A Neural Network Model of Normal and Impaired Calculation

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Introduction

In recent years a variety of computational models of arithmetic fact retrieval have been proposed, primarily associative network models. Most of these proposed models have not been implemented, and those that have utilize complicated network designs in an effort to simulate experimental data. The initial aim of this study was to design a simple neural network model of single-digit addition and multiplication that would exhibit a pattern of performance in line with the behavioral data of normal subjects. The prominent findings from this data are a problem-size effect, that is, arithmetic problems get more difficult as the operands increase in value, and also an apparent alternate representation for problems containing 0 or 1 as operands.

Another goal of this research was to take into consideration findings from cognitive neuropsychology. Of particular interest were findings from *acalculia* a disturbance in calculation and number processing due to brain injury. After the model showed a typical 'normal' pattern of performance, it would be systematically damaged and its output compared to the empirical results of acalculics on simple arithmetic problems.

A simple three-layer feedforward network was utilized in this study, with backpropagation of error used as the learning algorithm. 30 hidden units were fully connected to all 19 input units (9 units for each of the operands and 1 unit for the operator) and 18 output units (9 units representing the first digit of the answer and 9 representing the second).

Experiment 1

The first study examined the performance of the normal model on single-digit addition and multiplication problems.

Method

The network was trained on single-digit addition and multiplication facts, proportionate to their frequency in popular children's mathematics textbooks. Patterns of performance were examined at different points in training.

Results and Discussion

Substantial training was needed to achieve perfect performance. An analysis of the error rates during training indicated that, as expected, addition problems were learned much faster than multiplication problems. A distinct problem-size effect was observed, with problems with larger operands requiring more training to achieve perfect performance than those with smaller operands, mimicking the performance of normal subjects, and supporting the view that the problem-size effect is due to frequency of presentation. Problems with larger operands were typically less frequent in training than those with smaller operands. However, this is not the case for problems containing 0 or 1 as an operand. Despite the relative scarcity of these problems in the training set, the network quickly achieves excellent performance, suggesting that the network is able to extract a separate rule for these type of problems.

Experiment 2

The second study examined the types of errors made by a lesioned model, and compared these errors to those made by acalculics.

Method

The model was damaged to various degrees in three locations - the hidden units, the connections between input and hidden units, and to the connections between hidden and output units.

Results and Discussion

An examination of the error rates of the three different forms of damage indicate that damage to the hidden units produced the most 'acalculic-like' symptoms of the three. When these hidden units are damaged, the types and proportions of errors produced bear close similarity to the types and rates of calculation errors typically made by acalculics. Another pattern which emerged is that problems involving 0 or 1 as operands were very resistant to damage, a dissociation which has been observed in acalculic patients.

Conclusions

The model, despite its simplicity, displayed a pattern of results very similar to those exhibited by experimental subjects, both normal and impaired. These results suggest that it is plausible to posit a simple associative network as a basis for arithmetic fact retrieval. It also provided possible explanations for some empirical phenomena, such as the problem-size effect and the differential error rates for 0 and 1 operand problems, as well as suggesting that acalculics suffer from damage in the representation of arithmetic facts, as opposed to damage in the connections between input and output nodes.