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ABSTRACT

The Pomeranchuk (P) trajectory and residue near $t = 0$ are calculated through self-consistency requirements from properties of the P'. The calculation is based on a general multiperipheral model whose kernel is assumed to be nonsingular near $J = 1$, $t = 0$, apart from a small component associated with pomeron exchange and containing the corresponding AFS branch point. This small component of the kernel is treated by standard perturbation techniques. The self-consistent P trajectory turns out to have a slope smaller than normal and substantial positive curvature, whereas the logarithmic derivative of the residue is abnormally large. Although these effects result from proximity to the AFS branch point in the multiperipheral kernel, the corresponding branch point in the amplitude itself turns out to be relatively unimportant near $t = 0$.

DISCLAIMER

I. INTRODUCTION

It has recently been conjectured that the high-subenergy, pomeron component of the multiperipheral kernel, although much weaker than the low-subenergy, resonance component, may split the leading output pole at $t = 0$ into two poles, the P and the P'.[1] In such an event there occurs a partial bootstrap mechanism for the pomeron, since to some degree the output P is the result of the P input into the kernel. This pomeron bootstrap mechanism is less self-contained than that described in Ref. 2, which ignored the low-subenergy component of the kernel, but even a partial bootstrap may place significant constraints on the elastic diffraction mechanism. In this paper we confirm the conjecture of Ref. 1 that self-consistency of the "schizophrenic pomeron" hypothesis requires the slope of the pomeron near $t = 0$ to be smaller than that of "normal" trajectories. We also show the logarithmic derivative of the pomeron residue to be abnormally large. Although a complete explanation of the pomeron intercept cannot be claimed, it will be seen that proximity to $J = 1$ is a natural feature of the partial bootstrap.

Our analysis is based on a generalization and extension to non-zero momentum transfer of the model employed in Ref. 1. The generalization allows all linkage types to appear in the multiperipheral chain, no longer requiring that alternate exchange-links be pionic. It also will be possible to relax the factorizability requirement for the low-subenergy component of the kernel. We require factorizability only for the pomeron component, for which supporting arguments are given. A final improvement on Ref. 1 is that we do not here replace J cuts in the multiperipheral kernel by "equivalent" poles. Such an approximation, although sometimes justified and useful, tends to obscure the meaning of the results obtained.

The reader is referred to Refs. 1 and 3 for physical arguments showing that the pomeron component of the multiperipheral kernel can be no more than a small perturbation of the total kernel if the experimentally observed multiplicity of produced particles is to be achieved. We do not in this paper concern ourselves with production multiplicity, but the experimental facts in this connection should not be forgotten. They constitute a crucial motivation for the idea of a "weak pomeron."

II. GENERAL FORMALISM

Because we shall make no use of the detailed structure of the strong low-subenergy component of the kernel, it is possible to consider a rather general form of multiperipheral model. We employ the multi-Regge formalism developed by Ciafaloni, DeTar, and Misheloff,[4] assuming that in principle the inclusion of a sufficiently large number of poles in the kernel can represent the low-subenergy components. Regge cuts may be included as "continua" of poles.

We thus assume that after diagonalization in angular momentum, parity, isospin, etc. the (projected) imaginary part is given by the analytic continuation of a matrix function $B^{J,t}_{\gamma,\gamma'}$, which obeys the integral equation

$$B^{J,t}_{\gamma\gamma'} = V^{J,t}_{\gamma\gamma'} + \sum_{\gamma''} B^{J,t}_{\gamma\gamma''} \; S^{J,t}_{\gamma''} \; V^{J,t}_{\gamma''\gamma'} \; . \qquad \text{(II.1)}$$

In this symbolic notation J and t represent the set of diagonal variables, including the total (crossed channel) angular momentum and the square of the overall momentum transfer, while γ represents the nondiagonal variables. As indicated in Fig. 1, γ includes two partial momentum transfers (reggeon "masses"), designated t_u and t_ℓ, and a "channel" index to identify a particular pair of Regge trajectories. The vertex quantity $V^{J,t}_{\gamma\gamma'}$ corresponds to the emission of one stable particle, uniquely identified by the quantum numbers of the four attached trajectories. The quantity $S_\gamma{}^{J,t}$ can be characterized as a two-reggeon "propagator," since it contains the pole factor

$$\left[J - \alpha_\gamma{}^u(t_u) - \alpha_\gamma{}^\ell(t_\ell) + 1\right]^{-1} .$$

Being concerned with generation of the pomeron, we restrict attention to the equation with overall vacuum quantum numbers. Among the different possible channels (reggeon pairs) in this case is a pair of pomerons, which we denote by $\gamma = P$. Now there exists no stable particle with quantum numbers coupling P to P; in other words, $V_{PP}^{J,t} = 0$. By a contraction process it is then possible to eliminate the P channel from the space in which the equation is to be solved. That is, for $\gamma \neq P \neq \gamma'$, and temporarily suppressing the common indices J,t, one may show Eq. (II.1) to be equivalent to

$$B_{\gamma\gamma'} = \hat{V}_{\gamma\gamma'} + \sum_{\gamma'' \neq P} B_{\gamma\gamma''} S_{\gamma''} \hat{V}_{\gamma''\gamma'} , \qquad \text{(II.1')}$$

where

$$\hat{V}_{\gamma\gamma'} = V_{\gamma\gamma'} + v_{\gamma\gamma'} , \qquad \text{(II.2)}$$

with

$$v_{\gamma\gamma'} \equiv V_{\gamma P} S_P V_{P\gamma'} . \qquad \text{(II.3)}$$

This definition of $\hat{V}$ is illustrated in Fig. 2.

The two physical assumptions essential to the schizophrenic pomeron idea may now be stated in terms of the kernel

$$\begin{aligned} \hat{K} &= S \hat{V} \\ &= K + k , \end{aligned} \qquad \text{(II.4)}$$

where

$$\begin{aligned} K &= S V , \\ k &= S v . \end{aligned} \qquad \text{(II.5)}$$

(1) The predominantly low-subenergy kernel component K is strong, with a leading eigenvalue that corresponds to a trajectory passing near $J = 1$, $t = 0$, a neighborhood in which $K(J,t)$ is itself nonsingular since it contains no pomeron propagators. The "unperturbed" spectrum in this neighborhood thus consists of a single well-isolated pole.

(2) The kernel component k, constituting the pomeron input, is a weak perturbation, being of negligible consequence except near the singularity arising from the pomeron propagator S_P. To exhibit this crucial singularity we write out the definition of v in the weak coupling or high-subenergy approximation, keeping only the pole part of the propagator:[5]

$$v_{\gamma\gamma'}(J,t) = \int dt_u'' \, dt_\ell'' \, V^{J,t}_{\gamma P}(t_u, t_\ell, t_u'', t_\ell'') S_P^{J,t}(t_u'', t_\ell'') V^{J,t}_{P\gamma'}(t_u'', t_\ell'', t_u', t_\ell')$$

$$\approx \int \frac{dt_u'' \, dt_\ell''}{[-\lambda(t, t_u'', t_\ell'')]^{1/2}} \frac{\beta^*_{\gamma P}(t_u, t_u'') \beta_{\gamma P}(t_\ell, t_\ell'') \beta^*_{P\gamma'}(t_u'', t_u') \beta_{P\gamma'}(t_\ell'', t_\ell')}{J - \alpha_P(t_u'') - \alpha_P(t_\ell'') + 1} , \tag{II.6}$$

the functions β representing vertices with two reggeons and one particle and λ being the usual phase-space factor,

$$\lambda(x,y,z) = x^2 + y^2 + z^2 - 2(xy + xz + yz) .$$

The range of integration is such that $\lambda(t, t_u'', t_\ell'') \leq 0$. We are evidently dealing here with the familiar AFS branch point,[6] although for us it appears in the kernel, not in the amplitude itself.

The AFS branch point in $v(J,t)$, even though the discontinuity around the associated cut is small, introduces fine structure into the spectrum of the complete kernel $\hat{K} = K + k$. That is, a single pole in

the spectrum of the "unperturbed" kernel $K(J,t)$ occurring near the weak branch point of $k(J,t)$ becomes a pole in the perturbed spectrum on <u>each</u> sheet of the Riemann surface associated with the branch point.[7] The pole displacements from one sheet to the next, although relatively small, may nevertheless be observable, and the first two sheets have part of the physical region as a common boundary. It is therefore possible for the poles on these two sheets both to be of physical importance; we shall later identify one of these poles as the P and the other as the P'.

III. FIRST-ORDER PERTURBATION OF THE FREDHOLM SOLUTION

Our formal problem is to solve the integral equation

$$B = \hat{V} + B S \hat{V} , \tag{III.1}$$

where

$$\hat{V} = V + v , \tag{III.2}$$

given a knowledge of the solution to the "unperturbed" equation

$$B^{(0)} = V + B^{(0)} S V . \tag{III.3}$$

The slightly tricky point is that we are working close to a solution of the unperturbed homogeneous equation, so a straightforward expansion of B to first order in v is not adequate. We may, however, make a simple perturbation expansion of the separate Fredholm numerator and denominator functions. That is, if

$$B_{\gamma\gamma'} = \frac{N_{\gamma\gamma'}}{D} , \tag{III.4}$$

then to a good approximation

$$N_{\gamma\gamma'} \approx N^{(0)}_{\gamma\gamma'} + \delta N_{\gamma\gamma'} , \tag{III.5}$$

$$D \approx D^{(0)} + \delta D , \tag{III.6}$$

where $\delta N_{\gamma\gamma'}$ and δD are both small, of first order in v.

Recalling that all quantities depend on J and t, we see that Regge poles occur at points where $D(J,t)$ vanishes. Similarly an "unperturbed" Regge trajectory may be defined by $D^{(0)}(J,t) = 0$. Now we are assuming that the leading unperturbed trajectory, $J = \alpha_0(t)$, passes fairly close to the point $J = 1$, $t = 0$. This means that we are interested in the region where $D^{(0)}$ is small, so the perturbation of D is of

greater significance than that of $N_{\gamma\gamma'}$. There is, after all, no reason to expect $N^{(0)}_{\gamma\gamma'}$ to be small. We therefore adopt as our perturbed solution

$$B_{\gamma\gamma'} \approx \frac{N^{(0)}_{\gamma\gamma'}}{D^{(0)} + \delta D} . \tag{III.7}$$

The Fredholm denominator is a determinant

$$D = \det (1 - S \hat{V}) , \tag{III.8}$$

and to first order in v

$$\det\Big[1 - S(V + v)\Big] \approx \det(1 - SV)\Big\{ 1 - \mathrm{tr}\Big[Sv(1 - SV)^{-1} \Big]\Big\} . \tag{III.9}$$

This formula can be shown to become exact if v is factorizable, a property which we shall later argue to be roughly valid, but for the moment we may be content with (III.9) as a first-order perturbation result.

We now insert the additional information that J is close to α_0 , so close that $B^{(0)}$ may be approximated by a single pole, with its factorized residue,

$$B^{(0)}_{\gamma\gamma'}(J) \approx \frac{b_\gamma{}^{(0)}\, b^{(0)}_{\gamma'}}{J - \alpha_0} . \tag{III.10}$$

The motivation here rests heavily on the presumed absence of J singularities of the unperturbed kernel near α_0 , and the presumed consequent wide spacing between the leading unperturbed trajectory and the remainder of the unperturbed spectrum. Since

$$B^{(0)} = V(1 - S\,V)^{-1} , \tag{III.11}$$

it follows that $b_\gamma{}^{(0)}$ is an eigenfunction of SV, with eigenvalue unity, while

$$D^{(0)} \approx c[\, J - \alpha_0 \,], \tag{III.12}$$

where c is independent of J. One may then evaluate (III.9) as

$$D \approx c\left[J - \alpha_0 - \sum_{\gamma\gamma'} b_\gamma^{(0)} S_\gamma v_{\gamma\gamma'} S_{\gamma'} b_{\gamma'}^{(0)}\right]. \tag{III.13}$$

Comparing (III.7), (III.10), and (III.13), we see that our perturbed solution may be written

$$B_{\gamma\gamma'} \approx \frac{b_\gamma^{(0)} b_{\gamma'}^{(0)}}{J - \alpha_0 - \Delta(J)}, \tag{III.14}$$

where

$$\Delta(J) \equiv \sum_{\gamma\gamma'} b_\gamma^{(0)} S_\gamma v_{\gamma\gamma'}^{(J)} S_{\gamma'} b_{\gamma'}^{(0)}. \tag{III.15}$$

The J dependence of the denominator shift Δ, which will be of central concern to us, arises principally from the AFS branch point of $v_{\gamma\gamma'}$, as given by Formula (II.6). The propagator S_γ is nonsingular in the region of interest and, within this propagator, J may be set equal to α_0.

IV. DEPENDENCE ON t

The single unperturbed J-pole approximation (III.14) amounts to linearizing the J dependence of the unperturbed Fredholm determinant $D^{(0)}$, with a total neglect of the J dependence of the numerator. It would evidently have been equally plausible to so treat the t variable. The result then would be a t dependence of the pomeron residue that arises entirely from the denominator shift $\Delta(J,t)$ through $v_{\gamma\gamma'}(J,t)$, as given by (II.6). Such an approximation can be justified a posteriori if the t dependence generated by $\Delta(J,t)$ turns out to be "abnormally rapid." By this we mean, for example, that the logarithmic derivative of the pomeron residue with respect to t at $t = 0$ turns out to be substantially larger than the rate of variation characteristic of the unperturbed amplitude.

With the only t dependence residing in the Fredholm denominator, pole residues vary with t in a channel-independent manner. That is, if a pole of $B_{\gamma\gamma'}(J,t)$ occurs at $J = \alpha(t)$, where $D(\alpha(t),t) = 0$, the residue is

$$\frac{b_{\gamma}^{(0)}\, b_{\gamma'}^{(0)}}{\left(\frac{\partial D(J,t)}{\partial J}\right)_{J=\alpha(t)}} \quad . \tag{IV.3}$$

If we denote the factorized pomeron residue, in particular, by

$$g_{\gamma P}(t_u, t_\ell, t)\; g_{P\gamma'}(t_u', t_\ell', t) \quad , \tag{IV.4}$$

corresponding to Fig. 3, then the t independence of the numerator in (IV.3) implies

$$g_{\gamma P}(t_u, t_\ell, t) \approx g_\gamma(t_u, t_\ell)\, r_P^{1/2}(t) , \qquad \text{(IV.5)}$$

where the normalization of r_P is arbitrary.

The vertex functions $\beta_{\gamma P}$ appearing in Formula (II.6) are analytic continuations of $g_{\gamma P}$ to a point where one of the three reggeons has become a stable physical particle. If the form (IV.5) is maintained in this continuation, then

$$\begin{aligned} \beta_{\gamma P}(t_u, t_u'') &\approx \beta_\gamma(t_u) r_P^{1/2}(t_u''), \\ \beta_{P\gamma'}(t_u'', t_u') &\approx \beta_{\gamma'}(t_u')\, r_P^{1/2}(t_u'') , \end{aligned} \qquad \text{(IV.6)}$$

leading to

$$v_{\gamma\gamma'}(J,t) \approx \beta_\gamma^*(t_u)\, \beta_\gamma(t_\ell)\, \beta_{\gamma'}^*(t_u')\, \beta_{\gamma'}(t_\ell')\, \rho_P(J,t) , \qquad \text{(IV.7)}$$

where

$$\rho_P(J,t) = \int \frac{dt_u''\, dt_\ell''}{\left[-\lambda(t, t_u'', t_\ell'')\right]^{1/2}} \frac{r_P^*(t_u'')\, r_P(t_\ell'')}{J - \alpha_P(t_u'') - \alpha_P(t_\ell'') + 1} . \qquad \text{(IV.8)}$$

We choose now to normalize r_P in Eq. (IV.6) so that $\rho_P(J,0) \sim \frac{1}{J}$ as $J \to \infty$.

Inserting (IV.7) into (III.15), we may then write

$$\Delta(J,t) \approx \epsilon_P\, \rho_P(J,t) , \qquad \text{(IV.9)}$$

where

$$\epsilon_P \equiv \sum_{\gamma,\gamma'} b_\gamma^{(0)}\, S_\gamma\, \beta_\gamma^*(t_u)\, \beta_\gamma(t_\ell)\, \beta_{\gamma'}^*(t_u')\, \beta_{\gamma'}(t_\ell')\, S_{\gamma'}\, b_{\gamma'}^{(0)} , \qquad \text{(IV.10)}$$

the sum including integration over the four t's. The final form of our perturbed solution, with all t and J dependence

exhibited, thus becomes

$$B_{\gamma\gamma'}(J,t) \approx \frac{b_\gamma^{(0)}\, b_{\gamma'}^{(0)}}{J - a - bt - \epsilon_P\, \rho_P(J,t)}\,, \qquad \text{(IV.11)}$$

the important function $\rho_P(J,t)$ being given by (IV.8). Similar expressions, although usually with additional t dependence, have been previously derived on the basis of more explicit models,[1,3,7,8] including that of ABFST--which does not require representing the low-subenergy kernel by a Regge expansion.[3] We conjecture that <u>any</u> multiperipheral model with a weak pomeron input, including versions not yet studied, will lead to the approximate form (IV.11).

The two Eqs. (IV.8) and (IV.11) constitute our bootstrap conditions. The pomeron appears as the leading singularity of (IV.11) where it is determined in part by ρ_P, and in Eq. (IV.8) it is used to calculate ρ_P. Implicit in Eq. (IV.8) for $\rho_P(J,t)$ is the assumption that the pomeron "propagator" is adequately represented by a single Regge pole. Our approach is based on the identification and special treatment of singularities close to $J = 1$, $t = 0$, but this neighborhood will in general include singularities, such as the AFS branch point, in addition to the leading pole. In the Appendix a more general formula is given for $\rho_P(J,t)$, which includes all the singularities implied by Eq. (IV.11). We shall not attempt in this paper to solve the problem of self-consistency for all these singularities, but rather assume that the pole is dominant and hence employ Eq. (IV.8) for $\rho_P(J,t)$. The error introduced by neglect of the cut will be estimated in Sec. VII.

Gribov and Migdal,[9] using "reggeon calculus," have arrived at a form similar to (IV.11) in the course of "modifying" a "bare" pomeron

propagator. There may be no harm in referring to our unperturbed trajectory, $\alpha_0 = a + bt$, as "bare," although a connotation "fundamental" would be inappropriate, but other aspects of the Gribov-Migdal approach[9] differ from ours in an essential way. They assume that both the "bare" and the "modified" pomeron trajectories pass exactly through $J = 1$ at $t = 0$. Neither does so in our model. In order to achieve consistency near $t = 0$, furthermore, Gribov and Migdal assume their equivalent of $\rho_p(J,t)$ to vanish at $t = 0$. Nonvanishing at this point is a vital feature of our Formula (IV.8). Our approach is less flexible because we depend for motivation on unitarity as represented through the multiperipheral model.

V. PARTIAL BOOTSTRAP CONDITIONS

One of the partial bootstrap conditions resides in the requirement that the pomeron trajectory $\alpha_P(t)$, which appears in Formula (IV.8) for $\rho_P(J,t)$, be given by the leading pole of (IV.11). That is, we require

$$0 = \alpha_P(t) - a - bt - \epsilon_P \rho_P(\alpha_P(t),t) . \tag{V.1}$$

A second condition is that the pomeron vertex factor $r_P(t)$ in (IV.8) be proportional to the residue of this pole, that is, proportional to the reciprocal of the J derivative of $D(J,t)$, evaluated at $J = \alpha_P(t)$:

$$r^{-1}(t) \propto \left[1 - \epsilon_P \frac{\partial \rho_P(J,t)}{\partial J} \right]_{J=\alpha_P(t)} . \tag{V.2}$$

The constant of proportionality in this residue relation involves the unperturbed residue, so we cannot determine ϵ_P through our partial bootstrap.

In a future more complete bootstrap one may attempt to obtain ϵ_P from the solution to the full integral equation, but to do so will require a detailed treatment of the low-subenergy kernel. The simplest detailed model with substantial plausibility is that of ABFST, which has been discussed for $t = 0$ in Ref. 3, where an explicit Fredholm determinant estimate has been derived that can be compared with (III.13) and (IV.9). The result for the product $c\,\epsilon_P$ was

$$c\,\epsilon_P \approx \frac{3}{2} \frac{1}{16\pi^3} \Delta_\pi \sigma_{\pi\pi}^{e\ell} , \tag{V.3}$$

where Δ_π is the effective momentum-transfer upper limit for a pion link and $\sigma_{\pi\pi}^{e\ell}$ is the elastic $\pi\pi$ cross section just above the region of

prominent resonances. Estimating $\sigma_{\pi\pi}^{el} \approx 3$ mb and $\Delta_\pi \approx 1$ GeV^2 leads from (V.3) to $c\,\epsilon_P \approx 0.03$, whereas the same ABFST model estimates c to be of order unity. We shall see below that $\epsilon_P \approx 0.03$ is indicated by the results of this paper as well as the corresponding ones in Ref. 1, in agreement with Formula (V.3).

However, the low-subenergy ABFST kernel has less strength than required by experiment,[3,10] so the estimate (V.3) cannot be made a direct component of our bootstrap calculation. In any event the uncertainty about the momentum-transfer cutoff Δ_π will remain until the pion link itself is included in the bootstrap. It therefore seems unavoidable at present to regard ϵ_P as an arbitrary parameter.

The bootstrap equations (V.1), (V.2) and (IV.8) are supposed to be valid only near $t = 0$, and in this paper we require their satisfaction only to zeroth and first order in a power-series expansion in t about the origin. We begin by assuming that the pomeron residue does not vanish at or near $t = 0$ and can be represented by an exponential form for $r_P(t)$:

$$r_P(t) = \text{constant} \times e^{\gamma_P t}, \qquad \text{(V.4)}$$

the parameter γ_P to be determined from self-consistency requirements. It is evident that γ_P must be positive for the integral in (IV.8) to exist. Formula (IV.8) then becomes

$$\rho_P(J,t) = \frac{\displaystyle\iint \frac{dt_u\,dt_\ell}{[-\lambda(t,t_u,t_\ell)]^{1/2}} \; \frac{e^{\gamma_P(t_u+t_\ell)}}{J - \alpha_P(t_u) - \alpha_P(t_\ell) + 1}}{\displaystyle\iint \frac{dt_u\,dt_\ell}{[-\lambda(0,t_u,t_\ell)]^{1/2}} \; e^{\gamma_P(t_u+t_\ell)}}, \qquad \text{(V.5)}$$

if the normalization of r_P is adjusted so that $\rho_P(J,0) \sim \frac{1}{J}$ as $J \to \infty$. Let us next employ a linear representation of the pomeron trajectory,

$$\alpha_P(t) \approx 1 - x_P + b_P t , \tag{V.6}$$

the constants $1 - x_P$ and b_P corresponding, respectively, to the pomeron intercept and slope. We introduce the displacement of the intercept from $J = 1$, rather than the intercept itself, because we expect the displacement to be small. Substitution of (V.6) into (V.5) and the carrying out of one of the two integrations then give

$$\rho_P(J,t) = e^{(\gamma_P/2)t} \int_0^\infty d\xi \, \frac{e^{-\xi}}{J - \left[1 - 2x_P + \frac{b_P}{2} t - \frac{b_P}{\gamma_P} \xi\right]} , \tag{V.7}$$

leading to

$$\rho_P[\alpha_P(t),t] = e^{(\gamma_P/2)t} \int_0^\infty d\xi \, \frac{e^{-\xi}}{x_P + \frac{b_P}{2} t + \frac{b_P}{\gamma_P} \xi} , \tag{V.8}$$

and

$$\left[\frac{\partial \rho_P(J,t)}{\partial J}\right]_{J=\alpha_P(t)} = -e^{(\gamma_P/2)t} \int_0^\infty d\xi \, \frac{e^{-\xi}}{\left(x_P + \frac{b_P}{2} t + \frac{b_P}{\gamma_P} \xi\right)^2}$$

$$= \frac{\gamma_P}{b_P} \left[- \frac{e^{(\gamma_P/2)t}}{x_P + \frac{b_P}{2} t} + \rho_P[\alpha_P(t),t] \right] , \tag{V.9}$$

Now requiring that Eq. (V.1) be satisfied to zeroth and first powers of t, we find

$$1 - a - x_P = \epsilon_P \int_0^\infty d\xi \, \frac{e^{-\xi}}{x_P + \frac{b_P}{\gamma_P}\xi} \qquad (V.10)$$

and

$$b - b_P = \gamma_P \left\{ \frac{\epsilon_P}{2x_P} - 1 + a + x_P \right\} . \qquad (V.11)$$

At the same time, consistency of Formula (V.2) with (V.4) to a corresponding order in t implies that

$$\gamma_P = \frac{\epsilon_P \frac{\partial}{\partial t} \left[\left(\frac{\partial \rho_P(J,t)}{\partial J} \right)_{J=\alpha_P(t)} \right]_{t=0}}{1 - \epsilon_P \left[\left(\frac{\partial \rho_P(J,t)}{\partial J} \right)_{J=\alpha_P(t)} \right]_{t=0}}$$

$$= \frac{\gamma_P \left\{ \frac{\epsilon_P}{2x_P} \left(\frac{1}{x_P} - \frac{\gamma_P}{b_P} \right) - \frac{b - b_P}{b_P} \right\}}{\frac{b}{b_P} + \frac{\gamma_P \epsilon_P}{2x_P b_P}},$$

or

$$\frac{\gamma_P}{b_P} = \frac{1}{2x_P} - \frac{x_P}{\epsilon_P} \left(\frac{2b}{b_P} - 1 \right) . \qquad (V.12)$$

Thus, from the three requirements that the pomeron intercept, slope and residue logarithmic derivative be self-consistent, we have found three

equations (V.10, 11, 12) which determine these three parameters (i.e., x_P, b_P and γ_P) once values have been assigned to a, b, and ϵ_P.

Before we take up numerical aspects of the solution to these equations, certain qualitative observations are in order. Note, first of all, that considerations of reality in Eq. (V.10) require the $t = 0$ pomeron slope b_P to be nonnegative, while x_P must fall into the interval

$$0 < x_P < 1 - a . \tag{V.13}$$

The pomeron intercept, that is, must lie below $J = 1$ and above the intercept of the leading unperturbed trajectory. Furthermore if ϵ_P is small in comparison with $1 - a$, Eq. (V.10) requires x_P to lie either close to zero or close to $1 - a$.

Turning to Eq. (V.11), we see that for small ϵ_P the nonnegative requirement for b_P imposes a lower bound on x_P:

$$x_P > \frac{1}{2} \frac{\epsilon_P}{1 - a + b/\gamma_P} . \tag{V.14}$$

In order that γ_P be positive, on the other hand, we require from (V.12) that

$$x_P^2 < \frac{1}{2} \frac{\epsilon_P}{2 \frac{b}{b_P} - 1} , \tag{V.15}$$

ruling out the possibility that, as $\epsilon_P \to 0$, x_P might approach $1 - a$. When all these constraints are put together, it is plausible that, for small ϵ_P, x_P will be proportional to ϵ_P. Such is indeed the result of a complete numerical analysis, as we shall see in Sec. VII.

Further qualitative analysis of the three equations reveals that as ϵ_P and x_P together approach zero the quotient b_P/γ_P also becomes proportional to ϵ_P. In fact, if $\gamma_P \gg b$ and $\epsilon_P \ll 1$, one may deduce from (V.11) and (V.12) that

$$x_P \approx \frac{\epsilon_P}{2(1 - a)} \tag{V.16}$$

and

$$\frac{b_P}{\gamma_P} \approx 2x_P . \tag{V.17}$$

These two formulas turn out to be approximately valid in the domain of physical interest. Formula (V.17) suggests that the pomeron slope b_P will be small, although precisely how small is difficult to establish without a complete numerical solution. Before embarking on detailed numerical analysis we must first decide on the physically interesting choice for the parameters a, b, and ϵ_P.

VI. CHOICE OF THE PARAMETERS a, b, AND ϵ_P

The parameters a and b characterize the low-subenergy component of the kernel, giving the intercept and slope of the leading trajectory that would be generated by that component acting alone. Since we are not attempting in this paper to determine these parameters from self-consistency, we look to experiment for guidance in their choice.

The pomeron perturbation is expected to be substantial only near $J = 1$, where the weak pomeron coupling is compensated by the proximity of the kernel singularity, so it is plausible that near $J = 2$ the unperturbed trajectory, $a + bt$, may be a good approximation to the actual leading trajectory with vacuum quantum numbers. The lowest mass $J = 2$ meson with such quantum numbers is the $f_0(1250)$, so we adopt, as one condition,

$$a + b\, m_{f_0}^2 \approx 2 . \qquad \text{(VI.1)}$$

A second condition may be based on the observed energy dependence of elastic amplitudes near $t = 0$. Phenomenological analysis in terms of two or three Regge poles with vacuum quantum numbers leads to an intercept for the second vacuum pole, usually labeled P', at $J \approx 0.6$.[11] In our model only the leading pole occurs on the real axis of the physical J sheet; other poles must be reached by passing through the cut connecting $J = -\infty$ to the branch point at

$$\alpha_c(t) = \max\left[\alpha_P(t_u) + \alpha_P(t_\ell) - 1\right]$$
$$\approx 1 - 2x_P + \frac{b_P}{2} t . \qquad \text{(VI.2)}$$

Nevertheless, if we examine the cut discontinuity in (IV.11) we find that for small ϵ_P the discontinuity is small except in a narrow interval

near $J \approx a + bt$, where there is a sharp peak due to a close-lying second-sheet pole. (See Fig. 5.) The effect of the cut can thus be approximated by a single real-axis pole which we identify with the P'.[12] Roughly, then,

$$a \approx \alpha_{P'}(0) . \qquad \text{(VI.3)}$$

Choosing ϵ_P is more difficult. A rough upper bound is provided by the experimental requirement that the pomeron intercept be close to $J = 1$. Requiring $\alpha_P(0) > 0.9$ or $x_P < 0.1$, for example, leads from (V.16) with $a > 0.6$ to $\epsilon_P \lesssim 0.08$.

To obtain a lower bound on ϵ_P we may refer to the experimental indication that the pomeron residue at $t = 0$ is of the same order of magnitude as that of the P'. Using the fact that our model amplitude, as given by Formula (IV.11), satisfies an unsubtracted dispersion relation in J, with a pole at $J = \alpha_P$ and a cut running from $J = -\infty$ to $J = \alpha_c$, it is easy to demonstrate a sum rule that the integrated cut discontinuity plus the pole residue is equal to the residue of the unperturbed pole. A short calculation then gives the ratio of the sum of the integrated cut discontinuity and the pole residue to the pole residue alone, at $t = 0$, to be

$$\frac{g_P^2(0) + g_{P'}^2(0)}{g_P^2(0)} = \frac{b}{b_P} + \frac{\gamma_P \epsilon_P}{2 x_P b_P} . \qquad \text{(VI.4)}$$

We are here interpreting the integrated cut discontinuity as the effective residue of P', assuming that the peak in the cut discontinuity near $J = \alpha_{P'}$ exhausts most of the integral from $J = -\infty$ to $J = \alpha_c$.

This last assumption will be investigated in Sec. VII, but tentatively we see from (VI.4) that to avoid an unacceptably small pomeron residue we must not allow either b/b_P or $\gamma_P \epsilon_P / 2x_P b_P$ to become very large compared with unity. If we use the rough relations (V.16) and (V.17), the latter objective requires that the quantity

$$\frac{\epsilon_P}{4x_P^2} \approx \frac{(1 - a)^2}{\epsilon_P}$$

not be excessively large. Thus we cannot allow ϵ_P to be orders of magnitude smaller than $(1 - a)^2$.

Following the preceding guidelines, the range of ϵ_P to be studied in the following section will be $0.01 \lesssim \epsilon_P \lesssim 0.06$.

VII. NUMERICAL SOLUTION OF THE BOOTSTRAP EQUATIONS

We may for convenience use the reciprocal of the unperturbed trajectory slope b as our unit of squared momentum transfer. The results for b_P and γ_P with $b = 1$ are then to be understood as ratios b_P/b and γ_P/b. Figure 4 shows the solution of Eqs. (V.10, 11, 12) for $a = 0.7$ and the physically interesting range of ϵ_P. In addition to showing the values of x_P, b_P/b, and γ_P/b, we exhibit the ratio of P to P' residues, as given by (VI.4), and the P' intercept, the latter being defined as the value of J near $J = a$ where the real part of the denominator of (IV.11) vanishes. The cut discontinuity has its maximum at this point, as illustrated in Fig. 5, which corresponds to the special case $a = 0.7$ and $\epsilon_P = 0.03$, the choice for these parameters that was made in Ref. 1. We see that for this choice the P' intercept occurs at $J = 0.57$. Also note that the cut discontinuity near the branch point is small compared to the pomeron residue, justifying a posteriori the neglect of the cut in Eq. (IV.8).

Figure 4 shows that the pomeron slope b_P decreases as ϵ_P is made smaller, the ratio b_P/b becoming equal to $1/2$ at $\epsilon_P = 0.014$. Pomeron slopes much smaller than this are accompanied by unacceptably small values of the pomeron residue. (See Sec. VIII.) On the other hand, pomeron slopes close to the unperturbed slope are accompanied by a pomeron intercept excessively far below $J = 1$. Thus our model leads unavoidably to an "intermediate" value for the pomeron slope at $t = 0$, given the conflicting demands of residue and intercept.

The P' intercept is controlled mainly by the value of the parameter a and is relatively insensitive to ϵ_P. We therefore consider

a as being unambiguously fixed at a value near 0.7. The requirement (VI.1) then implies $b \approx 0.83$ GeV^{-2}, from the measured f_0 mass $m_{f_0} = 1.25$ GeV, and we may convert the content of Fig. 4 into more usual notation, as shown in Table I.

VIII. DISCUSSION OF RESULTS

Given a knowledge of the leading trajectory in the spectrum of the unperturbed low-subenergy kernel, a trajectory which we have roughly identified with the P', it has been possible to predict pomeron characteristics near $t = 0$ in terms of one additional parameter ϵ_P. What is known experimentally about these characteristics?

To approach this question we must establish the relation of our model to measured differential elastic and total cross sections. The elastic amplitude is obtained by inverting the projection that led to $B_{\gamma\gamma'}(J,t)$ and then continuing the reggeon mass variables t_u, t_ℓ, t_u', and t_ℓ' to points that correspond to physical particles. To avoid kinematic singularities one must understand $B_{\gamma\gamma'}(J,t)$ as a "reduced" partial-wave amplitude from which there has been removed a factor

$$\left[\frac{k_\gamma(t)\, k_{\gamma'}(t)}{s_0}\right]^J , \qquad \text{(VIII.1)}$$

where

$$k_\gamma^2(t) = \frac{-\lambda(t,\, t_u,\, t_\ell)}{4t} , \qquad \text{(VIII.2)}$$

and s_0 is an arbitrarily chosen constant of dimension energy squared.[13] Different choices for s_0 evidently lead to different t dependence for a Regge residue, but we suppose, with regard to the reasoning of Sec. IV, that choosing $s_0 = 1\ \text{GeV}^2$ will lead to a logarithmic derivative for the residue of the leading unperturbed trajectory that is smaller than or of the order of $1\ \text{GeV}^2$. We assume, in other words, that the characteristic dimension of the unperturbed kernel is of the order

1 GeV^2.[14] The treatment in Sec. IV is presumed to be justified if γ_P turns out large compared with 1 GeV^2, and by the same token the precise value of s_0 will not matter so long as this parameter is near 1 GeV^2.

With the understanding that the pomeron residue, Formula (IV.4), refers to the reduced partial-wave amplitude with $s_0 = 1$ GeV^2, it follows that the elastic cross section for particle A on particle B has the standard high energy form[13]

$$\frac{d\sigma_{el}^{AB}}{dt} \sim g_{AP}^2(t)\, g_{BP}^2(t) \left(\frac{s}{s_0}\right)^{2\alpha_P(t)-2} \left| i - \cot\frac{\pi\alpha_P(t)}{2} \right|^2 , \tag{VIII.3}$$

and the corresponding total cross section is

$$\sigma_{tot}^{AB} \sim \frac{1}{(\pi)^{1/2}}\, g_{AP}(0)\, g_{BP}(0) \left(\frac{s}{s_0}\right)^{\alpha_P(0)-1} . \tag{VIII.4}$$

We are ignoring spin, a simplification legitimate near $t = 0$. In view of the factorized t dependence of our model pomeron residue, Formula (IV.5), and the exponential shape (V.4) assumed for $r_P(t)$, we may write (VIII.3) in the simpler form

$$\frac{d\sigma_{el}^{AB}}{dt} \sim g_A^2\, g_B^2\, e^{2\gamma_P t} \left(\frac{s}{s_0}\right)^{2\alpha_P(t)-2} , \tag{VIII.3'}$$

the signature factor being omitted on the grounds that $\alpha_P(t)$ is close to 1.

In comparing these simple forms with finite-energy experiments, there arise the usual ambiguities due to lower-lying Regge singularities. In addition to singularities such as the P', that are in principle

included in our model, there may occur branch points associated with s-channel "absorption" that are not included.[15] Considering the roughness of our model, we shall therefore make no effort to correct for absorption. It is of course possible to correct for the P' by employing the results of phenomenological Regge pole analysis.[13] We are in fact directly interested in the ratio of P to P' residues.

The most significant experimental evidence concerning the pomeron slope near $t = 0$ comes from the 1969 pp elastic scattering measurement at Serpukhov,[16] which yields an uncorrected value

$$\alpha_P'(0) = 0.47 \pm 0.09 \text{ GeV}^{-2} . \qquad \text{(VIII.5)}$$

Taken at face value, this slope implies for our model, according to Fig. 4, that $\epsilon_P = 0.021 \pm 0.010$. Such a value for ϵ_P means from Fig. 4 that $\alpha_P(0) = 0.97 \pm 0.01$, a satisfactorily high pomeron intercept. For γ_P the model prediction is a bit too large but in the right neighborhood; from Fig. 4 we read off $\gamma_P = 6.0 \pm 2.0 \text{ GeV}^{-2}$. The uncorrected Serpukhov pp experiment[16] implies $\gamma_P = 3.4 \pm 0.2 \text{ GeV}^{-2}$ whereas the highest-energy Brookhaven πp elastic experiment[17] gives $\gamma_P = 3.8 \pm 0.2 \text{ GeV}^{-2}$, assuming the pomeron slope (VIII.5) and making no P' correction. Including the latter correction reduces the Brookhaven-measured γ_P to 3.0 GeV^{-2}.[11] It is gratifying that the experimental value of γ_P is similar in pp and πp scattering, rough channel-independence of the pomeron t behavior being a characteristic feature of our model.

For further discussion of the model let us adopt a slightly higher value of ϵ_P that is consistent with the experimental requirements for both $\alpha_P'(0)$ and γ_P. We see from Table I that $\epsilon_P = 0.03$ leads to

acceptable values for $\alpha_P(0)$, $\alpha_P'(0)$, and γ_P. Only the model-predicted ratio $g_P^2(0)/g_{P'}^2(0) = 0.33$ need cause concern. The experimental value of this ratio from πp total cross sections is 1.4, while from pp and $\bar{p}p$ total cross sections it is 1.1.[13] We are encouraged by the roughly channel-independent nature of the experimental ratio, a feature of our model, but the predicted theoretical ratio has too low a magnitude.

A different choice for s_0 can make some difference here. For example, were we to choose $s_0 = 0.5$ GeV^2, the experimental ratio would decrease by a factor $\approx\sqrt{2}$. It was found in the model of Ref. 1, furthermore, that the pomeron perturbation of the Fredholm numerator, an effect ignored in this paper, enhances the P residue with respect to the P'. It seems plausible that such will be the case in any version of the model. (The factor of enhancement in the particular model of Ref. 1 was 1.4.) We also of course may increase the P/P' residue ratio by further increasing ϵ_P, at the price of additional decrease in the pomeron intercept.[18] A glance at Table I with $\epsilon_P = 0.4$ illustrates the situation. Finally, we see from Fig. 5 that to associate the entire cut discontinuity with the P', as we have done, is somewhat unfair to the pomeron.

IX. CONCLUSION

With the foregoing generally favorable experimental picture, it would be premature at present to discard the schizophrenic pomeron model on the grounds of too small a P/P' residue ratio. This point must, however, be carefully watched. A closely related remarkable prediction of the model, as noted in Ref. 1, is that the rate of increase of multiplicity with the log of the energy will change from a value at moderate energies that is characteristic of the unperturbed residue to a significantly smaller value at extremely high energies that is characteristic of the pomeron residue.[19] If no such decrease is observed, the model will be in serious trouble.

Since we have assumed that the pomeron residue does not vanish at $t = 0$, the intercept unavoidably lies below $J = 0$ and the total cross section is correspondingly predicted to decrease asymptotically with increasing energy. This decrease is slow, however, and its onset may be delayed by absorptive effects not included in our model.

Another characteristic prediction of the schizophrenic pomeron model, albeit one not explored in the foregoing analysis, is that positive curvature will develop in the pomeron trajectory as, with negatively increasing t, it approaches the AFS branch point. If the pomeron slope at $t = 0$ is b_P, with intercept $1 - x_P$, then the branch point has slope $\frac{1}{2} b_P$, with intercept $1 - 2x_P$, and the two singularities will intersect at $t = -2x_P/b_P$ unless curvature develops. It can be seen from Eq. (V.1) that intersection is impossible (since ρ_P would at that point become infinite) and curvature is thus inevitable. With $\epsilon_P = 0.03$ the projected intersection would occur, according to Table I, at $t = -0.17$ GeV^2, so

the trajectory curvature is large.[20] We have not yet extended our search for a self-consistent solution beyond the linear region around $t = 0$, but it is plausible that for negative t the pomeron may never deviate from $J = 1$ by an amount large compared with ϵ_p, and may even approach $J = 1$ asymptotically as $t \to -\infty$. Such curious trajectory behavior may, of course, turn out to be of only academic interest if the pomeron residue continues its precipitous decrease as the separation grows between unperturbed and perturbed poles.

An important limitation of this paper has been the restriction to very small values of $|t|$. For a variety of reasons, the approximations employed break down as the pomeron pole-trajectory approaches the AFS branch point. In addition to the nonlinearity of the trajectory, the integrated cut discontinuity from the region just below the branch point will become larger and compete in importance with the pomeron residue. The pole dominance shown in Fig. 5, that is, is not guaranteed to persist. Pole and cut should then be combined in generating the pomeron perturbation of the kernel. This more difficult but perhaps not insoluble self-consistency problem remains for future work; the relevant equations are given in the Appendix.

It is interesting also to speculate on the pomeron's fate for positive t. Here the pole and branch point widen their separation, so it is plausible that the perturbation of the kernel becomes progressively smaller, i.e. that $\alpha_p(t)$ approaches $\alpha_0(t)$ as t increases positively. At the same time the P' trajectory remains on the next sheet of the J Riemann surface and passes the AFS branch point at $t \approx (1 - 2x_p - a)/(b - \frac{1}{2} b_p) \approx 0.4\ \text{GeV}^2$, thereafter being distant from

the physical region. Thus, as suggested in Ref. 1, by the time we reach $t = m_{f_0}^2$, $J = 2$, there will be only one physically important trajectory --the analytic continuation of the pomeron. The pomeron changes from normal status when $t \gtrsim 1$ GeV^2 to an abnormal status for $t \lesssim -1$ GeV^2. Near $t = 0$, where this paper has concentrated its attention, the pomeron is in transition. The speculations of this paragraph and the one preceding are illustrated in Fig. 6.

What if the pomeron residue vanishes at $t = 0$? The trajectory then might pass through $J = 1$ at $t = 0$, with the branch cut playing a mathematically prominent role.[22] We believe that, since we have been able to achieve approximate self-consistency in the model of this paper, the physical content of such an alternative would not differ greatly from our solution. (In either case, for example, the total cross section has a slow asymptotic decrease.) In bootstrap investigations, where a well-defined set of equations never exists, it is especially important to recognize the unattainability of absolute accuracy in the description of natural phenomena. The selection of equations unavoidably depends on a guess about the nature of their solution and can only be justified a posteriori in some approximate sense. By the same token it is pointless to make much ado about mathematical differences between "solutions" whose physical content lies within that range of uncertainty which is inherent in the original selection of equations.

ACKNOWLEDGMENT

The authors have benefited greatly from detailed exchange of information with J. Ball and G. Marchesini, who made calculations earlier than ours with a version of the ABFST model that turned out to have similar physical content.[21] We have learned from Ball and Marchesini that recent analysis of the results published in Ref. 21 show the presence in their model of two poles that may be identified as the P and P', a fact that escaped recognition at the time of publication. Many of the qualitative points emphasized in the present paper can now be seen in these earlier Ball-Marchesini results.

Discussions with W. R. Frazer have been influential in our work, as remarked in Footnote 7. We are also pleased to acknowledge help from M. Ciafaloni, C. DeTar, C. H. Mehta, M. Misheloff, and C. Risk.

APPENDIX: GENERALIZATION OF THE POMERON PROPAGATOR

Equation (IV.11) includes a branch point as well as the pomeron pole on the physical J-sheet, the relative magnitudes of the cut-discontinuity and the pole residue being channel-independent. Second sheet singularities, such as the P', also are included. All high energy amplitudes with vacuum quantum numbers are then approximately determined in our model, apart from normalization, by a pomeron "propagator,"

$$S_P(J,t) = \frac{1}{J - \alpha_0(t) - \epsilon_P \rho_P(J,t)} . \qquad (A.1)$$

This propagator satisfies an unsubtracted J-dispersion relation,

$$S_P(J,t) = \frac{r_P(t)}{J - \alpha_P(t)} + \frac{1}{\pi} \int_{-\infty}^{\alpha_c(t)} \frac{s(J',t)}{J - J'} \, dJ' , \qquad (A.2)$$

as also does the function $\rho_P(J,t)$:

$$\rho_P(J,t) = \frac{1}{\pi} \int_{-\infty}^{\alpha_c(t)} \frac{w(J',t)}{J - J'} \, dJ' . \qquad (A.3)$$

The discontinuity of S_P is related to that of ρ_P by

$$s(J,t) = \frac{\epsilon_P \, w(J,t)}{[J - \alpha_0(t) - \epsilon_P \, \mathrm{Re} \, \rho_P(J,t)]^2 + \epsilon_P^2 \, w^2(J,t)} , \qquad (A.4)$$

with the pomeron pole position and residue being given by Eqs. (V.1) and (V.2).

Formula (IV.8) corresponds to keeping only the pole in the propagator spectrum (A.2), neglecting the cut and ignoring the signature factor. The more general formula is

$$\rho_P(J,t) = \text{constant} \times \iint \frac{dt_u\, dt_\ell}{[-\lambda(t,t_u,t_\ell)]^{1/2}} \iint dJ_u dJ_\ell \frac{\bar{s}^*(J_u,t_u)\bar{s}(J_\ell,t_\ell)}{J - J_u - J_\ell + 1}, \tag{A.5}$$

where

$$\bar{s}(J,t) \equiv (i - \cot\tfrac{\pi J}{2}) \left\{ r_P(t)\, \delta[J - \alpha_P(t)] + \frac{1}{\pi} s(J,t)\theta[\alpha_c(t) - J] \right\}, \tag{A.6}$$

the normalizing constant to be adjusted, as before, so that $\rho_P(J,0) \sim \frac{1}{J}$ as $J \to \infty$.

The partial bootstrap still requires pre-assignment of $\alpha_0(t)$ and ϵ_P, but now one may attempt to make self consistent the entire pomeron propagator and not simply the leading pole. The possibility of $\alpha_P(0) = 1$ with $r_P(0) = 0$ remains open.

Table I: Self-consistent pomeron parameters for $a = 0.7$ and $b = 0.83\ \text{GeV}^{-2}$, corresponding to $m_{f_0} = 1.25$ GeV and $\alpha_{P'}(0) \approx 0.6$.

ϵ_P	$\alpha_P(0)$	$\alpha'_P(0)\text{GeV}^{-2}$	$\gamma_P\ \text{GeV}^{-2}$	$g_P^2(0)/g_{P'}^2(0)$
0.02	0.969	0.48	6.1	0.21
0.03	0.953	0.55	4.2	0.34
0.04	0.936	0.60	3.0	0.52

FOOTNOTES AND REFERENCES

* This work was supported in part by the U. S. Atomic Energy Commission.

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15. Because the multiperipheral integral equation represents part of the content of s-channel unitarity, certain absorptive effects would be included in a more complete multiperipheral bootstrap which requires self-consistency for the entire amplitude.

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FIGURE CAPTIONS

Fig. 1. Diagram defining the variables appearing in Eq. (II.1). Wiggly lines denote Regge trajectories.

Fig. 2. Diagram illustrating the definition of the modified vertex after contraction of the P channel.

Fig. 3. Diagram depicting the factorized pomeron residue.

Fig. 4. Numerical solution of the linearized bootstrap equations (V.10, 11, 12) for $a = 0.7$.

Fig. 5. The discontinuity of $B(J, t = 0)$ across the AFS cut for $a = 0.7$ and $\epsilon_P = 0.03$. For purposes of relative normalization the rectangle centered at $J = \alpha_P(0)$ has been given an area corresponding to the pomeron residue.

Fig. 6. Conjectured extrapolation away from $t = 0$ of the P and (real part of) P' trajectories for $\epsilon_P = 0.03$ and $a = 0.7$. The dotted line is the unperturbed trajectory.

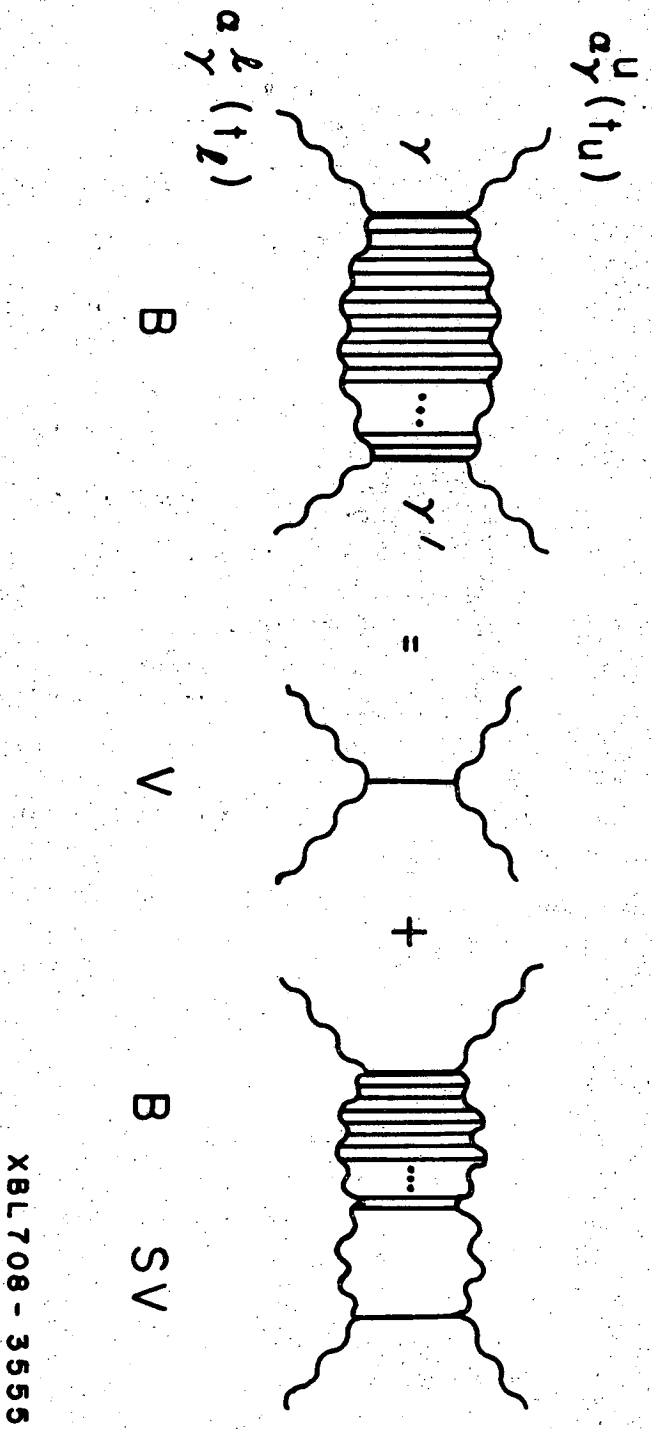
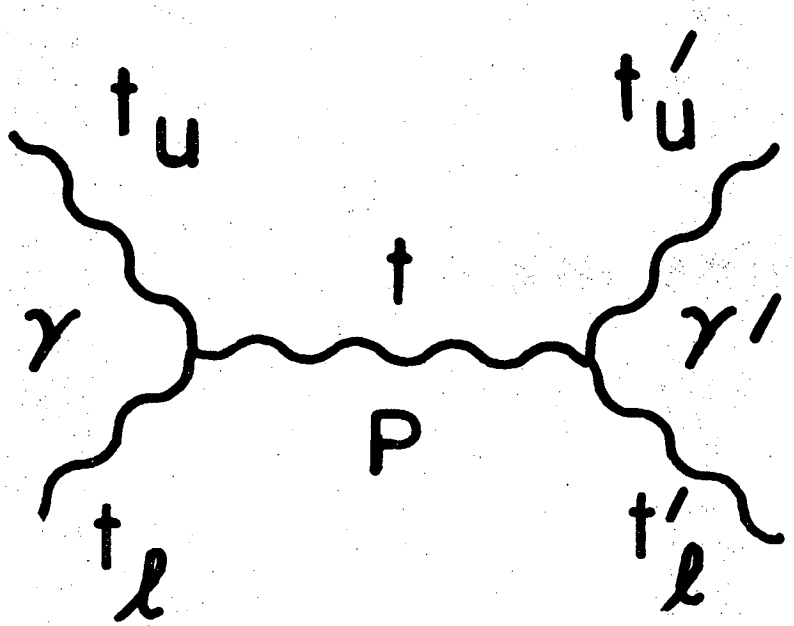


Fig. 1

XBL708-3556

Fig. 2



XBL708-3554

Fig. 3

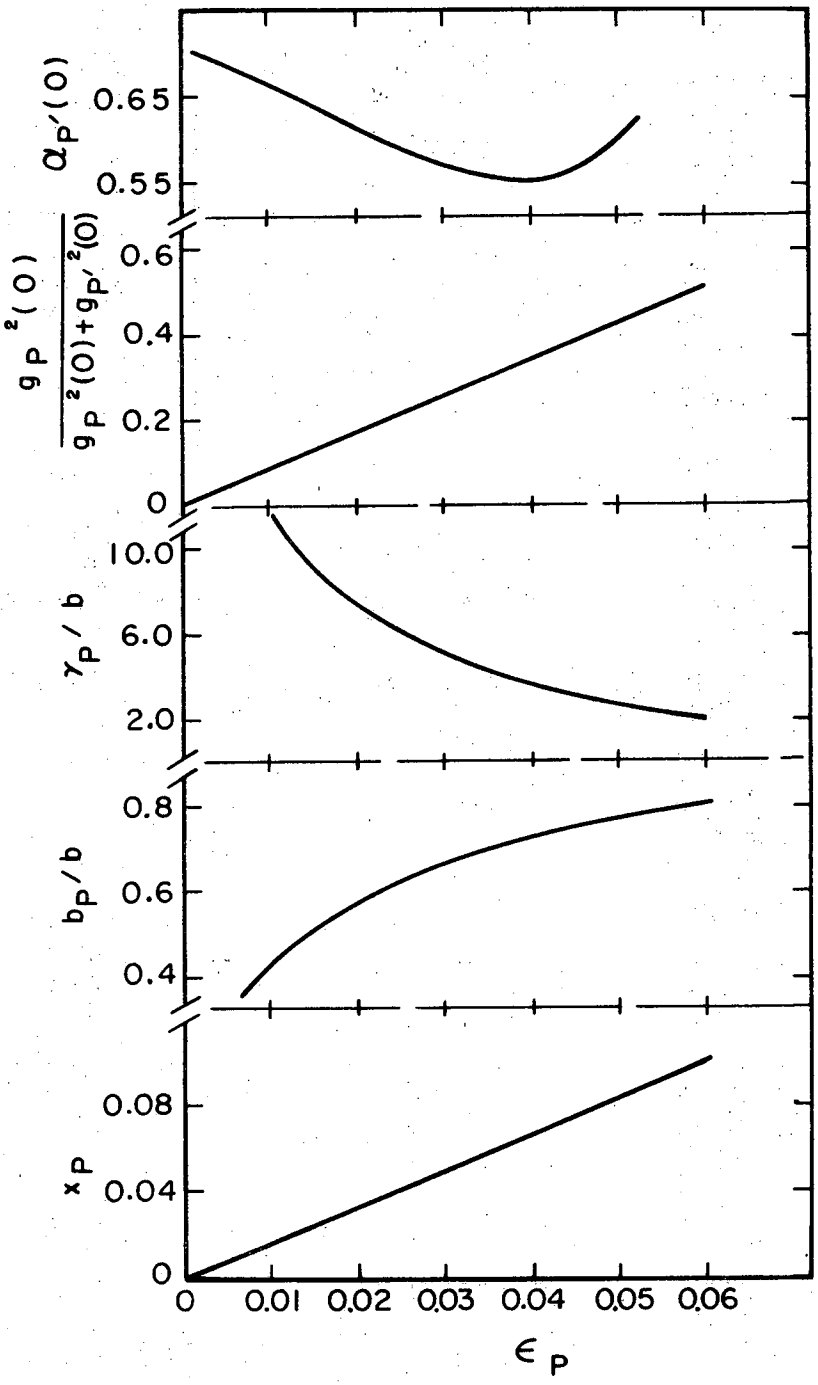

$\alpha_{P'}(0)$
0.65
0.55
$\frac{g_P^2(0)}{g_P^2(0)+g_{P'}^2(0)}$
0.6
0.4
0.2
0
γ_P / b
10.0
6.0
2.0
b_P / b
0.8
0.6
0.4
x_P
0.08
0.04
0
0
0.01
0.02
0.03
0.04
0.05
0.06
ϵ_P
XBL708-3559

Fig. 4

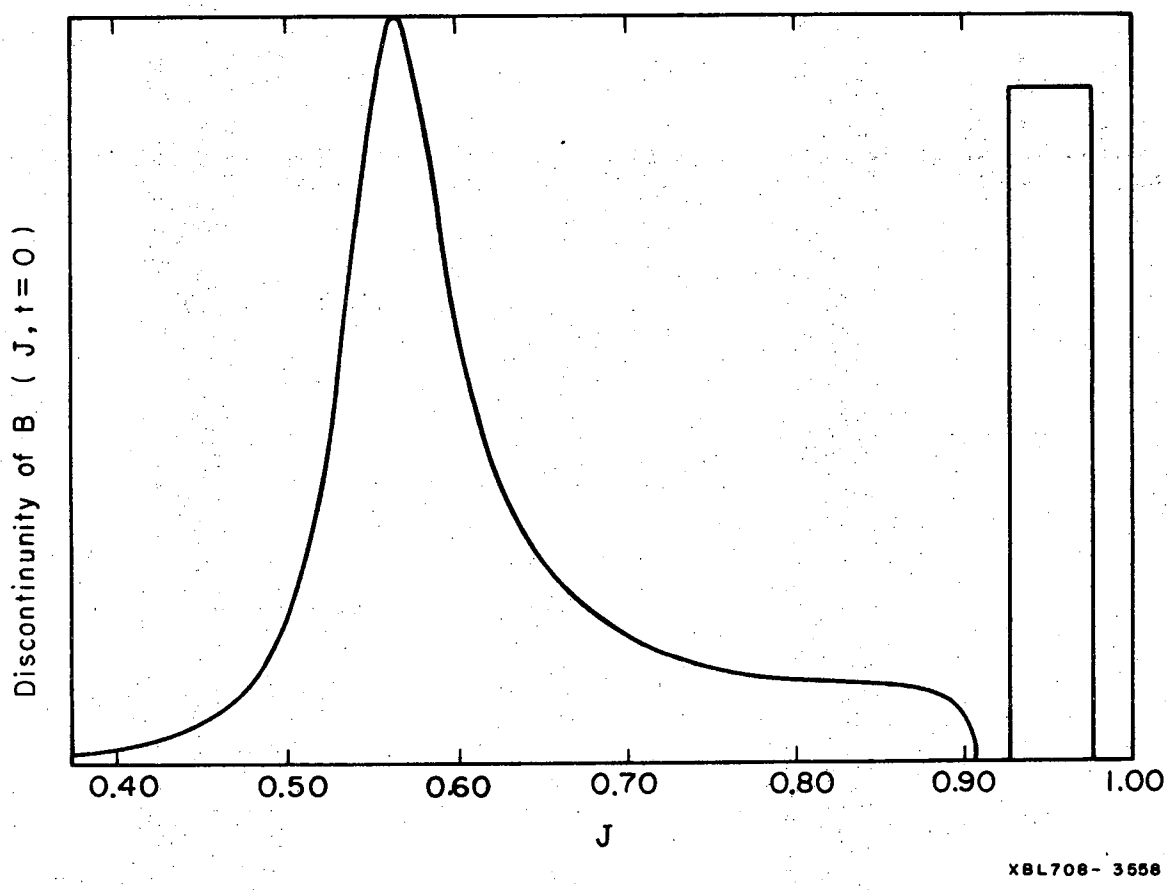

Discontinunity of B (J, t = 0)
0.40
0.50
0.60
0.70
0.80
0.90
1.00
J
XBL708- 3658

Fig. 5

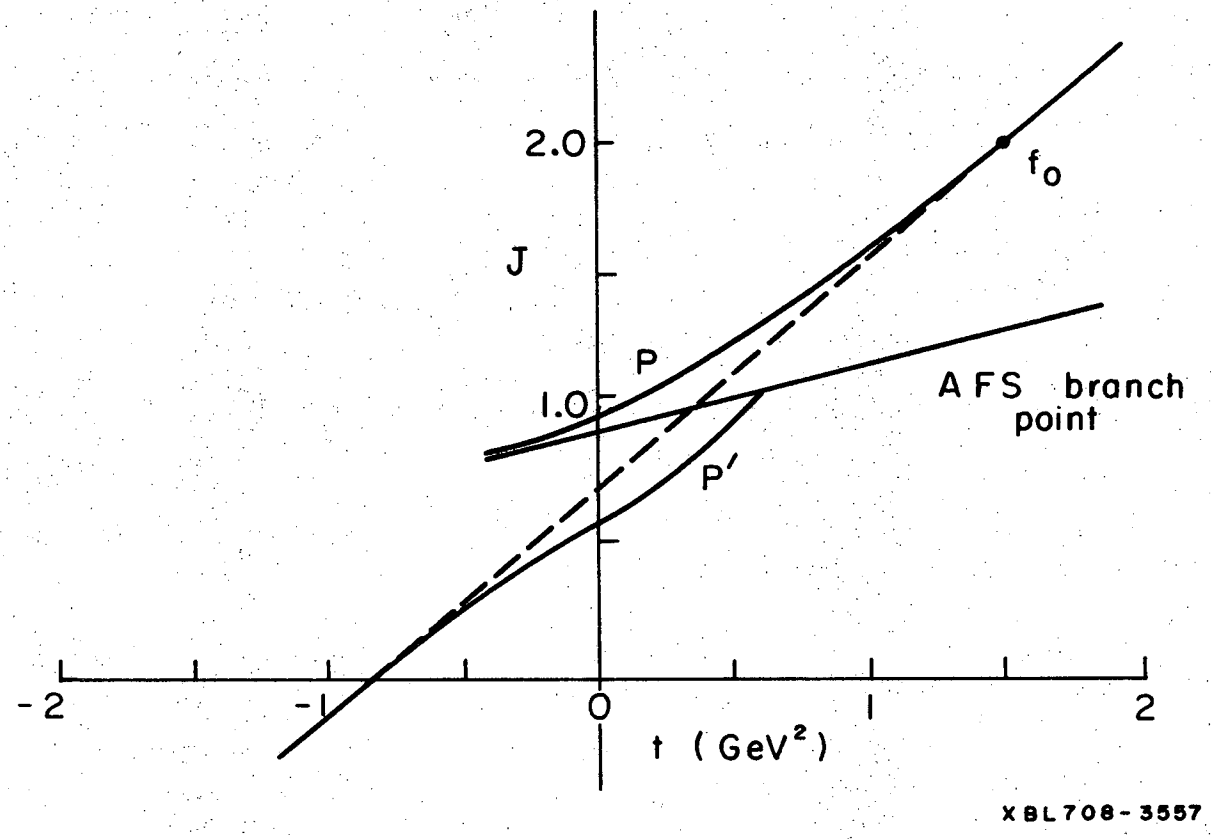


Fig. 6

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