(18c^2-\frac{27}{2}\right)M_W^2+\frac{9}{4}M_Z^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{C.25}$$

$$(B)_{ZZ} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2 c^2}\left[c^4\left(9M_W^2+\frac{19}{6}p^2\right)\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{C.26}$$

$$(C)_{ZZ} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2 c^2}\left[-\frac{3(c^2-s^2)^2}{2}M_{\rho^\pm}^2-\left(12c^2-\frac{15}{2}\right)M_W^2-\frac{3}{2}M_Z^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{C.27}$$

$$(D)_{ZZ} = \frac{ie^2 g^{\mu\nu}}{(4\pi) s^2 c^2}\left[-6c^4 M_W^2\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{C.28}$$

$$(E)_{ZZ} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2 c^2}\left[\frac{(c^2-s^2)^2}{4}\left(2M_{\rho^\pm}^2-\frac{1}{3}p^2\right)+(2c^2-1)M_W^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{C.29}$$

$$(F)_{ZZ} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2 c^2}\left[\frac{(c^2-s^2)^2}{4}\left(2M_W^2-\frac{1}{3}p^2\right)\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{C.30}$$

$$(G)_{ZZ} = \frac{ie^2}{(4\pi)^2 s^2 c^2}\left[-\frac{(c^2-s^2)^2}{2}M_{\rho^\pm}^2-(2c^2-1)M_W^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{C.31}$$

$$(H)_{ZZ} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2 c^2}\left[-\frac{(c^2-s^2)^2}{2}M_W^2\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{C.32}$$

$$(I)_{ZZ} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2 c^2}\left[-\frac{(c^2-s^2)^2}{2}M_{\rho^\pm}^2-M_W^2+\frac{1}{2}M_Z^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{C.33}$$

$$(J)_{ZZ} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2 c^2}\left[-2(c^4-2c^2+1)M_W^2\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{C.34}$$

$$(K)_{ZZ} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2 c^2}\left[\frac{(c^2-s^2)^2}{4}\left(-M_{\rho^\pm}^2+\frac{1}{6}p^2\right)-\left(2c^2-\frac{3}{2}\right)M_W^2-\frac{1}{4}M_Z^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{C.35}$$

$$(L)_{ZZ} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2 c^2}\left[c^4\left(-M_W^2+\frac{1}{6}p^2\right)\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{C.36}$$

$$(M)_{ZZ} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2 c^2}\left[\frac{1}{8}\left(M_{\rho^\pm}^2 - \frac{1}{3}p^2\right) - \left(c^2 - \frac{9}{8}\right)M_W^2 - \frac{1}{8}M_Z^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{C.37}$$

$$(N)_{ZZ} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2 c^2}\left\{\frac{1}{2}M_W^2\log\frac{\Lambda^2}{M_W^2} + \left[-\frac{1}{2}M_{\rho^\pm}^2 + \left(4c^2 - \frac{7}{2}\right)M_W^2 + \frac{1}{2}M_Z^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\right\}, \tag{C.38}$$

$$(O)_{ZZ} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2 c^2}\left[\frac{9}{4}M_W^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{C.39}$$

$$(P)_{ZZ} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2 c^2}\left[-\frac{1}{4}M_W^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{C.40}$$

where we have neglected terms of $\mathcal{O}(\alpha x^2 M_W^2)$ and used $M_W^2 \approx c^2 M_Z^2$ which is valid to leading order in x.

D. Feynman Graph Results: Charged Gauge Bosons

In this appendix, we present the results of each diagram contributing to the charged gauge-boson self-energy function Π_{WW}, as shown in Fig. 3.

$$(A)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[(1-c^2)\left(\frac{9}{2}M_W^2 + \frac{19}{6}p^2\right)\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{D.1}$$

$$(B)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[c^2\left(\frac{9}{2}M_W^2 + \frac{9}{2}M_Z^2 + \frac{19}{6}p^2\right)\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{D.2}$$

$$\begin{aligned}(C)_{WW} &= \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[\frac{1}{4}\left(\frac{9}{2}M_{\rho^\pm}^2 + \frac{9}{2}M_{\rho^0}^2 + \frac{19}{6}p^2\right) + \left(-9c^2 + 18\right)M_W^2 - \frac{9}{4}M_Z^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2} \\ &= \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[\frac{1}{4}\left(9M_{\rho^\pm}^2 + \frac{19}{6}p^2\right) + \left(-9c^2 + \frac{135}{8}\right)M_W^2 - \frac{9}{8}M_Z^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,,\end{aligned} \tag{D.3}$$

$$(D)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[-3M_W^2\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{D.4}$$

$$(E)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[-3c^2 M_Z^2\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{D.5}$$

$$(F)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[-\frac{3}{4}M_{\rho^\pm}^2 + 3(c^2-2)M_W^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{D.6}$$

$$\begin{aligned}(G)_{WW} &= \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[-\frac{3}{4}M_{\rho^0}^2 + \left(3c^2 - \frac{27}{4}\right)M_W^2 + \frac{3}{4}M_Z^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2} \\ &= \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[-\frac{3}{4}M_{\rho^\pm}^2 + 3(c^2-2)M_W^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \end{aligned}\tag{D.7}$$

$$(H)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[-\frac{(1-c^2)}{2}\left(M_W^2 - \frac{1}{3}p^2\right)\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{D.8}$$

$$(I)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[-\frac{c^2}{2}\left(M_W^2 + M_Z^2 - \frac{1}{3}p^2\right)\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{D.9}$$

$$\begin{aligned}(J)_{WW} &= \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[-\frac{1}{8}\left(M_{\rho^\pm}^2 + M_{\rho^0}^2 - \frac{1}{3}p^2\right) + \left(c^2-2\right)M_W^2 + \frac{1}{4}M_Z^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2} \\ &= \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[-\frac{1}{4}\left(M_{\rho^\pm}^2 - \frac{1}{6}p^2\right) + \left(c^2 - \frac{15}{8}\right)M_W^2 + \frac{1}{8}M_Z^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \end{aligned}\tag{D.10}$$

$$(K)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[\left(-c^2+2\right)M_W^2 - M_Z^2\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{D.11}$$

$$(L)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[-(1-c^2)M_W^2\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{D.12}$$

$$(M)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left\{\frac{1}{4}M_W^2\log\frac{\Lambda^2}{M_W^2} + \left[-\frac{1}{4}M_{\rho^\pm}^2 + \left(c^2 - \frac{5}{4}\right)M_W^2 + \frac{3}{4}M_Z^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\right\}, \tag{D.13}$$

$$(N)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[-\frac{1}{4}M_{\rho^\pm}^2 + \left(c^2-1\right)M_W^2 - \frac{1}{4}M_Z^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{D.14}$$

$$(O)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left\{\frac{1}{4}M_Z^2\log\frac{\Lambda^2}{M_W^2} + \left[-\frac{1}{4}M_{\rho^\pm}^2 + \left(c^2 + \frac{1}{4}\right)M_W^2 - \frac{3}{4}M_Z^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\right\}, \tag{D.15}$$

$$(P)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[-\frac{1}{4}M_{\rho^\pm}^2 + \left(c^2 - \frac{7}{4}\right)M_W^2 + \frac{1}{2}M_Z^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{D.16}$$

$$(Q)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[\frac{1}{4}\left(M_W^2 + M_Z^2 - \frac{1}{3}p^2\right)\right]\log\frac{\Lambda^2}{M_W^2}\,, \tag{D.17}$$

$$(R)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[\frac{1}{16}\left(M_{\rho^\pm}^2 - \frac{1}{3}p^2\right) - \left(\frac{1}{4}c^2 + \frac{1}{16}\right)M_W^2 + \frac{5}{16}M_Z^2\right]\log\frac{\Lambda^2}{M_{\rho^\pm}^2}\,, \tag{D.18}$$

$$(S)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[\frac{1}{16}\left(M^2_{\rho^0} + M^2_W - \frac{1}{3}p^2\right) - \left(\frac{1}{4}c^2 - \frac{7}{16}\right) M^2_W - \frac{1}{4}M^2_Z\right] \log\frac{\Lambda^2}{M^2_{\rho^\pm}}$$

$$= \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[\frac{1}{16}\left(M^2_{\rho^\pm} - \frac{1}{3}p^2\right) - \left(\frac{1}{4}c^2 - \frac{7}{16}\right) M^2_W - \frac{3}{16}M^2_Z\right] \log\frac{\Lambda^2}{M^2_{\rho^\pm}}, \quad \text{(D.19)}$$

$$(T)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[\frac{1}{4}\left(M^2_{\rho^\pm} + M^2_{\rho^0} - \frac{1}{3}p^2\right) - \left(2c^2 - \frac{7}{2}\right) M^2_W - \frac{1}{2}M^2_Z\right] \log\frac{\Lambda^2}{M^2_{\rho^\pm}}$$

$$= \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[\frac{1}{2}\left(M^2_{\rho^\pm} - \frac{1}{6}p^2\right) - \left(2c^2 - \frac{13}{4}\right) M^2_W - \frac{1}{4}M^2_Z\right] \log\frac{\Lambda^2}{M^2_{\rho^\pm}}, \quad \text{(D.20)}$$

$$(U)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[-\frac{1}{4}M^2_W\right] \log\frac{\Lambda^2}{M^2_W}, \quad \text{(D.21)}$$

$$(V)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[-\frac{1}{4}M^2_Z\right] \log\frac{\Lambda^2}{M^2_W}, \quad \text{(D.22)}$$

$$(W)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[-\frac{1}{4}M^2_{\rho^0} + \left(c^2 - 2\right) M^2_W + \frac{1}{2}M^2_Z\right] \log\frac{\Lambda^2}{M^2_{\rho^\pm}}$$

$$= \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[-\frac{1}{4}M^2_{\rho^\pm} + \left(c^2 - \frac{7}{4}\right) M^2_W + \frac{1}{4}M^2_Z\right] \log\frac{\Lambda^2}{M^2_{\rho^\pm}}, \quad \text{(D.23)}$$

$$(X)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[-\frac{1}{4}M^2_{\rho^\pm} + \left(c^2 - \frac{3}{2}\right) M^2_W\right] \log\frac{\Lambda^2}{M^2_{\rho^\pm}}, \quad \text{(D.24)}$$

$$(Y)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[\frac{9}{8}M^2_Z\right] \log\frac{\Lambda^2}{M^2_{\rho^\pm}}, \quad \text{(D.25)}$$

$$(Z)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[\frac{9}{8}M^2_W\right] \log\frac{\Lambda^2}{M^2_{\rho^\pm}}, \quad \text{(D.26)}$$

$$(\alpha)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[-\frac{1}{8}M^2_Z\right] \log\frac{\Lambda^2}{M^2_{\rho^\pm}}, \quad \text{(D.27)}$$

$$(\beta)_{WW} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s^2}\left[-\frac{1}{8}M^2_W\right] \log\frac{\Lambda^2}{M^2_{\rho^\pm}}, \quad \text{(D.28)}$$

where we have neglected terms of $\mathcal{O}(\alpha x^2 M^2_W)$ and used $M^2_W \approx c^2 M^2_Z$ which is valid to leading order in x. Note that, in Eqn. (5.5), we have used the relation $M^2_{\rho^0} \approx M^2_{\rho^\pm} + \frac{s^2}{c^2}M^2_W$ which follows from Eqns.(4.2) and (4.7) to leading order in $x = g_0/g_1$.

E. Pinch Contributions and γ-ρ, Z-ρ, W-ρ Mixing Amplitudes

As discussed in Section 6.2, the ρ-pinch contributions of Eqns. (6.29) – (6.33) arise diagramatically from the γ-ρ, Z-ρ and W-ρ mixing contributions to the scattering amplitudes of ordinary fermions. From Eqn. (6.26), we see that the couplings of the neutral ρ-meson may be written

$$J^\mu_\rho = \frac{e}{sx} J^{\mu\prime}_3 - sx\left(1 - \frac{x_1}{x^2}\right) J^\mu_A - x\frac{c^2 - s^2}{2c}\left(1 - \frac{2c^2}{c^2 - s^2}\frac{x_1}{x^2}\right) J^\mu_Z\ . \tag{E.1}$$

For ordinary fermions, whose couplings to the gauge-eigenstate V^μ at site 1 are suppressed by $x_1 = \mathcal{O}(x^2)$, the $\gamma - \rho$ and $Z - \rho$ mixing amplitudes give rise to corrections to four-fermion scattering amplitudes analogous to Eqns. (6.27) and (6.28):

$$\begin{aligned}
\mathcal{M}|^{\gamma-\rho} \propto\ & \mathcal{A}\cdot\frac{1}{p^2}\cdot\Pi_{A\rho}\cdot\frac{1}{p^2 - M^2_\rho}\cdot\left[-sx\left(1 - \frac{x_1}{x^2}\right)\mathcal{A}' - x\frac{c^2 - s^2}{2c}\left(1 - \frac{2c^2}{c^2 - s^2}\frac{x_1}{x^2}\right)\mathcal{Z}'\right] \\
& + (\mathcal{A},\mathcal{Z} \leftrightarrow \mathcal{A}',\mathcal{Z}')\ , \qquad \text{(E.2)} \\
\mathcal{M}|^{Z-\rho} \propto\ & \mathcal{Z}\cdot\frac{1}{p^2 - M^2_Z}\cdot\Pi_{Z\rho}\cdot\frac{1}{p^2 - M^2_\rho}\cdot\left[-sx\left(1 - \frac{x_1}{x^2}\right)\mathcal{A}' - x\frac{c^2 - s^2}{2c}\left(1 - \frac{2c^2}{c^2 - s^2}\frac{x_1}{x^2}\right)\mathcal{Z}'\right] \\
& + (\mathcal{A},\mathcal{Z} \leftrightarrow \mathcal{A}',\mathcal{Z}')\ . \qquad \text{(E.3)}
\end{aligned}$$

Comparing to Eqns. (6.7) and (6.15), we see that these corrections may be absorbed into redefinitions of the neutral boson self-energy contributions through

$$\Delta\Pi_{AA} = 2s\frac{x}{M^2_\rho}\left(1 - \frac{x_1}{x^2}\right)p^2\Pi_{A\rho}(0), \tag{E.4}$$

$$\Delta\Pi^\gamma_{ZA} = \frac{c^2 - s^2}{2c}\frac{x}{M^2_\rho}\left(1 - \frac{2c^2}{c^2 - s^2}\frac{x_1}{x^2}\right)(p^2 - M^2_Z)\Pi_{A\rho}(0), \tag{E.5}$$

$$\Delta\Pi^Z_{ZA} = s\frac{x}{M^2_\rho}\left(1 - \frac{x_1}{x^2}\right)p^2\Pi_{Z\rho}(0), \tag{E.6}$$

$$\Delta\Pi_{ZZ} = \frac{c^2 - s^2}{c}\frac{x}{M^2_\rho}\left(1 - \frac{2c^2}{c^2 - s^2}\frac{x_1}{x^2}\right)(p^2 - M^2_Z)\Pi_{Z\rho}(0)\ , \tag{E.7}$$

where we have assumed $p^2 \simeq M^2_{W,Z} \ll M^2_\rho$. Similar considerations (or, alternatively, taking the limit $s \to 0$ and $M_Z \to M_W$ in Eqn. (E.7)) lead to contributions to the charged-boson self-energy

$$\Delta\Pi_{WW} = \frac{x}{M^2_\rho}\left(1 - 2\frac{x_1}{x^2}\right)(p^2 - M^2_W)\Pi_{W\rho}(0)\ . \tag{E.8}$$

In this appendix, we present the results of the mixing amplitudes $\Pi_{A\rho}$, $\Pi_{Z\rho}$, and $\Pi_{W\rho}$ and we confirm that the relations Eqns. (E.4)–(E.8) reproduce the results of Eqns. (6.29) – (6.33).

E.1 Photon-ρ and Z-ρ Mixing Amplitudes $\Pi_{A\rho}$ and $\Pi_{Z\rho}$

The photon-ρ mixing amplitude $\Pi_{A\rho}$ and the Z-ρ mixing amplitude $\Pi_{Z\rho}$ arise from the

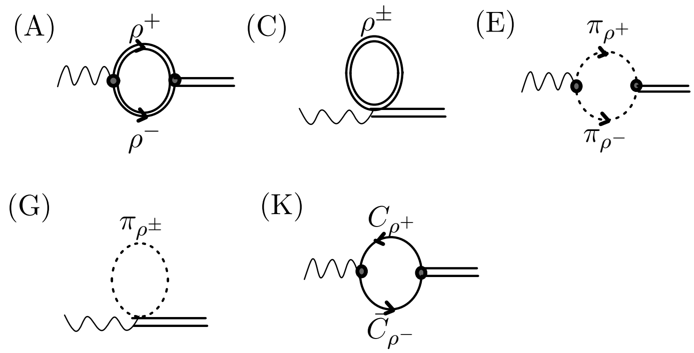

Figure 8: One-loop diagrams contributing to $Z-\rho$ mixing amplitude

diagrams illustrated in Fig. 8. We find

$$(A)_{\gamma\rho} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s}\left[\frac{9M_\rho^2}{x}\right]\log\frac{\Lambda^2}{M_\rho^2}\,, \tag{E.9}$$

$$(C)_{\gamma\rho} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s}\left[\frac{-6M_\rho^2}{x}\right]\log\frac{\Lambda^2}{M_\rho^2}\,, \tag{E.10}$$

$$(E)_{\gamma\rho} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s}\left[\frac{M_\rho^2}{x}\right]\log\frac{\Lambda^2}{M_\rho^2}\,, \tag{E.11}$$

$$(G)_{\gamma\rho} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s}\left[-\frac{M_\rho^2}{x}\right]\log\frac{\Lambda^2}{M_\rho^2}\,, \tag{E.12}$$

$$(K)_{\gamma\rho} = \frac{ie^2 g^{\mu\nu}}{(4\pi)^2 s}\left[-\frac{M_\rho^2}{x}\right]\log\frac{\Lambda^2}{M_\rho^2}\,, \tag{E.13}$$

for $\Pi_{A\rho}$ and

$$(A)_{Z\rho} = \frac{ie^2 (c^2-s^2) g^{\mu\nu}}{2(4\pi)^2 s^2 c}\left[\frac{9M_\rho^2}{x}\right]\log\frac{\Lambda^2}{M_\rho^2}\,, \tag{E.14}$$

$$(C)_{Z\rho} = \frac{ie^2 (c^2-s^2) g^{\mu\nu}}{2(4\pi)^2 s^2 c}\left[\frac{-6M_\rho^2}{x}\right]\log\frac{\Lambda^2}{M_\rho^2}\,, \tag{E.15}$$

$$(E)_{Z\rho} = \frac{ie^2 (c^2-s^2) g^{\mu\nu}}{2(4\pi)^2 s^2 c}\left[\frac{M_\rho^2}{x}\right]\log\frac{\Lambda^2}{M_\rho^2}\,, \tag{E.16}$$

$$(G)_{Z\rho} = \frac{ie^2 (c^2-s^2) g^{\mu\nu}}{2(4\pi)^2 s^2 c}\left[-\frac{M_\rho^2}{x}\right]\log\frac{\Lambda^2}{M_\rho^2}\,, \tag{E.17}$$

$$(K)_{Z\rho} = \frac{ie^2 (c^2-s^2) g^{\mu\nu}}{2(4\pi)^2 s^2 c}\left[-\frac{M_\rho^2}{x}\right]\log\frac{\Lambda^2}{M_\rho^2}\,, \tag{E.18}$$

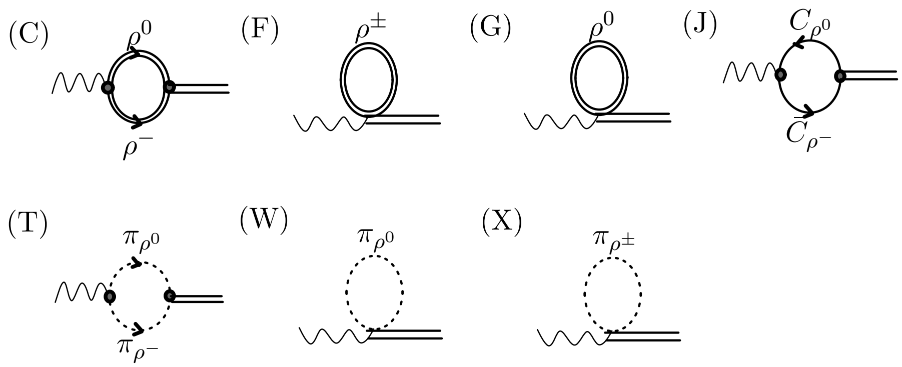

Figure 9: One-loop diagrams contributing to $W-\rho$ mixing amplitude

for $\Pi_{Z\rho}$. Here we keep only the leading M_ρ^2/x terms, and neglect other subleading contributions. Putting these contributions together, we find

$$\Pi_{A\rho}(0) = \frac{2e^2}{(4\pi)^2 s}\frac{M_\rho^2}{x}\log\frac{\Lambda^2}{M_\rho^2}\,, \tag{E.19}$$

$$\Pi_{Z\rho}(0) = \frac{e^2(c^2-s^2)}{(4\pi)^2 s^2 c}\frac{M_\rho^2}{x}\log\frac{\Lambda^2}{M_\rho^2}\,. \tag{E.20}$$

Combining Eqns. (E.19)–(E.20) with Eqns. (E.4)–(E.7) yield the results presented in Section 6.2.

E.2 W-ρ Mixing Amplitude $\Pi_{W\rho}$

The W-ρ mixing amplitude $\Pi_{W\rho}$ arises from the diagrams illustrated in Fig. 9. We find

$$(C)_{W\rho} = \frac{ie^2 g^{\mu\nu}}{2(4\pi)^2 s^2}\left[\frac{9M_\rho^2}{x}\right]\log\frac{\Lambda^2}{M_\rho^2}\,, \tag{E.21}$$

$$(F)_{W\rho} = \frac{ie^2 g^{\mu\nu}}{2(4\pi)^2 s^2}\left[-\frac{3M_\rho^2}{x}\right]\log\frac{\Lambda^2}{M_\rho^2}\,, \tag{E.22}$$

$$(G)_{W\rho} = \frac{ie^2 g^{\mu\nu}}{2(4\pi)^2 s^2}\left[-\frac{3M_\rho^2}{x}\right]\log\frac{\Lambda^2}{M_\rho^2}\,, \tag{E.23}$$

$$(J)_{W\rho} = \frac{ie^2 g^{\mu\nu}}{2(4\pi)^2 s^2}\left[-\frac{M_\rho^2}{x}\right]\log\frac{\Lambda^2}{M_\rho^2}\,, \tag{E.24}$$

$$(T)_{W\rho} = \frac{ie^2 g^{\mu\nu}}{2(4\pi)^2 s^2}\left[\frac{M_\rho^2}{x}\right]\log\frac{\Lambda^2}{M_\rho^2}\,, \tag{E.25}$$

$$(W)_{W\rho} = \frac{ie^2 g^{\mu\nu}}{2(4\pi)^2 s^2}\left[-\frac{M_\rho^2}{2x}\right]\log\frac{\Lambda^2}{M_\rho^2}\,, \tag{E.26}$$

$$(X)_{W\rho} = \frac{ie^2 g^{\mu\nu}}{2(4\pi)^2 s^2}\left[-\frac{M_\rho^2}{2x}\right]\log\frac{\Lambda^2}{M_\rho^2}\,. \tag{E.27}$$

Here we keep only the leading M_ρ^2/x terms, and neglect other subleading contributions. Putting these contributions together, we find

$$\Pi_{W\rho}(0) = \frac{e^2}{(4\pi)^2 s^2}\frac{M_\rho^2}{x}\log\frac{\Lambda^2}{M_\rho^2}\,. \tag{E.28}$$

The amplitude Eq. (E.28), used in Eq. (E.8), yields the results presented in Section 6.2.

F. Vertex Corrections from Fermion Delocalization Operator

From the forms of the x_1-dependent interactions of Eqn. (7.3), we see it is straightforward to compute the vertex correction amplitudes $J_L^3 - A$ and $J_L^3 - Z$. Note that the vertex correction arising from an interaction of the first term of line two in Eqn. (7.3) is suppressed by a factor of $(M_W^2/M_{\rho^\pm}^2)$ due to the $\pi_{W^\pm}$ loop; therefore, it may be neglected since we are concerned with the leading log contributions of the form $\log(\Lambda^2/M_{W,\rho}^2)$. The relevant Feynman graphs are shown in fig. 5.

By using the formulae of Feynman integral in Appendix B, the vertex correction amplitudes corresponding to diagrams (A)-(E) of figure 5 are evaluated to be, for an external photon

$$(B)_{J^3_L A} = \frac{2ie^3 g^{\mu\nu}}{(4\pi)^2 s^2} \left(\frac{x_1}{x^2}\right) \log \frac{\Lambda^2}{M^2_{\rho^\pm}} \,, \tag{F.1}$$

$$(C)_{J^3_L A} = -\frac{ie^3 g^{\mu\nu}}{(4\pi)^2 s^2} \left(\frac{x_1}{x^2}\right) \log \frac{\Lambda^2}{M^2_{\rho^\pm}} \,, \tag{F.2}$$

$$(E)_{J^3_L A} = \frac{ie^3 g^{\mu\nu}}{(4\pi)^2 s^2} \left(\frac{x_1}{x^2}\right) \log \frac{\Lambda^2}{M^2_{\rho^\pm}} \,, \tag{F.3}$$

and, for an external Z-boson

$$(A)_{J^3_L Z} = \frac{ie^3 g^{\mu\nu}}{(4\pi)^2 s^3 c} \left(\frac{x_1}{x^2}\right) \log \frac{\Lambda^2}{M^2_{\rho^\pm}} \,, \tag{F.4}$$

$$(B)_{J^3_L Z} = \frac{ie^3 (c^2 - s^2) g^{\mu\nu}}{(4\pi)^2 s^3 c} \left(\frac{x_1}{x^2}\right) \log \frac{\Lambda^2}{M^2_{\rho^\pm}} \,, \tag{F.5}$$

$$(C)_{J^3_L Z} = -\frac{ie^3 g^{\mu\nu}}{4(4\pi)^2 s^3 c} \left(\frac{x_1}{x^2}\right) \log \frac{\Lambda^2}{M^2_{\rho^\pm}} \,, \tag{F.6}$$

$$(D)_{J^3_L Z} = -\frac{ie^3 (c^2 - s^2) g^{\mu\nu}}{2(4\pi)^2 s^3 c} \left(\frac{x_1}{x^2}\right) \log \frac{\Lambda^2}{M^2_{\rho^\pm}} \,, \tag{F.7}$$

$$(E)_{J^3_L Z} = \frac{ie^3 (c^2 - s^2) g^{\mu\nu}}{2(4\pi)^2 s^3 c} \left(\frac{x_1}{x^2}\right) \log \frac{\Lambda^2}{M^2_{\rho^\pm}} \,. \tag{F.8}$$

It should be noticed that $(C)_{J^3_L A} + (E)_{J^3_L A} = 0$; the corresponding one-loop generated operator is written in the form,

$$\mathcal{L}_{(C)+(E)} = -\bar{\psi}_L \gamma^\mu (g_0 L_\mu - g_1 V_\mu) \psi_L \cdot \left[\frac{e^2}{(4\pi)^2 s^2} \left(\frac{x_1}{x^2}\right) \log \frac{\Lambda^2}{M^2_{\rho^\pm}} \right] , \tag{F.9}$$

in which the operator $(g_0 L_\mu - g_1 V_\mu)$ is orthogonal to the photon.

Combining Eqns.(F.4)-(F.8) and (F.1), we find that the vertex corrections are incorporated into the following operators:

$$\mathcal{L}_{\text{eff}} = \frac{e}{sc} \cdot G_1(M^2_{\rho^\pm}; x_1) \cdot J^{\mu 3}_L Z_\mu + e \cdot G_2(M^2_{\rho^\pm}; x_1) \cdot J^{\mu 3}_L A_\mu \,, \tag{F.10}$$

where

$$G_1(M^2_{\rho^\pm}; x_1) = \frac{e^2}{(4\pi)^2 s^2} \left(2c^2 - \frac{1}{4}\right) \left(\frac{x_1}{x^2}\right) \log \frac{\Lambda^2}{M^2_{\rho^\pm}} \,, \tag{F.11}$$

$$G_2(M^2_{\rho^\pm}; x_1) = \frac{2e^2}{(4\pi)^2 s^2} \left(\frac{x_1}{x^2}\right) \log \frac{\Lambda^2}{M^2_{\rho^\pm}} \,. \tag{F.12}$$

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