Reference.

M. Hirsch and W. Thurston, Foliated bundles, flat manifolds and invariant measures, Ann. Math. (to appear).

Address. M. Hirsch, Department of Mathematics, University of California, Berkeley, California 94720, U.S.A.

6. A stable analytic foliation with only exceptional minimal sets.

M. Hirsch.

Let M be a compact manifold admitting a foliation $\mathcal{F}$. A subset $\Lambda \subset M$ is said to be minimal if it is (1) compact and nonempty and (2) a union of leaves and if (3) no proper subset of Λ satisfies (1) and (2). Examples of minimal sets are a compact leaf and, if every leaf is dense, M itself. All other minimal sets are called exceptional. An exceptional minimal set must be nowhere dense.

Denjoy found a C^1 codimension 1 foliation of T^2 with an exceptional minimal set and showed that this cannot happen in class C^2. A corollary to the Poincaré-Bendixson theorem says that if M^2 is an open subset of S^2 then a foliation of M cannot have exceptional minimal sets. Raymond has an example of a C^∞ foliation on S^3 with an exceptional minimal set.

Theorem. For $n \geqslant 3$ $\exists$ a compact n-manifold M with a codimension 1 foliation $\mathcal{F}$ s.t.

(a) $\mathcal{F}$ is analytic.

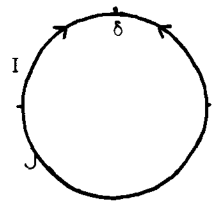
(b) $\mathcal{F}$ is stable under C^1-small perturbations of the tangent field $T\mathcal{F}$.

(c) $\mathcal{F}$ has a unique minimal set Λ and Λ is exceptional.

(d) for $n > 3$ $\pi_1(M)$ is solvable but not polycyclic.

Proof. We shall only describe this foliation in the case n = 3. For any immersion $f: S^1 \to S^1$ choose an embedding g of the solid torus $S^1 \times D^2$ into its interior so that $pg = fp$ where $p: S^1 \times D^2 \to S^1$ is the projection. Let $V = S^1 \times D^2 - \text{int}\, g(S^1 \times D^2)$. Then the boundary of V is two copies of $S^1 \times S^1$ and we define a closed 3-manifold M from V by identifying x with g(x) for each x on the boundary of $S^1 \times D^2$. Let $\pi: V \to M$ and $q = p|V$ be the projections. There is a unique foliation $\mathcal{F}$ of M that induces on V the fibration by q^{-1} of points.

For $x \in S^1$ we define the <u>extended f-orbit</u> of x, $E(x)$, as $\{y \in S^1;\ \exists\, m, n \geqslant 0$ with $f^m x = f^n y\}$, For $(x,z) \in V \subset S^1 \times D^2$, the leaf of $\mathcal{F}$ through $\pi(x,z)$ is $M_{E(x)}$ where, for $A \subset S^1$, we define $M_A = \pi q^{-1} A$. We say $A \subset S^1$ is minimal for f if it is compact, non-empty, invariant by f and f^{-1} and minimal with respect to these properties. The $A \subset S^1$ is minimal for f if and only if $M_A \subset M$ is minimal for $\mathcal{F}$.



Now we choose a particular structurally stable C^ω immersion of degree 2, $f: S^1 \to S^1$. f has one attracting fixed point δ and no other attracting periodic orbits. There are just two other fixed points and they divide S^1 into two semicircles I and J. $fI = I$ but fJ wraps around 1½ times with $f|J$ expanding. Such an f has a unique minimal set which is Γ, the complement of $W^s(\delta) = \bigcup f^{-n}(\text{int } I)$. Γ is a Cantor set and $f|\Gamma$ is conjugate to the one-sided 2-shift. So $\mathcal{F}$ has a unique minimal set M_Γ and M_Γ is clearly exceptional.

A foliation $\mathcal{G}$ of M that is C^1 close to $\mathcal{F}$ gives rise to a foliation $\mathcal{G}'$ of V meeting ∂V in circles and $\mathcal{G}'$ can be shown to fibre V over a map $S^1 \to S^1$ that is close to f. Thus the stability of f leads to the stability of $\mathcal{F}$.

A presentation of $\pi_1(M)$ can be obtained by considering the identification of the boundary of V and from this presentation it is not hard to see (d).

<u>Address.</u> M. Hirsch, Department of Mathematics, University of California, Berkeley, California 94720, U.S.A.

7. <u>Accessibility and foliations.</u>

<u>Peter Stefan.</u>

Let S be a set of C^q vector fields, $q \geqslant 1$, defined on open subsets of a finite dimensional paracompact manifold M. The S-orbit of a point $x \in M$ is $\{y \in M;$ x can be joined to y by a finite sequence of segments of integral curves of vector fields in S$\}$. What can be said about the partition $\mathcal{P}(S)$ of M into S-orbits ?