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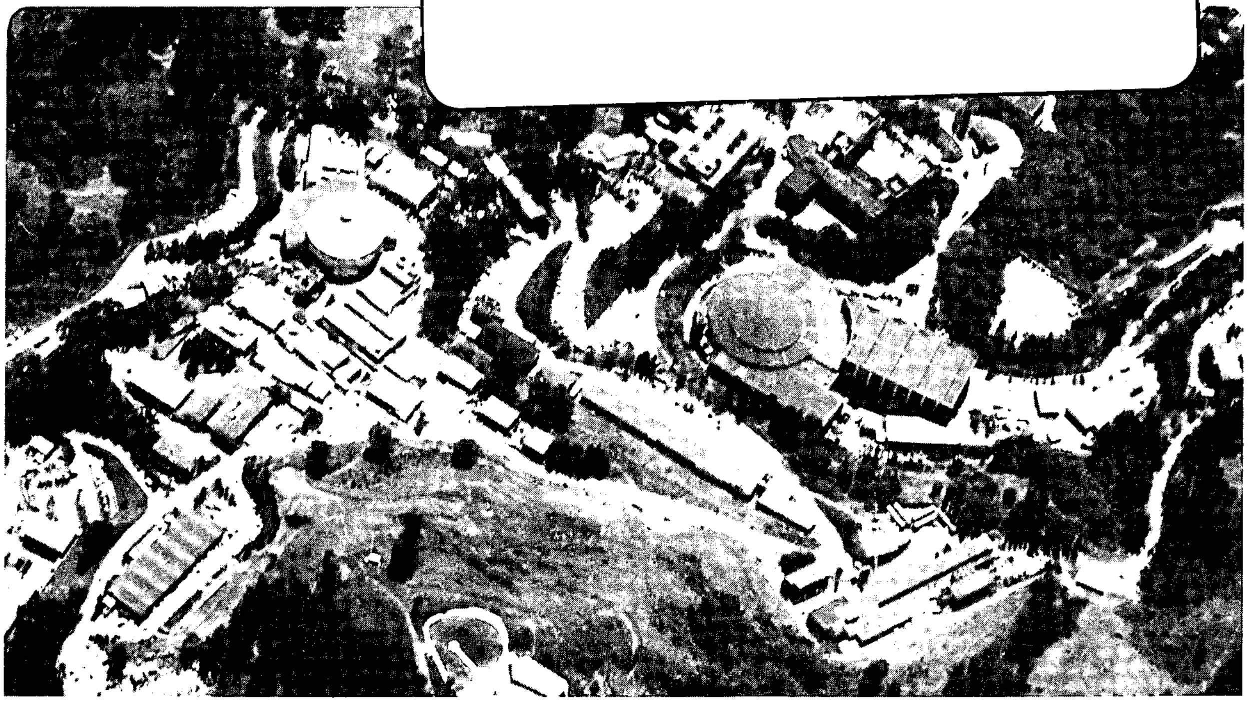
April 1988



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# CAN NEUTRINO–ELECTRON SCATTERING TELL US WHETHER NEUTRINOS ARE DIRAC OR MAJORANA PARTICLES? *

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## Abstract

There has recently been interest in the possibility that neutrino–electron scattering experiments could determine whether neutrinos are Dirac or Majorana particles by providing information on their electromagnetic structure. We try to explain why studies of neutrino electromagnetic structure actually cannot distinguish between Dirac and Majorana neutrinos.

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Can experiments on neutrino electron scattering tell us whether neutrinos are Majorana or Dirac particles? There has recently been interest in this question. In this talk I would like to try to clarify the answer.

Consider the scattering by electrons of highly-relativistic muon neutrinos produced by an accelerator. While the cross section for this process is dominated by $Z^0$ exchange, it also receives smaller contributions from other diagrams, such as photon exchange between the $\nu_\mu$ and the $e$. In the standard model, the photon exchange contribution is expected to be several percent. Suppose one could observe this contribution, a possibility that has been raised by Auriemma, Srivastava, and Widom,[1] and by Winter.[2] Then one would gain some knowledge of the electromagnetic structure of $\nu_\mu$; that is, of the matrix element $< \nu_\mu \left| J_\alpha^{EM} \right| \nu_\mu >$, where $J_\alpha^{EM}$ is the electromagnetic current.

Auriemma *et al.*[1] have argued that a search for the photon exchange contribution to $\nu_\mu e$ scattering would be a feasible way to learn whether muon neutrinos are Dirac or Majorana particles. They state that the electromagnetic form factors of a Majorana neutrino vanish; that is, that such a neutrino does not couple to a photon. Thus, if a photon exchange contribution to $\nu_\mu e$ scattering were detected, we would know that $\nu_\mu$ is a Dirac particle. They state further that in the standard model, a Dirac neutrino does couple to a photon through a non-vanishing electric charge radius, whose size is predicted by the model, and is large enough to make the photon exchange contribution to $\nu_\mu e$ scattering nearly observable with present experimental sensitivity. Thus, with somewhat improved sensitivity, we could observe the photon exchange contribution corresponding to a standard-model Dirac $\nu_\mu$, or, if we fail to observe it at the predicted level, we would have evidence that the $\nu_\mu$ is a Majorana particle.

This is a reasonable and interesting argument, but, unfortunately, it is not correct. The actual state of affairs is an example of what has been referred to as the "practical Majorana-Dirac confusion theorem".[3] This asserts that, so long as there are no right-handed currents, when the mass of a neutrino goes to zero, it gradually becomes impossible to tell whether the neutrino is a Majorana or a Dirac particle. In particular, it has been shown explicitly[3] that when a neutrino $\nu$ has a mass which is small compared to its energy, we cannot determine whether $\nu$ is a Majorana or a Dirac particle by measuring its electromagnetic matrix element, $< \nu \left| J_\alpha^{EM} \right| \nu >$. Let us briefly review the demonstration of this fact.[4]

For a Dirac neutrino $\nu^D$ (a neutrino that is not its own antiparticle), the most general expression for the electromagnetic matrix element is

$$\begin{aligned} < \nu^D( p_f, s_f) \left| J_\alpha^{EM} \right| \nu^D(p_i, s_i) >= & \, i\overline{u_f}[F_D\gamma_\alpha \\ & + G_D(q^2\gamma_\alpha - 2m_\nu iq_\alpha)\gamma_5 + M_D\sigma_{\alpha\beta}q_\beta + E_D i\sigma_{\alpha\beta}q_\beta\gamma_5]u_i. \end{aligned} \tag{1}$$

Here $p_i$ and $s_i$ are the momentum and spin-projection of the incoming $\nu^D$, and $p_f$ and $s_f$ are the corresponding quantities for the outgoing one. The quantities $u_i$ and $u_f$ are Dirac spinors, $q = p_i - p_f$, $m_\nu$ is the mass of the neutrino, and $F_D, G_D, M_D$, and $E_D$ are form factors which depend on $q^2$. For a relativistic neutrino, the $M_D$ and $E_D$ terms are helicity flipping, and the others helicity conserving.

By contrast, for a Majorana neutrino $\nu^M$ (a neutrino that is its own antiparticle), the most general expression for the electromagnetic matrix element is[3,5,6]

$$< \nu^M(p_f, s_f) \left| J_\alpha^{EM} \right| \nu^M(p_i, s_i) >= i\overline{u_f}[G_M(q^2\gamma_\alpha - 2m_\nu iq_\alpha)\gamma_5]u_i, \tag{2}$$

where $G_M$ is a form factor. While this expression appears to be very different from its analogue for $\nu^D$, we shall see that it is not possible to distinguish between the two experimentally by studying the available relativistic, left-handed neutrinos.

The electromagnetic structures of Dirac and Majorana neutrinos do differ. Only a Dirac neutrino can possess an $F$-type form factor. Crudely speaking, this form factor is the Fourier transform of the electric charge distribution, $\rho_{chg}(r)$:

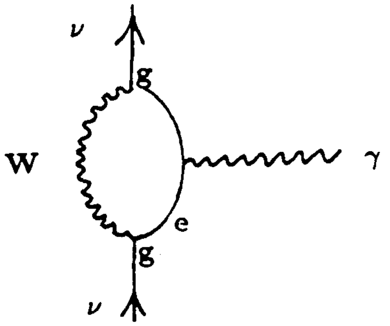
$$F_D \sim \int \rho_{chg}(r)e^{i\vec{q}\cdot\vec{r}}d^3r. \tag{3}$$

From this relation, it follows that the electric charge radius of the neutrino, $r_{chg}$, is given by

$$< r_{chg}^2 >= -6\frac{dF_D}{dq^2}|_{q^2=0}. \tag{4}$$

Indeed, Eq. (4) serves as a <u>definition</u> of the charge radius, even when Eq. (3) is not strictly applicable.

For a neutrino, a charge radius can arise from possesses such as



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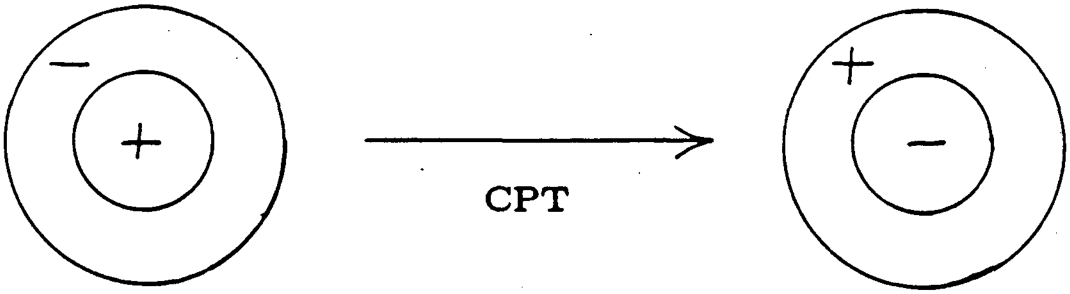
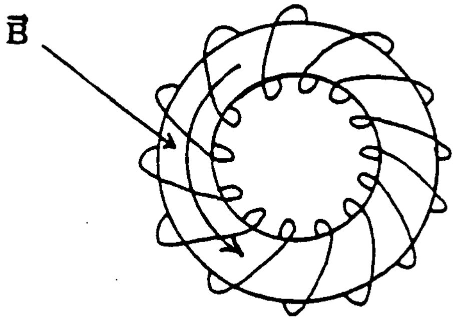
in which the neutrino breaks virtually into a pair of charged particles, whose distribution can be probed by a photon. We expect the charged particles to include the rather heavy $W$, as shown, so that, from the uncertainty principle, $r_{chg} \sim 1/m_W$. More precisely, if we include the semiweak couplings in the diagram, we expect that $< r^2_{chg} > \sim g^2/m^2_W \sim 10^{-32} cm^2$. Now, suppose that processes such as the one in the diagram impart to a (neutral) neutrino a charge distribution which consists, for example, of a positively–charged core surrounded by a compensating negatively–charged shell. This is shown on the left below. Under CPT, this



charge distribution transforms into a negative core surrounded by a positive shell, something quite different from its original self. By contrast, under CPT a Majorana neutrino goes into itself, apart from a spin reversal. Thus, a Majorana neutrino cannot contain the charge distribution under consideration. This illustrates why, more generally, a Majorana neutrino cannot have a charge radius. Only a Dirac neutrino can have one.

Nevertheless, a Majorana neutrino can couple to a photon, as we see from Eq. (2). Such a neutrino can have a G–type form factor, and the electromagnetic structure to which this type of form factor corresponds is pictured below.



The structure is a doughnut, in whose interior a magnetic field $\vec{B}$ such as would be produced by the indicated imaginary windings circulates.[6] It can be shown that the effect of CPT on this structure is to reverse the magnetic field. This is precisely the desired effect for a Majorana neutrino, since the doughnut must obviously surround the neutrino spin axis, and the effect of CPT on a Majorana neutrino is to reverse the spin.

Evidently, the electromagnetic structure of a $\nu^D$, which can include both a charge radius and a magnetic doughnut, is quite different from that of a $\nu^M$, which can involve only the latter. Nevertheless, experiments on relativistic, left-handed neutrinos, the only neutrinos available, are insensitive to this difference. To see why this is so, we note first that since a neutrino does not couple to a photon directly, the neutrino–photon effective coupling arises only from higher-order diagrams, such as the loop diagram drawn earlier. In any of these diagrams, the vertices involving the incoming and outgoing neutrinos themselves are (charged- or neutral-current) weak vertices. Thus, assuming that all weak currents involving a neutrino are left-handed, $< \nu^D \left| J^{EM}_\alpha \right| \nu^D > \xrightarrow[m_\nu \to 0]{} 0$ unless both the initial and final $\nu^D$ have negative helicity. Hence, in Eq. (1) we may neglect the helicity-flipping $\sigma_{\alpha\beta}$ terms compared to the others. In addition, the operator multiplying $G_D$ and $G_M$ in Eqs. (1) and (2) obviously simplifies when $m_\nu$ is negligible. Consequently, for left-handed relativistic incoming neutrinos, the surviving electromagnetic matrix elements in the Dirac and Majorana cases are

$$< \nu^D_- \left| J^{EM}_\alpha \right| \nu^D_- > \simeq i\overline{u_{f(-)}}[F_D\gamma_\alpha + G_D q^2\gamma_\alpha\gamma_5]u_{i(-)}, \tag{5}$$

and

$$< \nu^M_- \left| J^{EM}_\alpha \right| \nu^M_- > \simeq i\overline{u_{f(-)}}[G_M q^2\gamma_\alpha\gamma_5]u_{i(-)}. \tag{6}$$

Here the subscript "$-$" on the $\nu$ and the parenthetical "$(-)$" on the $u$ refer to helicity.

Now, for $m_\nu << |\vec{p}_\nu|$, the Dirac spinor for the incoming neutrino satisfies

$$\gamma_5 u_{i(\pm)} \simeq \mp u_{i(\pm)}. \tag{7}$$

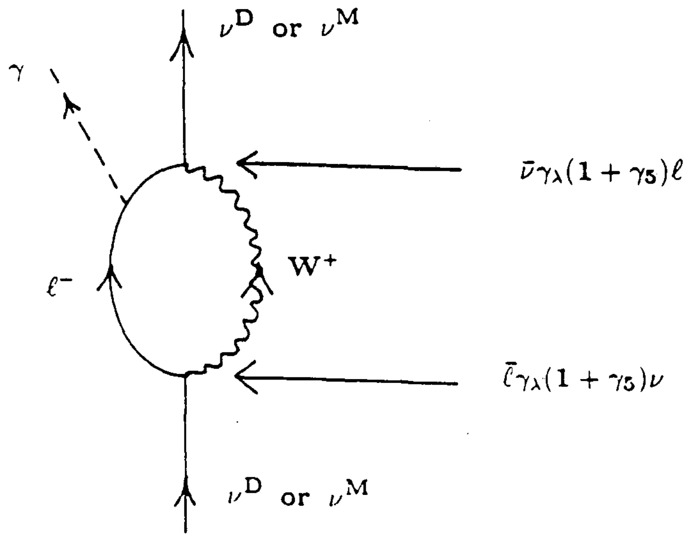
Thus, for neutrinos of negative helicity, the effects of the matrices $\gamma_\alpha$ and $\gamma_\alpha\gamma_5$ in Eqs. (5) and (6) are identical. Indeed, these equations show that if the Dirac and Majorana form factors obey the relation

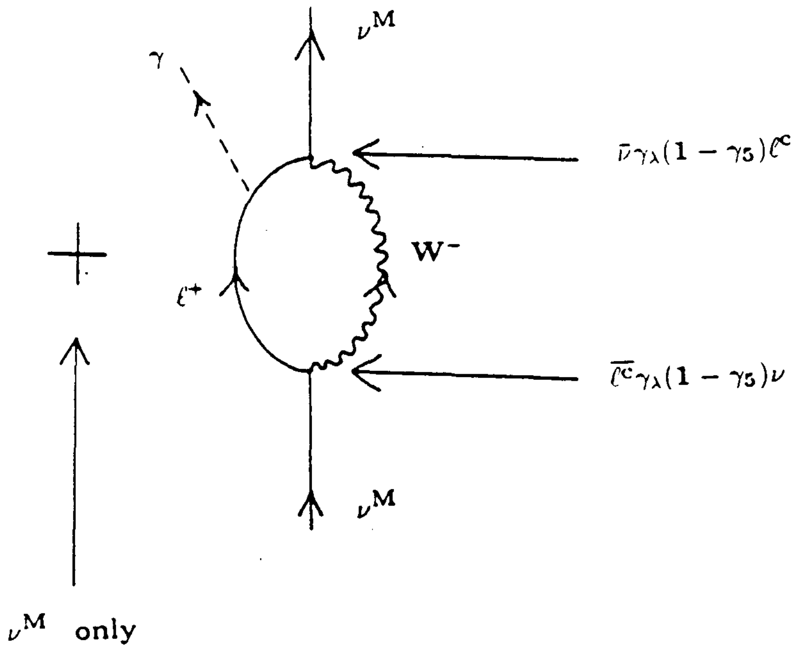
$$G_M q^2 = F_D + G_D q^2, \tag{8}$$

then the electromagnetic matrix elements of relativistic Dirac and Majorana neutrinos are completely indistinguishable.

To make clear the meaning of the relation (8), let us imagine a theory with given left–handed weak interactions, and with Dirac neutrinos possessing the electromagnetic form factors $F_D, G_D, M_D$, and $E_D$ that follow from the given weak interactions. Now suppose that, without changing the weak interactions, we modify the mass terms in the theory so as to change the Dirac neutrinos into Majorana ones. Each neutrino now possesses a single electromagnetic form factor $G_M$ which is determined by the given weak interactions. Equation (8) is a relation between this new form factor and the ones for the Dirac case.

One might think that it would take an accident for relation (8) to hold, but in reality no accident is required. Indeed, at the one–loop level, it is easy to verify that this relation is always obeyed. At this level, either a $\nu^D$ or a $\nu^M$ couples to a photon through the first of the loop diagrams below. In the diagram, $\ell^-$ is some charged lepton, and the currents which act at the weak vertices are indicated. The amplitude corresponding to this diagram is independent of whether the neutrino is of Dirac or Majorana character. However, if the neutrino is a Majorana particle, then, "confused" about whether it is a lepton or an antilepton, it also couples to a photon through the second diagram below. In this diagram, the left–handed currents which occur at the vertices have been rewritten in terms of the charge–conjugate field $\ell^c$. This step makes clear that here these currents act as if they were right–handed.





The only difference between the Dirac and Majorana cases is the presence of the second diagram in the latter case. However, since the currents in this diagram are effectively right-handed, the diagram vanishes for a neutrino of negative helicity as $m_\nu/|\vec{p}_\nu| \to 0$.

Thus, in this limit

$$< \nu_-^M \left| J_\alpha^{EM} \right| \nu_-^M > = < \nu_-^D \left| J_\alpha^{EM} \right| \nu_-^D >, \tag{9}$$

at least to one–loop order, and there is no need to prove relation (8) explicitly. It is, however, amusing to do so.

For an incoming $\nu^D$ of positive helicity, $< \nu_+^D \left| J_\alpha^{EM} \right| \nu_+^D > \xrightarrow[m_\nu \to 0]{} 0$ (assuming no right–handed currents), as already discussed. From Eq. (7) and the analogue of Eq. (5) for positive helicity, this implies that when $m_\nu$ is small,

$$F_D = G_D q^2. \tag{10}$$

Now, if the first of the two loop diagrams above yields for small $m_\nu$ a Dirac electromagnetic matrix element of the form (5), then the two diagrams together obviously yield a Majorana electromagnetic matrix element given by

$$\begin{aligned} < \nu_-^M \left| J_\alpha^{EM} \right| \nu_-^M > &= i\overline{u_{f(-)}}\Big[(F_D\gamma_\alpha + G_D q^2 \gamma_\alpha\gamma_5) \\ &\qquad - (F_D\gamma_\alpha - G_D q^2 \gamma_\alpha\gamma_5)\Big] u_{i(-)}. \end{aligned} \tag{11}$$

In this expression, the quantity in the second parenthesis is the contribution of the second diagram. It has an overall minus sign relative to the first diagram due to the $\gamma - \ell^+$ coupling, and a minus sign in front of the $\gamma_\alpha\gamma_5$ term due to the fact that the currents in the two diagrams have opposite signs in front of $\gamma_5$. Comparing Eq. (11) to Eq. (6), and using Eq. (10), we see that

$$G_M q^2 = F_D + G_D q^2. \tag{8}$$

That is, the relation which makes the electromagnetic matrix elements of Dirac and Majorana neutrinos indistinguishable is indeed satisfied.[7]

Even though the photon exchange contribution to $\nu_\mu e$ scattering cannot tell us whether the $\nu_\mu$ is a Dirac or a Majorana particle, it would still be interesting to observe this contribution. To do so will, however, be fairly difficult. In estimating the sensitivity required, it must be borne in mind that photon exchange is just one of several higher–order contributions to $\nu_\mu e$ scattering. A quantitative treatment must include all of them of the same order as the photon exchange, since, among other things, the latter by itself is not gauge–invariant in a gauge theory. (However, our analysis leading to the conclusion that $< \nu \left| J_\alpha^{EM} \right| \nu >$ is insensitive to whether $\nu$ is a Majorana particle or a Dirac one is valid in any gauge.)

An extensive treatment of the pertinent higher–order contributions implied by the standard model has been given by Marciano, Sarantakos, and Sirlin.[8,9] It is found that in 't–Hooft–Feynman gauge, there is a tendency towards cancellations among certain contributions, and the net higher–order correction to the $\nu_\mu e$ cross section is smaller than that from photon exchange alone. Indeed, the (gauge–invariant) net correction only causes a 1% difference between the value of the Weinberg mixing parameter $\sin^2 \theta_W \equiv 1 - m_W^2/m_Z^2$ that one would infer from the experimental value of $\sigma(\nu_\mu e)/\sigma(\bar{\nu}_\mu e)$ neglecting all higher–order contributions, and the true value. Observation of a difference much larger than 1% might be evidence for physics beyond the standard model.

In summary, while the electromagnetic structures of Dirac and Majorana neutrinos can be quite different, experiments performed with left–handed relativistic neutrinos are insensitive to this difference. This is true, in particular, of $\nu_\mu e$ scattering experiments performed with accelerator neutrinos. To be sure, detection of the higher–order correction to the $\nu_\mu e$ cross section, which includes a photon exchange contribution, would be quite interesting.

## ACKNOWLEDGMENTS

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