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Risk Neutral Investors
Do Not Acquire Information[*]

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Abstract

Give a risk neutral investor the choice to acquire a costly signal prior to
Walrasian asset market equilibrium. She refuses to pay for the signal. The
reason is that a risk neutral investor is indifferent between a risky stock
or a safe bond in equilibrium and expects the same return to her portfo-
lio *ex ante*, whether or not she acquires information. Risk neutral asset
pricing thus implies the absence of costly information from asset price,
unless non-Walrasian market conditions prevail. Non-Walrasian market
conditions, however, get reflected in price beyond the asset's fundamental
payoff value.

Keywords: information acquisition; risk neutrality; portfolio choice; ra-
tional expectations equilibrium

JEL classiciation: D81, D83, G14

Risk neutral investors play an important role in theoretical and empirical models of financial markets. Risk neutral investors are often assumed to be informed. On the theoretical side, Kyle (1985), Glosten and Milgrom (1985) and Back and Baruch (2004), for instance, consider risk neutral but informed investors; Leland (1992) and Repullo (1999) consider informed insiders. Risk neutral valuation of assets remains a benchmark pricing approach in finance (Stutzer 1996, Collin-Dufresne, Goldstein and Hugonnier 2004). Based on risk-neutral valuation models, empirical techniques attempt to extract market information from asset price (Jackwerth and Rubinstein 1996). Soderlind and Svensson (1997) survey the use of such techniques for policy analysis. In these frameworks, however, risk neutrality poses a considerable theoretical challenge.

Consider a standard model of portfolio and consumption choice, linear in consumption to reflect risk neutrality. Make one change: Add a choice of information acquisition *ex ante*. The risk neutral investor refuses to pay for information under general conditions (and irrespective of strategic uncertainty). So, risk neutral asset valuation implies the absence of costly information from asset price.

The intuitive reason is that a risk neutral investor is indifferent whether she holds a risky asset or a safe bond in her portfolio. Hence, she expects her actions upon signal realizations to yield the same return *ex ante* as uninformed actions do, which makes her indifferent to signals. She will accept signals for free, but refuse to incur any cost of information acquisition.

Risk neutral asset valuation typically relies on standard utility functions and Walrasian markets. Non-standard utility or non-Walrasian market conditions, however, are needed for a risk neutral investor to acquire information. Unless investors are credit constrained, or markets cease to clear, or equilibrium price fails to aggregate information, or utility is not intertemporally additive—unless these or similar conditions supersede Walrasian equilibrium, a risk neutral investor will neither spend money to obtain information nor sacrifice leisure to process information.

A theorem and its discussion below clarify which non-Walrasian assumptions are crucial for risk neutral investors to acquire information. Non-Walrasian market conditions can create incentives for risk neutral investors to buy information but those conditions are then reflected in asset price beyond the fundamental payoff value of the asset.

1   No acquisition of financial information

Consider two periods, today 0 and tomorrow 1, and two assets: one bond b with safe gross return $R = 1 + r$ and one stock x with risky payoff θ. No assumption is placed on the distribution of θ. So, the model applies to many types of risky

securities and derivative assets.

A signal s^i has been sent to investor i and informs her about the risky asset return tomorrow. A risk neutral investor maximizes the expected net present value of consumption

$$\mathbb{E}\left[C_0^i + \beta\, C_1^i | s^i\right],$$

where β is the discount factor and C_t^i consumption of investor i at time t. The budget constrains consumption today to be $C_0^i = q_0^i - (b_1^i + P_0 x_1^i)$, where b_1^i and x_1^i are investor i's choices of bond and stock holdings, P_0 is the price of the stock, and $q_0^i = b_0^i + P_0 x_0^i$ is investor i's initial portfolio endowment. Tomorrow, consumption will be $C_1^i = Rb_1^i + \theta x_1^i$. The first-order conditions for a risk neutral investor's optimal portfolio choice are

$$R = 1/\beta \qquad \text{and} \qquad P_0 = \beta\, \mathbb{E}\left[\theta | s^i\right] \tag{1}$$

for the bond and the stock.

The two single-period budget constraints imply the intertemporal budget constraint

$$C_0^i + \tfrac{1}{R}C_1^i = q_0^i + \tfrac{1}{R}\left(\theta_1 - RP_0\right) x_1^i, \tag{2}$$

which holds with certainty. The net present value of an investor's consumption stream equals the value of the initial endowment q_0^i plus the (discounted) excess return $\theta_1 - RP_0$ of the optimal stock holdings x_1^i beyond the opportunity cost of holding the bond.

Having received a signal realization s^i, investor i can assess the impact that the signal realization has on the expected net present value of her optimal consumption. By (2), she considers

$$C_0^{i*} + \tfrac{1}{R}\mathbb{E}\left[C_1^{i*}|s^i\right] = \mathbb{E}\left[q_0^i|s^i\right] + \tfrac{1}{R}\mathbb{E}\left[\theta_1 - RP_0\,|\,s^i\,\right] x_1^{i*} \tag{3}$$

to vary with s^i because her optimal asset demands b_1^{i*} and x_1^{i*} respond to the signal realization.

Suppose investor i is asked to pay for her signal s^i. How much will she pay? The signal realization s^i is still unknown to her (she would not pay for something known). To evaluate the signal, the investor rationally anticipates her expected asset-demand response to the signal S^i (if she did not anticipate to act on the signal it would have no value to begin with). By (3) and iterated expectations, she considers

$$\mathbb{E}_{S^i}\left[C_0^{i*}\right] + \tfrac{1}{R}\mathbb{E}_{S^i}\left[C_1^{i*}\right] = \mathbb{E}_{S^i}\left[q_0^i\right] + \tfrac{1}{R}\mathbb{E}_{S^i}[(\theta_1 - RP_0)\,x_1^{i*}] \tag{4}$$

before she learns the signal realization s^i.

The investor considers in particular that her presence in the asset market equalizes the safe return R to the inverse discount rate, and equalizes the risky

asset price P_0 to $\mathbb{E}[\theta|s^i]/R$ by (1). She knows that, otherwise, her risk neutral asset demand would be negative infinite or positive infinite and that unbounded demand cannot be an outcome in standard asset market equilibrium. So, $\mathbb{E}_{S^i}[(\theta_1 - RP_0)\,x_1^{i*}] = 0$ and investor i's expected utility before she receives the signal realization becomes

$$\mathbb{E}_{S^i}\left[C_0^i\right] + \beta\,\mathbb{E}_{S^i}\left[C_1^i\right] = \mathbb{E}_{S^i}\left[q_0^i\right].$$

She rationally anticipates that anything she finds out from the signal realization about the stock's excess return $\mathbb{E}[\theta_1|s^i] - RP_0$ will be wiped out in equilibrium because equilibrium price equals the expected return in her presence.

Her only benefit from a signal can come from an endowment revaluation $\mathbb{E}_{S^i}[q_0^i] = b_0^i + \mathbb{E}_{S^i}[P_0]\,x_0^i$ as her response to the signal changes asset price in market equilibrium. The expected equilibrium price level, however, is the same with and without a signal by iterated expectations: $\mathbb{E}_{S^i}[P_0] = \beta\mathbb{E}_{S^i}[\mathbb{E}[\theta|S^i]] = \beta\mathbb{E}[\theta]$ under (1). So, to a risk neutral investor, there is no expected benefit from a signal.

This is true more generally.

Theorem 1 *Suppose signals are costly. Then an investor with intertemporally separable von-Neumann-Morgenstern utility acquires a signal prior to Walrasian rational expectations equilibrium in financial markets only if she is not risk neutral.*

Proof. Investor i has intertemporally separable von-Neumann-Morgenstern utility

$$U_t^i = \mathbb{E}\left[\sum_{s=t}^{t+T}\beta^{s-t}u(C_s^i)\,\big|\,\mathcal{F}_t^i\right],$$

where instantaneous utility $u(C)$ is linear and strictly increases in C, she lives T periods (possibly $T\to\infty$) and $\mathcal{F}_t^i$ is her information set at t. Denote the ex-dividend price of the stock in period s with P_s. Then her intertemporal budget constraint is

$$b_{s+1}^i + P_s x_{s+1}^i = R_s b_s^i + (\theta_s + P_s)x_s^i - C_s^i.$$

Forward-iterate the budget constraint to find the net present value of consumption

$$\sum_{s=t}^{t+T} R_{t,s}^{-1}\,C_s^i = R_t q_t^i + \sum_{s=t}^{t+T} R_{t,s}^{-1}\left(\theta_s + P_s - R_s P_{s-1}\right)x_s^i - R_{t,T}^{-1}\,q_{t+T+1}^i, \qquad (5)$$

satisfied with certainty, where $R_{t,s}^{-1} \equiv (\Pi_{\tau=t+1}^s R_\tau)^{-1}$ and $q_t^i = b_t^i + P_t x_t^i$. In optimum, $q_{t+T+1}^{i*} = 0$ (for $T\to\infty$ by the transversality condition).

For a risk neutral investor, $u'(C)$ is a constant and the Euler equations for optimal portfolio choice become

$$R_{s+1} = 1/\beta \qquad \text{and} \qquad \mathbb{E}\left[P_s | \mathcal{F}_t^i\right] = \beta \, \mathbb{E}\left[\theta_{s+1} + P_{s+1} | \mathcal{F}_t^i\right]. \qquad (6)$$

The expected net present value of consumption, the expectation of (5), is equivalent to von-Neumann-Morgenstern utility, which turns into

$$U_t^i = \sum_{s=t}^{t+T} \beta^{s-t} \, \mathbb{E}\left[C_s^i | \mathcal{F}_t^i\right] = \mathbb{E}\left[q_t^i | \mathcal{F}_t^i\right]/\beta = b_t^i/\beta + \mathbb{E}\left[P_t | \mathcal{F}_t^i\right] x_t^i/\beta \qquad (7)$$

under Euler conditions (6). By iterated expectations, $\mathbb{E}\left[\mathbb{E}\left[P_t | \mathcal{F}_t^i\right]\right] = \mathbb{E}\left[P_t\right]$ so that expected utility $U_t^i = \mathbb{E}_{\mathcal{F}}[U_t^i]$ is identical in the presence and in the absence of the expected receipt of a signal. ∎

For a risk neutral investor, financial information has no utility value.

2 Discussion

Theorem 1 does not apply to risk aversion. For a risk averse investor, information offers an additional benefit because it reduces the *ex ante* expected variance of future consumption. By variance decomposition, $\mathbb{E}_S\left[\mathbb{V}\left(\theta | S\right)\right] = \mathbb{V}\left(\theta\right) - \mathbb{V}_S\left(\mathbb{E}\left[\theta | S\right]\right)$.

Theorem 1 is based on von-Neumann-Morgenstern utility and a set of assumptions on asset market equilibrium. Some assumptions are not necessary. The proof invoked the law of iterated expectations; but it's failure under strategic uncertainty would not make private information valuable to risk neutral investors. An investor's age was assumed to be known in the proof of Theorem 1; but uncertainty about life expectancy does not change the result. Other assumptions are crucial: utility is intertemporally additive, markets clear, and there is no excess demand for the risky asset in equilibrium. These assumptions are common in risk neutral valuation approaches to asset pricing (e.g. Stutzer 1996, Collin-Dufresne et al. 2004). They need not hold, however, and deserve some scrutiny. Models such as Jackson and Peck (1999) or Barlevy and Veronesi (2000), where risk neutral investors acquire information, remove at least one of the key assumptions.

2.1 Necessary assumptions for the no-acquisition result

Utility can be intertemporally non-separable in many forms. Consider a risk neutral investor i whose discount rate β^i is state dependent (it may depend on her uncertain state of health) and not revealed before the resolution of the asset

return. Signals inform her about both her expected utility parameters and the asset return. Then the expected net present value of her optimal consumption exhibits a correlation between her discount rate and asset return,

$$\mathbb{E}\left[q^i|s^i\right] + \mathbb{E}\left[\beta^i(\theta - RP)|s^i\right] x^{i*},$$

replacing the right-hand side of (4). If a signal reveals joint information on utility parameters and asset returns, it can have a positive utility value for a risk neutral investor. Risk neutral valuation techniques, however, do typically not deal with investor heterogeneity, which is now reflected in asset price.

Credit constrained investors value information even if they are risk neutral (Barlevy and Veronesi 2000). The credit constraint removes from the first-order condition its knife-edge property—by which asset price equals the expected return $P = \beta\mathbb{E}\left[\theta|s\right]$, else demand becomes positive or negative infinite. If investor i lacks resources to go long in the asset, she has to accept $P < \beta\mathbb{E}\left[\theta|s^i\right]$ (reflected in a strictly negative Lagrange-multiplier under a Kuhn-Tucker approach). If credit constraints happen to bind all risk neutral investors in equilibrium, a strictly positive excess return prevails. As a result, expected excess return $\mathbb{E}_{S^i}\left[(\theta - RP)x^{i*}\right]$ is non-zero *ex ante* and signal acquisition becomes worthwhile for a risk neutral investor. The asset price, however, now depends on the initial wealth distribution and reflects more than investors' market information.

Market clearing or Walrasian demand aggregation need not be satisfied. Froot, Scharfstein and Stein (1992) make information valuable to risk neutral investors by not permitting the market to clear immediately. Instead, half of the orders is randomly deferred to a future period. In Jackson and Peck (1999), risk neutral investors simultaneously submit bid and offer functions in a Shapley and Shubik (1977) market game. This disconnects asset price from information on the fundamental because investors submit bids based on their own information and the anticipated bids of others, without being able to condition on equilibrium price. The first order condition $P = \beta\mathbb{E}\left[\theta|s\right]$ fails in Froot et al. (1992) and Jackson and Peck (1999), so market clearing does not necessarily result in a price that reflects the expected value of the asset. As a consequence for risk neutral asset valuation, asset price omits market information.

2.2   Unrelated assumptions and the no-acquisition result

The law of iterated expectations can fail under strategic uncertainty (e.g. Morris and Shin 2002). Average market expectations across investors may differ from an individual investor's higher-order expectations (higher-order expectations are an investor's expectations of others' expectations of others' expectations, and so fourth, about the fundamental). The presence of strategic uncertainty, however,

does not alter a risk averse investor's valuation of a signal: the signal continues to have no value. Under strategic uncertainty, individual expectations of market price $\mathbb{E}^i \left[\bar{\mathbb{E}} \left[P \right] \right]$ (where superscript i is a shorthand for investor i's information set) do not simplify to the average of investors' expectations of market price $\bar{\mathbb{E}} \left[P \right]$. Yet, for a risk neutral investor, private information does not result in a utility improvement *ex ante* because a private signal only adds precision over publicly available information, leaving the expected equilibrium price unaffected (see appendix A for a formal proof). Risk neutral investors are neutral towards the precision of their information. So, even under strategic uncertainty, financial information has no utility value.

Lifetime T was taken to be certain in the proof of Theorem 1. As long as utility is additively separable, however, uncertainty about T does not alter the result. Irrespective of life time, the excess return of any future period is wiped out by the first-order condition (6). Whether alive or not, a risk neutral investor earns no expected excess return so that lifetime is irrelevant.

A competitive fringe of risk neutral traders or market makers is part of several microstructure models of financial information (e.g. Kyle 1989, Hirshleifer, Subrahmanyam and Titman 1994, Vives 1995). Market makers observe aggregate demand. One might argue that the costs of information acquisition for market makers are zero because information on aggregate demand is just a byproduct of their market making. If so, market makers' risk neutrality would not impede their information acquisition. Market makers' information on aggregate demand, however, is secondary information in that it derives from the primary information behind informed investors' demands. Those informed investors cannot be risk neutral, otherwise they would not acquire information.

3 Concluding remarks

How much income or leisure does a risk neutral investor give up to acquire information? Under general Walrasian market conditions conditions, the answer is no income and no leisure at all. A risk neutral investor is indifferent between holding a risky stock or a safe bond in Walrasian equilibrium. Hence, she expects her actions upon signal realizations to yield the same return *ex ante* as uninformed actions do. This makes signals useless to her.

Risk neutral asset valuation under Walrasian market conditions (Stutzer 1996, Collin-Dufresne et al. 2004) therefore suffers from the theoretical challenge that, under risk neutrality, asset price does not contain costly information. The no-acquisition result also suggests that findings in the literature on optimal experimentation (Bolton and Harris 1999, Cripps, Keller and Rady 2005), where agents are risk neutral, may be limited to specific markets. Relaxing necessary

conditions for the no-acquisition result clarifies the circumstances under which information acquisition can occur in models with risk neutral investors (Jackson and Peck 1999, Barlevy and Veronesi 2000).

When dropping Walrasian assumptions to make costly information acquisition consistent with risk neutrality, price starts to reflect more than the fundamental payoff value of the asset: under non-separable utility, asset price captures heterogeneity in investors' time preferences; under credit constraints, asset price depends on investors' initial wealth distribution; under non-Walrasian market clearing, asset price reflects only a fragment of investors' information on the asset's fundamental. When substituting risk neutrality for risk aversion, on the other hand, the asset price reflects the degree of risk aversion beyond the fundamental value of the asset. A purely payoff-based asset price that also reflects costly information does not exist.

Appendix

A   Strategic uncertainty

Consider a continuum of investors $i \in [0,1]$, private signals $s^i = \theta + \epsilon^i$ with precision $\alpha = 1/\sigma_\epsilon^2$, and a public signal $y = \theta + \eta$ with precision $\gamma = 1/\sigma_\eta^2$ as in Morris and Shin (2002). Investors receive these signals only once, in the initial period t. To make the multi-period setting of Theorem 1 comparable to the one-period setting of Morris and Shin (2002), suppose that nature draws one unique θ but adds white noise ν_s to the asset return every period s. Then, under multivariate normality, individual return expectations and average market expectations become

$$\mathbb{E}^i\left[\theta|s^i, y\right] = \frac{\alpha y + \gamma s^i}{\alpha + \gamma} \qquad \text{and} \qquad \bar{\mathbb{E}}\left[\theta\right] = \int_0^1 \mathbb{E}^i\left[\theta|s^i, y\right] \mathrm{d}i = \frac{\alpha y + \gamma \theta}{\alpha + \gamma}$$

for every period. So, an investor's expectation of market expectations and second-order market expectations are

$$\mathbb{E}^i\left[\bar{\mathbb{E}}\left[\theta\right]\right] = \frac{[(\alpha + \gamma)^2 + \gamma^2]y + \gamma s^i}{(\alpha + \gamma)^2} \quad \text{and} \quad \bar{\mathbb{E}}^2\left[\theta\right] = \frac{[(\alpha + \gamma)^2 + \gamma^2]y + \gamma \theta}{(\alpha + \gamma)^2}.$$

More generally, market expectations and individual expectations of market expectations of order k are (Morris and Shin 2002, Lemma 1)

$$\mathbb{E}^i\left[\bar{\mathbb{E}}^k\left[\theta\right]\right] = (1 - \mu^{k+1})y + \mu^{k+1}s^i \quad \text{and} \quad \bar{\mathbb{E}}^{k+1}\left[\theta\right] = (1 - \mu^{k+1})y + \mu^{k+1}\theta, \quad (8)$$

where $\mu \equiv \gamma/(\alpha + \gamma)$.

Asset price obeys first-order condition (6) in the presence of a risk neutral investor so that

$$
\begin{aligned}
\mathbb{E}^i\left[P_t\right] &= \beta \mathbb{E}^i\left[\theta_{t+1}\right] + \beta^2 \mathbb{E}^i\left[\bar{\mathbb{E}}\left[\theta_{t+2}\right]\right] + \beta^3 \mathbb{E}^i\left[\bar{\mathbb{E}}^2\left[\theta_{t+3}\right]\right] + \ldots \\
&= \beta \sum_{s=t+1}^{\infty} \beta^{s-(t+1)}\mathbb{E}^i\left[\bar{\mathbb{E}}^{s-(t+1)}\left[\theta_s\right]\right]
\end{aligned}
$$

by recursion and under the assumption that the order of expectations increases period by period. By (8) and stationarity, this implies

$$
\begin{aligned}
\mathbb{E}^i\left[P_t|y, s^i\right] &= \sum_{k=0}^{\infty} \beta^{k+1}\mathbb{E}^i\left[\bar{\mathbb{E}}^k\left[\theta\right]\Big|y, s^i\right] = \sum_{k=0}^{\infty} \beta^{k+1}\left[(1 - \mu^{k+1})y + \mu^{k+1}s^i\right] \\
&= \frac{\beta}{1 - \beta}\frac{\alpha\, y + (1 - \beta)\gamma\, s^i}{\alpha + (1 - \beta)\gamma}.
\end{aligned}
\qquad (9)
$$

When asked to pay for her signal s^i, investor i considers expected utility

$$U_t^i = b_t^i/\beta + \mathbb{E}^i\left[P_t|y, s^i\right] x_t^i/\beta,$$

as in (7), and takes the *ex ante* expectations over all possible signal realizations. Note that $\mathbb{E}_{S^i}\left[s^i|y\right] = \mathbb{E}\left[\theta|y\right] = y$. So, the *ex ante* expectation of asset price is

$$\mathbb{E}_{S^i}\left[\mathbb{E}^i\left[P_t|S^i\right]|y\right] = \mathbb{E}^i\left[P_t|y\right] = \frac{\beta}{1-\beta}y$$

by (9)—exactly what it would be if the investor acquired no private signal. So, even under strategic uncertainty, financial information has no utility value for a risk neutral investor.

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