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State estimation for unbalanced electric power distribution systems using AMI data

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State Estimation for Unbalanced Electric Power Distribution Systems Using AMI Data

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***Abstract*—The distribution system state estimation (DSSE) problem is very challenging due to complexities at the distribution network structure level and sensor level. This paper first develops a DSSE formulation similar to the real world settings. In particular, unbalanced single-phase and two-phase measurements are considered. In addition, constraints associated with zero injections and center tapped transformers are carefully modeled. An orthogonal elimination based DSSE algorithm is developed to solve the nonlinear optimization problem with equality constraints. The proposed algorithm has better numerical properties than the existing methods and does not require tunable parameters. The simulation results from modified IEEE 13-bus test feeder show that the proposed DSSE formulation and algorithm yield accurate state estimation results.**

***Index Terms*—Advanced Metering Infrastructure, Distribution System, Orthogonal Elimination, Smart Meter, State Estimation.**

I. Introduction

State estimation is one of the most important functions in modern energy management systems (EMS) of interconnected transmission networks [1]. Various key applications such as contingency analysis, preventive control, and corrective control all depend on state estimation solutions. The power system state estimation algorithm was first introduced by Schweppe [2] in 1970 and has since been implemented in almost every EMS of transmission networks around the world. However, the state estimation algorithm has not seen similar level of adoption and application in the distribution management systems (DMS).

An increasing amount of distributed energy resources (DERs) is being integrated into the electric power distribution systems. To proactively manage the large-scale and heterogeneous DERs, the distribution system operators first need a robust distribution system state estimator (DSSE). The installation of supervisory control and data acquisition (SCADA) system at the feeder level [3] and the widespread adoption of advanced metering infrastructure (AMI) have finally made the implementation of a reliable DSSE feasible.

It is not simple to extend the state estimation algorithm developed for the transmission system to the distribution system due to reasons on two levels [4]. Unlike the transmission systems, the distribution systems are typically radial and unbalanced at the network structure level. The unbalanceness is reflected in two ways. First, the electricity loads are unbalanced on three phases. Second, there is a mixture of single-, two-, and three-phase laterals in the distribution feeders. At the sensor level, the DMS typically only has access to low-frequency smart meter readings due to the bottleneck of communication systems. In addition, the level of measurement redundancy in distribution networks is much lower than that of transmission systems. Finally, the smart meter measurements are asynchronous as the built-in real-time clock of smart meters is only periodically synchronized with actual time [5]. The low level of measurement redundancy can cause the system to be unobservable. Asynchronous measurements make it difficult to interpret the state estimation results.

Several researchers have attempted to address the state estimation problem in the electric power distribution systems. Baran *et al.* pointed out that the voltage and electric load measurements at customer sites can be used for state estimation in the distribution systems [3]. As shown in [6], the access to accurate AMI data can improve the state estimation results in power distribution systems. In [7], the author provided a detailed discussion of three-phase distribution system modeling, measurement functions, and constraints. In terms of the DSSE algorithm design, early works in distribution system state estimation adopted the weighted least squares (WLS) method [8]. A current-based fast decoupled state estimation algorithm was developed for distribution systems [9]. A distribution system state estimation algorithm considering non-synchronized smart meter data was developed by modeling the load variations [10].

The previous literature did not provide a detailed measurement model for distribution networks with a mix of single-phase, two-phase, and three-phase laterals. In addition, previous literature is also missing the modeling for electrical connections among customers/smart meters, the center-tapped transformers, and the secondary feeders. Regarding the solution technique for state estimation problem, the previous algorithms rely on a tunable parameter to solve the linear equality constrained least square problems. However, it is difficult to find an appropriate parameter for all distribution feeders. This paper fills the knowledge gap by providing a detailed measurement model for all customer/smart meter connection types. This paper also develops an orthogonal elimination based method within the sequential quadratic programming

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framework to solve the nonlinear DSSE problem with equality constraints. The proposed DSSE algorithm no longer needs any tunable parameter and has better numerical stability.

The remainder of this paper is organized as follows. Section II presents the problem formulation. The measurement models and equality constraints for distribution systems are discussed in detail. Section III provides the solution techniques for distribution system state estimation problems. Section IV shows the simulation results on the IEEE 13-bus test feeder to validate the proposed DSSE algorithm. Section V gives concluding remarks.

II. Problem Formulation

In this section, we present an overview of a typical distribution system, the general formulation for state estimation problems, and the unique measurement functions associated with different smart meter connections and equality constraints in power distribution systems.

A. Distribution System Overview

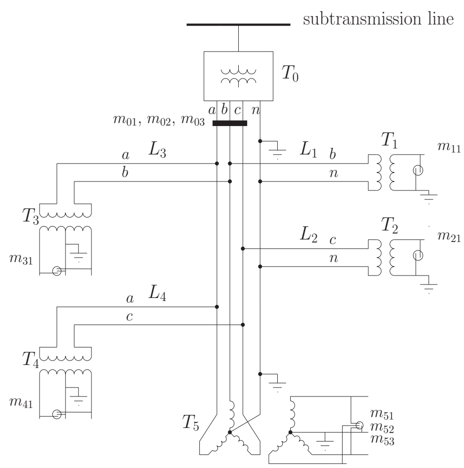

Figure 1. Illustration of a distribution system

The illustration of a typical electric power distribution system is shown in Fig.1. Labels a, b, and c represent the three phases. n represents the neutral wire. L stands for a lateral and T stands for a transformer. The laterals can be single-phase (L_1 and L_2), two-phase (L_3 and L_4), or three-phase. Residential customers can be served by either a single-phase transformer (T_1, T_2) or a center-tapped transformer (T_3, T_4). Commercial customers are typically served by a three-phase transformer (T_5).

There are one or multiple service transformers on each secondary feeder in the distribution network. Each service transformer serves one or multiple buildings which are equipped with smart meters. The smart meters measure the real power consumption (kWh) and voltage magnitudes of each building (e.g., ~240V for center-tapped transformers). The SCADA system at the distribution feeder level also measures the phase currents, neutral current, line-to-line voltage, and complex power flow on the secondary side of the substation transformer.

B. General Formulation for State Estimation Problems

The state estimation problem aims at finding the states of the electric power distribution systems, given the network connectivity information and measurement data. The state variable vector $\boldsymbol{x}$ is typically defined as the voltage angles and magnitudes at each node and each phase.

$$\boldsymbol{x} = \left[|V_1^a|, |V_1^b|, \cdots, |V_N^c|, \theta_1^b, \cdots, \theta_N^c\right]^{\mathrm{T}}$$

where $|V_i^p|$ denotes voltage magnitude of bus i with phase p. θ_i^p stands for the voltage angle of bus i with phase p. $\theta_1^a \equiv 0$ is chosen as the reference angle.

The measurement model can be written as follows:

$$\boldsymbol{z} = \boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{e} \tag{1}$$

where $\boldsymbol{h}(\cdot)$ is a system of nonlinear equations that map the state variables into the measurement space. It is assumed that all of the measurement noise terms $\boldsymbol{e}$ are additive and zero mean Gaussian.

In addition to measurement functions associated with measurement devices, there are equality constraints introduced by network topology or circuit elements. They can be written as:

$$\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{0} \tag{2}$$

The state estimation result is defined as the solution of Equations (1) and (2) in the maximum likelihood sense. Under the Gaussianity assumption, the estimation problem is equivalent to the following non-convex optimization problem.

$$\begin{aligned} \min_{x} \quad & (\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x}))^{\mathrm{T}}\mathbf{R}^{-1}(\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x})) \\ \text{s.t.} \quad & \boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{0} \end{aligned} \tag{3}$$

where $\mathbf{R}$ stands for the measurement error weighting matrix. The problem formulation (3) applies to unbalanced three-phase systems. The development of measurement functions and equality constraints for unbalanced systems are presented in the next two subsections.

C. Measurement Models

The development of measurement models and equality constraints are based on three-phase power flow equations:

$$P_i^p = |V_i^p| \sum_{k=1}^{N} \sum_{m \in \{a,b,c\}} |V_k^m| (g_{ik}^{pm}\cos(\theta_{ik}^{pm}) + b_{ik}^{pm}\sin(\theta_{ik}^{pm})) \tag{4}$$

$$Q_i^p = |V_i^p| \sum_{k=1}^{N} \sum_{m \in \{a,b,c\}} |V_k^m| (g_{ik}^{pm}\sin(\theta_{ik}^{pm}) - b_{ik}^{pm}\cos(\theta_{ik}^{pm})) \tag{5}$$

where P_i^p and Q_i^p are the real and reactive net injected power at bus i with phase p. V_i^p is the voltage at bus i with phase p. $\theta_{ik}^{pm} = \theta_i^p - \theta_k^m$ is the voltage angle differences between bus i with phase p and bus k with phase m. The distribution network admittance matrix is given by:

$$\mathbf{Y} = [\mathbf{Y}_{ik}] \quad \mathbf{Y}_{ik} = [y_{ik}^{pm}] = [g_{ik}^{pm} + jb_{ik}^{pm}]$$

where $i, k \in$ set of buses and $p, m \in \{a, b, c\}$. Each $[\mathbf{Y}_{ik}]$ is a 3×3 block with off-diagonal elements representing mutual magnetic coupling between phases.

Unlike transmission systems, there are two types of measurement models in electric power distribution systems: the

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single-phase and two-phase measurements. The single-phase measurements include single-phase voltage magnitudes and single-phase electricity loads. The two-phase measurements include line-to-line voltage magnitudes and two-phase electricity loads. The following assumptions are made in this paper about distribution system measurements:

- At the distribution substation, phase-to-neutral voltage magnitude, magnitude of current injection, and three-phase complex power injection measurements are available.
- At every distribution center-tapped transformer, two-phase voltage magnitude and aggregated real two-phase power injection measurements are available.
- At every single- or three-phase distribution transformer, phase-to-neutral voltage magnitudes and real power injection measurements are available.
- The center-tapped transformers and the single-phase transformers are assumed to be ideal. The series impedance and shunt admittance of the lines from the service transformers' secondary to the customers' buildings are negligible.

The single-phase measurement equations are listed below:

$$|V_i^p|_{meas} = |V_i^p| + e_{|V_i^p|} \tag{6}$$

$$P_{i\ meas}^p = P_i^p + e_{P_i^p} \tag{7}$$

$$Q_{1meas}^p = Q_1^p + e_{Q_1^p} \tag{8}$$

$$\begin{aligned} |I_1^p|_{meas} &= \left\| \begin{bmatrix} \mathrm{Re}(I_1^p) & \mathrm{Im}(I_1^p) \end{bmatrix}^{\mathrm{T}} \right\|_2 + e_{|I_1^p|} \\ \mathrm{Re}(I_1^p) &= \sum_{k=1}^{N} \sum_{m\in\{a,b,c\}} |V_k^m|(g_{1k}^{pm}\cos(\theta_k^m) - b_{1k}^{pm}\sin(\theta_k^m)) \\ \mathrm{Im}(I_1^p) &= \sum_{k=1}^{N} \sum_{m\in\{a,b,c\}} |V_k^m|(g_{1k}^{pm}\sin(\theta_k^m) + b_{1k}^{pm}\cos(\theta_k^m)) \end{aligned} \tag{9}$$

The two-phase measurement equations are listed below:

$$|V_i^{pm}|_{meas} = \sqrt{|V_i^p|^2 + |V_i^m|^2 - 2|V_i^p||V_i^m|\cos(\theta_i^p - \theta_i^m)} + e_{|V_i^{pm}|} \tag{10}$$

$$\begin{aligned} P_{i\ \ meas}^{pm} &= |V_i^p| \sum_{k=1}^{N} \sum_{n\in\{a,b,c\}} |V_k^n|(g_{ik}^{pn}\cos(\theta_{ik}^{pn}) + b_{ik}^{pn}\sin(\theta_{ik}^{pn})) \\ &- |V_i^m| \sum_{k=1}^{N} \sum_{n\in\{a,b,c\}} |V_k^n|(g_{ik}^{pn}\cos(\theta_{ik}^{mn}) + b_{ik}^{pn}\sin(\theta_{ik}^{mn})) + e_{P_i^{pm}} \end{aligned} \tag{11}$$

where $|V_i^{pm}| = |V_i^p - V_i^m|$ denotes line-to-line voltage magnitudes and $P_i^{pm} = \mathrm{Re}((V_i^p - V_i^m)I_i^{p*})$ stands for two-phase real power injections.

D. Equality Constraints

Two types of equality constraints are modeled in the distribution state estimation problems. The first type of equality constraints are associated with buses with neither load nor generation. These constraints are called zero injection constraints where the net injected power must be zero.

$$P_{\mathrm{tap},i}^p = 0 \tag{12}$$

$$Q_{\mathrm{tap},i}^p = 0 \tag{13}$$

The second type of equality constraints are associated with center-tapped transformers where the sum of currents flowing through the two phases must be zero.

$$\mathrm{Re}(I_i^p) + \mathrm{Re}(I_i^m) = 0 \tag{14}$$

$$\mathrm{Im}(I_i^p) + \mathrm{Im}(I_i^m) = 0 \tag{15}$$

The measurement functions and equality constraints are incorporated into the objective function and constraints of the optimization problem of the state estimation algorithm. Note that a distribution system node may have a combination of these constraints and measurements. For example, in Fig.2, node i has two measurement functions, two zero injection constraints, and two equality constraints associated with the center-tapped transformer. The measurement functions are real power $P_{i\ meas}^{bc}$ and line-to-line voltage magnitude $|V_i^{bc}|_{meas}$. The zero injection constraints are $P_i^a = 0$ and $Q_i^a = 0$. The equality constraints associated with center tapped transformers are $\mathrm{Re}(I_i^b) + \mathrm{Re}(I_i^c) = 0$ and $\mathrm{Im}(I_i^b) + \mathrm{Im}(I_i^c) = 0$.

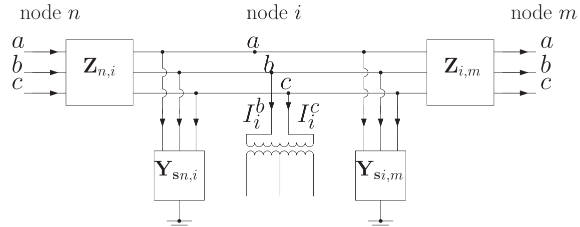

Figure 2. Constraints and measurements at node i

III. Technical Methods

In this section, the algorithm to solve the state estimation problem stated in (3) is presented. To solve this nonconvex optimization problem, the Lagrange multiplier theorem is invoked [11]. The theorem (expressed as equations (16) and (17)) states that the gradient of the Lagrangian must vanish at local minima or maxima.

$$\nabla_{\boldsymbol{x}}\mathcal{L} = -2\mathbf{H}^{\mathrm{T}}\mathbf{R}^{-1}\Delta\boldsymbol{z} - \mathbf{F}^{\mathrm{T}}\boldsymbol{\lambda} = \mathbf{0} \tag{16}$$

$$\nabla_{\boldsymbol{\lambda}}\mathcal{L} = -\boldsymbol{f}(\hat{\boldsymbol{x}}) = \mathbf{0} \tag{17}$$

where $\mathbf{H} = \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{x}}\Big|_{\boldsymbol{x}=\hat{\boldsymbol{x}}}$, $\mathbf{F} = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\Big|_{\boldsymbol{x}=\hat{\boldsymbol{x}}}$, and $\Delta\boldsymbol{z} = \boldsymbol{z} - \boldsymbol{h}(\hat{\boldsymbol{x}})$. $\hat{\boldsymbol{x}}$ denotes a local minimum or maximum. Equations (16) and (17) are the necessary but not sufficient conditions for local optimum solutions.

Equations (16) and (17) can be solved using the Newton-Raphson (N-R) method. First, we approximate the second order derivative of the original objective function (3) as:

$$\nabla_{\boldsymbol{xx}}(\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x}))^{\mathrm{T}}\mathbf{R}^{-1}(\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x})) \approx 2\mathbf{H}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{H} \tag{18}$$

Then, at each N-R iteration, a system of linear equations is constructed and solved. This linear equation is the same as the closed form solution of the system of equations corresponding to the KKT condition for the following problem [14]:

$$\begin{aligned} \min_{\Delta\boldsymbol{x}} \quad & (\Delta\boldsymbol{z} - \mathbf{H}\Delta\boldsymbol{x})^{\mathrm{T}}\mathbf{R}^{-1}(\Delta\boldsymbol{z} - \mathbf{H}\Delta\boldsymbol{x}) \\ \text{s.t.} \quad & \mathbf{F}\Delta\boldsymbol{x} = \Delta\boldsymbol{f} \end{aligned} \tag{19}$$

where $\Delta \boldsymbol{x} = \boldsymbol{x}^{k+1} - \boldsymbol{x}^k$ and $\Delta \boldsymbol{f} = -\boldsymbol{f}(\boldsymbol{x}^k)$. Jacobian matrices are evaluated at current iterate $\boldsymbol{x}^k$. Note that the approximation in (18) is reasonable when the measurement residuals and nonlinearity of the measurement models are small. The closed form solution to Equation (19) can be ill-conditioned, as the KKT matrix is indefinite. Therefore, the performance of different algorithms varies. A desirable algorithm to solve Equation (19) should:

1) have good numerical properties (e.g. small condition number for coefficient matrix)
2) preserve sparsity of the problem

Hachtel's augmented matrix method [12] satisfies the second condition. However, it requires a good tuning parameter to satisfy the first condition. It is difficult to develop an algorithm to find an appropriate tuning parameter for a general DSSE problem.

To eliminate the tuning process, we advocate the adoption of an orthogonal elimination based method [13]. Note that the linearized equality constraints define an affine space. The affine space has the following form:

$$\Delta \boldsymbol{x} = \boldsymbol{x}^p + \boldsymbol{x}^h \tag{20}$$

One possible particular solution can be $\boldsymbol{x}^p = \mathbf{F}^{\mathrm{T}}(\mathbf{F}\mathbf{F}^{\mathrm{T}})^{-1}\Delta \boldsymbol{f}$, this is the minimum 2-norm solution to the constraints. All the homogeneous solutions ($\boldsymbol{x}^h$ denotes any one of them) constitute the nullspace of constraints Jacobian, which can be expressed as arbitrary linear combinations of nullspace basis vectors:

$$\boldsymbol{x}^h = \mathbf{V}_2 \boldsymbol{\beta} \qquad \forall \boldsymbol{\beta} \tag{21}$$

Multiple matrix factorization methods can find such bases. For example the singular value decomposition gives an orthonormal set of basis vectors. However, It is preferable to have a sparse nullspace basis. In this paper, we adopt the work proposed by the authors of [14], who described a computationally efficient procedure for finding a sparse nullspace basis [15].

If we substitute $\Delta \boldsymbol{x} = \boldsymbol{x}^p + \boldsymbol{x}^h$ into the objective, then minimize it with respect to $\boldsymbol{\beta}$, we get the following necessary optimality condition:

$$\mathbf{V}_2^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{H}\mathbf{V}_2\boldsymbol{\beta} = \mathbf{V}_2^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{R}^{-1}(\Delta \boldsymbol{z} - \mathbf{H}\boldsymbol{x}^p) \tag{22}$$

In forming Equation (22), it is likely that the sparsity of the problem was destroyed. However one can show that, if there are no structural nonzeros in *both* $\mathbf{H}$ and $\mathbf{V}_2$, then the product $\mathbf{H}\mathbf{V}_2$ is still sparse in a sparsely connected distribution network. Equation (22) can be solved using numerical robust methods such as QR factorization:

$$\mathbf{R}^{-\frac{1}{2}}\mathbf{H}\mathbf{V}_2 = \mathbf{Q}\mathbf{U} = \begin{bmatrix}\mathbf{Q}_1 & \mathbf{Q}_2\end{bmatrix}\begin{bmatrix}\mathbf{U}_1 \\ \mathbf{0}\end{bmatrix} \tag{23}$$

The authors of [16] advocated the use of Givens rotation for computing the QR factorization when the matrix is large and sparse. After the QR factorization Equation (22) is reduced to:

$$\mathbf{U}_1\boldsymbol{\beta} = \mathbf{Q}_1^{\mathrm{T}}\mathbf{R}^{-\frac{1}{2}}(\Delta \boldsymbol{z} - \mathbf{H}\boldsymbol{x}^p) \tag{24}$$

where $\mathbf{U}_1$ is an upper triangular matrix. Note that in (24), forming the gain matrix $\mathbf{G}' = \mathbf{V}_2^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{H}\mathbf{V}_2$ was avoided. After $\Delta \boldsymbol{x}$ is obtained using (20), the overall iteration proceeds. When close to the optimal solution, $\Delta \boldsymbol{f} \to \mathbf{0}$, minimum 2-norm particular solution vanishes. In that case, substituting nullspace of constraints into the objective is sufficient for finding the optimal solution to the subproblem.

IV. Simulation Results

The proposed orthogonal elimination based DSSE algorithm is implemented on a modified IEEE 13-bus test feeder. Both state estimation results and numerical properties of the proposed method are presented in this section.

The IEEE 13-bus test feeder [17] is modified to include center-tapper transformers as follows.

- Node 650 is considered as the secondary side of the substation transformer. The primary side is assumed to be connected to an infinite bus. The voltage regulator is ignored.
- The circuit breaker between node 671 and 692 is removed.
- The uniformly distributed loads are removed from the feeder.
- Node 646 is assumed to be connected to a center-tapped transformer across phases b and c.

The simulations are set up as follows. First, load flow calculations are carried out using Newton-Raphson method. Noise corrupted measurements are then generated using the measurement models described in Section.II. Second, the measurements are fed into the DSSE algorithm. At last, the differences between the load flow results and the DSSE results are recorded. The simulations are repeated M times. The normalized root-mean-square errors of DSSE are computed.

$$|\tilde{V_i^p}| = \sqrt{\frac{\sum_{k=1}^{M}\left(\frac{|V_i^p| - |\hat{V_i^p}|_k}{|V_i^p|}\right)^2}{M}} \qquad \tilde{\theta_i^p} = \sqrt{\frac{\sum_{k=1}^{M}\left(\frac{\theta_i^p - \hat{\theta}_{i\,k}^p}{2\pi}\right)^2}{M}} \tag{25}$$

where ($|\hat{V}|$ $\hat{\theta}$) denote results of state estimation. The stopping criterion for the load flow calculation and the state estimation are set to be the same as follows: $\max_i |\Delta x_i| < 0.00001$ p.u..The measurement noise covariance matrix was set to be $\mathbf{R} = \mathrm{diag}\left[\sigma^2, \sigma^2, \cdots, \sigma^2\right]$. With $\sigma = 0.01$ p.u., the results of DSSE are shown in Table.I.

The simulation results show that under 0.01 p.u. measurement noise, most of the voltage magnitude estimation errors are less than 1%, and all of the voltage angle estimation errors are less than $1\% \cdot 2\pi$. Additional simulations are conducted to analyze the impact of measurement noise on state estimation errors. The standard deviation of the measurement noise σ is increased systematically from 0.0001% to 1.5%. Under each setting of σ, Monte Carlo simulations are conducted. The state estimation errors under each measurement noise setting are reported in Fig.3. In the figure, each curve represents the change of estimation error in response to measurement noise

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Table I
RMSE OF $|\hat{V}|$ AND $\hat{\theta}$ (PER UNIT)

Node \ Ph	$\lvert\tilde{V}\rvert$ a	$\lvert\tilde{V}\rvert$ b	$\lvert\tilde{V}\rvert$ c	$\tilde{\theta}$ a	$\tilde{\theta}$ b	$\tilde{\theta}$ c
ref	0.0092	0.0099	0.0089	0.0	0.0036	0.0056
650	0.0066	0.0074	0.0080	0.0020	0.0038	0.0029
646	-	0.0091	0.0061	-	0.0045	0.0031
645	-	0.0094	0.0061	-	0.0046	0.0031
632	0.0072	0.0058	0.0062	0.0021	0.0042	0.0031
633	0.0088	0.0069	0.0073	0.0022	0.0043	0.0033
634	0.0102	0.0088	0.0084	0.0022	0.0043	0.0033
611	-	-	0.0103	-	-	0.0044
684	0.0068	-	0.0067	0.0027	-	0.0038
671	0.0082	0.0073	0.0083	0.0025	0.0043	0.0034
675	0.0090	0.0087	0.0076	0.0032	0.0049	0.0040
652	0.0091	-	-	0.0039	-	-
680	0.0082	0.0073	0.0083	0.0025	0.0043	0.0034

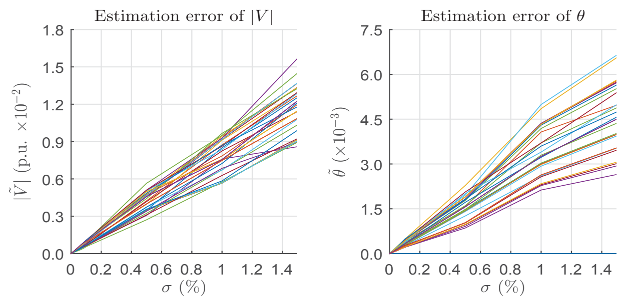

Figure 3. Estimation error v.s. measurement noise

for one node and one phase. As expected, the measurement errors increase as the measurement noise level increases.

The numerical stability and computational efficiency of the proposed algorithm is compared to that of the Hachtel's augmented matrix method. The numerical stability is evaluated by measuring the condition number of the coefficient matrix which is used in solving the linear equation (24). The computational efficiency of the proposed algorithm is evaluated by measuring the number of nonzero elements ($\mathbf{nnz}$) in the coefficient matrix. The $\mathbf{nnz}$ and condition number of coefficient matrices are reported in Tables II and III. As shown in Tables II and III, the coefficient matrix in the proposed algorithm has a smaller number of nonzero elements and lower condition number. This demonstrates that the proposed algorithm is more computational efficient and numerically stable than the Hachtel's augmented matrix method.

Table II
$\mathbf{nnz}$ OF COEFFICIENT MATRICES

	Augmented matrix	Orthogonal elimination
$\mathbf{nnz}$	1526	496

Table III
CONDITION NUMBER OF COEFFICIENT MATRICES ($\times 10^3$), $\sigma = 0.01$ P.U.

Method \ Iteration		$k=0$	$k=1$	$k=2$	$k=3$
Augmented matrix	$\alpha = 1$	18.628	21.871	24.747	23.678
	$\alpha = 0.1$	3.500	3.142	3.396	3.209
	$\alpha = 0.05$	6.998	4.574	4.727	5.058
Orthogonal elimination	-	0.937	1.069	1.073	1.029

V. CONCLUSION

In this paper, a distribution system state estimation problem is formulated considering unbalanced single-phase and two-phase measurements. Constraints associated with zero injections and center-tapped transformers are incorporated into the problem formulation. An orthogonal elimination based algorithm is developed to solve the DSSE problem. The proposed solution algorithm yields better numerical stability and computational efficiency than existing methods. The simulation results on a modified IEEE test feeder validated the accuracy, numerical stability, and computational efficiency of the proposed algorithm.

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