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### Publication Date

1999-03-01

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# ERNEST ORLANDO LAWRENCE BERKELEY NATIONAL LABORATORY

## Quantum Corrections from Nonresonant WW Scattering

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**Physics Division**

March 1999

To be published in the  
Lev Okun festschrift  
volume of *Physics Reports C*



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March 22, 1999

LBNL-43000

## Quantum corrections from nonresonant $WW$ scattering <sup>1</sup>

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### Abstract

An estimate is presented of the leading radiative corrections to low energy electroweak precision measurements from strong nonresonant  $WW$  scattering at the TeV energy scale. The estimate is based on a novel representation of nonresonant scattering in terms of the exchange of an effective scalar propagator with simple poles in the complex energy plane. The resulting corrections have the form of the corrections from the standard model Higgs boson with the mass set to the unitarity scale for strong  $WW$  scattering.

*To be published in the Lev Okun festschrift volume of Physics Reports C*

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<sup>1</sup>This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

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## Prologue

I first met Lev Okun at the 1976 “Rochester” conference held in the USSR, in Tbilisi, Georgia. Sakharov was under strong attack by the government for his human rights activities and was originally not invited but was permitted to attend after he protested the lack of an invitation to the Soviet Academy. Understandably even those Soviet physicists who were sympathetic to Sakharov and his ideas were cautious about associating with him during the meeting. While there may well have been others I did not observe, to me it was remarkable to see one Soviet physicist who did not hesitate to stroll openly with Sakharov on the streets of Tbilisi. This was of course Okun. His behavior then demonstrated the same simple idealism and courage that is reflected now in the decision he has taken since the dissolution of the USSR to remain in Moscow, to preserve the unique physics environment at ITEP, when he could easily have accepted more comfortable positions outside of Russia.

Not unrelated to his moral character is the clarity, depth and humanity with which Okun practices physics. This gives me a selfish reason for submitting the work presented here: I would like to have his view of it. It has a plausible conclusion reached by a strange method and raises questions I do not understand. It is based on a representation of an exactly unitary model of non-resonant  $WW$  scattering in terms of an effective scalar propagator with simple poles in the complex energy plane. The method was applied and verified for tree approximation amplitudes and is used here to estimate quantum corrections.

## Introduction

The electroweak symmetry may be broken by weakly coupled Higgs bosons below 1 TeV or by a new sector of quanta at the TeV scale that interact strongly with one another and with longitudinally polarized  $W$  and  $Z$  bosons. Precision electroweak data favors the first scenario[1, 2], but the conclusion is not definitive, because the relevant quantum corrections are open to contributions from many forms of new physics. Occam’s (an archaic spelling of Okun’s?) razor favors the simplest interpretation, which assumes that the only new physics contributing significantly are the quanta that directly form the symmetry breaking condensate. In that case the data do favor weak symmetry breaking by Higgs scalars. But nature may have dealt us a more complicated hand, with other,

probably related, new physics also contributing to the radiative corrections. Then the precision data tells us nothing about the symmetry breaking sector — unless we can “unscramble” the different contributions, which in general we do not know how to do — and implementation of the Higgs mechanism by strong, dynamical symmetry breaking remains a possibility. The nature of the symmetry breaking sector can only be established definitively by its direct discovery and detailed study in experiments at high energy colliders.

Strong  $WW$  scattering is a generic feature of strong, dynamical electroweak symmetry breaking.[3] The longitudinal polarization modes  $W_L$  scatter strongly above 1 TeV because the enforcement of unitarity is deferred to the mass scale of the heavy quanta that form the symmetry breaking condensate. To the extent that QCD might be a guide to dynamical symmetry breaking we expect the  $a_{00}$  partial wave to smoothly saturate unitarity between 1 and 2 TeV. Like the SM (standard model) Higgs boson, nonresonant strong  $WW$  scattering would also contribute to the low energy radiative corrections probed in precision electroweak measurements. This note presents an estimate of those corrections, based on a novel representation of nonresonant strong  $WW$  scattering as an effective-Higgs boson exchange amplitude.

Strong  $WW$  scattering models are customarily formulated in R-gauges. The effective-Higgs representation allows them to be reexpressed gauge invariantly and, in particular, in unitary gauge.[4, 5] It applies to the leading  $s$ -wave amplitudes with  $I = 0, 2$ . The effective-Higgs representation has a significant practical advantage: it predicts the experimentally important transverse momentum distributions of the final state quark jets and the  $WW$  diboson in the collider process  $qq \rightarrow qqWW$ , which cannot be obtained from the conventional method based on the effective  $W$  approximation.[6] The method has been verified numerically for tree amplitudes[4] and gauge (i.e., BRST) invariance has been demonstrated.[5]

The K-matrix model is a useful model of strong  $WW$  scattering which smoothly extrapolates the  $WW$  low energy theorems[3, 7] in a way that exactly satisfies elastic unitarity. The effective-Higgs representation of the K-matrix model has a surprisingly simple form: the singularities of the propagator are simple poles in the complex  $s$  plane, like an elementary scalar. It is then easy to compute the contribution to the  $W$  and  $Z$  vacuum polarization tensors from

which the ‘‘oblique’’ corrections[8, 9] are obtained.

The final result for the oblique parameters  $S$  and  $T$  is like the SM Higgs contribution with  $m_H$  replaced by a combination of the unitarity scales for strong scattering in the  $I = 0, 2$  channels, determined in turn by the low energy theorems as noted in [3].  $S$  and  $T$  are given by

$$S = \frac{1}{18\pi} \left[ \ln \left( \frac{16\pi v^2}{\mu^2} \right) + \frac{1}{2} \ln \left( \frac{32\pi v^2}{\mu^2} \right) \right] \quad (1)$$

$$T = \frac{-1}{8\pi \cos^2 \theta_W} \left[ \ln \left( \frac{16\pi v^2}{\mu^2} \right) + \frac{1}{2} \ln \left( \frac{32\pi v^2}{\mu^2} \right) \right] \quad (2)$$

where  $v^2 = (\sqrt{2}G_F)^{-1}$ ,  $\theta_W$  is the weak interaction mixing angle and  $\mu$  is the reference scale. For  $\mu = 1$  TeV the corrections are  $S \simeq 0.036$  and  $T \simeq -0.11$ . Similar results follow from the cut-off nonlinear sigma model when the unitarity scales are used for the cutoffs.[10]

In the following sections I review the K-matrix model, derive the effective scalar propagator, deduce the oblique corrections, raise some theoretical issues, and finally discuss the physical interpretation of the result.

#### K-matrix model for $WW \rightarrow ZZ$

In the SM the Higgs sector contribution to  $WW \rightarrow ZZ$  is given by just the  $s$ -channel Higgs pole. Therefore we use the K-matrix model for  $WW \rightarrow ZZ$  to abstract the effective-Higgs propagator. The model is summarized in this section.

As is conventional we use the ET[11] (equivalence theorem) to define the model in terms of the unphysical Goldstone bosons,  $w^\pm$  and  $z$ . Partial wave unitarity is conveniently formulated as

$$\text{Im} \frac{1}{a_{IJ}} = -1. \quad (3)$$

The K-matrix model is constructed to satisfy the low energy theorems and partial wave unitarity. It is defined by

$$\frac{1}{a_{IJ}^K} = \frac{1}{R_{IJ}} - i \quad (4)$$

where  $R_{IJ}$  are the real threshold amplitudes that follow from the low energy theorems,

$$R_{00} = \frac{s}{16\pi v^2} \quad (5a)$$

$$R_{20} = \frac{-s}{32\pi v^2}. \quad (5b)$$

The corresponding  $s$ -wave T-matrix amplitudes are

$$\mathcal{M}_I^K(s) = 16\pi a_{I0}^K \quad (6)$$

for  $I = 0, 2$ . Finally the  $ww \rightarrow zz$  amplitude is

$$\mathcal{M}^K(w^+w^- \rightarrow zz) = \frac{2}{3}(\mathcal{M}_0^K - \mathcal{M}_2^K) \quad (7)$$

### Effective-Higgs propagator

To obtain the effective-Higgs propagator we “transcribe” the K-matrix model from R-gauge to U-gauge.[4, 5] The heart of the matter is to find the contribution of the symmetry-breaking sector in U-gauge, which encodes the dynamics specified in the original R-gauge formulation of the model. This is accomplished using the ET as follows.

Suppose that the longitudinal gauge boson modes scatter strongly. At leading order in the weak gauge coupling  $g$  we write the amplitude  $W_L^+W_L^- \rightarrow ZZ$  as a sum of gauge-sector and Higgs-sector terms,

$$\mathcal{M}_{\text{Total}} = \mathcal{M}_{\text{Gauge}} + \mathcal{M}_{\text{SB}} \quad (8)$$

where SB denotes the symmetry breaking (i.e., Higgs) sector. Gauge invariance ensures that the contributions to  $\mathcal{M}_{\text{Gauge}}$  that grow like  $E^4$  cancel, leaving a sum that grows like  $E^2$ , given by

$$\mathcal{M}_{\text{Gauge}} = g^2 \frac{E^2}{\rho m_W^2} + \mathcal{O}(E^0, g^4) \quad (9)$$

where  $\rho = m_W^2/(\cos^2\theta_W m_Z^2)$ . The neglected terms of order  $E^0$  and of higher order in  $g^2$  include the electroweak corrections to the leading strong amplitude.

The order  $E^2$  term in equation (9) is the residual “bad high energy behavior” that is cancelled by the Higgs mechanism. It is also precisely the low energy theorem amplitude,

$$\mathcal{M}_{\text{LET}} = \frac{s}{\rho v^2} = \mathcal{M}_{\text{Gauge}} + \mathcal{O}(s^0, g^4) \quad (10)$$



using  $m_W = gv/2$  and  $s = 4E^2$ . Eqs. (8) and (9) may be used to derive the low energy theorem without invoking the ET.<sup>3</sup>

Now consider an arbitrary strong scattering model, designated as model “X”, formulated in the usual way in an R-gauge in terms of the unphysical Goldstone bosons,  $\mathcal{M}_{\text{Goldstone}}^X(ww \rightarrow zz)$ . The total gauge boson amplitude is gauge invariant and the ET tells us that for  $E \gg m_W$  it is approximately equal to the Goldstone boson amplitude, i.e.,

$$\mathcal{M}_{\text{Total}}^X(W_L W_L) \simeq \mathcal{M}_{\text{Goldstone}}^X(ww) \quad (11)$$

in the same approximation as eq. (9). Eq. (8) holds in any gauge. Specifying U-gauge we combine it with eqs. (9-11) to obtain the U-gauge Higgs sector contribution for model X,

$$\mathcal{M}_{\text{SB}}^X(W_L W_L) = \mathcal{M}_{\text{Goldstone}}^X(ww) - \mathcal{M}_{\text{LET}}. \quad (12)$$

The preceding result applies to any strong scattering amplitude. Now we specialize to  $s$ -wave  $WW \rightarrow ZZ$  scattering and use eq.(12) to obtain an effective-Higgs propagator with standard “Higgs”-gauge boson couplings. Neglecting  $m_W^2 \ll s$  and higher orders in  $g^2$  as always, the effective scalar propagator is

$$P_X(s) = -\frac{v^2}{s^2} \mathcal{M}_{\text{SB}}^X(W_L W_L) \quad (13)$$

Eqs.(10) and (12) with  $\rho = 1$  then imply

$$P_X(s) = -\frac{v^2}{s^2} \mathcal{M}_R^X(ww) + \frac{1}{s} \quad (14)$$

The term  $1/s$ , corresponding to a massless scalar, comes from  $\mathcal{M}_{\text{LET}}$  in eq. (12). It ensures good high energy behavior while the other term in eq. (14) expresses the model dependent strong dynamics.

Finally we substitute the K-matrix amplitude, eq. (7), into eq. (14) to obtain the effective propagator for the K-matrix model as the sum of two simple poles

$$P_K = \frac{2}{3} \left( \frac{1}{s - m_0^2} + \frac{1}{2} \frac{1}{s - m_2^2} \right) \quad (15)$$

---

<sup>3</sup> If the symmetry breaking force is strong, the quanta of the symmetry breaking sector are heavy,  $m_{SB} \gg m_W$ , and decouple in gauge boson scattering at low energy,  $\mathcal{M}_{SB} \ll \mathcal{M}_{\text{Gauge}}$ . Then the quadratic term in  $\mathcal{M}_{\text{Gauge}}$  dominates  $\mathcal{M}_{\text{Total}}$  for  $m_W^2 \ll E^2 \ll m_{SB}^2$ , which establishes the low energy theorem without using the ET.[7]

where  $m_0$  and  $m_2$  are

$$m_0^2 = -16\pi v^2 \quad (16)$$

and

$$m_2^2 = +32\pi v^2. \quad (17)$$

It is surprising to find such a simple expression involving only simple poles. It is not surprising that the poles are far from the real axis since they describe nonresonant scattering. Interpreted heuristically as Breit-Wigner poles they correspond to resonances with widths twice as big as their masses.

### Oblique corrections

The oblique corrections are evaluated from the vacuum polarization diagrams that in the SM include the Higgs boson.[8] In place of the SM propagator,  $P_{SM} = 1/(s - m_H^2)$ , we substitute  $P_K$  from eq. (15). Where the SM contribution depends on the log of the Higgs boson mass,  $L_{SM} = \ln(m_H^2/\mu^2)$ , we now find instead the combination  $L_K$ ,

$$L_{SM} = \ln\left(\frac{m_H^2}{\mu^2}\right) \rightarrow L_K = \frac{2}{3} \ln\left(\frac{m_0^2}{\mu^2}\right) + \frac{1}{3} \ln\left(\frac{m_2^2}{\mu^2}\right) \quad (18)$$

where  $m_{0,2}$  are complex masses defined in eqs. (16-17).

The results quoted in eqs. (1-2) follow from the usual expressions for  $S, T$  where we use the real part of  $L_K$  in place of  $L_{SM}$ ,

$$S = \frac{\text{Re}(L_K)}{12\pi} \quad (19)$$

and

$$T = \frac{-3 \text{Re}(L_K)}{16\pi \cos^2\theta_W} \quad (20)$$

The imaginary part of  $L_K$  is an artifact which we discard; it results from the fact that our approximation neglects the  $W$  mass, as in any application of the ET. At  $q^2 = 0$ , where the oblique corrections are computed, there is no contribution to the imaginary part of the vacuum polarization from the relevant diagrams.

Combining the  $I = 0$  and  $I = 2$  terms in eq. (18) we have

$$\text{Re}(L_K) = \ln\left(\frac{2^{1/3}16\pi v^2}{\mu^2}\right). \quad (21)$$

Evaluating eq. (21) we find that the oblique correction from the K-matrix model is like that of a Higgs boson with mass 2.0 TeV.

### Questions

The  $I = 2$  component of the effective propagator has peculiar properties, perhaps due to the fact that for the  $I = 2$  channel we are representing  $t$ - and  $u$ -channel dynamics by an effective  $s$ -channel exchange. The minus sign in the  $I = 2$  low energy theorem, eq. (5b), which may be thought of as arising from the identity  $t + u = -s$ , leads to interesting differences between the  $I = 0$  and  $I = 2$  components of the effective propagator  $P_K$ .

First, the  $I = 2$  component of the effective scalar propagator has a negative pole residue, which would correspond to a unitarity violating ghost if it described an asymptotic state (which it does not). In fact the sign is required to *ensure* unitarity, since it is needed to cancel the bad high energy behavior of the gauge sector amplitude which has a negative sign in the  $I = 2$  channel. In eq. (15) for  $P_K$  the  $I = 2$  pole appears with a positive sign because of a second minus sign from the isospin decomposition, eq. (7). Neither pole of the effective propagator has a negative (ghostly) residue. In any case the amplitude is exactly unitary by construction.

The sign difference between the pole positions,  $m_0^2$  and  $m_2^2$  in eqs. (16) and (17), may also be traced to the phases of the low energy theorems in eq. (5). The position of  $m_0^2$  on the negative imaginary axis of the complex  $s$  plane corresponds to poles in the fourth and second quadrants of the complex energy plane, consistent with causal propagation as in the conventional  $m^2 - i\epsilon$  prescription. But the position of  $m_2^2$  on the positive imaginary axis corresponds to poles in the first and third quadrants of the complex energy plane. This would imply acausal propagation if the poles are on the first sheet but not if they are on the second sheet. Working in the limit of massless external particles as we are it is not apparent on which sheet they occur.<sup>4</sup>

I conclude that the sign of the pole residue arising from the  $I = 2$  amplitude is not problematic but that the implications of the pole position requires better understanding.

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<sup>4</sup>I thank Henry Stapp for a discussion of this point.

### Physical interpretation

We have used a convenient representation of the K-matrix model to estimate the low energy radiative corrections from strong  $WW$  scattering. The result that the corrections are like those of a Higgs boson with mass at the unitarity scale is plausible and agrees with an earlier estimate using the cut-off nonlinear sigma model.[10] The estimate establishes a ‘default’ radiative correction from the strongly coupled longitudinal gauge bosons in theories of dynamical symmetry breaking. In general there will be additional contributions from other quanta in the symmetry breaking sector. Those contributions are model dependent as to magnitude and sign. In computing their effect it is important to avoid double-counting contributions that are dual to the contribution considered here.

Current SM fits to the electroweak data prefer a light Higgs boson mass of order 100 GeV with a 95% CL upper limit that I will conservatively characterize as  $\lesssim 300$  GeV.[2] Since the corrections computed here are equivalent to those of a Higgs boson with a mass of 2 TeV, they are excluded at 4.5 standard deviations. Therefore there must be additional, cancelling contributions to the radiative corrections from other quanta in the theory if strong  $WW$  scattering occurs in nature. This would not require fine-tuning although it would require a measure of serendipity.

There are good reasons for the widespread view that a light Higgs boson is likely and for the popular designation of SUSY (supersymmetry) as The People’s Choice. But SUSY also begins to require a measure of serendipity[12] to meet the increasing lower limits on sparticle and light Higgs boson masses. While the community of theorists has all but elected SUSY, the question is not one that can be decided by democratic processes. At the end of the day only experiments at high energy colliders can tell us what the symmetry breaking sector contains. Collider experiments, particularly those at the LHC, should be prepared for the full range of possibilities, including the capability to measure  $WW$  scattering in the TeV region.

*Acknowledgements:* I wish to thank David Jackson, and Henry Stapp for helpful discussions. This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contracts DE-AC03-76SF00098.

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