Information Recovery in Dynamic Economic Systems

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ABSTRACT:

To make predictions pertaining to the statistical equilibrium of a dynamic economic system, we investigate, from an ordinal point of view, a basis for extracting qualitative information from non-linear times ordered observations. In contrast to traditional system analysis, ordinal patterns are used to gain an insight into the underlying intrinsic information hidden in the dynamics of economic systems. When combined with information theoretic entropy based divergence methods this permits model free prediction of the ordinal accessible patterns-paths and a basis for quantifying the probability distribution associated with the statistical equilibrium of a dynamic economic system.

Keywords: Adaptive behavior, Causal entropy maximization; Information Theoretic Methods; Minimum Power Divergence; Permutation Entropy; Statistical Equilibrium.

JEL Classification: C1; C10; C24

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1. Introduction

Although economic systems are simple in nature, the underlying dynamics is complicated and not well understood. In spite of beautiful theoretical equilibrium models, the behavior based nondeterministic economic world we observe is a bit of a mess. Wherever we look there is a level of disorder, or as thermodynamic physicist might say, there is free energy. Given the nondeterministic nature of economic systems, the predictability of the resulting dynamic system must be considered in a stochastic context and the level of disorder measured in terms of the statistical equilibrium-disequilibrium of the system. Probabilistic dynamics, known either as a stochastic process or random dynamical system, provides a natural language for this purpose (for example see Qian, 2013). Since the focus is stochastic in nature, to recover the unknown probability-divergence based methods. As a status-criterion-optimality measure we use the entropic family of minimum power divergences that recognizes the connection between adaptive intelligent behavior and causal entropy maximization (see Wissner-Gross and Freer, 2013).. More precisely this stochastic process has power-divergence entropy functionals that are monotonic in the systems dynamics and can be interpreted as emerging adaptive behavior. This connection is one way to express self-organized equilibrium seeking behavior in an open dynamic economic system. Thus in contrast to theoretical tradition, economic systems may not be in determinate equilibrium, but are viewed as equilibrium-stationary state seeking.

2. The Permutation Entropy Concept

In this section we seek a way of describing, in probability density function-distribution form, the underlying state of a dynamic system. To provide a probability density function-distribution of the temporal dynamics that is linked to the sample space, Bandt and Pompe

(2002), proposed a method that takes time causality into account by comparing time related observations-ordinal patterns-paths in a time series. In contrast to traditional functional analysis, they consider the order relation between time series instead of the individual values. Permutation patterns-partitions-vectors are developed, by comparing the order of neighboring observations. A vector of the *D*-th subsequent values is constructed, where *D* is the embedding dimension that determines how much information is contained in each vector with which an ordinal pattern is associated. The values of each vector are sorted in ascending order and a permutation of *D* partitions are created. Patterns of occurrence may not have the same probability and thus information is revealed concerning the ordinal structure of a given one-dimensional time series and the underlying dynamic system. With each time ordered data series it is possible to associate a probability density function whose frequencies are the possible permutation patterns.

More specifically, to use the Bandt and Pompe (2002) methodology to evaluate the order dimensions associated with a time series dynamical system under study, we start by considering partitions of the pertinent *D*-dimensional space that will reveal relevant details of the ordinal structure of a given one-dimensional time series. We are interested in "ordinal patterns," of order *D*, which assigns to each time *period* the *D*-dimensional vector of values. Clearly, the greater the *D*-value, the more information of the past is incorporated into our vectors. As an example, to identify the statistical implications of dynamic economic equilibrium and disequilibrium situations, we make use of the concept of permutation entropy and a D! = 3 dimensional space. Consider a time series of length *T* with time delay t = 1. Within the full time series there will be $T-D+1$ vectors of length *D*. An illustration of the *D* ordinal binary patterns for 3-dimensional vectors-states is given in Figure 2.1.

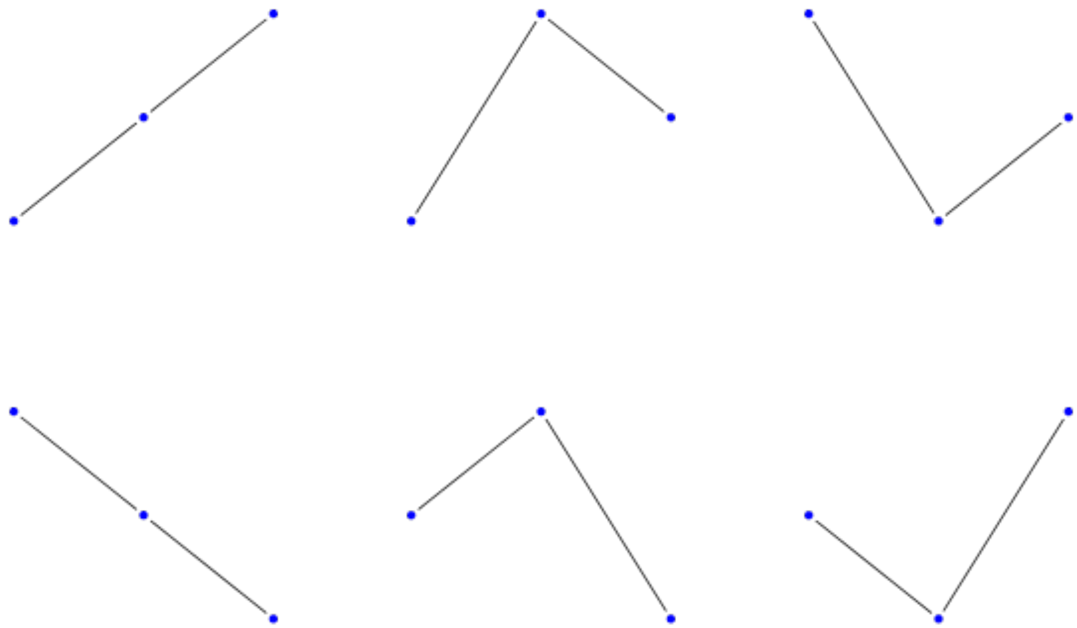

Figure 2.1. D = 3 Pattern of Vectors-States, S=1, 2,---, 6

Note in the D! = 3 case that the patterns do not reflect the *values* within the 3-dimensional binary vectors, but rather the *ordinal scale invariant relationships* between the values in the vectors. The permutation entropy (PE) method relies on the ordinal sequencing of the vectors, and the subsequent counts of these ordinal *D*-dimensional patterns. The relative frequencies of sub-sequences, reveals information about the underlying dynamics and the connected nature of the time dated observations. Alternatively, if we set the embedding dimension as *D*=4, the state space is divided into 4! Partitions, and 24 mutually exclusive permutation symbols are considered.

3. Divergence Measures and Recovering Fluctuations of a Statistical Equilibrium

To investigate the nature of the statistical equilibrium in terms of the probability distribution of the temporal dynamics that is linked to the sample space, we make use of the Cressie-Read (CR-1984 and RC 1988) multi parametric family of entropic power divergence measures:

$$I(\mathbf{p},\mathbf{q},\gamma)=\frac{1}{\gamma(\gamma+1)}\sum_{i=1}^{n}p_i\left[\left(\frac{p_i}{q_i}\right)^{\gamma}-1\right]. \qquad (3.1)$$

In (3.1), γ is a parameter that indexes members of the CR family, p_i's represent the subject probabilities and the q_i's, are interpreted as reference probabilities. All of the well known entropy divergences belong to the CR family. This is a more general measure of dis-similarity between distribution p_i's and q_i's, than the widely used Kullback-Leibler (KL) divergence. Being probabilities, the usual probability distribution characteristics of $p_i, q_i \in [0,1]\forall i$, $\sum_{i=1}^{n} p_i = 1$, and $\sum_{i=1}^{n} q_i = 1$ are assumed to hold. In (3.1) as γ varies, the resulting CR family of estimators that minimize power divergence, exhibit qualitatively different behavior. When gamma limits to zero, the information divergence of the two statistical equilibriums outcome distributions is defined as relative entropy-KL divergence. As γ varies the resulting CR family of estimators, that minimize power divergence, exhibits qualitatively different sampling behavior. This class of estimation procedures is referred to as *Minimum Power Divergence* (MPD) estimation (see Gorban, Gorban, and Judge 2010; Judge and Mittelhammer 2011, 2012). In identifying the probability space, the CR- entropy family of power divergences is defined through a class of additive convex functions, and the CR power divergence measure and leads to *a broad family of likelihood functions* test statistics.

To provide a probability distribution of the temporal dynamics that is linked to the sample space, the permutation entropy method takes time causality into account by comparing the time related observations-ordinal patterns. Thus we consider the order relation between time series instead of the individual values. Permutation patterns-partitions-vectors are developed, by comparing the order of neighboring observations and from this it is possible to create a probability distribution, whose elements are the frequencies associated with this i^{th} permutation pattern, i=1, 2, ---, D. Using, for example, the $\gamma \rightarrow 0$ member of the CR family with a uniform reference distribution, the information content of such a distribution for the D distinct assessable states may be defined as $P\text{E} = -\sum_{i=1}^{D!} p_i \ln p_i$, with normalized version $P\text{E}_{\text{norm}} = -\frac{1}{\ln D!}\sum_{i=1}^{D!} p_i \ln p_i$. For a review and applications of the Permutation Entropy method, see Zanin et al., (2012) and Kowalski et al., (2012). In terms of inference, simple consistent and powerful test of independence exist that provide a basis for using PE as a measure of serial dependence (see Mattila-Garcia and Marin (2008)).

4. Predicting a Distribution of the Patterns From Partial Information

Building on the PE and CR-entropy power divergence measures of Section 2 and 3, for expository purposes we again use the PE principle with D!=3. The result is a dynamic economic system consisting of six ordinal patterns-discrete dynamical states, **S**= S_1, S_2, ---, $S_{6,}$. Before the data are observed each state is equally likely to occur and may be identified with state values **S** = 1, 2, ---, 6. *After a large number of trials the observations associated with the permutation ordinal patterns, and thus the mean of the state distribution, $\bar{S}$, is observable.* Of course many probability distributions **p** are consistent with the observed mean of the state outcomes. The entropy family displayed in (3.1) provides a basis for computing the preferred probability density function-distribution over the microstates and predicting the relevant statistical equilibrium, for the corresponding dynamic economic system.

In the D = 3 binary pattern case there are an infinite number of probability distributions supported on *the state space* $\{1, 2, \ldots, 6\}$, that have a mean of $\bar{S}$. For a system in statistical equilibrium and a corresponding state mean of $\bar{S}$ *= 3.5*, each of the possible *state* outcomes are equally likely. Thus a uniform distribution of probabilities results and permits the prediction of the random fluctuations associated with a dynamic economic system in equilibrium. If however, the economic system is in an arbitrary statistical disequilibrium, we would expect the observed-empirical permutation entropy outcomes to yield a mean state outcome of $\bar{S} \neq 3.5$. For a system in arbitrary statistical disequilibrium, the resulting mean of the microstates permits estimation of the unknown pathway probabilities and the associated random fluctuations and deviations of the system. Thus the dynamic system economic statistical equilibrium question may be formulated as an extremum problem and solved for the statistical equilibrium fluctuations or the statistical disequilibrium fluctuations-deviations.

4.1 An Example

In this type of an ill posed inverse problem, if we use the, $\gamma \rightarrow 0$, member of the CR entropy family formulation (3.1) to recover the unknown probabilities, for dynamic problems of this type we may formulate the path entropy information recovery problem to select the probabilities that

Maximize $H(p) = -\sum_{i=1}^{N} p_i \ln(p_i)$ (4.1)

subject to the mean $\bar{S}$, where

$\sum_{i=1}^{N} p_i s_i = \bar{S}$ (4.2)

and the condition that the probabilities must sum to one

$\sum_{i=1}^{N} p_i = 1.$ (4.3)

The Lagrangian for the resulting extremum problem is

$$L = -\sum_{i=1}^{N} p_i \ln(p_i) + \lambda(\bar{S} - \sum_{i=1}^{N} p_i s_i) + \varphi(1 - \sum_{i=1}^{N} p_i) \qquad (4.4)$$

and results in a unique interior solution. Solving the first-order conditions yields the exponential probability distribution pathway result

$$p_i = \frac{\exp(-p_i\hat{\lambda})}{\sum_{i=1}^{N} \exp(-p_i\hat{\lambda})} \qquad (4.5)$$

where *p(λ)* is a member of a canonical exponential family with mean

$$\bar{S} = \sum_{i=1}^{N} p_i s_i. \qquad (4.6)$$

This information recovery framework provides one basis for thinking about alternative state mean-statistical situations and understanding the nature of the corresponding statistical equilibrium-disequilibrium and the economic system fluxes. Also the permutation pattern distribution from either the original discrete time series data' or from the first moment information developed in section 4, provides one basis for making predictions about future pattern outcomes and corresponding loss functions.

Some examples of the frequency distribution estimates for CR $\gamma \rightarrow 0$ and various values of $\bar{S}$, are presented in Table 4.1.

Table 4. 1. Estimated Distributions for Various $\bar{S}$

$\bar{S}$,	$\hat{p}_1$	$\hat{p}_2$	$\hat{p}_3$	$\hat{p}_4$	$\hat{p}_5$	$\hat{p}_6$	$H(\hat{p})$
2.0	0.381	0.295	0.210	0.124	0.038	0.021	1.367
3.0	0.247	0.207	0.174	0.146	0.123	0.103	1.748
3.5	0.167	0.167	0.167	0.167	0.167	0.167	1.792
4.0	0.103	0.123	0.146	0.174	0.207	0.247	1.748
5.0	0.021	0.038	0.072	0.136	0.255	0.478	1.367

$\bar{S}$ is the mean value of the distribution, $\hat{p}_i$, i=1,...,6 the frequencies, and H(p) the information function value

The maximum of the CR functional $H(\hat{p})$ and the minimum micro information occurs with a mean of $\bar{S} = 3.5$ and a uniform state distribution. Other monotonic distributions of the state space probabilities are, using the CR distance measure, in the form of exponential distributions.

5. Summary and Implications

In this paper we recognize that economic behavioral processes and systems are stochastic in nature, are seldom in equilibrium, and that new methods of modeling and information recovery are needed to explain the hidden dynamic world of interest. To make predictions pertaining to the statistical equilibrium of a dynamic economic system, we investigate from an ordinal viewpoint, a basis for extracting qualitative information from non-linear time orderd data observations. Ordinal patterns are used to describe the intrinsic patterns hidden in the dynamics of economic systems. Instead of the actual values, the concept of permutation entropy uses the order relation between the values of the time ordered data. This permits nonlinear model-free prediction of the ordinal accessible patterns and a basis for quantifying the dynamic characteristics of an economic system. In many time ordered economic data sets the limited number of observations may be a problem. In this case bootstrapping (Efron and Tibshirani, 1993) that relies on random sampling with replacement, may be used as a basis for increasing the number of observations and assigning measures of accuracy.

In the context of the information recovery methods of Sections 3 and 4 and their corresponding pattern probability ensembles, we note that:

i) If a dynamic system is in statistical equilibrium, it is only subject to random fluctuations and like the PE pattern of a Maximum Entropy-uniform distribution, predictions may be of little or no value.

ii) If the system is in an arbitrary statistical disequilibrium, random fluctuations that include transient deviations are in play. Consequently, like a loaded die the value of a prediction may be measured by the nature of the statistical disequilibrium. The farther the system is out of equilibrium, the greater the value of the prediction.

iii) When an economic system is in statistical disequilibrium, the permutation entropy concept and information theoretic methods provide a probability basis for identifying the deviation losses and the fluctuating dynamics of the system.

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