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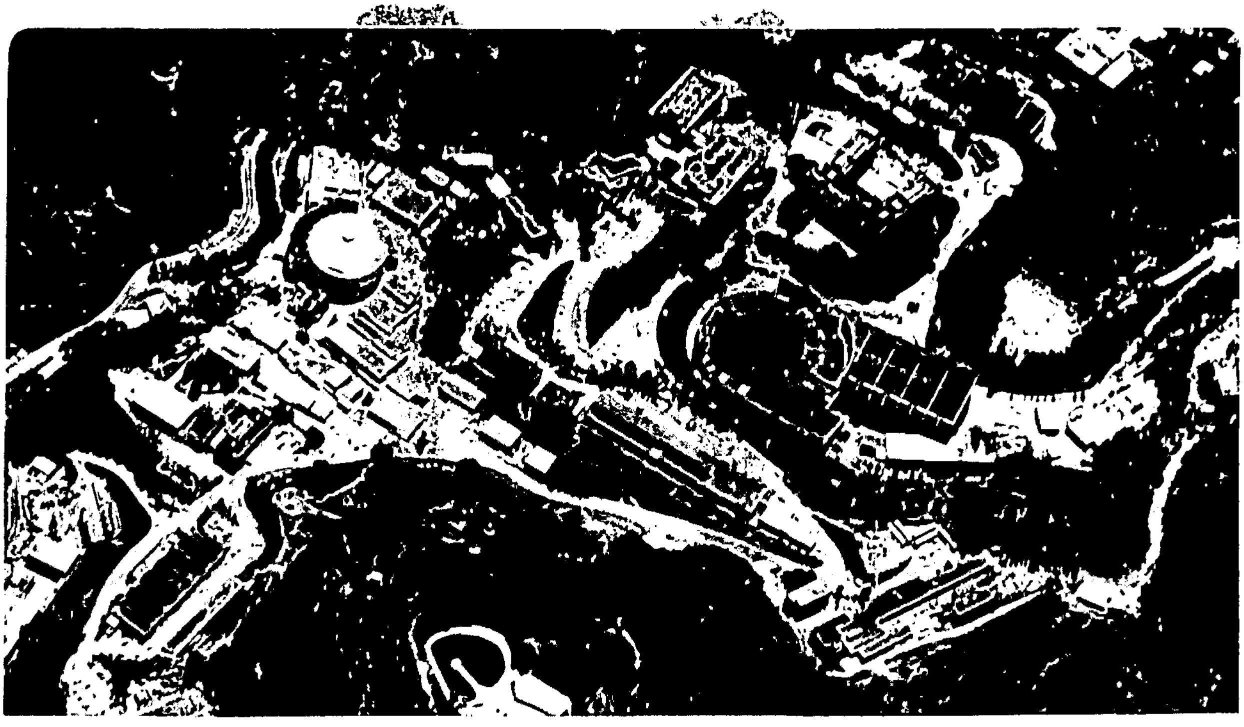

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SUPERGRAVITY IN U(1) SUPERSPACE WITH A TWO FORM GAUGE POTENTIAL*

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Abstract

N = 1 supergravity with an antisymmetric tensor gauge potential is formulated in U(1) superspace. This makes the structure of the theory most transparent. Minimal, new minimal, and 16+16 supergravity are derived from a reducible 20+20 multiplet, and the coupling to supersymmetric gauge theories is constructed. 16+16 supergravity can be coupled to gauge fields via the two form potential in complete analogy to ten-dimensional supergravity.

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1. Introduction

Up to now, four different N = 1 supergravity multiplets have been found: the minimal multiplet [1], the non-minimal multiplet [2], the new minimal multiplet [3], and the 16+16 multiplet [4]. The new minimal multiplet has a local U(1) invariance. Both the new minimal and the 16+16 multiplet contain an antisymmetric tensor gauge potential. The non-minimal multiplet has been shown to be reducible to either the new minimal or the 16+16 multiplet [5,4]. Therefore we will not consider it here.

The structure of supergravity theories becomes most transparent when they are formulated as geometrical theories in curved superspace. This has been done in a systematic way for all the different N = 1 multiplets [6,4]. Except for the minimal multiplet, however, the necessary calculational effort is still enormous. One purpose of this paper is to show that it can be considerably diminished by extending the structure group of superspace to $SL(2,\mathbb{C}) \times U(1)$. In particular, this is the most natural setting for the description of the new minimal multiplet. The second purpose is to emphasize the rôle of the two form gauge potential as a basic geometric object in superspace. It is treated here almost equally to the vielbein, connection, and U(1) potential.

In sect. 2 a short introduction into the geometry of U(1) superspace is given. Natural constraints in U(1) superspace are derived from those in Lorentz superspace. As for the torsion, they correspond to the constraints for minimal supergravity [7]. The Bianchi identities subject to these constraints are solved in sect. 3. The independent component fields form a reducible multiplet with 16 bosonic and 16 fermionic degrees of freedom. It can be considered as the minimal multiplet plus a vector multiplet.

Chiral superfields and chiral densities in U(1) superspace are defined in sect. 4. A special chiral density and an invariant action are constructed. This action leads to the lagrangians for the minimal and the 16+16 multiplet. For the new minimal multiplet it vanishes as a consequence of the vanishing volume of this kind of superspace [5]. It turns out, however, that a different action can be constructed in this special case.

In sect. 5 the superspace geometry of a two form gauge potential is briefly reviewed. Constraints are imposed on its field strength and the Bianchi identities are solved. The independent component fields enlarge the multiplet of sect. 3 to a reducible 20+20 multiplet. It can be reduced either to the new minimal multiplet (12+12) or, by breaking the U(1), to the 16+16 multiplet. The latter is parameterized by a real number n. For n = 0 it is further reduced to the minimal multiplet (12+12). For n = ¼ the 16+16 multiplet is the N = 1 limit of a N = 4 supergravity theory with antisymmetric tensor potential [8]. This has been shown in ref. [4] (our n differs from theirs).

In sects. 6 and 7 supersymmetric gauge theories are coupled to the various supergravity multiplets using a chiral basis of superspace. For minimal and new minimal supergravity the results are well-known [7,9]. A different mechanism for 16+16 supergravity has been found by R. Grimm [10]. The transformation law of the two form potential is modified such that its field strength contains the Chern-Simons three form of the gauge fields. The supergravity and Yang-Mills lagrangians are then given by one and the same action in superspace. We transfer these results to U(1) superspace and compute the component form of the combined lagrangian. It is remarkable that this mechanism, which is also known from ten-dimensional supergravity [11], yields a unified supergravity/Yang-Mills lagrangian only for the 16+16 multiplet.

Finally, the coupling to matter superfields is constructed in the usual way [7]. R-invariance is required for new minimal supergravity and simplifies the coupling to 16+16 supergravity. Even without R-invariance, however, no singularity has to be introduced. 16+16 supergravity coupled to matter could possibly be the low-energy limit of the superstring [12].

We use the conventions of the book of Wess and Bagger [7]. At least chap. XII of this book is presupposed in the following.

2. Geometry of U(1) superspace

To start with, we fix the structure group of superspace to be $SL(2,\mathbb{C}) \times U(1)$. This is the most general choice for N = 1 superspace. The group parameters are Lie algebra valued, i. e. they have the same matrix structure as the generators of the Lie group. For the Lorentz group this means

$$\begin{aligned} L_B{}^A &\sim (L_b{}^a, L_\beta{}^\alpha, L^{\dot\beta}{}_{\dot\alpha}) , \\ L_a{}^\alpha &= L_\alpha{}^a = L^{\dot\alpha}{}_{\dot\alpha} = 0 , \\ L_{\beta\dot\beta\alpha\dot\alpha} &= 2\varepsilon_{\dot\beta\dot\alpha} L_{\beta\alpha} - 2\varepsilon_{\beta\alpha} L_{\dot\beta\dot\alpha} , \end{aligned} \tag{1}$$

and for the U(1) we have $\Lambda = -\Lambda^+$. Under these transformations a n-form Ω^A changes infinitesimally by

$$\delta\Omega^A = \Omega^B L_B{}^A + w(\Omega)\,\Omega^A \Lambda , \tag{2}$$

where $w(\Omega)$ means the U(1) weight of Ω^A. The same transformation law holds for the covariant exterior derivative

$$\mathcal{D}\Omega^A = d\Omega^A + \Omega^B \phi_B{}^A + w(\Omega)\,\Omega^A A . \tag{3}$$

The connections $\phi_B{}^A = dz^M \phi_{MB}{}^A$ and $A = dz^M A_M$ for the Lorentz group, resp. U(1), are Lie algebra valued one forms and transform as follows:

$$\delta\phi_B{}^A = -\mathcal{D}L_B{}^A , \qquad \delta A = -d\Lambda . \tag{4}$$

Applying $\mathcal{D}$ to eq. (3) gives

$$\mathcal{D}\mathcal{D}\Omega^A = \Omega^B R_B{}^A + w(\Omega)\,\Omega^A F . \tag{5}$$

The curvatures

$$R_B{}^A = d\phi_B{}^A + \phi_B{}^C \phi_C{}^A \,, \qquad F = dA \tag{6}$$

are Lie algebra valued two forms,

$$\begin{aligned} R_B{}^A &= \tfrac{1}{2}\, dz^M dz^N R_{NMB}{}^A = \tfrac{1}{2}\, E^C E^D R_{DCB}{}^A \,, \\ F &= \tfrac{1}{2}\, dz^M dz^N F_{NM} = \tfrac{1}{2}\, E^A E^B F_{BA} \,, \end{aligned} \tag{7}$$

and satisfy the Bianchi identities

$$\mathcal{D} R_B{}^A = 0 \,, \qquad dF = 0 \,. \tag{8}$$

This was the affine structure of U(1) superspace. In addition, we impose a metric structure, i. e. we distinguish a set $E^A = dz^M E_M{}^A$ of one forms and call them orthonormal or vielbein. We assign the following U(1) weights:

$$w(E^\alpha) = 1 \,, \quad w(E_{\dot\alpha}) = -1 \,, \quad w(E^a) = 0 \,. \tag{9}$$

The covariant derivative of the vielbein,

$$\begin{aligned} T^A &= \mathcal{D} E^A \,, \\ T^A &= \tfrac{1}{2}\, dz^M dz^N T_{NM}{}^A = \tfrac{1}{2}\, E^B E^C T_{CB}{}^A \,, \end{aligned} \tag{10}$$

is the torsion two form, which satisfies the Bianchi identity

$$\mathcal{D} T^A = E^B R_B{}^A + w(E^A)\, E^A F \,. \tag{11}$$

We rewrite some equations, which will be needed later on, in a more explicit form. The covariant derivative (3) acts on a vector V_A as follows:

$$\mathcal{D}_M V_A = \partial_M V_A - \phi_{MA}{}^B V_B + w(V)\, A_M V_A \,. \tag{12}$$

The (anti)commutator of two covariant derivatives (5) gives

$$\begin{aligned} [\mathcal{D}_A, \mathcal{D}_B\} V_C &= - T_{AB}{}^D \mathcal{D}_D V_C - R_{ABC}{}^D V_D + w(V)\, F_{AB} V_C \,, \\ [\mathcal{D}_A, \mathcal{D}_B\} &= \mathcal{D}_A \mathcal{D}_B - (-)^{ab} \mathcal{D}_B \mathcal{D}_A \,. \end{aligned} \tag{13}$$

The structure equations (10) and (6) read explicitly

$$\begin{aligned} T_{NM}{}^A &= \mathcal{D}_N E_M{}^A - (-)^{nm} \mathcal{D}_M E_N{}^A \,, \\ R_{NMB}{}^A &= \partial_N \phi_{MB}{}^A - (-)^{nm} \partial_M \phi_{NB}{}^A \\ &\quad - \phi_{NB}{}^C \phi_{MC}{}^A + (-)^{nm} \phi_{MB}{}^C \phi_{NC}{}^A \,, \\ F_{NM} &= \partial_N A_M - (-)^{nm} \partial_M A_N \,, \end{aligned} \tag{14}$$

and the Bianchi identities (11) and (8) can be written as

$$\begin{aligned} &\oint_{DCB} \left(R_{DCB}{}^A + w(E^A)\, \delta_B^A F_{DC} - \mathcal{D}_D T_{CB}{}^A - T_{DC}{}^E T_{EB}{}^A \right) = 0 \,, \\ &\oint_{EDC} \left(\mathcal{D}_E R_{DCB}{}^A + T_{ED}{}^F R_{FCB}{}^A \right) = 0 \,, \\ &\oint_{CBA} \left(\mathcal{D}_C F_{BA} + T_{CB}{}^D F_{DA} \right) = 0 \,. \end{aligned} \tag{15}$$

Here $\oint$ denotes the graded cyclic sum. (The sign changes according to $AB = -(-)^{ab} BA$.)

Lorentz indices A, B, ... transform under the structure group, Einstein indices M, N, ... under general coordinate transformations with parameters $\delta z^M = \xi^M$. (This differs from [7] by sign.) The transformation laws of the vielbein, the connections, and an arbitrary vector V^A read

$$
\begin{aligned}
\delta V^A &= \xi^M \partial_M V^A + V^B L_B{}^A + w(V)\, V^A \Lambda , \\
\delta E_M{}^A &= \xi^N \partial_N E_M{}^A + (\partial_M \xi^N) E_N{}^A + E_M{}^B L_B{}^A + w(E^A)\, E_M{}^A \Lambda , \\
\delta \phi_{MB}{}^A &= \xi^N \partial_N \phi_{MB}{}^A + (\partial_M \xi^N) \phi_{NB}{}^A - \mathcal{D}_M L_B{}^A , \\
\delta A_M &= \xi^N \partial_N A_M + (\partial_M \xi^N) A_N - \partial_M \Lambda .
\end{aligned} \tag{16}
$$

The reparametrizations $L_B{}^A = \xi^M \phi_{MB}{}^A + L'_B{}^A$ and $\Lambda = \xi^M A_M + \Lambda'$ lead to the following covariant form of the transformation laws:

$$
\begin{aligned}
\delta V^A &= \xi^B \mathcal{D}_B V^A + V^B L'_B{}^A + w(V)\, V^A \Lambda' , \\
\delta E_M{}^A &= \xi^B T_{BM}{}^A + \mathcal{D}_M \xi^A + E_M{}^B L'_B{}^A + w(E^A)\, E_M{}^A \Lambda' , \\
\delta \phi_{MB}{}^A &= \xi^C R_{CMB}{}^A - \mathcal{D}_M L'_B{}^A , \\
\delta A_M &= \xi^A F_{AM} - \partial_M \Lambda' .
\end{aligned} \tag{17}
$$

In the next step we are going to impose covariant constraints in order to reduce the huge number of component fields contained in vielbein and connection. In ref. [6] a set of natural constraints has been found, which eliminate the Lorentz connection and the higher dimensional parts of the vielbein as independent variables. These constraints are

$$
\begin{aligned}
& T_{\gamma\beta}{}^a = 0 , \quad T_{\gamma\dot\beta}{}^a = 2i\, \sigma^a_{\gamma\dot\beta} , \\
& T_{\gamma\beta}{}^\alpha = (n+1)\, (\delta^\alpha_\beta T_\gamma + \delta^\alpha_\gamma T_\beta) , \\
& T_\gamma{}^{\dot\beta}{}_{\dot\alpha} = (n-1)\, \delta^{\dot\beta}_{\dot\alpha} T_\gamma , \quad T_{\gamma b}{}^a = 2n\, \delta^a_b T_\gamma , \\
& T_{cb}{}^a = 0 , \qquad\qquad (n \neq -\tfrac{1}{3})
\end{aligned} \tag{18}
$$

and their complex conjugates. However, there is still the freedom to redefine the vielbein:

$$
E'_{M\dot\alpha} = E_{M\dot\alpha} - 2ix\, E_M{}^a (\bar\sigma_a)_{\dot\alpha}{}^\alpha T_\alpha . \tag{19}
$$

This results in [6]

$$
n' = \frac{n+2x}{1-6x} , \qquad T'_\alpha = (1-6x)\, T_\alpha . \tag{20}
$$

The choice $x = -n/2$ gives $n' = 0$, $T'_\alpha = (3n+1)\, T_\alpha$, and the constraints take the form

$$
\begin{aligned}
& T_{\gamma\beta}{}^a = 0 , \quad T_{\gamma\dot\beta}{}^a = 2i\, \sigma^a_{\gamma\dot\beta} , \\
& T_{\gamma\beta}{}^\alpha = \delta^\alpha_\beta T_\gamma + \delta^\alpha_\gamma T_\beta , \\
& T_\gamma{}^{\dot\beta}{}_{\dot\alpha} = -\delta^{\dot\beta}_{\dot\alpha} T_\gamma , \quad T_{\gamma b}{}^a = 0 , \\
& T_{cb}{}^a = 0 .
\end{aligned} \tag{21}
$$

These equations might as well be called natural constraints in Lorentz superspace.

In U(1) superspace we have the additional freedom to change the U(1) connection. The redefinition $A'_{\alpha} = A_{\alpha} - T_{\alpha}$ leads to $T'_{\gamma\beta}{}^{\alpha} = T'_{\gamma}{}^{\dot\beta}{}_{\dot\alpha} = 0$, and a suitable redefinition of A_a gives $F_{\alpha\dot\alpha} = 0$. Thus the natural constraints in U(1) superspace are

$$T_{\gamma\beta}{}^{a} = T^{\dot\gamma\dot\beta a} = 0\,, \quad T_{\gamma\dot\beta}{}^{a} = 2i\,\sigma^{a}_{\gamma\dot\beta}\,,$$
$$T_{\underline{\gamma}\underline{\beta}}{}^{\underline{\alpha}} = 0\,, \quad T_{\underline{\gamma} b}{}^{a} = 0\,, \tag{22}$$
$$T_{cb}{}^{a} = 0\,, \quad F_{\alpha\dot\alpha} = 0\,.$$

Here $\underline{\alpha}$ denotes either α or $\dot\alpha$.

3. Solution of the Bianchi identities

We are going to solve the Bianchi identities (15) subject to the constraints (22) in order to determine the independent superfields contained in vielbein and connection. It has been shown [13], that it suffices to solve the first identity. The Bianchi identities for the curvatures are then automatically satisfied.

To anticipate the result, we will need the following superfields:

superfield	R	R^+	$G_{\alpha\dot\alpha}$	$W_{\alpha\beta\gamma}, W_\alpha$	$\bar W_{\dot\alpha\dot\beta\dot\gamma}, \bar W_{\dot\alpha}$
U(1) weight	2	-2	0	1	-1

(23)

$G_{\alpha\dot\alpha}$ is hermitian and $W_{\alpha\beta\gamma}$ is completely symmetric in its indices. W_α and $\bar W_{\dot\alpha}$ are defined as

$$W_\alpha = \tfrac{1}{2}\left(\mathcal{D}_\alpha R - \bar{\mathcal{D}}^{\dot\alpha} G_{\alpha\dot\alpha}\right),$$
$$\bar W_{\dot\alpha} = \tfrac{1}{2}\left(\bar{\mathcal{D}}_{\dot\alpha} R^+ - \mathcal{D}^{\alpha} G_{\alpha\dot\alpha}\right). \tag{24}$$

Therefore only three superfields are independent. They are, however, subject to the following conditions:

1) $\bar{\mathcal{D}}_{\dot\alpha} R = \mathcal{D}_\alpha R^+ = 0$

2) $\bar{\mathcal{D}}_{\dot\alpha} W_\alpha = \mathcal{D}_\alpha \bar W_{\dot\alpha} = 0$

3) $\mathcal{D}^\alpha W_\alpha = \bar{\mathcal{D}}_{\dot\alpha} \bar W^{\dot\alpha} \quad \Leftrightarrow \quad \mathcal{D}\mathcal{D}R - \bar{\mathcal{D}}\bar{\mathcal{D}}R^+ = 4i\,\mathcal{D}^a G_a$

4) $\bar{\mathcal{D}}_{\dot\alpha} W_{\alpha\beta\gamma} = \mathcal{D}_\alpha \bar W_{\dot\alpha\dot\beta\dot\gamma} = 0$

5) $\mathcal{D}^\gamma W_{\alpha\beta\gamma} = \sum_{\alpha\beta}\left(\frac{i}{2}\mathcal{D}_\alpha{}^{\dot\gamma} G_{\beta\dot\gamma} - \frac{1}{3}\mathcal{D}_\alpha W_\beta\right)$ *

$$\bar{\mathcal{D}}^{\dot\gamma} \bar W_{\dot\alpha\dot\beta\dot\gamma} = \sum_{\dot\alpha\dot\beta}\left(\frac{i}{2}\mathcal{D}^{\gamma}{}_{\dot\alpha} G_{\gamma\dot\beta} - \frac{1}{3}\bar{\mathcal{D}}_{\dot\alpha} \bar W_{\dot\beta}\right) \tag{25}$$

All the components of the torsion and the curvatures may be expressed in terms of the superfields (23) and their covariant derivatives. The results are summarized below.

Torsion

1) $T_{\dot\gamma\beta\dot\beta\alpha} = -2i\,\varepsilon_{\dot\gamma\dot\beta}\,\varepsilon_{\beta\alpha}\,R$

$T_{\gamma\beta\dot\beta\dot\alpha} = -2i\,\varepsilon_{\gamma\beta}\,\varepsilon_{\dot\beta\dot\alpha}\,R^+$

2) $T_{\gamma\beta\dot\beta\alpha} = -i\left(\varepsilon_{\gamma\beta}\,G_{\alpha\dot\beta} + \frac{1}{2}\varepsilon_{\gamma\alpha}\,G_{\beta\dot\beta}\right)$

$T_{\dot\gamma\beta\dot\beta\dot\alpha} = -i\left(\varepsilon_{\dot\gamma\dot\beta}\,G_{\beta\dot\alpha} + \frac{1}{2}\varepsilon_{\dot\gamma\dot\alpha}\,G_{\beta\dot\beta}\right)$

---

* $\sum_{\alpha\beta} \mathcal{D}_\alpha W_\beta$ means $\mathcal{D}_\alpha W_\beta + \mathcal{D}_\beta W_\alpha$.

3) $T_{\gamma\dot\gamma\,\beta\dot\beta\,\alpha} = \varepsilon_{\dot\gamma\dot\beta}\, t_{\underset{\smile}{\gamma\beta}\alpha} + \varepsilon_{\gamma\beta}\, t_{\underset{\smile}{\dot\gamma\dot\beta}\alpha}$

$$t_{\gamma\beta\alpha} = -2\, W_{\gamma\beta\alpha} + \sum_{\gamma\beta} \varepsilon_{\gamma\alpha} \left(\tfrac{1}{2} \mathcal{D}_\beta R + \tfrac{1}{3} W_\beta \right)$$

$$t_{\dot\gamma\dot\beta\alpha} = \tfrac{1}{2} \sum_{\dot\gamma\dot\beta} \bar{\mathcal{D}}_{\dot\gamma}\, G_{\alpha\dot\beta}$$

4) $T_{\gamma\dot\gamma\,\beta\dot\beta\,\dot\alpha} = \varepsilon_{\gamma\beta}\, t_{\underset{\smile}{\dot\gamma\dot\beta}\dot\alpha} + \varepsilon_{\dot\gamma\dot\beta}\, t_{\underset{\smile}{\gamma\beta}\dot\alpha}$

$$t_{\dot\gamma\dot\beta\dot\alpha} = -2\, \bar W_{\dot\gamma\dot\beta\dot\alpha} + \sum_{\dot\gamma\dot\beta} \varepsilon_{\dot\gamma\dot\alpha} \left(\tfrac{1}{2} \bar{\mathcal{D}}_{\dot\beta} R^+ + \tfrac{1}{3} \bar W_{\dot\beta} \right)$$

$$t_{\gamma\beta\dot\alpha} = \tfrac{1}{2} \sum_{\gamma\beta} \mathcal{D}_\gamma\, G_{\beta\dot\alpha} \qquad (26)$$

Curvature

1) $R_{\delta\gamma\beta\alpha} = 4\,(\varepsilon_{\delta\beta}\,\varepsilon_{\gamma\alpha} + \varepsilon_{\delta\alpha}\,\varepsilon_{\gamma\beta})\, R^+$

$$R_{\dot\delta\dot\gamma\dot\beta\dot\alpha} = 4\,(\varepsilon_{\dot\delta\dot\beta}\,\varepsilon_{\dot\gamma\dot\alpha} + \varepsilon_{\dot\delta\dot\alpha}\,\varepsilon_{\dot\gamma\dot\beta})\, R$$

2) $R_{\delta\gamma\,\dot\beta\dot\alpha} = R_{\dot\delta\dot\gamma\,\beta\alpha} = 0$

3) $R_{\delta\dot\gamma\,\beta\alpha} = - \sum_{\beta\alpha} \varepsilon_{\delta\beta}\, G_{\alpha\dot\gamma}$

$$R_{\delta\dot\gamma\,\dot\beta\dot\alpha} = - \sum_{\dot\beta\dot\alpha} \varepsilon_{\dot\gamma\dot\beta}\, G_{\delta\dot\alpha}$$

4) $R_{\delta\gamma\dot\gamma\,\beta\alpha} = i\,\varepsilon_{\delta\gamma}\, t_{\beta\alpha\dot\gamma} + \tfrac{i}{2} \sum_{\beta\alpha} \varepsilon_{\delta\beta}\, (t_{\gamma\alpha\dot\gamma} + \varepsilon_{\gamma\alpha}\, t_{\dot\gamma\dot\beta}{}^{\dot\beta})$

$$R_{\dot\delta\gamma\dot\gamma\,\dot\beta\dot\alpha} = i\,\varepsilon_{\dot\delta\dot\gamma}\, t_{\dot\beta\dot\alpha\gamma} + \tfrac{i}{2} \sum_{\dot\beta\dot\alpha} \varepsilon_{\dot\delta\dot\beta}\, (t_{\dot\gamma\dot\alpha\gamma} + \varepsilon_{\dot\gamma\dot\alpha}\, t_{\gamma\beta}{}^{\beta})$$

5) $R_{\dot\delta\gamma\dot\gamma\,\beta\alpha} = -2i\,\varepsilon_{\dot\delta\dot\gamma}\, t_{\beta\alpha\gamma} + \tfrac{i}{2} \sum_{\beta\alpha} \varepsilon_{\gamma\beta}\, (t_{\dot\delta\dot\gamma\alpha} + \varepsilon_{\dot\delta\dot\gamma}\, t_{\alpha\delta}{}^{\delta})$

$$R_{\delta\gamma\dot\gamma\,\dot\beta\dot\alpha} = -2i\,\varepsilon_{\delta\gamma}\, t_{\dot\beta\dot\alpha\dot\gamma} + \tfrac{i}{2} \sum_{\dot\beta\dot\alpha} \varepsilon_{\dot\gamma\dot\beta}\, (t_{\delta\gamma\dot\alpha} + \varepsilon_{\delta\gamma}\, t_{\dot\alpha\dot\delta}{}^{\dot\delta})$$

6) $R_{\delta\dot\delta\,\gamma\dot\gamma\,\beta\dot\beta\,\alpha\dot\alpha} = \varepsilon_{\dot\delta\dot\gamma}\,\varepsilon_{\dot\beta\dot\alpha}\,\chi_{\underset{\smile}{\delta\gamma}\,\underset{\smile}{\beta\alpha}} + \varepsilon_{\dot\delta\dot\gamma}\,\varepsilon_{\beta\alpha}\,\varphi_{\underset{\smile}{\delta\gamma}\,\underset{\smile}{\dot\beta\dot\alpha}}$

$$+ \varepsilon_{\delta\gamma}\,\varepsilon_{\dot\beta\dot\alpha}\,\bar\varphi_{\underset{\smile}{\dot\delta\dot\gamma}\,\underset{\smile}{\beta\alpha}} + \varepsilon_{\delta\gamma}\,\varepsilon_{\beta\alpha}\,\bar\chi_{\underset{\smile}{\dot\delta\dot\gamma}\,\underset{\smile}{\dot\beta\dot\alpha}}$$

$$\chi_{\delta\gamma\beta\alpha} = - \oint_{\delta\gamma\beta\alpha} \mathcal{D}_\delta\, W_{\gamma\beta\alpha} + \tfrac{1}{6}\,(\varepsilon_{\delta\beta}\,\varepsilon_{\gamma\alpha} + \varepsilon_{\delta\alpha}\,\varepsilon_{\gamma\beta})\, R_{ab}{}^{ab}$$

$$\bar\chi_{\dot\delta\dot\gamma\dot\beta\dot\alpha} = \oint_{\dot\delta\dot\gamma\dot\beta\dot\alpha} \bar{\mathcal{D}}_{\dot\delta}\, \bar W_{\dot\gamma\dot\beta\dot\alpha} + \tfrac{1}{6}\,(\varepsilon_{\dot\delta\dot\beta}\,\varepsilon_{\dot\gamma\dot\alpha} + \varepsilon_{\dot\delta\dot\alpha}\,\varepsilon_{\dot\gamma\dot\beta})\, R_{ab}{}^{ab}$$

$$R_{ab}{}^{ab} = -\tfrac{3}{2}\,(\mathcal{D}\mathcal{D} R + \bar{\mathcal{D}}\bar{\mathcal{D}} R^+) - 2\,\mathcal{D}^\alpha W_\alpha + 48\, R R^+ + 6\, G^a G_a$$

$$\varphi_{\delta\gamma\dot\beta\dot\alpha} = \bar\varphi_{\dot\beta\dot\alpha\delta\gamma} = \tfrac{1}{4} \sum_{\delta\gamma} \sum_{\dot\beta\dot\alpha} \big(\mathcal{D}_\delta\, \bar{\mathcal{D}}_{\dot\beta}\, G_{\gamma\dot\alpha} - \bar{\mathcal{D}}_{\dot\beta}\, \mathcal{D}_\delta\, G_{\gamma\dot\alpha}$$

$$- 2\, G_{\delta\dot\beta}\, G_{\gamma\dot\alpha} \big) \qquad (27)$$

Field strength

1) $F_{\underline{\beta\alpha}} = 0$

2) $F_{\beta\alpha\dot\alpha} = 2i\,\varepsilon_{\beta\alpha}\,\bar W_{\dot\alpha}$

$$F_{\dot\beta\alpha\dot\alpha} = 2i\,\varepsilon_{\dot\beta\dot\alpha}\,W_\alpha$$

3) $F_{\beta\dot\beta\alpha\dot\alpha} = \varepsilon_{\dot\beta\dot\alpha}\, f_{\underline{\beta\alpha}} - \varepsilon_{\beta\alpha}\, f_{\underline{\dot\beta\dot\alpha}}$

$$f_{\beta\alpha} = -\frac{1}{2}\sum_{\beta\alpha}\mathcal{D}_\beta W_\alpha \ , \qquad f_{\dot\beta\dot\alpha} = \frac{1}{2}\sum_{\dot\beta\dot\alpha}\bar{\mathcal{D}}_{\dot\beta}\bar W_{\dot\alpha} \tag{28}$$

With several redefinitions, this solution can also be obtained from the more general solution in ref. [6].

The U(1) field strength is entirely given in terms of the chiral superfield W_α. Therefore W_α contains all the gauge invariant component fields of the U(1) connection. To derive an explicit relation, we write F = dA in the form

$$F_{BA} = \mathcal{D}_B A_A - (-)^{ba}\mathcal{D}_A A_B + T_{BA}{}^C A_C \tag{29}$$

and obtain

$$\begin{aligned}
&\sum_{\beta\alpha}\mathcal{D}_\beta A_\alpha = \sum_{\dot\beta\dot\alpha}\bar{\mathcal{D}}_{\dot\beta}\bar A_{\dot\alpha} = 0 \ , \\
&A_{\alpha\dot\alpha} = \frac{i}{2}\left(\mathcal{D}_\alpha \bar A_{\dot\alpha} + \bar{\mathcal{D}}_{\dot\alpha} A_\alpha\right) \ , \\
&W_\alpha = -\frac{i}{4}\left(\bar{\mathcal{D}}^{\dot\alpha} A_{\alpha\dot\alpha} - \mathcal{D}_{\alpha\dot\alpha}\bar A^{\dot\alpha}\right) + A_\alpha R - \frac{5}{8}\bar A^{\dot\alpha} G_{\alpha\dot\alpha} \ , \\
&\bar W_{\dot\alpha} = -\frac{i}{4}\left(\mathcal{D}^{\alpha} A_{\alpha\dot\alpha} - \mathcal{D}_{\alpha\dot\alpha} A^{\alpha}\right) - \bar A_{\dot\alpha} R^+ + \frac{5}{8} A^{\alpha} G_{\alpha\dot\alpha} \ .
\end{aligned} \tag{30}$$

It is now straightforward to find the independent component fields of the vielbein and the connections. The transformation laws (17) show that most of the $\theta = \bar\theta = 0$ components may be gauged away using the θ and $\bar\theta$ components of the parameters ξ^A, $L'_B{}^A$, and Λ'. We are left with

$$E_M{}^A\Big|_{\theta=\bar\theta=0} \sim \begin{pmatrix} e_m{}^a & \frac{1}{2}\psi_m{}^\alpha & \frac{1}{2}\bar\psi_{m\dot\alpha} \\ 0 & \delta_\mu^\alpha & 0 \\ 0 & 0 & \delta_{\dot\alpha}^{\dot\mu} \end{pmatrix} \ , \tag{31}$$

$$\phi_{MB}{}^A\Big| \sim \left(\omega_{mB}{}^A, 0, 0\right) \ , \quad A_M\Big| \sim \left(i a_m, 0, 0\right) \ .$$

The graviton $e_m{}^a$ and the gravitino $\psi_m{}^\alpha$ are the physical fields of N = 1 supergravity. The Lorentz connection ω is not an independent field. As a consequence of the constraint $T_{cb}{}^a = 0$, it may be expressed in terms of e and ψ :

$$\begin{aligned}
\omega_{nml} = &-\frac{1}{2}\left(e_{la}\partial_n e_m{}^a - e_{na}\partial_m e_l{}^a + e_{ma}\partial_l e_n{}^a\right) \\
&+\frac{i}{4}\left(\psi_n\sigma_l\bar\psi_m - \psi_m\sigma_n\bar\psi_l + \psi_l\sigma_m\bar\psi_n\right) - (m\leftrightarrow l) \ .
\end{aligned} \tag{32}$$

The remaining component fields are defined as follows:

$$\begin{aligned}
&R\big| = -\frac{1}{6}M \ , \quad R^+\big| = -\frac{1}{6}\bar M \ , \quad G_a\big| = -\frac{1}{3}b_a \ , \\
&W_\alpha\big| = -i\lambda_\alpha \ , \quad \bar W_{\dot\alpha}\big| = i\bar\lambda_{\dot\alpha} \ , \quad \mathcal{D}^\alpha W_\alpha\big| = -2D \ .
\end{aligned} \tag{33}$$

Altogether, we obtain the multiplet

$$\left(e_m{}^a, \psi_m{}^\alpha, a_m, M, b_a, \lambda_\alpha, D\right) \tag{34}$$

with 16 bosonic and 16 fermionic degrees of freedom. It is reducible, since it contains the minimal multiplet ($e_m{}^a$, $\psi_m{}^\alpha$, M, b_a) as well as the vector multiplet (a_m, λ_α, D).

The component fields (34) are all independent. It remains to be shown that they form a closed set of fields, i. e. transform into each other under supergravity transformations. The parameters of these transformations have the $\theta = \bar\theta = 0$ components

$$\zeta^A| \sim (0, \zeta^\alpha, \bar\zeta_{\dot\alpha}) \,, \quad L'_B{}^A| = 0 \,, \quad \Lambda'| = 0 \,. \tag{35}$$

In addition, they have non-vanishing θ and $\bar\theta$ components in order to preserve the gauge (31). Inserting (35) into (17) yields the following transformation laws:

$$\begin{aligned}
\delta e_m{}^a &= i\,(\zeta\sigma^a\bar\psi_m + \bar\zeta\bar\sigma^a\psi_m) \,, \\
\delta\bar\psi_{m\dot\alpha} &= 2D_m\bar\zeta_{\dot\alpha} + \tfrac{i}{3}(\zeta\sigma_m)_{\dot\alpha}\bar M + \tfrac{i}{3}\bar\zeta_{\dot\alpha}b_m - \tfrac{i}{3}(\bar\zeta\bar\sigma_m\sigma^a)_{\dot\alpha}b_a \,, \\
\delta a_m &= i\,(\zeta\sigma_m\bar\lambda + \bar\zeta\bar\sigma_m\lambda) \,, \\
\delta\bar M &= 4i\,\bar\zeta\bar\lambda - 4\,\bar\zeta^{\dot\alpha}\,t_{\dot\alpha\dot\beta}{}^{\dot\beta}| \,, \\
\delta b_{\alpha\dot\alpha} &= -4i\,(\zeta_\alpha\bar\lambda_{\dot\alpha} + \bar\zeta_{\dot\alpha}\lambda_\alpha) - \zeta_\alpha\,t_{\dot\alpha\dot\beta}{}^{\dot\beta}| + \bar\zeta_{\dot\alpha}\,t_{\alpha\beta}{}^{\beta}| \\
&\quad - 3\,\zeta^\beta\,t_{\beta\alpha\dot\alpha}| + 3\,\bar\zeta^{\dot\beta}\,t_{\dot\beta\dot\alpha\alpha}| \,, \\
\delta\bar\lambda_{\dot\alpha} &= -i\,\bar\zeta_{\dot\alpha}D + i\,\bar\zeta^{\dot\beta}f_{\dot\beta\dot\alpha}| \,, \\
\delta D &= (\bar\zeta\bar\sigma^a)^\alpha\hat D_a\lambda_\alpha - (\zeta\sigma^a)_{\dot\alpha}\hat D_a\bar\lambda^{\dot\alpha} - \tfrac{i}{2}(\zeta\sigma^a\bar\lambda + \bar\zeta\bar\sigma^a\lambda)\,b_a \,, \\
\hat D_a\bar\lambda_{\dot\alpha} &= e_a{}^m\left(D_m\bar\lambda_{\dot\alpha} + \tfrac{i}{2}\bar\psi_{m\dot\alpha}D - \tfrac{i}{2}\bar\psi_m{}^{\dot\beta}f_{\dot\beta\dot\alpha}|\right) \,.
\end{aligned} \tag{36}$$

The $\theta = \bar\theta = 0$ components of $T_{cb}{}^{\alpha}$ and F_{ba} are related to the Rarita-Schwinger and U(1) field strengths through

$$\begin{aligned}
t_{\dot\alpha\dot\beta}{}^{\dot\beta}| &= -\tfrac{1}{2}(\bar\psi_{mn}\bar\sigma^{mn})_{\dot\alpha} + \tfrac{i}{4}(\psi_m\sigma^m)_{\dot\alpha}\bar M - \tfrac{i}{4}\bar\psi_{m\dot\alpha}b^m \,, \\
t_{\beta\alpha\dot\alpha}| &= -\tfrac{1}{2}(\sigma^{mn})_{\beta\alpha}\bar\psi_{mn\dot\alpha} - \tfrac{i}{24}\sum_{\beta\alpha}\big(2\,\psi_{\alpha\dot\alpha\beta}\bar M \\
&\quad + \bar\psi_{\alpha\dot\alpha}{}^{\dot\beta}b_{\beta\dot\beta} - 3\,\bar\psi_{\beta\dot\beta}{}^{\dot\beta}b_{\alpha\dot\alpha}\big) \,, \\
f_{\dot\beta\dot\alpha}| &= -i\,(\bar\sigma^{mn})_{\dot\beta\dot\alpha}\mathcal{F}_{mn} - \tfrac{1}{2}\sum_{\dot\beta\dot\alpha}\big(\psi_{\beta\dot\beta}{}^{\beta}\bar\lambda_{\dot\alpha} + \bar\psi_{\beta\dot\beta\dot\alpha}\lambda^\beta\big) \,, \\
\psi_{mn}{}^{\alpha} &= D_m\psi_n{}^{\alpha} - D_n\psi_m{}^{\alpha} \,, \quad \mathcal{F}_{mn} = \partial_m a_n - \partial_n a_m \,.
\end{aligned} \tag{37}$$

4. Chiral superfields and chiral densities

Chiral superfields in U(1) superspace are subject to the condition

$$\bar{\mathcal{D}}_{\dot\alpha}\phi = 0 \,. \tag{38}$$

Their component fields are defined as

$$A = \phi| \,, \quad \chi_\alpha = \mathcal{D}_\alpha\phi| \,, \quad F = -\tfrac{1}{4}\mathcal{D}\mathcal{D}\phi| \,. \tag{39}$$

Under supergravity transformations the component fields transform as follows:

$$\begin{aligned}
\delta A &= \zeta\chi \,, \\
\delta\chi_\alpha &= 2\zeta_\alpha F + 2i\,(\sigma^a\bar\zeta)_\alpha\hat D_a A \,, \\
\delta F &= \tfrac{1}{3}\zeta\chi\bar M - \tfrac{1}{6}(\bar\zeta\bar\sigma^a\chi)\,b_a + i\,(\bar\zeta\bar\sigma^a)^\alpha\hat D_a\chi_\alpha \\
&\quad + 2i\,w(\phi)\,\bar\zeta\bar\lambda A \,.
\end{aligned} \tag{40}$$

The supercovariant derivatives $\hat{D}_a$ in x-space are defined by

$$\begin{aligned} \hat{D}_a A &= e_a{}^m \left(D_m A - \tfrac{1}{2} \psi_m \chi \right) , \\ \hat{D}_a \chi_\alpha &= e_a{}^m \left(D_m \chi_\alpha - \psi_{m\alpha} F - i (\sigma^b \bar{\psi}_m)_\alpha \hat{D}_b A \right) . \end{aligned} \tag{41}$$

The transformation laws (40) can be written in the short form

$$\delta \Phi = \eta^M D_M \Phi + w(\Phi) \Lambda \Phi , \tag{42}$$

where

$$\Phi = A + \Theta \chi + \Theta\Theta F \tag{43}$$

and

$$\begin{aligned} \eta^m &= 2i\, \Theta \sigma^m \bar{\zeta} + \Theta\Theta \left(\bar{\zeta} \bar{\sigma}^n \sigma^m \bar{\psi}_n \right) , \\ \eta^\mu &= \zeta^\mu + \tfrac{1}{3} \Theta\Theta \left(\zeta^\mu \bar{M} - \tfrac{1}{2} \bar{\zeta}_{\dot{\alpha}} b^{\mu\dot{\alpha}} \right) - \tfrac{1}{2} \eta^m \psi_m{}^\mu , \\ \Lambda &= 2i\, \Theta\Theta \bar{\zeta} \bar{\lambda} . \end{aligned} \tag{44}$$

Here Θ^α are new anticommuting variables and $D_M \sim (D_m, \delta^\alpha_\mu \frac{\partial}{\partial \Theta^\alpha})$.

Chiral superfields may be constructed out of general superfields by means of a chiral projection operator. In U(1) superspace this is

$$\bar{\Delta} = \bar{\mathcal{D}} \bar{\mathcal{D}} - 8R . \tag{45}$$

For any Lorentz scalar V we have

$$\bar{\mathcal{D}}_{\dot{\alpha}} \bar{\Delta} V = 0 . \tag{46}$$

Chiral densities in U(1) superspace are defined by the transformation law

$$\begin{aligned} \delta \Delta &= (-)^m D_M (\eta^M \Delta) + w(\Delta) \Lambda \Delta , \\ \Delta &= a + \Theta \rho + \Theta\Theta f . \end{aligned} \tag{47}$$

The product of Δ and a chiral superfield Φ is again a chiral density with U(1) weight $w(\Delta\Phi) = w(\Delta) + w(\Phi)$. A special chiral density $\mathcal{E}$ can be constructed from the lowest component

$$a = e = \det e_m{}^a . \tag{48}$$

Comparing the transformation laws (47) and (36), we find

$$\begin{aligned} \rho_\alpha &= i e \left(\sigma^m \bar{\psi}_m \right)_\alpha , \\ f &= - e \left(\bar{M} + \bar{\psi}_m \bar{\sigma}^{mn} \bar{\psi}_n \right) . \end{aligned} \tag{49}$$

Eq. (47) yields for the highest component of $\mathcal{E}$

$$\delta f = - 4 D_m \left(e \bar{\zeta} \bar{\sigma}^{mn} \bar{\psi}_n \right) + 2 i e\, w(\mathcal{E}) \bar{\zeta} \bar{\lambda} . \tag{50}$$

Comparing this with the explicit variation of f gives $w(\mathcal{E}) = -2$.

The integral of a chiral density Δ over x and Θ is an invariant action, provided that the U(1) weight of Δ vanishes. This condition is satisfied for

$$\mathcal{L} = \int d^4x\, d^2\Theta\, \mathcal{E} R + \text{h.c.} . \tag{51}$$

Using

$$R_{ab}{}^{ab}| - 2i\,(\bar\psi_m \bar\sigma^m)^\alpha t_{\alpha\beta}{}^\beta| - 2i\,(\bar\sigma^m \psi_m)^{\dot\alpha} t_{\dot\alpha\dot\beta}{}^{\dot\beta}|$$

$$= \mathcal{R} - \tfrac{1}{2}\,\varepsilon^{klmn}(\bar\psi_k \bar\sigma_l \psi_{mn} - \psi_k \sigma_l \bar\psi_{mn}) \qquad (52)$$

$$-(\bar\psi_m \bar\sigma^{mn} \bar\psi_n) M - (\psi_m \sigma^{mn} \psi_n)\bar M \;,$$

$$\mathcal{R} = e_a{}^m e_b{}^n R_{mn}{}^{ab}| \;,$$

we obtain the following component form of $\mathcal{S}$:

$$\mathcal{S} = \int d^4x\, \frac{e}{3}\Big[\tfrac{1}{2}\mathcal{R} - 2D - \tfrac{1}{4}\varepsilon^{klmn}(\bar\psi_k \bar\sigma_l \psi_{mn} - \psi_k \sigma_l \bar\psi_{mn}) \qquad (53)$$

$$+ \psi_m \sigma^m \bar\lambda - \bar\psi_m \bar\sigma^m \lambda + \tfrac{1}{3}(M\bar M - b^a b_a)\Big] \;.$$

This invariant is not yet a consistent action for supergravity. It leads, however, to consistent lagrangians if the multiplet (34) is reduced. As a first application, we consider the reduction to the

Minimal multiplet

This multiplet is obtained by eliminating the U(1): A = F = 0 $\Rightarrow W_\alpha = 0 \Rightarrow \lambda_\alpha = D = 0$. We are left with the component fields

$$(e_m{}^a,\ \psi_m{}^\alpha,\ M,\ b_a). \qquad (54)$$

The invariant (53) yields the supergravity lagrangian

$$\mathcal{L}_{min} = -3\int d^2\Theta\, \mathcal{E} R + h.c. = -\tfrac{1}{2}\, e\, \mathcal{R} \qquad (55)$$

$$+ \tfrac{1}{4}\, e\, \varepsilon^{klmn}(\bar\psi_k \bar\sigma_l \psi_{mn} - \psi_k \sigma_l \bar\psi_{mn}) - \tfrac{1}{3}\, e\,(M\bar M - b^a b_a) \;.$$

Another reduction of the multiplet (34) is given by the constraint R = 0. This leads to the new minimal multiplet. Since this multiplet contains an antisymmetric tensor potential, we will consider it in the next section.

5. Two form gauge potential

We introduce a two form gauge potential $B = \frac{1}{2}\, dz^M dz^N B_{NM}$ with the transformation law

$$\delta B = d\omega \;. \qquad (56)$$

Here $\omega = dz^M \omega_M$ is a one form gauge parameter. The field strength

$$G = dB \;, \qquad (57)$$

$$G = \tfrac{1}{3!}\, dz^M dz^N dz^L G_{LNM} = \tfrac{1}{3!}\, E^A E^B E^C G_{CBA} \;,$$

satisfies the Bianchi identity

$$dG = 0 \;. \qquad (58)$$

The above equations read explicitly

$$\delta B_{NM} = \xi^L \partial_L B_{NM} + (\partial_N \xi^L) B_{LM} - (-)^{nm} (\partial_M \xi^L) B_{LN} \qquad (59)$$

$$+ \partial_N \omega_M - (-)^{nm} \partial_M \omega_N \;,$$

$$G_{LNM} = \oint_{LNM} \partial_L B_{NM} \;, \qquad (60)$$

$$E^A E^B E^C E^D (\mathcal{D}_D G_{CBA} + \tfrac{3}{2}\, T_{DC}{}^F G_{FBA}) = 0 \;. \qquad (61)$$

The transformation law (59) can be written in the covariant form

$$\delta B_{NM} = \xi^A G_{ANM} + \partial_N \omega'_M - (-)^{nm} \partial_M \omega'_N \;, \qquad (62)$$

where $\omega'_M = \omega_M + \xi^N B_{NM}$.

Now we impose the following constraints on the field strength G:

$$G_{\underline{\gamma\beta\alpha}} = 0 ,$$
$$G_{\gamma\beta a} = G^{\dot\gamma\dot\beta}{}_{a} = 0 , \tag{63}$$
$$G_{\gamma\dot\beta a} = 2i\,(\sigma_a)_{\gamma\dot\beta}\, e^{U} .$$

U is a hermitian superfield with vanishing U(1) weight. The remaining components of G are then determined by the Bianchi identities (61). We find

$$G_{\gamma\beta\dot\beta\alpha\dot\alpha} = -2\,\varepsilon_{\dot\beta\dot\alpha} \sum_{\beta\alpha} \varepsilon_{\gamma\beta}\, \mathcal{D}_\alpha e^{U} ,$$
$$G_{\dot\gamma\beta\dot\beta\alpha\dot\alpha} = -2\,\varepsilon_{\beta\alpha} \sum_{\dot\beta\dot\alpha} \varepsilon_{\dot\gamma\dot\beta}\, \bar{\mathcal{D}}_{\dot\alpha} e^{U} , \tag{64}$$

$$G_{\gamma\dot\gamma\beta\dot\beta\alpha\dot\alpha} = 2i\,(\varepsilon_{\gamma\alpha}\varepsilon_{\dot\gamma\dot\beta}\tilde G_{\beta\dot\alpha} - \varepsilon_{\gamma\beta}\varepsilon_{\dot\gamma\dot\alpha}\tilde G_{\alpha\dot\beta}) ,$$
$$\tilde G_{\alpha\dot\alpha} = 2\,e^{U} G_{\alpha\dot\alpha} - \tfrac{1}{2}\,[\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}]\, e^{U} . \tag{65}$$

Here the dual field strength vector $\tilde G_a$ is defined as $\tilde G^a = (1/3!)\,\varepsilon^{abcd}\, G_{bcd}$. Moreover, we obtain the condition

$$(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8R)\, e^{U} = (\mathcal{D}\mathcal{D} - 8R^{+})\, e^{U} = 0 . \tag{66}$$

Defining

$$T_\alpha = \mathcal{D}_\alpha U , \quad \bar T_{\dot\alpha} = \bar{\mathcal{D}}_{\dot\alpha} U , \tag{67}$$

we find as an immediate consequence

$$\sum_{\beta\alpha} \mathcal{D}_\beta T_\alpha = \sum_{\dot\beta\dot\alpha} \bar{\mathcal{D}}_{\dot\beta} \bar T_{\dot\alpha} = 0 . \tag{68}$$

Eqs. (65) and (66) may be rewritten in the form

$$\tilde G_{\alpha\dot\alpha} = e^{U}\,(2G_{\alpha\dot\alpha} + \tfrac{1}{2}\bar{\mathcal{D}}_{\dot\alpha} T_\alpha - \tfrac{1}{2}\mathcal{D}_\alpha \bar T_{\dot\alpha} - T_\alpha \bar T_{\dot\alpha}) ,$$
$$R = \tfrac{1}{8}\,(\bar{\mathcal{D}} + \bar T)_{\dot\alpha} \bar T^{\dot\alpha} , \quad R^{+} = \tfrac{1}{8}\,(\mathcal{D} + T)^{\alpha} T_\alpha . \tag{69}$$

Thus we are left with U, $G_{\alpha\dot\alpha}$, and $W_{\alpha\beta\gamma}$ as independent superfields.

In the next step we determine the independent component fields of the two form potential. The transformation law (60) allows to choose the gauge

$$B_{MN}\Big| \sim \begin{pmatrix} t_{mn} & 0 \\ 0 & 0 \end{pmatrix} \tag{70}$$

for the $\theta = \bar\theta = 0$ components. The remaining component fields are defined by

$$U\big| = C , \quad T_\alpha\big| = \varphi_\alpha , \quad \bar T_{\dot\alpha}\big| = \bar\varphi_{\dot\alpha} . \tag{71}$$

They enlarge the multiplet (34) to the reducible 20+20 multiplet

$$(e_m{}^a, \psi_m{}^\alpha, a_m, t_{mn}, C, \varphi_\alpha, M, b_a, \lambda_\alpha, D). \tag{72}$$

This set of fields is smaller than the 24+24 multiplet proposed in ref. [6]. Nevertheless it contains all the irreducible multiplets of N = 1 supergravity.

The transformation laws of the component fields (70) and (71) under supergravity transformations ($\omega'_M\big| = 0$) are

$$\delta t_{mn} = 2e^{C}\,(\zeta\sigma_{mn}\varphi + \bar\zeta\bar\sigma_{mn}\bar\varphi)$$
$$- ie^{C}\,[\zeta\sigma_m\bar\psi_n + \bar\zeta\bar\sigma_m\psi_n - (m \leftrightarrow n)] ,$$

$$\delta C = \zeta \varphi + \bar{\zeta} \bar{\varphi} \ ,$$

$$\delta \varphi_\alpha = \tfrac{2}{3} \zeta_\alpha \bar{M} - (\sigma^a \bar{\zeta})_\alpha \left(\tfrac{2}{3} b_a + e^{-C} \tilde{G}_a \big| - i \hat{D}_a C \right)$$

$$- (\zeta \varphi + \bar{\zeta} \bar{\varphi}) \varphi_\alpha \ ,$$

$$\hat{D}_a C = e_a{}^m \left(\partial_m C - \tfrac{1}{2} \psi_m \varphi - \tfrac{1}{2} \bar{\psi}_m \bar{\varphi} \right) . \qquad (73)$$

The $\theta = \bar{\theta} = 0$ component of $\tilde{G}_a$ is related to the field strength of the antisymmetric tensor through

$$\tilde{G}^a \big| = e_m{}^a g^m - i e^C (\varphi \sigma^{ab} \psi_b - \bar{\varphi} \bar{\sigma}^{ab} \bar{\psi}_b)$$

$$+ \tfrac{i}{2} \varepsilon^{abcd} e^C (\psi_b \sigma_c \bar{\psi}_d) \ , \qquad (74)$$

$$g^k = \tfrac{1}{2} \varepsilon^{klmn} \partial_l t_{mn} \ .$$

We consider now the possible reductions of the multiplet (72) and distinguish two cases: with and without U(1) symmetry. Leaving the U(1) unbroken leads to the

New minimal multiplet

To obtain this multiplet, we impose the constraint

$$U = 0 \ . \qquad (75)$$

This gives $T_\alpha = R = 0$, $G_a = \frac{1}{2} \tilde{G}_a$, $W_\alpha = - t_{\alpha\beta}{}^\beta$, and $\mathcal{D}^\alpha W_\alpha = -\frac{1}{2} R_{ab}{}^{ab} + (3/4) \tilde{G}^a \tilde{G}_a$. The dim-2 Bianchi identity for the field strength G becomes $\mathcal{D}^a \tilde{G}_a = 0$, which can also be found from the condition (25.3). The above superfield equations show that the component fields C, φ_α, M, b_a, λ_α, and D are eliminated from the multiplet (72). We are left

with the 12+12 multiplet

$$(e_m{}^a ,\ \psi_m{}^\alpha ,\ a_m ,\ t_{mn}) . \qquad (76)$$

As a consequence of the constraint R = 0, the action (51) vanishes. In this special case, however, there exists another chiral superfield which can be used to construct an invariant action:

$$S = \bar{\mathcal{D}}_{\dot{\alpha}} \bar{A}^{\dot{\alpha}} \ , \quad \bar{\mathcal{D}}_{\dot{\alpha}} S = 0 \ , \quad w(S) = 2 \ . \qquad (77)$$

The lagrangian for new minimal supergravity turns out to be

$$\mathcal{L}_{new} = - \tfrac{1}{2} \int d^2\Theta \, \mathcal{E} S + h.c. \ . \qquad (78)$$

To compute its component form, one has to fix a covariant gauge for the higher θ components of A_α. The result, which does not depend on this gauge, is

$$\mathcal{L}_{new} = - \tfrac{1}{2} e \mathcal{R} + \tfrac{1}{4} e \varepsilon^{klmn} (\bar{\psi}_k \bar{\sigma}_l \psi_{mn} - \psi_k \sigma_l \bar{\psi}_{mn}$$

$$- 4i a_k \psi_l \sigma_m \bar{\psi}_n) + \tfrac{3}{4} e \tilde{G}^a \tilde{G}_a \big| - 2ie A^a \tilde{G}_a \big| \ . \qquad (79)$$

The redefinition $A'_a = A_a + (3i/8) \tilde{G}_a$ leads to the following simple form of the lagrangian:

$$\mathcal{L}_{new} = - \tfrac{1}{2} e \mathcal{R} + \tfrac{1}{4} e \varepsilon^{klmn} (\bar{\psi}_k \bar{\sigma}_l \psi'_{mn} - \psi_k \sigma_l \bar{\psi}'_{mn}$$

$$+ 4 a'_k \partial_l t_{mn}) \ . \qquad (80)$$

16+16 multiplet

The second possibility to reduce the multiplet (72) is by breaking the U(1) symmetry. The most general way to do this is to impose the constraint $A_\alpha = (n+im)\, T_\alpha$, where n and m are real parameters. However, m can be absorbed by the redefinition $E'_M{}^\alpha = e^{imU} E_M{}^\alpha$. This is a U(1) transformation with parameter $\Lambda = imU$, which leaves the constraints (22) and (63) invariant, but changes the above condition into $A'_\alpha = A_\alpha - \mathcal{D}_\alpha \Lambda = n\, T_\alpha$. Therefore it is sufficient to require

$$A_\alpha = n\, T_\alpha \, , \quad \bar{A}_{\dot\alpha} = -n\, \bar{T}_{\dot\alpha} \quad . \tag{81}$$

Inserting this into (30) gives after some manipulations

$$\begin{aligned}
A_{\alpha\dot\alpha} &= \tfrac{i}{2} n \left(\bar{\mathcal{D}}_{\dot\alpha} T_\alpha - \mathcal{D}_\alpha \bar{T}_{\dot\alpha} \right) \\
&= i n \left(e^{-U} \tilde{G}_{\alpha\dot\alpha} - 2 G_{\alpha\dot\alpha} + T_\alpha \bar{T}_{\dot\alpha} \right) , \\
(4n-3) W_\alpha &= -4n\, t_{\alpha\beta}{}^\beta + 3in\, \mathcal{D}_{\alpha\dot\alpha} \bar{T}^{\dot\alpha} \\
&\quad + \tfrac{3}{2} n\, \bar{T}^{\dot\alpha} \left(e^{-U} \tilde{G}_{\alpha\dot\alpha} - 3 G_{\alpha\dot\alpha} + i \mathcal{D}_{\alpha\dot\alpha} U + T_\alpha \bar{T}_{\dot\alpha} \right) , \\
(4n-3)\, \mathcal{D}^\alpha W_\alpha &= -2n\, R_{ab}{}^{ab} + 6n \left(T^\alpha t_{\alpha\beta}{}^\beta - \bar{T}^{\dot\alpha} t_{\dot\alpha\dot\beta}{}^{\dot\beta} \right) \\
&\quad - 3in \left(T^\alpha \mathcal{D}_{\alpha\dot\alpha} \bar{T}^{\dot\alpha} + \bar{T}^{\dot\alpha} \mathcal{D}_{\alpha\dot\alpha} T^\alpha \right) - 3n\, (\mathcal{D}^a U) \mathcal{D}_a U \\
&\quad - 6n\, \mathcal{D}^a \mathcal{D}_a U + 3n\, e^{-2U} \tilde{G}^a \tilde{G}_a - 6n\, e^{-U} T^\alpha \bar{T}^{\dot\alpha} \tilde{G}_{\alpha\dot\alpha} \\
&\quad + 9n\, T^\alpha \bar{T}^{\dot\alpha} G_{\alpha\dot\alpha} - \tfrac{9}{2} n\, T T \bar{T} \bar{T} \quad .
\end{aligned} \tag{82}$$

These equations show two things. First, the value n = 3/4 has to be excluded, because it leads to equations of motion for the graviton and the gravitino. Secondly, the component fields a_m, λ_α , and D are eliminated from the reducible multiplet (72) and we are left with the 16+16 multiplet

$$(e_m{}^a,\ \psi_m{}^\alpha ,\ t_{mn},\ C,\ \varphi_\alpha ,\ M,\ b_a) . \tag{83}$$

The lagrangian for 16+16 supergravity can be obtained from the action (51). It is found to be

$$\mathcal{L}_{16} = (4n-3) \int d^2\Theta\, \mathcal{E} R + h.c. \quad . \tag{84}$$

Inserting the $\theta = \bar\theta = 0$ components of (82) into (53) and integrating by parts yields the following component form of the lagrangian:

$$\begin{aligned}
\tfrac{1}{e} \mathcal{L}_{16} &= -\tfrac{1}{2} \mathcal{R} + \tfrac{1}{4} \varepsilon^{klmn} \left(\bar\psi_k \bar\sigma_l \psi'_{mn} - \psi_k \sigma_l \bar\psi'_{mn} \right) \\
&\quad - in \left(\bar\varphi \bar\sigma^m D'_m \varphi + \varphi \sigma^m D'_m \bar\varphi \right) - n\, (\partial^m C)\, \partial_m C \\
&\quad + n\, e^{-2C} \hat{g}^m \hat{g}_m + \tfrac{1}{9} (4n-3) \left(M \bar{M} - \hat{b}^a \hat{b}_a \right) \\
&\quad - n \left(\psi_n \sigma^m \bar\sigma^n \varphi + \bar\psi_n \bar\sigma^m \sigma^n \bar\varphi \right) \partial_m C \\
&\quad + in\, e^{-C} \left(\psi_n \sigma^m \bar\sigma^n \varphi - \bar\psi_n \bar\sigma^m \sigma^n \bar\varphi \right) \hat{g}_m \\
&\quad + 2n(n-1)\, e^{-C} (\varphi \sigma^m \bar\varphi)\, \hat{g}_m - \tfrac{n}{2} (n-1)(4n-3)\, \varphi\varphi\bar\varphi\bar\varphi \\
&\quad - in(n-1) \left[(\psi_m \sigma^m \bar\varphi) \varphi\varphi + (\bar\psi_m \bar\sigma^m \varphi) \bar\varphi\bar\varphi \right] \\
&\quad + \tfrac{n}{2} \left[(\psi^m \psi_m) \varphi\varphi + (\bar\psi^m \bar\psi_m) \bar\varphi\bar\varphi \right] \\
&\quad + \tfrac{n}{2} \left(\psi_m \sigma^m \bar\psi_n - \bar\psi_m \bar\sigma^m \psi_n \right) (\varphi \sigma^n \bar\varphi) \quad .
\end{aligned} \tag{85}$$

Here we have used the definitions

$$\begin{aligned} \hat{b}_{\alpha\dot\alpha} &= b_{\alpha\dot\alpha} + 3n\,\varphi_\alpha \bar\varphi_{\dot\alpha} \,, \\ \hat{\mathcal{G}}^k &= \tfrac{1}{2}\,\varepsilon^{klmn}\left(\partial_l t_{mn} + i e^c \psi_l \sigma_m \bar\psi_n\right) \,, \\ \psi'_{mn} &= D'_m \psi_n - D'_n \psi_m \,, \end{aligned} \qquad (86)$$

where D'_m is the Lorentz covariant derivative in x-space. We have checked explicitly that the above expression corresponds with the lagrangian given in ref. [4]. The redefinitions are specified in appendix A. The fact that our lagrangian results from the geometry in a simpler and almost diagonal form, clearly indicates that the approach from U(1) superspace is the more natural one.

We conclude this section with some remarks concerning the range of the real parameter n. For n = 0 the lagrangian (85) reduces to the minimal lagrangian (55). For this value of n the 16+16 multiplet splits into the minimal multiplet and a 4+4 multiplet containing the antisymmetric tensor. - The kinetic terms of the lagrangian (85) have the correct sign for n > 0. Negative values of n are therefore excluded. - For n = ¼ the 16+16 multiplet is the N = 1 limit of a N = 4 supergravity theory with antisymmetric tensor potential [8]. - The value n = 3/4 is excluded.

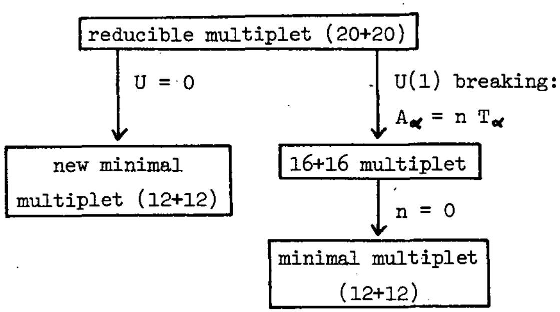

Fig. 1. Relations between the various supergravity multiplets (cf. the corresponding diagram in [10]).

6. Yang-Mills coupling

In the remaining two sections we construct the coupling of supersymmetric gauge theories to the various supergravity multiplets. Gauge theories are characterized by a compact semisimple Lie group $\mathcal{G}$. The infinitesimal transformation law of a n-form Ω in some representation of $\mathcal{G}$ reads

$$\delta_{\tilde\Lambda}\,\Omega = \Omega\,\tilde\Lambda \,, \qquad (87)$$

where $\tilde\Lambda = -\tilde\Lambda^+$ are the Lie algebra valued parameters of $\mathcal{G}$. (The tilde distinguishes the non-abelian from the abelian quantities.) The gauge covariant exterior derivative is given by

$$\tilde{\mathcal{D}}\Omega = \mathcal{D}\Omega + \Omega\tilde{A} \,. \qquad (88)$$

The Yang-Mills potential $\tilde A$ is a Lie algebra valued one form and transforms as follows:

$$\delta\tilde{A} = -\tilde{\mathcal{D}}\tilde\Lambda = -d\tilde\Lambda - [\tilde\Lambda, \tilde A] \,. \qquad (89)$$

The product of two covariant derivatives yields the Yang-Mills field strength $\tilde F$:

$$\begin{aligned} \tilde{\mathcal{D}}\tilde{\mathcal{D}}\Omega &= \mathcal{D}\mathcal{D}\Omega + \Omega\tilde{F} \,, \\ \tilde{F} &= d\tilde{A} + \tilde{A}\tilde{A} \,. \end{aligned} \qquad (90)$$

$\tilde F$ is a Lie algebra valued two form and satisfies the Bianchi identity

$$\tilde{\mathcal{D}}\tilde{F} = 0 \,. \qquad (91)$$

We restrict $\tilde F$ by the usual Yang-Mills constraints $\tilde F_{\underline{\beta\alpha}} = 0$. The solution of the Bianchi identity (91) subject to these constraints corresponds with the abelian solution (28) if one replaces $F \to \tilde F$, $W \to \tilde W$, and $\mathcal{D} \to \tilde{\mathcal{D}}$. The superfields $\tilde W_\alpha$ and $\tilde W_{\dot\alpha}$ are subject to the conditions (25.2) and (25.3).

Now we are going to couple the Yang-Mills fields to 16+16 supergravity via the two form gauge potential. This mechanism is known from ten-dimensional supergravity [11] and has first been applied to N = 1 superspace by R. Grimm [10]. The transformation law (56) of the two form potential is extended to

$$\delta B = d\omega + k\ \mathrm{tr}\,(\tilde{\Lambda}\, d\tilde{A})\ , \tag{92}$$

where k is a real parameter of (mass) dimension -2. The invariant field strength

$$G = dB + k\ \mathrm{tr}\,(\tilde{A}\tilde{F} - \tfrac{1}{3}\tilde{A}^3) \tag{93}$$

contains a superspace generalization of the Chern-Simons three form and satisfies the Bianchi identity

$$\begin{aligned} &dG = k\ \mathrm{tr}\,(\tilde{F}\tilde{F})\ , \\ &E^A E^B E^C E^D \left[\tfrac{1}{3}\mathcal{D}_D G_{CBA} + \tfrac{1}{2} T_{DC}{}^E G_{EBA} - \tfrac{k}{2}\mathrm{tr}\,(\tilde{F}_{DC}\tilde{F}_{BA}) \right] = 0\ . \end{aligned} \tag{94}$$

The solution of these identities subject to the constraints (63) gives the same results as before except for additional terms in eqs. (65) and (66). We find

$$\begin{aligned} &\tilde{G}_{\alpha\dot\alpha} = 2e^V G_{\alpha\dot\alpha} - \tfrac{1}{2}[\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}]e^V - 2k\ \mathrm{tr}\,(\tilde{W}_\alpha \tilde{W}_{\dot\alpha})\ , \\ &(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8R)\, e^V = 2k\ \mathrm{tr}\,(\tilde{W}^\alpha \tilde{W}_\alpha)\ , \\ &(\mathcal{D}\mathcal{D} - 8R^+)\, e^V = 2k\ \mathrm{tr}\,(\tilde{W}_{\dot\alpha}\tilde{W}^{\dot\alpha})\ . \end{aligned} \tag{95}$$

The lagrangian for 16+16 supergravity coupled to gauge fields is given by eq. (84), i. e. in terms of superfields it is equal to the pure supergravity lagrangian. To compute its component form, one has

to take into account that eqs. (69) and (82) pick up additional terms as a consequence of (95). The component fields of the Yang-Mills multiplet can be defined in complete analogy to (31) and (33) and the tilde may be omitted after eliminating the U(1). The final result is

$$\mathcal{L} = (4n-3)\int d^2\Theta\, \mathcal{E} R + \text{h.c.} = \mathcal{L}_{16} + \mathcal{L}_{YM}\ , \tag{96}$$

where $\mathcal{L}_{16}$ is given by eq. (85) with the modified definitions

$$\begin{aligned} \hat{b}_{\alpha\dot\alpha} &= b_{\alpha\dot\alpha} + 3n\,\varphi_\alpha\bar\varphi_{\dot\alpha} + 6nk\, e^{-C}\,\mathrm{tr}\,(\lambda_\alpha\bar\lambda_{\dot\alpha})\ , \\ \hat{g}^k &= \tfrac{1}{2}\varepsilon^{klmn}\left[\partial_l t_{mn} - k\ \mathrm{tr}\,(a_l \mathcal{F}_{mn} + \tfrac{2}{3} i\, a_l a_m a_n) \right. \\ &\qquad \left. + i e^C \psi_l \sigma_m \bar\psi_n \right]\ , \end{aligned} \tag{97}$$

and

$$\begin{aligned} \mathcal{L}_{YM} = nk\, e\cdot e^{-C}\,\mathrm{tr}\,\Big[&\mathcal{F}^{mn}\mathcal{F}_{mn} \\ &+ 2i\,(\bar\lambda\bar\sigma^m \tilde{D}'_m\lambda + \lambda\sigma^m\tilde{D}'_m\bar\lambda) - 2\hat{D}\hat{D} \\ &+ 2(2n-1)\,e^{-C}(\lambda\sigma^m\bar\lambda)\,\hat{g}_m - 2(\varphi\sigma^{mn}\lambda + \bar\varphi\bar\sigma^{mn}\bar\lambda)\,\mathcal{F}_{mn} \\ &- 2i\,(\psi^m\sigma^n\bar\lambda + \bar\psi^m\bar\sigma^n\lambda)\,\mathcal{F}_{mn} \\ &+ \varepsilon^{klmn}(\psi_k\sigma_l\bar\lambda - \bar\psi_k\bar\sigma_l\lambda)\,\mathcal{F}_{mn} \\ &+ \text{a lot of quartic spinor terms}\ \Big] \end{aligned} \tag{98}$$

with the definitions

$$\begin{aligned} \mathcal{F}_{mn} &= \partial_m a_n - \partial_n a_m - i[a_m, a_n]\ , \\ \hat{D} &= D - \tfrac{i}{2}(\varphi\lambda - \bar\varphi\bar\lambda)\ . \end{aligned} \tag{99}$$

For $n \neq 0$ the choice $k = -1/8ng^2$ gives the correct normalization of the kinetic terms of $\mathcal{L}_{YM}$ (see [14] for conventions).

The Yang-Mills lagrangian (98) vanishes for n = 0. Therefore this coupling is not possible for minimal supergravity and we have to add by hand the term [7]

$$\mathcal{L}_{YM} = \frac{1}{8g^2} \int d^2\Theta\, \mathcal{E}\, \mathrm{tr}\,(\tilde{W}^\alpha \tilde{W}_\alpha) + h.c. \quad . \tag{100}$$

For the new minimal multiplet the situation is more complicated. Inserting the constraint U = 0 into (95) gives $R = -(k/4)\,\mathrm{tr}(\tilde{W}^\alpha \tilde{W}_\alpha)$. As a consequence, the superfield S (77) is no longer chiral. In order to construct an action for supergravity we have to introduce a (complex) prepotential V with the property $\bar{\mathcal{D}}_{\dot\alpha} V = \bar{A}_{\dot\alpha}$ and the pregauge transformations $V \rightarrow V + \chi$, $\bar{\mathcal{D}}_{\dot\alpha}\chi = 0$. The "gauged" supergravity lagrangian is found to be

$$\mathcal{L}_{new} = -\frac{1}{2} \int d^2\Theta\, \mathcal{E}\, (\bar{\Delta} V + 6R) + h.c. \quad , \tag{101}$$

and the Yang-Mills lagrangian is given by the integral over $\mathcal{E}$ R, which is equivalent to the expression (100) for the minimal multiplet.

7. Matter coupling

Matter (scalar) superfields are chiral superfields, i. e. they are subject to the condition $\bar{\mathcal{D}}_{\dot\alpha}\phi = 0$. In flat superspace the most general renormalizable lagrangian involving only matter fields is given by

$$\mathcal{L}_\phi = \int d^2\theta \left(-\frac{1}{8}\phi \bar{D}\bar{D}\phi^+ + \vartheta(\phi)\right) + h.c. \quad , \tag{102}$$

where the superpotential ϑ is a polynomial of degree three [7]. This expression can easily be generalized to curved superspace by replacing $d^2\theta \rightarrow d^2\Theta\,\mathcal{E}$ and $\bar{D}\bar{D} \rightarrow \bar{\Delta}$:

$$\mathcal{L}_\phi = \int d^2\Theta\, \mathcal{E} \left(-\frac{1}{8}\phi\bar{\Delta}\phi^+ + \vartheta(\phi)\right) + h.c. \quad . \tag{103}$$

It would now be straightforward to identify $\mathcal{E}$ with the chiral density (48,49) and $\bar{\Delta}$ with the chiral projection operator (45) in U(1) superspace. However, there are some complications due to the U(1) weight of $\mathcal{E}$. We discuss this in the following for each of the three supergravity multiplets.

In the case of the minimal multiplet the parameter $\Lambda(x,\Theta)$ (44) vanishes after eliminating the U(1). Therefore the U(1) weights play no rôle at all and the lagrangian (103) may be coupled to minimal supergravity without any restriction.

In the case of the new minimal multiplet the U(1) weight of $\mathcal{E}$, $w(\mathcal{E}) = -2$, has to be cancelled by the kinetic term and the superpotential of the matter fields. This condition is satisfied for the kinetic term in (103). For the superpotential it means that it must be possible to assign a U(1) weight (R-weight) to each matter field ϕ such that $\vartheta(\phi)$ has the uniform R-weight $w(\vartheta) = 2$. Thus only R-invariant lagrangians may be coupled to new minimal supergravity.

For the 16+16 multiplet we distinguish two cases: with and without R-invariance. If the coupling is R-invariant, we can use $\mathcal{E}$ and $\bar{\Delta}$ from U(1) superspace. If it is not R-invariant, a chiral density and a chiral projector in Lorentz superspace have to be constructed. This is done in appendix B. The matter lagrangian is then given by (103) with the chiral density $\mathcal{E}'$ (115) and the chiral projection operator $\bar{\Delta}''$ (117). - We remark that $\mathcal{E}'$ may also be used to construct an action for 16+16 supergravity. The proper chiral superfield is $\bar{\Delta}''1 = -8\,e^{-2nU} R$ and the supergravity lagrangian is given by

$$\mathcal{L}_{16} = (4n-3) \int d^2\Theta\, \mathcal{E}'\, e^{-2nU} R + h.c. \quad . \tag{104}$$

It is not hard to verify that this expression is identical with the lagrangian (84), resp. (96) in U(1) superspace.

Finally we consider the case that both Yang-Mills and matter fields are coupled to supergravity. The matter superfields are then defined by the modified condition

$$\tilde{\mathcal{D}}_{\dot\alpha}\,\phi = 0\,. \qquad (105)$$

Moreover, the kinetic part of the lagrangian (103) has to be gauged. This turns out to be extremely simple. Since $\phi\phi^+$ is invariant under the gauge transformations (87), it suffices to replace the derivatives $\mathcal{D}$ by gauge covariant derivatives $\tilde{\mathcal{D}}$. The chiral projection operator $\tilde\Delta$ obtained this way satisfies $\tilde{\mathcal{D}}_{\dot\alpha}\,\tilde\Delta\, V = 0$ for any Lorentz scalar V. This is due to the constraint $\tilde F_{\beta\dot\alpha} = 0$, which is required for the consistency of (105). - It should be noted here that the transformation law $\delta_{\tilde\Lambda}\,\phi = \phi\tilde\Lambda$, $\tilde\Lambda = -\tilde\Lambda^+$, is not inconsistent with the chirality of ϕ. Like $\delta_\xi\,\phi = \xi^A\tilde{\mathcal{D}}_A\phi$, it has to be understood in terms of a Θ-expansion.

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Appendix A

The lagrangian (85) for 16+16 supergravity can be obtained from the lagrangian given in ref. [4] by the following redefinitions:

$$
\begin{aligned}
&n = \frac{3n'+1}{4n'}\,, \qquad \frac{1}{4n-3} = n'\,,\\
&\omega_{nml} = \hat\omega_{nml} - 2n'^2\,\varepsilon_{nmlk}\,(T\sigma^k\bar T)\,,\\
&\psi_m{}^\alpha = \hat\psi_m{}^\alpha\,, \qquad \varphi_\alpha = -4n'\,T_\alpha\,, \qquad\qquad (106)\\
&C = -4n'\,\Psi\,, \qquad t_{mn} = N_{mn}\,,\\
&M = 3n'\,\hat{\bar S}\,, \qquad \hat b_a = -3\,\hat G_a\,.
\end{aligned}
$$

Here the quantities of [4] are written on the r.h.s. of the equations and the parameter n of [4] is denoted by n'.

Appendix B

Chiral density in Lorentz superspace

We are going to construct a chiral density for 16+16 supergravity in Lorentz superspace. For the sake of generality we include the case of Yang-Mills coupling via the two form potential. The transition from U(1) to Lorentz superspace is described by

$$\mathcal{D}\Omega = \mathcal{D}'\Omega + w(\Omega)\,\Omega A \qquad (\Omega \ n\text{-form}) \ ,$$
$$\mathcal{D}_A V_B = \mathcal{D}'_A V_B + w(V)\, A_A V_B \ , \tag{107}$$

where $\mathcal{D}'$ denotes the Lorentz covariant derivative. The superfields A_A are given by (81) and

$$A_{\alpha\dot\alpha} = \frac{i}{2} n \left(\bar{\mathcal{D}}'_{\dot\alpha} T_\alpha - \mathcal{D}'_\alpha \bar T_{\dot\alpha} - 2n\, T_\alpha \bar T_{\dot\alpha} \right)$$
$$= in\left(e^{-V} \tilde G_{\alpha\dot\alpha} - 2 G_{\alpha\dot\alpha} + T_\alpha \bar T_{\dot\alpha} \right) + 2ink\, e^{-V}\, tr\left(\tilde W_\alpha \tilde W_{\dot\alpha}\right) . \tag{108}$$

In addition, we will need the equation

$$\mathcal{D}'^\alpha T_\alpha = 8R^+ + (n-1)\, T^\alpha T_\alpha + 2k\, e^{-V}\, tr\left(\tilde W_{\dot\alpha} \tilde W^{\dot\alpha}\right) . \tag{109}$$

The torsion $T'^A = \mathcal{D}' E^A$ in Lorentz superspace is obtained from

$$T^A = T'^A + w(E^A)\, E^A A \ . \tag{110}$$

Explicitly, we find

$$T'_\gamma{}^{\dot\beta}{}_{\dot\alpha} = n\, \delta^{\dot\beta}_{\dot\alpha} T_\gamma \ , \qquad T'^{\dot\gamma}{}_\beta{}^\alpha = n\, \delta^\alpha_\beta \bar T^{\dot\gamma} \ ,$$
$$T'_{\gamma\beta}{}^\alpha = -n\left(\delta^\alpha_\beta T_\gamma + \delta^\alpha_\gamma T_\beta \right) ,$$
$$T'^{\dot\gamma\dot\beta}{}_{\dot\alpha} = -n\left(\delta^{\dot\beta}_{\dot\alpha} \bar T^{\dot\gamma} + \delta^{\dot\gamma}_{\dot\alpha} \bar T^{\dot\beta} \right) ,$$

$$T'_{\gamma b}{}^\alpha = T_{\gamma b}{}^\alpha + \delta^\alpha_\gamma A_b \ ,$$
$$T'^{\dot\gamma}{}_{b\dot\alpha} = T^{\dot\gamma}{}_{b\dot\alpha} - \delta^{\dot\gamma}_{\dot\alpha} A_b \ . \tag{111}$$

All other components remain unchanged.

Chiral superfields in Lorentz superspace are defined by the condition $\bar{\mathcal{D}}'_{\dot\alpha}\phi = 0$. The transformation law (42) under supergravity transformations becomes

$$\delta'\phi = \eta'^M D'_M \phi \tag{112}$$

with the parameters

$$\eta'^m = \eta^m + \frac{i}{2} n\, \Theta\Theta\left(\bar\zeta \bar\sigma^m \varphi\right) ,$$
$$\eta'^\mu = \eta^\mu + n\,\Theta^\mu\left(\zeta\varphi - \bar\zeta\bar\varphi\right) + \frac{n}{2}(\Theta\zeta)\varphi^\mu$$
$$+ \Theta\Theta\Big[\frac{n}{8}\zeta^\mu\left(\mathcal{D}'^\alpha T_\alpha| - \frac{n}{2}\varphi\varphi\right) - \frac{i}{4}\left(\bar\zeta\bar\sigma^a\right)^\mu\left(3A_a| - n\hat D_a C\right)$$
$$- \frac{i}{4} n \left(\bar\zeta\bar\sigma^m\right)^\mu\left(\psi_m\varphi - 2\bar\psi_m\bar\varphi\right)\Big] . \tag{113}$$

Chiral densities in Lorentz superspace are defined by the transformation law

$$\delta'\Delta = (-)^m D'_M\left(\eta'^M \Delta\right) ,$$
$$\Delta = a + \Theta\rho + \Theta\Theta f \ . \tag{114}$$

A special chiral density $\mathcal{E}'$ can be constructed analogously to the density $\mathcal{E}$ in U(1) superspace. However, the lowest component has to be different from (48). After a lengthy calculation, we find the following components of $\mathcal{E}'$:

$$
\begin{aligned}
a &= e \cdot e^{2nC} \,, \\
\rho_\alpha &= a\left[\tfrac{7}{2} n \varphi_\alpha + i (\sigma^m \bar{\psi}_m)_\alpha \right] , \\
f &= a\left[\left(\tfrac{4}{3}n - 1\right) \bar{M} - n(4n-1)\varphi\varphi + 2in\, \bar{\psi}_m \bar{\sigma}^m \varphi \right. \\
&\quad \left. - \bar{\psi}_m \bar{\sigma}^{mn} \bar{\psi}_n + 2nk\, e^{-C}\, \mathrm{tr}(\bar{\lambda}\bar{\lambda}) \right] .
\end{aligned} \tag{115}
$$

At last we construct a chiral projection operator in Lorentz superspace. Comparing the definition $\bar{\mathcal{D}}'_{\dot\alpha} \bar{\Delta}' V = 0$ with (46) shows that $\bar{\Delta}' V = \bar{\Delta} V$ if $w(V) = -2$. Thus $\bar{\Delta}'$ can be computed from the chiral projector (45) in U(1) superspace. The result is

$$\bar{\Delta}' = \bar{\mathcal{D}}'\bar{\mathcal{D}}' + 3n\, \bar{T}\bar{\mathcal{D}}' - 8R + 2n(\bar{\mathcal{D}}'\bar{T}) + 2n^2\, \bar{T}\bar{T} \,. \tag{116}$$

This, however, is not the only possible projection operator. The simplest and most useful one is

$$\bar{\Delta}'' = \bar{\Delta}' e^{-2nU} = e^{-2nU} \left(\bar{\mathcal{D}}'\bar{\mathcal{D}}' - n\, \bar{T}\bar{\mathcal{D}}' - 8R \right) . \tag{117}$$

The factor $e^{-2nU}|$ exactly cancels the factor e^{2nC} in front of the chiral density $\mathcal{E}'$.

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