Title
Scalar Mass Relations and Flavor Violations in Supersymmetric Theories

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Publication Date
1996-05-01

LBL-38566
UC-414

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Physics Division

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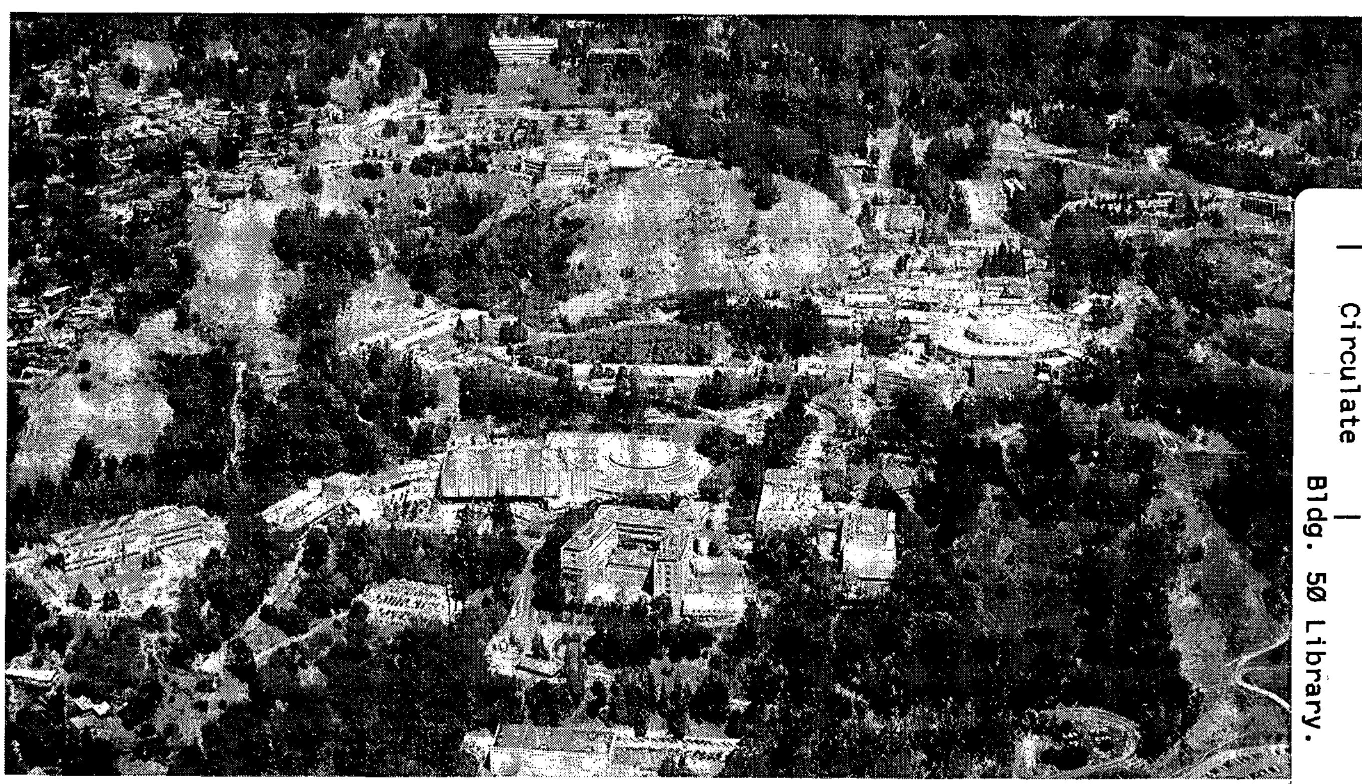
(Ph.D. Thesis)

May 1996



Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098

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May 9, 1996

LBL-38566
UCB-PTH-96/11

Scalar Mass Relations and Flavor Violations in Supersymmetric Theories*

(Ph.D. Dissertation, University of California at Berkeley)

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Abstract

Supersymmetry provides the most promising solution to the gauge hierarchy problem. For supersymmetry to stablize the hierarchy, it must be broken at the weak scale. The combination of weak scale supersymmetry and grand unification leads to a successful prediction of the weak mixing angle $\sin^2\theta_W$ to within 1% accuracy. If supersymmetry is a symmetry of nature, future experiments will discover many new particles, in particular the superpartners of all the quarks and leptons. The mass spectrum and the flavor mixing pattern

*This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

of these scalar quarks and leptons will provide important information about a more fundamental theory at higher energies.

We studied the scalar mass relations which follow from the assumption that at high energies there is a grand unified theory which leads to a significant prediction of the weak mixing angle. Two intragenerational mass relations for each of the light generations are derived. In addition, a third mass relation is found which relates the Higgs masses and the masses of all three generation scalars. These relations will serve as important tests of grand unified theories.

The gauge interactions of any supersymmetric extension of the standard model involve new flavor mixing matrices. In a realistic supersymmetric grand unified theory, nontrivial flavor mixings are expected to exist at all gaugino vertices. This could lead to important contributions to the neutron electric dipole moment, the decay mode $p \rightarrow K^0\mu^+$, weak scale radiative corrections to the up-type quark masses, and lepton flavor violating signals such as $\mu \rightarrow e\gamma$. These also provide important probes of physics at high energy scales.

Supersymmetric theories involving a spontaneously broken flavor symmetry can provide both a solution to the supersymmetric flavor-changing problem and an understanding of the fermion masses and mixings. We studied the possibilities and the general conditions under which some fermion masses and mixings can be obtained radiatively. We also constructed theories of flavor in which the first generation fermion masses arise from radiative corrections while flavor-changing constraints are satisfied.

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Acknowledgements

I would like to express my most sincere gratitude to my adviser, professor Lawrence J. Hall, for his help and encouragement throughout the years of my graduate study. His perceptiveness in physics and warm attitude have always inspired me in my work of research. It was such a joyful experience to work under his guidance. Any of my progress during this period and in particular this work should attribute mostly to his enthusiatic and knowledgeable instruction.

I would like to thank professors Mahiko Suzuki, Hitoshi Murayama, for teaching me a lot of things and answering many questions. Thanks to postdocs Chris Carone, Jonathan Feng and Takeo Moroi for much useful discussion and help. Special thanks go to my fellow student, Nima Arkani-Hamed, with whom most of my work is collaborated with. We had such a wonderful collaboration that I wish it could last forever.

I am grateful to the University of California and Lawrence Berkeley National Laboratory for providing such a nice atmosphere for me to complete my graduate study. I also want to thank Betty Moura, Barbara Gordon, Luanne Neumann from the LBNL and Anne Takizawa, Donna Sakima from the Physics Department for their help during the years.

Finally, I want to thank my parents Sih-Chong Cheng and Shu-Chen Liu, my brother Hsin-Wei Cheng for their love and support, and all my friends for their encouragement.

This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

Chapter 1

Preamble

Two fundamental questions of the standard model of particle physics are the origin of the electroweak symmetry breaking and the pattern of fermion masses and mixings. In standard model, the electroweak symmetry breaking occurs because the scalar Higgs doublet acquires a nonzero vacuum expectation value (VEV). However, in quantum field theory, the loop corrections to scalar particle masses are quadratically divergent, so the scale of the electroweak symmetry breaking is unstable against radiative corrections, and therefore the huge hierarchy between the electroweak symmetry breaking scale and the Planck scale is a mystery. Weak scale supersymmetry [1] offers the most promising solution to this problem. In supersymmetric theories, the quadratic divergences cancel between fermion loops and boson loops. Therefore, the extension of standard model with supersymmetry softly broken at the weak scale does not suffer from the problem of unstable hierarchy. In addition, the electroweak symmetry breaking is triggered by the dynamics of a heavy top quark [2] instead of being put in by hand. Also, the grand unification prediction for the weak mixing angle is highly successful if there are superpartners at the weak scale. The experimental discovery of superpartners would represent enormous progress in understanding the electroweak symmetry breaking. It could also offer us a great handle of physics at very high energies through studying these superparticles.

In any supersymmetric extension of the standard model, the superpartners of the quarks and leptons must be given masses. There will be many new flavor parameters in the scalar quark and lepton masses and mixings in addition to those in the quarks and leptons. If these parameters are arbitrary, in general, they will cause flavor-changing processes which are severely constrained by experiments. Hence, these parameters must be constrained by physics at higher energies. That is, physics at high energies will leave some imprints on these low energy parameters. Studying these scalar quark and lepton masses and mixings will provide us an important probe of physics at more fundamental levels.

Unification of fundamental physics has always been a goal for physicists. In grand unified theories (GUT's), the standard model gauge group $SU(3) \times SU(2) \times U(1)$ is beautifully unified into a simple group $SU(5)$ or $SO(10)$. The fermions of a single family fit exactly into a $\bar{\mathbf{5}}$ and a $\mathbf{10}$ representations of $SU(5)$, or with an additional right-handed neutrino, into a single $\mathbf{16}$ spinorial representation of $SO(10)$. Supersymmetry, which solves the hierarchy problem, allows us to extrapolate physics up to very high energies, and hence makes the dicussions about unification at extremely high energies sensible. In fact, the combination of weak scale supersymmetry and grand unification give a successful prediction of the weak mixing angle. This is a strong hint that supersymmetric unification might be realized in nature. However, it is only one prediction and it could be an accident. In chapters 2 and 3 we study other signals of supersymmetric unification, which will serve as evidence for or against grand unification.

Fermion masses and mixings have been studied to provide tests of grand unified theories [3, 4, 5]. However, chiral and guage symmetry breaking effects can mask the grand unified symmetry relations for the fermions. On the other hand, scalar particles can have symmetry-preserving masses and therefore provide a more reliable test of supersymmetric unification. In chapter 2, we study the scalar quark and lepton spectrum in supersymmetric theories with the assumption that there is a GUT at high energies with the successful prediction of weak mixing angle preserved. Because the scalar particles belonging to the same multiplet have the same mass at the GUT scale and the scaling from the GUT scale to the weak scale is known, these scalar masses at low energy are related to each other. We derive two intragenerational relations for each of the light generations. In addition, a third mass relation is found which relates the Higgs masses and the masses of all three generation scalars. After the scalar quarks and leptons are discovered, verification of these mass relations will provide an important test for grand unification.

Flavor and CP violations can also provide important probes of supersymmetric grand unified theories [6, 7, 8, 9]. Because the leptons and the quarks lie in the same multiplets in unified theories, there will be new flavor and CP violating effects in the lepton sector as well as in the quark sector. In chapter 3, we study these new flavor and CP violating effects in a realistic supersymmetric unified theories. In constrast with the minimal models, there are non-trivial flavor mixings in the up quark sector. This can lead to important contributions to the neutron electric dipole moment and to the proton decay mode $p \rightarrow K^0 \mu^+$ and suggests that there may be important weak scale radiative corrections to the up-type quark mass matrices. The lepton flavor violating signal $\mu \rightarrow e\gamma$ is studied in the large $\tan\beta$ (defined in later chapters) scenario and puts a strong constraint on the slepton masses.

As mentioned above, if the scalar masses and mixings are arbitrary, they will induce unacceptable flavor-changing effects, this is known as the supersymmetric flavor problem. A complete supersymmetric theory of flavor should address both this problem and the fermion mass problem. The smallness of supersymmetric contributions to the flavor-changing effects could be related to the smallness of the light generation Yukawa couplings. Flavor symmetries should forbid Yukawa couplings of the light generation fermions. After the flavor symmetries are broken, the light generation fermions can acquire small Yukawa couplings from the symmetry breaking effects. One usual way to get small Yukawa couplings is the Froggatt-Nielsen mechanism [10], which generates small numbers from the ratio of two different energy scales. Another attractive way is to generate the light fermion masses through radiative corrections, which automatically incorporates the small number $\frac{1}{16\pi^2}$ from the loop factor. In chapter 4, we show that supersymmetry has the right ingredients for radiative fermion masses. We study the possibility that some of the light fermion masses arise from the loop corrections due to the flavor-violations in scalar masses, while the flavor-changing constraints are still satisfied. We also construct a supersymmetric theory of flavor in which the fermion mass and mixing pattern arise naturally and the first generation fermion masses come from radiative corrections.

Chapter 2

Scalar mass relations in grand unified theories

2.1 Introduction

If supersymmetry (SUSY) is a symmetry of nature, broken only at the weak scale, then future experiments will discover many extra particles, in particular the superpartners of all the quarks and leptons. The masses of these scalar quarks and leptons will provide extra clues about a more fundamental theory at higher energies. However, whereas the quark and lepton masses provide information on how chiral and flavor symmetries are broken, the squark and slepton masses will provide a window to the structure of supersymmetry breaking.

It may be that the squark and slepton spectrum will show no clear pattern or regularities, and the origin of the spectrum will become a major puzzle, rather like the present situation with quark and lepton masses. However, much attention has been focussed on a single theory, the minimal supersymmetric standard model (MSSM), in which a very clear pattern emerges in the scalar spectrum. By the MSSM we will mean the supersymmetric extension of the standard model with minimal field content, which has a boundary condition near the Planck scale that the soft supersymmetry breaking mass parameters for the scalars are all equal. In this model, the physical masses of the 14 squarks and sleptons of the lighter two generations are given in terms of just 5 unknown parameters: the universal scalar masses at the Planck scale, m_0^2, the three gaugino masses, M_a, and the ratio of electroweak breaking VEV's, $\tan\beta = v_2/v_1$. Due to effects of large Yukawa couplings, the physical squark and slepton masses of the heaviest generation depend on one further parameter, the triscalar coupling A. Although these effects are well understood and can easily be added, for simplicity, we consider only the lightest two generations. Thus the MSSM has many relations amongst the scalar masses. However, the question as to why all scalars are assumed degenerate at the Planck scale becomes extremely important. If experiments are done to check the validity of the scalar mass relations of the MSSM [11], what is the fundamental principle which is being tested?

Flavor-changing processes provide considerable experimental constraints on the form of the squark and slepton mass matrices [12, 13, 14]. However, these constraints are intimately connected with flavor violation and provide constraints between the masses of scalars of different generations. For a given generation there are five independent gauge invariant squark and slepton masses: $m_Q, m_{U^c}, m_{D^c}, m_L$ and m_{E^c}, where Q and L represent $SU(2)$ doublet squarks and sleptons, while U, D and E are $SU(2)$ singlet squarks and sleptons. Certainly the flavor-changing constraints do not constrain the ratios $m_Q : m_U : m_D : m_L : m_E$, and it is largely these ratios which will be addressed in this chapter.

The assumption of a universal scalar mass at high energies originated from studies of $N = 1$ supergravity theories in which supersymmetry is broken in a hidden sector. The scalar mass was found to be universal in particular models [15, 16] and also in a wide class of models [17]. However, the universal mass is not a general property of supergravity models, and involves an assumption about the form of the Kähler potential. If there are N fields in the observable sector of the theory, an $SU(N)$ invariance of the Kähler potential guarantees the universality of the scalar masses at the Planck scale [17]. However, this symmetry is clearly broken elsewhere in the theory, and so the universality of the scalar masses can only be understood as a special property of certain supergravity theories. If the scalar mass relations of the MSSM were violated, it might simply mean that the Kähler potential does not possess this $SU(N)$ invariance.

In this chapter we study squark and slepton mass relations which follow from two assumptions, which have nothing to do with supergravity.

(1) The standard model is unified into a grand unified theory.

It is well known that a grand unified symmetry, together with supersymmetry, has yielded a successful relation amongst the gauge couplings of the standard model [18]. Much attention has also been given to quark and lepton mass relations which can follow from a grand unified symmetry. It therefore seems well worthwhile studying what squark and slepton mass relations might follow purely from grand unification.

(2) The generation changing entries in the squark and slepton masses (in a basis where the quark and lepton masses are diagonal) are sufficiently small not to affect the scalar mass eigenvalues at a level of accuracy to which the mass relations will be experimentally tested.

In fact, the latter is hardly an assumption, such large flavor-changing effects[1]

[1] The flavor-changing effects are studied in later chapters.

are almost certainly experimentally excluded. Since the grand unified symmetry acts within a generation, we expect relations amongst squark and slepton masses of the same generation, we do not expect any relations between masses of particles in different generations.

We begin section 2.2 by writing down the mass relations between squarks and sleptons of a given generation which occur in the MSSM. We then list the assumptions which a supersymmetric grand unified theory (SGUT) must satisfy for a successful weak mixing angle prediction to occur at the 1% level. Finally, we show that, with these assumptions, we are able to derive two intragenerational scalar mass relations. The mass relation of the MSSM which relates the masses of the two charged sleptons within a generation may be violated. This is a particularly important mass relation since it is likely that the squarks will be much heavier than the sleptons, and this will be the first mass relation of the MSSM to be tested. In section 2.3 we study the extent to which this mass relation is expected to follow if the GUT gauge groups includes $SO(10)$. While this slepton mass relation is generically expected as a consequence of the $SO(10)$ gauge symmetry, we find that radiative corrections and additional D term contributions to the scalar masses, beyond those of the MSSM, may lead to its violation. In section 2.4 we show that even if the additional D term contributions do not arise at tree level, they could be generated by radiative corrections. In section 2.5 we show that these extra D^2 interactions found in $SO(10)$ could lead to an easing of the fine tunning problem which has been found when the MSSM has large $\tan\beta$ and the universal scalar mass boundary condition.

2.2 Scalar mass relations in a class of grand unified theories

Before studying grand unified theories, we give the well known predictions for the scalar masses in the MSSM, taken to have universal scalar masses m_0^2 at the Planck scale. Mass splittings arise from renormalization group scaling from Planck to weak scales [2, 20], and the renormalization group equations (RGE's) are given by

$$\begin{aligned} \frac{d}{d\ln\mu} m_i^2(\mu) &= \frac{1}{16\pi^2}[-8C_2(R_a^i)g_a^2(\mu)M_a^2(\mu) + \frac{6}{5}Y_i g_1^2(\mu)S(\mu) \\ &\quad + \sum_{j,k} |\lambda_{ijk}|^2 (m_i^2 + m_j^2 + m_k^2 + A_{ijk}^2)], \qquad (2.2.1) \\ \frac{d}{d\ln\mu} S(\mu) &= \frac{b_1}{2\pi}\alpha_1(\mu)S(\mu), \qquad (2.2.2) \\ S(\mu) &= \sum_i Y_i m_i^2(\mu), \qquad (2.2.3) \end{aligned}$$

where $a = 1, 2, 3$ represents $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$,[2] i represents the species of the scalar and Y_i is the corresponding hypercharge, A_{ijk}'s are the soft SUSY breaking trilinear scalar couplings, and λ_{ijk}'s are the superpotential couplings. $C_2(R_a^i)$ is the second Casimir invariant of the gauge group a for the species i, $C_2 = \frac{N^2-1}{2N}$ for the fundamental representation of $SU(N)$, $\frac{3}{5}Y_i^2$ for $U(1)_Y$. The S term is zero under the assumption of universal scalar masses and hence does not contribute.

[2]The $SU(5)$ GUT normalization, $g_1^2 = \frac{5}{3}g'^2$, is used for the $U(1)$ coupling.

For the lightest two generations, whose superpotential coupling contributions are negligible, the mass splittings involve only contributions from the gauginos, which have masses M_{0a} at the Planck scale. Mass splittings also arise from the D^2 terms of the potential due to $SU(2)_L \times U(1)_Y$ interactions. These are proportional to $M_Z^2 \cos 2\beta$. The result is

$$m_i^2(\mu) = m_0^2 + \sum_a f_{ai} M_{0a}^2 + (T_{3_i} - Q_i \sin^2\theta_W) M_Z^2 \cos 2\beta, \qquad (2.2.4)$$

where i runs over the seven types of squark and slepton: U, D, U^c, D^c, E, N and E^c, and it is understood that the two light generations have identical scalar spectra. The renormalization constants f_{ai} are

$$f_{ai}(\mu) = \frac{2}{b_a} C_2(R_a^i) \left(\frac{\alpha_a^2(\mu)}{\alpha_a^2(M_p)} - 1 \right). \qquad (2.2.5)$$

where b_a is the one-loop beta function coéfficient, and μ should be taken equal to the scalar mass, m_i.

Suppose that β is known, for example from a Higgs mass measurement, then the seven values of m_i^2 depend only on four unknown parameters, m_0 and M_{0a} yielding three intragenerational mass relations for the MSSM [19]. Two further relations follow if M_{0a} is independent of a. In the following the scalar masses are scaled to the same renormalization point so that these mass relations can be displayed in simpler forms,

Two of these relations have only to do with $SU(2)$ breaking and are

$$m_U^2 - m_D^2 = m_N^2 - m_E^2 = M_Z^2 \cos 2\beta \cos^2\theta_W. \qquad (2.2.6)$$

These splittings arise because of the differing T_3 quantum numbers of the upper and lower components of the doublets $Q = (U, D)$ and $L = (N, E)$. It is convenient to define m_Q^2 and m_L^2 as the average squared mass of the doublet representation, thus $m_Q^2 = \frac{1}{2}(m_U^2 + m_D^2)$ and $m_L^2 = \frac{1}{2}(m_N^2 + m_E^2)$. In the rest of this chapter it is the masses $m_I^2, I = 1...5$ of the five types of multiplet $Q, U^c, D^c L$, and E^c which will interest us. In the MSSM, these are:

$$m_I^2 = m_0^2 + \sum_a f_{aI} M_{0a}^2 - Y_I \sin^2\theta_W M_Z^2 \cos 2\beta, \tag{2.2.7}$$

where Y_I is the hypercharge of multiplet I $(Q = T_3 + Y)$.

The mass predictions of (2.2.7) are based on several strong assumptions. The universal scalar mass is a speculative assumption about the form of the interactions in supergravity, and has been questioned, particularly by those working on string-inspired models [21]. The mass formula of equation (2.2.4) assumes the minimal particle content beneath the Planck scale. If there are extra gauge interactions then the index a $= 1, 2, 3, 4...$, yielding extra terms. If there are extra chiral fields with gauge quantum number then the b_a of equation (2.2.5) will change. Furthermore, if these extra chiral fields allow further superpotential interactions of strength λ involving quark and lepton fields, then additional terms proportional to λ^2 will contribute to $m_i^2(\mu)$.

In this chapter we study the scalar mass relations which follow from certain assumptions about grand unification. The assumptions appear to us to be better motivated than those listed above for the MSSM, since they are based on the successful supersymmetric GUT prediction for $\sin^2\theta_W$ [18], the weak mixing angle. The precision measurement results for $\sin^2\theta_W$ from LEP, SLD and CDF+D0 are in good agreement, and the combined global fit gives $\sin^2\theta_W(M_Z) = 0.2314\pm0.0002\pm0.0002$, with $m_t = 171 \pm 12\text{GeV}$ [22]. The experimental numbers should be compared with the supersymmetric GUT central prediction of $\sin^2\theta_W(SGUT) = 0.2342 \pm 0.0014$, where the only uncertainty shown is that due to $\alpha_s(M_Z) = 0.120 \mp 0.005$. In addition, simple models could have uncertainties of 0.0030 from threshold corrections at the GUT and weak scales. The weak mixing angle therefore provides the only successful theoretical prediction at the 1% level of any parameter of the standard model. This suggests that we take the assumptions which are *sufficient* to get this prediction and use them to make predictions for the squark and slepton masses. These assumptions are

1. At some scale M_G the gauge group is $SU(5) \times G$, where $SU(5)$ contains the entire standard model gauge group.

2. At mass scales below M_G the gauge group is $SU(3)_c \times SU(2)_L \times U(1)_Y \times G'$.

3. At mass scales below M_G the only particles coupling to the standard model gauge interactions are those of the MSSM.[3]

These assumptions are not a necessary requirement for an acceptable value of $\sin^2\theta_W$. Acceptable values can be obtained in very many ways, for example in

[3] In fact the prediction of $\sin^2\theta_W$ is not altered if extra complete, degenerate $SU(5)$ multiplets occur beneath M_G. We assume these to be absent; it could be worth studying the extent to which such representations affect the scalar mass relations.

non-supersymmetric $SU(5)$ theories with extra multiplets which are not $SU(5)$ degenerate [23]. However, it is these assumptions which uniquely produce a significant prediction. All the other schemes have a free parameter which can be chosen to fit $\sin^2 \theta_W$.[4]

What scalar mass relations follow from these assumptions? The first assumption imposes the boundary condition (which is taken to be at M_G now) on scalar masses within the same generation[5]:

$$m_{Q_0} = m_{E_0^c} = m_{U_0^c} = m_{10}, \tag{2.2.8}$$

$$m_{L_0} = m_{D_0^c} = m_{\bar{5}}, \tag{2.2.9}$$

because Q, E^c and U^c all lie in a 10 dimensional representation, and L and D^c lie in the $\bar{\mathbf{5}}$. There is no boundary condition relating masses of particles in different generations, and hence no such mass relations will result.

Let us study a particular generation, and suppose that in the $SU(5) \times G$ theory it lies in representation $(\mathbf{10}, R_1) + (\bar{\mathbf{5}}, R_2)$. If R_1 and R_2 are non-trivial and if G breaks to G' which is non-trivial, then the G' gauginos can renormalize the squark and slepton masses. However, since all members of the **10** have the same G' quantum numbers, this renormalization is common, and can simply be absorbed

[4]In the MSSM the scale of supersymmetry breaking is not a free parameter - it is determined to be of order the weak scale by radiative electroweak symmetry breaking.

[5]Mixings between the three low energy generations and some heavy vector-like particles at the GUT scale may spoil these relations[24]. However, large mixings are not expected since the KM matrix is very close to unity.

into the unknown parameter m_{10}. An identical situation applies to the $\bar{\mathbf{5}}$. Hence the common mass m_0^2 in the formula (2.2.7) should be replaced by $m_0^2 \to m_{I_0}^2$ which take on the two possible values shown in (2.2.8) and (2.2.9) according to whether I lies in a $\mathbf{10}$ or $\bar{\mathbf{5}}$ representation. In addition, the S term, which vanishes under the universal boundary condition assumption, is now given by

$$S(M_G) = \sum_i Y_i m_i^2(M_G) = m_{H_2}^2(M_G) - m_{H_1}^2(M_G). \tag{2.2.10}$$

Since H_2 and H_1 lie in different representations of $SU(5)$, $m_{H_2}^2(M_G)$ and $m_{H_1}^2(M_G)$ are not necessarily equal. From (2.2) it follows that S scales as α_1,

$$\frac{S(\mu)}{S(\mu_0)} = \frac{\alpha_1(\mu)}{\alpha_1(\mu_0)}. \tag{2.2.11}$$

The contributions of the S term can be written as

$$\delta_S m_i^2(\mu) = Y_i T, \tag{2.2.12}$$

where

$$\begin{aligned} T &= -\tfrac{3}{5b_1}(S(M_G) - S(\mu)) = -\tfrac{3}{5b_1}S(M_G)(1 - \tfrac{\alpha_1(\mu)}{\alpha_1(M_G)}) \\ &= -\tfrac{3}{5b_1}S(\mu)(\tfrac{\alpha_1(M_G)}{\alpha_1(\mu)} - 1). \end{aligned} \tag{2.2.13}$$

Among the five masses (2.2.7) of each light generation, there are three combinations independent of $m_{I_0}^2$:

$$m_Q^2 - m_{U^c}^2 = (C_2 - \frac{15}{36}C_1)M_0^2 - \frac{5}{6}\sin^2\theta_W M_Z^2 \cos 2\beta + \frac{5}{6}T, \tag{2.2.14a}$$

$$m_Q^2 - m_{E^c}^2 = (C_3 + C_2 - \frac{35}{36}C_1)M_0^2 + \frac{5}{6}\sin^2\theta_W M_Z^2 \cos 2\beta - \frac{5}{6}T, \tag{2.2.14b}$$

$$m_{D^c}^2 - m_L^2 = (C_3 - C_2 - \frac{5}{36}C_1)M_0^2 - \frac{5}{6}\sin^2\theta_W M_Z^2 \cos 2\beta + \frac{5}{6}T, \tag{2.2.14c}$$

where we have written $f_{3i} = C_3$ for a color triplet, $f_{2i} = C_2$ for a weak doublet and $f_{1i} = Y_i^2 C_1$, M_0 is the gaugino mass at the GUT scale, and the $\alpha_a(M_p)$ in f_{ai} should be replaced by $\alpha_a(M_G)$. By rearranging the above equations, we arrive at the following two mass relations independent of T:

$$2m_Q^2 - m_{U^c}^2 - m_{E^c}^2 = (C_3 + 2C_2 - \frac{25}{18}C_1)M_0^2, \qquad (2.2.15a)$$

$$m_Q^2 + m_{D^c}^2 - m_{E^c}^2 - m_L^2 = (2C_3 - \frac{10}{9}C_1)M_0^2, \qquad (2.2.15b)$$

and also an expression for T:

$$T = \frac{3}{10}(m_Q^2 - 2m_{U^c}^2 + m_{D^c}^2 + m_{E^c}^2 - m_L^2 + \frac{10}{3}\sin^2\theta_W M_Z^2 \cos 2\beta). \qquad (2.2.16)$$

Since $S(M_G)$ is only proportional to the difference $m_{H_2}^2(M_G) - m_{H_1}^2(M_G)$ and $b_1 = \frac{33}{5}$, we have $|T| < \frac{1}{11}|m_{H_2}^2(M_G) - m_{H_1}^2(M_G)|$. If the splitting between $m_{H_2}^2(M_G)$ and $m_{H_1}^2(M_G)$ is not too large, then T is small and these mass relations of (2.2.14), with $T = 0$, are approximately true. Alternatively, one can use (2.2.3), (2.2.13) and (2.2.16) to get

$$\begin{aligned}
&(m_Q^2 - 2m_{U^c}^2 + m_{D^c}^2 + m_{E^c}^2 - m_L^2 + \tfrac{10}{3}\sin^2\theta_W M_Z^2 \cos 2\beta)_{3rd\ generation} \\
&+(m_{H_2}^2 - m_{H_1}^2) = S(\mu) - \tfrac{20}{3}T = -(\tfrac{11\alpha_1(\mu)}{\alpha_1(M_G)-\alpha_1(\mu)})T - \tfrac{20}{3}T \\
&= -(\tfrac{20\alpha_1(M_G)+13\alpha_1(\mu)}{3\alpha_1(M_G)-3\alpha_1(\mu)})T. \qquad (2.2.17)
\end{aligned}$$

This combination does not suffer from the renormalization effects of the large third generation Yukawa couplings. Using T from (2.2.16) in (2.2.17) gives a third (intergenerational) mass relation:

$$
\begin{aligned}
& (m^2_{Q} - 2m^2_{U^c} + m^2_{D^c} + m^2_{E^c} - m^2_{L} + \tfrac{10}{3}\sin^2\theta_W M_Z^2 \cos 2\beta)_{1st\ or\ 2nd\ gen.} \\
= \quad & -\{(m^2_{Q} - 2m^2_{U^c} + m^2_{D^c} + m^2_{E^c} - m^2_{L} + \tfrac{10}{3}\sin^2\theta_W M_Z^2 \cos 2\beta)_{3rd\ gen.} \\
& +(m^2_{H_2} - m^2_{H_1})\} \times \tfrac{10\alpha_1(M_G) - 10\alpha_1(\mu)}{20\alpha_1(M_G)+13\alpha_1(\mu)}. \qquad (2.2.18)
\end{aligned}
$$

The MSSM provides 4 mass relations within each generation: those of (2.2.14) with $T = 0$ together with

$$
m^2_L - m^2_{E^c} = (C_2 - \frac{3}{4}C_1)M_0^2 + \frac{3}{2}\sin^2\theta_W M_Z^2 \cos 2\beta, \qquad (2.2.19)
$$

and also predicts identical spectra for each of the light generations.

In this section we have shown that two of these mass relations follow from a completely different boundary condition assumption than the one of universal scalar masses used for the MSSM. We have found that, in any GUT where the successful prediction of the weak mixing angle at the 1% accuracy level is preserved, two of the four mass relations of the MSSM for each light generation is preserved and a third one can be recovered provided that the third generation scalar masses and Higgs masses are also measured.

2.3 An extra mass relation in SO(10)?

The mass relation (2.2.19) can be reformulated as a relation between the two charged slepton masses of a given generation:

$$
m^2_E - m^2_{E^c} = (C_2 - \frac{3}{4}C_1)M_0^2 + (-\frac{1}{2} + 2\sin^2\theta_W)M_Z^2 \cos 2\beta. \qquad (2.3.1)
$$

In the following we will not include the contributions from the S term. It is assumed to be small or can be obtained from (2.2.16) or (2.2.17), then be substracted from the scalar masses. It is also assumed that M_0 is known (e.g. from the chargino mass measurement). This relation is particularly important because:

(a) The super-QCD interactions tend to increase the masses of the squarks above the sleptons, hence we expect this to be the first scalar mass relation of the MSSM to be tested.

(b) We have shown that this relation is precisely the one which cannot be deduced from $SU(5)$ unification. This is clearly because E and E^c are in different representations of $SU(5)$.

If the gauge group is extended to include $SO(10)$, such that a single generation lies entirely in a 16 dimensional spinor representation, then it is tempting to think that this slepton mass relation will be recovered, perhaps one can view this particular mass relation as a low energy signature of $SO(10)$. In this section, we explore in more detail the extent to which this is true.

We will make the three assumptions, given in the last section, necessary for the GUT to yield a significant $\sin^2\theta_W$ prediction. In addition we add the 4th assumption:

4. At energy scales greater than M_{10}, which is greater than or equal to M_G, the gauge group contains a factor which includes the usual $SO(10)$ gauge group.

This assumption provides the extra boundary condition which sets $m_L(\mu)$ and

$m_{E^c}(\mu)$ equal at $\mu \geq M_{10}$. The crucial question now is: are there any additional effects which could split these masses other than those of the $SU(2)_L \times U(1)_Y$ gaugino contributions and the $SU(2)_L \times U(1)_Y$ D^2 interactions, shown in (2.2.19) and (2.3.1)?

There are four such effects , which could break the slepton mass relation in an important way [25, 26, 27]:

(a) Radiative contributions from the gauge couplings and gaugino masses between M_{10} and M_G,

(b) Radiative contributions from the superpotential couplings between M_{10} and M_G,

(c) Tree level D term contributions,

(d) Radiatively generated D term contributions.

Suppose that M_{10} is higher than M_G, and that beneath M_{10} $SO(10)$ breaks down to $SU(5)$ (or $SU(5) \times U(1)_X$). The two charged sleptons of a given generation belong to $\bar{\mathbf{5}}$ and $\mathbf{10}$ representations of $SU(5)$ respectively and therefore their masses receive different radiative corrections. The radiative correction contributions from the $SU(5)$ gaugino mass is

$$\delta m^2(R) = \frac{2}{b_5} C_2(R)(1 - \frac{\alpha_5^2(M_{10})}{\alpha_5^2(M_G)}) M_5^2(M_G), \tag{2.3.2}$$

where $C_2(\bar{\mathbf{5}}) = \frac{12}{5}$ and $C_2(\mathbf{10}) = \frac{18}{5}$. Therefore we have $\frac{\delta m^2(\mathbf{10})}{\delta m^2(\bar{\mathbf{5}})} = \frac{3}{2}$. If $U(1)_X$ survives beneath M_{10}, the $U(1)_X$ gaugino mass also contributes to the radiative corrections and reduces this ratio ($X_{10} = -1,\ X_{\bar{5}} = 3$), but in general its contributions are smaller.

If this is the only source which violates the slepton mass relation, then we have

$$1 \leq \frac{m_{10}}{m_{\bar{5}}} \leq \frac{3}{2}, \tag{2.3.3}$$

and the violation should be small if gaugino mass is found to be small unless the gauge coupling increases very rapidly above M_G.

In addition to the radiative corrections from the gauge couplings, if the sleptons have some superpotential coupling of strength λ with fields which acquire masses $\mathcal{O}(M_G)$, then there are radiative corrections to the slepton masses between M_{10} and M_G at order λ^2. In order to generate significant violations of the slepton mass relation, λ has to be large, probably $\gtrsim \frac{1}{3}$, but such a large superpotential coupling could also destroy the degeneracy of scalar masses of different generations and induce unacceptable flavor changing effects unless there is a horizontal symmetry above M_G which keeps the scalar masses of the two lighter generations degenerate.

D term contributions to scalar masses can arise when the rank of the gauge group is reduced. To see this, consider the following situation. Suppose the $U(1)_X$ subgroup of $SO(10)$ ($SO(10) \supset SU(5) \times U(1)_X$) is broken by the VEV's of N and $\overline{N}$ fields which lie in **16** and $\overline{\mathbf{16}}$ representations of $SO(10)$. The $U(1)_X$ gauge interaction contains a piece

$$\frac{1}{2} g_X^2 (X_N\,|N|^2 - X_N|\overline{N}|^2 + \sum_i X_i\,|\phi_i|^2)^2, \tag{2.3.4}$$

where X_i is the X charge of the ϕ_i field. When the VEV's of N and $\overline{N}$ fields are not equal, it gives extra contributions to the squared masses of scalar fields of nonzero X charges. This happens if the soft SUSY breaking masses of N and $\overline{N}$ are different

[26, 28, 29]. The relevant part of the scalar potential for these fields we take to be

$$\begin{aligned} V(N, \overline{N}) &= \frac{1}{2} g_X^2 (X_N |N|^2 - X_N |\overline{N}^2|)^2 \\ &\quad + m_N^2 |N|^2 + m_{\overline{N}}^2 |\overline{N}^2| + \lambda^2 |N\overline{N} - \mu^2|^2, \end{aligned} \tag{2.3.5}$$

where m_N^2 and $m_{\overline{N}}^2$ are the soft SUSY breaking masses of the N and $\overline{N}$ fields, and they are of the order of the SUSY breaking scale m_S. The last term is to give large VEV's ($\simeq \mu$) to N and $\overline{N}$ fields.[6] Defining $\Sigma \equiv |N|^2 + |\overline{N}^2|$, $\Delta \equiv |N|^2 - |\overline{N}^2|$, $m_\Sigma^2 \equiv \frac{1}{2}(m_N^2 + m_{\overline{N}}^2)$ and $m_\Delta^2 \equiv \frac{1}{2}(m_N^2 - m_{\overline{N}}^2)$, we can rewrite V as

$$V = \frac{1}{2} g_X^2 (X_N \Delta)^2 + m_\Sigma^2 \Sigma + m_\Delta^2 \Delta + \lambda^2 |\frac{1}{2}\sqrt{\Sigma^2 - \Delta^2} - \mu^2|^2. \tag{2.3.6}$$

Minimizing the potential with respect to Δ we obtain

$$\Delta = -\frac{m_\Delta^2}{X_N^2 g_X^2} + \mathcal{O}(\frac{m_S^3}{\mu}), \tag{2.3.7}$$

This shifts the mass of the scalar particle with charge X_i by the amount

$$\delta m_i^2 = g_X^2 X_i X_N \Delta \simeq -\frac{X_i}{X_N} m_\Delta^2. \tag{2.3.8}$$

Therefore any scalar particle which carries $U(1)_X$ charge will receive a tree level D term contribution which is proportional to its $U(1)_X$ charge and the difference of the soft-breaking masses m_N^2 and $m_{\overline{N}}^2$. Since N and $\overline{N}$ lie in different representations of $SO(10)$, $SO(10)$ allows m_N^2 to be very different from $m_{\overline{N}}^2$, and also X_{10} and $X_{\overline{5}}$ are different ($X_{10} = -1$, $X_{\overline{5}} = 3$), this provides a large breaking of the slepton relation (2.2.19), (2.3.1).

[6]Different ways of stablizing the VEV's of N and $\overline{N}$ do not change the basic result, they only give corrections to the higher order terms in equation (2.3.7).

From the above discussion it follows that a significant violation of the slepton mass relation by the D term requires a large difference between m_N^2 and $m_{\overline{N}}^2$ (of the same order of the slepton masses). If some symmetry of the Kähler potential guarantees that m_N^2 and $m_{\overline{N}}^2$ are equal at the tree level, a large difference between them can still be generated by radiative corrections, especially if $U(1)_X$ is broken by the same radiative corrections at some much lower energy. We consider such a model in the next section.

2.4 Large D term corrections from radiative breaking of $\mathrm{U(1)_X}$

If the scalar masses are universal at the Planck scale because of some symmetry of the Kähler potential, the difference between m_N^2 and $m_{\overline{N}}^2$ can still be generated by radiative corrections below the Planck scale if N and $\overline{N}$ couple to other fields differently. An interesting case is that the $U(1)_X$ is also broken by the same radiative corrections which modify m_N^2 and $m_{\overline{N}}^2$, i.e., N and $\overline{N}$ fields get VEV's when $m_\Sigma^2 = \frac{1}{2}(m_N^2 + m_{\overline{N}}^2)$ is renormalized to negative. In this case, $m_\Delta^2 = \frac{1}{2}(m_N^2 - m_{\overline{N}}^2) \simeq m_N^2$ which is presumably comparable to the masses of the squarks and sleptons, then the D term correction to the sparticle spectrum can be quite large. In what follows we consider a simple model which will demonstrate this case.

We assume, for simplicity, $M_{10} = M_G$, and beneath M_G, the particle contents are the usual ones in the MSSM with three right-handed neutrinos, the additional $U(1)_X$ gauge field, an N and an $\overline{N}$ fields discussed above which break the $U(1)_X$ when they get nonzero VEV's, and three gauge singlets S_k, $k = 1, 2, 3$. The N and $\overline{N}$ belong to the **16** and $\bar{\mathbf{16}}$ representations of $SO(10)$ at the GUT scale with all other components getting superheavy masses and decoupled below the GUT scale. This can be achieved by a **45** Higgs with VEV's in the hypercharge direction[30]. The two low energy Higgs doublets H_1 and H_2 are assumed to belong to the **10** representations of $SO(10)$ and their X charges are -2 and 2 respectively. The X charges of all chiral fields are shown in Table 2.1. Note that we only add the stan-

field:	q_L	${u_R}^c$	${d_R}^c$	l_L	${e_R}^c$	${\nu_R}^c$	H_1	H_2	N	$\overline{N}$	S
X	-1	-1	3	3	-1	-5	-2	2	-5	5	0

Table 2.1: The $U(1)_X$ charges of different fields

dard model gauge group singlets to the MSSM so that the successful prediction of $\sin^2\theta_W$ in the SGUTs is retained.

We consider a superpotential given by

$$W = \boldsymbol{Q\lambda_U U^c} H_2 + \boldsymbol{Q\lambda_D D^c} H_1 + \boldsymbol{L\lambda_E E^c} H_1 + \boldsymbol{L\lambda_\nu \nu_R^c} H_2 + \mu H_1 H_2 + \sum_{k=1}^{3} \lambda_k \nu_{R_k}^c S_k \overline{N}. \tag{2.4.1}$$

Other possible interactions, such as $NS_k\overline{N}$, mS_k^2 and S_k^3, could vanish either because S_k's are embedded in some non-trivial representations of $SO(10)$, or because of some discrete symmetry. (For example, a parity whose lepton fields change sign and S_k and $\overline{N}$ are multiplied by i.) The scalar potential involving N and $\overline{N}$ fields is given by [7]

$$V = \frac{1}{2}g_X^2(X_N\,|N|^2 + X_{\overline{N}}|\overline{N}^2| + \sum_i X_i\,|\phi_i|^2)^2 + \sum_{k=1}^{3}\left|\lambda_k \tilde{\nu}_R{}^c{}_k \overline{N}\right|^2$$
$$+\sum_{k=1}^{3}\left|\lambda_k S_k \overline{N}\right|^2 + m_N^2\,|N|^2 + m_{\overline{N}}^2|\overline{N}^2| + \sum_{k=1}^{3} A_k\lambda_k \tilde{\nu}_R{}^c{}_k S_k \overline{N}$$
$$= \frac{1}{2}g_X^2(X_N\Delta + \sum_i X_i\,|\phi_i|^2)^2 + m_\Sigma^2\Sigma + m_\Delta^2\Delta$$
$$+\sum_{k=1}^{3}\left|\lambda_k \tilde{\nu}_R{}^c{}_k \overline{N}\right|^2 + \sum_{k=1}^{3}\left|\lambda_k S_k \overline{N}\right|^2 + \sum_{k=1}^{3} A_k\lambda_k \tilde{\nu}_R{}^c{}_k S_k \overline{N}, \qquad (2.4.2)$$

where Σ, Δ, m_Σ^2, and m_Δ^2 are defined as before. When m_Σ^2 is driven negative by the Yukawa interactions $\lambda_k \nu_{Rk}^c S_k \overline{N}$ at some intermediate mass scale M_I, (λ_k's are assumed to be $\mathcal{O}(1)$,) N and $\overline{N}$ fields will get nonzero VEV's and break the $U(1)_X$. The difference of the squares of their VEV's Δ is given by $\Delta = -\frac{m_\Delta^2}{X_N^2 g_X^2}$ by minimizing V with respect to Δ, and the sum Σ is fixed by the one-loop correction

$$\Delta V = \frac{1}{64\pi^2} StrM^4[\ln\frac{M^2}{\mu^2} - \frac{3}{2}] \qquad (2.4.3)$$

to the scalar potential [31], $\Sigma \sim M_I^2$ where M_I is the scale at which $m_\Sigma^2(M_I) = m_N^2(M_I) + m_{\overline{N}}^2(M_I) = 0$ [28]. Fig. 2.1 shows the evolutions of the soft breaking masses of N, $\overline{N}$, S_k, and $\tilde{\nu}_R{}^c{}_k$ fields. For simplicity, we have assumed that the soft SUSY breaking parameters are universal at M_G and the parameters are chosen to be $\lambda_{t0} = \lambda_{\nu_\tau 0} = 1.5$, $\lambda_{b0,\tau 0} \ll 1$, $\lambda_{k0} = 1$, $k = 1,2,3$, and the universal soft breaking trilinear couplings $A_0 = 3m_0$. The $m_{\tilde{\nu}_{R3}}^2$ is also driven negative at low energies because of the large λ_{ν_3} coupling. However, the terms $\sum_{k=1}^{3}\left|\lambda_k \tilde{\nu}_R{}^c{}_k \overline{N}\right|^2$

[7] We use S and N to represent both the superfields and their scalar components. It should be clear which one they represent.

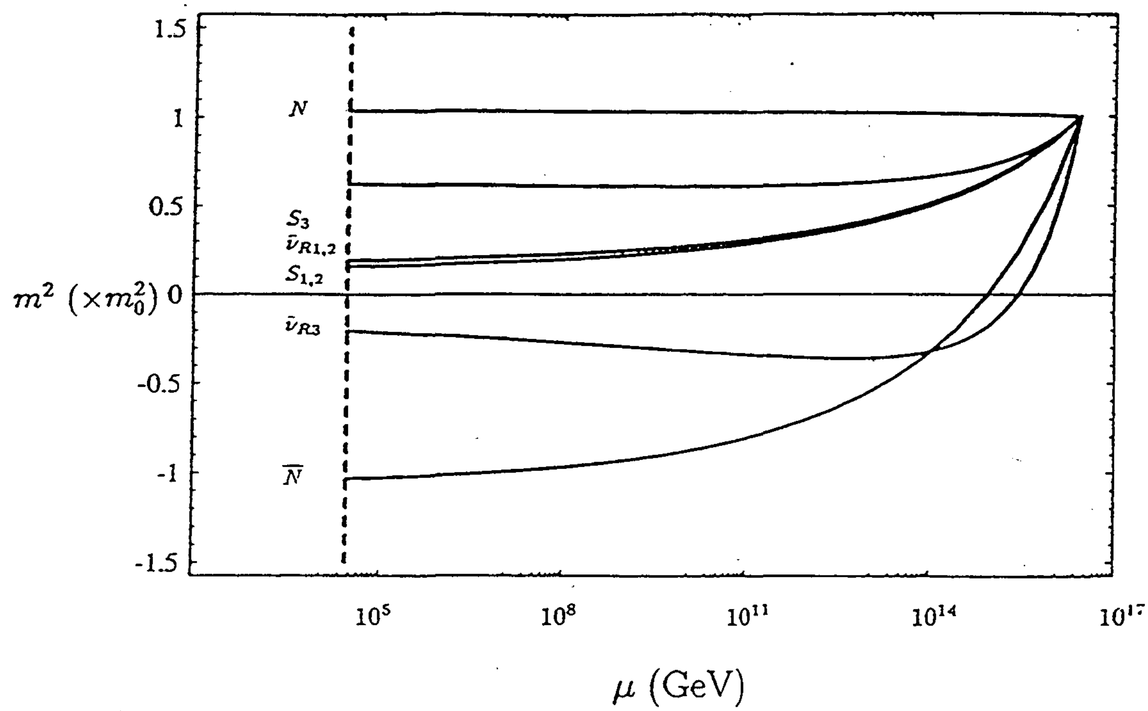


Figure 2.1: The evolutions of the soft breaking masses of N, $\overline{N}$, S_k, and $\tilde{\nu}_R{}^c{}_k$ fields from GUT scale (2.7×10^{16} GeV) to $U(1)_X$ breaking scale (30 TeV).

An universal soft breaking mass m_0 is assumed at GUT scale and the parameters are chosen to be $\lambda_{t0} = \lambda_{\nu_\tau 0} = 1.5, \lambda_{b0,\tau 0} \ll 1, \lambda_{k0} = 1, k = 1, 2, 3$, and the universal soft breaking trilinear couplings $A_0 = 3m_0$.

in the scalar potential V (equation(2.4.2)) prevent both $\overline{N}$ and $\tilde{\nu}_R{}^c$ from getting non-zero VEV's. After $U(1)_X$ is broken, the mass square of $\tilde{\nu}_{R3}^c$ gets a large positive contribution from the $\overline{N}$ VEV and $\langle\tilde{\nu}_{R3}^c\rangle$ remains zero.

The present bounds on the mass of the $U(1)_X$ gauge boson Z_X are $M_{Z_X} > 320$ GeV (direct) and > 670 GeV (indirect) [32]. The primordial nucleosynthesis may put a more stringent limit on M_{Z_X}, taking $N_\nu < 3.5$, M_{Z_X} has to be greater than $\mathcal{O}$(TeV) [33] because of the extra massless states present in our model. Cosmological constraints also put an upper limit on M_I. The flaton (a linear combination of N and $\overline{N}$ which corresponds to the quasi-flat direction) decays into light particles through the heavy intermediate states of $\mathcal{O}(M_I)$ after the phase transition of $U(1)_X$ breaking. The decay rate must be fast enough in order not to affect the primordial nucleosynthesis or over-dilute the baryon asymmetry. This gives an upper bound on M_I [34]. With these considerations, we will take M_I to be in the range of 10^3GeV to 10^7GeV.

Compared with MSSM, the scalar masses contain two extra contributions: the $U(1)_X$ gaugino contribution and the $U(1)_X$ D term. For the first two generations where the Yukawa couplings are negligible, the scalar masses are given by

$$\begin{aligned} m_i^2 &= m_0^2 + \sum_{a=1}^{3} f_{ai} M_0^2 + f_{Xi} M_0^2 \\ &\quad + (T_{3i} - Q_i \sin^2\theta_W) M_Z^2 \cos 2\beta - \frac{X_i}{X_N} m_\Delta^2, \end{aligned} \tag{2.4.4}$$

where m_0 and M_0 are the scalar mass and gaugino mass at M_G respectively, f_{ai}, $a =$

1, 2, 3 are the same as before and f_{Xi} is given by

$$f_{Xi} = \frac{2}{b_X}\frac{X_i^2}{40}\left[\frac{\alpha_X^2(M_I)}{\alpha_X^2(M_G)} - 1\right], \tag{2.4.5}$$

In this simple model, m_Δ^2 can also be expressed in terms of m_0 and M_0,

$$m_\Delta^2 = m_N^2 = m_0^2 + f_{XN} M_0^2, \tag{2.4.6}$$

then we have

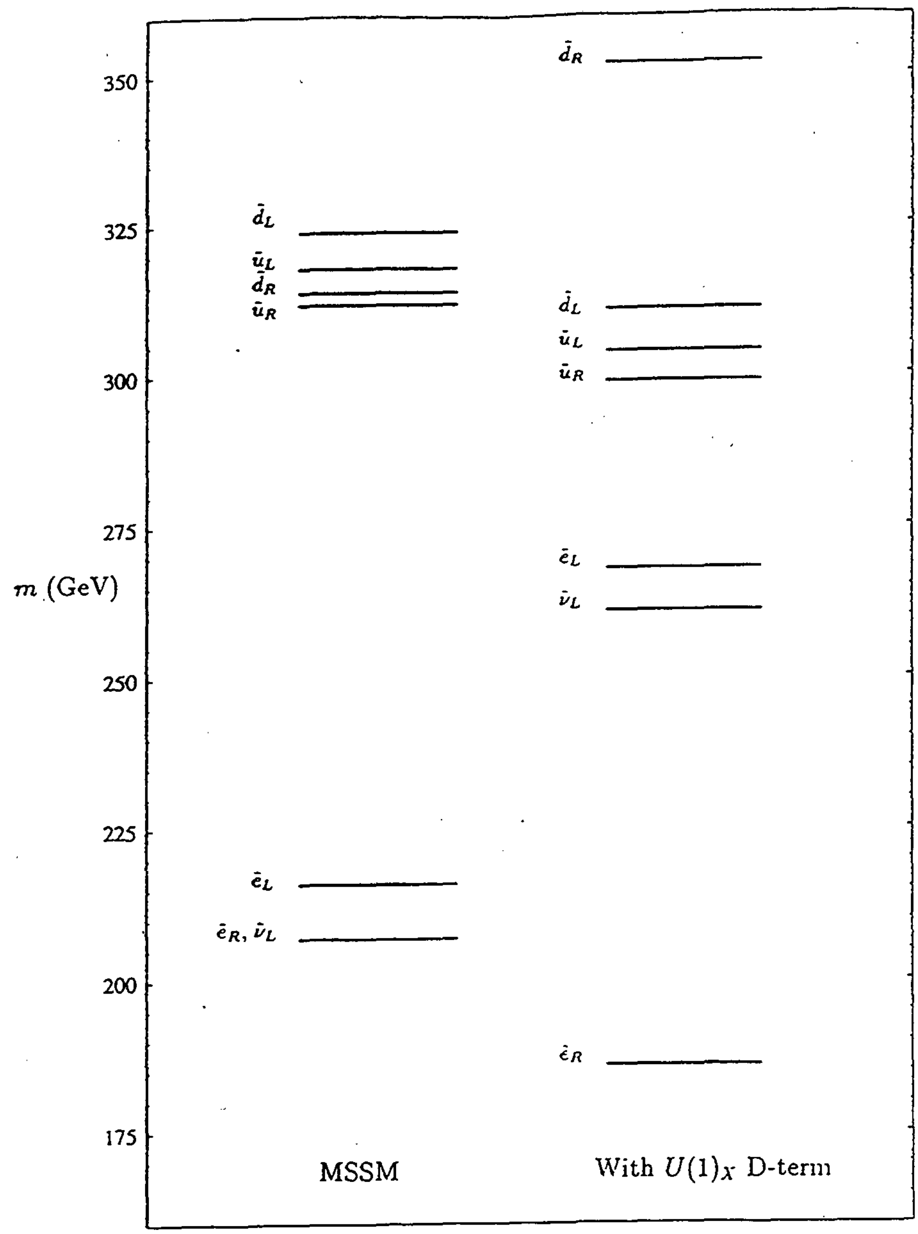
$$\begin{aligned} m_i^2 &= (1 - \frac{X_i}{X_N}) m_0^2 + \sum_{a=1}^{3} f_{ai} M_0^2 + (f_{Xi} - \frac{X_i}{X_N} f_{XN}) M_0^2 \\ &+ (T_{3i} - Q_i \sin^2\theta_W) M_Z^2 \cos 2\beta. \end{aligned} \tag{2.4.7}$$

The corrections $-\frac{X_i}{X_N} m_0^2 + (f_{Xi} - \frac{X_i}{X_N} f_{XN}) M_0^2$ to the masses of squarks and sleptons compared to the MSSM can be as large as 60% for $X_i = 3$ in the limit $m_0 \gg M_0$. Fig. 2.2 shows the comparison of the scalar spectra with and without the $U(1)_X$ D term corrections for a set of m_0 and M_0. We see that the corrections are more significant for the sleptons than for the squarks because of the smaller gaugino mass contributions to the sleptons than to the squarks. Now the slepton mass relation (2.3.1) is modified to be

$$\begin{aligned} m_E^2 - m_{E^c}^2 &= (C_2 - \frac{3}{4} C_1) M_0^2 + 8 C_X M_0^2 + \frac{4}{5} m_\Delta^2 \\ &+ (-\frac{1}{2} + 2\sin^2\theta_W) M_Z^2 \cos 2\beta, \end{aligned} \tag{2.4.8}$$

where $f_{Xi} = X_i^2 C_X$. In a more general $SO(10)$ theory there is no simple relation between m_Δ^2 and m_0^2 and m_Δ^2 has to be treated as a parameter.

Before going to the next section, we have three comments on this model.



$m_0 = 200\text{GeV},\ M_0 = 100\text{GeV},\ \tan\beta = 2$

Figure 2.2: Comparison of the scalar particle spectra with and without the $U(1)_X$ D term corrections for a set of m_0, M_0 and $\tan\beta$.

(1) S term contributions: When $U(1)_X$ is broken at intermediate energy M_I, $S(M_I)$ is also shifted by $\delta m^2_{H_2} - \delta m^2_{H_1} = \frac{4}{5} m^2_\Delta$. Then the equations (2.2.13), (2.2.17) and (2.2.18) are not valid. Therefore, if (2.2.15a), (2.2.15b) hold but (2.2.18) does not, it may be a hint of an $U(1)_X$ breaking at intermediate energy scale and providing a shift of the S term.

(2) Neutrino masses: In our simplest model, there are three heavy Dirac neutrinos and three massless neutrinos because of the three singlet states we introduced [35, 36]. We can see them from the mass terms of the neutrinos (for simplicity, we only consider one family here and drop the family indices)

$$m_D \nu_L \nu^c_R + M_D S \nu^c_R, \tag{2.4.9}$$

where $m_D = \lambda_\nu \langle H^0_2 \rangle \sim \mathcal{O}(m_{u,c,t})$, and $M_D = \lambda \langle \overline{N} \rangle \sim \mathcal{O}(M_I)$. One linear combination of ν_L and S, $\nu_L \sin\theta + S\cos\theta$, where $\tan\theta = \frac{m_D}{M_D}$, is married with ν_R and gets a large mass $\sqrt{m^2_D + M^2_D} \sim \mathcal{O}(M_I)$, which is consistent with experimental constraints [36], and the other combination $\nu_L \cos\theta - S\sin\theta$ is left massless. However, it is possible to give the three light neutrinos small majorana masses which are favored to solve the solar neutrino problem by just adding some extra interactions to the superpotential of the model. For example, if we add to the superpotential the non-renormalizable interaction $\frac{1}{M_G} S^2 N \overline{N}$ which gives a small majorana mass term $m_S S^2 = \frac{1}{M_G} \langle N \rangle \langle \overline{N} \rangle S^2$ to S, then the mass matrix of the fields ν_L, ν^c_R, and S

becomes

$$\mathcal{M} = \begin{pmatrix} 0 & m_D & 0 \\ m_D & 0 & M_D \\ 0 & M_D & m_S \end{pmatrix}. \tag{2.4.10}$$

The product of the three mass eigenvalues is given by $\det \mathcal{M} = -m_D^2 m_S$, and the two larger masses are approximately equal to M_D, so the mass of the light neutrino is approximate

$$m_{light} \simeq \frac{m_D^2 m_S}{M_D^2} \sim \frac{m_D^2 M_I^2}{M_D^2 M_G} \sim \frac{m_D^2}{M_G} \tag{2.4.11}$$

which is similar to that generated by the usual see-saw mechanism.

(3) b-τ Yukawa unification: Because the $U(1)_X$ is broken at low energy, there are extra interactions surviving at low energies compared with the MSSM. Especially the τ-neutrino Yukawa coupling λ_{ν_τ} which should be about the same as λ_t at the GUT scale enters the RGE's of many parameters. The RGE for the b-τ mass ratio R is modified to be

$$\frac{dR}{d\ln\mu} = \frac{R}{16\pi^2}\left(-\frac{16}{3}g_3^2 + \frac{4}{3}g_1^2 + \lambda_t^2 - \lambda_{\nu_\tau}^2 + 3\lambda_b^2 - 3\lambda_\tau^2\right). \tag{2.4.12}$$

In the small $\tan\beta$ case where λ_b and λ_τ can be neglected, the unification of b and τ Yukawa couplings in SGUT requires a large top Yukawa coupling to compensate the contribution from the $SU(3)$ gauge coupling. In our model the contribution of λ_t is largely cancelled out by λ_{ν_τ}, making it difficult to achieve the b-τ unification for the top Yukawa coupling staying in the perturbative regime at the GUT scale. However, since the b- and τ- Yukawa couplings are small, they do not necessarily come from a single renormalizable interaction of the form $\mathbf{16}_3\,\mathbf{10}\,\mathbf{16}_3$ in $SO(10)$ and therefore their unification is not mandatory. In the large $\tan\beta$ case where λ_b and λ_τ are comparable to λ_t (which we will discuss in the next section), the terms $3\lambda_b^2 - 3\lambda_\tau^2$ in the RGE for R also contribute and make up the negative contribution from λ_{ν_τ} ($\lambda_b > \lambda_\tau$ below the GUT scale). In addition, the couplings between b and H_2 through the bottom squark-gluino loops and top squark-chargino loops [37, 38, 39] could also give a significant contribution to R if $\tan\beta$ is large. Therefore, the b-τ unification is possible in this case.

2.5 Fine-tuning problem in the Yukawa unification scenario

Recently, the large $\tan\beta$ scenario in which the tau lepton and the bottom and top quark Yukawa couplings unify at the grand unification scale has drawn considerable interest [38, 40, 41, 42]. This happens in an $SO(10)$ GUT if the two light Higgs doublets lie predominantly in a single **10** representation of the gauge group $SO(10)$ and the t, b, and τ masses originate in the renormalizable Yukawa interactions of the form $\mathbf{16}_3\,\mathbf{10}\,\mathbf{16}_3$. In this case, the top quark mass can also be predicted and it was predicted to be heavy [38]. In fact, such a heavy top quark has be found by the CDF and D0 collaborations at the Fermilab[43, 44], $m_t = 180 \pm 12$ GeV[22]. The problem with this scenario is that radiative electroweak symmetry breaking is hard to achieve although significant progress has already been made [39, 42, 45, 46]. The masses of the up- and down-type Higgs are the same at M_{10}

because they lie in the same representation and run almost in parallel because of the boundary condition $\lambda_t(M_{10}) = \lambda_b(M_{10})$. Usually one relies on heavy gauginos to amplify the small hypercharge-induced difference in the running of $m_{H_1}^2$ and $m_{H_2}^2$. However, all these attemps require severe fine tuning of the parameters which we will explain below.

The relevant part of the Higgs potential is given by

$$\mu_1^2|H_1^0|^2 + \mu_2^2|H_2^0|^2 + B\mu(H_1^0 H_2^0 + \text{h.c.}) + \frac{1}{8}(g^2 + g'^2)(|H_1^0|^2 - |H_2^0|^2)^2. \quad (2.5.1)$$

Minimizing the Higgs potential we obtain the following conditions,

$$\frac{\mu_1^2 - \tan^2\beta\mu_2^2}{\tan^2\beta - 1} = \frac{M_Z^2}{2}, \quad (2.5.2)$$

$$\frac{-\mu B}{\mu_1^2 + \mu_2^2} = \frac{1}{2}\sin 2\beta. \quad (2.5.3)$$

In the case of $\lambda_t(M_{10}) = \lambda_b(M_{10})$, $\tan\beta \simeq \frac{m_t}{m_b} \sim \mathcal{O}(50) \gg 1$. We see that $\mu_2^2 \simeq -\frac{M_Z^2}{2}$ for μ_1^2 not too large, then

$$m_A^2 = \mu_1^2 + \mu_2^2 \simeq (\mu_1^2 - \mu_2^2) - M_Z^2 < \mu_1^2 - \mu_2^2 \equiv \epsilon_c m_S^2, \quad (2.5.4)$$

where m_A is the CP-odd scalar mass , m_S^2 is the typical supersymmetric particle mass scale, $m_S \sim \max(m_0, M_0)$, and ϵ_c represents the custodial symmetry breaking effects. Equation (2.5.4) tells us that both m_A^2 and M_Z^2 are smaller than $\epsilon_c m_S^2$, so there is an $\mathcal{O}(\epsilon_c)$ fine-tuning of the Z mass. In addition, writing $m_A^2 = \epsilon m_S^2$, $\epsilon < \epsilon_c \ll 1$, we have

$$\frac{-\mu B}{\epsilon m_S^2} = \frac{1}{2}\sin 2\beta \simeq \frac{1}{\tan\beta} \Rightarrow -\mu B \simeq \frac{\epsilon}{\tan\beta} m_S^2. \quad (2.5.5)$$

While μ is typically of the order m_S in order to satisfy $\mu_2^2 = m_{H_2}^2 + \mu^2 \simeq -\frac{M_Z^2}{2}$, the B parameter which receives contributions from the gaugino masses and the soft SUSY-breaking trilinear scalar coupling A and therefore is also naturally of the order m_S has to be fine-tuned to $\mathcal{O}(\frac{\epsilon}{\tan\beta} m_S)$. The fine-tuning is at least one part in 10^3 and is much worse than the naive expectation $\frac{1}{\tan\beta}$.

The $U(1)_X$ D term which gives the opposite contributions to $m_{H_1}^2$ and $m_{H_2}^2$ provides the desired ingredient to solve this problem [30, 39, 47]. One can either simply have $m_N^2 \neq m_{\overline{N}}^2$ at tree level [39] or have the difference m_Δ^2 generated by radiative corrections as described in the last section. However, the simple model discussed in the previous section gives a positive contribution to $m_{H_2}^2$ and a negative contribution to $m_{H_1}^2$ which is incompatible with the fact that $\mu_1^2 > \mu_2^2$. We thus modify the model so that it has interactions $\lambda_k' \nu_{Rk}^c S_k' N$, $k = 1,2,3$, instead of $\lambda_k \nu_{Rk}^c S_k \overline{N}$. The S_k''s are still standard model gauge group singlets, but carry $U(1)_X$ charge $+10$ (they may belong to the $\overline{\mathbf{126}}$ of SO(10)). We also have to add $\overline{S_k}'(X = -10)$ to the model in order to cancel the anomaly and we assume that they only have the $U(1)_X$ gauge interaction. Then, the m_N^2, instead of $m_{\overline{N}}^2$, is driven negative by the Yukawa interactions. The $m_\Delta^2 = \frac{1}{2}(m_N^2 - m_{\overline{N}}^2)$ becomes negative in this case and therefore it gives the correct-sign D term contributions to $m_{H_1}^2$ and $m_{H_2}^2$. Let δm_H^2 be the difference between $m_{H_1}^2$ and $m_{H_2}^2$ generated by the renormalization group from M_{GUT} to m_S without the D term correction.

$$\delta m_H^2 = m_{H_1}^2 - m_{H_2}^2 = \epsilon_c m_S^2 \ll m_S^2. \tag{2.5.6}$$

The parameter $m_A^2 = \mu_1^2 + \mu_2^2$ is now given by

$$m_A^2 = \mu_1^2 + \mu_2^2 = (\mu_1^2 - \mu_2^2) + 2\mu_2^2 \simeq D + \delta m_H^2 - M_Z^2, \tag{2.5.7}$$

where $D = (-\frac{2}{5}m_\Delta^2) - (+\frac{2}{5}m_\Delta^2) = -\frac{4}{5}m_\Delta^2 \sim m_S^2$. For m_S larger than M_Z, m_A^2 is naturally of the order m_S^2 and the problem of a light m_A^2 can be avoided. The fine-tuning problem of μB is also relieved though not totally eliminated as we can see from equation(2.5.5) that a fine tune of $\frac{1}{\tan\beta} \sim \mathcal{O}(\frac{1}{50})$ is still required. However, it should be generic since a large pure number $\tan\beta$ has to be generated.

2.6 Conclusions

It is well known that quark and lepton mass and mixing angle relations may provide evidence for grand unification. Although squarks and sleptons have yet to be discovered, mass relations amongst scalars provide a much more reliable test of unification than do the relations involving fermion masses. This is because chiral and gauge symmetry breaking effects mask the grand unified symmetry relations for the fermions, but are not present for the scalars. In this chapter we have derived several scalar mass relations which follow directly from the grand unified symmetry, and we have studied the reliability of such relations as a probe of supersymmetric unification.

The small size of flavor-changing processes suggests that in models with weak-scale supersymmetry the squarks of a given charge should be approximately degenerate. This has led to the speculation that squarks and sleptons of different charge might also be degenerate. Although only a speculation, such a boundary condition of universal scalar masses has become a ubiquitous feature of supersymmetric models and is incorporated in the minimal supersymmeric standard model. Since there are five types of quark and leptons, the quark and lepton weak doublets Q and L and the weak singlets U^c,D^c and E^c, such a boundary condition leads to four relations between the scalar masses. However, the origin of these relations is more a matter of simplicity than of any underlying fundamental principle.

In this chapter we have derived mass relations, between scalars of a given generation, which result from the most general possible boundary condition that respects a grand unified symmetry. With $SU(5)$ unification, the five types of quarks and leptons are unified into two irreducible representations (Q, U^c, E^c) and (L, D^c), leading to the expectation of three mass relations, which are given in equation (2.2.14). However, these three relations involve a quantity T, which depends on the mass splitting of the Higgs scalars at the unification mass. It is likely that this mass splitting is small enough that the relations (2.2.14) with $T = 0$ will result. However, if the mass splitting is very large there are only two mass relations between the scalar mass parameters of each of the light generations. These relations are given by eliminating T, and are given in equations (2.2.15). We believe that these relations must be correct in any grand unified theory which incorporates the usual $SU(5)$ group. If these relations are found to be incorrect, then it is unlikely that grand unification is correct. Although extra particles and interactions could be added to a grand unified theory to invalidate these mass relations, such particles and interactions will lead to extra renormalizations of the weak mixing angle, upsetting the outstanding agreement between the theoretical prediction and the experimental value.

Even if the parameter T is large, a third mass relation can be derived because T can be evaluated by measuring the Higgs boson and third generation scalar masses. This mass relation is given in equation (2.2.18).

If the quark and leptons are further unified, so that all five species of a generation are unified in a single representation, as occurs in $SO(10)$ theories, a fourth mass relation is to be expected. This is written, ignoring T, in equation (2.3.1), as a relation between the masses of the two charged sleptons. This mass relation is likely to be the first which is subject to precise experimental test. If it were verified it would provide striking support for SO(10) unification. However, unlike the two mass relations mentioned above, it is not a necessary consequence of $SO(10)$ unification. We have shown in this chapter that it is possible to have large corrections to this mass relation from $U(1)_X$ D^2 interactions, either at tree level or by radiative corrections.

Chapter 3

Flavor mixing signal for realistic supersymmetric unification

3.1 Introduction

It has recently been demonstrated that flavor and CP violations provide an important new probe of supersymmetric grand unified theories [6, 7, 8, 9]. These new signals, such as $\mu \to e\gamma$ and the electron electric dipole moment d_e, are complementary to the classic tests of proton decay, neutrino masses and quark and charged lepton mass relations. The classic tests are very dependent on the flavor interactions and symmetry breaking sector of the unified model: it is only too easy to construct models in which these signals are absent or unobservable. However, they are insensitive to the hardness scale, Λ_H, of supersymmetry breaking.[1] On the other hand, the new flavor and CP violating signals are relatively insensitive to the form of the flavor interactions and unified gauge symmetry breaking, but are absent if the hardness scale, Λ_H, falls beneath the unified scale, M_G. The signals are generated by the unified flavor interactions leaving an imprint on the form of the soft supersymmetry breaking operators [25], which is only possible if supersymmetry breaking is present in the unified theory at scales above M_G.

The flavor and CP violating signals have been computed in the minimal $SU(5)$ and $SO(10)$ models for leptonic [6, 7, 8] and hadronic processes [9], for moderate values of $\tan\beta$, the ratio of the two Higgs vacuum expectation values. While rare muon decays provide an important probe of $SU(5)$, it is the $SO(10)$ theory which is most powerfully tested. If the hardness scale for supersymmetry breaking is large

[1]This is the highest scale at which supersymmetry breaking squark and gluino masses appear in the theory as local interactions.

enough, as in the popular supergravity models, it may be possible for the minimal $SO(10)$ theory to be probed throughout the interesting range of superpartner masses by searches for $\mu \to e\gamma$ and d_e.

The flavor-changing and CP-violating probes of $SO(10)$ are sufficiently powerful to warrant an exploration of consequences for non-minimal models, which is the subject of this chapter. In particular, we study $SO(10)$ theories in which

(I) *The Yukawa interactions are non-minimal.*

In the minimal model the quarks and leptons lie in three **16**'s and the two Higgs doublets H_U and H_D lie in two 10 dimensional representations $\mathbf{10}_U$ and $\mathbf{10}_D$. The quark and charged lepton masses are assumed to arise from the interactions $\mathbf{16}\boldsymbol{\lambda}_U\mathbf{16}\ \mathbf{10}_U + \mathbf{16}\boldsymbol{\lambda}_D\mathbf{16}\ \mathbf{10}_D$. This model is a useful fiction: it is very simple to work with, but leads to the mass relation $m_e/m_\mu = m_d/m_s$, which is in error by an order of magnitude. It is clearly necessary to introduce a mechanism to insert $SO(10)$ breaking into the Yukawa interactions. The simplest way to achieve this is to assume that at the unification scale, M_G, some of the Yukawa interactions arise from higher dimensional operators involving fields A which break the $SO(10)$ symmetry group. This implies that $\boldsymbol{\lambda}_{U,D} \to \boldsymbol{\lambda}_{U,D}(A)$. Every realistic model of $SO(10)$ which has been constructed has this form; hence one should view this generalization of the minimal model as a necessity.

(II) *The ratio of electroweak VEV's,* $\tan\beta = v_U/v_D$, *is allowed to be large,* $\approx m_t/m_b$.

This is certainly not a necessity; to the contrary, a simple extrapolation of the results of [7] to such large values of $\tan\beta$ suggests that it is already excluded by the present limit on $\mu \to e\gamma$. The case of large $\tan\beta$ in $SO(10)$ has received much attention [5, 38, 39, 40, 41] partly because it has important ramifications for the origin of $m_t/m_b = (\lambda_t/\lambda_b)\tan\beta$. To what extent is this puzzling large ratio to be understood as a large hierarchy of Yukawa couplings, and to what extent in terms of a large value for $\tan\beta$? If the third generation masses arise from a single interaction of the form $\mathbf{16}_3\mathbf{16}_3\mathbf{10}$ it is possible to predict m_t using m_b and m_τ as input [40], providing the theory is perturbative up to M_G. The prediction is 175 ± 10 GeV [38], and requires $\tan\beta \approx m_t/m_b$. In this chapter we investigate whether this intriguing possibility is excluded by the $\mu \to e\gamma$ signal; or, more correctly, we determine whether it requires a soft origin for supersymmmetry breaking, making it incompatible with the standard supergravity scenario [15, 16, 17].

In the next section we show that $SO(10)$ models with $\boldsymbol{\lambda} \to \boldsymbol{\lambda}(A)$ possess new gaugino mixing matrices in the up-quark sector, which did not arise in the minimal models. In section 3.3 we set our notation for the supersymmetric standard model with arbitrary gaugino mixing matrices, and we show which mixing matrices are expected from unified models according to the gauge group and the value of $\tan\beta$. In section 3.4 we describe the new phenomenological signatures which are generated by the gaugino mixing matrices in the up sector; these signatures are generic to all models with Yukawa interactions generated from higher dimensional operators. The consequences of large $\tan\beta$ for the flavor and CP violating signatures are analyzed analytically in section 3.5 and numerically in section 3.6. The analysis of the first five sections applies to a wide class of models. In section 3.7 we illustrate the results in the particular models introduced by Anderson *et al.* [5]. As well as providing illustrations, these models have features unique to themselves. Conclusions are drawn in section 3.8.

3.2 New flavor mixing in the up sector

In [6, 7, 8, 9] flavor and CP violating signals are studied in minimal $SU(5)$ and $SO(10)$ models with moderate $\tan\beta$. In these models the radiative corrections to the scalar mass matrices are dominated by the top quark Yukawa coupling λ_t of the unified theory, so the scalar mass matrices tend to align with the up-type Yukawa coupling matrix and all non-trivial flavor mixing matrices are simply related to the KM matrix. However, as mentioned above, the minimal models do not give realistic fermion masses. One has to insert $SO(10)$ breaking into the Yukawa interactions. The simplest way to achieve this is to assume that the light fermion masses come from the non-renormalizable operators

$$\lambda'_{ij}\mathbf{16}_i\frac{A_1}{M_1}\frac{A_2}{M_2}\cdots\frac{A_\ell}{M_\ell}\mathbf{10}\frac{A_{\ell+1}}{M_{\ell+1}}\cdots\frac{A_n}{M_n}\mathbf{16}_j, \tag{3.2.1}$$

where the $\mathbf{16}_i$'s contain the three low energy families, $\mathbf{10}$ contains the Higgs doublets, and A's are adjoint fields with VEV's which break the $SO(10)$ gauge group. After substituting in the VEV's of the adjoints, they become the usual Yukawa interactions with different Clebsch factors associated with Yukawa couplings of fields with different quantum numbers. For example in the models introduced by Anderson *et al.* [5], (hereafter referred to as ADHRS models)

$$\lambda_U = \begin{pmatrix} 0 & z_u C & 0 \\ z'_u C & y_u E & x_u B \\ 0 & x'_u B & A \end{pmatrix}, \lambda_D = \begin{pmatrix} 0 & z_d C & C \\ z'_d & y_d E & x_d B \\ 0 & x'_d B & A \end{pmatrix}, \lambda_E = \begin{pmatrix} \underline{0} & z_e C & 0 \\ z'_e C & y_e E & x_e B \\ 0 & x'_e B & A \end{pmatrix}, \tag{3.2.2}$$

where the x, y, z's are Clebsch factors arising from the VEV's of the adjoint fields. Thus realistic fermion masses and mixings can be obtained.

The radiative corrections to the soft SUSY-breaking operators above M_G are now more complicated. From the interactions (3.2.1) the following soft supersymmetry breaking operators are generated:

$$\lambda^{\dagger}_{ik}(A) m^2_{k\ell}(A) \lambda_{\ell j}(A)\ \phi^{\dagger}_i \phi_j, \tag{3.2.3}$$

where ϕ_i, ϕ_j are scalar components of the superfields, and $\lambda_{ij}(A)$ are adjoint dependent couplings, $\lambda(A) = \lambda' \frac{A_1}{M_1} \cdots \frac{A_n}{M_n}$. After the adjoints take their VEV's, the $m^2_{k\ell}(A)$ become the usual soft scalar masses. If we ignore the wavefunction renormalization of the adjoint fields (which is valid in the one-loop approximation), this is the same as if we had replaced the adjoints by their VEV's all the way up to the ultraheavy scale where the ultraheavy fields are integrated out, and treated these nonrenormalizable operators as the usual Yukawa interactions and scalar mass operators. This is a convenient way of thinking and we will use it in the rest of the chapter.

Above the GUT scale, in addition to the Yukawa interactions which give the fermion masses

$$Q\lambda_U U^c H_U,\ Q\lambda_D D^c H_D,\ E^c\lambda_E L H_D, \tag{3.2.4}$$

the operators (3.2.1) also lead to

$$\begin{aligned} &Q\lambda_{qq}QH_{U_3},\ E^c\lambda_{eu}U^cH_{U_3},\ N\lambda_{nd}D^cH_{U_3}, \\ &Q\lambda_{q\ell}LH_{D_3},\ U^c\lambda_{ud}D^cH_{D_3}, N\lambda_{n\ell}LH_U, \end{aligned} \tag{3.2.5}$$

where H_{U_3}, H_{D_3} are the triplet partners of the two Higgs doublets H_U and H_D. Each Yukawa matrix has different Clebsch factors associated with its elements, so they can not be diagonalized in the same basis. The scalar mass matrices receive radiative corrections from Yukawa interactions of both (3.2.4) and (3.2.5), which, in the one-loop approximation, take the form

$$\begin{aligned} \Delta\mathbf{m}_Q^2 &\propto \lambda_U\lambda_U^\dagger + \lambda_D\lambda_D^\dagger + 2\lambda_{qq}\lambda_{qq}^\dagger + \lambda_{q\ell}\lambda_{q\ell}^\dagger, \\ \Delta\mathbf{m}_U^2 &\propto 2\lambda_U^\dagger\lambda_U + \lambda_{eu}^\dagger\lambda_{eu} + 2\lambda_{ud}\lambda_{ud}^\dagger, \\ \Delta\mathbf{m}_D^2 &\propto 2\lambda_D^\dagger\lambda_D + \lambda_{nd}^\dagger\lambda_{nd} + 2\lambda_{ud}^\dagger\lambda_{ud}, \\ \Delta\mathbf{m}_L^2 &\propto \lambda_E^\dagger\lambda_E + 3\lambda_{q\ell}^\dagger\lambda_{q\ell} + \lambda_{n\ell}^\dagger\lambda_{n\ell}, \\ \Delta\mathbf{m}_E^2 &\propto 2\lambda_E\lambda_E^\dagger + 3\lambda_{eu}\lambda_{eu}^\dagger. \end{aligned} \tag{3.2.6}$$

In the minimal $SO(10)$ model, scalar mass renormalizations above M_G arise from a single matrix λ_U. It is therefore possible to choose a "U-basis" in which the scalings are purely diagonal. This is clearly not possible in the general models. All scalar mass matrices and Yukawa matrices are in general diagonalized in different bases.

Therefore, flavor mixing matrices should appear in all gaugino vertices, including in the up-quark sector (where they are trivial in the minimal models studied in [6, 7, 8, 9]). The up-type quark-squark-gaugino flavor mixing is a novel feature of the general models. Its consequences will be discussed in Sec. 3.4. Also, the flavor mixing matrices are no longer simply the KM matrix. They are model dependent and are different for different types of quarks and charged leptons, and are fully described in the next section.

3.3 Flavor mixing matrices in general supersymmetric standard models.

In this section we set our notation for the gaugino flavor mixing matrices in the supersymmetric theory below M_G, taken to have minimal field content. We also give general expectations for these matrices in a wide variety of unified theories.

The most general scalar masses are 6×6 matrices for squarks and charged sleptons and 3×3 matrix for sneutrinos,

$$\begin{aligned}
\mathbf{m}_U^2 &= \begin{pmatrix} \mathbf{m}_{U_L}^2 & (\zeta_U + \lambda_U \mu \cot\beta) v_U \\ (\zeta_U^\dagger + \lambda_U^\dagger \mu \cot\beta) v_U & \mathbf{m}_{U_R}^2 \end{pmatrix}, \\
\mathbf{m}_D^2 &= \begin{pmatrix} \mathbf{m}_{D_L}^2 & (\zeta_D + \lambda_D \mu \tan\beta) v_D \\ (\zeta_D^\dagger + \lambda_D^\dagger \mu \tan\beta) v_D & \mathbf{m}_{D_R}^2 \end{pmatrix}, \\
\mathbf{m}_E^2 &= \begin{pmatrix} \mathbf{m}_{E_L}^2 & (\zeta_E + \lambda_E \mu \tan\beta) v_D \\ (\zeta_E^\dagger + \lambda_E^\dagger \mu \tan\beta) v_D & \mathbf{m}_{E_R}^2 \end{pmatrix}, \\
\mathbf{m}_\nu^2 &= \left(\mathbf{m}_{\nu_{ij}}^2\right),
\end{aligned} \tag{3.3.1}$$

where $\mathbf{m}^2_{U_L}, \mathbf{m}^2_{D_L}, \mathbf{m}^2_{U_R}, \mathbf{m}^2_{D_R}, \mathbf{m}^2_{E_L}, \mathbf{m}^2_{E_R}$ are 3×3 soft SUSY-breaking mass matrices for the left-handed and right-handed squarks and sleptons, and $\zeta_U, \zeta_D, \zeta_E$ are the trilinear soft SUSY-breaking terms. To calculate flavor-violating processes, such as $\mu \to e\gamma$, one can diagonalize the mass matrix $\mathbf{m}^2_{\tilde{E}}$ by the 6×6 unitary rotation matrix V_E and $\mathbf{m}^2_{\tilde{\nu}}$ by the 3×3 unitary rotation V_ν,

$$\mathbf{m}^2_{\tilde{E}} = V_E \overline{\mathbf{m}}^2_{\tilde{E}} V_E^\dagger, \qquad \mathbf{m}^2_{\tilde{\nu}} = V_\nu \overline{\mathbf{m}}^2_{\tilde{\nu}} V_\nu^\dagger, \tag{3.3.2}$$

where $\overline{\mathbf{m}}^2_{\tilde{E}}, \overline{\mathbf{m}}^2_{\tilde{\nu}}$ are diagonal. The amplitude for $\mu \to e\gamma$ is given by the diagrams in Fig. 3.1, summing up all the internal scalar mass eigenstates.

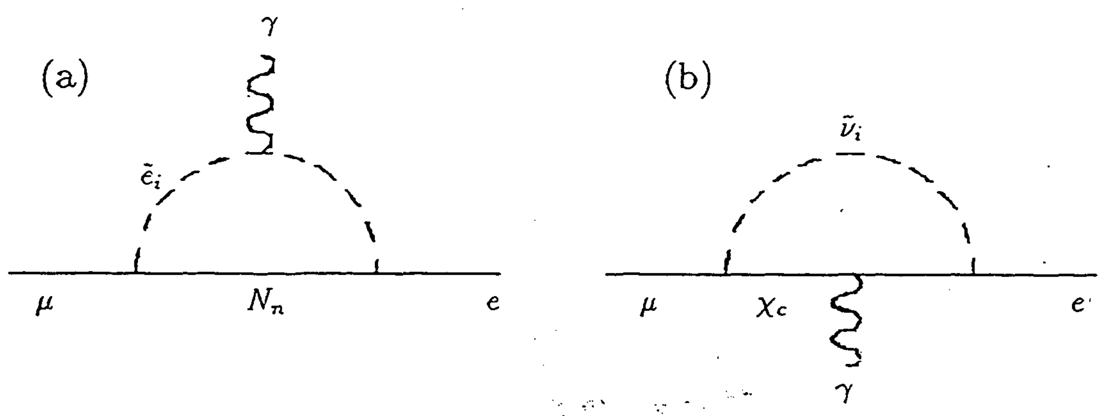


Figure 3.1: Feynman diagrams contributing to $\mu \to e\gamma$.

If the entries in the scalar mass matrices are arbitrary, they generally give unacceptably large rates for flavor-violating processes. From the experimental limits one expects that the first two generation scalar masses should be approximately degenerate and the chirality-changing mass matrices ζ_A should be approximately proportional to the corresponding Yukawa coupling matrices $\boldsymbol{\lambda}_A$. In this chapter we treat the chirality-conserving mass matrices and chirality-changing mass matrices separately, i.e., the mass eigenstates are assumed to be purely left-handed or right-handed, and the chirality-changing mass terms are treated as a perturbation. This may not be a good approximation for the third generation where the Yukawa couplings are large, the correct treatment will be used in the numerical studies of Sec. 3.6. The superpotential contains

$$W \supset Q^T \lambda_U U^c H_U + Q^T \lambda_D D^c H_D + E^{cT} \lambda_E L H_D, \tag{3.3.3}$$

where $\lambda_U, \lambda_D, \lambda_E$ are the Yukawa coupling matrices which are diagonalized by the left and right rotations,

$$\begin{aligned} \lambda_U &= V_{U_L}^* \overline{\lambda}_U V_{U_R}^\dagger, \\ \lambda_D &= V_{D_L}^* \overline{\lambda}_D V_{D_R}^\dagger, \\ \lambda_E &= V_{E_R}^* \overline{\lambda}_E V_{E_L}^\dagger. \end{aligned} \tag{3.3.4}$$

The soft SUSY-breaking interactions contain

$$\begin{aligned} \widetilde{Q}^\dagger \mathbf{m}_Q^{2*} \widetilde{Q} &+ \widetilde{U}^{c\dagger} \mathbf{m}_U^2 \widetilde{U}^c + \widetilde{D}^{c\dagger} \mathbf{m}_D^2 \widetilde{D}^c + \widetilde{L}^\dagger \mathbf{m}_L^2 \widetilde{L} + \widetilde{E}^{c\dagger} \mathbf{m}_E^{2*} \widetilde{E}^c \\ &+ \widetilde{Q}^T \zeta_U \widetilde{U}^c H_U + \widetilde{Q}^T \zeta_D \widetilde{D}^c H_D + \widetilde{E}^{cT} \zeta_E \widetilde{L} H_D. \end{aligned} \tag{3.3.5}$$

Because the trilinear terms should be approximately proportional to the Yukawa couplings, we write

$$\zeta = \zeta_0 + \Delta\zeta = A\lambda + \Delta\zeta, \tag{3.3.6}$$

where the universal A term (and also the μ parameter) are assumed to be real in order to avoid the SUSY CP problem. The soft-breaking mass matrices are diagonalized by:[2]

$$\mathbf{m}_Q^{2*} = U_Q \overline{\mathbf{m}}_Q^{2*} U_Q^\dagger, \;\; \mathbf{m}_U^2 = U_U \overline{\mathbf{m}}_U^2 U_U^\dagger, \;\; \mathbf{m}_D^2 = U_D \overline{\mathbf{m}}_D^2 U_D^\dagger,$$

$$\mathbf{m}_L^2 = U_L \overline{\mathbf{m}}_L^2 U_L^\dagger, \;\; \mathbf{m}_E^{2*} = U_E \overline{\mathbf{m}}_E^{2*} U_E^\dagger, \tag{3.3.7}$$

$$\Delta\zeta_U = V_{U_L}'^{*} \Delta\overline{\zeta}_U V_{U_R}'^{\dagger}, \;\; \Delta\zeta_D = V_{D_L}'^{*} \Delta\overline{\zeta}_D V_{D_R}'^{\dagger}, \;\; \Delta\zeta_E = V_{E_R}'^{*} \Delta\overline{\zeta}_E V_{E_L}'^{\dagger}. \tag{3.3.8}$$

In the mass eigenstate basis the rotation matrices V, U appear in the gaugino couplings,

$$\begin{aligned}
\mathcal{L}_g &= \sqrt{2}g' \sum_{n=1}^{4} \Big[-\frac{1}{2}\overline{e}_L W_{E_L}^\dagger \widetilde{e}_L N_n (H_{n\widetilde{B}} + \cot\theta_W H_{n\widetilde{w}_3}) + \overline{e}_L^c W_{E_R}^\dagger \widetilde{e}_R N_n H_{n\widetilde{B}} \\
&+ \frac{1}{2}\cot\theta_W \overline{\nu}_L \widetilde{\nu}_L N_n H_{n\widetilde{w}_3} \\
&+ \overline{u}_L W_{U_L}^\dagger \widetilde{u}_L N_n (\frac{1}{6} H_{n\widetilde{B}} + \frac{1}{2}\cot\theta_W H_{n\widetilde{w}_3}) + \overline{d}_L W_{D_L}^\dagger \widetilde{d}_L N_n (\frac{1}{6} H_{n\widetilde{B}} - \frac{1}{2}\cot\theta_W H_{n\widetilde{w}_3}) \\
&- \frac{2}{3}\overline{u}_L^c W_{U_R}^\dagger \widetilde{u}_R N_n H_{n\widetilde{B}} + \frac{1}{3}\overline{d}_L^c W_{D_R}^\dagger \widetilde{d}_R N_n H_{n\widetilde{B}} + h.c.\Big] \\
&+ g\sum_{c=1}^{2} [\overline{e}_L W_{E_L}^\dagger \widetilde{\nu}_L (\chi_c K_{c\widetilde{w}}) + \overline{\nu}_L \widetilde{e}_L (\chi_c^\dagger K_{c\widetilde{w}}^*) \\
&+ \overline{d}_L W_{D_L}^\dagger \widetilde{u}_L (\chi_c K_{c\widetilde{w}}) + \overline{u}_L W_{U_L}^\dagger \widetilde{d}_L (\chi_c^\dagger K_{c\widetilde{w}}^*) + h.c.] \\
&+ \sqrt{2}g_3 [\overline{u}_L W_{U_L}^\dagger \widetilde{u}_L \widetilde{g} + \overline{d}_L W_{D_L}^\dagger \widetilde{d}_L \widetilde{g} + \overline{u}_L^c W_{U_R}^\dagger \widetilde{u}_R \widetilde{g} + \overline{d}_L^c W_{D_R}^\dagger \widetilde{d}_R \widetilde{g} + h.c.],
\end{aligned} \tag{3.3.9}$$

where[3] the neutralino and chargino mass eigenstates are related to the gauge eigenstates by e.g. $\widetilde{B} = \sum_{n=1}^{4} H_{n\widetilde{B}} N_n$, $\widetilde{w}_3 = \sum_{n=1}^{4} H_{n\widetilde{w}_3} N_n$, $\widetilde{w}^+ = \sum_{c=1}^{2} K_{c\widetilde{w}} \chi_c$, and

$$W_{E_L} = U_L^\dagger V_{E_L}, \;\; W_{E_R} = U_E^\dagger V_{E_R}, \;\; W_{U_L} = U_Q^\dagger V_{U_L}, \;\; W_{D_L} = U_Q^\dagger V_{D_L},$$

[2]Here we ignore the $SU(2) \times U(1)$ breaking contribution to the scalar masses. Otherwise there should be different rotation matrices diagonalizing up and down left-handed squark mass matrices.

[3]Neutrino masses are not discussed here and we choose the neutrino to be in the sneutrino mass eigenstate basis.

$$W_{U_R} = U_U^\dagger V_{U_R},\ W_{D_R} = U_D^\dagger V_{D_R}.$$

There are also non-diagonal chirality-changing mass terms

$$\begin{aligned} -\mathcal{L}_m^{n.d} &= \tilde{e}_R^T W_{E_R}^* (A_E + \mu \tan\beta) \lambda_E W_{E_L}^\dagger \tilde{e}_L v_D + \tilde{e}_R^T U_E^T \Delta\zeta_E U_L \tilde{e}_L v_D \\ &+ \tilde{d}_L^T W_{D_L}^* (A_D + \mu \tan\beta) \lambda_D W_{D_R}^\dagger \tilde{d}_R v_D + \tilde{d}_L^T U_Q^T \Delta\zeta_D U_D \tilde{d}_R v_D \\ &+ \tilde{u}_L^T W_{U_L}^* (A_U + \mu \cot\beta) \lambda_U W_{U_R}^\dagger \tilde{u}_R v_U + \tilde{u}_L^T U_Q^T \Delta\zeta_U U_U \tilde{u}_R v_U \\ &+ h.c. \end{aligned} \tag{3.3.10}$$

The lepton flavor-violating (LFV) couplings are summarized in Fig. 3.2.

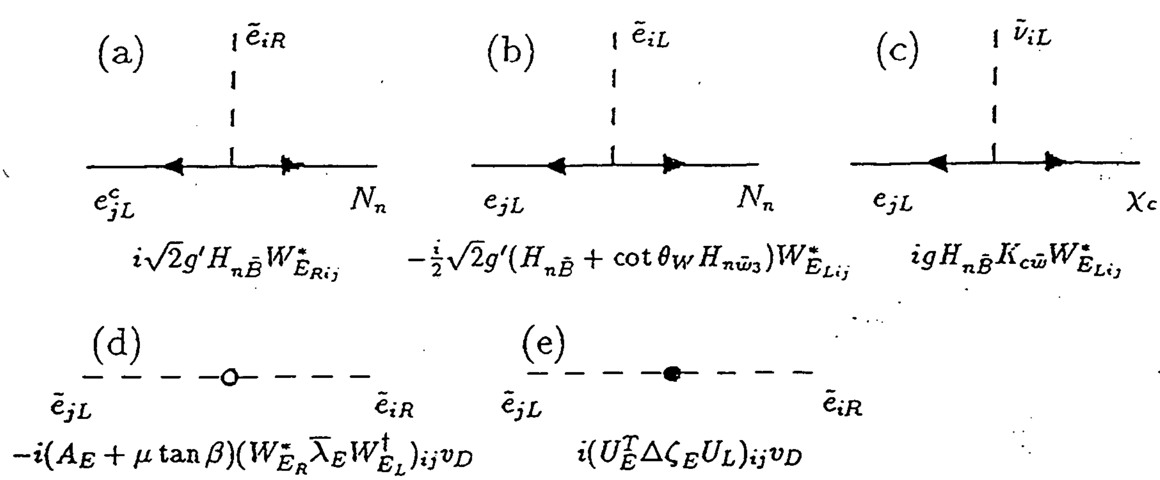


Figure 3.2: Lepton flavor-violating couplings in general supersymmetric standard models.

In the rest of this section we discuss the flavor mixing matrices in the minimal supersymmetric standard model, minimal and general $SU(5)$ and $SO(10)$ models, with moderate or large $\tan\beta$. The results are summarized in Table 3.1.

For the minimal supersymmetric standard model, the radiative corrections to the soft masses only come from the Yukawa interactions of the MSSM:

$$\begin{aligned}
\Delta\mathbf{m}_Q^2 &\propto \boldsymbol{\lambda}_U\boldsymbol{\lambda}_U^\dagger + \kappa\boldsymbol{\lambda}_D\boldsymbol{\lambda}_D^\dagger,\\
\Delta\mathbf{m}_U^2 &\propto 2\boldsymbol{\lambda}_U^\dagger\boldsymbol{\lambda}_U,\\
\Delta\mathbf{m}_D^2 &\propto 2\boldsymbol{\lambda}_D^\dagger\boldsymbol{\lambda}_D,\\
\Delta\mathbf{m}_L^2 &\propto \boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E,\\
\Delta\mathbf{m}_E^2 &\propto 2\boldsymbol{\lambda}_E\boldsymbol{\lambda}_E^\dagger.
\end{aligned} \tag{3.3.11}$$

We have assumed a boundary condition on the scalar mass matrices $\mathbf{m}_A^2 \propto I$ at M_{PL}, and $\kappa \neq 1$ represents the possibility that the proportionality constants are not universal. For moderate $\tan\beta, \lambda_t \gg \lambda_b$ so that the radiative corrections are dominated by λ_t. Thus one can neglect the $\boldsymbol{\lambda}_D$ contribution and the only nontrivial mixing is W_{D_L}. For large $\tan\beta, \lambda_t$ and λ_b are comparable, so $\mathbf{m}_Q^2$ will lie between $\boldsymbol{\lambda}_U\boldsymbol{\lambda}_U^\dagger$ and $\boldsymbol{\lambda}_D\boldsymbol{\lambda}_D^\dagger$. Therefore both W_{U_L} and W_{D_L} are non-trivial.

For the minimal $SU(5)$ model, there are only two Yukawa matrices, $\boldsymbol{\lambda}_U = \boldsymbol{\lambda}_{10}$, $\boldsymbol{\lambda}_D = \boldsymbol{\lambda}_E = \boldsymbol{\lambda}_5$, and

$$\begin{aligned}
\Delta\mathbf{m}_Q^2 &\propto 3\boldsymbol{\lambda}_U\boldsymbol{\lambda}_U^\dagger + 2\kappa\boldsymbol{\lambda}_D\boldsymbol{\lambda}_D^\dagger,\\
\Delta\mathbf{m}_U^2 &\propto 3\boldsymbol{\lambda}_U^\dagger\boldsymbol{\lambda}_U + 2\kappa\boldsymbol{\lambda}_D^\dagger\boldsymbol{\lambda}_D,\\
\Delta\mathbf{m}_D^2 &\propto 4\boldsymbol{\lambda}_D^\dagger\boldsymbol{\lambda}_D,\\
\Delta\mathbf{m}_L^2 &\propto 4\boldsymbol{\lambda}_D^\dagger\boldsymbol{\lambda}_D,
\end{aligned}$$

$$\Delta\mathbf{m}_E^2 \propto 3\boldsymbol{\lambda}_U\boldsymbol{\lambda}_U^\dagger + 2\kappa\boldsymbol{\lambda}_D\boldsymbol{\lambda}_D^\dagger. \qquad (3.3.12)$$

For moderate $\tan\beta, \lambda_t \gg \lambda_b$, we have non-trivial mixings for W_{D_L} and W_{E_R}, as found in [6, 7]. For large $\tan\beta$, $\boldsymbol{\lambda}_D$ can not be ignored, giving non-trivial mixings for W_{U_L} and W_{U_R}.

For the minimal $SO(10)$ model considered in [7, 8],

$$\begin{aligned}
\Delta\mathbf{m}_Q^2 &\propto 5\boldsymbol{\lambda}_U\boldsymbol{\lambda}_U^\dagger + 5\kappa\boldsymbol{\lambda}_D\boldsymbol{\lambda}_D^\dagger, \\
\Delta\mathbf{m}_U^2 &\propto 5\boldsymbol{\lambda}_U^\dagger\boldsymbol{\lambda}_U + 5\kappa\boldsymbol{\lambda}_D^\dagger\boldsymbol{\lambda}_D, \\
\Delta\mathbf{m}_D^2 &\propto 5\boldsymbol{\lambda}_U^\dagger\boldsymbol{\lambda}_U + 5\kappa\boldsymbol{\lambda}_D^\dagger\boldsymbol{\lambda}_D, \\
\Delta\mathbf{m}_L^2 &\propto 5\boldsymbol{\lambda}_U^\dagger\boldsymbol{\lambda}_U + 5\kappa\boldsymbol{\lambda}_D^\dagger\boldsymbol{\lambda}_D, \\
\Delta\mathbf{m}_E^2 &\propto 5\boldsymbol{\lambda}_U\boldsymbol{\lambda}_U^\dagger + 5\kappa\boldsymbol{\lambda}_D\boldsymbol{\lambda}_D^\dagger. \qquad (3.3.13)
\end{aligned}$$

We have non-trivial mixings $W_{D_L}, W_{D_R}, W_{E_L}$, and W_{E_R} for moderate $\tan\beta$ and non-trivial mixings for all W's for large $\tan\beta$.

For the general $SU(5)$ or $SO(10)$ models, defined in the last section, we get non-trivial mixings for all mixing matrices in general. However, in $SU(5)$ models with moderate $\tan\beta$, the splittings among $\mathbf{m}_D^2$ and $\mathbf{m}_L^2$ are too small (because they are generated by the small $\boldsymbol{\lambda}_5(A)$) to give significant flavor-changing effects.

One might expect that the mixing in the W_U's are smaller than those in the W_D's because of the larger hierarchy in $\boldsymbol{\lambda}_U$ compared with $\boldsymbol{\lambda}_D$. However, a given W is the product of a $U^\dagger$ (which diagonalizes the scalar mass matrix) and a V (which diagonalizes the Yukawa matrix). Even if the mixings in V_U's are smaller than those in V_D's because of the larger hierarchies in $\boldsymbol{\lambda}_U$, we do not have a general argument for the size of mixings in U matrices. This is because U diagonalizes (appropriate combinations of) known Yukawa matrices and unknown Yukawa matrices appearing above the GUT scale, (3.2.5). The mixings in $U^\dagger$ and V can add up or cancel each other. Our only general expectation is that these new Yukawa matrices have similar hierarchical patterns as $\boldsymbol{\lambda}_U$ or $\boldsymbol{\lambda}_D$. Without a specific model, one can at most say that all non-trivial W's are expected to be comparable to V_{KM}; the argument that the mixings in W_U's should be smaller than is W_D's is not valid.

In the minimal models at moderate $\tan\beta$, the leading contributions to flavor-changing processes, such as $\mu \rightarrow e\gamma$, involve diagrams with a virtual scalar of the third generation. Although such contributions are highly suppressed by mixing angles, they dominate because they have large violations of super-GIM[48]: the top Yukawa coupling makes $m_{\tilde{\tau}}$ very different from $m_{\tilde{e}}, m_{\tilde{\mu}}$. At large $\tan\beta$, the strange/muon Yukawa couplings get enhanced, so the splitting between $m_{\tilde{e}}$ and $m_{\tilde{\mu}}$ increases, leading to potentially competitive contributions to flavor changing processes which do not involve the third generation. The importance of these new diagrams can be estimated by comparing the contributions to Δm^2_{21} (in a basis where gaugino vertices are diagonal) when the super-GIM cancellation is between scalars of the first two generations (2-1) and third generations (3-1):

$$\frac{\Delta m^2_{21}(2\text{-}1)}{\Delta m^2_{21}(3\text{-}1)} \simeq \frac{V_{cd}\lambda_2^2}{V_{td}V_{ts}\lambda_t^2} \simeq \begin{cases} 10^{-2}, & \text{for } \lambda_2 = \lambda_c, \\ \left(\frac{\tan\beta}{60}\right)^2, & \text{for } \lambda_2 = \lambda_s. \end{cases} \tag{3.3.14}$$

We can see that for large $\tan\beta$ (or any $\tan\beta$ with small λ_s coming from the

		$SU(5)$		$SO(10)$	
	MSSM	Minimal	general	minimal	general
δm_3^2	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
δm_2^2	•	•	∘	•	∘
W_{U_L}	•	•	$\surd$	•	$\surd$
W_{D_L}	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
W_{U_R}	—	•	$\surd$	•	$\surd$
W_{D_R}	—	—	$\surd^*$	$\surd$	$\surd$
W_{E_L}	—	—	$\surd^*$	$\surd$	$\surd$
W_{E_R}	—	$\surd$	$\surd$	$\surd$	$\surd$

Table 3.1: Summary table for the flavor mixing matrices.

δm_3^2 : important effects due to some third generation scalars not degenerate with those of first two generations.

δm_2^2 : non-negligible effects due to nondegeneracy of the first two generation scalars.

W_i : fermion i and scalar $\tilde{i}$ are rotated differently to get to mass basis.

$\surd$: present for any value of $\tan\beta$.

• : present only for large $\tan\beta$.

∘ : present for large $\tan\beta$, but model dependent for moderate $\tan\beta$.

–: not present.

$*$: although present, its effect for moderate $\tan\beta$ on flavor violation is small due to the small non-degeneracy among different generation scalars.

mixing of Higgs at M_G i.e., $\lambda_s(M_G) = \frac{\tan\beta}{60}\lambda_2(M_G)$), this could be comparable to the flavor violating effects from the large splitting of the third generation scalar masses. However, for the $\mu \to e\gamma$ in $SO(10)$ models, it does not contribute to diagrams which are proportional to m_τ, (because it does not involve the third generation scalars), the dominant contributions are still those diagrams considered in [7]. For flavor-changing processes which do not need chirality flipping, such as $K-\overline{K}$ mixing, and all flavor-changing processes in $SU(5)$ models, this non-degeneracy between the first two generations is important. The above discussion is summarized in Table 3.1.

3.4 Phenomenology from up-type mixing

As discussed in the previous section, unlike the minimal models with moderate $\tan\beta$ studied in [6, 7, 8, 9] in generic GUT's (for any $\tan\beta$) and even for minimal GUT's (at large $\tan\beta$), we expect mixing matrices in the up sector. Having motivated an origin for non-trivial up mixing matrices $W_{U_{L(R)}} \neq 1$, we consider some effects they produce. In the following we simply assume some $W_{u_{L(R)}}$ at the weak scale and consider their phenomenological consequences. (See however section 3.5 and the appendix A for a discussion of the scaling of mixing matrices from GUT to weak scales.) In particular we discuss $D-\bar{D}$ mixing, corrections to up-type quark masses, contributions to the neutron electric dipole moment (e.d.m.) and the possibility of different dominant proton decay modes than those expected from minimal models.

3.4.1 $D - \bar{D}$ mixing

To get an idea for the contribution of up-type mixing matrices to $D - \bar{D}$ mixing, we follow [49, 50] and employ the mass insertion approximation. The bounds obtained from $D - \bar{D}$ mixing on the 6×6 up-squark mass matrix

$$m_U^2 = \begin{pmatrix} m^2_{U_{LL}} & m^2_{U_{LR}} \\ m^2_{U_{RL}} & m^2_{U_{RR}} \end{pmatrix}$$

(in the basis where gluino and Yukawa couplings are diagonal) are summarized in [50]. For average up-squark mass of $\widetilde{m} = 1$ TeV, they are

$$\sqrt{\frac{m^2_{U_{LL12}}}{\widetilde{m}^2}\frac{m^2_{U_{RR12}}}{\widetilde{m}^2}} \le 0.04, \tag{3.4.1}$$

$$\frac{m^2_{U_{LR12}}}{\widetilde{m}^2} \le 0.06. \tag{3.4.2}$$

Consider first (3.4.1). In the last section we estimated that the contribution to m^2_{12} from the slight non-degeneracy between the first two generation scalars is generically at most comparabale to that from the non-degeneracy between the first two and third generation scalars. Thus, for our calculation, we only consider the contribution from the splitting between first two and third generation scalars. Then, for $A = L, R$

$$\left|\frac{m^2_{AA12}}{\widetilde{m}^2}\right| = \left|W_{U_{A13}} W^\dagger_{U_{A32}}\right| \left|\frac{m^2_{\tilde{t}_A} - m^2_{\tilde{u}_A}}{\widetilde{m}^2}\right| \le \left|W_{U_{A13}} W^\dagger_{U_{A32}}\right|. \tag{3.4.3}$$

We see that for W's of the same size as the corresponding KM matrix elements, the left hand side of (3.4.1) is of order 4×10^{-4}, and the bound is easily satisfied. Turning to (3.4.2), note that if $\zeta_U = A\lambda_U$, $m^2_{U_{LR12}} = 0$. However, we expect $\zeta_U = A\lambda_U + \Delta\zeta_U$, with $\Delta\zeta_U$ induced in running from M_{PL} to M_G having primarily a third generation component in the gauge eigenstate basis. If all relevant mixing matrix elements are of order the KM matrix elements, we expect $\left|\frac{m^2_{U_{LR12}}}{m^2}\right| = O\left(\left|\frac{Am_t}{m^2} V_{td} V_{ts}\right|\right)$. Again, we see that the bound (3.4.2) is generically easily satisfied, and thus we do not in general expect significant contributions to $D - \bar{D}$ mixing.

3.4.2 Weak-scale corrections to up-type quark masses

It is well known that there are important weak-scale radiative corrections to the down quark mass matrix proportional to $\tan\beta$ [37, 38, 39, 51, 52]. In general unified models, with non-zero W_U, there are also important weak scale corrections to the up quark mass matrix.

From the diagram in Fig. 3.3, we have a contribution to up-type masses proportional to m_t. We find, again assuming degeneracy between the scalars of the first two generations,

$$\begin{aligned} \Delta m_u^{ij} &= \frac{8}{3}\left(\frac{\alpha_s}{4\pi}\right) m_t \left(\frac{A + \mu\cot\beta}{M_{\tilde{g}}}\right) W_{U_{L3i}} W_{U_{R3j}} \left[h(x_{t_L}, x_{t_R}) - h(x_{t_L}, x_{u_R})\right. \\ &\quad \left. - h(x_{u_L}, x_{t_R}) + h(x_{u_L}, x_{u_R})\right], \end{aligned} \tag{3.4.4}$$

where

$$x_i \equiv \frac{\widetilde{m}_i^2}{M_{\tilde{g}}^2}, \quad h(x,y) = \frac{1}{x-y}\left[\frac{x\ln x}{1-x} - \frac{y\ln y}{1-y}\right]. \tag{3.4.5}$$

The largest fractional change in the mass occurs for the up quark. If $W_{U_{L(R)31}}$ is comparable to the corresponding KM matrix element, the contribution to $\frac{\Delta m_u}{m_u}$ is

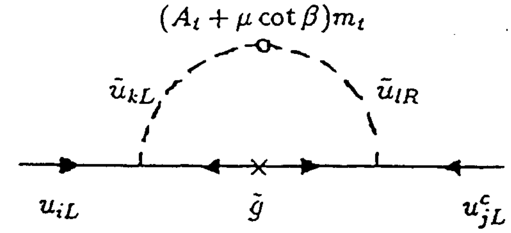


Figure 3.3: Corrections to the up-type quark mass matrix, proportional to m_t.

not significant. However, if each of the $W_{U_{L(R)31}}$ are a factor 3 larger than the corresponding KM elements we can get sizable contributions. In Fig. 3.4, we plot $\frac{\Delta m_u}{m_u}$ in $\frac{m_{\tilde{u}}}{M_{\tilde{g}}} - \frac{m_{\tilde{t}}}{m_{\tilde{u}}}$ space, where we have assumed $m_{\tilde{u}_L} = m_{\tilde{u}_R} \equiv m_{\tilde{u}}$; $m_{\tilde{t}_L} = m_{\tilde{t}_R} \equiv m_{\tilde{t}}$, and we have put $|W_{U_{L31}}| = |W_{U_{R31}}| = 1/30$, $(A + \mu \cot\beta)/m_{\tilde{t}} = 3$. Any deviations from these values can simply be multiplied in $\Delta m_u / m_u$. In some regions of the parameter space it is possible to get the entire up quark mass as a radiative effect. More on radiatively generated fermion masses will be discussed in the next chapter.

3.4.3 Neutron e.d.m.

If we attach a photon in all possible ways to the diagram giving the contribution to u-quark mass, we get a contribution to the u-quark e.d.m., which is proportional to m_t for any value of $\tan\beta$. Evaluating the diagram, we find

$$d^u = e|F| \sin\phi_u \tag{3.4.6}$$

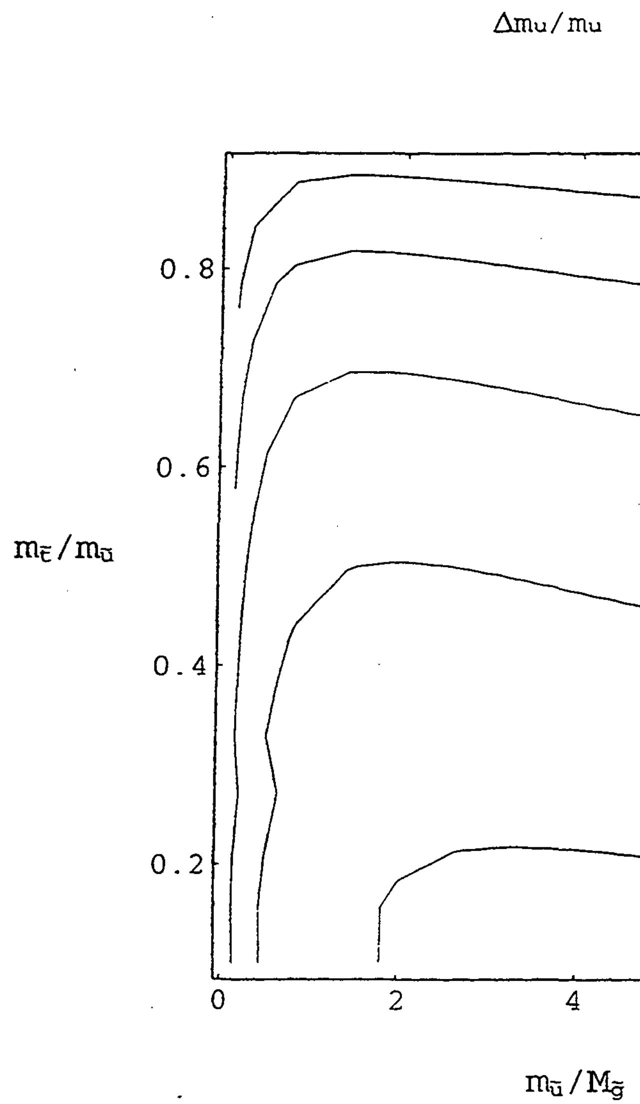


Figure 3.4: Contours for $\frac{\Delta m_u}{m_u}$ in $\frac{m_{\tilde{u}}}{M_{\tilde{g}}}$ - $\frac{m_{\tilde{t}}}{m_{\tilde{u}}}$ plane, assuming $m_{\tilde{u}_L} = m_{\tilde{u}_R} \equiv m_{\tilde{u}}$, $m_{\tilde{t}_L} = m_{\tilde{t}_R} \equiv m_{\tilde{t}}$, $W_{U_{L31}} = W_{U_{R31}} = \frac{1}{30}$, $\frac{A+\mu\cot\beta}{m_{\tilde{t}}} = 3$.

where

$$F = \frac{8}{3}\left(\frac{\alpha_s}{4\pi}\right) m_t \frac{A + \mu \cot\beta}{M_{\tilde{g}}^3} W_{U_{L31}} W^*_{U_{L33}} W_{U_{R31}} W^*_{U_{R33}}$$
$$\times \left[\widetilde{G}_2(x_{t_L}, x_{t_R}) - \widetilde{G}_2(x_{t_L}, x_{u_R}) - \widetilde{G}_2(x_{t_R}, x_{u_L}) + \widetilde{G}(x_{u_L}, x_{u_R})\right], \quad (3.4.7)$$

$$\widetilde{G}_2(x,y) = \frac{g(x) - g(y)}{x - y}, \quad g(x) = \frac{1}{2(x-1)^3}[x^2 - 1 - 2x \ln x], \quad (3.4.8)$$

and

$$\mathrm{Im}\left[m_t W_{U_{L31}} W^*_{U_{L33}} W_{U_{R31}} W^*_{U_{R33}}\right] \equiv |m_t W_{U_{L31}} W^*_{U_{R33}} W_{U_{R31}} W^*_{U_{R33}}| \sin\phi_u. \quad (3.4.9)$$

In general we expect a large non-zero sin ϕ_u. If the combination of W's appearing in the above is comparable to the combination giving a down quark e.d.m., the u-quark contribution will dominate over the d-quark contribution to the neutron e.d.m. considered in [8] by a factor $\frac{m_t}{4m_b \tan\beta}$, (the factor 4 comes from the quark model result $d_n = 4/3d_d - 1/3d_u$). Hence, the neutron e.d.m. may be competitive with $\mu \to e\gamma$ and d_e as the most promising flavor-changing signal for supersymmetric unification.

3.4.4 Proton decay

Finally we turn briefly to the relevance of up-type mixing matrices for proton decay; in particular to the important question of the charge of the lepton in the final state. We know that upon integrating out the superheavy Higgs triplets we can generate the baryon number violating operators $\frac{1}{2M_H}(QQ)(QL)$ and $\frac{1}{M_H}(EU)(DU)$ in the superpotential. These operators must subsequently be dressed at the weak scale in order to obtain four-fermion operators leading to proton decay. The dressing may be done with neutralinos, charginos or gluinos where possible. Since the dressed operator grows with gauge couplings and vanishes for vanishing neutralino/chargino/gluino mass, one might naively expect gluino dressing to be most important. However, if the up-type mixing matrices are trivial, gluino dressed operators can only lead to proton decay with a neutrino in the final state. To see this, we examine each operator separately: $(eu_a)(d_b u_c)\epsilon^{abc}$ (where a, b, c are color indices) must involve u's from two different generations because of the ϵ^{abc}. One of them has to be a u, so the other is a c or a t. If there is no up mixing, the up flavor does not change in the dressing process, so the final state would have to contain a c or a t. Since $m_t, m_c > m_p$, this can not happen. Next, consider $(QQ)(QL) = u_L^a d_L^b (u_L^c e_L - d_L^c \nu_L)\epsilon_{abc}$. By exactly the same argument as the above, the $u_L^a d_L^b u_L^c e_L \epsilon_{abc}$ operator can not contribute to proton decay. Thus, we see that in the absence of mixing in the up sector, gluino dressing can only give neutrinos in the final state. However, the above arguments break down if up-mixing matrices are non-trivial, since gluino dressed diagrams give a significant contribution to the branching ratio for charged lepton modes in proton decay. A detailed study of flavor mixing in the up sector [53] concludes that, whether the wino or gluino dressings are dominant, the muon final state in proton decay is of greatly enhanced importance. Without the mixings, one expects $\frac{\Gamma(p \to K^\circ \mu^+)}{\Gamma(p \to K^+ \bar{\nu})} \approx 10^{-3}$. The up mixing in general models increases this by $O(100)$ making the mode $p \to K^\circ \mu^+$ a favorable one for discovery of proton decay.

3.5 Large $\tan\beta$: analytic treatment

The large $\tan\beta$ scenario is interesting for a number of reasons. For moderate $\tan\beta$, the only way to understand $m_t \gg m_b, m_\tau$ is to have $\lambda_t \gg \lambda_b, \lambda_\tau$ at the weak scale. This gives us little hope of attributing a common origin to third generation Yukawa couplings at a higher scale. However, for large $\tan\beta \sim O\left(\frac{m_t}{m_b}\right)$, the weak scale $\lambda_t, \lambda_b, \lambda_\tau$ are comparable and the above hope is restored. (In fact it is realized in $SO(10)$ models like the ADHRS example outlined in section 3.7). For us, this is sufficient motivation to study the large $\tan\beta$ case in more detail. Also, this case was not studied in [7]. We shall see that unexpected new features arise in the large $\tan\beta$ limit.

The largest contribution to the $\mu \to e\gamma$ amplitude comes from the diagram with $L-R$ scalar mass insertion (Fig. 3.5). In the $L-R$ insertion approximation,

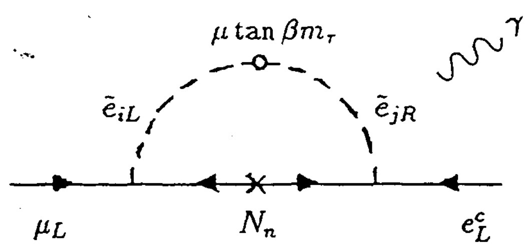


Figure 3.5: The diagram which gives the dominant contribution to $\mu \to e\gamma$ in the large $\tan\beta$ limit. A photon is understood to be attached to the diagram in all possible ways.

the amplitude for $\mu_{L(R)}$ decay is

$$\begin{aligned} F_{L(R)} &= \frac{\alpha}{4\pi\cos^2\theta_W} m_\tau W_{E_{L(R)32}} W_{E_{R(L)31}} W^*_{E_{L(R)33}} W^*_{E_{R(L)33}} (A_E + \mu\tan\beta) \\ &\times \left[G_2(m^2_{\tilde{\tau}_L}, m^2_{\tilde{\tau}_R}) - G_2(m^2_{\tilde{e}_L}, m^2_{\tilde{\tau}_R}) - G_2(m^2_{\tilde{\tau}_L}, m^2_{\tilde{e}_R}) + G_2(m^2_{\tilde{e}_L}, m^2_{\tilde{e}_R}) \right], \end{aligned}$$

where

$$\begin{aligned} G_2(m_1^2, m_2^2) &= \frac{G_2(m_1^2) - G_2(m_2^2)}{m_1^2 - m_2^2}, \\ G_2(m^2) &= \sum_{n=1}^{4} \frac{H_{n\tilde{B}}}{M_n} (H_{n\tilde{B}} + \cot\theta_W H_{n\tilde{w}_3}) g\left(\frac{m^2}{M_n^2}\right). \end{aligned} \tag{3.5.1}$$

Note, however, that for large $\tan\beta$ the $L-R$ insertion approximation may be a bad one, since the chirality-changing mass for the third generation becomes comparable to the chirality conserving masses. A correct treatment will be used for the numerical analysis in the next section. We still expect, however, that the amplitude to be proportional to $W_{E_{32}} W_{E_{31}}$ because of the unitarity of the mixing matrices: the sum of contributions from the first two generations is proportional to $W_{1i}W^*_{1j} + W_{2i}W^*_{2j} = -W_{3i}W^*_{3j}$ for $i \neq j$, and the contribution from the third generation is itself proportional to $W_{3i}W^*_{3j}$.

Two simplifications in the dependence of the $\mu \to e\gamma$ rate on parameter space occur for large $\tan\beta$. First, since the dominant diagram involves the $L-R$ insertion $(A+\mu\tan\beta)m_t$, and since $\tan\beta$ is large, the amplitude does not depend on the weak scale parameter A. Second, in the large $\tan\beta$ limit, the chargino mass matrix is

$$M_\chi = \begin{pmatrix} M_2 & \sqrt{2}M_W\sin\beta \\ \sqrt{2}M_W\cos\beta & -\mu \end{pmatrix} \longrightarrow \begin{pmatrix} M_2 & \sqrt{2}M_W \\ 0 & -\mu \end{pmatrix}, \tag{3.5.2}$$

and the parameters M_2, μ have a direct interpretation as the chargino masses. (Note that this assures us that $\mu \tan\beta$ will likely always be much bigger than A; for a $\tan\beta$ of 50, the LEP lower bound on chargino mass of 45 GeV tells us that $\mu \tan\beta > 2$ TeV, so for A to be comparable to $\mu \tan\beta$ we must have $A > 2$ TeV.)

In considering $\mu \to e\gamma$ for large $\tan\beta$, two factors come immediately to mind which tend to (perhaps dangerously) enhance the rate over the case with moderate $\tan\beta$.

(i) As we have already mentioned, the dominant contribution to $\mu \to e\gamma$ grows with $\tan\beta$; the diagram in Fig. 3.5 is proportional to $\tan\beta$, a factor of 900 in the rate for $\tan\beta = 60$ compared to $\tan\beta = 2$.

(ii) For large $\tan\beta$, λ_τ can be $O(1)$ and we can not neglect its contribution to the running of the slepton mass matrix from M_G to M_S (soft SUSY breaking scale). This scaling generally splits the third generation slepton mass even further from the first two generations, meaning a less effective super-GIM mechanism and a larger amplitude for $\mu \to e\gamma$.

While both of the above effects certainly exist, there are also two sources of *suppression* of the amplitude for large $\tan\beta$, which can together largely compensate for the above factors:

(i)′ Large $\tan\beta$ allows λ_t to be smaller than for moderate $\tan\beta$. There are two reasons for this. First, large $\tan\beta$ allows v_U to be larger and so λ_t can be smaller to reproduce the top mass. Secondly, $b - \tau$ unification [3] is achieved with a smaller

λ_τ in the large $\tan\beta$ regime [38, 39]. Since λ_t is smaller, a smaller non-degeneracy between the third and first two generations is induced in running from M_{PL} to M_G, suppressing the amplitude compared to the moderate $\tan\beta$ case.

(ii)′ In comparing large and moderate tanβ, we must know how the mixing matrices $W_{L,R_{3i}}$ (appearing at the vertices of the diagrams responsible for $\mu \to e\gamma$) compare in these two cases. In the moderate tanβ minimal models discussed in [7], $W_{L,R_{3i}}$ were equal to the corresponding KM matrix elements V_{KM3i} at M_G, and this equality was approximately maintained in running form M_G to M_S. As discussed in the previous sections, for more general models one expects that the $W_{L(R)_{3i}}$ at M_G are equal to V_{KM3i} at M_G up to some combination of Clebsches. One might then expect (as in the minimal models) that this relationship continues to approximately hold at lower scales. In fact for large $\tan\beta$ this expectation is false. We find that often, the $W_{L(R)3i}$ *decrease* from M_G to M_S, overcompensating for the increased non-degeneracy between the third and first two generation slepton masses induced by large λ_τ (point (ii) above).

In the following, we examine the scaling of these mixing matrices in detail. Consider first the lepton sector. The RGE for $\boldsymbol{\lambda}_E$ (in the following $t = \frac{\ln(\mu/M_S)}{16\pi^2}$) is

$$\frac{d\boldsymbol{\lambda}_E}{dt} = \boldsymbol{\lambda}_E[3\boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E + Tr(3\boldsymbol{\lambda}_D^\dagger\boldsymbol{\lambda}_D + \boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E) - 3g_2^2 - \frac{9}{5}\, g_1^2] \tag{3.5.3}$$

giving

$$\frac{d}{dt}\boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E = 6\boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E + 2\boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E Tr(3\boldsymbol{\lambda}_D^\dagger\boldsymbol{\lambda}_D + \boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E) - (6g_2^2 + \frac{18}{5}\, g_1^2)\boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E \tag{3.5.4}$$

$$\frac{d}{dt}\boldsymbol{\lambda}_E\boldsymbol{\lambda}_E^\dagger = 6\boldsymbol{\lambda}_E\boldsymbol{\lambda}_E^\dagger + 2\boldsymbol{\lambda}_E\boldsymbol{\lambda}_E^\dagger Tr(3\boldsymbol{\lambda}_D^\dagger\boldsymbol{\lambda}_D + \boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E) - (6g_2^2 + \frac{18}{5}\, g_1^2)\boldsymbol{\lambda}_E\boldsymbol{\lambda}_E^\dagger. \tag{3.5.5}$$

These in turn imply that the basis in which $\boldsymbol{\lambda}_E^\dagger \boldsymbol{\lambda}_E$ is diagonal, and the (in general different) basis where $\boldsymbol{\lambda}_E \boldsymbol{\lambda}_E^\dagger$ is diagonal, do not change with scale. Consider now the evolution of the left handed slepton mass matrix $\mathbf{m}_L^2$. The RGE for $\mathbf{m}_L^2$ is

$$\frac{d}{dt}\mathbf{m}_L^2 = (\mathbf{m}_L^2 + 2m_{H_d}^2)\boldsymbol{\lambda}_E^\dagger \boldsymbol{\lambda}_E + 2\boldsymbol{\lambda}_E^\dagger \mathbf{m}_E^2 \boldsymbol{\lambda}_E + \boldsymbol{\lambda}_E^\dagger \boldsymbol{\lambda}_E \mathbf{m}_L^2 + 2\zeta_E^\dagger \zeta_E - \text{ gaugino terms.} \tag{3.5.6}$$

In the basis where $\boldsymbol{\lambda}_E^\dagger \boldsymbol{\lambda}_E$ is diagonal, keeping only the λ_τ contribution, the $3i$ entry ($i \neq 3$) becomes:

$$\frac{d}{dt} m_{L3i}^2 = \lambda_\tau^2 m_{L3i}^2 + 2(\zeta_E^\dagger \zeta_E)_{3i}. \tag{3.5.7}$$

In this basis, we have $\mathbf{m}_L^2 = W_L^\dagger \overline{\mathbf{m}}_L^2 W_L$. (Here and in the remainder of this section, we abbreviate $W_{E_{L(R)}} \rightarrow W_{L(R)}$). Assuming degeneracy between scalars of the first two generations, $m_{L_{3i}}^2 = W_{L3i} W_{L33}^\dagger (m_{\tau_L}^2 - m_{e_L}^2) \equiv W_{L3i} W_{L33}^\dagger \Delta m_L^2$. Then (3.5.7) becomes

$$\frac{d}{dt}(W_{L3i} W_{L33}^\dagger \Delta m_L^2) = \lambda_\tau^2 (W_{L3i} W_{L33}^\dagger \Delta m_L^2) + 2(\zeta_E^\dagger \zeta_E)_{3i}. \tag{3.5.8}$$

For now, we ignore the $(\zeta_E^\dagger \zeta_E)_{3i}$ term in (3.5.8), yielding the solution:

$$(W_{L3i} W_{L33}^\dagger \Delta m_L^2)(M_S) = e^{-I_\tau}(W_{L3i} W_{L33}^\dagger \Delta m^2)(M_G), \tag{3.5.9}$$

where

$$I_i \equiv \int_0^{t_G} dt \lambda_i^2(t), \;\; t_G = \frac{\ln(M_G/M_S)}{16\pi^2}. \tag{3.5.10}$$

Thus,

$$W_{L3i} W_{L33}^\dagger (M_S) = e^{-I_\tau} \frac{\Delta m_L^2(M_G)}{\Delta m_L^2(M_S)} W_{L3i}^\dagger W_{L_{33}}(M_G). \tag{3.5.11}$$

Similarly, we find

$$W_{R3i}W^\dagger_{R33}(M_S) = e^{-2I_\tau}\frac{\Delta m^2_R(M_G)}{\Delta m^2_R(M_S)}W_{R3i}W^\dagger_{R_{33}}(M_G). \qquad (3.5.12)$$

Note that, generically the quantities $\frac{\Delta m^2_{L(R)}(M_G)}{\Delta m^2_{L(R)}(M_S)}$ are smaller than one, since the third generation mass gets split even further from the first two generations in running from M_G to M_S. Thus, we find that the $W_{L(R)_{3i}}$ get smaller in magnitude as we scale from M_G to M_S, in contrast with the KM matrix elements V_{KM3i}, which scale as

$$V_{KM3i}(M_S) = e^{(I_t+I_b)}V_{KM3i}(M_G). \qquad (3.5.13)$$

Suppose that at M_G the $W_{L(R)}$ are related to V_{KM} through some combination of Clebsches determined by the physics above the GUT scale.

$$W^\dagger_{L(R)33}W_{L(R)3i}(M_G) = z_{iL(R)}V_{KM3i}(M_G). \qquad (3.5.14)$$

This relationship is not maintained at lower scales; instead we have:

$$W^\dagger_{L33}W_{L3i}(M_S) = \frac{\Delta m^2_L(M_G)}{\Delta m^2_L(M_S)}e^{-(I_\tau+I_t+I_b)}z_{i_L}V_{KM3i}(M_S), \qquad (3.5.15)$$

$$W^\dagger_{R33}W_{R3i}(M_S) = \frac{\Delta m^2_R(M_G)}{\Delta m^2_R(M_S)}e^{-(2I_\tau+I_t+I_b)}z_{i_R}V_{KM3i}(M_S). \qquad (3.5.16)$$

The dominant contribution to the $\mu \to e\gamma$ rate is proportional to

$|W^\dagger_{L33}W_{L32}W^\dagger_{R33}W_{R31}(M_S)|^2 + |W^\dagger_{L33}W_{L31}W^\dagger_{R33}W_{R32}(M_S)|^2$, giving

$$\begin{aligned}B(\mu \to e\gamma) &= \left[\frac{\Delta m^2_L(M_G)}{\Delta m^2_L(M_S)}\frac{\Delta m^2_R(M_G)}{\Delta m^2_R(M_S)}\right]^2 e^{-(6I_\tau+4I_t+4I_b)} \times (|z_{2_L}z_{1_R}|^2 + |z_{1_L}z_{2_R}|^2) \\ &\quad \times B(\mu \to e\gamma, W_{L(R)3i}W^\dagger_{L(R)33}(M_S) \to V_{KM3i}(M_S)) \\ &\equiv \epsilon(|z_{2_L}z_{1_R}|^2 + |z_{1_L}z_{2_R}|^2) \times B(\mu \to e\gamma, W^\dagger_{L(R)33}W_{L(R)33}(M_S) \to V_{KM3i}(M_S)).\end{aligned}$$

(3.5.17)

This ϵ represents a possibly significant suppression of the rate for large $\tan\beta$.

At this point, the reader may object: it is true that the $W_{L(R)3i}$ decrease from M_G to M_S, but as already mentioned, the non-degeneracy between the third and first two generations is increasing. Which effect wins? We argue that in general there is a net suppression. This is easiest to see if in computing the $\mu \to e\gamma$ amplitude, we use the mass insertion approximation rather than mixing matrices at the the vertices (Fig. 3.6). Although this may be a poor approximation, it serves to illustrate our

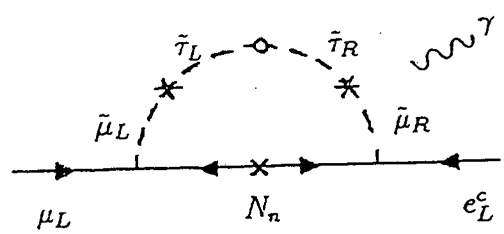


Figure 3.6: The dominant diagram (for $\mu \to e\gamma$) in the mass insertion approximation.

point. (Of course no such approximation is made in our numerical work.) From the diagram it is clear that the amplitude is proportional to $m^2_{L32}m^2_{R31}(M_S)$. From (3.5.7), we see that the rate scales as

$$\left(m^2_{L32}m^2_{R31}\right)^2(M_S) = e^{-(6I_\tau+4I_t+4I_b)}\left(m^2_{L_{32}}m^2_{R_{31}}\right)(M_G), \tag{3.5.18}$$

a net suppression. In the mass insertion approximation, then, the terms $\frac{\Delta m^2(M_G)}{\Delta m^2(M_S)}$ in (3.5.17) serve to exactly compensate for the increased non-degeneracy between $m^2_{\tilde{e}_L}$ and $m^2_{\tilde{\tau}_L}$; what remains is still a suppression. This, together with (i)$'$ invalidates the naive expectation that the theory is ruled out in most regions of parameter space due to the enhancing factors (i) and (ii), (although there are still stringent constraints on the parameter space).

The above analysis suggests that individual lepton number conservation is an infrared fixed point of the MSSM (whereas individual quark number conservation is an ultraviolet fixed point). A more complete analysis of scaling for the lepton sector and a discussion of scaling in the quark sector is presented in the appendix A.

3.6 Large $\tan\beta$: numerical results

The amplitude for $\mu \to e\gamma$ depends on the 6×6 slepton mass matrix $\mathbf{M}^2$. In the basis where $\mathbf{m}_L^2, \mathbf{m}_E^2$ are diagonal, we have

$$\mathbf{M}^2 = \begin{pmatrix} \overline{\mathbf{m}}_{E_L}^2 + \mathbf{D}_L & \mathbf{k} \\ \mathbf{k}^\dagger & \overline{\mathbf{m}}_{E_R}^2 + \mathbf{D}_R \end{pmatrix} \tag{3.6.1}$$

where in the large $\tan\beta$ limit, $D_i = -(T_{3i} - Q_i \sin^2\theta_W) M_Z^2$ is the D term contribution, and $k_{ij} = \mu m_\tau \tan\beta W_{L3i} W_{R3j}$. The amplitude from Fig. 3.1 for μ_L decay is

$$F_L = \frac{\alpha}{4\pi \cos^2\theta_W} W_{Li2}^\dagger G_2(\mathbf{M}^2)_{LRij} W_{Rj1}^\dagger$$

where

$$G_2(\mathbf{M}^2) \equiv \begin{pmatrix} G_2(M^2)_{LL} & G_2(M^2)_{LR} \\ G_2(M^2)_{RL} & G_2(M^2)_{RR} \end{pmatrix}, \tag{3.6.3}$$

In [7], $\mathbf{M}^2$ was approximately diagonalized by the $\mu m_\tau \tan\beta$ insertion approximation, and $G_2(\mathbf{M}^2)$ was calculated using this approximate diagonalization. Since here

$\tan\beta$ is large, we wish to avoid making such an approximation, and numerically diagonalize the full 6×6 $\mathbf{M}^2$.

Faced with a rather large parameter space, we must decide which parameters to use in our numerical work. We have firstly decided to do our analysis only for large $\tan\beta$, since the moderate $\tan\beta$ scenario has been covered in [7]. Secondly, we choose to present our results in a different way than in [7], where the rates for $\mu \rightarrow e\gamma$ were plotted against a combination of Planck scale and weak scale parameters. In our work, we compute $\mu \rightarrow e\gamma$ entirely in terms of weak scale parameters. In particular, we assume that the necessary condition for a significant $\mu \rightarrow e\gamma$ rate exist at the weak scale, namely non-trivial mixing matrix $W_{L,R_{3i}}$ and non-degeneracy between third and first two generation slepton masses. In the previous sections, we have shown a possible way in which these ingredients may be produced. Our plots for $\mu \rightarrow e\gamma$ rates are made against low energy parameters, and we separately plot the regions in low energy parameter space predicted by our particular scenario for generating $\mu \rightarrow e\gamma$. This way, our plots are in terms of experimentally accessible quantities and can be thought of as constraining the parameter space of the effective 3-2-1 softly broken supersymmetric theory resulting from the spontaneous breakdown of a GUT. (We use the GUT to relate weak scale gaugino masses.) Our low energy plots have no dependence on the physics above the GUT scale, all the model dependence comes into the predictions for low energy parameters the GUT makes. If the predicted region of low energy parameters corresponds to a $\mu \rightarrow e\gamma$ rate exceeding experimental bounds, the theory is ruled out.

There is a more practical reason for working directly with low-energy parameters specific to large $\tan\beta$: the well known difficulty in achieving electroweak symmetry breaking in this regime. Working with high energy parameters, and imposing universal scalar masses necessitates a fine-tune to achieve $SU(2) \times U(1)$ breaking. However, we have nowhere in our analysis made the assumption of universal scalar masses, hence the Higgs masses and squark/slepton masses are independent in our analysis, and therefore the μ parameter is not tightly constrained by squark/slepton masses. Working with weak scale parameters allows us to assume that the desired breaking has occurred without having to know the details of the breaking.

With the aforementioned assumption about the existence of a GUT, and assuming degeneracy between the first two generations, the rate for $\mu \to e\gamma$ depends on the weak scale parameters $\mu, \tan\beta, M_2, m^2_{\tilde{e}_L}, m^2_{\tilde{\tau}_L}, m^2_{\tilde{e}_R}, m^2_{\tilde{\tau}_R}$, W_{L3i}, W_{R3i}. We know that the amplitude depends on $W_{L(R)3i}$ simply through the product $W_{L3i}W_{R3j}$, so for normalization in our plots we put $W_{L(R)3i} = V_{KM3i}$. Any deviation from this can be simply multiplied into the rate. We also fix $\tan\beta = 60$, and put $m_{\tilde{\tau}_{L(R)}} = m_{\tilde{e}_{L(R)}} - \Delta_{L(R)}$. Next, we use some high energy bias to relate $m_{\tilde{e}_L}$ and $m_{\tilde{e}_R}$: we assume that their difference is proportional to M_2 (as would be the case if they started out degenerate and were split only through different gauge interactions), so we put $m_{\tilde{e}_L} = m_{\tilde{e}_R} - rM_2$. In all specific models we have looked at, r is small (less than about 0.2). We find that, as long as r is small, the rate has little dependence on its exact value, so we put $r = 0, m_{\tilde{e}_L} = m_{\tilde{e}_R} \equiv \overline{m}_{\tilde{e}}$. We also found that as long as $\frac{\Delta_L}{\Delta_R}$ is close to 1, there is little dependence on its actual value either, so we put $\Delta_L = \Delta_R \equiv \Delta$.

Now, the $\mu \rightarrow e\gamma$ rate depends only on $\mu, M_2, \overline{m}_{\tilde{e}}$ and Δ, and we have the large $\tan\beta$ interpretation of μ and M_2 as chargino masses. Fixing $\overline{m}_{\tilde{e}} = 300$ GeV, we make contour plots of $B(\mu \rightarrow e\gamma)$. The rate scales roughly as $\overline{m}_{\tilde{e}}^{-4}$ and μ^2 for scalar masses heavy compared with gaugino masses. In Fig. 3.7, we fix μ and plot in $M_2 - \Delta$ space. In Fig. 3.8, we fix Δ and plot in $\mu - M_2$ space. In Fig. 3.9, we plot the values of Δ predicted by the GUT against M_2, for various values of $\lambda_t(M_G)$ and $A_e(M_S)$ and for two values of b_5, the gauge beta function coefficient above the GUT scale. In Fig. 3.10, we plot the suppression factor ϵ for the same parameter set as in Fig. 3.9. We see that, over a significant region in parameter space, ϵ is small, between 0.2 and 0.01.

It is clear from Fig. 3.7 that, with no suppression, a typical value for Δ of 0.3 ($\times$300GeV) would give rise to rates above the current bound of $B(\mu \rightarrow e\gamma) < 4.9 \times 10^{-11}$[54]. However, from Fig. 3.10, the suppression from ϵ is seen to be typically 20, allowing Δ's of up to 0.45 ($\times$300GeV). We see that ϵ is crucial in giving the GUT more breathing room, as Δ's of less than 0.45 are more common. From Fig. 3.8 it is also clear that regions of small μ and M_2 (that is, light chargino masses) are preferred. Smaller μ is preferred because it decreases the L-R mass $\mu m_\tau \tan\beta$, small M_2 is preferred because in the limit that the neutralino mass tends to zero, the diagrams Fig. 3.5 vanish. We also note that smaller μ, M_2 are preferred for electroweak symmetry breaking[38, 39].

If μ and M_2 are both small, the lightest supersymmetric particle (LSP) can be quite light, (but where it has significant Higgsino component, it must be heavier than 45 GeV in order to be consistent with the precise measurement of the Z width), and it annihilates (primarily through its Higgsino components) through a Z into fermion antifermion pairs much like a heavy neutrino. The contribution of the LSP to energy density of the universe Ωh^2 then just depends on its mass, and the size of its Higgsino components, both of which only depend on μ and M_2 in the large $\tan\beta$ limit. In Fig. 3.11, we make a plot of Ωh^2 in $\mu - M_2$ space. We see that it is possible to get $\Omega \sim O(1)$ in some region of the parameter space.

3.7 The example of ADHRS models

In this section, we study the ADHRS models [5] which are known to give realistic fermion masses and mixing patterns. These models are specific enough for us to do calculations and make some real predictions. Although not necessarily correct, they are good representatives of general GUT models. We believe that by studying them, one can see in detail the general features of generic realistic GUT models and the differences between them and the minimal $SU(5)$ or $SO(10)$ models.

As mentioned in Sec. 3.2, in ADHRS models, the three families of quarks and leptons lie in three 16 dimensional representations of $SO(10)$, and the two low energy Higgs doublets lie in a single 10 dimensional representation. Only the third

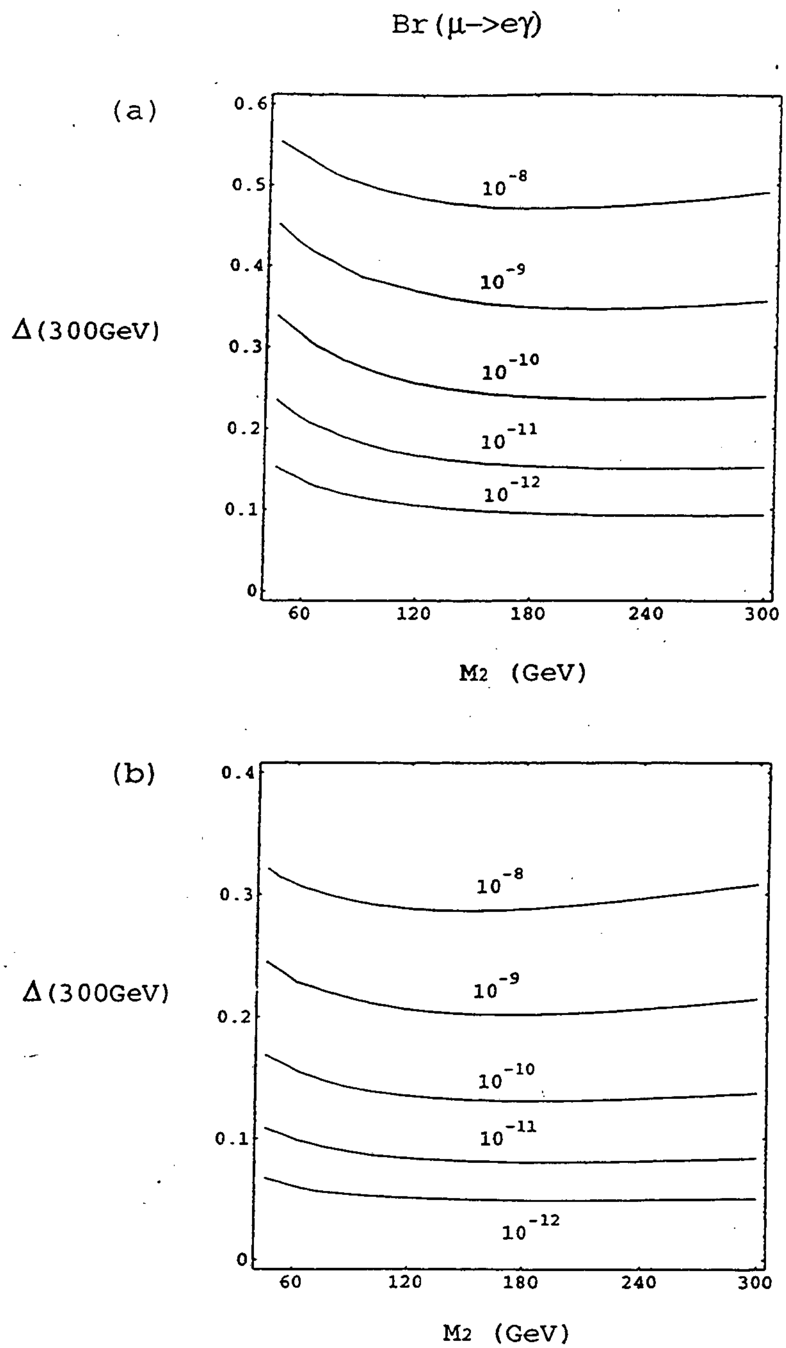


Figure 3.7: Contours for $B(\mu \to e\gamma)$ in $M_2 - \Delta$ plane with $m_{\tilde{e}_{L(R)}} = 300\text{GeV}$, $\Delta \equiv m_{\tilde{e}_{L(R)}} - m_{\tilde{\tau}_{L(R)}}$, $W_{E_{L(R)32}} = 0.04$, $W_{E_{L(R)31}} = 0.01$, for $\mu =$ (a) 100GeV, (b) 300GeV. Contours for negative μ are virtually identical. To get $B(\mu \to e\gamma)$ prediction from a GUT, multiply by appropriate Clebsch, and ϵ factor (Fig. 3.10).

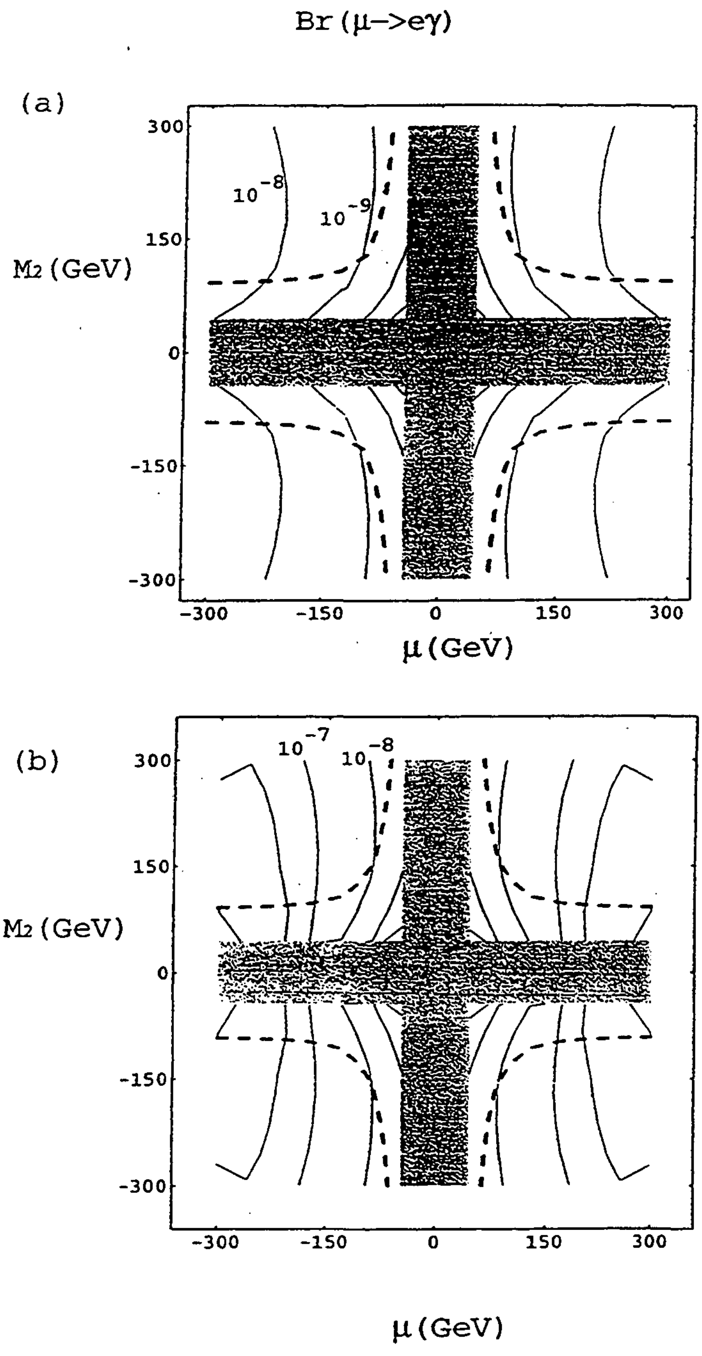


Figure 3.8: Contours for $B(\mu \to e\gamma)$ in $\mu - M_2$ plane for (a) $\Delta = 0.25$, (b) $\Delta = 0.5$, with other parameters same as in Fig. 3.7.

The blacked out regions are ruled out by the LEP bound of 45 GeV on chargino masses. The thick dashed lines are contours for a 45 GeV LSP mass.

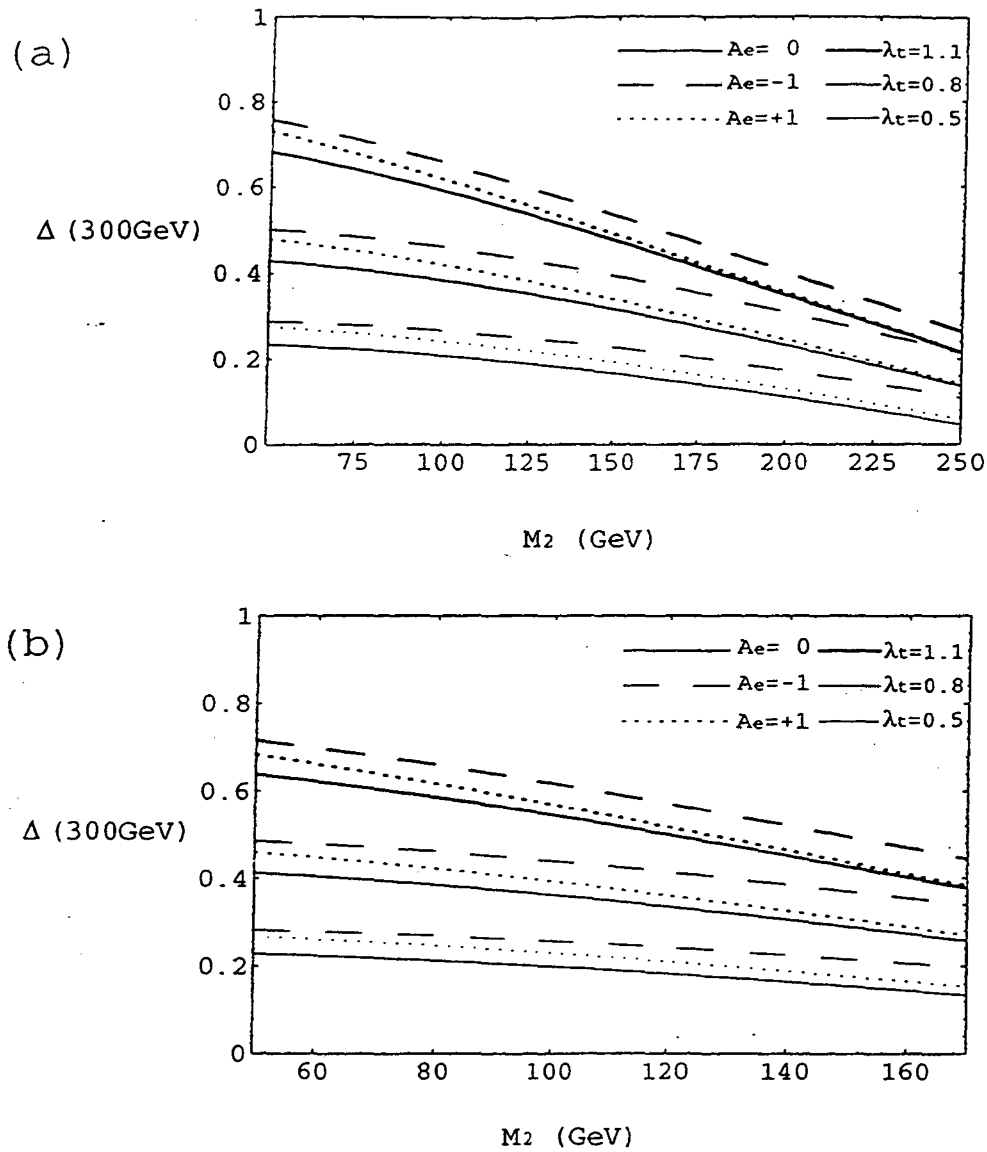


Figure 3.9: Plots of the averaged difference between the third and the first two generations charged slepton masses $\Delta \equiv \frac{\Delta_L+\Delta_R}{2}$, $\Delta_{L(R)} \equiv m_{\tilde{e}_{L(R)}}$ (at M_S), against M_2, for $\frac{1}{2}(m^2_{\tilde{e}_L} + m^2_{\tilde{e}_R}) = (300\text{ GeV})^2$, $\lambda_t = \lambda_b = \lambda_\tau$ (at M_G) $= 0.5, 0.8, 1.1$, $A_e(M_S) = 1, 0, -1$, two values of the gauge beta function coefficient b_5 between M_G and M_{PL}, (a) $b_5 = 3$ (asymptotically free), (b) $b_5 = -20$. Scalar masses are assumed degenerate at $M_{PL} = 2.4 \times 10^{18}$ GeV. M_G is taken to be 2.7 $\times 10^{16}$ GeV.

(a)

(b)

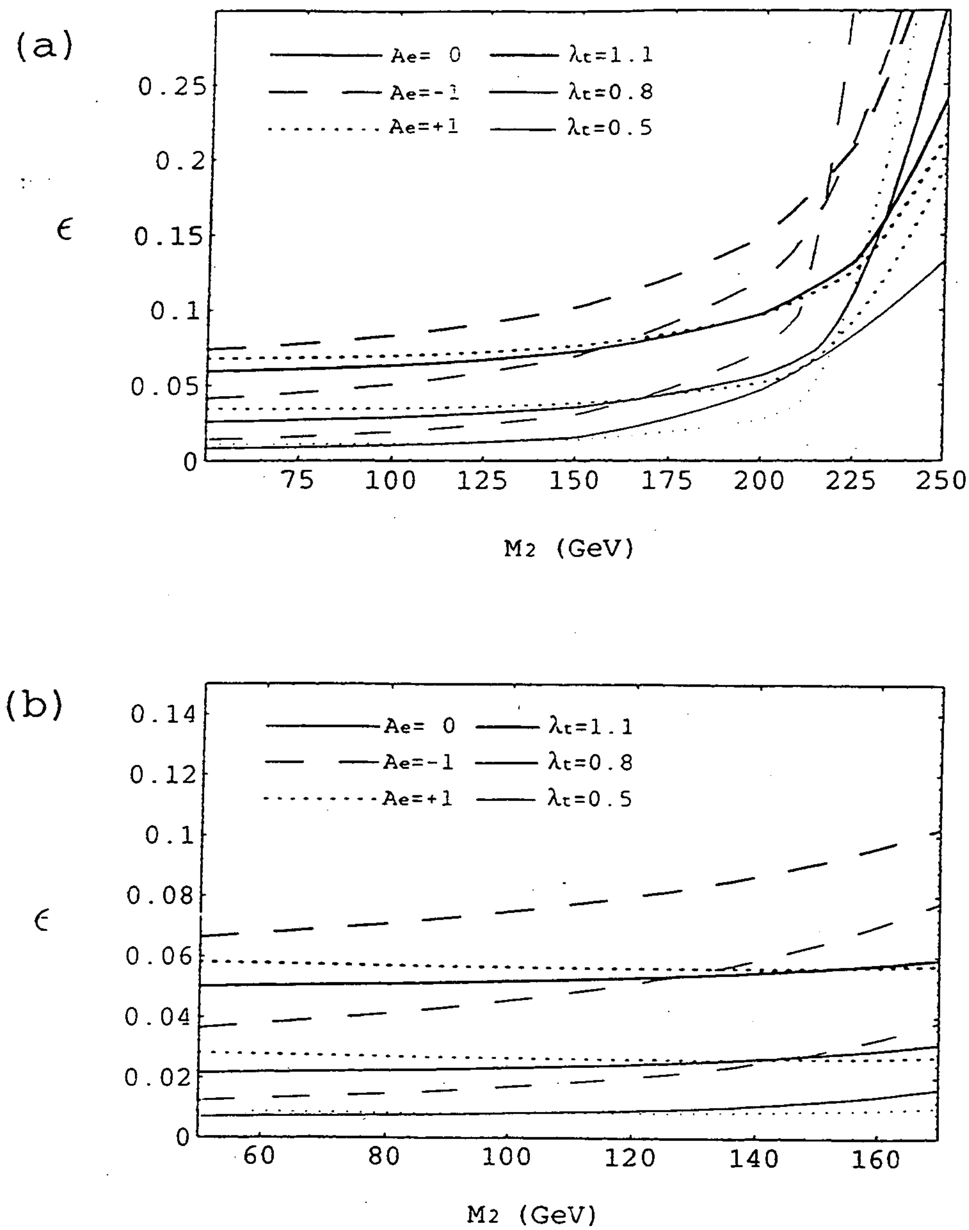


Figure 3.10: Plots of the suppression factor ϵ against M_2, with the same parameters as in Fig. 3.9.

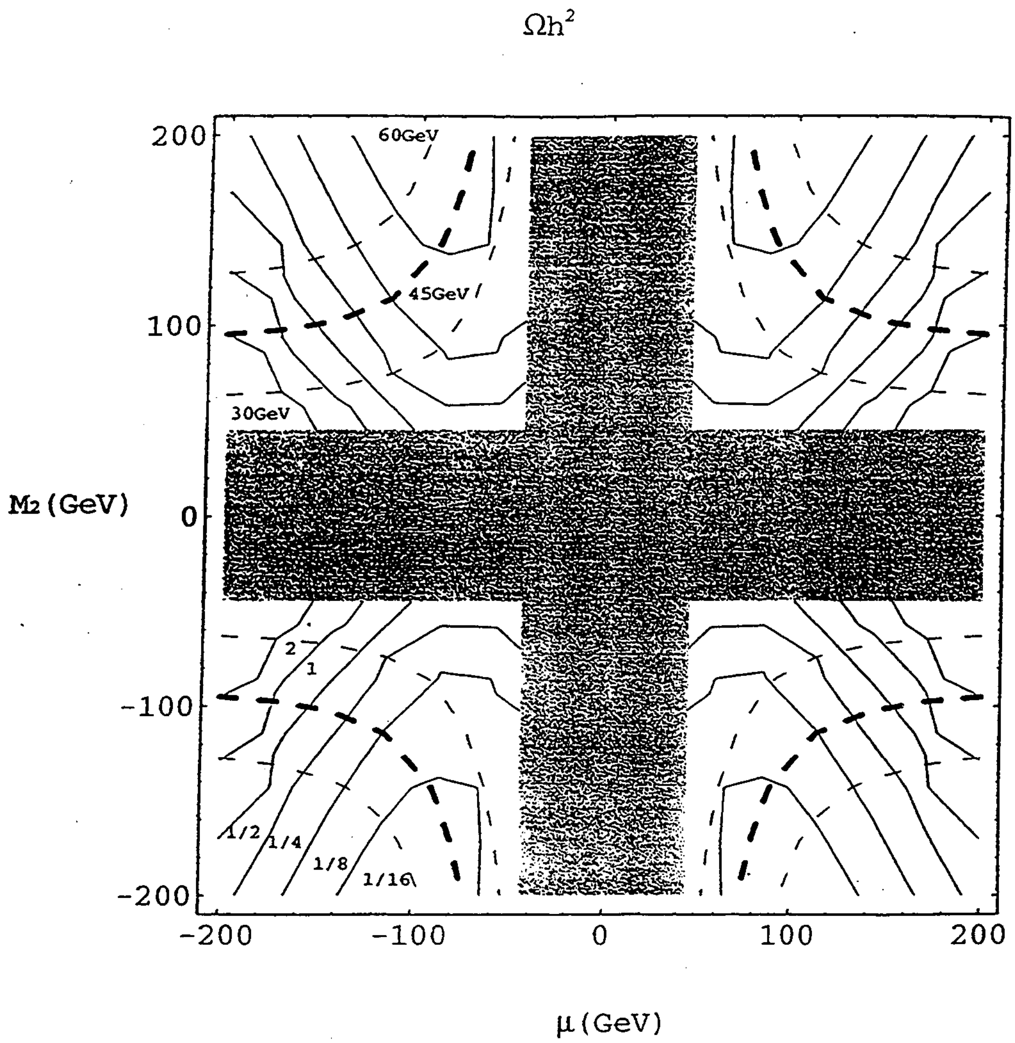


Figure 3.11: Contours for Ωh^2 in $\mu - M_2$ plane in the large $\tan\beta$ limit. Dashed lines are LSP mass contours of 30, 45, and 60 GeV. For all regions of $m_{LSP} < 45$ GeV in this plot, the Higgsino components of LSP are too big and therefore they are ruled out by the Z width.

generation Yukawa couplings come from a renormalizable interaction

$$\lambda_{33}\mathbf{16}_3\mathbf{16}_3\mathbf{10}. \tag{3.7.1}$$

All other small Yukawa couplings come from nonrenormalizable interactions after integrating out the heavy fields. These interactions can be written in general as

$$\mathbf{16}_i\lambda_{ij}(A_a)\mathbf{16}_j\mathbf{10}. \tag{3.7.2}$$

The A_a's are fields in the adjoint representation of $SO(10)$ and their VEV's break $SO(10)$ down to the standard model gauge group. Therefore, these Yukawa couplings can take different values for fermions of the same generation with different quantum numbers under $SU(3)\times SU(2)\times U(1)$ and a realistic fermion mass pattern and nontrivial KM matrix can be generated. In ADHRS models, the minimal number (four) of operators is assumed to generate the up, down-type quark and charged lepton Yukawa coupling matrices $\boldsymbol{\lambda}_U, \boldsymbol{\lambda}_D$ and $\boldsymbol{\lambda}_E$, and they take the form at M_G

$$\boldsymbol{\lambda}_U = \begin{pmatrix} 0 & z_uC & 0 \\ z'_uC & y_uE & x_uB \\ 0 & x'_uB & A \end{pmatrix}, \boldsymbol{\lambda}_D = \begin{pmatrix} 0 & z_dC & 0 \\ z'_dC & y_dE & x_dB \\ 0 & x'_dB & A \end{pmatrix}, \boldsymbol{\lambda}_E = \begin{pmatrix} 0 & z_eC & 0 \\ z'_eC & y_eE & x_eB \\ 0 & x'_eB & A \end{pmatrix}, \tag{3.7.3}$$

where the x, y, z's are Clebsch factors arising from the VEV's of the adjoint Higgs fields A_a. This form is known to give the successful relations $V_{ub}/V_{cb} = \sqrt{m_u/m_c}$ and $V_{td}/V_{ts} = \sqrt{m_d/m_s}$ [55] so it is well motivated. Strictly speaking, the interaction (3.7.2) become the usual Yukawa form only after the adjoints A_a take their VEV's at the GUT scale. However, as we explained in Sec. 3.2, they can be treated as the usual Yukawa interactions up to the ultraheavy scale (which we will assume to be M_{PL}) where the ultraheavy fields are integrated out if the wavefunction renormalizations of A_a's are ignored. In the one-loop approximation which we use later in calculating radiative corrections from M_{PL} to M_G, they give the same results, because the wavefunction renormalizations of the adjoints A_a only contribute at the two-loop order. This makes our analysis much easier. Above the GUT scale, in addition to the Yukawa interactions (3.2.4) which give the fermion masses, we have the interactions (3.2.5) as well. Each Yukawa matrix has different Clebsch factors x, y, z associated with its elements. All the Yukawa matrices have the ADHRS form

$$\lambda_I = \begin{pmatrix} 0 & z_I C & 0 \\ z'_I C & y_I E & x_I B \\ 0 & x'_I B & A \end{pmatrix}, \quad I = qq, eu, ud, q\ell, nd, n\ell. \tag{3.7.4}$$

If each entry of the Yukawa matrices is generated dominantly by a single operator, like in the ADHRS models, then the phases of the same entries of all Yukawa matrices are identical. One can remove all but the λ_{22} phases by rephasing the operators. After phase redefinition only E is complex and is responsible for CP violation. In order to generate the realistic fermion mass and mixing pattern, one expects the following hierarchies,

$$\begin{aligned} \frac{B}{A} &\sim V_{cb} \quad \sim \epsilon^2, \\ \frac{E}{A} &\sim \frac{m_s}{m_b} \quad \sim \epsilon^2, \\ \frac{C}{E} &\sim \sin\theta_c \sim \epsilon, \quad \text{where } \epsilon \sim 0.2. \end{aligned} \tag{3.7.5}$$

The hierarchical Yukawa matrices can be diagonalized approximately [55], the unitary rotation matrices which diagonalize them at the GUT scale can be approximately written as

$$\lambda = \begin{pmatrix} 0 & zC & 0 \\ z'C & y\,|E|\,e^{i\tilde{\phi}} & xB \\ 0 & x'B & A \end{pmatrix} = V_F^* \bar{\lambda} V_B^\dagger, \tag{3.7.6}$$

$$V_F \simeq \begin{pmatrix} e^{i\phi} & S_{F_1} e^{i\phi} & 0 \\ -S_{F_1} & 1 & S_{F_2} \\ S_{F_1} S_{F_2} & -S_{F_2} & 1 \end{pmatrix}, \tag{3.7.7}$$

$$V_B \simeq \begin{pmatrix} 1 & S_{B_1} & 0 \\ -S_{B_1} e^{-i\phi} & e^{-i\phi} & S_{B_2} \\ S_{B_1} S_{B_2} e^{-i\phi} & -S_{B_2} e^{-i\phi} & 1 \end{pmatrix}, \tag{3.7.8}$$

where

$$\begin{aligned} S_{F_2} &= \frac{xB}{A},\ S_{B_2} = \frac{x'B}{A},\ S_{F_1} = \frac{zC}{E'},\ S_{B_1} = \frac{z'C}{E'}, \\ E' &= |yE - S_{F2} S_{B2} A| = \left| y|E|e^{i\tilde{\phi}} - \frac{x'xB^2}{A} \right|, \\ \tilde{\phi} &= arg(E),\ \phi = arg(yE - \frac{x'xB^2}{A}). \end{aligned}$$

The soft SUSY-breaking scalar masses for the three low energy generations and trilinear A terms are assumed to be universal[4] at Planck scale M_{PL} as in [7]. Beneath

[4]If the nonrenormalizable operators already appear in the superpotential of the underlying supergravity theory, the A terms will be different for different dimensional operators, and will induce unacceptably large $\mu \to e\gamma$ rate because the triscalar interactions and the Yukawa interactions can not be diagonalized in the same basis for the first two generations. In theories where the nonremormalizable operators come from integrating out heavy fields at M_{PL} and all the relevant interactions have the same A term, the resulting nonrenormalizable operators will also have the same A term.

M_{PL}, the radiative corrections from the Yukawa couplings destroy the universalities and render the mixing matrices non-trivial. In the one-loop approximation, the radiative corrections to the soft SUSY-breaking parameters at M_G are simply related to the Yukawa coupling matrices and therefore the relations between general mixing matrix elements and KM matrix elements are also simple. This allows us to see the similar hierarchies in the general mixing matrices and the KM matrix very clearly. Although the one-loop approximation may not be a good approximation for quantities involving third generation Yukawa couplings, we will be satisfied with it since it simplifies things a lot and the uncertainties in other quantities such as Clebsch factors are probably much bigger than the errors made in the one-loop approximation. The RGE's, for $\mathbf{m}_E^2$ as an example, from M_{PL} to M_G are

$$\begin{aligned}\frac{d}{dt}\mathbf{m}_E^2 &= 2(2\boldsymbol{\lambda}_E\mathbf{m}_L^2\boldsymbol{\lambda}_E^\dagger + 2\boldsymbol{\lambda}_E m_{H_D}^2\boldsymbol{\lambda}_E^\dagger + \mathbf{m}_E^2\boldsymbol{\lambda}_E\boldsymbol{\lambda}_E^\dagger + \boldsymbol{\lambda}_E\boldsymbol{\lambda}_E^\dagger\mathbf{m}_E^2 + 2\zeta_E\zeta_E^\dagger)\\ &\quad + 3(2\boldsymbol{\lambda}_{eu}\mathbf{m}_U^2\boldsymbol{\lambda}_{eu}^\dagger + 2\boldsymbol{\lambda}_{eu} m_{H_{U_3}}^2\boldsymbol{\lambda}_{eu}^\dagger + \mathbf{m}_E^2\boldsymbol{\lambda}_{eu}\boldsymbol{\lambda}_{eu}^\dagger + \boldsymbol{\lambda}_{eu}\boldsymbol{\lambda}_{eu}^\dagger\mathbf{m}_E^2 + 2\zeta_{eu}\zeta_{eu}^\dagger)\\ &\quad - \text{gaugino mass contribution.}\end{aligned} \tag{3.7.9}$$

In the one-loop approximation, the gaugino mass contributions are diagonal and the same for all three generations, so they can be absorbed into the common scalar masses and do not affect the diagonalization. The corrections to scalar masses at M_G have the following leading flavor dependence

$$\Delta\mathbf{m}_E^2 \propto 2\boldsymbol{\lambda}_E\boldsymbol{\lambda}_E^\dagger + 3\boldsymbol{\lambda}_{eu}\boldsymbol{\lambda}_{eu}^\dagger$$

$$= 5 \begin{pmatrix} \overline{z_e^2} C^2 & \overline{z_e y_e} C E^* & \overline{z_e x_e} C B \\ \overline{z_e y_e} C E & \overline{z_e'^2} C^2 + \overline{y_e^2} |E|^2 + \overline{x_e^2} B^2 & \overline{y_e x_e'} E B + \bar{x}_e B A \\ \overline{z_e x_e'} C B & \overline{y_e x_e'} E^* B + \bar{x}_e B A & \overline{x_e'^2} B^2 + A^2 \end{pmatrix}, \quad (3.7.10)$$

where the overline represents the weighted average of the Clebsch factors, $\overline{z_e^2} = \frac{1}{5}(2z_e^2 + 3z_{eu}^2)$ and so on. Because $\Delta \mathbf{m}_E^2$ is hierarchical, assuming no big x, y Clebsches (ADHRS models have some big z Clebsches), the rotation matrix which diagonalizes it can be given approximately as

$$U_E(M_G) \simeq \begin{pmatrix} 1 & \bar{S}_{E_1} & \bar{S}_{E3} \\ -\bar{S}_{E_1} e^{-i\widetilde{\phi}} & e^{-i\widetilde{\phi}} & \bar{S}_{E2} \\ \bar{S}_{E_1} \bar{S}_{E_2} e^{-i\widetilde{\phi}} - \bar{S}_{E3} & -\bar{S}_{E2} e^{-i\widetilde{\phi}} & 1 \end{pmatrix}, \quad (3.7.11)$$

where

$$\bar{S}_{E_2} = \frac{\bar{x}_e B}{A}, \; \bar{S}_{E_3} = \frac{\overline{z_e x_e'} C B}{A^2}$$

$$\bar{S}_{E_1} = \frac{\overline{z_e y_e} C |E|}{\overline{z_e'^2} C^2 + \overline{y_e^2} |E|^2 + (\overline{x_e^2} - \bar{x}_e^2) B^2}, \quad e^{-i\widetilde{\phi}} = \frac{E}{|E|}.$$

Similarly, for other scalar masses the leading flavor dependent corrections at M_G are

$$\Delta \mathbf{m}_L^2 \propto \boldsymbol{\lambda}_E^\dagger \boldsymbol{\lambda}_E + 3\boldsymbol{\lambda}_{q\ell}^\dagger \boldsymbol{\lambda}_{q\ell} + \boldsymbol{\lambda}_{n\ell}^\dagger \boldsymbol{\lambda}_{n\ell},$$

$$\Delta \mathbf{m}_Q^2 \propto \boldsymbol{\lambda}_U \boldsymbol{\lambda}_U^\dagger + \boldsymbol{\lambda}_D \boldsymbol{\lambda}_D^\dagger + 2\boldsymbol{\lambda}_{qq} \boldsymbol{\lambda}_{qq}^\dagger + \boldsymbol{\lambda}_{q\ell} \boldsymbol{\lambda}_{q\ell}^\dagger,$$

$$\Delta \mathbf{m}_U^2 \propto 2\boldsymbol{\lambda}_U^\dagger \boldsymbol{\lambda}_U + \boldsymbol{\lambda}_{eu}^\dagger \boldsymbol{\lambda}_{eu} + 2\boldsymbol{\lambda}_{ud} \boldsymbol{\lambda}_{ud}^\dagger,$$

$$\Delta \mathbf{m}_D^2 \propto 2\boldsymbol{\lambda}_D^\dagger \boldsymbol{\lambda}_D + \boldsymbol{\lambda}_{nd}^\dagger \boldsymbol{\lambda}_{nd} + 2\boldsymbol{\lambda}_{ud}^\dagger \boldsymbol{\lambda}_{ud}, \quad (3.7.12)$$

and the rotation matrices which diagonalize them are given by expressions similar to (3.7.11) with Clebsches replaced by the appropriate ones. Then, the mixing matrices appearing at the lepton-slepton-gaugino vertices are given by

$$W_{E_L}(M_G) = U_L^\dagger V_{E_L}$$
$$\simeq \begin{pmatrix} 1 & S_{E_{L1}} - S_{L1} e^{i(\widetilde{\phi}-\phi_e)} & \bar{S}_{L_1}(\bar{S}_{L_2} - S_{E_{L_2}})e^{i\widetilde{\phi}} - \bar{S}_{L_3} \\ (\bar{S}_{L_1} - \bar{S}_{E_{L_1}})e^{-i(\widetilde{\phi}-\phi_e)} & e^{i(\widetilde{\phi}-\phi_e)} & -(\bar{S}_{L_2} - S_{E_{L_2}})e^{i\widetilde{\phi}} \\ -S_{E_{L_1}}(\bar{S}_{L_2} - S_{E_{L_2}})e^{-i\phi_e} + \bar{S}_{L_3} & (\bar{S}_{L_2} - S_{E_{L_2}})e^{-i\phi_e} & 1 \end{pmatrix}, \tag{3.7.13}$$

$$W_{E_L}(M_G) = U_E^\dagger V_{E_R}$$
$$\simeq \begin{pmatrix} e^{i\phi_e} & S_{E_{R1}} e^{i\phi} e - \bar{S}_{E_1} e^{i\widetilde{\phi}} & \bar{S}_{E_1}(\bar{S}_{E_2} - S_{E_{R2}})e^{i\widetilde{\phi}} - \bar{S}_{E_3} \\ \bar{S}_{E_1} e^{i\phi_e} - S_{E_{R1}} e^{i\widetilde{\phi}} & e^{i\widetilde{\phi}} & -(\bar{S}_{E_2} - S_{E_{R_2}})e^{-i\phi} \\ -S_{E_{R1}}(\bar{S}_{E_2} - S_{E_{R2}}) + \bar{S}_{E_3} e^{i\phi_e} & \bar{S}_{E_2} - S_{E_{R2}} & 1 \end{pmatrix}, \tag{3.7.14}$$

where

$$S_{E_{L_1}} = \frac{z'_e C}{E'_e},\ S_{E_{R_1}} = \frac{z_e C}{E'_e},\ E'_e = |y_e E - \frac{x'_e x_e B^2}{A}|,$$
$$S_{E_{L_2}} = \frac{x'_e B}{A},\ S_{E_{R_2}} = \frac{x_e B}{A},\ \phi_e = arg\left(y_e |E| e^{i\widetilde{\phi}} - \frac{x'_e x_e B^2}{A}\right)$$
$$\simeq \widetilde{\phi}\ \text{(if } y_e \sim x_e, x'_e),$$
$$\bar{S}_{E_1} = \frac{\overline{z_e y_e} C|E|}{\overline{z'^2_e} C^2 + \overline{y_e^2}\,|E|^2 + (\overline{x_e^2} - \overline{x}_e^2)B^2},\ \bar{S}_{E_2} = \frac{\overline{x}_e B}{A},\ \bar{S}_{E_3} = \frac{\overline{z_e x'_e} CB}{A^2},$$
$$\bar{S}_{L_1} = \frac{\widehat{z'_e y_e} C|E|}{\widehat{z_e^2} C^2 + \widehat{y_e^2}|E|^2 + (\widehat{x_e^2} - \widehat{x}_e^2)B^2},\ \bar{S}_{L_2} = \frac{\widehat{x'_e} B}{A},\ \bar{S}_{L_3} = \frac{\widehat{z'_e x_e} CB}{A^2},$$

and

$$\bar{x}_e = \frac{1}{5}(2x_e + 3x_{eu}), \ \widehat{x'_e} = \frac{1}{5}(x'_e + 3x'_{g\ell} + x'_{n\ell}) \ \text{etc..}$$

Note that

$$\bar{S}_{L_3} \sim \text{ Clebsch } \times \frac{CB}{A^2}, \ S_{E_{L_1}}(\bar{S}_{L_2} - \bar{S}_{E_{L_2}}) = \text{Clebsch} \times \frac{CB}{EA}. \qquad (3.7.15)$$

If there is no very big or small Clebsch involved and no accidental cancellation, $\bar{S}_{L_3}, \bar{S}_{E_3}$ can be neglected in W's.

Compared with V_{KM},

$$V_{KM}(M_G) = V^{\dagger}_{U_L} V_{D_L}$$
$$\simeq \begin{pmatrix} 1 & S_{D_{L_1}} - S_{U_{L_1}} e^{-i(\phi_d - \phi_u)} & -S_{U_{L_1}}(S_{D_{L_2}} - S_{U_{L_2}})e^{i\phi_u} \\ S_{U_{L_1}} - S_{D_{L_1}} e^{-i(\phi_d - \phi_u)} & e^{-i(\phi_d - \phi_u)} & (S_{D_{L_2}} - S_{U_{L_2}})e^{i\phi_u} \\ S_{D_{L_1}}(S_{D_{L_1}} - S_{U_{L_2}})e^{-i\phi_d} & -(S_{D_{L_2}} - S_{U_{L_2}})e^{-i\phi_d} & 1 \end{pmatrix}, \qquad (3.7.16)$$

where

$$S_{U_{L1}} = \frac{z_u C}{E'_u}, \ E'_u = \left| y_u E - \frac{x_u x'_u B^2}{A} \right|, \phi_u = arg\left(y_u E - \frac{x_u x'_u B^2}{A} \right),$$
$$S_{D_{L1}} = \frac{z_d C}{E'_d}, \ E'_d = \left| y_d E - \frac{x_d x'_d B^2}{A} \right|, \ \phi_d = arg\left(y_d E - \frac{x_d x'_d B^2}{A} \right),$$
$$S_{U_{L2}} = \frac{x_u B}{A},$$
$$S_{D_{L2}} = \frac{x_d B}{A},$$

we can see that the W's and V_{KM} do have similar hierarchical patterns, but have different Clebsch factors associated with their entries.

When a specific model is given, one can calculate all the Clebsch factors and make some definite predictions for that particular model. For example, the ADHRS

Model 6, which gives results in good agreement with the experimental data, has the following four effective fermion mass operators

$$
\begin{aligned}
O_{33} &= \mathbf{16}_3\, \mathbf{10}\, \mathbf{16}_3, \\
O_{23} &= \mathbf{16}_2\, \frac{A_Y}{A_X}\, \mathbf{10}\, \frac{A_Y}{A_X} \mathbf{16}_3, \\
O_{22} &= \mathbf{16}_2 \frac{A_X}{M}\, \mathbf{10}\, \frac{A_{B-L}}{A_X}\, \mathbf{16}_2 \quad \text{or other 5 choices}, \\
O_{12} &= \mathbf{16}_1 \left(\frac{A_X}{M}\right)^3 \mathbf{10} \left(\frac{A_X}{M}\right)^3 \mathbf{16}_2,
\end{aligned} \tag{3.7.17}
$$

where A_X, A_Y, A_{B-L} are adjoint's of $SO(10)$ with VEV's in the $SU(5)$ singlet, hypercharge, and $B-L$ directions. There are six choices of O_{22} operators which give the same predictions for the fermion masses and mixings, but different Clebsches for other operators appearing above M_G. Fortunately, they do not enter the leading terms of the most important mixing matrix elements $W_{E_{L32}}, W_{E_{L31}}, W_{E_{R32}}, W_{E_{R31}}$, which appear in the leading contributions to the amplitudes of LFV processes and the electric dipole moment.

The magnitude of the mixing matrix elements $V_{KM32}, V_{KM31}, W_{E_{L32}}, W_{E_{L31}}, W_{E_{R32}}, W_{E_{R31}}$, and the relevant Clebsch factors are listed in Tables 3.2 and 3.3.

In ADHRS models $\tan\beta$ is large. The $\mu \to e\gamma$ rate for large $\tan\beta$ has been calculated in Sec. 3.5 and 3.6 for $W_{E_{L32}} = W_{E_{R32}} = V_{ts}$ and $W_{E_{L31}} = W_{E_{R31}} = V_{td}$. To obtain the predictions of ADHRS models we only have to multiply the results by the suitable Clebsch factors. The relevant Clebsch factors for Model 6 are listed in Table 3.3. For a generic realistic GUT model with small $\tan\beta$, for example the modified ADHRS models in which the down type Higgs lies predominantly

	u	d	e	eu	$q\ell$	$n\ell$
x	-1	$-\frac{1}{6}$	$\frac{3}{2}$	-6	$\frac{1}{4}$	0
x'	-1	$-\frac{1}{6}$	$\frac{3}{2}$	-6	$\frac{1}{4}$	0
y	0	1	3	-	-	-
z	$-\frac{1}{27}$	1	1	$-\frac{1}{27}$	1	125
z'	$-\frac{1}{27}$	1	1	$-\frac{1}{27}$	1	125
$\widehat{x'_e} = \frac{9}{20}$, $\overline{x_e} = -3$						

Table 3.2: Clebsch factors for Yukawa coupling matrices in ADHRS model 6.

in some fields which do not interact with the three low energy generations and contain only a small fraction of the doublets in the **10** which interact with the low energy generations [56], most of analysis should still hold. In this case the leading contributions to $\mu \to e\gamma$ are the same ones as in the minimal $SO(10)$ model of Ref. [7] (Fig. 3.10 $b_{L,R}, c_{L,R}, c'_{L,R}$ of [7]). The diagrams c_{LR}, c'_{LR} involve the corrections to the trilinear scalar couplings.

In the one-loop approximation the leading corrections to ζ_E at M_G contain pieces proportional to $\boldsymbol{\lambda}_E$, $\boldsymbol{\lambda}_E(\boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E + 3\boldsymbol{\lambda}_{q\ell}^\dagger\boldsymbol{\lambda}_{q\ell} + \boldsymbol{\lambda}_{n\ell}^\dagger\boldsymbol{\lambda}_{n\ell})$, $(2\boldsymbol{\lambda}_E\boldsymbol{\lambda}_E^\dagger + 3\boldsymbol{\lambda}_{eu}\boldsymbol{\lambda}_{eu}^\dagger)\boldsymbol{\lambda}_E$ respectively. The piece proportional to $\boldsymbol{\lambda}_E$ can be absorbed into ζ_{E_0} by a redefinition of A_E, the other two pieces are proportional to the product of $\boldsymbol{\lambda}_E$ and the corrections to the scalar masses,

$$\Delta\zeta_E = \Delta\zeta_{E_R} + \Delta\zeta_{E_L}$$

	ADHRS models	Model 6	Relevant proc.
$\left\|W_{E_{L32}}/V_{ts}\right\|$	$\left\|\frac{\hat{x}'_e-x'_e}{x_d-x_u}\right\|$	1.26	
$\|W_{E_{R32}}/V_{ts}\|$	$\left\|\frac{\bar{x}_e-x_e}{x_d-x_u}\right\|$	5.4	
$\|W_{E_{L31}}/V_{td}\|$	$\left\|\frac{z'_e y_d(\hat{x}'_e-x'_e)}{z_d y_e(x_d-x_u)}\right\|$	0.42	
$\|W_{E_{R31}}/V_{td}\|$	$\left\|\frac{z_e y_d(\bar{x}_e-x_e)}{z_d y_e(x_d-x_u)}\right\|$	1.8	
$\left\|\frac{W_{E_{L32}}W_{E_{R31}}}{V_{ts}V_{td}}\right\|$	$\left\|\frac{z_e y_d(\bar{x}_e-x_e)(\hat{x}'_e-x'_e)}{z_d y_e(x_d-x_u)^2}\right\|$	2.268	$\mu \to e\gamma$ amp.
$\left\|\frac{W_{E_{R32}}W_{E_{L31}}}{V_{ts}V_{td}}\right\|$	$\left\|\frac{z'_e y_d(\bar{x}_e-x_e)(\hat{x}'_e-x'_e)}{z_d y_e(x_d-x_u)^2}\right\|$	2.268	$\mu \to e\gamma$ amp.
$\left\|\frac{\sqrt{2}W_{E_{L31}}W_{E_{R32}}/Vtd}{\sqrt{\|W_{E_{L32}}W_{E_{R31}}\|^2+\|W_{E_{R32}}W_{E_{L31}}\|^2/V_{ts}}}\right\|$	$\left\|\frac{\sqrt{2}z'_e z_e y_d}{\sqrt{(z_e^2+z_e'^2)z_d y_e}}\right\|$	$\frac{1}{3}$	d_e

Table 3.3: The relevant Clebsch factors for $\mu \to e\gamma$ and d_e in ADHRS model 6.

$$\begin{aligned}\Delta\zeta_{E_R} &= \frac{1}{\mu_{E_R}}\Delta\mathbf{m}_E^2\,\boldsymbol{\lambda}_E \\ \Delta\zeta_{E_L} &= \frac{1}{\mu_{E_L}}\boldsymbol{\lambda}_E\,\Delta\mathbf{m}_L^2\end{aligned} \tag{3.7.18}$$

where μ_{E_L}, μ_{E_R} are proportional constants ($\mu_{E_R} = \mu_{E_L} = \frac{6m_0^2+A_0^2}{3A_0}$ in one-loop approximation). The LFV couplings in Fig. 3.2(e), $\tilde{e}_R^T U_E^T \Delta\zeta_E U_L \tilde{e}_L v_D$, now can be written as

$$\begin{aligned}&\frac{1}{\mu_{E_R}}\tilde{e}_R^T U_E^T \Delta\mathbf{m}_E^2\boldsymbol{\lambda}_E U_L\tilde{e}_L v_D + \frac{1}{\mu_{E_L}}\tilde{e}_R^T U_E^T\boldsymbol{\lambda}_E\Delta\mathbf{m}_L^2 U_L\tilde{e}_L v_D \\ &= \frac{1}{\mu_{E_R}}\tilde{e}_R^T\Delta\overline{\mathbf{m}}_E^2 W_{E_R}^*\bar{\boldsymbol{\lambda}}_E W_{E_L}^\dagger\tilde{e}_L v_D + \frac{1}{\mu_{E_L}}\tilde{e}_R^T W_{E_R}^*\bar{\boldsymbol{\lambda}}_E W_{E_L}^\dagger\Delta\overline{\mathbf{m}}_L^2\tilde{e}_L v_D,\end{aligned} \tag{3.7.19}$$

where the overline means that the matrix is diagonal. Again, the amplitudes are given by the same formulas as in [7] (eqns. 29, 30), except that $V_{32}^e V_{31}^e (V_{33}^{e*})^2$ has to be replaced by $W_{E_{L32}} W_{E_{R31}} W_{E_{L33}}^* W_{E_{R33}}^*$, and $W_{E_{R32}} W_{E_{L31}} W_{E_{R33}}^* W_{E_{L33}}^*$, and $\frac{5}{7}I'_G$ by

$\frac{1}{\mu_{E_R}}\Delta\overline{m}^2_{E33}$ and $\frac{1}{\mu_{E_L}}\Delta\overline{m}^2_{L33}$. The results in [7] are only modified by some multiplicative factors and therefore represent the central values for the LFV processes.

It was pointed out in [7, 8] that the electric dipole moment of the electron (d_e) constitutes an independent and equally important signature for the $SO(10)$ unified theory as $\mu \to e\gamma$ does. The diagrams which contribute to the electric dipole moment of the electron are the same as the ones which contribute to $\mu \to e\gamma$, with $\mu_L(\mu_L^c)$ replaced by $e_L(e_L^c)$. Thus a simple relation between d_e and the $\mu \to e\gamma$ rate was obtained in the minimal $SO(10)$ model [7],

$$\Gamma(\mu \to e\gamma) = \frac{\alpha}{2} m_\mu^3 |F_2|^2, \tag{3.7.20}$$

$$|d_e| = e|F_2| \left|\frac{V_{td}}{V_{ts}}\right| \sin\phi = e\sqrt{\frac{2\Gamma(\mu \to e\gamma)}{\alpha m_\mu^3}} \left|\frac{V_{td}}{V_{ts}}\right| \sin\phi, \tag{3.7.21}$$

where ϕ is an unknown new CP-violating phase defined by

$$\mathrm{Im}[m_\tau (V_{31}^e)^2 (V_{33}^{e*})^2] = |m_\tau (V_{31}^e)^2 (V_{33}^{e*})^2| \sin\phi.$$

In a more generic $SO(10)$ model, such as the ADHRS model, we still have this simple relation but the mixing matrix elements have to be replaced by the W's:

$$|d_e| = e\sqrt{\frac{2\Gamma(\mu \to e\gamma)}{\alpha m_\mu^3}} \frac{\sqrt{2}|W_{E_{L31}} W_{E_{R31}}|}{\sqrt{|W_{E_{L32}} W_{E_{R31}}|^2 + |W_{E_{R31}} W_{E_{L31}}|^2}} \sin\phi', \tag{3.7.22}$$

where ϕ' is defined by

$$\mathrm{Im}[m_\tau W_{E_{L31}} W_{E_{R31}} W^*_{E_{R33}} W^*_{E_{L33}}] = |m_\tau W_{E_{L31}} W_{E_{R31}} W^*_{E_{L33}} W^*_{E_{R33}}| \sin\phi'.$$

In particular, in ADHRS models there is only one CP-violating phase, so the phase ϕ' can be related to the phase appeared in the KM matrix of the standard model.

From eqns. (3.7.13) (3.7.14) (3.7.16) we can see that $\phi' \approx \phi_e, \phi_e \approx \phi_d \approx \tilde{\phi}, \phi_u = 0$ (because $y_u = 0$). The rephrase invariant quantity J of the KM matrix is given by

$$\begin{aligned} J &= \operatorname{Im} V_{ud} V_{tb} V_{td}^* V_{ub}^* \\ &\simeq -S_{U_{L1}} S_{D_{L1}} (S_{D_{L2}} - S_{U_{L2}})^2 \sin \phi_d. \end{aligned} \tag{3.7.23}$$

Therefore the CP-violating phase appeared in d_e related to the CP violation in the standard model by

$$\sin \phi' \simeq \frac{J}{|V_{td}||V_{ub}|}. \tag{3.7.24}$$

Finally, as mentioned in the Sec. 3.3, we consider the possibility that the slight non-degeneracy between the first two generation scalar masses could give a significant contribution to the flavor changing processes because of the larger mixing matrix elements. We still use ADHRS models as an example to estimate this contribution to the LFV process $\mu \rightarrow e\gamma$. For an order of magnitude estimate, the mass insertion approximation in the super-KM basis employed in [6] will serve as a convenient method. After rotating the $\Delta \mathbf{m}_E^2$ in eqn. (3.7.10) to the charged lepton mass eigenstate basis, the contribution from the first two generations to $\Delta m_{E_{21}}^2$ is

$$\begin{aligned} \Delta m_{E_{21}}^2(2\text{-}1) &\simeq V_{E_{R22}} V_{E_{R21}}^* \Delta m_{E22}^2 + V_{E_{R12}} V_{E_{R11}}^* \Delta m_{E_{11}}^2 + V_{E_{R22}} V_{E_{R11}}^* \Delta m_{E_{21}}^2 \\ &\simeq [-\frac{z_e C}{E_e'} \frac{\overline{z_e'^2} C^2 + \overline{y_e^2}|E|^2 + \overline{x_e^2} B^2}{A^2} + \frac{z_e C}{E_e'} \frac{\overline{z_e^2} C^2}{A^2} + e^{-i\phi_e} \overline{z_e y_e} \frac{CE}{A^2}] \Delta m_{E_{33}}^2 \\ &\simeq -\frac{z_e}{y_e} \left[\frac{\overline{y_e^2} C|E| + \overline{x_e^2} \frac{CB^2}{|E|} + \overline{z_e y_e} C|E|}{A^2} \right] \Delta m_{E_{33}}^2 \end{aligned}$$

$$\text{(assume } z_e = z_e' \text{ as in ADHRS model).} \tag{3.7.25}$$

Compared with the result found in [6] for minimal $SU(5)$:

$$\begin{aligned}\Delta m^2_{E_{21}}(\mathrm{BH}) &= V^*_{ts}V_{td}\Delta m^2_{E_{33}} \\ &\simeq -\frac{z_d C}{E'_d}\frac{(x_d - x_u)^2 B^2}{A^2}\Delta m^2_{E_{33}} \\ &\simeq -\frac{z_d}{y_d}\frac{(x_d - x_u)^2 \frac{CB^2}{|E|}}{A^2}\Delta m^2_{E_{33}},\end{aligned} \tag{3.7.26}$$

we can see that if the Clebsch factors are $O(1)$, this contribution is comparable to that of the minimal $SU(5)$ model. In order for this contribution to be competitive with the dominant diagrams (Fig. 10 $b_{L,R}, c_{L,R}, c'_{L,R}$ of [7]) which are enhanced by $\frac{m_\tau}{m_\mu}$, large Clebsch factors are required. While it is possible to have large Clebsch factors, we consider them as model dependent, not generic to all realistic unified theories.

3.8 Conclusions

In supersymmetric theories, the Yukawa interactions which violate flavor symmetries not only generate the quark and lepton mass matrices, but necessarily also lead to radiative breaking of flavor symmetries in the squark and slepton mass matrices, leading to a variety of flavor signals. While such effects have been well studied in the MSSM and, more recently, in minimal unified models, the purpose of this chapter has been to explore these phenomena in a wide class of grand unified models which have realistic fermion masses.

We have argued that, if the hardness scale Λ_H is above M_G, the expectation for all realistic grand unified supersymmetric models is that non-trivial flavor mixing matrices should occur at *all* neutral gaugino vertices. These additional, weak scale, flavor violations are expected to have a form similar to the Kobayashi-Maskawa (KM) matrix. However, the precise values of the matrix elements are model dependent and have renormalization group scalings which differ from those of the Kobayashi-Maskawa matrix elements.

It is the non-triviality of the flavor mixing matrices of neutral gaugino couplings in the up quark sector which strongly distinguishes between the general and minimal unified models, as shown in Table 3.1. Although the minimal unified models provide a simple approximation to flavor physics, they are not realistic, so we stress the important new result that flavor mixing in the up sector couplings of neutral gauginos is a necessity in unified models. this leads to four important phenomenological consequences. While the $D^0 - \bar{D}^0$ mixing induced by this new flavor mixing is generally not close to the present experimental limit, it could be much larger than that predicted in the standard model.

The new mixing in the up-quark sector implies that there may be significant radiative contributions to the up quark mass matrix which arise when the superpartners are integrated out of the theory. This is illustrated in Fig. 3.4, where the new mixing matrix elements have been taken to be a factor of three larger than the corresponding Kobayashi-Maskawa matrix elements. In this case the entire up quark mass could be generated by such a radiative mechanism: above the weak scale the violation of up quark flavor symmetries lies in the squark mass matrix.

The electric dipole moment of the neutron, d_n, is a powerful probe of the neutral gaugino flavor mixing induced by unified theories. In the minimal $SO(10)$ theory, d_n arises from the flavor mixing in the down sector, which leads to a down quark dipole moment, d_d. However, in realistic models the flavor mixing in the up quark sector leads to a d_u which typically provides the dominant contribution to d_n. Thus the neutron electric dipole moment is a more powerful probe of unified supersymmetric theories than previously realized.

The presence of flavor mixing in the up sector plays a very important role in determining the branching ratio for a proton to decay to $K^0\mu^+$. In the minimal models, without such mixings, this branching ratio is expected to be about 10^{-3}: the charged lepton mode will not be seen and experimental efforts must concentrate on the mode containing a neutrino, $K^+\nu$. However, including these mixings the charged lepton branching ratio is greatly increased to about 0.1. While this number is very model dependent, we nevertheless think that this effect greatly changes the importance of searching for the charged lepton mode.

These four phenomenologocal consequences are sufficiently interesting that we stress once more that they appear as a necessity in a wide class of unified theories. The absence of mixing in the up sector is a special feature of the minimal models. Since the flavor sectors of the minimal models must be augmented to obtain realistic fermion masses, any conclusions based on the absence of flavor mixings in the up sector are specious.

A second topic addressed in this chapter is the effect of large $\tan\beta$ on the lepton process, $\mu \to e\gamma$ which is expected in unified supersymmetric $SO(10)$ models. The amplitude for this process has a contribution proportional to $\tan\beta$. In this chapter, we have found that the naive expectation that large $\tan\beta$ in supersymmetric $SO(10)$ is excluded by $\mu \to e\gamma$ is incorrect, at least for some values of the superpartner masses of interest. Contour plots for the $\mu \to e\gamma$ branching ratio are shown in Figs. 3.7 and 3.8. It depends sensitively on the parameter Δ, which is the mass splitting between the scalar electron and scalar tau, and is plotted in Fig. 3.9. Lower values of the top quark Yukawa coupling, which for large $\tan\beta$ still give allowed predictions for the b/τ mass ratio, give a much reduced value for Δ, thereby reducing the $\mu \to e\gamma$ rate and partially compensating the $\tan^2\beta$ enhancement. A further significant suppression of an order of magnitude is induced by the renormalization group scaling of the leptonic flavor mixing angles, and is shown in Fig. 3.10. The net effect is that while the case of $\tan\beta \approx m_t/m_b$ is not excluded in SO(10), the $\mu \to e\gamma$ rate is still typically larger than for moderate $\tan\beta$, so that this process provides a more powerful probe of the theory as $\tan\beta$ increases.

For large $\tan\beta$, μ and M_2 become the physical masses of the two charginos. The $\mu \to e\gamma$ contours of Fig. 8 show that μ and M_2 should not be too large, providing an important limit to the chargino masses in the large $\tan\beta$ limit. Furthermore, this constrains the LSP mass to be quite small. We find that in this region it is still possible for the LSP to account for the observed dark matter, and even to critically close the universe, as can be seen from Fig. 3.11. However, the requirement that the LSP mass be larger than 45 GeV suggests that the two light charginos will not be light enough to be discovered at LEP II.

As an example of theories with both a realistic flavor sector and large $\tan\beta$ we studied the models introduced by Anderson *et al.*. The flavor sectors of these theories are economical: the free parameters can all be fixed from the known quark and lepton masses and mixings. Hence the flavor mixing matrices at all neutral gaugino vertices can be calculated. These are shown for the lepton sector of model 6 in Table 3.3. The Clebsch factors enhance the $\mu \to e\gamma$ amplitude by a factor of 2.3, and suppress d_e by a factor of 3. Even taking the top quark Yukawa coupling to have its lowest value the rate for $\mu \to e\gamma$ in this theory is very large. Another interesting feature of these theories is that the flavor sectors contain just a single CP violating phase. This means that the phase which appears in the result for d_n and d_e can be computed: since it is closely related to the phase of the Kobayashi-Maskawa matrix it is not very small. That which appears in d_e is given in eqn. (3.7.24) and is numerically about 0.2. We have computed the radiative corrections to m_u in the ADHRS models and have found that the new mixing matrices in the up sector are not large enough to yield sizable contributions: thus the ADHRS analysis of the quark mass matrices is not modified. Furthermore, due to a cancellation special to these theories, there is no contribution to d_n from the up quark at one loop.

Chapter 4

A supersymmetric theory of flavor with radiative fermion masses

4.1 Introduction

A complete supersymmetric theory of flavor must address both the fermion mass problem and the flavor-changing problem [12]. An early proposal to address the flavor-changing problem by invoking a $U(N)$ flavor symmetry of the Kähler potential in supergravity [17] was very incomplete; it did not address how the symmetry could be broken to get the fermion mass interactions of the superpotential. By studying broken flavor symmetries, one can study both issues simultaneously [57], opening the door to a new field of flavor model building. Although there is considerable freedom in the choice of the flavor symmetry group and the pattern of symmetry breaking, the enterprise is nevertheless constrained by the direct link between the flavor-changing and fermion mass problems. Many candidate theories of fermion masses are excluded by flavor-changing phenomenology. In this chapter we study the possibility that some fermion masses arise radiatively, which requires large flavor changing interactions of the squarks or sleptons. Hence theories of flavor, based on spontaneously broken flavor symmetries, which involve radiative fermion masses, are very highly constrained by flavor-changing phenomenology.

Flavor symmetries should forbid Yukawa couplings of the light fermions. After the flavor symmetries are broken, the light generation fermions should acquire small Yukawa couplings. Many models of fermion masses use the Froggatt-Nielsen mechanism [10] to generate small Yukawa couplings: assuming a flavor symmetry is broken by the VEV of some fields $\langle\phi\rangle$, and after integrating out heavy states of mass M, one can get light generation Yukawa couplings suppressed by $\frac{\langle\phi\rangle}{M}$. This mechanism can naturally generate second generation Yukawa couplings, but in order to ensure small enough first generation Yukawa couplings one usually has to assume contrived representations of the flavor group and/or contrived patterns of flavor breaking. There is, however, another possibility for generating small Yukawa couplings: if generated radiatively, they are suppressed by the loop factor $\frac{1}{16\pi^2}$. This intriguing possibility has been extensively studied in the literature[58]. A universal feature of all models must be that an "accidental" chiral symmetry is present in the Yukawa sector to force a zero Yukawa coupling at tree level, while this symmetry must be broken in another sector of the theory in order for the Yukawa coupling to be radiatively generated. As pointed out in [59, 60], supersymmetric theories can provide a natural way for this to happen: the constraints of holomorphy can force the superpotential to have accidental symmetries not shared by the D terms. Given that the supersymmetric extension of the standard model is of interest for other reasons, we are naturally led to explore the idea of radiative fermion masses in supersymmetric models. To be specific, we consider supersymmetric $SU(3) \times SU(2) \times U(1)$ theories with minimal low energy field content, i.e. we do not consider extra Higgses or extra families etc. We will find that, with this assumption, the set of possibilities for radiative fermion masses is highly constrained, and yields robust experimental predictions.

The outline of this chapter is as follows. In section 4.2 we consider general possibilities for radiative fermion masses in supersymmetric theories with minimal low-energy field content, and conclude that, quite generally, only the lightest generation can be obtained radiatively. In section 4.3 we discuss phenomenological constraints and consequences which follow from generating the lightest generation radiatively. In the subsequent sections, we consider issues related to building models which naturally implement radiative fermion Yukawa couplings for the first generation: In section 4.4, we discuss some general properties such models should have; and in section 4.5 we first present a model for leptons and then extend it to the quark sector. Our conclusions are drawn in section 4.6.

4.2 General possibilities for radiative fermion masses

We now consider general possibilities for radiatively generated Yukawa couplings in supersymmetric theories with minimal low energy field content. We know that, in the limit of exact supersymmetry, a Yukawa coupling which is zero at tree level will never be generated radiatively. Thus, in order to have radiative Yukawa couplings, we need soft supersymetry breaking operators which, further, must explicitly break the chiral symmetries associated with the zero Yukawa couplings of the superpotential. Also, the particles in the radiative loop must be at the weak scale: since the generated Yukawa coupling λ is dimensionless and vanishes in the limit m_S (the supersymmetry breaking scale) goes to zero, we must have $\lambda \sim \frac{1}{16\pi^2}\frac{m_S}{M}$, where M is a typical mass for the particles in the loop. Thus, M must be near the weak scale (rather than the GUT or Planck scale) in order to generate large enough

Yukawa couplings.

Thus, we see that the breaking of the flavor symmetries associated with the zero Yukawa couplings must lie in the weak scale soft supersymmetry breaking operators: the trilinear scalar A terms and the soft scalar masses. In this chapter we make the plausible assumptions that the flavor symmetry is not an R symmetry and that supersymmetry breaking fields are flavor singlets. Then, the A terms must respect the same flavor symmetries as the the Yukawa couplings, since any flavor symmetry forbidding $\int d^2\theta f(\phi)$ (where $f(\phi)$ is some function of the superfields ϕ in the theory) will also forbid $\int d^2\theta\theta^2 f(\phi)$. Hence, all the flavor symmetry breaking responsible for generating radiative fermion masses resides in the scalar mass matrices. (However, in appendix B we repeat the analysis without this assumption. Requiring our vacuum to be the global minimum of the potential and using constraints from flavor-changing neutral currents (FCNC), the A terms are such that the conclusions of this section are not greatly altered.)

For simplicity, let us work in the lepton sector, and consider the possibility of radiatively generating K lepton masses for K = 3,2,1 in turn.

$\boldsymbol{K}$**=3.** In this case, we have a vanishing tree level Yukawa matrix which has a large $U(3)_\ell \times U(3)_e$ symmetry. By our assumption that the flavor symmetry is not an R symmetry and that supersymmetry breaking fields do not carry flavor, the A terms must also vanish. But then, all the soft scalar mass matrices can be simultaneously diagonalized, leaving an independent, unbroken $U(1)$ symmetry acting on every superfield, preventing the radiative generation of any Yukawa couplings.

***K*=2.** Here, we only have the third generation Yukawa coupling at tree level. This case is more interesting. We shall find that, although it is possible to generate two Yukawa eigenvalues radiatively, strong constraints from FCNC force the ratio of the (radiatively generated) first to second generation Yukawa couplings to a value too small to be compatible with experiment.

Let us work in a basis where the Yukawa matrix $\boldsymbol{\lambda}_E$ is diagonal,

$$\boldsymbol{\lambda}_E = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \lambda \end{pmatrix}. \tag{4.2.1}$$

Since $\boldsymbol{\lambda}_E$ is invariant under independent rotations of the first two generation left and right handed lepton superfields, we can make these rotations on the left and right handed scalar masses $\mathbf{m}^2_{L(R)}$,

$$\mathbf{m}^2_{L(R)} \to U_{L(R)} \mathbf{m}^2_{L(R)} U^\dagger_{L(R)}, \tag{4.2.2}$$

where the $U_{L(R)}$ are unitary rotations in the upper 2×2 block,

$$U_{L(R)} = \begin{pmatrix} u_{L(R)} & 0 \\ 0 & 1 \end{pmatrix}. \tag{4.2.3}$$

If we write

$$\mathbf{m}^2_L = \begin{pmatrix} m^2_{2\times2} & m^2_{2\times1} \\ m^{2\dagger}_{2\times1} & m^2_{33} \end{pmatrix}, \tag{4.2.4}$$

then under U_L we have

$$\mathbf{m}^2_L \to \begin{pmatrix} u_L m^2_{2\times2} u^\dagger_L & u_L m^2_{2\times1} \\ m^{2\dagger}_{2\times1} u_L{}^\dagger & m^2_{33} \end{pmatrix}, \tag{4.2.5}$$

and we can choose u_L so that

$$u_L m^2_{2\times 1} = \begin{pmatrix} 0 \\ \Delta m^2_{23} \end{pmatrix}. \tag{4.2.6}$$

Thus, we can choose a basis where the 1-3 and 3-1 entries of $\mathbf{m}^2_L$ are 0, and similarly for $\mathbf{m}^2_R$; the scalar masses have the form

$$\mathbf{m}^2_{L(R)} = \begin{pmatrix} m_1^2 & \delta m^2_{12} & 0 \\ \delta m^{2*}_{12} & m_2^2 & \Delta m^2_{23} \\ 0 & \Delta m^{2*}_{23} & m_3^2 \end{pmatrix}_{L(R)} . \tag{4.2.7}$$

The 1-2 entries, δm^2_{12}, are constrained to be very small compared to m_1^2 and m_2^2 from FCNC considerations. Suppose we put just one of the δm^2_{12}, say δm^2_{12L}, equal to zero. Then, we have a $U(1)$ symmetry acting on the left-handed lepton superfield of the first generation, which will prevent the generation of any Yukawa coupling for the first generation. Hence, the radiatively generated first generation Yukawa coupling will be suppressed relative to the second generation one by roughly

$$\frac{\lambda_1}{\lambda_2} \sim \frac{\delta m^2_{12L}}{m_L^2}\frac{\delta m^2_{12R}}{m_R^2}, \tag{4.2.8}$$

where the $m^2_{L,R}$ are typical scalar masses for the first two generations.

Let us make a more careful estimate for the size of this suppression. For simplicity, we work in the mass insertion approximation where m_1^2, m_2^2, m_3^2 are taken to be degenerate and equal to m^2. We find the radiatively generated Yukawa matrix for the upper 2×2 block is

$$\lambda_{2\times 2} = \begin{pmatrix} \frac{\delta m^2_{12L}}{m^2}\frac{\delta m^2_{12R}}{m^2} f(7)x & \frac{\delta m^2_{12L}}{m^2} f(6)x \\ \frac{\delta m^2_{12R}}{m^2} f(6)x & f(5)x \end{pmatrix}, \tag{4.2.9}$$

where

$$f(n) = m^{2n-4} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m^2)^{n-1}(k^2 - M^2)}, \; x = \text{const} \times M \frac{\Delta m^2_{23L}}{m^2} \frac{\Delta m^2_{23R}}{m^2},$$

and M is the gaugino mass. Since $f(n)$ is only logarithmically sensitive to the ratio $\frac{M^2}{m^2}$, we put $M^2 = m^2$. Then, $f(n) = \frac{1}{(n-2)(n-1)}$ and we have

$$\lambda_{2\times2} = \begin{pmatrix} \frac{1}{30} \frac{\delta m^2_{12L}}{m^2} \frac{\delta m^2_{12R}}{m^2} x & \frac{1}{20} \frac{\delta m^2_{12L}}{m^2} x \\ \frac{1}{20} \frac{\delta m^2_{12R}}{m^2} x & \frac{1}{12} x \end{pmatrix}. \tag{4.2.10}$$

Diagonalizing the above matrix, we find the ratio of the first to second generation eigenvalues to be

$$\frac{\lambda_1}{\lambda_2} \sim \frac{1}{25} \frac{\delta m^2_{12L}}{m^2} \frac{\delta m^2_{12R}}{m^2}. \tag{4.2.11}$$

We see that it is impossible to generate large enough first generation Yukawa couplings consistent with FCNC constraints (unless the scalars are taken to be unacceptably heavy), which require (for 300 GeV sleptons and 500 GeV squarks)

$$\begin{aligned} &\frac{1}{25} \frac{\delta m^2_{12\ell}}{m^2} \frac{\delta m^2_{12e}}{m^2} < 2 \times 10^{-4} \; (\mu \to e\gamma) \\ &\frac{1}{25} \frac{\delta m^2_{12q}}{m^2} \frac{\delta m^2_{12d}}{m^2} < 1 \times 10^{-6} \; (K_1 - K_2 \text{ mixing}) \\ &\frac{1}{25} \frac{\delta m^2_{12q}}{m^2} \frac{\delta m^2_{12d}}{m^2} < 6 \times 10^{-5} \; (D_1 - D_2 \text{ mixing}). \end{aligned} \tag{4.2.12}$$

We are left with the case K=1, where Yukawa couplings for two generations occur at tree level, while the remaining Yukawa couplings, which necessarily correspond to the lightest generation, are radiatively generated. In the next section, we study the phenomenological constraints on this scenario in detail.

4.3 Phenomenological constraints

In this section, we discuss the phenomenology of obtaining the first generation Yukawa coupling radiatively. Recall that we are relying on the scalar mass matrices to break the chiral symmetries associated with the Yukawa matrices; in particular, then, the scalar mass matrices cannot be diagonalized in the same basis as the Yukawa matrices. Thus, if we work in the mass eigenstate basis for all fields, we will have non-trivial mixing matrices at the gaugino vertices as we showed in last chapter.

Using the notation defined in chapter 3, we now consider the dominant radiative contributions to the lepton, up and down mass matrices given in Fig. 4.1. In the

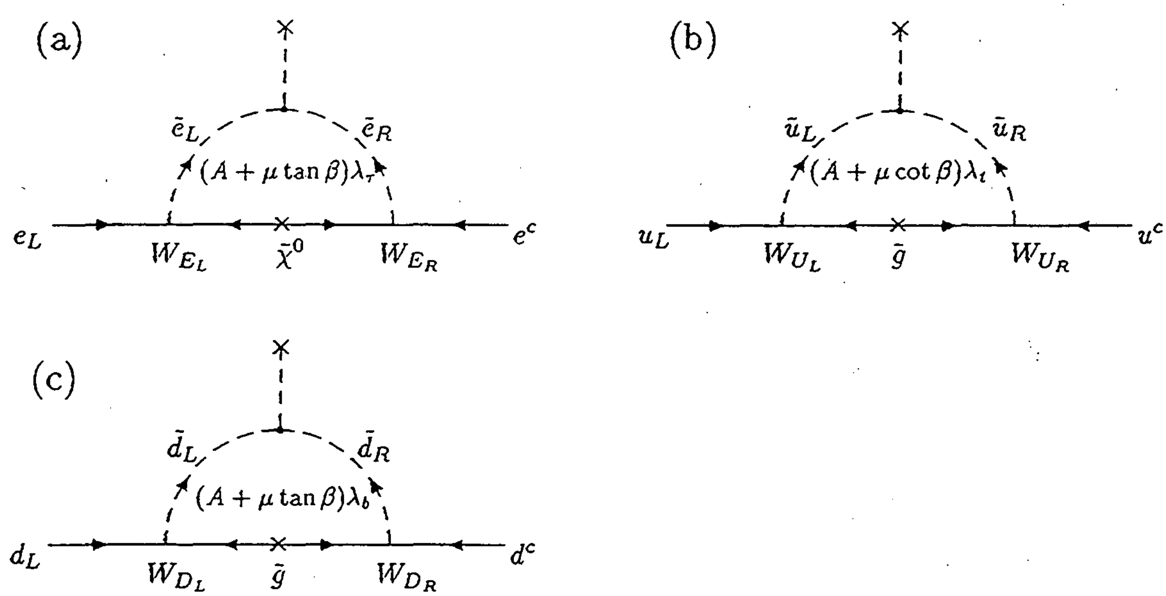


Figure 4.1: The dominant radiative contributions to the fermion masses: (a) charged leptons, (b) up-type quarks, (c) down-type quarks.

following, we assume that the first two generation scalars are degenerate, since we know from the previous section that the contribution to the mass matrix from the non-degeneracy between the first two generations is negligible. Evaluating the diagrams, we find (keeping only the contribution from the third generation tree-level mass) [25] :

$$\Delta \mathbf{m_e}_{\alpha\beta} = \sum_{n=1}^{4} \frac{H_{n\tilde{B}}}{M_n}(H_{n\tilde{B}} + \cot\theta_W H_{n\tilde{w}_3})$$

$$\times \frac{\alpha m_\tau}{4\pi\cos^2\theta_W}(A+\mu\tan\beta) \times \{W_{E_L 3\alpha}W^*_{E_L 33}\, W_{E_R 3\beta}W^*_{E_R 33}$$

$$[h(x_{3L_n}, x_{3R_n}) - h(x_{3L_n}, x_{1R_n}) - h(x_{1L_n}, x_{3R_n}) + h(x_{1L_n}, x_{1R_n})]$$

$$+ W_{E_L 3\alpha}W^*_{E_L 33}\,\delta_{3\beta}[h(x_{3L_n}, x_{1R_n}) - h(x_{1L_n}, x_{1R_n})]$$

$$+ \delta_{\alpha 3}\, W_{E_R 3\beta}W^*_{E_R 33}[h(x_{1L_n}, x_{3R_n}) - h(x_{1L_n}, x_{1R_n})]$$

$$+ \delta_{\alpha 3}\delta_{3\beta} h(x_{1L_n}, x_{1R_n})\}, \tag{4.3.1}$$

$$\Delta \mathbf{m_u}_{\alpha\beta} = \frac{8}{3}\frac{\alpha_s m_t}{4\pi}(\frac{A+\mu\cot\beta}{M_{\tilde{g}}}) \times \{W_{U_L 3\alpha}W^*_{U_L 33}\, W_{U_R 3\beta}W^*_{U_R 33}$$

$$[h(x_{3L}, x_{3R}) - h(x_{3L}, x_{1R}) - h(x_{1L}, x_{3R}) + h(x_{1L}, x_{1R})]$$

$$+ W_{E_L 3\alpha}W^*_{E_L 33}\delta_{3\beta}[h(x_{3L}, x_{1R}) - h(x_{1L}, x_{1R})]$$

$$+ \delta_{\alpha 3}\, W_{E_R 3\beta}W^*_{E_R 33}[h(x_{1L}, x_{3R}) - h(x_{1L}, x_{1R})]$$

$$+ \delta_{\alpha 3}\,\delta_{3\beta}\, h(x_{1L}, x_{1R})\}, \tag{4.3.2}$$

where $x_{3L(R)_n} = \frac{m^2_{\tilde{\tau}_{L(R)}}}{M_n^2}$, $x_{1L(R)_n} = \frac{m^2_{\tilde{e}_{L(R)}}}{M_n^2}$ in the lepton sector, $x_{3L(R)} = \frac{m^2_{\tilde{t}_{L(R)}}}{M_{\tilde{g}}^2}$, $x_{1L(R)} = \frac{m^2_{\tilde{u}_{L(R)}}}{M_{\tilde{g}}^2}$ and $\Delta\mathbf{m_d}_{\alpha\beta}$ is the same as $\Delta\mathbf{m_u}_{\alpha\beta}$ with the replacements $\cot\beta \to$

$\tan\beta, m_t \to m_b$ and $\tilde{t}, \tilde{u} \to \tilde{b}, \tilde{d}$, and where

$$
\begin{aligned}
h(x,y) &= \frac{f(x)-f(y)}{x-y}, \\
f(x) &= \frac{x\ln x}{1-x}.
\end{aligned}
\tag{4.3.3}
$$

Let us begin our phenomenological discussion with the lepton sector. The above expression for the radiative contribution to the lepton mass matrix is rather unwieldy; while we can use it for numerical work, in order to get an approximate feeling for the size of the radiative electron mass, we simply look at the 11 entry of the radiative correction matrix $m_e \approx \Delta\mathbf{m}_{e11}$. For simplicity, we assume that one of the neutralinos is pure bino, that the scalar tau's are degenerate with mass m and much lighter than the selectrons. Then we find as in [59]

$$
m_e = \frac{\alpha m_\tau}{4\pi\cos^2\theta_W}\frac{(A+\mu\tan\beta)}{M_1} \times W_{E_L 31} W_{E_R 31} h(x_3, x_3), \tag{4.3.4}
$$

where M_1 is the bino mass, $h(1,1) = 1/2$, and we have assumed $W_{E33} \simeq 1$. As explained in [59], we must work in the large $\tan\beta$ regime, and so we can neglect the A term contribution above. If we set $\tan\beta = 60$ and $\mu = M_1 = m$, equation (4.3.4) reproduces the electron mass if the product $W_{E_R 31} W_{E_L 31} \simeq 0.01$. This is roughly speaking a lower bound for this product. In this calculation we have taken the selectron to be much heavier than the stau so that the super-GIM cancellation in the loop can be ignored. In fact, however, for selectrons moderately heavier than the staus, there will be a super-GIM cancellation and $W_{E_R 31} W_{E_L 31}$ will be correspondingly larger. In Fig. 4.2, we give a plot for the relevant super-GIM suppression factor. Assuming left and right handed scalars degenerate, scalars of the first two generations degenerate, and the third generation scalar degenerate with the gaugino, we plot the super-GIM factor against the ratio of first two generation to the third generation scalar masses. This implies that each of $W_{E_{R31}}$,$W_{E_{L31}}$ should be at least 0.1. In the following we will explore the consequences of having such large mixing angles.

$-\mu \to e\gamma$: One immediate observation is that, if in the diagram of Fig. 4.1(a) we replace one of the external electrons with a muon and attach a photon to the graph, we get a potentially dangerous contribution to the rare process $\mu \to e\gamma$. How dangerous is this effect? In appendix C, we present the FCNC constraints on the elements of the mixing matrices W. Requiring the $\mu \to e\gamma$ rate to be smaller than current experimental bound constrains $W_{E_{L(R)32}} W_{E_{R(L)31}}$ to be smaller than $\sim 10^{-4}$. Since we know that we need $W_{E_{L(R)31}} \sim 0.1$ in order to generate the electron mass radiatively, we must have that $W_{E_{L(R)32}} \lesssim 10^{-3}$ in order to avoid a dangerous $\mu \to e\gamma$ rate. It may seem strange that $W_{E_{L(R)}31}$ and $W_{E_{L(R)}32}$ have such disparate sizes; any theory of lepton flavor with radiatively generated electron mass must naturally explain why $W_{E_{L(R)}32}$ is so much smaller than $W_{E_{L(R)}31}$. Speaking more loosely, if the electron mass is radiative, muon number must be very nearly conserved.

$-\tau \to e\gamma$: What about the decay $\tau \to e\gamma$? Since it is a 3-1 transition, it is directly related to $W_{E_{L(R)31}}$. Under the same set of assumptions that went into the

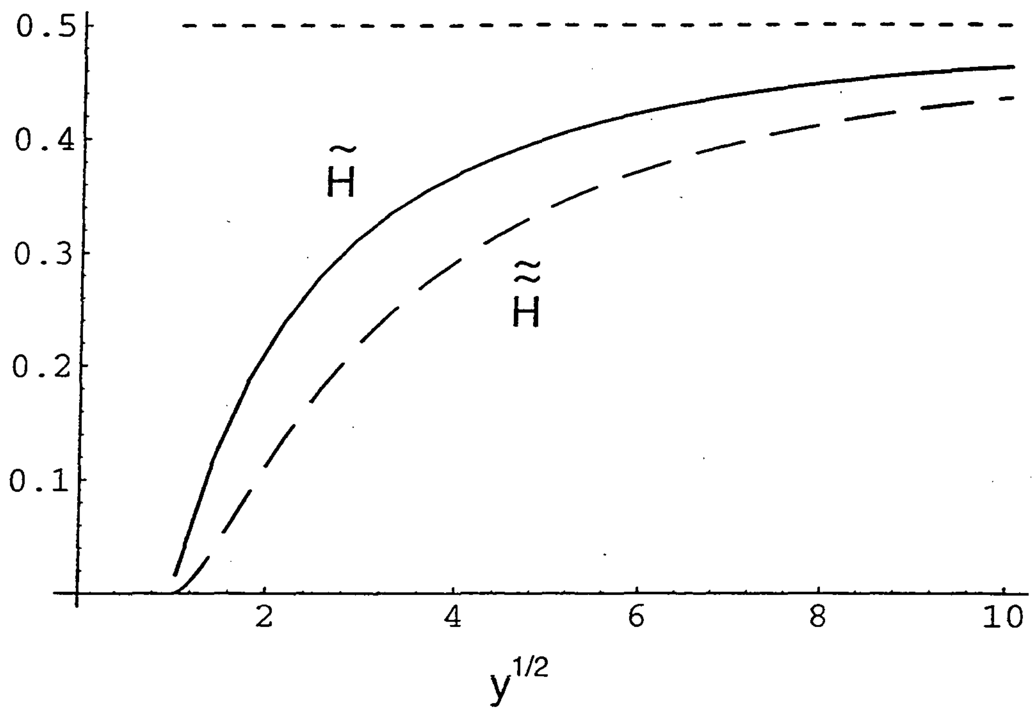


Figure 4.2: Plots of the super-GIM factor $\widetilde{\widetilde{H}} \equiv h(x_3, x_3) - h(x_3, x_1) - h(x_1, x_3) + h(x_1, x_1)$ and $\widetilde{H} \equiv h(x_3, x_3) - h(x_3, x_1)$ versus the ratio between the first two generation and the third generation scalar masses $\sqrt{y}$.

$y \equiv \tilde{m_1}^2/\tilde{m_3}^2 = x_1/x_3$, with $x_3 = 1$, $(M_g = \tilde{m_3})$.

$\frac{\Delta \mathrm{m}_{e\alpha\beta}}{m_\tau} = 2.4 \times 10^{-2}(\frac{\mu}{m_{\tilde{\tau}}})(\frac{\tan\beta}{60})(\frac{\widetilde{\widetilde{H}}}{0.5})\sqrt{x_3} W_{E_{L3\alpha}} W_{E_{R3\beta}}$,

$\frac{\Delta \mathrm{m}_{u\alpha\beta}}{m_t} = 1.2 \times 10^{-2}(\frac{A}{m_{\tilde{t}}})(\frac{\widetilde{\widetilde{H}}}{0.5})\sqrt{x_3} W_{U_{L3\alpha}} W_{U_{R3\beta}}$,

$\frac{\Delta \mathrm{m}_{d\alpha\beta}}{m_b} = 0.7(\frac{\mu}{m_{\tilde{b}}})(\frac{\tan\beta}{60})(\frac{\widetilde{\widetilde{H}}}{0.5})\sqrt{x_3} W_{D_{L3\alpha}} W_{D_{R3\beta}}$,

for α, $\beta = 1$, 2, and $\widetilde{\widetilde{H}}$ has to be replaced by $\widetilde{H}$ if one of the α, β is 3.

simplified equation (4.3.4), the amplitude for $\tau_{L(R)}$ decay is

$$F_{L(R)} = \frac{\alpha m_\tau}{4\pi \cos^2\theta_W} \frac{(A + \mu\tan\beta)}{M_1^3} \times W_{E_{L(R)}31} g(x_3, x_3), \tag{4.3.5}$$

where

$$\begin{aligned} g(x,y) &= \frac{f'(x) - f'(y)}{x - y}, \\ f'(x) &= \frac{x^2 - 2x\ln x - 1}{2(x-1)^3}, \end{aligned} \tag{4.3.6}$$

and $g(1,1) = \frac{1}{12}$. The branching ratio for $\tau \to e\gamma$ is proportional to $|W_{E_L31}|^2 + |W_{E_R31}|^2 \geq 2|W_{E_L31}W_{E_R31}|$, which is the product constrained by the requirement of obtaining radiative electron mass. Putting $\mu = M_1 = m = 300$ GeV gives $B(\tau \to e\gamma) \approx 10^{-6}$, a factor of 100 beneath the current bound. We make a more careful analysis as follows. Assuming that the left and right scalars, as well as the scalars of the first two generations are degenerate, both the radiatively generated m_e and the $\tau \to e\gamma$ rate depend on the following parameters (other than the mixing angles) in the large $\tan\beta$ regime: $(\mu, M_1, M_2, m_{\tilde{\tau}}^2, m_{\tilde{e}}^2, \tan\beta)$ Putting $\tan\beta = 60$ and assuming the grand unification relation $M_2 \sim 2M_1$, the dependence is reduced to only $(\mu, M_1, m_{\tilde{\tau}}^2, m_{\tilde{e}}^2)$. Specifying these parameters determines what the product $|W_{E_L31}W_{E_R31}|$ should be to obtain the correct electron mass, and this in turn provides us with a lower bound on $B(\tau \to e\gamma)$. In Fig. 4.3, we give a representative contour plot for this lower bound on $B(\tau \to e\gamma)$. Over a significant portion of the parameter space, the rate is only 10-100 times smaller than the current bound $B(\tau \to e\gamma) \lesssim 1.2 \times 10^{-4}$ [61].

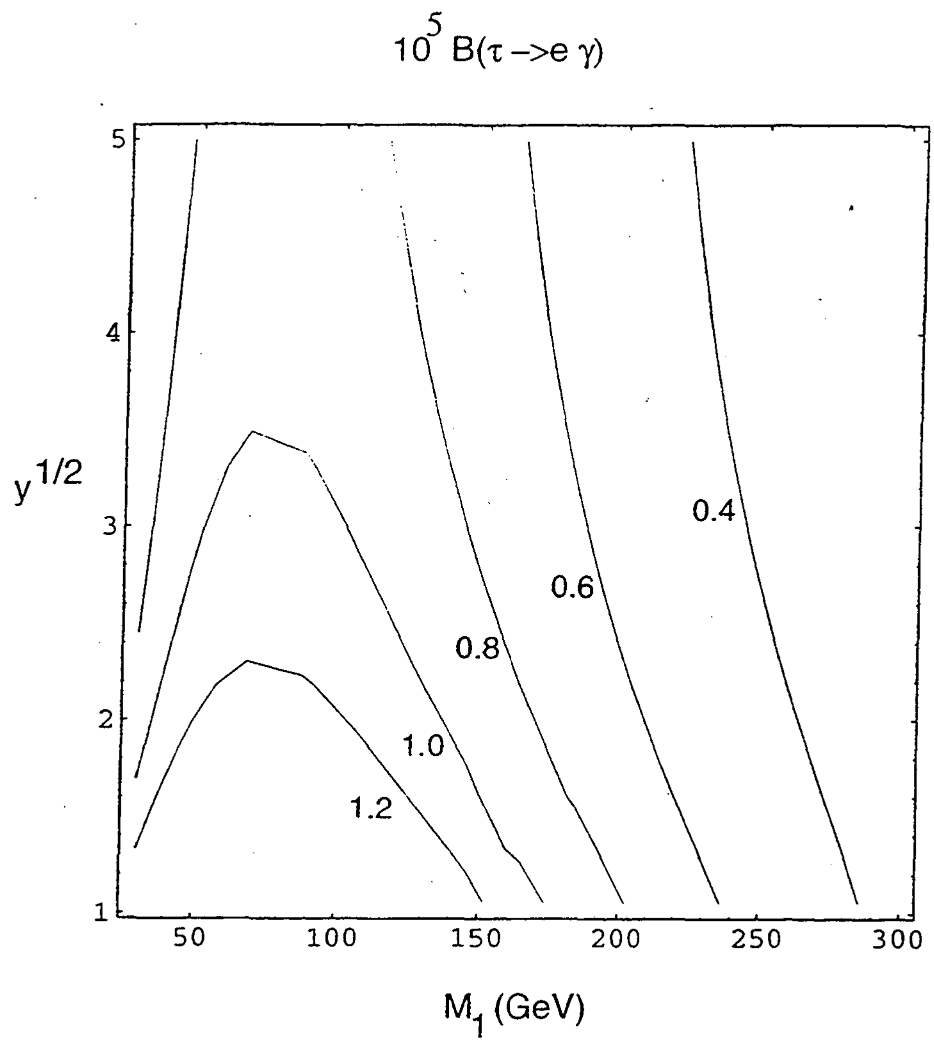


Figure 4.3: Contour plot of $B(\tau \to e\gamma)$, where the mixing angles are fixed by requiring a radiative electron mass.

We have put $\tan\beta = 60.$, $\mu = m_{\tilde{\tau}}$=200 GeV, and plot in the M_1 - $\sqrt{y}$ plane where M_1 is the bino mass and we have assumed the GUT relation $M_2 \sim 2M_1$; $y = \frac{m_{\tilde{\chi}}^2}{m_{\tilde{\tau}}^2}$. We also assume that the left and right handed mixing angles are equal, giving us a lower bound on $B(\tau \to e\gamma)$. The branching ratio scales as $\frac{\mu \tan\beta}{m_{\tilde{\tau}}^4}$.

$-d_e$: If there are CP-violating phases in the theory, we have further considerations. First, we note that if there is no mixing with the second generation (as seems to be required for avoiding dangerous $\mu \to e\gamma$), then we can choose a basis where the mixing matrices $W_{E_{L(R)}}$ are real: the only potentially complex coupling is $(\tilde{e}_L^* m_{13L}^2 \tilde{\tau}_L + h.c. + L \to R)$. Since the tree level electron Yukawa coupling is zero, we can independently rephase the superfields $e_{L(R)}$ to make $m_{13L(R)}^2$ real. Thus, the only sources of CP violation are the phases in the A and μ parameters. Ordinarily, (when no fermion masss are generated radiatively), the phases of A and μ are constrained to be small, since arbitrary phases lead to large electric dipole moments via diagrams proportional to the tree level first generation Yukawa couplings. Does the situation change when we generate the lightest generation Yukawa coupling radiatively? To answer this question, let us look at the lepton mass matrix and dipole moment matrix in the two dimensional space of the first and third generation (the second generation has no mixing and is thus irrelevant). For simplicity, we again consider taking the first two generation scalars much heavier than those of the third generation so that they are decoupled, and we set $\mu = M_1 = m$. Then, we have

$$\frac{\mathbf{m_e}}{m_\tau} \simeq \begin{pmatrix} .02\, W_{E_L 31} W_{E_R 31} e^{i\theta} & .02\, W_{E_L 31} e^{i\theta} \\ .02\, W_{E_R 31} e^{i\theta} & 1 \end{pmatrix},$$

$$\frac{\mathbf{d_e}}{e} \simeq 1.5 \times 10^{-21} \mathrm{cm} \times \left(\frac{300\mathrm{GeV}}{M_1}\right)^2$$

$$\times \begin{pmatrix} W_{E_L 31} W_{E_R 31} & W_{E_L 31} \\ W_{E_R 31} & 1 \end{pmatrix} e^{i\theta}, \tag{4.3.7}$$

where θ is the phase of $A + \mu \tan\beta$. We can approximately diagonalize the lepton mass matrix as follows

$$\frac{\mathbf{m_e}}{m_\tau} \simeq V_{E_L}^* \begin{pmatrix} 0.02\, W_{E_L 31} W_{E_R 31} & 0 \\ 0 & 1 \end{pmatrix} V_{E_R}^\dagger,$$

$$V_{E_{L(R)}} \simeq \begin{pmatrix} e^{-i\theta/2} & 0.02\, W_{E_{L(R)} 31} e^{-i\theta} \\ -0.02\, W_{E_{L(R)} 31} e^{i\theta/2} & 1 \end{pmatrix}. \tag{4.3.8}$$

In the basis where the lepton mass matrix is diagonal with real eigenvalues, the electric dipole moment matrix is $\mathbf{d}'_e = V_{E_L}^T \mathbf{d}_e V_{E_R}$, and the electric dipole moment of the electron is $d_e = \mathrm{Im}(\mathbf{d}'_{e11})$. We find with $M_1 = 300$ GeV and $W_{E_L 31} W_{E_R 31} \sim 0.01$ (as required to generate the electron mass),

$$\frac{d_e}{e} = 6 \times 10^{-24} \mathrm{cm} \times \sin\theta. \tag{4.3.9}$$

Thus, $\sin\theta$ must be smaller than $\sim 7 \times 10^{-4}$ for $\frac{d_e}{e}$ not to exceed the experimental limit of 4×10^{-27} cm. So, we have not made any progress on the supersymmetric CP problem. However, as we have already mentioned, if we assume that $\sin\theta$ is sufficiently suppressed, there are no other CP-violating contributions when muon number is conserved.

What if the electron mass is not all radiative in origin and has some small tree level contribution? If there is an $O(1)$ phase mismatch between the tree and radiative parts of the electron mass, there will be a phase in the electron electric dipole moment of order $\frac{m_e^{tree}}{m_e}$ even if A and μ are taken to be real. This would again give too large a dipole moment unless $\frac{m_e^{tree}}{m_e} \lesssim 10^{-3}$. (Of course, in deriving this result, we assume that most of the electron mass is radiative, otherwise there is no reason for the $W_{E_{L(R)}31}$ to be big enough to cause trouble with the dipole moment). We conclude that if there are large CP-violating phase differences in the theory, the electron mass must either be nearly all radiative or nearly all tree level.

In the quark sector, in addition to the first generation quark masses, we are also interested in the possibility of generating KM mixing angles by finite radiative corrections. Table 4.1 shows the relevant ratios of quark masses and mixing angles.

The constraints on SUSY FCNC have been studied in [13, 14], and the results are given in terms of $\delta_{ij} = \frac{\delta\tilde{m}_{ij}^2}{M_{\tilde{q}}^2}$, where $\delta\tilde{m}_{ij}^2$ is the off-diagonal squark mass in the super-KM basis and $M_{\tilde{q}}$ is the "universal squark mass". However, in order to generate the light generation quark masses entirely by radiative corrections, the splitting between scalar masses of the first two and the third generations must be quite large so that the super-GIM cancellation is not effective. As we can see from Fig. 4.2, this typically requires $\frac{\tilde{m}_1}{\tilde{m}_3} \gtrsim 3$. Then it is not clear which scalar mass should be used for $M_{\tilde{q}}$. In appendix C, we translate thes results obtained in [13, 14] into constraints directly on the mixing matrix elements, which are more suitable for our dicussions.

When $\tan\beta$ is large, some of the one-loop diagrams for the down type quark Yukawa couplings are enhanced by $\tan\beta$ (Figs. 4.1(c), 4.4(a)(b)). They can give

$\frac{m_c}{m_t}$	3.6×10^{-3}	$\frac{m_s}{m_b}$	2.7×10^{-2}
$\frac{m_u}{m_t}$	1×10^{-5}	$\frac{m_d}{m_b}$	1.3×10^{-3}
$\frac{\sin\theta_c m_c}{m_t}$	8×10^{-4}	$\frac{\sin\theta_c m_s}{m_b}$	6×10^{-3}
$\frac{V_{cb} m_t}{m_t}$	4×10^{-2}	$\frac{V_{cb} m_b}{m_b}$	4×10^{-2}
$\frac{V_{ub} m_t}{m_t}$	4×10^{-3}	$\frac{V_{ub} m_b}{m_b}$	4×10^{-3}
$\frac{V_{td} m_t}{m_t}$	1×10^{-2}	$\frac{V_{td} m_b}{m_b}$	1×10^{-2}

Table 4.1: The relevant ratios of quark masses and mixing angles with all quantities taken at the scale of top quark mass.

The values of quark masses, mixing angles, and the RG mass enhancement factors η_i are taken as follows: $m_t(m_t) = 168\,\text{GeV}$, $m_b(m_b) = 4.15\,\text{GeV}$, $m_c(m_c) = 1.27\,\text{GeV}$, $m_s(1\,\text{GeV}) = 180\,\text{MeV}$, $m_d(1\,\text{GeV}) = 8\,\text{MeV}$, $m_u(1\,\text{GeV}) = 4\,\text{MeV}$, $\eta_b = 1.5$, $\eta_c = 2.1$, $\eta_{u,d,s} = 2.4$, $\sin\theta_c = 0.22$, $V_{cb} = 4 \times 10^{-2}$, $V_{ub} = 4 \times 10^{-3}$, $V_{td} = 1 \times 10^{-2}$.

significant corrections to the down type quark masses and KM matrix elements[51]. Here we are interested in the possibility that some of the light generation quark masses and mixing angles are entirely generated by these loop corrections. Because of the large $\tan\beta$ enhancement, it is easier to generate KM mixing angles in the down sector than in the up sector. In fact, we can see from Table 4.1 that it is impossible to generate V_{cb} in the up sector, while generating V_{ub} and θ_c requires $W_{U_L 31}$ to be greater than about 0.4 and 0.2 respectively. W_{U_L} is linked to W_{D_L} by the KM matrix: $V_{KM} \simeq W_{U_L}{}^{\dagger} W_{D_L}$. To get the correct V_{ub}, $W_{U_L 31}$ has to be

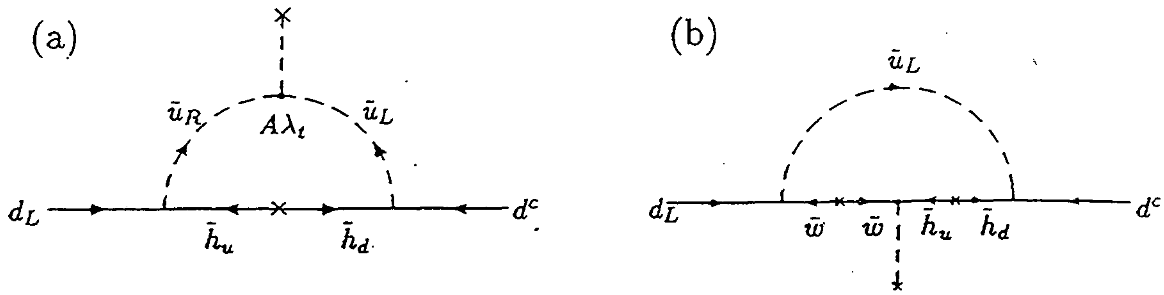


Figure 4.4: Chargino diagrams which contribute to radiative down-type quark masses and are enhanced by large $\tan\beta$.

canceled by the mixing angles of the same size in W_{D_L}, which will violate the FCNC constraints listed in Table C.1. Therefore, we will only consider generating KM mixing angles in the down sector.

The flavor diagonal gluino diagram could give large corrections to the down quark masses if the corresponding Yukawa couplings already exist at tree level. It does not generate fermion masses if they are absent at tree level, but gives large uncertainties in the tree level bottom Yukawa coupling λ_b^0, which appears in these gluino diagrams. The flavor-changing gluino diagram (through $m_b^0 \mu \tan\beta$) can give sizable down quark mass matrix elements involving light generations and therefore generate m_d and KM mixing angles. The first chargino diagram (Fig. 4.4(a)) only gives significant contributions when one of the external leg is b_R, i. e., it contributes to λ_{D13}, λ_{D23}, λ_{D33}. With some unification assumptions at high scales, one usually finds the chargino contribution to the bottom quark mass is smaller than and opposite to the gluino contribution [37, 38]. Here we do not make assumptions about physics at high scales so both contributions lead to uncertainties in the tree level λ_b^0. The contributions to λ_{D13} and λ_{D23} are proportional to V_{td} and V_{ts} respectively, so they can only give corrections to the already existing mixing angles but not generate them entirely. The second chargino diagram (Fig. 4.4(b)) is supressed by the weak coupling constant compared with other diagrams and will be ignored. In the following we will concentrate on the possibilities that the light fermion masses and mixing angles are generated by the flavor-changing gluino diagram.

–m_u: The possibility that m_u comes from radiative corrections by mixing with the third generation has been pointed out in [62]. We can see from Fig. 4.2 and $\frac{m_u}{m_t}$ in Table 4.1 that if $W_{U_{L31}} W_{U_{R31}} \sim 10^{-3}$, m_u can be generated entirely from radiative corrections. There is no direct constraint on the 1-3 mixing. The induced splitting between the first two generation left-handed squark masses could contribute to $K - \bar{K}$ mixing. However, this constraint is easily satisfied, so it is possible that m_u is entirely radiative.

–m_d: From Fig. 4.2 and Table 4.1. we can see that to generate m_d requires $W_{D_{L31}} W_{D_{R31}} \sim 2\times10^{-3}$. Compared with the constraints derived from $B-\bar{B}$ mixing in Table C.1(a), this requires the sfermion masses to be in the TeV range, which is somewhat uncomfortably large. In addition, if m_d does get its mass from radiative corrections, we also generate the 1-3 entry for the down Yukawa matrix. Their ratio is:

$$\frac{\Delta\lambda_{D11}}{\Delta\lambda_{D13}} = \frac{W_{D_{L31}}W_{D_{R31}}\widetilde{\widetilde{H}}}{W_{D_{L31}}W_{D_{R33}}\widetilde{H}} < \frac{W_{D_{R31}}}{W_{D_{R33}}} \lesssim 0.1, \tag{4.3.10}$$

for $m_{\tilde{b}} \sim 1\,\text{TeV}$, assuming $W_{D_{R33}} \simeq 1$, where $\widetilde{\widetilde{H}} = h(x_{3L}, x_{3R}) - h(x_{3L}, x_{1R}) - h(x_{1L}, x_{3R}) + h(x_{1L}, x_{1R})$, $\widetilde{H} = h(x_{3L}, x_{3R}) - h(x_{1L}, x_{3R})$, and h, $x_{1(3)L(R)}$ are defined in (4.3.2), (4.3.3). On the other hand, $\frac{m_d}{V_{ub}m_b} \simeq 0.3$. We see that the generated $\Delta\lambda_{D13}$ gives a too big contribution to V_{ub} which has to be canceled by a tree level λ_{D13}.

We now discuss the possibilities for radiative generation of KM elements. We take the independent parameters of the KM matrix to be V_{us}, V_{ub}, V_{cb} and the CP violating phase.

-θ_c: To generate θ_c we need $W_{D_{L31}}W_{D_{R32}} \sim 10^{-2}$, assuming $W_{D_{L(R)33}} \simeq 1$. From $B-\bar{B}$ mixing and $b \to s\gamma$ decay, or $K-\bar{K}$ mixing alone, the sfermion masses are also required to be $\gtrsim 1\,\text{TeV}$ in order to satisfy these constraints. Furthermore the phase of $W_{D_{L31}}W_{D_{R32}}$ has to be small ($< 10^{-1}$) from the ϵ parameter of CP violation. Similar to the case of m_d, generating θ_c radiatively may also give a too big contribution to V_{ub}. If we try to generate m_d, θ_c, and V_{ub} all by radiative corrections, ignoring the difference between $\widetilde{H}$ and $\widetilde{\widetilde{H}}$, we obtain the following ratio for the mixing matrix elements from Table 4.1:

$$W_{D_{R33}} : W_{D_{R32}} : W_{D_{R31}} \simeq V_{ub}m_b : \sin\theta_c m_s : m_d \simeq 4 : 6 : 1.3. \tag{4.3.11}$$

By unitarity we obtain

$$W_{D_{R33}} \simeq 0.55,\ W_{D_{R32}} \simeq 0.82,\ W_{D_{R31}} \simeq 0.18. \tag{4.3.12}$$

(Taking into account that $\widetilde{H} > \widetilde{\widetilde{H}}$ gives larger $W_{D_{R32}}$, $W_{D_{R31}}$.) From Table C.1, we can see that $m_{\tilde{b}}$ has to be pushed above 2 TeV (even higher for the first two generations) to satisfy the constraints from both ΔM_K and $b \to s\gamma$. If there are $O(1)$ phases in these W's, the ϵ constraints raise the lower limit of the squark masses to ~ 20 TeV, which is unacceptably large. Furthermore, it is unnatural for models to have such a large $W_{D_{R32}}$ mixing. Therefore, it is unlikely that all KM matrix elements can be generated by radiative corrections.

$-V_{ub}$: To generate V_{ub} we need $W_{D_{L31}} \sim 5\times10^{-3}$, which easily satisfies the $B-\bar{B}$ mixing constraints. Hence V_{ub} can be generated radiatively, but as we learned from above, V_{ub} and θ_c cannot both come from radiative corrections, and neither can V_{ub} and m_d.

$-V_{cb}$: Attaching a photon to the diagram which generates Δm_{D23} gives a diagram contributing to the decay $b \to s\gamma$. Hence one can write down the following simple relation between gluino diagram contributions to V_{cb} and to the Wilson coefficient $c_7(M_W)$ [63] for $b \to s\gamma$,

$$\Delta c_7(M_W) = q_D \frac{4\pi}{\alpha} \sin^2\theta_W \frac{M_W^2}{m_{\tilde{g}}^2} \frac{\widetilde{G}}{\widetilde{\widetilde{H}}} \frac{\Delta m_{D23}}{V_{cb} m_b}, \tag{4.3.13}$$

$$\Rightarrow \frac{\eta^{16/23}\Delta c_7(M_W)}{c_7(m_b)_{\text{SM}}} \simeq (\frac{8m_W}{m_{\tilde{g}}})^2 (\frac{5\widetilde{G}}{\widetilde{\widetilde{H}}}) (\frac{\Delta m_{D23}}{V_{cb} m_b}). \tag{4.3.14}$$

where $\widetilde{G} = g(x_{3L}, x_{3R}) - g(x_{1L}, x_{3R})$, and g is defined in (4.3.6). The gluino diagram contribution to $b \to s\gamma$ interferes constructively with the Standard Model contribution if V_{cb} is generated by the similar gluino diagram. Therefore, generating V_{cb} radiatively requires heavy gluino and squark masses ($\gtrsim 1$ TeV) or cancellation between the chargino diagram contributions to $b \rightarrow s\gamma$ and other contributions.

–CP-violating phases: From the above discussion we found that it is very difficult to generate all KM mixing matrix elements by radiative corrections. This means that a non-trivial KM matrix should occur at tree level. There is one physical CP-violating phase in V_{KM}, and several more in the quark-squark-gaugino mixing matrices. The number of CP-violating phases in the quark sector (not including the possible phases of the parameters A and μ) is counted as in the following. There are four unitary mixing matrices W_{U_L}, W_{U_R}, W_{D_L}, W_{D_R}, (V_{KM} is related to ${W_{U_L}}^{\dagger}$, W_{D_L} and hence is not independent,) connecting 7 species of quark and squark fields u_L, d_L, u_R, d_R, $\tilde{Q}$, $\tilde{U}$, $\widetilde{D}$. Among the phases of these fields, 6 are fixed by the 6 eigenvalues of the Yukawa matrices $\boldsymbol{\lambda}_U$ and $\boldsymbol{\lambda}_D$ (if there are no zero eigenvalues), one overall phase is irrelevant, so we can remove 14 of the 24 phases in the W's by phase redefinition of the quark and squark fields. Each massless quark removes one more phase by allowing independent phase rotations on the left and right quark fields. Each pair of degenerate quarks or squarks of the same species removes one phase as well. Assuming m_u and m_d massless at the tree level, and degeneracies between the first two generation squarks, we can remove 5 more phases and there are still 5 independent phases left. One of them cannot be moved to the W_U's and it can give significant contributions to the CP violation effects in the K and B systems.

4.4 Guidelines for model building

In the introduction we indicated some general features effective theories of flavor should have in order to generate radiative fermion masses. In particular, we pointed out that, in supersymmetric theories, an accidental superpotential symmetry is needed to ensure that the first generation is massless at tree-level, while this symmetry must be broken by D terms in order to obtain radiative masses. For instance, in the effective lepton models considered in [59], all holomorphic and flavor symmetric operators possess an accidental $U(1)_{\ell_1} \times U(1)_{e_1}$ which is violated by the D terms. From the point of view of an effective theory, then, it is representation content and holomorphy which are responsible for accidental symmetries for every possible superpotential operator, thereby forbidding some Yukawa couplings. However, this is by no means a necessary condition for the existence of tree level massless fermions: We do not always generate every operator consistent with symmetries when we integrate out heavy states. Thus, the condition that every effective operator in the superpotential possess an accidental symmetry is clearly too strong; we only need an accidental symmetry to exist for those operators induced by integrating out heavy states. For this reason, it seems that a deeper understanding of the accidental symmetries lies in examining the full theory, including superheavy states. This is our purpose in this section. We will find simple, sufficient conditions for guaranteeing the existence of tree level massless states after integrating out heavy states. We will also describe (in view of later application to the quark sector) the structure of the tree-level KM matrix. These conditions will serve as convenient guides for the explicit models we construct in the next section.

We begin by considering sufficient conditions for the existence of tree level massless states. Consider the lepton sector for simplicity. In Froggatt-Nielsen schemes, we have fields ℓ_α, e_α ($\alpha = 1,2,3$) which would be the three low energy left and right handed lepton fields in the flavor symmetric limit. However, there are also superheavy states with which ℓ and e mix after flavor symmetry breaking. In general, we have vector-like superheavy states $(L_i \oplus \bar{L}_i)$ and $(E_a \oplus \bar{E}_a)$, $(i = 4, ..., n+3,\ a = 4, ..., m+3)$, with L, E having the same gauge quantum numbers as ℓ, e respectively, and with the barred fields having conjugate gauge quantum numbers. We also have a set of gauge singlet fields ϕ with VEV's $\langle\phi\rangle$ breaking the flavor group G_f. In the superpotential, we have bare mass terms for the $(L, \bar{L})$ and the $(E, \bar{E})$ fields, as well as trilinear couplings mixing ϕ's with light and superheavy states. We also have a large Yukawa matrix $\boldsymbol{\Lambda}_{IA}$ ($I = 1, ..., n+3$, $A = 1, ..., m+3$), connecting the down-type Higgs h_d to the (ℓ_α, L_i) and (e_α, E_a),

$$W \supset \begin{pmatrix} \ell & L \end{pmatrix} \boldsymbol{\Lambda} \begin{pmatrix} e \\ E \end{pmatrix} h_d. \tag{4.4.1}$$

Once the fields ϕ develop VEV's, we will have mass terms like, $\ell \langle\phi\rangle \bar{L}$ mixing light and heavy states. In order to diagonalize the bare mass matrix and go from the flavor basis to the mass basis (where "light" and "heavy" are correctly identified), we must make appropriate $\langle\phi\rangle$ dependent unitary rotations on the fields:

$$\begin{pmatrix} \ell' \\ L' \end{pmatrix} = U_L(\langle\phi\rangle) \begin{pmatrix} \ell \\ L \end{pmatrix}, \quad \bar{L}' = U_{\bar{L}}(\langle\phi\rangle)\bar{L},$$

$$\begin{pmatrix} e' \\ E' \end{pmatrix} = U_E(\langle\phi\rangle) \begin{pmatrix} e \\ E \end{pmatrix}, \quad \bar{E}' = U_{\bar{E}}(\langle\phi\rangle)\bar{E}. \tag{4.4.2}$$

In this basis, the mass terms are $\sum_{i=4}^{n+3} M_i \bar{L}'_i L'_i + \sum_{a=4}^{m+3} M_a \bar{E}'_a E'_a$, and the Yukawa matrix becomes

$$\boldsymbol{\Lambda}'_{IA} = U^*_L(\langle\phi\rangle)_{IJ} \boldsymbol{\Lambda}_{JB} U^\dagger_E(\langle\phi\rangle)_{BA}, \tag{4.4.3}$$

where summation over J and B is understood. In order to integrate out the (now correctly identified) heavy states at tree level, we simply throw out any coupling involving them. The Yukawa matrix $\boldsymbol{\lambda}$ for the three low energy generation leptons is then

$$\boldsymbol{\lambda}_{\alpha\beta} = U^*_L(\langle\phi\rangle)_{\alpha J} \boldsymbol{\Lambda}_{JB} U^\dagger_E(\langle\phi\rangle)_{B\beta}, \ (\alpha, \beta = 1, 2, 3). \tag{4.4.4}$$

We would now like to understand circumstances under which we can have a certain number of zero eigenvalues for $\boldsymbol{\lambda}$. For $\boldsymbol{\lambda}$ to have $k \leq 3$ zero eigenvalues, its rank must be $3 - k$. To see when this is possible, we make the simple observation that each row (or alternatively each column) of $\boldsymbol{\Lambda}$ contributes at most one rank to $\boldsymbol{\lambda}$. Consider for instance the contribution to $\boldsymbol{\lambda}$ from the J_0'th row of $\boldsymbol{\Lambda}$. Defining

$$x_\alpha = U^*_{L_{\alpha J_0}}, \ y_\beta = \boldsymbol{\Lambda}_{J_0 B} U^\dagger_{E_{B\beta}},$$

we have

$$\boldsymbol{\lambda}^{\text{from row } J_0}_{\alpha\beta} = x_\alpha y_\beta, \tag{4.4.5}$$

which is manifestly rank 1 if it is not identically zero. Define a non-zero row (column) of $\boldsymbol{\Lambda}$ to be a row (column) with at least one non-zero entry. Then, it is clear that a *sufficient* condition for $\boldsymbol{\lambda}$ to have rank equal or less than $3-k$ is that the number of non-zero rows (or the number of non-zero columns) of $\boldsymbol{\Lambda}$, up to rotations, equal $3-k$, i. e., $\boldsymbol{\Lambda}$ also has rank $3-k$; since in this case $\boldsymbol{\lambda}$ is of the form

$$\boldsymbol{\lambda}_{\alpha\beta} = \sum_{J=1}^{3-k} x_\alpha^J y_\beta^J, \tag{4.4.6}$$

which is manifestly rank $3-k$ or less (the case of interest to us is $k=1$). We will make use of this criterion in the following section.

We next turn to examining the tree-level KM matrix in the quark sector. In analogy to the lepton sector, we have Yukawa matrices $\boldsymbol{\Lambda}_D$ and $\boldsymbol{\Lambda}_U$,

$$W \supset \begin{pmatrix} q & Q \end{pmatrix} \boldsymbol{\Lambda}_U \begin{pmatrix} d \\ D \end{pmatrix} h_d + \begin{pmatrix} q & Q \end{pmatrix} \boldsymbol{\Lambda}_D \begin{pmatrix} u \\ U \end{pmatrix} h_u, \tag{4.4.7}$$

where all new fields are in obvious analogy with the lepton sector. Let us assume that the general condition stated above, ensuring the existence of a massless eigenvalue for $\boldsymbol{\lambda}_D$ and $\boldsymbol{\lambda}_U$, is realized by $\boldsymbol{\Lambda}_D$ and $\boldsymbol{\Lambda}_U$. Then, we can write

$$\boldsymbol{\lambda}_{D\alpha\beta} = x_\alpha^1 y_\beta + x_\alpha^2 z_\beta, \; \boldsymbol{\lambda}_{U\alpha\beta} = x_\alpha^{\prime 1} y_\beta' + x_\alpha^{\prime 2} z_\beta', \tag{4.4.8}$$

Suppose in particular that $\boldsymbol{\Lambda}_D$ and $\boldsymbol{\Lambda}_U$ have nontrivial entries in the same two rows, in which case we can choose $x_\alpha^i = x_\alpha^{\prime i}$, $i = 1, 2$. Then, the resulting KM matrix has non-zero entries only in the 2-3 sector. The reason is that, since the first generation is massless, we can always choose a basis where the first generation quark doublet has no component of superheavy quark doublets with Yukawa couplings, and so both $\boldsymbol{\lambda}_D$ and $\boldsymbol{\lambda}_U$ are only non-zero in the lower 2×2 block. We can see this more explicitly as follows. First note that we can make a rotation on the left handed quarks to make x^1_α point in the 3 direction, and make independent rotations on the right-handed up and down quarks to make y_β and y'_β also point in the 3 direction. In this basis, we have

$$\boldsymbol{\lambda}_{D\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \eta \end{pmatrix}_{\alpha\beta} + x^2_\alpha z_\beta,\ \boldsymbol{\lambda}_{U\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \eta' \end{pmatrix}_{\alpha\beta} + x^2_\alpha z'_\beta \qquad (4.4.9)$$

However, we can always make rotations on the upper 2×2 block so that x^2, z, z' have 0 entries in the first component. Using equation (4.4.9), we easily see that both $\boldsymbol{\lambda}_D$ and $\boldsymbol{\lambda}_U$ are only non-zero in the lower 2×2 block, and KM mixing only occurs in the 23 sector, as claimed. Thus, in order to have, for example, a tree level θ_c or V_{ub} (as is necessary from our discussion in section 4.3), we must ensure that Λ_D and Λ_U do not have entries in the same two rows. Other than this case, we expect generically that all elements of the KM matrix exist at tree level.

In this section we have shown that if the Higgs couples in only 2 rows or 2 columns of the full Yukawa matrix to matter, then there will be a light generation which is massless at tree level. The required sparseness of Higgs couplings is due to G_f and holomorphy.

4.5 Realistic models for radiative fermion masses

In [59], some explicit lepton models of flavor with radiative electron mass are presented, which naturally fulfilled the phenomenological requirements of Sec. 4.3; namely, the electron is massless at tree level, the muon picks up a tree level mass upon integrating out heavy states, muon number is conserved, and D terms yields $e-\tau$ mixing which generates a radiative electron mass. In this section, we begin by presenting the lepton model most readily extended to the quark sector, the full model with flavor group $G_f = SU(2)_\ell \times SU(2)_e \times U(1)_A$, then give an extension to the quark sector.

4.5.1 The lepton model

The lepton model is based on the flavor group $G_f = SU(2)_\ell \times SU(2)_e \times U(1)_A$. The fields are categorized as light/heavy and matter/Higgs in Table 4.2.

	Light	**Heavy**
Matter	$\ell_3(0), \ell_I(+1)$	$L(+2), L_I(+1), \bar{L}(-2), \bar{L}^I(-1)$
	$e_3(0), e_i(-1)$	$E(-2), E_i(-1), \bar{E}(+2), \bar{E}^i(+1)$
Higgs	$h(0)$	$\phi_{\ell I}(+1), \phi_{ei}(-1), S(0)$

Table 4.2: Field content and G_f transformation properties for the lepton model. I, i are $SU(2)_\ell$ and $SU(2)_e$ indices respectively, the numbers in brackets are the $U(1)_A$ charges.

We require the theory to be invariant under matter-parity ($Matter \to -Matter$) and heavy-parity ($Heavy \to -Heavy$). Here, matter-parity is crucial to avoid dangerous R-parity violating couplings, but the heavy-parity is imposed only for simplicity.[1] Requiring these discrete symmetries and G_f invariance gives us the following renormalizable superpotential (where all dimensionless couplings are $O(1)$)

$$\begin{aligned} W = & \lambda_3 l_3 e_3 h + \lambda_4 LEh \\ & + f_1 l_3 \bar{L}^I \phi_{\ell I} + f_2 \ell_I \bar{L}^I S + f_3 \ell_I \epsilon^{IJ} \phi_{\ell J} \bar{L} \\ & + f'_1 e_3 \bar{E}^i \phi_{ei} + f'_2 e_i \bar{E}^i S + f'_3 e_i \epsilon^{ij} \phi_{ej} \bar{E} \\ & + M_L \bar{L} L + M_{L_I} \bar{L}^I L_I + M_E \bar{E} E + M_{E_i} \bar{E}^i E_i. \end{aligned} \tag{4.5.1}$$

Note that this superpotential has only two Yukawa couplings λ_3 (for the τ) and λ_4 (for the superheavy L, E). Therefore, using the results of the last section, we are guaranteed to have a tree-level massless state after we integrate out the heavy fields;[2] we identify this state with the electron.

The fields ϕ_ℓ, ϕ_e and S take VEV's which break the flavor symmetries. We can assume without loss of generality that $\langle \phi_\ell \rangle = (v_\ell, 0), \langle \phi_e \rangle = (v_e, 0)$. As described generally in the previous section, these VEV's mix the light and heavy states and

[1]However, both of these parities are automatic in the $SU(3)_\ell \times SU(3)_e$ models considered in [59]. The $U(1)_A$ factor in G_f also finds a natural explanation in these theories. We do not use the $SU(3)$ theories here as a starting point here because the requisite modifications to go to the quark sector are more difficult to see than in the $SU(2)_\ell \times SU(2)_e \times U(1)_A$ model we are considering.

[2]Actually, in this theory the existence of a massless state can already be seen in the effective theory as described in [59].

we must rotate to the mass basis where "light" and "heavy" are properly identified. An approximation to the resulting rotation on the Yukawa matrix is shown in Fig. 4.5, and we generate the following superpotential term for the light fields:

$$\begin{aligned}\Delta W &= \left(\frac{f_3 \ell_I \epsilon^{IJ} \langle \phi_{\ell_J} \rangle}{M_L}\right) \lambda_4 h \left(\frac{f_3' e_i \epsilon^{ij} \langle \phi_{e_j} \rangle}{M_E}\right) \\ &= \lambda_4 \left(\frac{f_3 v_\ell}{M_L}\right)\left(\frac{f_3' v_e}{M_E}\right) \ell_2 e_2 h, \end{aligned} \tag{4.5.2}$$

so, we can identify (ℓ_2, e_2) with the muon and (ℓ_1, e_1) with the electron.

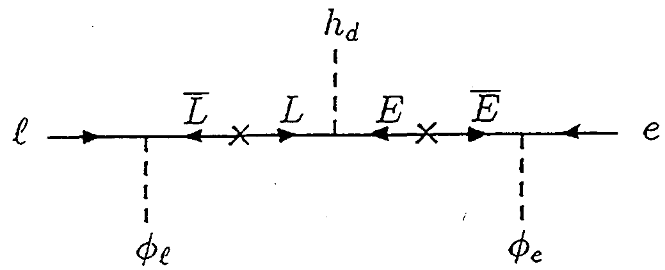


Figure 4.5: The diagram which generates the second generation masses.

Let us look at the above rotation more directly [24]. Setting ϕ_ℓ, ϕ_e, S to their VEV's gives the follwing mass terms in the superpotential:

$$W_{mass} = M_L \bar{L}(L + \epsilon_\ell \ell_2) + M_{L_I} \bar{L}^1 (L_1 + \epsilon_\ell' \ell_3 + \epsilon_\ell'' \ell_1) + M_{L_I} \bar{L}^2 (L_2 + \epsilon_\ell'' \ell_2), \tag{4.5.3}$$

plus similar terms for the E's, where $\epsilon_\ell = -\frac{f_3 v_\ell}{M_L}$, $\epsilon_\ell' = \frac{f_1 v_\ell}{M_{L_I}}$, $\epsilon_\ell'' = \frac{f_2 \langle S \rangle}{M_{L_I}}$. Thus, the mass basis is related to the flavor basis via $\ell' = U_\ell \ell$, where $\ell'^T = (\ell_1, \ell_2, \ell_3, L, L_1, L_2)'$.

To a first approximation, we have

$$U_\ell = \begin{pmatrix} 1 & 0 & 0 & 0 & -\epsilon_\ell''^* & 0 \\ 0 & 1 & 0 & -\epsilon_\ell^* & 0 & -\epsilon_\ell''^* \\ 0 & 0 & 1 & 0 & -\epsilon_\ell'^* & 0 \\ 0 & \epsilon_\ell & 0 & 1 & 0 & 0 \\ \epsilon_\ell'' & 0 & \epsilon_\ell' & 0 & 1 & 0 \\ 0 & \epsilon_\ell'' & 0 & 0 & 0 & 1 \end{pmatrix}. \qquad (4.5.4)$$

Completely similar statements hold for the e's. Now, in the original flavor basis, the Yukawa matrix $\boldsymbol{\Lambda}$ is

$$\boldsymbol{\Lambda} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \qquad (4.5.5)$$

After rotating to the mass basis, we have

$$\boldsymbol{\Lambda}' = U_\ell^* \boldsymbol{\Lambda} U_e^\dagger = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \epsilon_\ell \epsilon_e \lambda_4 & 0 & -\epsilon_\ell \lambda_4 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 & \epsilon_e' \lambda_3 & 0 \\ 0 & -\epsilon_e \lambda_4 & 0 & \lambda_4 & 0 & 0 \\ 0 & 0 & \epsilon_\ell' \lambda_3 & 0 & \epsilon_\ell' \epsilon_e' \lambda_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \qquad (4.5.6)$$

Dropping all couplings to the heavy states, we obtain the low energy Yukawa matrix $\boldsymbol{\lambda}$,

$$\boldsymbol{\lambda} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon_\ell \epsilon_e \lambda_4 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}, \tag{4.5.7}$$

just as we found earlier.

Note that the VEV's $\langle \phi_\ell \rangle$ and $\langle \phi_e \rangle$ do not completely break G_f; the generator

$$T_\mu = T_{U(1)_A} - 2(T_\ell^3 - T_e^3) \tag{4.5.8}$$

annihilates both $\langle \phi_\ell \rangle$ and $\langle \phi_e \rangle$, and corresponds to the muon number:[3]

$$e^{i\theta T_\mu} \begin{pmatrix} \ell_1 \\ \ell_2 \end{pmatrix} = \begin{pmatrix} \ell_1 \\ e^{2i\theta} \ell_2 \end{pmatrix}, \; e^{i\theta T_\mu} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} e_1 \\ e^{-2i\theta} e_2 \end{pmatrix}. \tag{4.5.9}$$

We now have most of what we want; we need only show that the required mixing between the τ and e is generated in the scalar mass matrix. We can generate D term mixings upon integrating out heavy states [24]. The diagram in Fig. 4.6 gives

$$\frac{f_2 \ell_I \langle S \rangle}{M_{L_I}} \frac{f_1^* \ell_3^\dagger \langle \phi_{\ell 1}^\dagger \rangle}{M_{L_I}} = \frac{f_2 \langle S \rangle}{M_{L_I}} \frac{f_1^* v_\ell^*}{M_{L_I}} \ell_1 \ell_3^\dagger. \tag{4.5.10}$$

Note that this term explicitly breaks the $U(1)_{\ell_1}$ chiral symmetry associated with the zero tree-level Yukawa coupling of the electron, so we expect the required mixing

[3]The $U(1)_A$ factor in G_f can be replaced with its Z_4 subgroup and still avoid dangerous muon number violating processes; after the VEV's are taken there is a symmetry under $(\ell_2, e_2) \rightarrow (-\ell_2, -e_2)$ which still forbids mixing between the scalar μ and τ, e, therefore avoiding the dangerous $\mu \rightarrow e\gamma$ decay.

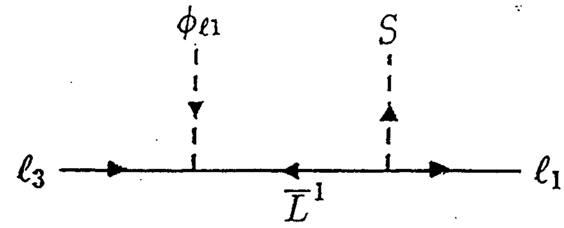


Figure 4.6: D term mixing between the first and the third generations.

between $\tilde{\tau}$ and $\tilde{e}$ to occur. Let us check it more explicitly. The D term part of the lagrangian is $\int d^4\theta(\phi^\dagger\phi + \theta^2\bar{\theta}^2\phi^\dagger m^2\phi)$, where ϕ is a collection of all the fields and m^2 is the soft supersymmetry breaking scalar mass matrix. When we rotate to the mass basis, we send $\phi \rightarrow U\phi$. Under this rotation, $\phi^\dagger\phi$ is invariant, but $m^2 \rightarrow Um^2U^\dagger$.[4] In our example, the scalar mass term for the left-handed lepton fields is $\ell^\dagger m_\ell^2 \ell$, with $m_\ell^2 = \mathrm{diag}(m_{\ell_I}^2, m_{\ell_I}^2, m_{\ell_3}^2, m_L^2, m_{L_I}^2, m_{L_I}^2)$. The scalar mass matrix for the three low

[4]This is not strictly speaking correct, since supersymmetry breaking can affect the rotation to the mass basis. For instance, in Fig. 4.6, we could attach s purions θ^2 and $\bar{\theta}^2$ to the superpotential vertices, obtaining a direct contribution to the scalar mass matrix of order $|A|^2$, where A is the trilinear soft term associated with the superpotential vertex. Put another way, we can have spurions θ^2 in the rotation matrix U, and get contributions to the scalar masses from rotating $\phi^\dagger\phi$. These contributions are of the same order as the ones we are discussing, but do not affect any of our results.

energy generations is then

$$\begin{aligned}
\mathrm{m}^{2\,(3\times3)}_{\ell_{\alpha\beta}} &= (U_\ell m_\ell^2 U_\ell^\dagger)_{\alpha\beta} \\
&= \begin{pmatrix} m_{\ell_3}^2 + |\epsilon'_\ell|^2 m_{L_I}^2 & 0 & \epsilon'^*_\ell \epsilon''_\ell m_{L_I}^2 \\ 0 & m_{\ell_I}^2 + |\epsilon_\ell|^2 m_L^2 + |\epsilon''_\ell|^2 m_{L_I}^2 & 0 \\ \epsilon'_\ell \epsilon''^*_\ell m_{L_I}^2 & 0 & m_{\ell_I}^2 + |\epsilon''_\ell|^2 m_{L_I}^2 \end{pmatrix}.
\end{aligned} \tag{4.5.11}$$

The zero entries in the above matrix are a consequence of the unbroken $U(1)_\mu$ symmetry of the theory. We can explicitly see the 1-3 entry generated in the scalar mass matrix, which, together with the corrseponding 1-3 entry in the the right-handed scalar mass matrix, is responsible for generating the radiative electron mass.

There are two difficulties when we try to extend the lepton model for radiative electron mass to the quark sector. First, the radiative down quark mass is severely constrained by $B - \bar{B}$ mixing as we showed in Sec. 4.3. This can be resolved if the SUSY-breaking masses are heavy enough ($\gtrsim$ 1 TeV). The other problem is that in addition to the quark masses, we also have to get the correct KM mixing matrix. As we have shown in Sec. 4.3, it is very difficult to generate all KM mixing matrix elements: squark masses have to be pushed up to unacceptably high scales and unnatural flavor mixing gaugino interactions are needed. Excluding that possibility, one has to put in some mixing angles at tree level. In subsection 4.5.2 we present a model in which all first generation fermion masses come from radiative corrections. In subsection 4.5.3 we construct a model in which m_e and m_u come from radiative corrections while m_d and θ_c appear at tree level with the prediction $\sin\theta_c = \sqrt{m_d/m_s}$. We show that this model can be naturally embedded in the flipped $SU(5)$ grand unified theory.

4.5.2 A complete model for radiative first generation fermion masses

The complete model for quarks and leptons is based on the same flavor group $G_f = SU(2)_l \times SU(2)_r \times U(1)_A$ as in the lepton model. However, a minimal direct extension of the lepton model to the quark sector does not give tree level KM mixing angles. Following the guidelines to generate tree level θ_c and V_{ub} in Sec. 4.4, we need to introduce two heavy left-handed $SU(2)_l$ singlet quarks Q, Q' (and their conjugates $\bar{Q}$, $\bar{Q}'$).[5] Their $U(1)_A$ charges are assigned such that Q only couples to the up-type Higgs but not the down-type Higgs and vice versa for Q'. In addition, there cannot be an unbroken $U(1)$ left in the quark sector, so we introduce a second $SU(2)_l$ doublet ϕ'_l, and a second $SU(2)_r$ doublet ϕ'_r, whose VEV's are in different directions from the directions of ϕ_l and ϕ_r VEV's, breaking G_f completely. The field content and G_f transformation properties of the quark sector are shown in Table 4.3. We also impose matter-parity and heavy-parity. The VEV's of ϕ, ϕ' and

[5]Second pairs of heavy U', $\bar{U}'$ and D', $\bar{D}'$ are not included in our discussion. They can be added as long as their $U(1)_A$ charge assignments forbid their Yukawa interactions with the Q's and Higgses.

		Heavy		
	Light			
	$u_3(0)$, $u_i(-1)$	$U(-2), \bar{U}(+2)$,		$U_i(-1), \bar{U}^i(+1)$
Matter	$q_3(0)$, $q_I(+1)$	$Q(+2), \bar{Q}(-2)$,	$Q'(0), \bar{Q}'(0)$,	$Q_I(+1), \bar{Q}^I(-1)$
	$d_3(0)$, $d_i(+1)$		$D(0), \bar{D}(0)$,	$D_i(+1), \bar{D}^i(-1)$
Higgs	$h_u(0)$, $h_d(0)$	$\phi_{lI}(+1), \phi_{ri}(-1)$,	$\phi'_{lI}(-1), \phi'_{ri}(+1)$,	$S(0)$

Table 4.3: Field content and G_f transformation properties of the quark sector. I and i are $SU(2)_l$ and $SU(2)_r$ doublet indices and the numbers in brackets are $U(1)_A$ charges.

S are assumed to take the most general form:[6]

$$
\begin{aligned}
\langle \phi_{lI} \rangle &= \begin{pmatrix} v_{l0} \\ 0 \end{pmatrix}, \ \langle \phi_{ri} \rangle = \begin{pmatrix} v_{r0} \\ 0 \end{pmatrix}, \\
\langle \phi'_{lI} \rangle &= \begin{pmatrix} v_{l1} \\ v_{l2} \end{pmatrix}, \ \langle \phi'_{ri} \rangle = \begin{pmatrix} v_{r1} \\ v_{r2} \end{pmatrix}, \ \langle S \rangle = v_s. \qquad (4.5.12)
\end{aligned}
$$

Because we are dealing with a full theory, we restrict ourselves to renormalizable interactions only and all possible renormalizable interactions consistent with the symmetries are included. Nonrenormalizable interactions are assumed to be absent or suppressed enough so that they can be ignored. The G_f transformation properties of the up sector are identical to those of the lepton model so the analysis is exactly

[6] ϕ_{lI}, ϕ_{ri} can be put in this form by $SU(2)_l$ and $SU(2)_r$ rotations, then ϕ'_{lI}, ϕ'_{ri} VEV's will take the general directions if there are no alignments between ϕ'_{lI}, ϕ'_{ri} and ϕ_{lI}, ϕ_{ri}. Here we will not specify the origin of these VEV's.

the same as in the lepton model. The superpotential for the up sector is

$$\begin{aligned} W_u = &\lambda_{u3} q_3 h_u u_3 + \lambda_{u4} Q h_u U \\ &+ f_{q1} q_3 \bar{Q}^I \phi_{lI} + f_{q2} q_I \bar{Q}^I S + f_{q3} \epsilon^{IJ} q_I \bar{Q} \phi_{lJ} \\ &+ f_{u1} u_3 \bar{U}^i \phi_{ri} + f_{u2} u_i \bar{U}^i S + f_{u3} \epsilon^{ij} u_i \bar{U} \phi_{rj} \\ &+ M_U \bar{U} U + M_{U_i} \bar{U}^i U_i + M_Q \bar{Q} Q + M_{Q_I} \bar{Q}^I Q_I. \end{aligned} \tag{4.5.13}$$

Note that although we introduce another pair of G_f breaking fields ϕ'_{lI} and ϕ'_{ri}, they do not have renormalizable interactions with the up sector and the lepton sector. The only such G_f invariant interactions

$$L\bar{L}^I \phi'_{lI},\ E\bar{E}^i \phi'_{ri},\ Q\bar{Q}^I \phi'_{lI},\ U\bar{U}^i \phi'_{ri} \tag{4.5.14}$$

are forbidden by heavy-parity. Therefore, we do not generate muon number violating operators even though G_f is completely broken.

The superpotential of the down sector is given by

$$\begin{aligned} W_d = &\lambda_{d3} q_3 h_d d_3 + \lambda_{d4} Q' h_d D \\ &+ f'_{q3} \epsilon^{IJ} q_I \bar{Q}' \phi'_{lJ} + f'_{q4} q_3 \bar{Q}' S \\ &+ f_{d1} d_3 \bar{D}^i \phi'_{ri} + f_{d2} d_i \bar{D}^i S + f_{d3} \epsilon^{ij} d_i \bar{D} \phi_{rj} + f_{d4} d_3 \bar{D} S \\ &+ M_D \bar{D} D + M_{D_i} \bar{D}^i D_i + M_{Q'} \bar{Q}' Q'. \end{aligned} \tag{4.5.15}$$

The f_{d1} and f_{d2} couplings are responsible for the D term mixing between d_3 and d_i, $i = 1, 2$ (with intermediate $\bar{D}^i$). f_{d3}, f_{d4} mix d_2, d_3 with D, f'_{q3}, f'_{q4} mix q_1, q_2, q_3

with Q' and they are responsible for generating tree level Yukawa couplings among d_2, d_3, and q_1, q_2, q_3 with h_d. After integrating out the heavy states, we obtain the following tree level Yukawa matrices for the up quarks and down quarks:

$$\boldsymbol{\lambda}_U = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon_{q2}\epsilon_{u2}\lambda_{u4} & 0 \\ 0 & 0 & \lambda_{u3} \end{pmatrix}, \boldsymbol{\lambda}_D = \begin{pmatrix} 0 & \epsilon'_{q1}\epsilon_{d2}\lambda_{d4} & \epsilon'_{q1}\epsilon_{d3}\lambda_{d4} \\ 0 & \epsilon'_{q2}\epsilon_{d2}\lambda_{d4} & \epsilon'_{q2}\epsilon_{d3}\lambda_{d4} \\ 0 & \epsilon'_{q3}\epsilon_{d2}\lambda_{d4} & \epsilon'_{q3}\epsilon_{d3}\lambda_{d4} + \lambda_{d3} \end{pmatrix}, \quad (4.5.16)$$

where,

$$\begin{aligned} \epsilon_{q2} &= \frac{f_{q3}v_{l0}}{M_Q}, \ \epsilon_{u2} = \frac{f_{u3}v_{r0}}{M_U}, \\ \epsilon'_{q1} &= -\frac{f'_{q3}v_{l2}}{M_{Q'}}, \ \epsilon'_{q2} = \frac{f'_{q3}v_{l1}}{M_{Q'}}, \ \epsilon'_{q3} = \frac{f'_{q4}v_s}{M_{Q'}}, \\ \epsilon_{d2} &= \frac{f_{d3}v_{r0}}{M_D}, \ \epsilon_{d3} = \frac{f_{d4}v_s}{M_D}. \end{aligned}$$

Both matrices are of rank 2, as suggested by the theorem of Sec. 4.4, (although this cannot be seen from the effective theory point of view). Now we have a massless state in each of the up and down sectors and all mixing angles are generated at tree level. m_u and m_d are then generated radiatively by the mixings between the first and the third generations induced by f_{q1}, f_{q2}, f_{u1}, f_{u2}, and f_{d1}, f_{d2} with intermediate $\bar{Q}'$, $\bar{U}$, and $\bar{D}$ states. f_{d3}, f_{d4}, f'_{q3}, f'_{q4} also induce the D term mixings among generations with intermediate $\bar{D}$ and $\bar{Q}'$ states. For example, the mixing between q_3 and q_2 is $\sim \epsilon'_{q3}\epsilon'_{q2}$, which is about the same size as the corresponding KM mixing angle. For large $\tan\beta$ they can give sizable corrections $[\mathcal{O}(50\%)]$ to the KM matrix elements. Since we do not know the exact size and the sign of these corrections, if we just take m_s, $\sin\theta_c$ and V_{cb} to be approximately equal to the tree level results, then we have [within $\mathcal{O}(50\%)$ accuracy]

$$\begin{aligned}
V_{cb} &\simeq \epsilon'_{q2}\epsilon_{d3}\frac{\lambda_{d4}}{\lambda_{d3}} \simeq 4\times 10^{-2}, \\
\frac{m_s}{m_b} &\simeq \epsilon'_{q2}\epsilon_{d2}\frac{\lambda_{d4}}{\lambda_{d3}} \simeq 2.7\times 10^{-2}, \\
\sin\theta_c &\simeq \frac{\epsilon'_{q1}\epsilon_{d2}}{\epsilon'_{q2}\epsilon_{d2}} \simeq 0.22
\end{aligned} \tag{4.5.17}$$

Combining the above relations, we obtain the approximate tree level V_{ub}

$$V_{ub}^{\rm tree} \simeq \epsilon'_{q1}\epsilon_{d3}\frac{\lambda_{d4}}{\lambda_{d3}} \simeq \sin\theta_c V_{cb} \simeq 9\times 10^{-3}, \tag{4.5.18}$$

which is about a factor of 2 bigger than the central value. However, as we found in Sec. 4.3, when we generate m_d by radiative corrections, we also generate $V_{ub}^{\rm rad}$ bigger than the central value by about a factor of 3, which has to be cancelled by the tree level $V_{ub}^{\rm tree}$. If the sign is right, (4.5.18) is just in the range which can cancel against the radiative contribution to produce the correct V_{ub}. Therefore, realistic values for all quark masses and KM mixing angles can be obtained. Naively, one might expect that it is difficult to have massless first generation quarks at tree level because of the Cabibbo angle. Here we showed, with the help of the theorem of Sec. 4.4 for the rank of the Yukawa matrices, that one can naturally get massless up and down quarks at tree level, while having nonzero $\sin\theta_c$.

4.5.3 A model of radiative m_u, m_e, and tree level m_d

As we have mentioned, a radiative m_d is only barely consistent with $B-\bar{B}$ mixing with very heavy SUSY-breaking masses. In this subsection, we present a model in which m_d is nonzero at tree level, while m_u and m_e arise purely from radiative effects. The flavor group is $G_f = SU(2)_T \times SU(2)_F \times Z_4$. The reason for the subscripts of the $SU(2)$ groups will be clear later. $U(1)_A$ is replace by its subgroup Z_4. Matter-parity and the heavy-parity are imposed as well. The field content is shown in Table 4.4, where I, i are $SU(2)_F$ and $SU(2)_T$ indices respectively, and the numbers in brackets are the Z_4 charges with n and (n mod 4) identified. ϕ_{Ti}, ϕ_{FI}, S and X have nonzero VEV's:

$$\langle \phi_{Ti} \rangle = \begin{pmatrix} v_T \\ 0 \end{pmatrix}, \ \langle \phi_{FI} \rangle = \begin{pmatrix} v_F \\ 0 \end{pmatrix}, \ \langle S \rangle = v_s, \ \langle X \rangle = v_x, \tag{4.5.19}$$

which break G_f completely. In this model there is only one pair of $SU(2)_{T,F}$ breaking

	Light	Heavy		
	$e_3(0), e_i(-1)$	$E(-2), \bar{E}(+2)$	$E_i(-1), \bar{E}^i(+1)$	
	$\ell_3(0), \ell_I(+1)$	$L(+2), \bar{L}(-2)$	$L_I(+1), \bar{L}^I(-1)$	
Matter	$u_3(0), u_I(+1)$	$U(+2), \bar{U}(-2)$,	$U_I(+1), \bar{U}^I(-1)$	
	$q_3(0), q_i(-1)$	$Q(-2), \bar{Q}(+2)$,	$Q_i(-1), \bar{Q}^i(+1)$,	$Q'_i(+1), \bar{Q}'^i(-1)$
	$d_3(0), d_i(-1)$	$D(-2), \bar{D}(+2)$,	$D_i(-1), \bar{D}^i(+1)$,	$D'_i(+1), \bar{D}'^i(-1)$
Higgs	$h_u(0), h_d(0)$	$\phi_{Ti}(-1), \phi_{FI}(+1)$,	$S(0)$, $X(2)$	

Table 4.4: Field content and G_f transformation properties of the model with radiative m_u, m_e, and tree level m_d.

fields ϕ_{Ti}, ϕ_{FI}. The tree level massless electron and up quark can be easily seen in an effective theory point of view[59], because the only $SU(2)_{T,F}$ invariant holomorphic combinations of the two light generations and fields with nonzero VEV's for the lepton and the up quark sectors are $\epsilon^{ij} e_i \phi_{Tj}$, $\epsilon^{IJ} \ell_I \phi_{FJ}$, $\epsilon^{IJ} u_I \phi_{FJ}$, and $\epsilon^{ij} q_i \phi_{Tj}$, which cannot give Yukawa couplings to both light generations with h_u and h_d. In the down sector, q's and d's have the same G_f transformation properties. One can write down the effective operator

$$\epsilon^{ij} q_i h_d d_j X S, \tag{4.5.20}$$

which generates the 12 and 21 entries of the down Yukawa matrix with equal size and opposite signs. Hence we can obtain both θ_c and m_d at tree level with the experimentally successful relation $\sin\theta_c \simeq \sqrt{m_d/m_s}$.

Compared with the lepton model discussed earlier in this section, the extra X field is required to break the left over "second generation parity" in order to generate V_{cb} and V_{us} but it may also induce a too big $\mu \to e\gamma$ rate, which will be discussed later. The Q'_i, $\bar{Q}'^i$, D'_i, $\bar{D}'^i$ are responsible for generating the operator (4.5.20). They can be omitted if nonrenormalizable operators are allowed and are sufficiently large. In fact, because this model can be analyzed in the effective theory point of view, including nonrenormalizable interactions will not affect our results. However, for simplicity and completeness, we will analyze the full theory and restrict ourselves to renormalizable interactions.

The lepton sector and the up quark sector are similar to the previous models. We will not repeat the detailed analysis. The only difference is that with the additional X field, we can have the following extra interactions:

$$f_{e5}Xe_3\bar{E},\ f_{\ell 5}X\ell_3\bar{L},\ f_{u5}Xu_3\bar{U},\ f_{q5}Xq_3\bar{Q}. \tag{4.5.21}$$

They mix the third generation with the heavy $SU(2)_{T(F)}$ singlet generation. In combination with $\epsilon^{ij}\phi_{Ti}e_j\bar{E}$, $\epsilon^{IJ}\phi_{FI}\ell_J\bar{L}$, $\epsilon^{IJ}\phi_{FI}u_J\bar{U}$, and $\epsilon^{ij}\phi_{Ti}q_j\bar{Q}$, they generate the 23 and 32 entries of the Yukawa matrices and also the D term mixing between the second and the third generations. For the up quark sector, the $D-\bar{D}$ mixing constraints are very weak and hence easily satisfied. However, for the lepton sector the constraint from the $\mu \to e\gamma$ rate requires the 2-3 mixing to be no bigger than $\mathcal{O}(10^{-3})$, while the naive expectation of 2-3 mixing in this model is of the order V_{cb}. Therefore, one has to assume that the couplings of the X field to the lepton sector are small, or prevented by some extra symmetries. We will see that this is possible to achieve later.

In the down quark sector, in addition to the usual interactions,

$$\begin{aligned} W_d = {} & \lambda_{d3}q_3h_dd_3 + \lambda_{d4}Qh_dD \\ & + f_{q1}q_3\bar{Q}^i\phi_{Ti} + f_{q2}q_i\bar{Q}^iS + f_{q3}\epsilon^{ij}q_i\bar{Q}\phi_{Tj} \\ & + f_{d1}d_3\bar{D}^i\phi_{Ti} + f_{d2}d_i\bar{D}^iS + f_{d3}\epsilon^{ij}d_i\bar{D}\phi_{Tj} \\ & + M_D\bar{D}D + M_{D_i}\bar{D}^iD_i + M_Q\bar{Q}Q + M_{Q_i}\bar{Q}^iQ_i, \end{aligned} \tag{4.5.22}$$

which give the tree level b and s quark masses and 1-3 D term mixing, we have the following interactions as well,

$$W'_d = f_{q5}q_3\bar{Q}X + f_{d5}d_3\bar{D}X$$

$$+ f_{q6} q_i \bar{Q}'^i X + f_{d6} d_i \bar{D}'^i X$$

$$+ \lambda_{d5} \epsilon^{ij} Q'_i h_d D_j + \lambda_{d6} \epsilon^{ij} Q_i h_d D'_j$$

$$+ M_{D'_i} \bar{D}'^i D'_i + M_{Q'_i} \bar{Q}'^i Q'_i. \tag{4.5.23}$$

As we have discussed before, the f_{q5}, f_{d5} couplings induce the 23 and 32 entries of the Yukawa matrix and the 2-3 D term mixing, so that V_{cb} can be generated. f_{q6}, f_{d6}, λ_{d5}, λ_{d6} together with f_{q2}, f_{d2} couplings generate the operator (4.5.20), which gives θ_c and m_d, and the successful relation $\sin\theta_c = \sqrt{m_d/m_s}$. The tree level down quark mass matrix takes the following form,

$$\begin{pmatrix} 0 & C & 0 \\ -C & E & B \\ 0 & B' & A \end{pmatrix}, \tag{4.5.24}$$

while the tree level up quark and lepton mass matrices have nonzero entries in the lower 2×2 block. In addition to m_u and m_e, V_{ub} is also generated by radiative corrections from the 3-1 mixing $W_{D_L 31}$. The required size of $W_{D_L 31}$ is much smaller than that required for generating m_d radiatively, so the phenomenological constraints are easier to satisfy as we have discussed in Sec. 4.3.

Looking at the G_f transformation properties of the fields, one can see that this model can be embedded into the flipped $SU(5)$ grand unified theory[64]: q and d (and the not discussed right-handed neutrino n) belong to the **10** representation of flipped $SU(5)$, u and ℓ belong to the $\bar{\mathbf{5}}$ and e is a singlet **1** under flipped $SU(5)$. $SU(2)_T$ is a flavor group for the **10**'s and $SU(2)_F$ is a flavor group for the $\bar{\mathbf{5}}$'s. In

Table 4.4, the e's are assigned to transform under $SU(2)_T$. Here one can either have them transform under a different $SU(2)_S$, or simply identify $SU(2)_S$ with $SU(2)_T$.

One nice feature of embedding this model into flipped $SU(5)$ is that the X field can be assigned to the **75** of $SU(5)$. Because only the $\mathbf{10} \times \overline{\mathbf{10}}$ contains **75** and the $\mathbf{5} \times \bar{\mathbf{5}}$, $\mathbf{1} \times \mathbf{1}$ do not, the X field can only couple to q and d but not the lepton sector. Then the μ-τ mixing and hence the troublesome $\mu \rightarrow e\gamma$ decay rate can be removed.

After flipped $SU(5)$ is broken, we do not expect the couplings and the mixings to be the same for fields belonging to the same representations of the flipped $SU(5)$.[7] But if we assume that they are of the same order, the radiative m_e, m_u and V_{ub} are also consistent: radiative V_{ub} does not need a big $W_{D_{L31}}$ ($\sim 10^{-2}$), then $W_{U_{R31}}$ has to be quite big ($\gtrsim 10^{-1}$) for generating m_u; but so is its flipped $SU(5)$ partner $W_{E_{L31}}$ for generating m_e. On the other hand, λ_U, λ_D, and λ_E are independent in flipped $SU(5)$ models. They can take suitable values so that all the tree level quantities come out correctly.

4.6 Conclusions

In this chapter, we have considered the possibility of generating some of the light fermion masses through radiative corrections. Any theory of radiative fermion

[7]If flipped $SU(5)$ were not broken, the tree level 12 and 21 entries of the down quark mass matrix would not be generated, because $\epsilon^{ij}\mathbf{10}_i\mathbf{10}_j h_d X S$ vanishes. However, since the flipped $SU(5)$ is broken, q's and d's can have different mixings so that $\epsilon^{ij} q_i d_j h_d X S$ can be nonzero.

masses must have an accidental symmetry for the Yukawa sector guaranteeing the absence of tree level masses, while this symmetry must be broken elsewhere in the theory for any mass to be generated radiatively. In our discussion, supersymmetry has been crucial in naturally implementing this scenario: supersymmetric theories automatically have two sectors (the superpotential and D terms) which need not have the same symmetries; because of holomorphy the superpotential may have accidental symmetries not shared by the D terms. Furthermore, the particles in the radiative loop generating the fermion masses are just the superpartners of known particles, and must be near the weak scale if supersymmetry is to solve the hierarchy problem. Thus, supersymmetric theories of radiative fermion masses can lead to testable predictions. Working with supersymmetric theories with minimal low energy field content, we found (with the plausible assumption that the accidental flavor symmetries of the tree level Yukawa matrix are only broken by soft scalar masses) that FCNC constraints allow only the first generation fermion masses to have a radiative origin.

In the lepton sector, a rather large mixing between the selectron and stau is needed in order to generate the electron mass. This implies that mixing with the smuon must be highly suppressed in order to avoid too large a rate for $\mu \to e\gamma$. The large selectron-stau mixing also gives rise to a significant rate for $\tau \to e\gamma$ which is only a factor 10-100 lower than the current experimental limit.

In the quark sector, in addition to the quark masses, the KM mixing matrix must also be obtained. The FCNC constraints strongly limit the possibilities of generating light quark masses and mixing angles. We found that m_u and V_{ub} can be generated by radiative corrections, while radiatively generating any of m_d, θ_c, and V_{cb} requires heavy scalar masses ($\sim$ 1TeV). Further, it is very difficult to generate m_d, θ_c, and V_{ub} together radiatively unless the scalar masses are between 2 and 20 TeV, which we view as unacceptably high. These constraints cause the principle difficulties in constructing a model of quark flavor with radiative masses.

We introduced a lepton model with flavor group $SU(2)_\ell \times SU(2)_e \times U(1)_A$ and then extended it to the quark sector. The lepton model has a number of nice features: the $SU(2)$ breaking ϕ VEV's are responsible for both D term mixing between the first and the third generation and generation of the second generation mass, so the ratio between the radiatively generated first generation mass and the second generation mass is naturally of the order $1/(16\pi^2)$. Further, muon number is conserved so that the dangerous rate for $\mu \to e\gamma$ is avoided. A direct extension of this model to the quark sector cannot generate the correct KM mixings, which requires the addition of more fields and flavor symmetry breakings to the theory.

We presented two complete models with radiative fermion masses. In the first model, all first generation fermion masses come from radiative corrections, and there are also tree level contributions to θ_c and V_{ub} as required by the FCNC constraints. First generation fermions are guaranteed to be massless at tree level by requiring the "big" Yukawa matrices of the full theory to be rank 2. Requiring a tree level θ_c and V_{ub} forces us to add another heavy left-handed quark Q' and its conjugate $\bar{Q}'$, and another pair of $SU(2)_{l,r}$ flavor symmetry breaking fields $\phi'_{l,r}$. Muon number is still conserved as a consequence of the field content and charge assignments of the theory. With these minimal extensions, we obtain a complete theory of radiative first generation fermion masses with successful values for KM mixing angles.

In view of the fact that a radiative m_d and $B-\bar{B}$ mixing are only compatible for very heavy scalar masses, we also constructed a second model in which m_u and m_e come from radiative corrections but m_d and θ_c arise at tree level with the successful relation $\sin\theta_c = \sqrt{m_d/m_s}$. The dangerous $\mu \to e\gamma$ rate can be naturally suppressed if we embed this model into the flipped $SU(5)$ grand unified theory.

Appendix A

In this appendix, we first give a more complete treatment of mixing matrix scaling in the lepton sector, and then give a treatment for the quark sector.

Let us return to (3.5.7) and consider the effect of including the $(\zeta_E^\dagger \zeta_E)_{3i}$ term. In general the scaling from M_{PL} to M_G will generate a $\zeta_E^\dagger \zeta_E$ not diagonal in the same basis as $\lambda_E^\dagger \lambda_E$, so we expect some non-zero $(\zeta_E^\dagger \zeta_E)_{3i}$. From the RGE for ζ_E, neglecting gauge couplings,

$$\frac{d}{dt}\zeta_E = \zeta_E[5\lambda_E^\dagger \lambda_E + \mathrm{Tr}(3\lambda_D^\dagger \lambda_D + \lambda_E^\dagger \lambda_E)] + \lambda_E[4\lambda_E^\dagger \zeta_E + \mathrm{Tr}(6\zeta_D \lambda_D^\dagger + 2\zeta_E \lambda_E^\dagger)]. \quad (A.1)$$

We have

$$\begin{aligned}\frac{d}{dt}(\zeta_E^\dagger \zeta_E) &= 5[\zeta_E^\dagger \zeta_E \lambda_E^\dagger \lambda_E + \lambda_E^\dagger \lambda_E \zeta_E^\dagger \zeta_E] + 2\,\mathrm{Tr}(3\lambda_D^\dagger \lambda_D + \lambda_E^\dagger \lambda_E)\zeta_E^\dagger \zeta_E \\ &\quad + 8\zeta_E^\dagger \lambda_E \lambda_E^\dagger \zeta_E + (\zeta_E^\dagger \lambda_E + \lambda_E^\dagger \zeta_E)\,\mathrm{Tr}(6\zeta_D \lambda_D^\dagger + 2\zeta_E \lambda_E^\dagger). \qquad (A.2)\end{aligned}$$

Then, to first order in the off diagonal parts of $\zeta_E^\dagger \zeta_E$ and $\zeta_E \zeta_E^\dagger$, and keeping only third generation Yukawa couplings we have

$$\frac{d}{dt}(\zeta_E^\dagger \zeta_E)_{3i} = (\zeta_E^\dagger \zeta_E)_{3i}[17\lambda_\tau^2 + 6\lambda_b^2 + 6\eta\lambda_b\lambda_\tau], \qquad (A.3)$$

where $\eta \equiv \frac{\zeta_{D33}}{\zeta_{E33}}$. Because of the large numerical coefficient in front of $\lambda_\tau^2, \lambda_b^2$ in the above equation, $(\zeta_E^\dagger \zeta_E)_{3i}$ is driven to zero more rapidly than W_{L3i}, after which it ceases to have any effect on the running of W_{L3i}. More explicitly, from (3.5.7) we have that

$$\frac{d}{dt}(m_{L3i}^2(t) e^{\int_t^{t_G} dt' \lambda_\tau^2(t')}) = 2(\zeta_E^\dagger \zeta_E)_{3i}(t) e^{\int_t^{t_G} dt' \lambda_\tau^2(t')}. \qquad (A.4)$$

Solving (A.3) for $(\zeta_E^\dagger \zeta_E)_{3i}(t)$ and inserting into (A.4) we get

$$\frac{d}{dt}\left(m_{L3i}^2(t) e^{\int_t^{t_G} dt' \lambda_\tau^2(t')}\right) = 2(\zeta_E^\dagger \zeta_E)_{3i}(M_G) e^{-\int_t^{t_G} dt' [16\lambda_\tau^2 + 6\lambda_b^2 + 6\eta\lambda_b\lambda_t](t')}. \qquad (A.5)$$

Integrating (A.5), we find

$$\begin{aligned} -m_{L3i}^2(M_S) e^{I_\tau} + m_{L3i}^2(M_G) &= 2\int_0^{t_G} dt e^{-\int_t^{t_G} dt' [16\lambda_\tau^2 + 6\lambda_b^2 + 6\eta\lambda_b\lambda_\tau](t')} \\ &\quad \times (\zeta_E^\dagger \zeta_E)_{3i}(M_G) \\ &\equiv \delta(\zeta_E^\dagger \zeta_E)_{3i}(M_G). \end{aligned} \qquad (A.6)$$

So, we have

$$m_{L3i}^2(M_S) = e^{-I_\tau}[m_{L3i}^2(M_G) - \delta(\zeta_E^\dagger \zeta_E)(M_G)]. \qquad (A.7)$$

We expect m_{L3i}^2 and $(\zeta_E^\dagger \zeta_E)_{3i}$ to be related by some combination of Clebsches x at M_G as follows:

$$(\zeta_E^\dagger \zeta_E)_{3i} = \frac{A_0^2}{m_0^2} x m_{L3i}^2 \qquad (A.8)$$

Where A_0, m_0^2 are the universal A parameter and scalar mass at M_{PL}, respectively. Then, we have from (A.7)

$$W_{L33}^\dagger W_{L3i}(M_S) = e^{-I_\tau} \frac{\Delta m^2(M_G)}{\Delta m^2(M_S)}[1 + \delta \frac{A_0^2}{m_0^2} x] W_{L33}^\dagger W_{L3i}(M_G). \qquad (A.9)$$

Clearly if $\delta\frac{A_0^2}{m_0^2}x \ll 1$, inclusion of the $(\zeta_E^\dagger\zeta_E)_{3i}$ term in (3.5.7) does not change any of our results. If $\delta\frac{A_0^2}{m_0^2}x \sim 1$ or $\gg 1$, we can still of course use (A.9), but the suppression effect may disappear. A simple estimate shows, however, that δ itself is already small $\sim \frac{1}{10}$, and so we are only in trouble if $\frac{A_0^2}{m_0^2}x$ is big. To see this, replace λ_τ, λ_b and η by some average values $\bar{\lambda}_\tau, \bar{\lambda}_b$ and $\bar{\eta}$ in the expression (A.6) for δ. Then,

$$\begin{aligned} \delta &= 2\int_0^{\frac{1}{16\pi^2}\ln\frac{M_G}{M_S}} e^{-t(16\bar{\lambda}_\tau^2+6\bar{\lambda}_b^2+6\bar{\lambda}_b\bar{\lambda}_t\bar{\eta})} \\ &= \frac{1}{8\bar{\lambda}_t^2+3(\bar{\lambda}_b^2+\bar{\eta}\bar{\lambda}_b\bar{\lambda}_\tau)}\left[e^{-\frac{1}{16\pi^2}\ln\frac{M_G}{M_S}(16\bar{\lambda}_\tau^2+6\bar{\lambda}_b^2+6\bar{\eta}\bar{\lambda}_b\bar{\lambda}_\tau)}\right]. \end{aligned} \tag{A.10}$$

So,

$$|\delta| < \frac{1}{8\bar{\lambda}_\tau^2+3(\bar{\lambda}_b^2+\bar{\eta}\bar{\lambda}_b\bar{\lambda}_\tau).} \tag{A.11}$$

For the $\bar{\lambda}$'s between 0.5 and 1, and $\bar{\eta} \sim 1, |\delta|$ ranges from $\frac{1}{3}$ to $\frac{1}{15}$.

How can we qualitatively understand the above results for the scaling of mixing matrices? The renormalization group equations try to align the soft supersymmetry breaking flavor matrices with whatever combination of flavor matrices responsible for their renormalization. However, because a given coupling can only be renormalized by harder couplings, there is a hierarchy in which flavor matrices affect the running of others. The Yukawa matrices, being dimensionless, can only be affected by other Yukawa matrices. In the lepton sector, this is the reason that the basis in which e.g. $\boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E$ is diagonal does not change. Next, the soft trilinear terms, having mass dimension one, can only be affected by other trilinear terms and Yukawa couplings. Again in the lepton sector this means that e.g. $\zeta_E^\dagger\zeta_E$ tries to align itself with $\boldsymbol{\lambda}_E^\dagger\boldsymbol{\lambda}_E$. Finally, the scalar mass, having dimension two, are affected by everything: $\mathbf{m}_L^2$ tries to align with $\lambda_E^\dagger\lambda_E$, but suffers interference from $\zeta_E^\dagger\zeta_E$, unless $\zeta_E^\dagger\zeta_E$ is diagonal in the same basis as $\lambda_E^\dagger\lambda_E$. Even if $\zeta_E^\dagger\zeta_E$ is not diagonal in the same basis as $\lambda_E^\dagger\lambda_E$, it is trying to align itself with $\lambda_E^\dagger\lambda_E$, so $\mathbf{m}_L^2$ will still tend to align with $\lambda_E^\dagger\lambda_E$.

From the above discussion, it is clear that the situation is slightly complicated in the quark sector. In the lepton sector, there was a fixed direction in flavor space given by λ_E, with which the soft matrices aligned. In the quark sector, we have both λ_U and λ_D, and $\lambda_U\lambda_U^\dagger$, $\lambda_D\lambda_D^\dagger$ are misaligned ($V_{KM} \neq 1$). This complicates the analysis for W_{U_L}, W_{D_L} so we discuss them last. Let us now examine the scaling of W_{U_R}, W_{D_R}. (Throughout the following, we assume degeneracy between first two generation scalar masses, we neglect all Yukawa coupling matrix eigenvalues except those of the third generation, and we do not include the effect of trilinear soft terms in the scaling. The last assumption is made for simplicity; we can make similar arguments about the importance of these neglected trilinear terms as we did above in the lepton sector.)

First, we show that the basis in which $\lambda_U^\dagger\lambda_U$ is diagonal remains fixed. The RGE for $\lambda_U^\dagger\lambda_U$ is

$$\frac{d}{dt}\lambda_U^\dagger\lambda_U = 6(\lambda_U^\dagger\lambda_U)^2+2\lambda_U^\dagger\lambda_D\lambda_D^\dagger\lambda_U+2(3\,\mathrm{Tr}\,\lambda_U\lambda_U^\dagger-\frac{16}{3}g_3^2-3g_2^2-\frac{13}{15}g_1^2)\lambda_U^\dagger\lambda_U. \quad (A.12)$$

Working in a basis where $\lambda_U^\dagger\lambda_U$ is diagonal, let us see if $\frac{d}{dt}\lambda_U^\dagger\lambda_U$ has off-diagonal components. We have, (recalling that in this basis $\lambda_D^\dagger\lambda_D = V_{KM}\bar{\lambda}_D^2 V_{KM}^\dagger$),

$$\frac{d}{dt}(\lambda_U^\dagger\lambda_U)_{\substack{ij\\ i\neq j}} = 2(\bar{\lambda}_U V_{KM}\bar{\lambda}_D^2 V_{KM}^\dagger\bar{\lambda}_U)_{ij}$$

$$= 2\bar{\lambda}_{U_i} V_{KM_{i\ell}} \bar{\lambda}^2_{D_\ell} V^\dagger_{KM_{\ell j}} \bar{\lambda}_{U_j}$$

$$= 0 \text{ for } i, j \neq 3 \qquad (A.13)$$

since we neglect all Yukawa's except the third generation. Similarly, the basis in which $\boldsymbol{\lambda}_D^\dagger \boldsymbol{\lambda}_D$ is diagonal does not change. Thus, the discussion for the scaling of W_{U_R}, W_{D_R} is completely analogous to that in the lepton sector, and we find

$$W_{U_{R3i}} W^\dagger_{U_{R33}}(M_S) = e^{-2I_t} \frac{\Delta m^2_U(M_G)}{\Delta m^2_U(M_S)} W_{U_{R3i}} W^\dagger_{U_{R33}}(M_G), \qquad (A.14)$$

$$W_{D_{R3i}} W^\dagger_{D_{R33}}(M_S) = e^{-2I_b} \frac{\Delta m^2_D(M_G)}{\Delta m^2_D(M_S)} W_{D_{R3i}} W^\dagger_{D_{R3}}(M_G). \qquad (A.15)$$

We now turn to W_{U_L}, W_{E_L}. Let $V^*_{U_L}(t)$ be the matrix diagonalizing $\boldsymbol{\lambda}_U \boldsymbol{\lambda}_U^\dagger(t)$:

$$\boldsymbol{\lambda}_U \boldsymbol{\lambda}_U^\dagger(t) = V^*_{U_L}(t) \bar{\lambda}^2_U(t) V^{*\dagger}_{U_L}(t). \qquad (A.16)$$

In the superfield basis in which $\boldsymbol{\lambda}_U \boldsymbol{\lambda}_U^\dagger$ is diagonal, the squark mass matrix is $\widetilde{\mathbf{m}}^{2*}_{Q3i} = V^\dagger_{U_L} \mathbf{m}^{2*}_Q V_{U_L}$. Note as before that $\widetilde{\mathbf{m}}^{2*}_{Q3i} = (W^\dagger_{U_L} \overline{\mathbf{m}}^{2*}_Q W_{U_L})_{3i} = W_{U_{L3i}} W^\dagger_{U_{L33}} \Delta m^2_Q$, so we are interested in $\frac{d}{dt}\widetilde{\mathbf{m}}^{2*}_{Q3i}$. Now,

$$\frac{d}{dt}\widetilde{\mathbf{m}}^{2*}_Q = \frac{d}{dt}(V^\dagger_{U_L} \mathbf{m}^{2*}_Q V_{U_L}) = \left(\frac{d}{dt} V^\dagger_{U_L}\right) \mathbf{m}^{2*}_Q V_{U_L} + V^\dagger_{U_L} \frac{d}{dt}\mathbf{m}^{2*}_Q V_{U_L} + V^\dagger_{U_L} \mathbf{m}^{2*}_Q \frac{d}{dt} V_{U_L}$$

$$= \left[\widetilde{\mathbf{m}}^{2*}_Q, V^\dagger_{U_L} \frac{d}{dt} V_{U_L}\right] + V^\dagger_{U_L} \frac{d}{dt}\mathbf{m}^{2*}_Q V_{U_L}. \qquad (A.17)$$

The second term is the analogue of what we have already seen in the lepton and right-handed quark sector; using the RGE for $\mathbf{m}^{2*}_Q$ we find to leading order

$$\left(V^\dagger_{U_L} \frac{d}{dt}\mathbf{m}^{2*}_Q V_{U_L}\right)_{3i} = (\lambda_t^2 + \lambda_b^2)\widetilde{\mathbf{m}}^2_{Q3i}. \qquad (A.18)$$

Now, $V^\dagger_{U_L} \frac{d}{dt} V_{U_L}$ is obtained from the RGE for $\boldsymbol{\lambda}_U \boldsymbol{\lambda}_U^\dagger$. Actually, note that

$$V^\dagger_{U_L} \left(\frac{d}{dt}\boldsymbol{\lambda}_U \boldsymbol{\lambda}_U^\dagger\right) V_{U_L} = \left[V^\dagger_{U_L} \frac{d}{dt} V_{U_L}, \bar{\lambda}^2_U\right] + \frac{d}{dt}\bar{\lambda}^2_U, \qquad (A.19)$$

so that only $[V_{U_L}^\dagger \frac{d}{dt} V_{U_L}, \bar{\boldsymbol{\lambda}}_U^2]$ is determined. (This is a reflection of the fact that V_{U_L} is not unique: let $X(t)$ be any unitary transformation leaving $\overline{\mathbf{m}}_Q^2(t)$ invariant: $\overline{\mathbf{m}}_Q^2(t) = X^\dagger(t)\overline{\mathbf{m}}_Q^2(t)X(t)$. In our case, $X(t)$ is most generally a $U(2)$ matrix in the first two generation subspace. Then, if V_{U_L} diagonalizes $\mathbf{m}_Q^{2*}$, so does $V_{U_L}X$. Under this change, $V_{U_L}^\dagger \frac{d}{dt} V_{U_L}$ is not invariant, but $[V_{U_L}^\dagger \frac{d}{dt} V_{U_L}, \bar{\boldsymbol{\lambda}}_U^2]$ *is* invariant). Further, since we neglect first two generations Yukawa eigenvalues, $[V_{U_L}^\dagger \frac{d}{dt} V_{U_L}, \bar{\boldsymbol{\lambda}}_U^2]_{ij} = 0$ for $i,j = 1,2$, and only $[V_{U_L}^\dagger \frac{d}{dt} V_{U_L}, \bar{\boldsymbol{\lambda}}_U^2]_{3i(i3)} = (\mp)\lambda_t^2 V_{U_L}^\dagger \frac{d}{dt} V_{U_{Li3(3i)}}^\dagger$ is determined, and we can choose all other components of $V_{U_L}^\dagger \frac{d}{dt} V_{U_L}$ to vanish. From the RGE for $\boldsymbol{\lambda}_U \boldsymbol{\lambda}_U^\dagger$,

$$\begin{aligned}\frac{d}{dt}(\boldsymbol{\lambda}_U \boldsymbol{\lambda}_U^\dagger) &= 6(\boldsymbol{\lambda}_U \boldsymbol{\lambda}_U^\dagger)^2 + 2(3\,\mathrm{Tr}\,\boldsymbol{\lambda}_U \boldsymbol{\lambda}_U^\dagger - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2)\boldsymbol{\lambda}_U \boldsymbol{\lambda}_U^\dagger \\ &+ \{\boldsymbol{\lambda}_U \boldsymbol{\lambda}_U^\dagger, \boldsymbol{\lambda}_D \boldsymbol{\lambda}_D^\dagger\}, \qquad (A.20)\end{aligned}$$

we find

$$\begin{aligned}\left(V_{U_L}^\dagger (\frac{d}{dt}\boldsymbol{\lambda}_U \boldsymbol{\lambda}_U^\dagger) V_{U_L}\right)_{3i} &= \{\bar{\boldsymbol{\lambda}}_U^2, V_{KM} \bar{\boldsymbol{\lambda}}_D^2 V_{KM}^\dagger\}_{3i} \\ &= \lambda_t^2 \lambda_b^2 V_{KM33} V_{KM3i}^\dagger, \qquad (A.21)\end{aligned}$$

and thus

$$\left(V_{U_L}^\dagger \frac{d}{dt} V_{U_{L3i}}\right) = \lambda_b^2 V_{KM3i}^\dagger V_{KM33}. \qquad (A.22)$$

Thus to leading order

$$[V_{U_L}^\dagger \mathbf{m}_Q^2 V_{U_L}, V_{U_L}^\dagger \frac{d}{dt} V_{U_L}]_{3i} = \Delta m_Q^2\, \lambda_b^2\, V_{KM3i}^\dagger V_{KM33}, \qquad (A.23)$$

and finally we have

$$\frac{d}{dt}(W_{U_{L3i}} W_{U_{L33}}^\dagger \Delta m_Q^2) = (\lambda_t^2 + \lambda_b^2) W_{U_{L3i}} W_{U_{L33}}^\dagger \Delta m_Q^2 + \lambda_b^2 V_{KM3i}^\dagger V_{KM33} \Delta m_Q^2. \quad (A.24)$$

Similarly we find

$$\frac{d}{dt}(W_{D_{L3i}}W^\dagger_{D_{L33}}\Delta m_Q^2) = (\lambda_t^2+\lambda_b^2)W_{D_{L3i}}W^\dagger_{D_{L33}}\Delta m_Q^2 + \lambda_t^2 V^\dagger_{KM3i}V_{KM33}\Delta m_Q^2. \quad (A.25)$$

We can formally solve the above equations, e.g.

$$W_{U_{L3i}}W^\dagger_{U_{L33}}(M_S) = e^{-\left(I_t+I_b+\int_0^{t_G}dt'\lambda_b^2\frac{V^\dagger_{KM3i}V_{KM33}}{W_{U_{L3i}}W^\dagger_{U_{L33}}}\right)}\frac{\Delta m_Q^2(M_G)}{\Delta m_Q^2(M_S)}W_{U_{L3i}}W^\dagger_{U_{L33}}(M_G), \quad (A.26)$$

and, to a good approximation, given that $W_{U_{L3i}}$ does not scale very significantly, we can replace

$$\int_0^{t_G}dt'\lambda_b^2\frac{V^\dagger_{KM3i}V_{KM33}}{W_{U_{L3i}}W^\dagger_{U_{L33}}} \approx I_b\frac{V^\dagger_{KM3i}V_{KM33}}{W_{U_{L3i}}W^\dagger_{U_{L33}}}(M_G). \quad (A.27)$$

So, an approximate solution of the RGE for W_{U_L}, W_{D_L} is

$$W_{U_{L3i}}W^\dagger_{U_{L33}}(M_S) \approx e^{-\left(I_t+I_b\left(1+\frac{V^\dagger_{KM3i}V_{KM33}}{W_{U_{L3i}}W^\dagger_{U_{L33}}}(M_G)\right)\right)}\frac{\Delta m_Q^2(M_G)}{\Delta m_Q^2(M_S)}W_{D_{L3i}}W^\dagger_{U_{L33}}(M_G), \quad (A.28)$$

and similarly

$$W_{D_{L3i}}W^\dagger_{D_{L33}}(M_S) \approx e^{-\left(I_b+I_t\left(1+\frac{V^\dagger_{KM3i}V_{KM33}}{W_{U_{L3i}}W^\dagger_{U_{L33}}}(M_G)\right)\right)}\frac{\Delta m_Q^2(M_G)}{\Delta m_Q^2(M_S)}W_{D_{L3i}}W^\dagger_{D_{L33}}(M_G). \quad (A.29)$$

The above results are in agreement with qualitative expectations; the extra terms in the exponential of (A.28) and (A.29) are a reflection of the fact that the bases in which $\boldsymbol{\lambda}_U\boldsymbol{\lambda}_U^\dagger$ and $\boldsymbol{\lambda}_D\boldsymbol{\lambda}_D^\dagger$ are diagonal change with scale. For moderate $\tan\beta$, however, we expect that the basis in which $\boldsymbol{\lambda}_U\boldsymbol{\lambda}_U^\dagger$ is diagonal should not change with scale, and in this limit the extra term drops out of (A.28).

Appendix B

In this appendix, we consider the possibility that the soft supersymmetry breaking trilinear A terms do not respect the chiral symmetries of the Yukawa matrix [65, 66]. Before beginning the discussion of radiative fermion masses in this scenario, let us consider the constraints imposed on the form of the A matrix by requiring the desired vacuum to be the global minimum of the potential. (The extent to which this is a neccesity is discussed at the end of this appendix). Consider the lepton sector for simplicity (identical arguments hold for the quark sector). Let us work in a basis where the lepton Yukawa matrix is diagonal and has K zeros. There are D-flat directions in field space where the right and left handed lepton fields and the down type Higgs are nonzero. If we restrict ourselves to the K massless generations, there are no quartic terms in the potential along the D-flat directions; all we have are the cubic A terms and the scalar masses. But, if the A terms are non-zero in the $K \times K$ block of the massless generations, there will be directions in field space where the cubic terms become indefinitely negative and cannot be stabilized by the quadratic mass terms. This can only be avoided if the A terms are zero in the $K \times K$ block of the K massless generations. This constraint is in itself quite powerful. For instance, if $K = 3$, we must have that the A matrix is zero, and the argument that one cannot generate any radiative masses goes through exactly as in section 4.2. Next, let us consider the case $K = 2$. In this case, the A matrix must be zero in the upper 2×2 block. Note that we can make a rotation on the first two generation scalars to make A_{i3}, A_{3i} zero for either $i = 1$ or $i = 2$. Now, the potential is no longer unbounded below, but there is still a local minimum along the D-flat directions for the first two generations where both left and right handed fields aquire VEV's, breaking electric charge. We require that the energy of this minimum is greater than that of the usual minimum, which is $-\frac{1}{4}M_Z^2 v^2$. For scalars much heavier than $(M_Z v)^{\frac{1}{2}} = 150$ GeV, we can approximate this requirement by demanding that the electric charge breaking minimum has energy greater than zero. A straightforward calculation analogous to that in [67] then gives us the following constraint, where we assume that all relevant scalars are degenerate with mass m:

$$\frac{1}{3}(|A_{33}| + |A_{3i}| + |A_{i3}|) \lesssim \lambda_3 m. \tag{B.1}$$

There are corrections to this inequality due to the fact that the true vacuum energy is not zero but $-\frac{1}{4}M_Z^2 v^2$; still assuming $m \gtrsim 150$ GeV the correction takes the form:

$$\frac{1}{3}(|A_{33}| + |A_{3i}| + |A_{i3}|) \lesssim \lambda_3 m(1 + \frac{1}{2\lambda_3}\frac{M_Z v}{m^2}). \tag{B.2}$$

With these constraints in hand, we begin the phenomenological analysis. Suppose that the scalar masses did not break the chiral symmetries of the Yukawa matrix. Then, since one of $A_{31,13}, A_{32,23}$ can be chosen to be zero by rotations, one generation would remain massless to all orders of perturbation theory. Thus, in order to generate both generations radiatively, we must have that both the A terms and the scalar masses break the chiral symmetries of the Yukawa sector. In the following, we consider the possibility that the A terms generate one mass radiatively while the scalar masses generate the other mass. It is easy to see that this is impossible in the lepton sector: the muon mass is too big to be generated radiatively, and even if we could, we would generate too large a rate for $\tau \to \mu\gamma$. Moving on to the quark sector, we have four cases to consider:

(1) m_d from scalar masses and m_s from A terms: In the mass insertion approximation, assuming for simplicity that all scalars are degenerate with mass m, we have in the large tanβ limit

$$\frac{m_s}{m_t} = \frac{\alpha_s}{18\pi}(\frac{\mu M_{\tilde{g}}}{m^2})\frac{(A^d_{23}v_d)}{m^2}\frac{(A^d_{32}v_d)}{m^2}. \qquad (B.3)$$

From equation (B.1), however, we must have that $\frac{(A^d_{23,32}v_d)}{m^2} \lesssim \frac{m_b}{m}$, so

$$\frac{m_s}{m_t} \lesssim 2 \times 10^{-3}(\frac{\mu M_{\tilde{g}}}{m^2})\frac{m_b^2}{m^2} \qquad (B.4)$$

which, even for m=100GeV, gives too small a value for m_s by a factor of $\sim$ 100.

(2) m_d from A terms and m_s from scalar masses: The same argument as in case (1) suggests that the generated mass for m_d will be too small by a factor of $\sim$ 10. Perhaps this factor can be overcome for some choice of parameters. However, the scalars are so light that the required mixing in the scalar mass matrix to generate m_s, together with the A terms responsible for m_d, gives unacceptable contributions

to $K - \bar{K}$ mixing, and, if there are CP-violating phases, even more unacceptable contributions to ϵ.

(3) m_u from scalar masses and m_c from A terms: The general problem with the up sector is that m_c seems to be too heavy to be radiative. In the case we are considering, we find analogously to equation (B.4)

$$\frac{m_c}{m_t} \lesssim 2 \times 10^{-3} (\frac{M_{\tilde{g}}}{m})(\frac{m_t}{m})^2 \qquad (B.5)$$

and so to generate large enough m_c we must again have fairly light squarks.

(4) m_u from A terms and m_c from scalar masses: In this case again it is difficult to get a large enough mass for the charm. In analogy to equation (4.3.10) we have, (in the limit where we decouple the first two generations, minimizing the super-GIM cancellation and so maximizing the generated charm mass)

$$\frac{m_c}{m_t} = \frac{2\alpha_s}{3\pi} \frac{A^u_{33}}{M_{\tilde{g}}} \times W_{U_L 31} W_{U_R 31} W^*_{U_L 33} W^*_{U_R 33} I(\frac{m^2_{\tilde{t}}}{M^2_{\tilde{g}}}). \qquad (B.6)$$

The maximium value of $W_{U_L 31} W_{U_R 31} W^*_{U_L 33} W^*_{U_R 33}$ consistent with the unitarity of the W matrices is $\frac{1}{4}$. Then, we have

$$\frac{m_c}{m_t} \lesssim 5 \times 10^{-3} \frac{A^u_{33}}{M_{\tilde{g}}} I(\frac{m^2_{\tilde{t}}}{M^2_{\tilde{g}}}). \qquad (B.7)$$

Recalling that $I(1) = \frac{1}{2}$, we see that, even with maximal mixing angles, the radiative charm mass is too small or perhaps right on the edge. However, having such large mixing in the left handed up 32 sector also implies large mixing in the left handed down 32 sector, which violates the bounds from $b \to s\gamma$ unless the third generation scalars are pushed above 1 TeV. This then makes it difficult to generate a large enough up mass, since the A term contribution is suppressed by $(\frac{m_t}{m})^2$ from (B.1). We find

$$\frac{m_u}{m_t} \lesssim 2 \times 10^{-5} (\frac{M_{\tilde{g}}}{m})(1 + \frac{\mu \cot \beta}{A_{33}})(\frac{1.7\text{TeV}}{m})^2 \qquad (B.8)$$

which is also on the edge. Another difficulty with having such large 32 mixing is that it disturbs the degeneracy between the scalar masses of the first two generations for both left handed up and down squarks, and this could again give problems with $K - \bar{K}$ mixing and ϵ.

The above arguments certainly do not rule out the possibility of generating both light generations radiatively; there may be regions of parameter space where our rough bounds are evaded. Indeed, it may even be the case that requiring the desired vacuum to have lower energy than the charge breaking minima is not necessary, perhaps the lifetime of the false vacuum can be long enough for the universe to have stayed in it up to the present; this remains to be seen. However, these arguments, together with the fact that for the A terms not to share the same chiral symmetries as the Yukawa matrices we must entangle flavor symmetry breaking and supersymmetry breaking, provide us with sufficient motivation to restrict our detailed treatment to the scenario considered in this chapter.

Appendix C

In [13, 14], the SUSY FCNC constraints are expressed in terms of the ratios of the off-diagonal scalar masses and the "universal squark or slepton masses". For example, the supersymmetric contribution to the $B-\bar{B}$ mixing is given by:[1]

$$\begin{aligned}\Delta M_B^{SUSY} = & \frac{\alpha_s^2}{216M_{\tilde{q}}^2}\frac{2}{3}f_B^2 m_B\{(\delta_{13}^d)_{LL}^2[-66\tilde{f}_6(x)-24xf_6(x)] \\ & +(\delta_{13}^d)_{RR}^2[-66\tilde{f}_6(x)-24xf_6(x)] \\ & +(\delta_{13}^d)_{LL}(\delta_{13}^d)_{RR}[-12\tilde{f}_6(x)-456xf_6(x)] \\ & +(\delta_{13}^d)_{LR}^2[132xf_6(x)]+(\delta_{13}^d)_{RL}^2[132xf_6(x)] \\ & +(\delta_{13}^d)_{LR}(\delta_{13}^d)_{RL}[228\tilde{f}_6(x)\}, \qquad (C.1)\end{aligned}$$

where,

$$\begin{aligned}f_6(x) &= \frac{1}{6(1-x)^5}(-6\ln x-18x\ln x-x^3+9x^2+9x-17), \\ \tilde{f}_6(x) &= \frac{1}{3(1-x)^5}(-6x^2\ln x-6x\ln x+x^3+9x^2-9x-1) \qquad (C.2)\end{aligned}$$

[1] We use the notation and the formula in [13], corrected by [14]

are the Feynman loop intrgrals defined in [13], and

$$x = \frac{M_{\tilde{g}}^2}{M_{\tilde{q}}^2},\ (\delta_{ij}^d)_{LL} = \frac{\delta \tilde{m}^2_{\tilde{d}_L b_L}}{M_{\tilde{q}}^2},\ \text{and so on.}$$

Demanding that each term is no bigger than the experimental value of ΔM_B gives the constraints on δ_{ij}^d. However, with large splitting in scalar masses of the first two and the third generations, it is better to have constraints directly on the mixing matrix elements because of the ambiguity of what $M_{\tilde{q}}$ should be. In this appendix, we will convert the constraints on δ_{ij} into constraints on the mixing matrix elements W_{ij} directly.

We assume degeneracy between the left-handed and the right-handed scalar masses, and also the first two generation scalar masses (denoted by m_1). To reduce the number of parameters, we also assume that the relevant gaugino mass is degenerate with the third generation scalar mass (denoted by m_3). We also take the chirality-changing scalar masses much smaller than the chirality-conserving ones, so that the eigenstates and eigenvalues are not disturbed significantly. Now we can express the SUSY FCNC contributions by the mixing matrix elements and the two parameters m_3 and $y \equiv \frac{m_1^2}{m_3^2}$. For example, the first term in (C.1) becomes

$$\begin{aligned}&\frac{\alpha_s^2}{216 M_{\tilde{q}}^2}\frac{2}{3} f_B^2 m_B \left(\frac{W_{D_L 31}(m_1^2 - m_3^2)}{m_3^2}\right)^2 [-66\tilde{f}_6(y) + 24 f_6(y)]\\ =\ &\frac{\alpha_s^2}{216 M_{\tilde{q}}^2}\frac{2}{3} f_B^2 m_B (W_{D_L 31})^2 (y-1)^2 [-66\tilde{f}_6(y) + 24 f_6(y)]. \end{aligned} \tag{C.3}$$

Demanding it to be smaller the ΔM_B^{EXP} gives the constraint on $W_{D_L 31}$,

$$\sqrt{\mathrm{Re}|W_{D_L 31}^{\ 2}|} < \frac{18 m_3}{\alpha_s f_B}\sqrt{\frac{\Delta M_B}{m_B}}(y-1)^{-1}[-66\tilde{f}_6(y) + 24 f_6(y)]^{-\frac{1}{2}}. \tag{C.4}$$

Similarly, we can obtain constraints on other mixing matrix elements from the other terms. The constraints from $B - \bar{B}$ mixing are shown in Table C.1(a).

For $K - \bar{K}$ mixing, $\Delta m^2_{LL(RR)_{21}}$ can have two contributions. One comes from the splitting between the first two generation scalar masses, $W_{D_{L(R)21}}(m_1^2 - m_2^2)$. We can use the constraints in [13, 14] in this case because the first two generation scalar masses have to be degenerate to a high degree and there is no ambiguity in what $M_{\tilde{q}}$ is. The other comes from the large splitting of the third generation scalar mass, $W^*_{D_{L(R)32}} W_{D_{L(R)31}}(m_1^2 - m_3^2)$. This part can be treated in the same way as in the $B - \bar{B}$ mixing described above. The terms proportional to the left-right mass insertions are a little more complicated because they involve new integrals. These terms are proportional to $[m_b\,(A + \mu \tan\beta)]^2$. For our purpose, we always work in the large $\tan\beta$ scenario. Hence the corresponding constraints scale as $\frac{m_3^3}{\mu \tan\beta}$, versus m_3 in the case of chirality-conserving terms. The results are listed in Table C.1(b) for Δm_K and Table C.1(c) for ϵ. The ϵ' parameter could put constraints on $|\,\mathrm{Im}\, W_{D_{L(R)32}}^{\,*} W_{D_{L(R)31}}|$ and $|\,\mathrm{Im}\, W_{D_{L(R)32}} W_{D_{R(L)31}}|$. The first one is weaker than the constraints from other places, the second one is enhanced by $\tan\beta$ and is listed in Table C.1(d). The numbers are obtained by requiring its contribution to ϵ' smaller than $3 \times 10^{-3}\epsilon$.

The mixing matrix elements $W_{D_{L(R)32}}$ are constrained by the $b \to s\gamma$ decay. The $b \to s\gamma$ branching ratio has been measured to be $(2.32 \pm 0.57 \pm 0.35) \times 10^{-4}$ by CLEO [68], which is consistent with the Standard Model prediction $(2.8 \pm 0.8) \times 10^{-4}$[69]. In supersymmetric models there are many other contributions. The gluino diagram contributions depend on the mixing matrix elements $W_{D_{L(R)32}}$ so they can be used to constrain $W_{D_{L(R)32}}$. Unlike other contributions, the gluino diagrams give significant contributions to both $\bar{s}_L\sigma^{\mu\nu}b_RF_{\mu\nu}$ and $\bar{s}_R\sigma^{\mu\nu}b_LF_{\mu\nu}$ operators. The former can interfere constructively or destructively with other contributions and the latter does not. In Table C.1(e) we list the constraints on $W_{D_{L32}}$ and $W_{D_{R32}}$ by requiring that each gluino diagram alone does not exceed the Standard Model contribution.

The up mixing matrices W_U's are constrained by $D-\bar{D}$ mixing, and the results are shown in Table C.1(f).

In the lepton sector, the most stringent constraints come from $\mu \to e\gamma$ decay. In the large $\tan\beta$ scenario in which we are interested, the amplitude of the dominant contribution is given in Ref. [62]. Requiring that the rate does not exceed the experimental limit, $B(\mu \to e\gamma) < 4.9\times10^{-11}$[54] give constraints on $W_{E_{L(R)32}}W_{E_{R(L)31}}$, which are shown in Table C.1(g). Because we are interested in generating m_e by radiative corrections which requires sizable mixing between the first and the third generations, $W_{E_{L(R)31}}$, the $\tau \to \mu\gamma$ decay does not give stronger constraints on $W_{E_{L(R)32}}$ than those from the $\mu \to e\gamma$ decay.

Table C.1

(a) Δm_B

$\sqrt{y}$	$\sqrt{\lvert \mathrm{Re}(W_{D_L 31})^2 \rvert}$	$\sqrt{\lvert \mathrm{Re} W^*_{D_L 31} W_{D_R 31} \rvert}$
2	1.0×10^{-1}	3.1×10^{-2}
3	6.5×10^{-2}	2.4×10^{-2}
5	4.9×10^{-2}	2.0×10^{-2}

(b) Δm_K

$\sqrt{y}$	$\sqrt{\lvert \mathrm{Re}(W^*_{D_L 32} W_{D_L 31})^2 \rvert}$	$\sqrt{\lvert \mathrm{Re} W^*_{D_L 32} W_{D_L 31} W_{D_R 32} W^*_{D_R 31} \rvert}$	$\sqrt{\lvert \mathrm{Re}(W_{D_L 32} W_{D_R 31})^2 \rvert}^{\#}$
2	4.7×10^{-2}	5.6×10^{-3}	7.4×10^{-2}
3	3.0×10^{-2}	4.2×10^{-3}	4.7×10^{-2}
5	2.2×10^{-2}	3.6×10^{-3}	3.7×10^{-2}

(c) ϵ

$\sqrt{y}$	$\sqrt{\lvert \mathrm{Im}(W^*_{D_L 32} W_{D_L 31})^2 \rvert}$	$\sqrt{\lvert \mathrm{Im} W^*_{D_L 32} W_{D_L 31} W_{D_R 32} W^*_{D_R 31} \rvert}$	$\sqrt{\lvert \mathrm{Im}(W_{D_L 32} W_{D_R 31})^2 \rvert}^{\#}$
2	3.7×10^{-3}	4.6×10^{-4}	6.0×10^{-3}
3	2.4×10^{-3}	3.4×10^{-4}	3.8×10^{-3}
5	1.8×10^{-3}	2.9×10^{-4}	3.0×10^{-3}

(d) ϵ'

$\sqrt{y}$	$\lvert \mathrm{Im} W_{D_L 32} W_{D_R 31} \rvert^{\#}$
2	1.4×10^{-3}
3	7.7×10^{-4}
5	5.4×10^{-4}

(e) $\boldsymbol{b \longrightarrow s\gamma}$

$\sqrt{y}$	$\|W_{D_L 32}\|^{\#}$
2	6.9×10^{-2}
3	5.3×10^{-2}
5	4.7×10^{-2}

(f) $\boldsymbol{\Delta m_D}$

$\sqrt{y}$	$\sqrt{\|\mathrm{Re}(W_{U_L 32}^{*} W_{U_L 31})^2\|}$	$\sqrt{\|\mathrm{Re} W_{U_L 32}^{*} W_{U_L 31} W_{U_R 32} W_{U_R 31}^{*}\|}$	$\sqrt{\|\mathrm{Re}(W_{U_L 32} W_{U_R 31})^2\|}^{@}$
2	9.5×10^{-2}	3.0×10^{-2}	3.9×10^{-1}
3	6.3×10^{-2}	2.3×10^{-2}	2.5×10^{-1}
5	4.7×10^{-2}	1.9×10^{-2}	2.0×10^{-1}

(g) $\boldsymbol{\mu \longrightarrow e\gamma}$

$\sqrt{y}$	$\|W_{E_L 32}^{*} W_{E_L 31}\|^{\#}$	$\|W_{E_L 32} W_{E_R 31}\|^{\#}$
2	2.4×10^{-3}	2.2×10^{-4}
3	1.8×10^{-3}	1.3×10^{-4}
5	1.6×10^{-3}	1.0×10^{-4}

Table C.1: Constraints on the fermion-sfermion flavor mixing matrix elements. The reference values are taken as: $\tilde{m}_3 = M_g = 500\,\mathrm{GeV}$, $\mu = 500\,\mathrm{GeV}$, $\tan\beta = 60$, and $\sqrt{y} \equiv \frac{\tilde{m}_1}{\tilde{m}_3}$. Same constraints also apply for $L \leftrightarrow R$. The ones with # scale as $(\frac{\tilde{m}_3}{500GeV})^3(\frac{500GeV}{\mu})(\frac{60}{\tan\beta})$, the one with @ scales as $(\frac{\tilde{m}_3}{500GeV})^3(\frac{500GeV}{A})$, others scale as $\frac{\tilde{m}_3}{500GeV}$.

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