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Reference.

M. Hirsch and W. Thurston, Foliated bundles, flat manifolds and invariant measures, Ann. Math. (to appear).

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6. A stable analytic foliation with only exceptional minimal sets.

M. Hirsch.

Let M be a compact manifold admitting a foliation \mathcal{F} . A subset $\Lambda \subset M$ is said to be minimal if it is (1) compact and nonempty and (2) a union of leaves and if (3) no proper subset of Λ satisfies (1) and (2). Examples of minimal sets are a compact leaf and, if every leaf is dense, M itself. All other minimal sets are called exceptional. An exceptional minimal set must be nowhere dense.

Denjoy found a C^1 codimension 1 foliation of T^2 with an exceptional minimal set and showed that this cannot happen in class C^2 . A corollary to the Poincaré-Bendixson theorem says that if M^2 is an open subset of S^2 then a foliation of M cannot have exceptional minimal sets. Raymond has an example of a C^∞ foliation on S^3 with an exceptional minimal set.

Theorem. For $n \geq 3$ \exists a compact n -manifold M with a codimension 1 foliation \mathcal{F} s.t.

- (a) \mathcal{F} is analytic.
- (b) \mathcal{F} is stable under C^1 -small perturbations of the tangent field $T\mathcal{F}$.
- (c) \mathcal{F} has a unique minimal set Λ and Λ is exceptional.
- (d) for $n > 3$ $\pi_1(M)$ is solvable but not polycyclic.

Proof. We shall only describe this foliation in the case $n = 3$. For any immersion $f: S^1 \rightarrow S^1$ choose an embedding g of the solid torus $S^1 \times D^2$ into its interior so that $pg = fp$ where $p: S^1 \times D^2 \rightarrow S^1$ is the projection. Let $V = S^1 \times D^2 - \text{int } g(S^1 \times D^2)$. Then the boundary of V is two copies of $S^1 \times S^1$ and we define a closed 3-manifold M from V by identifying x with $g(x)$ for each x on the boundary of $S^1 \times D^2$. Let $\pi: V \rightarrow M$ and $q = p|_V$ be the projections. There is a unique foliation \mathcal{F} of M that induces on V the fibration by q^{-1} of points.

For $x \in S^1$ we define the extended f -orbit of x , $E(x)$, as $\{y \in S^1; \exists m, n \geq 0 \text{ with } f^m x = f^n y\}$. For $(x, z) \in V \subset S^1 \times D^2$, the leaf of \mathcal{F} through $\pi(x, z)$ is $M_{E(x)}$ where, for $A \subset S^1$, we define $M_A = \pi q^{-1} A$. We say $A \subset S^1$ is minimal for f if it is compact, non-empty, invariant by f and f^{-1} and minimal with respect to these properties. The $A \subset S^1$ is minimal for f if and only if $M_A \subset M$ is minimal for \mathcal{F} .

Now we choose a particular structurally stable C^ω immersion of degree 2, $f: S^1 \rightarrow S^1$. f has one attracting fixed point δ and no other attracting periodic orbits. There are just two other fixed points and they divide S^1 into two semicircles I and J . $fI = I$ but fJ wraps around $1\frac{1}{2}$ times with $f|_J$ expanding. Such an f has a unique minimal set which is Γ , the complement of $W^s(\delta) = \cup f^{-n}(\text{int } I)$. Γ is a Cantor set and $f|_\Gamma$ is conjugate to the one-sided 2-shift. So \mathcal{F} has a unique minimal set M_Γ and M_Γ is clearly exceptional.

A foliation \mathcal{G} of M that is C^1 close to \mathcal{F} gives rise to a foliation \mathcal{G}' of V meeting ∂V in circles and \mathcal{G}' can be shown to fibre V over a map $S^1 \rightarrow S^1$ that is close to f . Thus the stability of f leads to the stability of \mathcal{F} .

A presentation of $\pi_1(M)$ can be obtained by considering the identification of the boundary of V and from this presentation it is not hard to see (d).

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7. Accessibility and foliations.

Peter Stefan.

Let S be a set of C^q vector fields, $q \geq 1$, defined on open subsets of a finite dimensional paracompact manifold M . The S -orbit of a point $x \in M$ is $\{y \in M; x \text{ can be joined to } y \text{ by a finite sequence of segments of integral curves of vector fields in } S\}$. What can be said about the partition $\mathcal{P}(S)$ of M into S -orbits?