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Authors

Mioc, F
Capolino, F
Sabbadini, M
[et al.](#)

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HIGH-FREQUENCY DESCRIPTION OF THE KIRCHHOFF-TYPE MODAL COUPLING BETWEEN OPEN ENDED WAVEGUIDES

F. MIOC¹, F. CAPOLINO¹, M. SABBADINI², S. MACI³

¹College of Engineering, Univ. of Siena, Via Roma 56, 53100, Siena, Italy

²Electrical Department, ESA-ESTEC, 2200 AG Noordwijk, The Netherlands.

³Dept. of Elec. Eng., Univ. of Florence, Via S.Marta 3, Florence, Italy

1. INTRODUCTION

The Kirchhoff-type aperture integration (AI) is the simplest way to calculate the radiation from an open-ended waveguide (OEW). Recently, a rigorous equivalence between the field predicted by AI and that radiated by the Physical Optics (PO) wall-current was demonstrated [1], in which the PO currents are defined as that associated to the unperturbed mode. By using this equivalence, a method for asymptotically reducing the AI into a line integration (LI) of incremental diffraction coefficients along the waveguide edge was presented [2]. A LI representation of the aperture field is well suited for introducing a fringe contribution as provided by the Physical Theory of Diffraction (PTD) [3] or by the Incremental Theory of Diffraction (ITD) [4], [5]. In this paper, the equivalence between PO and AI [1] is extended to evaluate the coupling between two OEWs of arbitrary cross-section. Furthermore, a Kirchhoff-type coupling coefficient is derived in terms of a double line integration of incremental coupling coefficients. This may provide a useful tool when the mutual impedance of two modal distributions has to be calculated in the framework of a Method of Moments (MoM) procedure which is formulated in terms of mode-shaped basis functions.

2. RECIPROCITY OF THE EQUIVALENCE BETWEEN PO AND AI.

Let us consider a receiving open ended waveguide (OEW1) of arbitrary cross section, with its axis along the z axis of a reference system and its aperture on the $x-y$ plane; suppose that OEW1 extends for $z < 0$ and denote by \vec{e}_1^n , \vec{h}_1^n (n integer), an arbitrary normalized mode propagating into the waveguide toward negative z . The normalization constant may be chosen in such a way that the integration of $\vec{e}_1^n \cdot \vec{h}_1^m$ on the aperture is equal to δ_{nm} (Kronecker's delta). Furthermore, suppose that the waveguide is illuminated by an incident field (\vec{E}_2 , \vec{H}_2) produced by an external source (\vec{J}_2 , \vec{M}_2). By invoking the Kirchhoff approximation, the excitation coefficient C^n of the n -th mode onto the OEW1 can be calculated as

$$C_1^n = - \iint_{A_1} [\vec{E}_2 \cdot \vec{j}_1^n - \vec{H}_2 \cdot \vec{m}_1^n] da_1 \quad (1)$$

in which $\vec{m}_1^n = \vec{e}_1^n \times \hat{z}$, $\vec{j}_1^n = \hat{z} \times \vec{h}_1^n$, and the total aperture field has been approximated by the incident field. The minus sign depends from the fact that the mode has been assumed as propagating toward negative z . Applying the reciprocity principle, yields

$$C_1^n = - \iiint_{V_2} [\vec{E}_1^n \cdot \vec{J}_2 - \vec{H}_1^n \cdot \vec{M}_2] dV_2 \quad (2)$$

where $(\vec{E}_1^n, \vec{H}_1^n)$ is the field produced by the aperture distribution $(\vec{j}_1^n, \vec{m}_1^n)$ and V_2 is a volume containing (\vec{J}_2, \vec{M}_2) . Since $(\vec{E}_1^n, \vec{H}_1^n)$, is equal to the field produced by the radiation in free space of the PO currents [1], a further application of the reciprocity principle leads to

$$C_1^n = - \int \int_{S_1} \vec{E}_2 \cdot \vec{j}_{1w}^n dS_1 \quad (3)$$

where the surface S_1 denotes the waveguide wall and $\vec{j}_{1w}^n = \hat{n}_{w1} \times \vec{h}_1^n$ (where \hat{n}_{w1} is the internal normal to the wall). Equations (1) and (2) express the equivalence between AI and PO for a receiving OEW.

3. MODAL COUPLING

Let us now consider a second, arbitrarily shaped open-ended waveguide (OEW2) which is fed by the normalized modal field \vec{e}_2^m, \vec{h}_2^m propagating toward the positive z axis (Fig. 1). An aperture field distribution $\vec{m}_2^m = \vec{e}_2^m \times \hat{z}, \vec{j}_2^m = \hat{z} \times \vec{h}_2^m$ is associated to the unperturbed incident mode, that radiates the Kirchhoff-type field $(\vec{E}_2^m, \vec{H}_2^m)$. Using (1), the Kirchhoff-type excitation coefficient of the n -th mode into OEW1 due to the Kirchhoff-type aperture radiation of OEW2 is

$$C_{12}^{nm} = - \int \int_{A_1} [\vec{E}_2^m \cdot \vec{j}_1^n - \vec{H}_2^m \cdot \vec{m}_1^n] dA_1 dA_2 \quad (4)$$

This quantity involves a four folded integral. Although for circular and rectangular waveguides, it can be reduced to a single spectral integral [Bird], this cannot be done for general shape.

By invoking the equivalence between PO and AI, $(\vec{E}_2^m, \vec{H}_2^m)$ may also be thought as produced by the currents $\vec{j}_2^m = \hat{n}_{w2} \times \vec{h}_2^m$. Applying eq. (3) leads to

$$C_{12}^{nm} = - \int \int_{S_1} \int \int_{S_2} \vec{j}_{1w}^n(\vec{r}_1) \cdot \vec{G}^E(|\vec{r}_1 - \vec{r}_2|) \cdot \vec{j}_{2w}^m(\vec{r}_2) dS_1 dS_2 \quad (5)$$

where $\vec{G}^E(|\vec{r}_2 - \vec{r}_1|)$ is the free-space dyadic Green's function pertinent to the electric source and the electric field, S_i and \vec{r}_i are the wall-surfaces of OEW i and the position vector on it, respectively. By integrating along strips parallel to the z axis (Fig. 1) belonging to the two waveguides, (2) is rearranged as

$$C_{12}^{nm} = \int \int_{\ell', \ell''} c(\ell', \ell'') d\ell' d\ell'' \quad (6)$$

where $c(\ell', \ell'')$ is an incremental coupling coefficient that represents the interaction between the two elementary strip

$$c(\ell', \ell'') = \int_{-\infty}^0 \int_0^{\infty} g(z', z'', \ell', \ell'') \frac{e^{-jkR(z', z'', \ell', \ell'')}}{4\pi R(z', z'', \ell', \ell'')} e^{-jk_z z'} e^{jk_z z''} dz' dz'' d\ell' d\ell'' \quad (7)$$

in which

$$g(z', z'', \ell', \ell'') = jk_z \vec{J}_{1w}^n(\ell'') \cdot (\hat{\Theta}\hat{\Theta} + \hat{\Psi}\hat{\Psi}) \cdot \vec{J}_{2w}^m(\ell'), \quad (8)$$

k and ζ are the free-space wavenumber and impedance, respectively, $R(z', z'', \ell', \ell'')$ is the distance between two generic points of the two interacting strips (Fig. 1), and \bar{J}_{iw}^n is equal to \bar{J}_{iw}^n calculated at $z=0$. In (7)-(8), only the first term of the asymptotic expansion of the dyadic Green's function has been retained.

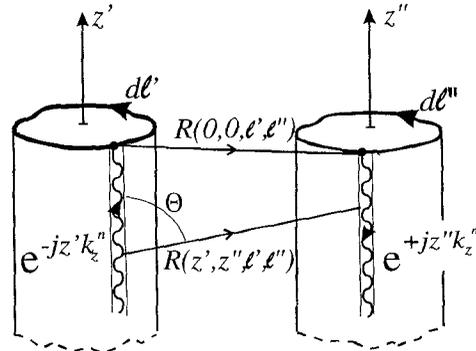


Fig. 1. Geometry for the problem of coupling between OEWs of arbitrary cross-section.

4. ASYMPTOTIC EXPRESSION OF THE INCREMENTAL COUPLING COEFFICIENTS

The integral in (7) has four critical points from which the asymptotic dominant contributions arise; i.e., the double end-point (EP) $(z', z'')=(0, 0)$ the double stationary-phase point (SPP) $(z', z'') \equiv (z_{s_1}, z_{s_2})$, two E-SSPs $(z', z'') \equiv (\bar{z}_1, 0)$ and $(z', z'') \equiv (0, \bar{z}_2)$. Only the contribution $c_{00}(\ell', \ell'')$ from the double EP is retained in the asymptotic approximation. Indeed, the other contributions asymptotically vanish when integrating along the double rim, as the contributions involving a SPP reconstruct the coupling with the field of an infinite waveguide, which is zero outside the waveguide itself. The contribution at the double end-point can be evaluated as in [2], leading to

$$c_{00}(\ell', \ell'') = g(0, 0, \ell', \ell'') \frac{e^{-jkR_{00}}}{4\pi k R_{00}} \frac{1}{\cos\theta_{t1} \cos\theta_{t2}} \frac{\cos\theta_{t2} \mathcal{F}(\delta_1) - \cos\theta_{t1} \mathcal{F}(\delta_2)}{\cos\theta_{t2} - \cos\theta_{t1}} \quad (9)$$

where $R_{00}=R(0, 0, \ell', \ell'')$, $\mathcal{F}(x)$ is the Fresnel transition function of the Uniform Theory of Diffraction (UTD) [7] with argument

$$\delta_i = 2kR_{00} \cos^2\left(\frac{\theta_{ti}}{2}\right) \quad i=1, 2 \quad (10)$$

and $\theta_{t1} = \cos^{-1}k_z^n/k$, $\theta_{t2} = \cos^{-1}k_z^m/k$, are the ray-mode angles in the waveguide 1 and 2, respectively.

5. NUMERICAL RESULTS

Results are presented for the coupling coefficient C_{12}^{nm} , calculated with eq. (4) (four-folded integrals), and with eq. (3) is compared with the full numerical integration (4) (two-folded integrals). In Fig. 2, C_{12}^{nm} is presented

for two circular TE_{11} modes versus the distance d between two circular OEWs with the same radius. The geometry and the polarisation of the modes are shown in the inset. Two different radii of the OEWs are considered; i.e., $a=0.3\lambda$, and $a=0.5\lambda$. Continues lines refer to the double LI solution (eq.6), and the dotted lines refer to the the double AI solution (eq. 4). In spite of the moderate size of the apertures, the agreement has been found quite satisfactory over all the dynamic range.

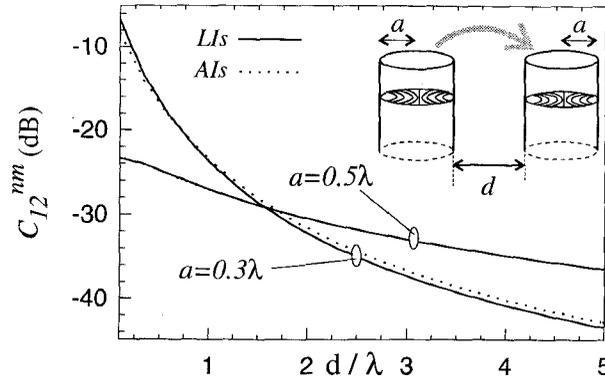


Fig. 2. Coupling coefficient between two TE_{11} modes, versus the distance d .

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