Title

MULTIPLICITY FLUCTUATIONS IN FINITE RAPIDITY WINDOWS: INTERMITTENCY OR QUANTUM STATISTICAL CORRELATION?

Authors

Carruthers, P.
Friedlander, E.M.
Shih, C.C.

Publication Date

1989

LBL-26202
Preprint

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Submitted to Physics Letter B

Multiplicity Fluctuations in Finite Rapidity Windows: Intermittency or Quantum Statistical Correlation?

P. Carruthers, E.M. Friedlander, C.C. Shih, and R.M. Weiner

January 1989

Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098.

Multiplicity Fluctuations in Finite Rapidity Windows: Intermittency or Quantum Statistical Correlation?

P.Carruthers[1], E.M.Friedlander[2], C.C.Shih[1,3] and R.M.Weiner[4]

1. Department of Physics and Center for the Study of Complex Systems, University of Arizona, Tucson AZ 85721
2. Nuclear Science Division, Lawrence Berkeley Laboratory, 1 Cyclotron Road, Berkeley CA 94720
3. Department of Physics,University of Tennessee, Knoxville TN 37916
4. Department of Physics, University of Marburg, 3550 Marburg, F.R. of Germany

Abstract

It is shown that the experimentally observed dependence of moments of multiplicity distributions on the width of the rapidity window can be interpreted by simple quantum statistical properties of the emitting system(s) and does not necessarily imply evidence for intermittency.

Recently, a great deal of attention has focused on the possibility that the distribution in rapidity of particles produced in high-energy hadronic collisions may show signals for "intermittency" [1,2,3]. It is essential to ask, whether these interesting signals can be understood within the framework of our present understanding, or whether new physics is needed. In this paper, we shall present results based on quantum statistical considerations. It is shown that a hadronic analogue of intensity correlations similar to those encountered in the Hanbury Brown-Twiss effects[4] is sufficient to interpret the behavior of data which appear to provide evidence of "intermittency". Such data are derived from high-energy nuclear collisions, specifically from the factorial moments of order q :

$$F_q \equiv < n(n-1)...(n-q+1) > \tag{1}$$

of the distribution of multiplicities n of particles observed in rapidity windows of finite width δy (for details see below). Currently, the evidence for intermittency has been inferred from a power law dependence of the ϕ_q on δy, where ϕ_q are suitably defined normalized factorial moments

$$\phi_q \equiv F_q/F_1^q. \tag{2}$$

The study of such "local" distributions of n has been suggested and used for the first time in ref.[5], and has since been widely applied (see e.g. [6,7]). In the context discussed here, the dependence of the ϕ_q on the scale of resolution may be a power law, as suggested, e.g., in the theory of turbulence with fractal dimension (with or without intermittency) [8]. Various parton branching-type models for multiparticle production leading to such a behavior have also been discussed at recent conferences [1,9,10,11,12]. A series of experiments [9,13,14] looked for and have observed initial linear increases of $\ln \phi_q$ with $-\ln \delta y$, followed by a saturation (at very narrow window width δy). Different definitions for the ϕ_q, however, have been applied for investigating this behavior.

To be precise with the formulation, we start from the number $n_{b,e}$ of charged particles observed in rapidity bin number b ($b \leq b_m$) of event number e ($e \leq e_m$). Having available for analysis a single cosmic ray event, ($e_m = 1$,) Bialas and Peschanski[1], then introduced the normalized moments ϕ_q, in terms of what could be called a "horizontal average" (over bins) :

$$F_{q,e}^{[h]} = \frac{1}{b_m}\sum_{b=1}^{b_m} n_{b,e}(n_{b,e}-1)...(n_{b,e}-q+1) \quad (3)$$

$$\phi_{q,e}^{[h]} = F_{q,e}^{[h]}/(F_{1,e}^{[h]})^q \quad ,e=1. \quad (4)$$

For experiments in which many ($e_m >> 1$) events are available, , the average over bins can be followed, [12] by an average over many events, so that

$$\phi_q^{[h]} = \frac{1}{e_m}\sum_{e=1}^{e_m}\phi_{q,e}^{[h]} \quad (5)$$

This is to be compared with the alternative "vertical average" also suggested [1] ,

$$F_{q,b}^{[v]} = \frac{1}{e_m}\sum_{e=1}^{e_m} n_{b,e}(n_{b,e}-1)...(n_{b,e}-q+1) \quad (6)$$

$$\phi_{q,b}^{[v]} = F_{q,b}^{[v]}/(F_{1,b}^{[v]})^q \quad (7)$$

$$\phi_q^{[v]} = \frac{1}{b_m}\sum_{b=1}^{b_m}\phi_{q,b}^{[v]}. \quad (8)$$

Another variation of the "vertical average" is also used [9]. There, $F_{1,b}^{[v]}$ in Eq. 7, is replaced by $F_1^{[v]}$, the average of $F_{1,b}^{[v]}$ over all the bins:

$$F_1^{[v]} = \frac{1}{b_m}\sum_{b=1}^{b_m} F_{1,b}^{[v]} = < n > /b_m \quad (9)$$

where$< n >$ is the average total number of particles (in all the bins), so that

$$\phi_q^{[v']} = \frac{1}{e_m b_m}\sum_{e=1}^{e_m}\sum_{b=1}^{b_m} n_{b,e}(n_{b,e}-1)...(n_{b,e}-q+1)/(\frac{< n >}{b_m})^q \quad (10)$$

Notice that by changing the order of averaging between bins and events Eq. 5, Eq. 8 and Eq. 10 are in general not equivalent, both because the fluctuations of $n_{e,b}$ come in with different weights and because the shape of the rapidity distribution influences the different ϕ_q in different ways. In either formulation, the behavior of ϕ_q has been considered as strong evidence of intermittency[9].

In what follows, we shall assume that physics is stationary throughout the interval of rapidity where intermittency is considered. That is to say, the inclusive multiplicity distribution is independent of the location of the bin. This enables us to use *the single interval inclusive distribution* Eq. 7 without considering the joint distribution of the individual bins. Within the approximation of stationarity, the single bin approximation is equivalent to both vertical averages Eq. 8 and Eq. 10 (but not to the horizontal average Eq. 5). The assumption of stationarity also allows us to use existing expressions to demonstrate essential features of the quantum statistical formulation [15]. General non-stationarity effects will be considered with other corrections in a different paper[16].

We shall now present[17] an alternative explanation of the $(\phi_q, \delta y)$ dependence, namely that it follows in a natural way from the quantum statistical correlation properties of partially coherent emitting systems [4,15,19]. To visualize this we shall consider the rapidity y as time, and the multiplicity fluctuations as intensity fluctuations similar to those appearing in the Hanbury, Brown-Twiss effect [15]. Any finite rapidity window δy leads,in the presence of a finite coherence length ξ to an "effective" number $\delta y/\xi$ "cells" of y-space [18], and the multiplicity distribution should approximate negative binomials or equivalent partial coherent distributions [19]. An effective power law dependence of $(\phi_q, \delta y)$ may therefore be only a reflection of the changing number of "cells" $\delta y/\xi$.

To be more specific, we shall consider one of the simplest models of quantum statistical ensembles of this kind, where the correlation in y is emphasized. Many detailed features of a more realistic model should eventually be built in as correlations [4]. Consider multiparticle production arising from a mixture of chaotic fields π_{chao} and coherent fields π_{coh}, such that the fraction of the *number n* of secondaries originating from the chaotic component of the field is p. With a finite coherence length ξ of the chaotic field π_{chao} introduced through

$$< \pi_{chao}(y)^{+}\pi_{chao}(y') >= exp(-\frac{\mid y-y' \mid}{\xi}) \tag{11}$$

the *normalized factorial cumulants* of a multiplicity distribution due to k emitting sources are given by: [15,20]

$$\mu_q = (q-1)!(p/k)^{q-1}[pB_q(\beta) + q(1-p)\overline{B}_q(\beta)] \tag{12}$$

where only the ratio

$$\beta \equiv \frac{\delta y}{\xi}, \tag{13}$$

depends on δy. This leads to a behavior called "β - scaling" of multiplicity distributions for finite δy [4].

The normalized factorial moments ϕ_q are then related in a simple way to the corresponding μ_q [4]. These cumulants μ_q are really more sensitive diagnostic tools for the dynamics of the interaction than the F_q, as we shall elaborate elsewhere. However, so far, all experimental results claiming intermittency have been expressed in terms of the ϕ_q. We shall therefore also use the ϕ_q for the sake of comparison with existing experiments.

The functions $B_q(\beta)$ and $\overline{B}_q(\beta)$ have been defined in refs. [15,20] and analytical expressions for them up to $q = 8$ have been given in [4,21].

To illustrate the simplest case of $q = 2$ we reproduce here for the reader's convenience:

$$\phi_2 = 1 + \mu_2, \tag{14}$$

$$B_2(\beta) = (e^{-2\beta} + 2\beta - 1)/2\beta^2, \tag{15}$$

and

$$\overline{B}_2(\beta) = 2(e^{-\beta} + \beta - 1)/\beta^2. \tag{16}$$

For very small rapidity windows $\beta << 1$; then the distribution law $P(n)$ of n tends towards the Glauber-Lachs distribution [22] and the ϕ_q are given by (see e.g. [24])

$$\phi_q = q!(\frac{p}{k})^q L_q^{k-1}(-x) \tag{17}$$

where L_q^{k-1} are generalized Laguerre polynomials and

$$x \equiv k\frac{(1-p)}{p}. \tag{18}$$

For $p \to 0$ the field becomes totally coherent, $P(n)$ becomes Poissonian and all $\ln \phi_q \to 0$. Conversely if $p \to 1$ i.e the field is totally chaotic, $P(n)$ turns into the negative binomial distribution and

$$\phi_q \to \frac{(q+k-1)!}{k^q (k-1)!}. \tag{19}$$

Since no squeezed states are allowed in this formulation[23], the widest fluctuations correspond to the negative binomial distribution with $k = 1$ (one-cell Bose-Einstein distribution). This yields an upper bound for the $\ln \phi_q$ which can be observed at very small δy:

$$\ln \phi_q \leq \ln(q!). \tag{20}$$

Thus, $\ln \phi_2 \leq 0.693$, $\ln \phi_3 \leq 1.782$, etc. !

It is interesting to note that, thus far, no experimental data violating these bounds have been presented.

In the above formulations of Eq. 12, stationarity of the quantum statistical expression is assumed throughout a wide interval of rapidity. It is a reasonable assumption for the central region. (e.g. $|y| < 1.5$ for the NA22 energy). The assumption of stationarity is crucial for the simplicity of side-stepping the additional "horizontal" average. It enables us to use the single interval average of Eq. 12 for the more complicated vertical averages Eq. 8 and Eq. 10 without considering the joint distribution of the individual bins. Use of the single interval inclusive Eq. 12 for "horizontal" averages (averaging over bins within one event), is more approximate in nature. We notice first that for each event with large n, the finite $< n >$ effect may not be serious. Furthermore, if the statistical average of Eq. 5 is not performed between events with dramatically different n, Eq. 12 may provide a reasonable approximation[19]. The same procedure has been applied to e^+e^- data [9]. A discussion of these data and details of the more complicated QS formalism implied will be published elsewhere [16].

As will be shown below, we do not find that the assumption of intermittency is necessary to interpret the y dependence of multiplicity distributions in hadronic reactions, since the quantum correlation model, as defined above, can explain this dependence [1].

[1] It should be noted that the good description of *the bulk of the events* by the quantum

We now proceed to analyze a few of the available experimental data in the light of the model described above.

In Fig.1a and 1b we show ($\ln \phi_q$,$-\ln \delta y$) plots for symmetrical pseudorapidity windows recorded in the UA5 streamer chamber [6] for $p\overline{p}$ at $\sqrt{s} = 540GeV$ and in the NA22 bubble chamber [7] for pp at $\sqrt{s} = 22GeV$, respectively, for q=2, 3, and 4. The "chaoticity" p was fitted to the "plateau" of ϕ_q at small δy for q=2; then ξ was adjusted to mimic the slope at large δy at the same q. The value of k is taken to be 1, since data at the smallest rapidity window are not compatible with $k > 1$. [2] At this stage, a more rigorous fitting procedure was not deemed useful, especially in view of the high degree of correlation between adjoining points on the graphs. It is remarkable how the curves, (computed with the numerical values of the parameters mentioned in the figure captions) fit not only ϕ_2 but the curves for the higher q - values, as well.

In Fig.2 a, and b we analyze in the same way the curves for $\ln < \phi_q >$ for δy intervals sweeping a wide range of y in the NA22 experiment on pp collisions ([7]) and for "central" $^{32}S + Au$ collisions at 200 A GeV in nuclear emulsion chambers from the data of the EMU01 Collaboration [14]. The latter experiment, in spite of its (as yet preliminary) low statistics, enjoys (because of the particular geometry of the emulsion chambers) a very high spatial resolution, with measuring errors on δy of the order of 0.01. This ensures the reality of the saturation observed at large $-\ln \delta y$, contrary to the conjecture [12] that it might be due to lack of spatial resolution (as is the case in streamer chambers and, especially in electronic detectors). Once more the agreement of the moments of higher order q with fits made on $q = 2$, only, is remarkable, given the approximations underlying both the experimental analysis [25] and finer details of the theory (see [4] for a discussion).

A few comments are in order, regarding these results and their inter-

statistical properties of the emitting system(s) does not exclude contributions to the ϕ_q from rare events with very large local multiplicity fluctuations ("spikes" [9,27]) which could signal intermittency and/or approach to phase transitions.

[2] There is, however, an inherent ambiguity of parametrization between k and p, for all the larger rapidity windows [28]. The decrease of the ϕ_q as a function of increasing δy can be partially due to an increase of the cell number k [28]. The value of p determined by the small δy behavior may therefore be slightly overestimated[4].

pretation:

1. Comparing the values of the parameters p and ξ needed to fit the data of Fig.1a and 1b we notice that both the chaoticity and the coherence length increase with $\sqrt{s}$, in agreement with an earlier analysis [4] where the forward-backward correlations were used to pinpoint ξ.

2. In pp reactions at $\sqrt{s}$=22GeV points close to the kinematic limit on δy tend to deviate from the fitted curve. This can be due partly because of the phase space constraints (which tend to narrow $P(n)$ and hence lower the ϕ_q, and partly to the fact that the condition of stationarity in rapidity (required by for the validity of the expressions given above for the factorial cumulants) is violated when one goes too far away from the central (cms) rapidity region (e.g. $\frac{\partial p}{\partial y} \neq 0$). It can be shown [16] that, for reasonable assumptions about $\frac{\partial p}{\partial y}$ (e.g., guided by the results of ref.[26]), the only change occurs in the expressions for the $B_q(\beta)$ and $\overline{B}_q(\beta)$ and implies a small correction in the right direction.

3. The same effect of non-stationarity in y must manifest itself in the different values of p and ξ fitting the data shown in Fig.1b and Fig.2a which, after all, come from the same data set. The difference lies in the fact that for Fig.1a the center of the rapidity window was kept fixed (at $y_{cms} = 0$) whereas the data presented in Fig.2a are averaged over the whole rapidity interval and variations of p with y influence the ϕ_q with different weights. This effect should be taken into account every time comparisons are drawn between what could be termed as "horizontal" analysis (ϕ_q measured first for *each* event and then averaged over all available events) and "vertical" analysis (when the ϕ_q are deduced from a given window in rapidity from *all* events).

4. It is noteworthy that nucleus-nucleus collisions ([14], Fig.2b) show much lower values of p, corresponding to very narrow $P(n)$. This may be a reflection of the need for a very large k. It may also be expected if the special sample analyzed in ref.[14] mainly involves an independent superposition of nucleon-nucleus collisions. Indeed,

taking into account the ratio of mean multiplicities observed in the data of Figs.2a and 2b (at practically the same energy per nucleon) the drop of the "effective" p-value can be easily understood. Different methods of averaging are particularly sensitive to the stationarity of the rapidity plateau (or the lack thereof) in nucleus-nucleus collisions. A simple fluctuation in rapidity plateau width from event to event can contribute to the variation of the observed moments with δy. A comparison with nucleus-nucleus events of lower multiplicity and/or centrality appears thus highly desirable.

To summarize, we have provided a alternative description of the variation of multiplicity moments not requiring intermittency or fractal properties of the rapidity distribution. More detailed studies of correlations among rapidity bins are being conducted to further test the various models currently under consideration.

P.C. was supported in part by the U. S. Department of Energy and by the Lounsbery Foundation; E.M.F. was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Nuclear Physics Division of the U.S. Department of Energy under Contract DE-AC03-76SF00098; C.C.S. was supported in part by the National Science Foundation; R.M.W. was supported by the Bundesministerium fuer Forschung und Technologie, F.R. of Germany. E.M.F. and R.M.W. would like to acknowledge the hospitality of the University of Arizona, Tucson,and C.C.S. that of the Nuclear Science Division, Lawrence Berkeley Laboratory, where parts of this work were carried out.

References

[1] A. Bialas and R. Peschanski, Nucl.Phys. B273 ,703 (1986); Phys. Lett 207B, 59 (1988); A. Bialas and K. Fialkowski and R. Peschanski, preprint, Institute of Physics, Jagellonian University, Cracow, Poland. (1988)

[2] J. Dias De Deus, Phys. Lett. 194B, 297 (1987).

[3] I. M. Dremin, JETP Lett. 30, 152 (1980); Sov. J. Nucl. Phys. 33, 726 (1981).

[4] G. N. Fowler et al., Univ. of Marburg preprint 1986, and R. M. Weiner. Proceedings of the Conference "Hadronic Matter in Collision-Lesip II", Santa Fe 1986. P. Carruthers and D Strottman, editors. G. N. Fowler, E. M. Friedlander, F. W. Pottag, R. M. Weiner J. Wheeler and G. Wilk, Phys. Rev. D37, 3127 (1988), C. C. Shih , submitted to Phys.Rev.D.

[5] G. N. Fowler, E. M. Friedlander and R. M. Weiner, Phys.Lett. 104B, 239 (1981).

[6] J. Alner et.al. UA5 Collaboration Phys. Lett. 160B 193 (1985).

[7] M. Adamus, EHS/NA22 Collaboration, Z. Phys. C 37, 215 (1988).

[8] P. Carruthers and Minh Duong-Van, Los Alamos preprint (1983) LA-UR-83-2419 (unpublished). See also P. Carruthers in the Proceedings of Perugia Workshop on Multiparticle dynamics (to be published).

[9] W. Kittel, University of Nijmegen preprint HEN-307 (1988) and Mini-Rapporteur's Review at the XXIV Int. Conf on High Energy Physics, Munich August (1988).

[10] A. Bialas, in Proceedings of International Workshop on Multiparticle Dynamics, Perugia (1988) (to be published).

[11] L. Van Hove, in Proceedings of the Conference on Hadronic Matter in Collision, Tucson (1988) (World Scientific,to be published).

[12] R. Peschanski, ibid.

[13] H. G. Ritter et al.(WA80 Collaboration), ibid.

[14] E. Stenlund et.al. (EMU01 Collaboration) in the Proceeding of the Conference on Quark Matter'88, Lennox, September (1988) and University of Lund report LUIP 8807 (prel).

[15] G. N. Fowler and R. M. Weiner, Phys. Rev. D17, 3118 (1978).

[16] E. M. Friedlander , C. C. Shih and R. M. Weiner, to be submitted for publication (1988).

[17] E. M. Friedlander, in Proceedings of the Conference on Hadronic Matter in Collision, Tucson (1988) (to be published).

[18] B.Durand and L.O'Raifeartaigh, Phys.Rev. D13, 99 (1976)

[19] P. Carruthers and C. C. Shih, Phys. Lett. 137B 425 (184); Mod. Phys. Lett. A2, 89 (1987).

[20] A. K. Jaiswal and C. L. Mehta, Phys. Rev. A2, 168 (1970).

[21] C. C. Shih University of Arizona, preprint submitted to Phys. Rev. A (1988).

[22] R. J. Glauber, Phys. Rev. 131, 1766 (1963), and G. Lachs, Phys. Rev. 138, 1012 (1965).

[23] A. Vourdas and R. M. Weiner, Phys. Rev. D38, 2009 (1988), C. C. Shih, Phys. Rev. D34, 2720 (1986).

[24] G. N. Fowler, E. M. Friedlander, R. M. Weiner, and G. Wilk, Phys. Rev. Lett. 56, 870 (1986).

[25] E. M. Friedlander,Lawrence Berkeley Laboratory report 26203, October 1988, to be submitted for publication.

[26] G. N. Fowler, E. M. Friedlander, R. M. Weiner and G. Wilk, Phys. Rev. Lett. 57, 2119 (1986).

[27] E. M. Friedlander et al.,EMU01 Collaboration, in Proceedings of the Conference on Hadronic Matter in Collision, Tucson (1988) (to be published).

[28] P.Carruthers and C.C. Shih, J. Modern Phy. A2, 1544 (1987).

Figure Captions

Fig.1 Normalized factorial moments ϕ_q of order q in finite (pseudo) rapidity windows of width δy centered around $y_{cms} = 0$, plotted against δy or $\delta\eta$ for:

a) $p\bar{p}$ collisions at $\sqrt{s} = 540GeV$,(η) and

b) pp collisions at $\sqrt{s} = 22GeV$ (y).

The order q is indicated next to the curves. These have been computed using Eq.(12) with : a) $p = 0.44$ and $\xi = 4.0$ and b) $p = 0.32$ and $\xi = 1.0$

Fig.2 *Averages* of normalized factorial moments ϕ_q of order q in finite rapidity windows of width δy shifted across a wide rapidity range in each event, plotted against δy or $\delta\eta$ for:

a) pp collisions at $\sqrt{s} = 22GeV$. (y),and

b) $^{32}S + Au$ collisions at 200 AGeV (η).

The order q is indicated next to the curves. These have been computed using Eq.(12) with : a) $p = 0.20$ and $\xi = 0.9$ and b) $p = 0.015$ and $\xi = 0.2$

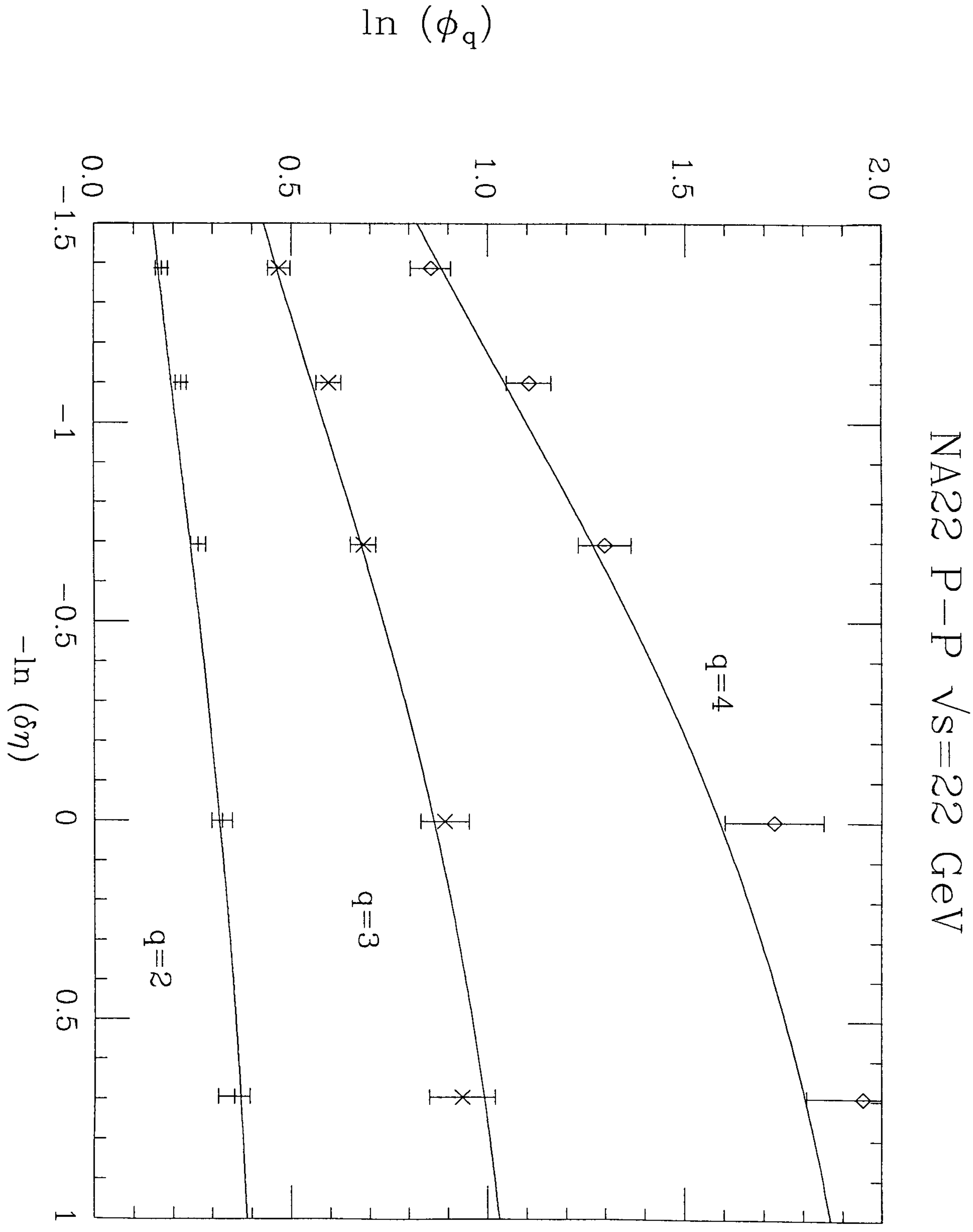
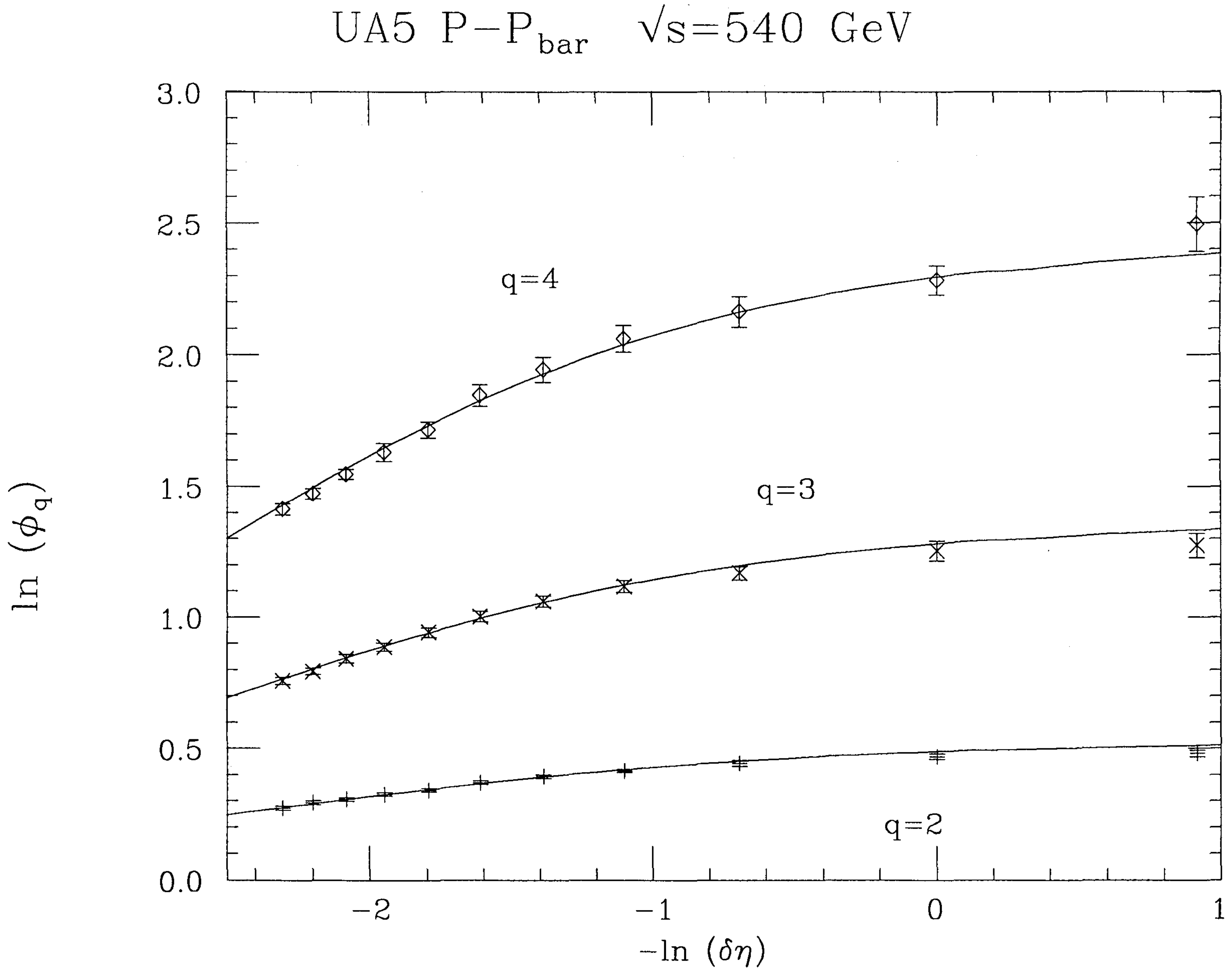

NA22 P–P √s=22 GeV
ln (ϕ_q)
−ln ($\delta\eta$)
q=4
q=3
q=2
0.0
0.5
1.0
1.5
2.0
−1.5
−1
−0.5
0
0.5
1

Fig. 1



UA5 P−P$_{bar}$ √s=540 GeV
3.0
2.5
2.0
1.5
1.0
0.5
0.0
ln (ϕ_q)
q=4
q=3
q=2
−2
−1
0
1
−ln ($\delta\eta$)

Fig. 2

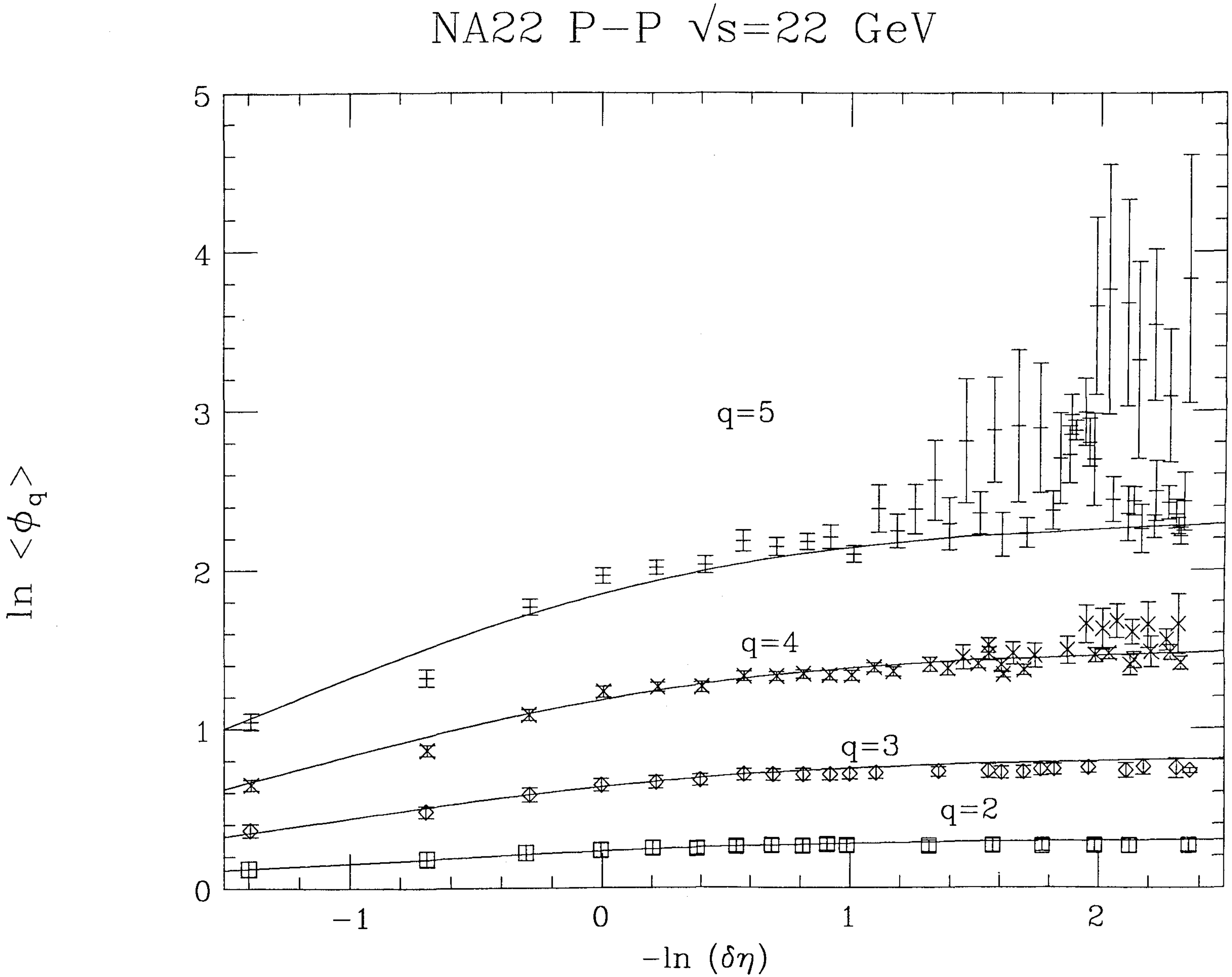


Fig. 3

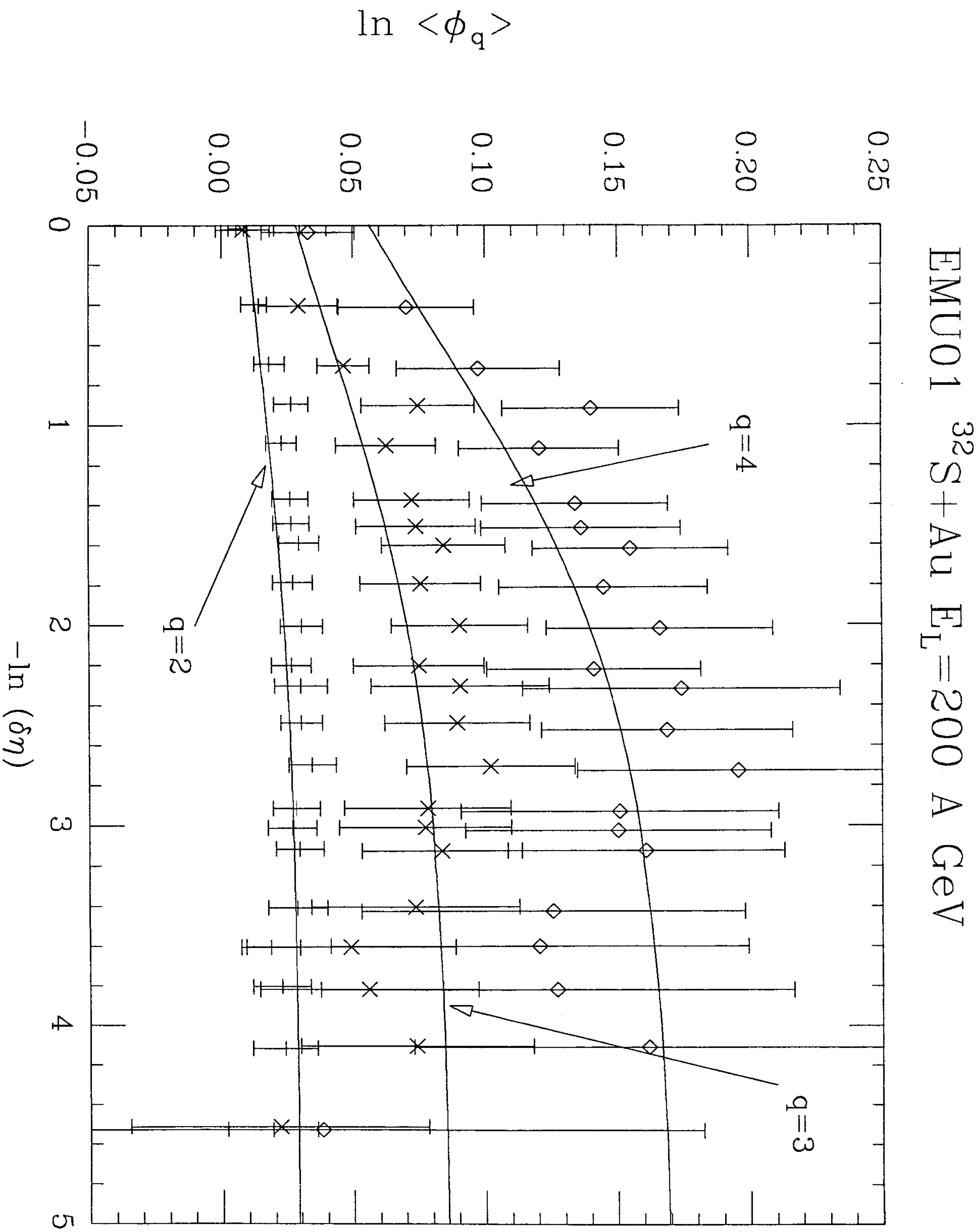

EMU01 ^{32}S+Au E_L=200 A GeV
ln $\langle\phi_q\rangle$
$-\ln(\delta\eta)$
q=2
q=3
q=4
-0.05
0.00
0.05
0.10
0.15
0.20
0.25
0
1
2
3
4
5

Fig. 4