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Publication Date

1993

**CALIFORNIA PATH PROGRAM
INSTITUTE OF TRANSPORTATION STUDIES
UNIVERSITY OF CALIFORNIA, BERKELEY**

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**PATH Working Paper
UCB-ITS-PWP-93-2**

This work was performed as part of the California PATH Program of the University of California, in cooperation with the State of California Business, Transportation, and Housing Agency, Department of Transportation; and the United States Department of Transportation, Federal Highway Administration.

June 1993

ISSN 1055-1425

A PROBABILISTIC MODEL AND A SOFTWARE TOOL FOR AVCS LONGITUDINAL COLLISION/SAFETY ANALYSIS

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ABSTRACT

This paper develops a probabilistic model and a software **tool** for analyzing longitudinal collision/safety between two automated vehicles. The input **parameters are** the length of the gap between the two vehicles, the common ***speed*** prior to the failure, the reaction delay of the following vehicle and a bivariate joint distribution of the deceleration rates of **the** two vehicles. **The** output includes the probability of a collision and **also** the probability distribution of the relative speed **at** collision time.

We will **use** this model **to** compare the safety consequences associated with the platooning and "free-agent" vehicle-following rules. We will **also** demonstrate that the free-agent vehiclefollowing rule implemented with a potential technology of fast and accurate emergency deceleration, under some reasonable conditions, can virtually avoid collisions while offering a high freeway capacity previously thought possible only under the platooning rule.

KEY WORDS: AVCS, Collision Speed Distribution, Platooning, Free-Agent

(1) INTRODUCTION

An Advanced Vehicle Control System (AVCS) consists of two major components: vehicle automation technology and freeway operating strategy. A full-automation technology integrates the communication technology between vehicles and between vehicles and roadside, sensing technology and sophisticated automatic vehicle control. An operating strategy is a collection of operating rules that govern the movement of automated vehicles based on their capability and reliability. Five major categories of operating rules are vehicle following, lane change, lane selection, automated access and automated egress.

Two primary objectives of Advanced Vehicle Control Systems (AVCS) are enhancements of highway capacity and safety. Capacity gain is achieved by reducing the average spacing, longitudinal and lateral, between vehicles. Safety improvement comes from the removal of human errors, which currently account for more than 90% of the total number of roadway accidents. However, automation may introduce new kinds of safety hazards through possible failures of the additional equipment, the roadside control system and the communication system. Any of these failures may lead to collisions of a vehicle with other vehicles or with objects on the roadway. Although a vehicle may be functioning perfectly, its behavior may still be hazardous because it has received incorrect insbuctions from the roadside control system. To simplify the discussion in this paper, we will always refer to the vehicle with hazardous behavior as the ***failed vehicle.*** For the same reason, the deceleration of the failed vehicle and that of its immediate follower will be referred to as ***failure deceleration*** and ***emergency deceleration*** respectively.

For a given automation technology, different operating strategies for AVCS will result in different degrees of capacity and safety enhancements. Among the five categories of operating rules, the vehicle-following category has been the focal point of recent studies because of its direct impact on both the capacity and safety enhancements. Central to any vehicle-following rule is the longitudinal spacing, i.e. the length of the gap between two adjacent vehicles. This paper concentrates on the safety consequences of this spacing. A vehicle failure under different vehiclefollowing rules will result in collisions of different severity with different probability. We will use ***collision speed,*** i.e. relative ***speed*** between two colliding vehicles at the time of collision, as a *surrogate* for collision severity.

A Probabilistic Model

Shladover *[6]* pointed out that different investigators made radically different assumptions about acceptable safety levels and derived radically different capacity estimates for the automated freeway. The differences in safety assumptions generally revolve around different answers to the following questions:

(a) How rapidly does a failed vehicle decelerate?

(b) How long does it take a vehicle to detect a failure of its predecessor?

(c) How rapidly can a following vehicle decelerate using its brakes to avoid a collision?

(d) Are low-impact-velocity collisions tolerable or not?

(e) Should the spacing between vehicles be determined based on safe accommodation of a single failure or must it accommodate combinations of multiple failures (such as an overspeed or brake failure of the following vehicle when the failed leading vehicle is decelerating)?

Clearly, answering these questions involves a *high degree of uncertainty* and the crucial issue about AVCS safety ***requires probabilistic analyses.*** We will, in this paper, develop a probabilistic model and a software tool for obtaining the probability of collision and the distribution of collision speed given the occurrence of a vehicle failure, in relation to the first three of the factors above. This model and tool can be used to help determine safe distances between two vehicles, the target rate of deceleration of the vehicle following a failed vehicle, the specification for the accuracy and the response time of the braking system, the specification for the response time of the communication system.

The input parameters considered in our probabilistic model are the spacing between the two vehicles, the common speed prior to the failure, the deterministic reaction delay of the following vehicle and a bivariate joint distribution of the deceleration rates of the two vehicles. The bivariate deceleration distribution is needed to allow possible correlation between the two random deceleration rates due to the common driving condition, e.g. slippery road condition on a rainy day. The bivariate deceleration distribution can be *any* discrete bivariate probability distribution over *any* possible finite state space.

The output will be the probability of a collision and also the probability distribution of the collision speed (Δv, i.e. the relative speed at the time of collision). The collision speed distribution will be expressed as a histogram. Parametric study can be conducted by varying the input parameters and examining the resulting collision probability and Δv distribution.

The development of this model and computer tool was motivated by many important questions regarding the safety consequences of longitudinal collisions. We give two simple examples:

Example 1 A Lower Bound for the Minimum Longitudinal Safety Spacing.

Assume that when a vehicle decelerates due to a failure, all the trailing vehicles within a reasonable range receive simultaneously the distress signal and start decelerating at a common and constant target rate (after a common delay). A possible minimum safety requirement is that the minimum safety spacing be long enough so that the probability of an initial collision between any two adjacent trailing vehicles is no larger than a given number. Assuming that the actual deceleration rate of the trailing vehicles is constant but random due to mechanical limitation, we need to calculate the collision probability.

Example 2 Determination of Target Emergency Deceleration Rate for the Trailing Vehicles.

Given an estimate of the deceleration rate of the failed vehicle, we may wish to determine the target emergency deceleration rate for its immediate follower and the rest of the trailing vehicles. Since different target deceleration rates may be realized with different errors and larger errors tend to lead to collisions among the trailing vehicles, to determine the target deceleration rate, we also need to analyze the interaction between the failed vehicle and its immediate follower.

The most complicated input to the model is the bivariate distribution for the two deceleration rates. To justify particular selections for it in the absence of data on the future technology or simply to facilitate the complex task of its determination, we will use the *Principle of Maximum Entropy* to derive a discrete bivariate distribution that satisfies user-specified marginal expectations, marginal standard deviations and coefficient of correlation. This distribution can be determined by solving a convex mathematical programming problem with linear equality constraints. The theoretical justification of this principle actually translates into the conservativeness appropriately ***required*** in the safety study of **this** kind. **The associated** computer **programs are** also included **as part** of **the** software tool. The adoption of this principle together with **the** discrete representation of the joint distribution of the **two** deceleration rates enable realistic and efficient parametric probabilistic studies of AVCS longitudinal ***safety.***

Two Basic Vehicle Following Rules

Two basic vehiclefollowing rules ***are*** the ***platooning rule*** and **the** ***free-agent rule.*** The platooning rule was first proposed and studied by Shladover in the late 70's [5] and **has** received renewed attention in the last few years. Under **this** rule, two adjacent vehicles in the same **lane are** kept either very close to or very far from each other. ***As*** a result, vehicles **are** organized in a clustered formation. Each cluster of vehicles **is** called a ***platoon.*** All vehicles within a platoon, except the leader, **are** under continuous feedback control which maintains a very close spacing with **the** vehicle in front while adjacent **platoons** in the same lane **are separated** with a large spacing. To predict the freeway capacity increase provided by platooning, Shladover [5] made a number of assumptions, including a common vehicle length of **3.05** meters and a capacity reservation of **20%** for lane changes. Based on the calculations for platoon sizes of 1, 2, 5, **10 and 20,** he showed that the capacity **increases** significantly with platoon size. **This** rule fully utilizes the fact that, when a failure **occurs,** it is safer if **the** vehicles **are** either very close to each other **or** very far **apart.** The large inter-platoon separation can minimize the probability of any collision between platoons in the same lane and the close intra-platoon spacing ensures that any collision within a platoon will have a small relative ***speed.*** Under the free-agent rule, vehicles move without any clustered formation and the minimum longitudinal spacing is significantly longer than typical intra-platoon spacings, but significantly shorter than typical inter-platoon spacings.

The validity of the platooning concept hinges upon the crucial assumption that a failure would lead to only low relative **speed** collisions between vehicles in one lane. If **this** assumption proves to be true, then the platooning rule should be safer than the free-agent rule. However, ***so*** far very little is known about what other collisions may occur after the initial low-relative-speed collision. Could this initial collision lead **to** vehicles' skidding, spinning **or** swaying into other lanes? Could the low-relative-speed collisions cause some of the sensors or other on-board automation devices to malfunction and the vehicle to become out-of-control? Tongue [7] is investigating the consequences of low-relative-speed collisions using the technique of computer simulation. The sufficiency of his simulation study for determining the validity of the assumption is yet to be determined. Hitchcock [3] proposed the idea of a "lane barrier" to prevent the spilling of a traffic accident from one lane to another and the idea of "gates" to allow lane changes. Even with these barriers, a low-relative-speed collision may lead to skidding and spinning, which in turn may lead to collisions between vehicles and the barriers and to other dangerous situations. *Also,* spilling over is still possible at the gates. The major weakness of the free-agent rule is that in the event of a collision, Δv tends to be more severe in comparison to the platooning rule. Advantages of the free-agent rule include simplified control protocols and more stable traffic flow.

The above uncertainties suggest that we should not rule out the free-agent rule. In addition, the possibility of fast emergency deceleration, which has the potential of avoiding collisions with short spacing, has not been fully explored in the literature.

A probabilistic Comparison Between the Platooning and Free-Agent Rules

We will use the probabilistic model to compare these two basic vehicle-following rules. This probabilistic comparison extends the deterministic analysis by Shladover [5] and other authors. We will further demonstrate that, with ***fast and accurate emergency bruking*** and under some other assumptions about the automation technology of the future, the free-agent rule might guarantee no collision after a failure while offering the high capacity thought possible with platooning.

Organization of the Paper

This paper is organized as follows: Section 2 explains our probabilistic approach. Section 3 contains the solution to this general problem. Section 4 briefly discusses the concept of maximum entropy and its role in our approach. The computer tool and the major modules are discussed in Section 5. Section 6 is devoted to the comparison between the two basic vehiclefollowing rules. Section 7 contains the concluding remarks.

(2) A PROBABILISTIC APPROACH

The goal is to provide the collision probability and distribution of collision speed for any given combination of the four input quantities:

(I1) common speed prior to deceleration,

(I2) spacing between the two vehicles,

(I3) reaction delay of the rear vehicle,

(I4) correlated bivariate distribution of the two deceleration rates.

The first three are deterministic and hence trivial to represent. The difficult input is the fourth one, the bivariate distribution. The ideal would be to allow any possible probability distribution as an input. But this is impractical because their representation and manipulation are intractable. Also, the use of most standard probability density functions cannot be convincingly justified for our problem. Therefore, we choose to discretize the domain of possible deceleration rates (i.e. select a finite number of possible deceleration rates as the only possible rates) and use the set of all possible discrete probability distributions over this domain to represent the input distributions. Note that discretization is a powerful tool because it can be used to approximate all probability distributions to any desired accuracy.

The assumptions of our model are:

(A1) Two vehicles are moving on a straight lane at a common speed prior to the failure.

(A2) The failed vehicle decelerates at a constant but random rate.

(A3) The following vehicle decelerates at a constant but random rate after a reaction delay (if it has not already collided with the failed vehicle).

(A4) The two rates are possibly correlated.

We use a two-dimensional coordinate system to represent the position of the two vehicles as a function of time. The horizontal axis represents the time and the time of failure is the origin, i.e. the deceleration of the front vehicle occurs at time zero. The vertical axis represents vehicle position, with the origin set at the position of the rear end of the front vehicle at the time when the front vehicle fails. We now introduce some notation, which is depicted in Figure 1:

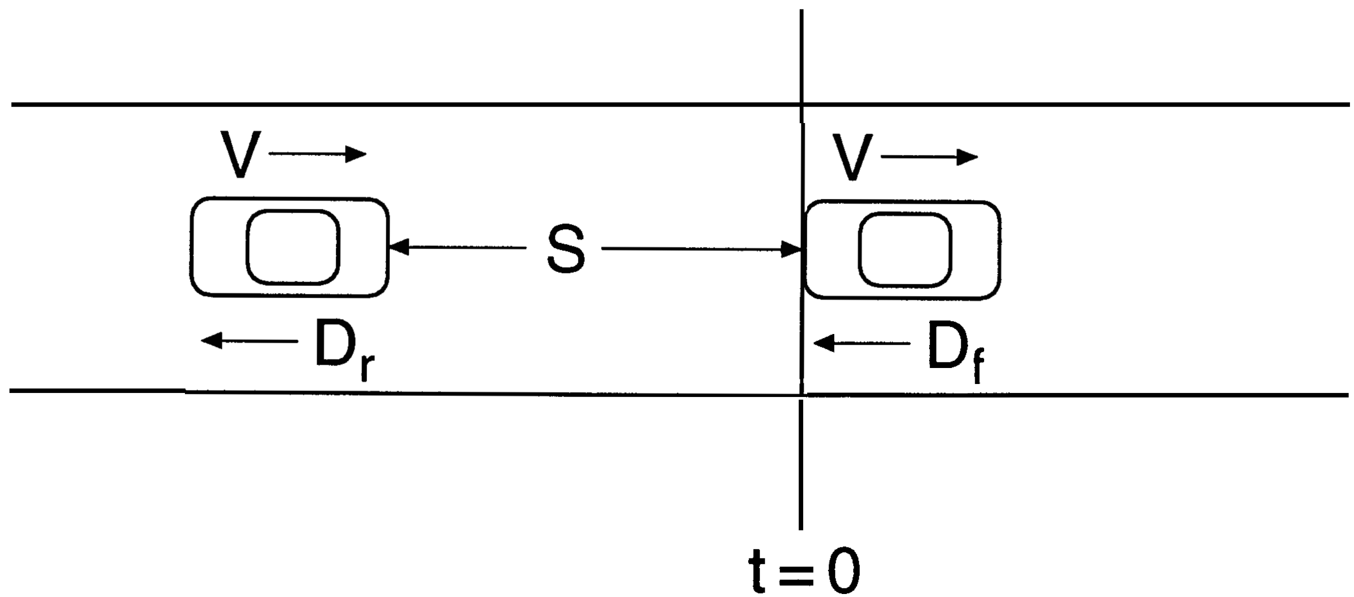

Figure 1: Initial Condition at t=0

$V \equiv$ known common speed prior to failure.

$S \equiv$ spacing between the two vehicles; more precisely, the distance between the rear end of the front vehicle and the front end of the rear vehicle.

$T \equiv$ the reaction time (delay), i.e. the time between start of deceleration of the front vehicle and the start of deceleration of the rear vehicle (if it has not already collided with the failed vehicle).

$D_f \equiv$ the random deceleration rate of the front vehicle.

$D_r \equiv$ the random deceleration rate of the rear vehicle.

$D \equiv$ the set of all possible deceleration rates (for both vehicles).

$p(d_f, d_r) \equiv$ the probability of $D_f = d_f$ and $D_r = d_r$; this defines the bivariate distribution of the two deceleration rates.

$\Delta v(d_f, d_r, S, V) \equiv$ the speed difference at collision given d_f, d_r, S and V. For ease of notation, this will simply be abbreviated as Δv.

$t \equiv$ the elapsed time after the start of the front vehicle's deceleration. In particular, the failure occurs at $t=0$.

$x_f(t) \equiv$ the position of the rear end of the front vehicle at time t, in absence of collision. In particular, $x_f(0)=0$.

$x_r(t) \equiv$ the position of front end of the rear vehicle vehicle at time t, in absence of collision.

$v_f(t) \equiv$ the speed of the front vehicle at time t.

$v_r(t) \equiv$ the speed of the rear vehicle at time t.

To find the probability distribution of Δv, we first determine, given a particular pair of deceleration rates $D_f = d_f$ and $D_r = d_r$, if the two vehicles collide at all and, if so, when they do. We can then determine their respective speeds and the difference. Finally, adding up the probabilities associated with the pairs (d_f, d_r) that lead to the same collision speed produces the desired distribution, which will be expressed as a histogram.

To determine if the two deceleration rates d_f and d_r would lead to a collision, we use the following approach. Since a collision can only take place while the rear vehicle is moving, and the rear vehicle stops at $t = T + \frac{V}{d_r}$ in absence of collision, we need only pay attention to the time period $(0, T + \frac{V}{d_r})$. We will refer to this period as *the relevant interval.* It is obvious that *the two vehicles would collide if and only if the two curves defined by $x_f(t)$ and $x_r(t)$ intersect in the relevant interval.* If they intersect multiple times, the earliest crossing time is the collision time.

In absence of collision, the trajectory for the front vehicle is:

$$x_f(t) = Vt - \frac{1}{2} d_f t^2 \quad \text{if } t \in [0, \frac{V}{d_f}];$$

$$\frac{1}{2} \frac{V^2}{d_f} \quad \textit{otherwise.}$$

In absence of collision, the trajectory for the rear vehicle is:

$$x_r(t) = Vt - S \quad \text{if } t \in [0, T]$$

$$Vt - \frac{1}{2} d_r (t-T)^2 - S \quad \text{if } t \in [T, T + \frac{V}{d_r}]$$

$$VT + \frac{1}{2} \frac{V^2}{d_r} - S \quad \text{if } t \in [T + \frac{V}{d_r}, \infty).$$

For convenience of discussion, the curve $x_f(t)$ will also be referred to as the front trajectory while $x_r(t)$ the rear trajectory. An example $x_r(t)$ is shown in Figure 2. Note that the first piece of the curve is a straight line covering the time period $(0, T)$, and this results from the fact that the rear vehicle maintained the speed and has not started to decelerate yet. The second piece is a quadratic curve reflecting

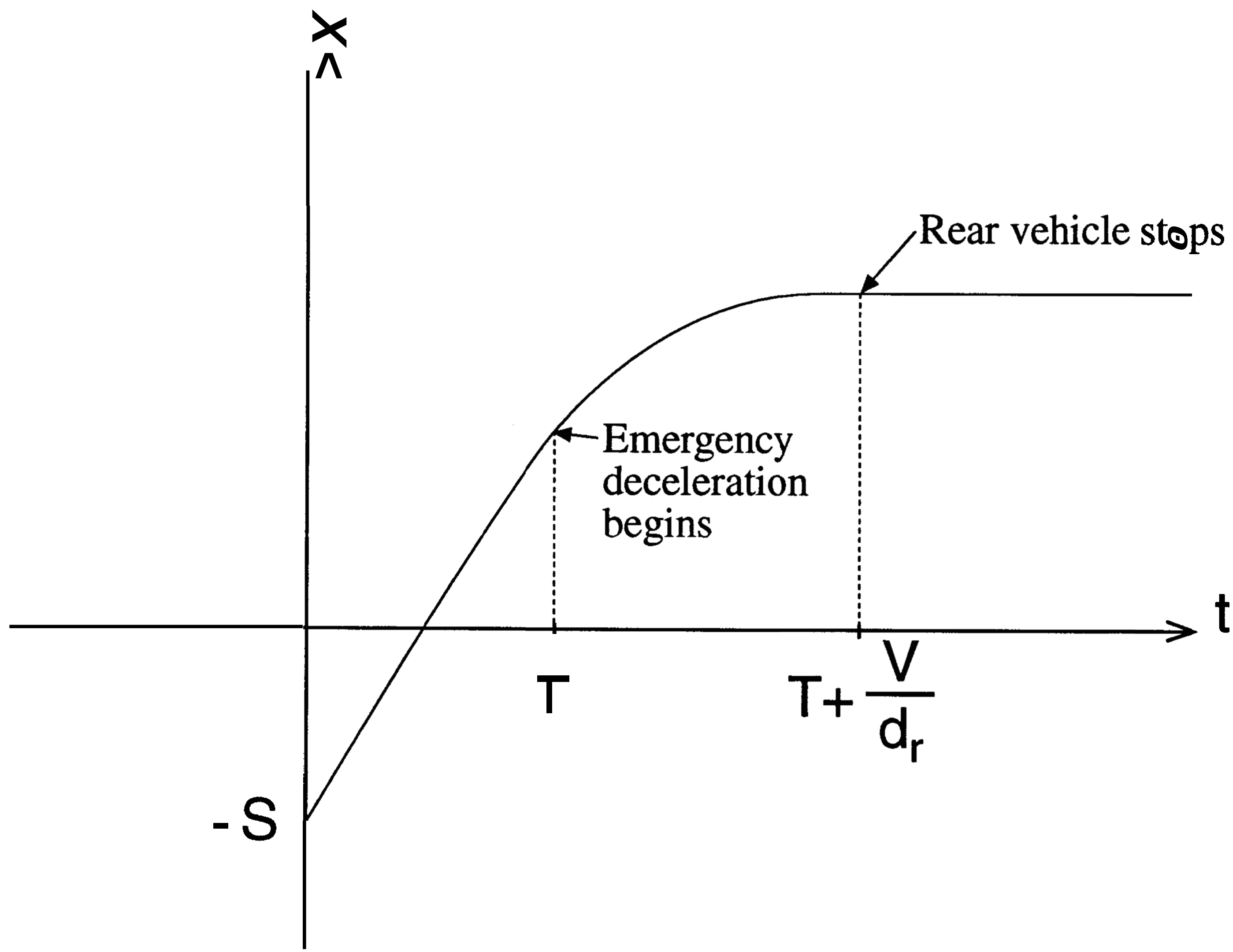

Figure 2: Trajectory $x_r(t)$ of the rear vehicle

the fact that the rear vehicle is decelerating at the constant rate of d_r. The third piece of the curve for the rear vehicle is a constant function and describes the stopping position of the rear vehicle if no collision had occurred. As mentioned before, this piece of trajectory will not play any role in the calculations because collision cannot occur after the rear vehicle has stopped. The curve $x_f(t)$ for the front vehicle may have two pieces in the relevant interval. The front vehicle starts to decelerate at time 0 and it stops at time $\frac{V}{d_f}$ if there is no collision. If $\frac{V}{d_f} \le T + \frac{V}{d_r}$, then the curve has two pieces in the relevant interval. Otherwise, there is only one. Note that these two curves can intersect more than once. Figure 3 shows a possible combination of the two curves in which they intersect only once. Figure 4 gives an example in which they cross twice.

In terms of timing, there are only four possible ways for the collision to occur:

(C1) During the reaction period but before the front vehicle has stopped;

(C2) During the reaction period but after the front vehicle has stopped;

(C3) When both vehicles are decelerating;

(C4) *After* the front vehicle has stopped and while the rear vehicle is decelerating.

(3) PROBLEM SOLUTION

We now summarize the derivation of the collision probability and collision speed given any specific pair of deceleration rates $D_f = d_f$ and $D_r = d_r$. Let t^* denote a crossing time.

For (C1) to occur, t^* must be on the first piece of the front trajectory and also on the first piece of the rear trajectory. Therefore, the prerequisites are $t^* \in [0, \frac{V}{d_r}]$ and $t^* \in [0, T]$. To determine the possible crossing times, solve:

$$Vt^* - \frac{1}{2} d_f t^{*2} = Vt^* - S.$$

The solutions are:

$$t^* = \left[\frac{2S}{d_f}\right]^{1/2} \text{ and } -\left[\frac{2S}{d_f}\right]^{1/2}.$$

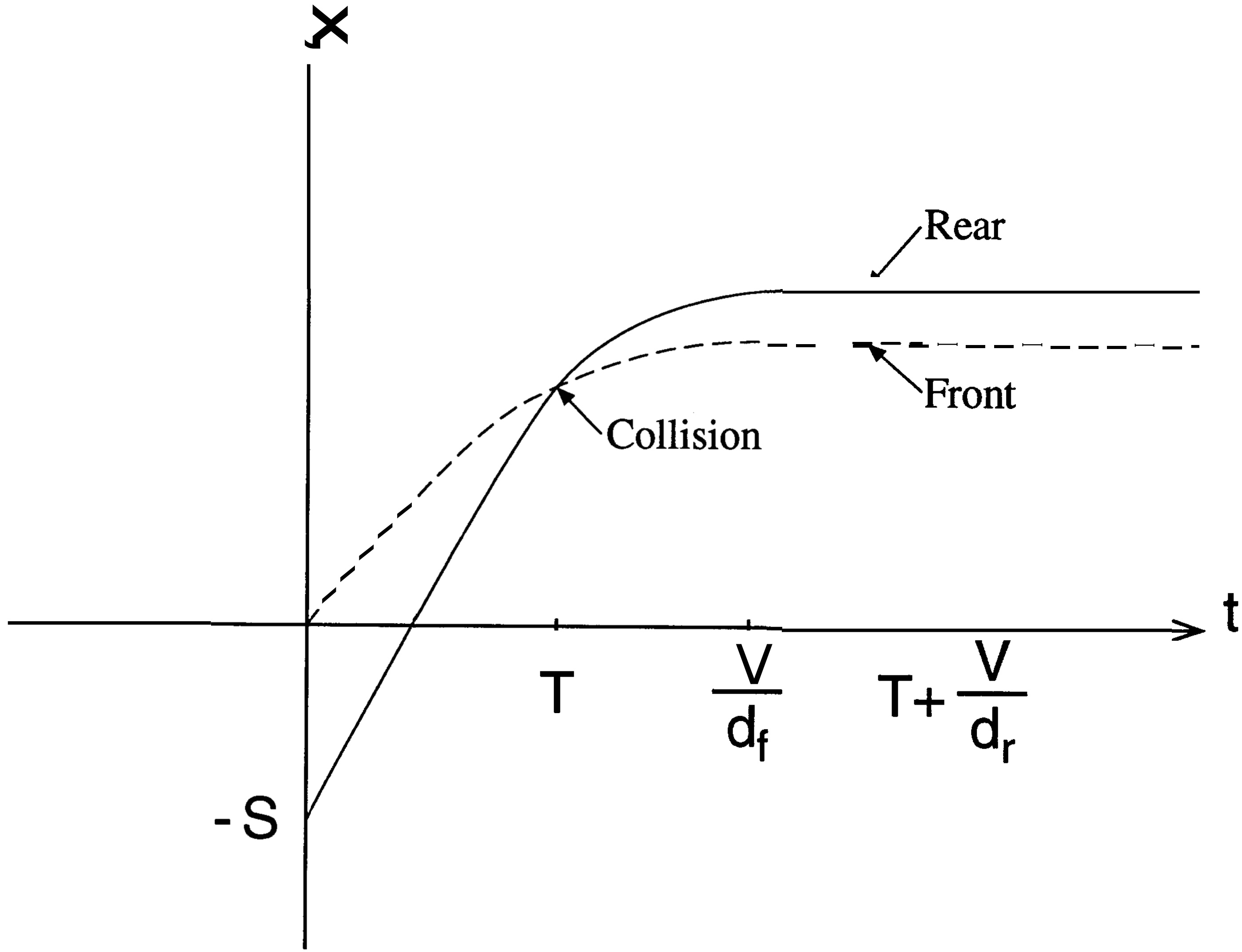

Figure 3: Two trajectories crossing only once

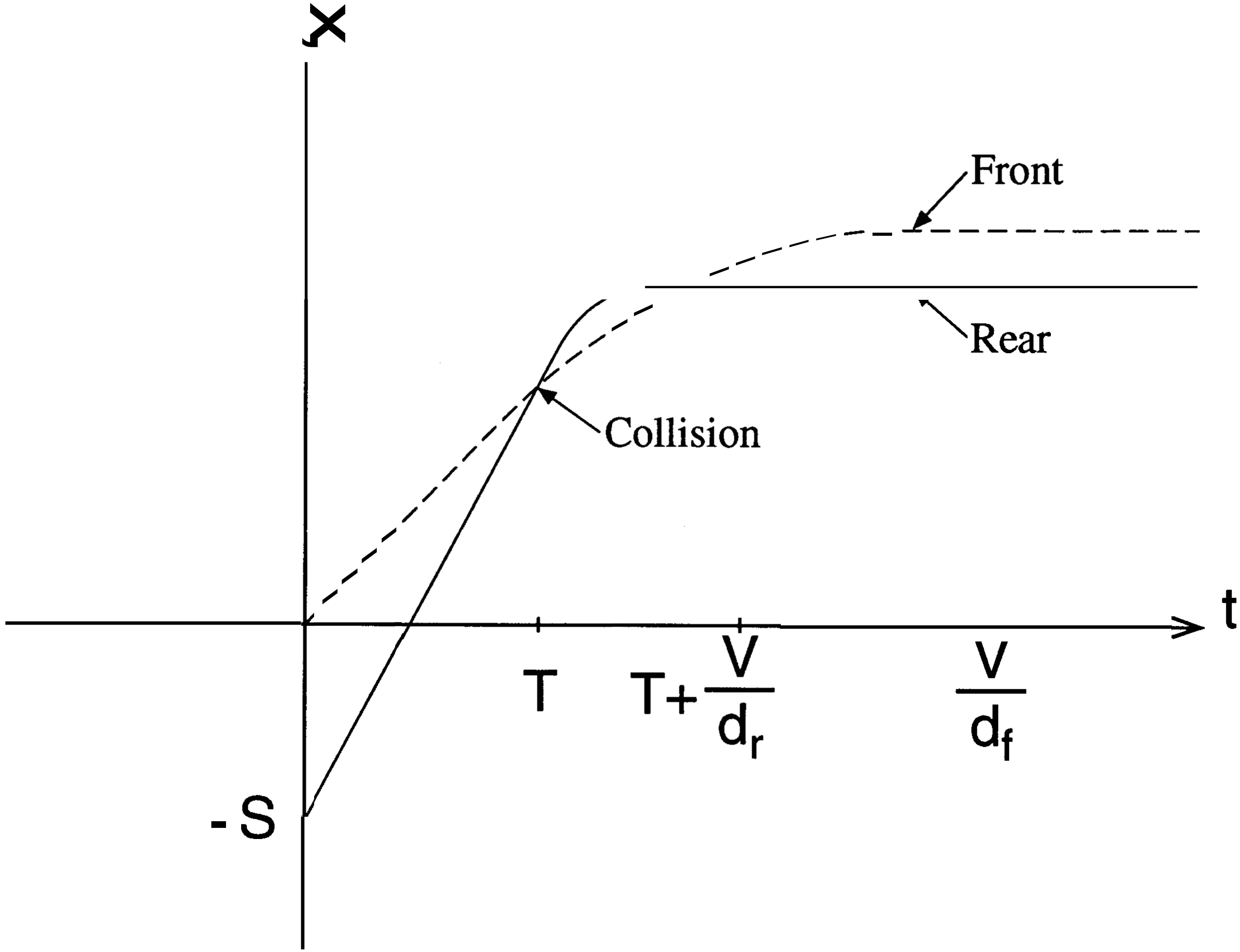

Figure 4: Two trajectories crossing twice

Clearly, the first crossing time is not acceptable because it does not meet the prerequisites. The *speed* difference, if t^* indeed falls in the required interval, will be

$$\Delta v = t^* d_f \ .$$

In order for (C2) to occur, a prerequisite is $\frac{V}{d_f} \le T$. Also, t^* must be on the second piece of the front trajectory and the first piece of the rear trajectory, i.e. $t^* \ge \frac{V}{d_r}$ and $t^* \in [0,T]$. For the possible crossing times, solve:

$$\frac{1}{2}\frac{V^2}{d_f} = Vt^* - S.$$

The solution is:

$$t^* = \frac{\frac{1}{2}\frac{V^2}{d_f} + S}{V} \ .$$

If t^* satisfies the prerequisites, then the *speed* difference would simply be

$$\Delta v = V$$

For (C3) to occur, a prerequisite is $T \le \frac{V}{d_f}$. *Also,* t^* must be on the first piece of front trajectory and the second piece of the rear trajectory, i.e. $t^* \in [0, \frac{V}{d_f}]$ and $t^* \in [T, T + \frac{V}{d_r}]$. To obtain the crossing times, we solve the following equation:

$$Vt^* - \frac{1}{2} d_f t^{*2} = Vt^* - \frac{1}{2} d_r (t^* - T)^2 - S.$$

The solutions are:

$$t^* = \frac{d_r T + \left[d_r^2 T^2 - 2(d_r - d_f)(\frac{1}{2} d_r T^2 + S) \right]^{1/2}}{d_r - d_f} \text{ and } \frac{d_r T - \left[d_r^2 T^2 - 2(d_r - d_f)(\frac{1}{2} d_r T^2 + S) \right]^{1/2}}{d_r - d_f} \ ,$$

if $d_r^2T^2-2(d_r-d_f)(\frac{1}{2}d_rT^2+S)\geq 0$ and $d_r\neq d_f$. If $d_r=d_f$,

$$t^* = \frac{\frac{1}{2}d_rT^2+S}{d_rT} .$$

The speed difference, if t^* satisfies the prerequisites, is:

$$\Delta v = d_f t^* - d_r t^* + d_r T.$$

Finally, in order for (C4) to occur, a prerequisite is $T+\frac{V}{d_r}\geq\frac{V}{d_f}$. Also, t^* must be on the second piece of the front trajectory and the second piece of the rear trajectory, i.e. $t^*\geq\frac{V}{d_f}$ and $t^*\in[T,T+\frac{V}{d_r}]$. To obtain the crossing times, solve:

$$\frac{1}{2}\frac{V^2}{d_f} = Vt^* - \frac{1}{2}d_r(t^*-T)^2-S.$$

The solutions are:

$$t^* = \frac{(d_rT+V)-\left[(d_rT+V)^2-d_r(d_rT^2+2S+\frac{V^2}{d_f})\right]^{1/2}}{d_r} \text{ and } \frac{(d_rT+V)+\left[(d_rT+V)^2-d_r(d_rT^2+2S+\frac{V^2}{d_f})\right]^{1/2}}{d_r},$$

if $(d_rT+V)^2-d_r(d_rT^2+2S+\frac{V^2}{d_f})\geq 0$. The speed difference, if the interval requirements are satisfied, will be

$$\Delta v = V - d_r(t^*-T) .$$

Adding up the probabilities associated with the pairs (d_f, d_r) that lead to the Same collision speed produces the desired results.

(4) MAXIMUM ENTROPY MODEL

Recall that we need a bivariate distribution for the two deceleration rates as part of the model input and note that the number of probabilities needed grows quadratically with the size of D, the set of all possible deceleration rates considered for both vehicles. To justify particular selections for the distribution in absence of data on the future technology, we propose to use the maximum entropy principle (MAXENT) to generate the bivariate probability distribution. In addition, when the size of D is large, the task of determining a bivariate distribution may become unwieldy. Therefore, a probability generator would be very desirable.

The MAXENT technique can determine a unique distribution, univariate or multivariate (with correlation), discrete or continuous, that satisfies any "linear constraints" on the probability distribution, using only a small number of "parameters" for the distribution (to be determined and supplied by the user). Such linear constraints can be used to express almost all common constraints on distributions, e.g. expected value, percentage quantile, the variance and correlation when the expected value is given, etc.

Entropy of a probabillity distribution on a finite domain, p_i, i=1,2,...,n, is defined by $-\sum_{i=1}^{n} p_i \ln p_i$. It can be interpreted as a measure of uncertainty contained in the distribution and the negation of entropy can be interpreted as a measure of information. The maximum-entropy distribution has the least "other information" out of all the distributions that satisfy the linear constraints. In other words, it picks the one that is "maximally non-committal". For example, the maximum-entropy distribution on any finite state space without any constraints is the uniform distribution, which can be viewed as the distribution containing the maximum amount of uncertainty or the least amount of information. For an analysis like ours where information about the exact distribution is limited, the selected distribution should be as non-committal as possible. Therefore, adoption of this principle is especially appropriate. One final note about the maximum-entropy approach is that there exist very robust and efficient algorithms for solving these distribution determination problems. For references on the subject of maximum entropy and detail of the algorithm, see [1].

(5) A COMPUTER TOOL

In addition to providing a MAXENT solver (a computer program for solving any general linearlyconstrained maximum entropy problem), we have also coded a special MAXENT problem generator. This generator takes the users' input on (i) the expected values of two marginal distributions, (ii) the standard deviations of the two marginal distributions, and (iii) the correlation coefficient between these two random variables, and then generates a special MAXENT problem for the MAXENT solver to produce a bivariate distribution. In short, the *computer fool has three major modules:*

(M1) **A MAXENT** problem generator

Given the two expected values, two standard deviations and the correlation, it generates a **MAXENT** problem whose solution is the desired bivariate distribution for the two deceleration rates.

(M2) **A MAXENT** solver:

It solves any linearly-constrained maximum entropy problem, including the one generated by the above **MAXENT** problem generator.

(M3) **A** collision probability and speed solver:

It takes the bivariate distribution and other input parameters and calculates the probability of a collision and the distribution of the collision speed.

The model for module (M3) has been described in detail in previous sections. (M1) and (M2) are briefly discussed in the Appendix.

(6) A COMPARISON BETWEEN PLATOONING AND FREE-AGENT RULES

In this section, we first itemize the assumptions of comparison. We then use the model and the software tool to produce the collision probability and collision speed distribution for a set of failure/reaction scenarios. Note that we are not attempting a complete comparison, which involves, among many other things, the failure probability (i.e. frequency), traffic disruption due to collisions (fatal, injury or property-damage only), complexity of vehicle control algorithm and protocol, complexity of operating strategy, and stability of traffic flow.

To set the stage for the comparison, we itemize the additional assumptions as follows:

(A1) Both deceleration rates are random. The randomness of the failure deceleration rate is due to chance. A target emergency deceleration rate has been *preset* for responding to vehicle failures; but, due to inaccuracy of the braking system, the actual emergency deceleration rate is random.

(A2) The distributions of these two rates are independent.

(A3) We set the common speed prior to the failure at 25 meters/second, which is approximately 55 miles/hour.

(A4) The reaction delay, including the communication delay and the brake actuation delay, is set at 100 milliseconds (0.1 second). This choice of the reaction delay is consistent with the current and planned automatic control technology.

The other input parameters, the spacing and the two deceleration distributions, will be varied to obtain collision probabilities and collision speed distributions. The spacings for the two rules are chosen so that the two resulting capacities are identical. We consider two different platooning scenarios: (i) 20-vehicle platoon with 1-meter intra-platoon spacing and 61-meter inter-platoon spacing, and (ii) 5-vehicle platoon with 1-meter intra-platoon spacing and 31-meter inter-platoon spacing. With the vehicle length set at 5 meters, their free-agent and identical-flow counterparts would have a common inter-vehicle spacing of 4 meters and 7 meters respectively (not counting the vehicle length). At the speed of 25 meters/second and with 20% capacity reserved for lane-change maneuvers, the two capacities are 8,000 and 6,000 vehicles per lane per hour respectively.

The possible deceleration rates, for both the failed vehicle and its immediate follower, are $i \times 0.5 m/s^2$, i=1,2,...,20. We choose to use two parameters, the expected value and the standard deviation, for determining a deceleration rate distribution. We vary these parameters for both deceleration rate distributions. For failure deceleration, we select two distributions: (i) with expected value 5.0 meters/sec^2 and standard deviation 1.0 meters/sec^2, and (ii) with expected value 3.0 meters/sec^2 and standard deviation 1.0 meters/sec^2. For emergency deceleration, we consider many more distributions with the expected values ranging from 3.0 meters/sec^2 to 8.0 meters/sec^2 and the standard deviation ranging from 0.1 meters/sec^2 to 1.0 meter/sec^2.

To illustrate the characteristics of Maximum Entropy distributions, five such distributions with different expected values and standard deviations are displayed in Figure 5. All these distributions will be used in the probabilistic comparison between the two major vehicle-following rules.. For a clearer comparison, we put these five distributions on the same scale but do not show the five histograms themselves. Instead, for each distribution we connect the points $(d_i, prob(d_i))$, where d_i is a possible deceleration rate and $prob(d_i)$ is the maximum entropy probability associated with d_i.

The collision speed distribution is expressed as a histogram with 15 intervals. These intervals, with the unit of *meters/second*, are: $((i-1)\times 0.5, i\times 0.5]$, for $i=1,2,...,14$, and $(7.0,\infty)$. For example, if the mean and standard deviation of the front deceleration are 5 meters/sec^2 and 1 meter/sec^2 respectively, those of the rear deceleration are 3 meters/sec^2 and 0.5 meter/sec^2 respectively, and the platooning rule is adopted, the probability of a collision with relative speed between 2.5 and 3.0 meters/second is 0.1046.

The result of our probabilistic comparison is tabulated in 6 tables. Table 1 contrasts the difference between the two rules for the case of a 20-vehicle platoon where the failure deceleration rate obeys a maximum entropy distribution with an expected value of 5 meters/sec^2 and a standard deviation of 1 meter/sec^2. Table 2 contrasts the same difference for the same case except that the expected value is 3 meter/sec^2. Tables 3 and 4 contain the same contrast as in Tables 1 and 2 respectively except that the platoon size is 5. Tables 5 and 6 provide succinct summaries of Tables 1 and 3 respectively.

Since a collision speed of 8 miles/hour (3.55 meters/sec) or below is considered safe while 16 m/h (7.1 meters/sec) or above is considered dangerous, in terms of injury and fatality, by some safety experts [4], we choose to display the probabilities of collision speed *greater* than 0 meters/sec, 3.5 meters/sec and 7.0 meters/sec in the two summary tables. Note that under the platooning rule, the failed vehicle may be at the very end of a platoon, in which case the collision probability should be minute but the collision speed, given the occurrence of a collision, may be high. This fact has been considered in all the probability calculations for the platooning rule.

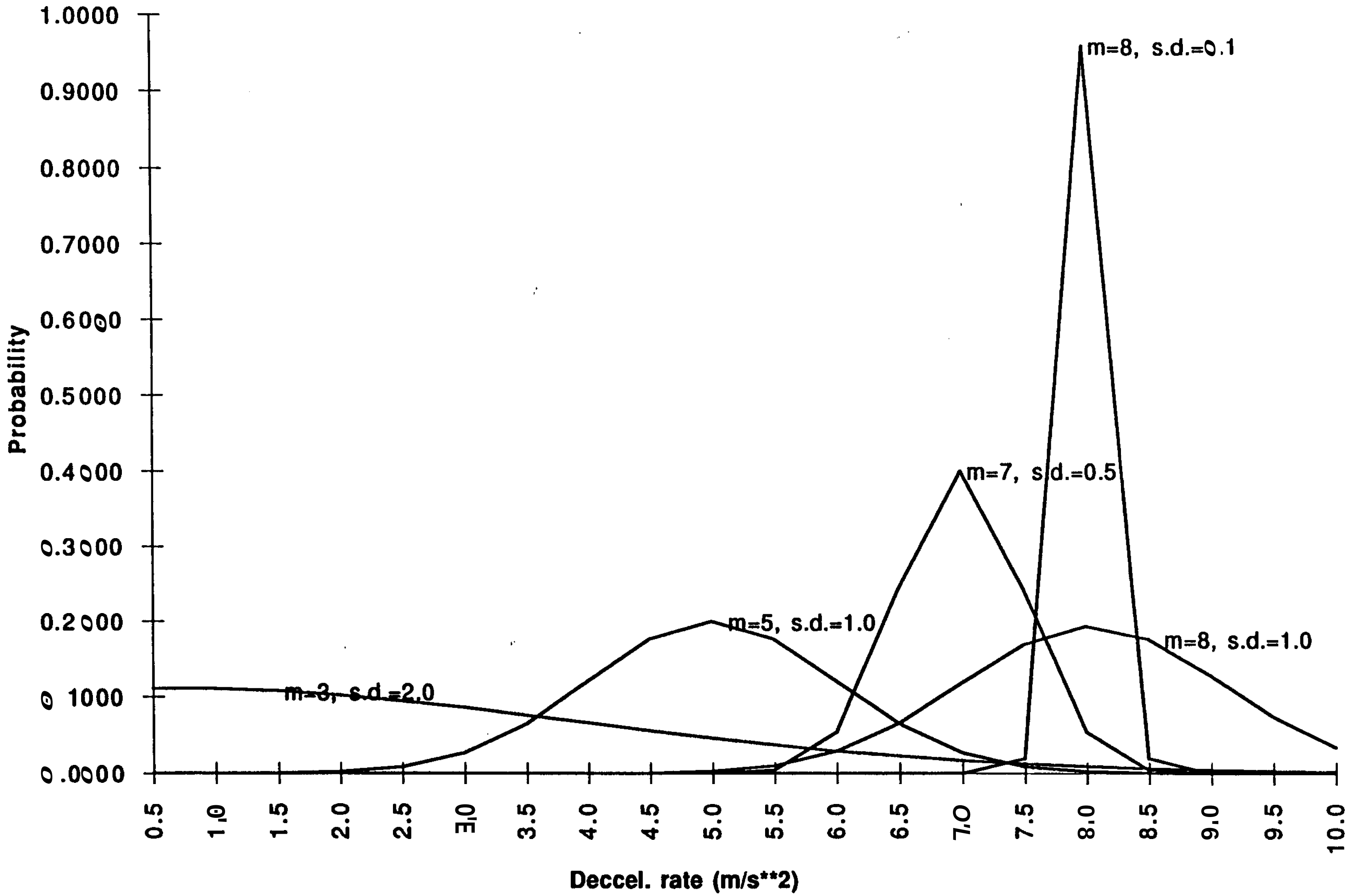

Figure 5: Five Maximum Entropy Distributions

Rear Decel.		Mode	Prob. of	Prob. of Collision at speed =														
mean	s.d.		Collision	<0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	3.0-3.5	3.5-4.0	4.0-4.5	4.5-5.0	5.0-5.5	5.5-6.0	6.0-6.5	6.5-7.0	>7.0
3	0.5	Platooning	.9407	.0342	.0000	.1825	.1534	.4356	.1046	.0201	.0001	.0024	.0002	.0000	.0002	.0000	.0021	.0054
		Free Agent	.9428	.0000	.0000	.0000	.0000	.0725	.1196	.1609	.0005	.3362	.1232	.0725	.0360	.0195	.0016	.0001
4	0.5	Platooning	.8270	.1129	.0008	.3164	.1598	.2045	.0309	.0016	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
		Free Agent	.7506	.0000	.0000	.0000	.0000	.1614	.1784	.1285	.0329	.1593	.0689	.0146	.0049	.0016	.0001	.0000
5	0.5	Platooning	.5597	.1225	.0469	.2048	.1311	.0484	.0059	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
		Free Agent	.4108	.0000	.0000	.0000	.0000	.1614	.1196	.0104	.0621	.0366	.0190	.0013	.0003	.0001	.0000	.0000
6	0.5	Platooning	.2369	.0110	.1026	.0697	.0474	.0059	.0003	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
		Free Agent	.1298	.0000	.0000	.0000	.0000	.0725	.0360	.0000	.0146	.0049	.0016	.0001	.0000	.0000	.0000	.0000
7	0.5	Platooning	.0544	.0000	.0342	.0139	.0059	.0003	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
		Free Agent	.0212	.0000	.0000	.0000	.0000	.0146	.0049	.0000	.0013	.0003	.0001	.0000	.0000	.0000	.0000	.0000
8	0.5	Platooning	.0062	.0000	.0046	.0013	.0003	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
		Free Agent	.0017	.0000	.0000	.0000	.0000	.0013	.0003	.0000	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
8	0.1	Platooning	.0027	.0000	.0022	.0004	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
		Free Agent	.0005	.0000	.0000	.0000	.0000	.0005	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
8	1	Platooning	.0255	.0007	.0140	.0068	.0036	.0004	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
		Free Agent	.0114	.0000	.0000	.0000	.0000	.0071	.0029	.0000	.0010	.0003	.0001	.0000	.0000	.0000	.0000	.0000

Table 1: 20–Vehicle Platooning:

Front Decel. Mean = 5 meters/sec^2

s.d. = 1 meters/sec^2

Rear Decel. mean	s.d.	Mode	Prob. of Collision	Prob. of Collision at speed = <0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	3.0-3.5	3.5-4.0	4.0-4.5	4.5-5.0	5.0-5.5	5.5-6.0	6.0-6.5	6.5-7.0	>7.0
3	0.5	Platooning	.5591	.1682	.0000	.2647	.0689	.0538	.0017	.0001	.0000	.0003	.0000	.0000	.0005	.0000	.0001	.0008
		Free Agent	.4096	.0000	.0000	.0000	.0000	.1599	.1188	.0725	.0000	.0515	.0051	.0014	.0003	.0001	.0000	.0000
4	0.5	Platooning	.2373	.1128	.0000	.1034	.0144	.0065	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
		Free Agent	.1310	.0000	.0000	.0000	.0000	.0725	.0364	.0144	.0006	.0063	.0005	.0001	.0000	.0000	.0000	.0000
5	0.5	Platooning	.0555	.0324	.0022	.0181	.0024	.0004	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
		Free Agent	.0220	.0000	.0000	.0000	.0000	.0150	.0051	.0006	.0008	.0003	.0001	.0000	.0000	.0000	.0000	.0000
6	0.5	Platooning	.0066	.0017	.0031	.0014	.0003	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
		Free Agent	.0018	.0000	.0000	.0000	.0000	.0014	.0003	.0000	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
7	0.5	Platooning	.0004	.0000	.0003	.0001	.0000	.0000	,0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
		Free Agent	.0001	.0000	.0000	.0000	.0000	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
8	0.5	Platooning	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
		Free Agent	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
8	0.1	Platooning	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
		Free Agent	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
8	1	Platooning	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
		Free Agent	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

Table 2 20–Vehicle Platooning:

Front Decel. Mean = 3 meters/sec^2

s.d. = 1 meters/sec^2

Rear Decel. mean	Rear Decel. s.d.	Mode	Prob. of Collision	Prob. of Collision at speed = <0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	3.0-3.5	3.5-4.0	4.0-4.5	4.5-5.0	5.0-5.5	5.5-6.0	6.0-6.5	6.5-7.0	>7.0
3	0.5	Platooning	.9236	.0288	.0000	.1537	.1292	.3664	.0880	.0170	.0008	.0000	.0000	.0085	.0000	.0175	.0000	.1138
		Free Agent	.9428	.0000	.0000	.0000	.0000	.0000	.0725	.0000	.1196	.0000	.1614	.1784	.1614	.1181	.0015	.1298
4	0.5	Platooning	.7332	.0950	.0006	.2664	.1346	.1721	.0260	.0013	.0052	.0001	.0000	.0085	.0000	.0041	.0000	.0191
		Free Agent	.7506	.0000	.0000	.0000	.0005	.0000	.1609	.0000	.1784	.0000	.1614	.1196	.0725	.0230	.0130	.0212
5	0.5	Platooning	.4730	.1032	.0395	.1725	.1104	.0408	.0050	.0001	.0007	.0001	.0000	.0002	.0000	.0003	.0000	.0003
		Free Agent	.4072	.0000	.0000	.0000	.0293	.0000	.1285	.0000	.1196	.0000	.0725	.0360	.0146	.0003	.0046	.0017
6	0.5	Platooning	.1995	.0093	.0864	.0587	.0399	.0050	.0003	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
		Free Agent	.0969	.0000	.0000	.0000	.0293	.0000	.0104	.0000	.0360	.0000	.0146	.0049	.0013	.0000	.0003	.0001
7	0.5	Platooning	.0458	.0000	.0288	.0117	.0050	.0003	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
		Free Agent	.0071	.0000	.0000	.0000	.0005	.0001	.0000	.0000	.0048	.0000	.0013	.0003	.0001	.0000	.0000	.0000
8	0.5	Platooning	.0053	.0000	.0039	.0011	.0003	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
		Free Agent	.0003	.0000	.0000	.0000	.0000	.0001	.0000	.0000	.0002	.0000	.0001	.0000	.0000	.0000	.0000	.0000
8	0.1	Platooning	.0023	.0000	.0018	.0004	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
		Free Agent	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
8	1	Platooning	.0215	.0006	.0118	.0058	.0030	.0003	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
		Free Agent	.0062	.0000	.0000	.0000	.0012	.0001	.0007	.0000	.0028	.0000	.0010	.0003	.0001	.0000	.0000	.0000

Table 3: 5–Vehicle Platooning:
Front Decel. Mean = 5 meters/sec^2
s.d. = 1 meters/sec^2

Rear Decel. mean	Rear Decel. s.d.	Mode	Prob. of Collision	Prob. of Collision at speed = <0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	3.0-3.5	3.5-4.0	4.0-4.5	4.5-5.0	5.0-5.5	5.5-6.0	6.0-6.5	6.5-7.0	>7.0
3	0.5	Platooning	.5068	.1416	.0000	.2229	.0580	.0453	.0014	.0020	.0000	.0000	.0000	.0004	.0001	.0136	.0000	.0214
		Free Agent	.4096	.0000	.0000	.0000	.0000	.0000	.1599	.0000	.1188	.0000	.0725	.0364	.0150	.0051	.0000	.0018
4	0.5	Platooning	.2019	.0950	.0000	.0871	.0121	.0055	.0001	.0000	.0000	.0000	.0000	.0004	.0000	.0009	.0000	.0008
		Free Agent	.1310	.0000	.0000	.0000	.0000	.0000	.0725	.0000	.0364	.0000	.0150	.0051	.0014	.0003	.0000	.0001
5	0.5	Platooning	.0468	.0273	.0019	.0153	.0020	.0003	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
		Free Agent	.0220	.0000	.0000	.0000	.0006	.0000	.0144	.0000	.0051	.0000	.0014	.0003	.0001	.0000	.0000	.0000
6	0.5	Platooning	.0055	.0014	.0026	.0012	.0003	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
		Free Agent	.0016	.0000	.0000	.0000	.0006	.0000	.0006	.0000	.0003	.0000	.0001	.0000	.0000	.0000	.0000	.0000
7	0.5	Platooning	.0003	.0000	.0003	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
		Free Agent	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
8	0.5	Platooning	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
		Free Agent	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
8	0.1	Platooning	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
		Free Agent	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
8	1	Platooning	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
		Free Agent	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

Table 4: 5–Vehicle Platooning:
Front Decel. Mean = 3 meters/sec^2
s.d. = 1 meters/sec^2

Rear Decel.		Rule	P(Col. Spd > 0)	P(Col. Spd > 3.5)	P(Col.Spd > 7.0)
mean	s.d.				
3	0.5	Platooning	.9407	.0104	.0054
		Free Agent	.9428	.5897	.0001
4	0.5	Platooning	.8270	.0002	.0001
		Free Agent	.7506	.2823	.0000
5	0.5	Platooning	.5597	.0000	.0000
		Free Agent	.4108	.1194	.0000
6	0.5	Platooning	.2369	.0000	.0000
		Free Agent	.1298	.0212	.0000
7	0.5	Platooning	.0544	.0000	.0000
		Free Agent	.0212	.0017	.0000
8	0.5	Platooning	.0062	.0000	.0000
		Free Agent	.0017	.0001	.0000
8	0.1	Platooning	.0027	.0000	.0000
		Free Agent	.0005	.0000	.0000
8	1	Platooning	.0255	.0000	.0000
		Free Agent	.0114	.0015	.0000

Table 5: 20–Vehicle Platooning:
Front Decel. Mean = 5 meters/sec^2
s.d. = 1 meters/sec^2

R [illegible]		[illegible]	[illegible]	[illegible]	[illegible] > 7.0)
3	0.5	Platooning	.9236	.1406	.1138
		Free Agent	.9428	.8702	.1298
4	0.5	Platooning	.7332	.0370	.0191
		Free Agent	.7506	.5892	.0212
5	0.5	Platooning	.4730	.0016	.0003
		Free Agent	.4072	.2494	.0017
6	0.5	Platooning	.1995	.0000	.0000
		Free Agent	.0969	.0572	.0001
7	0.5	Platooning	.0458	.0000	.0000
		Free Agent	.0071	.0065	.0000
8	0.5	Platooning	.0053	.0000	.0000
		Free Agent	.0003	.0002	.0000
8	0.1	Platooning	.0023	.0000	.0000
		Free Agent	.0000	.0000	.0000
8	1	Platooning	.0215	.0000	.0000
		Free Agent	.0062	.0043	.0000

Table 6 5–Vehicle Platooning:
Front Decel. Mean = 5 meters/sec^2
s.d. = 1 meters/sec^2

In any type of probabilistic analysis like ours, one needs to compare probability distributions and such comparisons often involve some kind of ordering among different distributions. We will use the concept of ***stochastic larger {smaller).*** However, we are not interested in any exact ordering of the input or output distributions. Therefore, for ease of discussion, we will use the terms ***larger*** and ***smaller*** as abbreviations and only use them in an approximate sense in the rest of this section. *Also,* since we are using the collision speed only as a surrogate of collision severity and the exact relationship between them has not been established, we are not ready to rigorously compare the two basic rules. But, for convenience of discussion, we will nevertheless use the term *safer* to loosely express our intuition.

It is apparent from the tables that when the emergency deceleration rate is smaller than the failure deceleration rate, platooning is safer because its collision probability is not much different from its free-agent counterpart while its collision *speed* is smaller. *Also,* when the two rates are comparable, platooning seems safer because of the Same reason. However, when the emergency deceleration rate is significantly larger than the failure deceleration, the free-agent rule seems safer because its collision probability is significantly smaller while its collision *speed* distribution is not significantly larger. When the emergency deceleration rate is much larger than the failure deceleration rate and the accuracy of emergency deceleration is also high, the collision probability can be eliminated for very small vehicle-following spacing under the freeagent rule. For example, (i) the emergency deceleration rate with an expected value of 8 meters/sec^2 and standard deviation of 0.1 meter/sec^2, (ii) the failure deceleration rate with an expected value of 5 meters/sec^2 and standard deviation of 1 meter/sec^2, (iii) reaction delay of 0.1 second, and (iv) longitudinal gap (between the read end of the front vehicle and the front end of the rear vehicle) of 7 meters would virtually guarantee no collision after the failure. (See Table 3.) **Note** that the qualifier ***virtually*** is used **because** of potential numerical inaccuracy or possible insufficiency of the discrete approximation of a continuous distribution. In this particular example, the collision probability after a vehicle failure is 0.00001864, a very small probability that is less than 1% of its platooning counterpart.

Regarding the validity of these assumptions, Hedrick [Hedrick, 19921 is optimistic that a braking system capable of 0.8g (approximately 8 meters/sec^2) or higher deceleration under normal driving conditions can be successfully developed in the future. With better tire design, pavement and braking technology, fast emergency deceleration Seems feasible. The distribution of failure deceleration rate depends on the possible failure modes of the automated vehicle and also the failure probabilities, both of which in turn depend on the future automation technology and, perhaps more importantly, the design specifications of the future AVCS systems. An apparent design objective is to lower the failure deceleration rate as much as the cost considerations allow. Although there is no concrete data to support the validity of the selected failure deceleration rate in this example, it seems quite conservative. (See Figure 5.)

A point worth noting is that the contrasts tabulated in the six tables do not account for the fact that the inter-platoon spacing should be a function of the achievable emergency deceleration rate. For example, with the fast and accurate braking system described in the previous paragraph, there is no need to separate two platoons by 60 meters. However, these Tables do show that, ***with fast and accurate emergency bruking,*** free-agent vehicle-following rule can indeed provide the high capacity achievable by the very platooning concept that has stimulated interest in AVCS technology among the IVHS research community.

Our parametric probabilistic study suggests that the merit of any vehiclefollowing rule in terms of the collision probability and the collision speed depends heavily not only on the three factors (a) through (c) pointed out by Shladover (and cited earlier in this paper), but also on the ***accuracy*** of the braking system. As for the relative merits of the two basic vehicle-following rules, the most fundamental question is what other collisions may occur after the initial collision. If vehicles would not deviate from their longitudinal trajectory after low-relative-@ collisions in the Same lane, e.g. by having a powerful automated steering system that remains operational after the longitudinal collisions, then the platooning rule seems to be safer than the free-agent rule for a given common flow requirement. However, if low-relative-@ collisions prove to be unsafe or their consequences unsure and ***fast and accurate emergency deceleration*** becomes feasible, technologically and economically, then the free-agent rule may be the safer way.

(7) CONCLUSION

We have proposed a model for calculating the probability of a two-vehicle collision and the resulting collision *speed* distribution after the front vehicle abruptly decelerates. Robust probabilistic modeling is possible only by using the proposed discrete representation of the joint deceleration distribution. The adoption of the maximum entropy principle made possible the task of determining the input distribution conservatively and efficiently. The availability of the software tool enabled efficient parametric studies of the safety consequences of a vehicle failure under various vehicle-following rules.

We have used the model to compare the safety consequences of a vehicle failure under the platooning and the free-agent vehicle-following rules in terms of the fundamental trade-off between the probability of collision and the magnitude of the collision speed. This extends the deterministic analysis of collision speed by Shladover [5] and adds the dimension of collision probability. Our comparison suggests that a vehicle failure would cause far more initial collisions under platooning. If a small fraction of these low-relative-speed collisions lead to major collisions, then the platooning rule would actually be less safe. We also demonstrated that the free-agent vehicle-following rule imple mented with a potential technology of fast and accurate emergency deceleration, under some plausible conditions, might avoid any collisions after a vehicle failure while offering the high freeway capacity thought possible with platooning.

Although we have discussed the probabilistic model in the context of vehicle failure, it can be used in any context in which a vehicle needs to decelerate abruptly. For example, a vehicle may need to decelerate because a foreign object is detected to be lying ahead. Moreover, this model is applicable to any analysis of initial collision in which longitudinal vehicle control is employed, e.g. Autonomous Intelligent Cruise Control (AICC).

Future research should extend the model and the software tool to accommodate the following: multiple vehicle collisions in the same lane and more accurate model for the curve of *speed* after deceleration (perhaps by a differential equation).

REFERENCES

[1] Fang, S.C. and Tsao, H.-S.J., "A Quadratically Convergent Global Algorithm for the Linearly-Constrained Minimum Cross-Entropy Problem," presented at 1992 Orlando ORSA/TIMS and to appear in European Journal of Operations Research.

[2] Hedrick, K., Department of Mechanical Engineering, The University of California, Berkeley, private discussion, August, 1992.

[3] Hitchcock, A., "A Specification of an Automated Freeway", PATH Research Report UCB-ITS-PRR-91-0808-2, 1991.

[4] Hitchcock, A., Institute of Transportation Studies, University of California, Berkeley, private discussion, Oct., 1992.

[5] Shladover, S, "Operation of Automated Guideway Transit Vehicles in Dynamically Reconfigured Trains and Platoons," (Extended Summary, Vol. I & II), UMTA-MA-06-0085-79-1, UMTA-MA-06-0085-79-2 and UMTA-MA-06-0085-79-3, U.S. Department of Transportation, Urban Mass Transportation Administration, Washington, D.C., July, 1979.

[6] Shladover, S., "Potential Freeway Capacity Effects of Advanced Vehicle Control Systems," Second International Conference on Applications of Advanced Technologies in Transportation Engineering, Minneapolis, Minnesota, August 18-21, 1991.

[7] Tongue, B., Department of Mechanical Engineering, The University of California, Berkeley, "Platoon Collision Dynamics and Emergency Maneuvering," current PATH research project, to be completed in August, 1993.

APPENDIX

A linearly-constrained maximum entropy problem is defined as the following mathematical programming problem:

$$maximize \; -\sum_{j=1}^{n} x_j \ln x_j$$

$$s.t. \;\; \sum_{j=1}^{n} a_{ij} = b_i, \;\; i=1,...,m,$$

$$x_j \geq 0 \;\; , j=1,...,n.$$

Note that the variables of this problem do not have to form a probability distribution. Let x_{ij} be the probability of $d_f = d_i$ and $d_r = d_j$, m_1 and m_2 be the expected values of the two marginal distributions, v_1 and v_2 be the variances of the two marginal distributions and cov be the covariance of the bivariate distribution. Then, the linearly-constrained maximum entropy problem far determining the bivariate distribution of deceleration rates, given the parameters, is:

$$maximize \;\; -\sum_{i=1}^{s}\sum_{j=1}^{s} x_{ij} \ln x_{ij}$$

$$s.t. \;\; \sum_{i=1}^{s} d_i \left(\sum_{j=1}^{s} x_{ij}\right) = m_1$$

$$\sum_{j=1}^{s} d_j \left(\sum_{i=1}^{s} x_{ij}\right) = m_2$$

$$\sum_{i=1}^{s} (d_i - m_1)^2 \left(\sum_{j=1}^{s} x_{ij}\right) = v_1$$

$$\sum_{j=1}^{s} (d_j - m_2)^2 \left(\sum_{i=1}^{s} x_{ij}\right) = v_2$$

$$\sum_{i=1}^{s}\sum_{j=1}^{s} (d_i d_j - m_1 m_2) x_{ij} = cov$$

$$\sum_{i=1}^{s}\sum_{j=1}^{s} x_{ij} = 1$$

$$x_{ij} \geq 0, i=1,...,s, j=1,...,s.$$

The last two constraints form the probability constraint, or the simplex constraint. For actual computer programming, the rest of the constraints can be simplified and the double-indexed variables can be represented by a single-indexed variable array. After reindexing and simplification, this problem can be posed as a linearly-constrained maximum entropy problem defined above.

This formulation is the basis for the module (M1). The user is expected to supply the 5 parameters and this module (M1) will generate the special MAXENT problem for the MAXENT solver (M2) to compute the bivariate dishibution, which in turn will be used as a part of the input to the main module (M3).