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Operational Issues and Network Effects in Vaccine Markets

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Abstract

One of the most important concerns for managing public health is the prevention of infectious diseases. Although vaccines provide the most effective means for preventing infectious diseases, there are two main reasons why it is often difficult to reach a socially optimal level of vaccine coverage: (i) the emergence of operational issues (such as yield uncertainty) on the supply side, and (ii) the existence of negative network effects on the consumption side. In particular, uncertainties about production yield and vaccine imperfections often make manufacturing some vaccines a risky process and may lead the manufacturer to produce below the socially optimal level. At the same time, negative network effects provide incentives to potential consumers to free ride off the immunity of the vaccinated population. In this research, we consider how a central policy-maker can induce a socially optimal vaccine coverage through the use of incentives to both consumers and the vaccine manufacturer. We consider a monopoly market for an imperfect vaccine; we show that a fixed two-part subsidy is unable to coordinate the market, but derive a two-part *menu* of subsidies that leads to a socially efficient level of coverage.

Keywords: Vaccine coverage, negative network effect, random yield, vaccine pricing, vaccine subsidy.

1 Introduction

Prevention, cure, and control of infectious diseases are important concerns for modern health care systems. The World Health Organization (1999) reports that infectious diseases are the world's biggest killer of young adults and children, and describes them as a "crisis of global proportions... threatening hard-won gains in health and life expectancy." Every one in two deaths in developing countries is caused by an infectious disease; globally, infectious diseases account for over 13 million deaths every year. In 2000, HIV/AIDS, TB, and malaria, together, killed 5.7 million people and caused debilitating illnesses in millions more (World Health Organization, 2002). Influenza and pneumonia claim about 250,000 to 500,000 lives every year globally, and about 36,000 deaths in the US alone (World Health Organization, 2005). In addition to the emergence of new infectious diseases, such as Severe Acute Respiratory Syndrome or SARS (World Health Organization, 2003), some of the diseases that were once eliminated resurface in populations earlier considered free of the disease. Old infections, such as tuberculosis and diphtheria, have occurred in large epidemics

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in Europe and other industrialized countries. The 1996 outbreak of polio in Albania, Greece, and the then Federal Republic of Yugoslavia indicates that an infectious disease can easily resurface, if immunization coverage is allowed to drop (World Health Organization, 1999).

Facing these grim realities, variety of epidemic control models have been proposed and used in practice including vaccination programs, prevention programs (such as changing risky behavior), and treatment programs (such as quarantine and use of antivirals); see (Brandeau et al. 2004) for more details. Among a variety of intervention strategies, we focus on *vaccination* programs in this research, as they are well-known for both their efficiency and cost-effectiveness. In fact, the World Health Organization (2000) considers vaccination to be the "ultimate weapon against infection and drug resistance." For influenza (or flu, more commonly), Germann et al. (2006) argue that vaccination, among other control programs such as social distancing, school closure, and use of antivirals, remains the most effective tool to eliminate the epidemic. They show that even if a flu vaccine is poorly matched to the circulating strains, it can still drastically slow down the spread of the disease.

The global vaccination market is a large one, with annual sales of US$21.7 and US$25.3 billion in 2009 and 2010, respectively, and is projected to grow at a compound annual rate of 9.3%, reaching an extraordinary figure of US$39.5 billion in 2015 (Carlson 2011, 2012). Despite this seemingly large size of the global market and the clear benefits of vaccination programs, vaccine uptakes in populations have typically been low (Blue 2008). Such undesirably low vaccine coverage can be attributed to two main factors:

Supply-Side Issues: Operational issues on the supply side, such as yield uncertainty, often lead profit-maximizing manufacturers to under-produce, resulting in supply shortages, shortage of influenza vaccines being a prime example. In fact, several papers in the operations management literature argue that the operational risk borne by a manufacturer is one of the main reasons for the shortage of influenza vaccine in the market (Chick et al. 2008, Deo and Corbett 2009).

Negative Network Effect: A relatively low vaccine coverage can also arise from the *negative network effect* faced by consumers. As the fraction of vaccinated individuals grows, the chance of contacting an infection diminishes. Therefore, the willingness to pay for the vaccine reduces, an effect contrary to the positive network effect, in which the willingness to pay for a product increases with the size of the network (Katz and Shapiro 1985).

In the end, a traditional *free market* for vaccines may not be socially efficient. On one hand, left to its profit-maximizing ways, a supplier may under-produce, resulting in shortages. On the other, an individual may choose to free ride off the herd immunity, and, consequently, voluntary vaccination may not reach a socially optimal level in the population. We are, therefore, interested in studying the following research questions that arise in this context:

- In the face of yield uncertainty and network effects, what is the socially optimal level of vaccine coverage?

- Is it possible, from a contract theory perspective, to induce the manufacturer and the consumers to achieve that level?
- How does the effectiveness of a vaccine influence the above decisions?
- What roles can governments or global non-profit organizations play here in coordinating the market?

Each of the above issues has been examined in an isolated manner in prior research. Some research in the operations management area has been devoted to the role of supply uncertainty and its effect on the supply chain outcome. For example, Chick et al. (2008) argue that the production risk due to unknown vaccine yields, which is assumed by vaccine manufacturers, is the primary reason for an insufficient supply of influenza vaccine in the market. They design a variant of a cost sharing contract which provides incentives to manufacturers, as well as to governments that purchase vaccines, so that a supply chain achieves an optimal balance between for-profit manufacturer incentives and public health incentives. Deo and Corbett (2009) examine the role of production yield in explaining the limited number of players in the influenza vaccine market. However, these works do not consider the negative network effect and the consumers' willingness to pay for the vaccine. On the other hand, the demand-side issue has started receiving some attention in more recent literature in operations management. For example, Cho (2010) studies consumers' willingness to pay, and Arifoglu et al. (2012) address network externality effects in a similar context. However, the objective of these articles are quite different from the current one, as we are concerned with designing contracts to coordinate the vaccine market.

Epidemic modeling and health economics literature has primarily focused on the negative network externality facing the consumers as a key driver for vaccination levels that are lower than what is socially desirable. For example, Bauch and Earn (2004) provide a game-theoretic analysis of vaccine coverage that considers the negative network effect as well as the necessary epidemiological details. They show that a voluntary vaccination program without any government intervention fails to achieve the vaccination level necessary to eliminate the epidemic. Brito et al. (1991) provide a more comprehensive model in their economic analysis of the situation, but lack the necessary epidemiological details. Geoffard and Philipson (1997) also consider vaccine subsidies with the goal of complete eradication of the disease, but do not consider the cost of production and do not discuss an efficient coordination of the vaccine market. They find that a subsidy program would not be effective at achieving eradication because of network externalities. In a more recent article, Althouse et al. (2010) study how public subsidies can help bring the vaccine uptake to a more efficient level in the case of a disease that is subject to network externalities,[1] but they consider only the consumers—their subsidy is designed to maximize the aggregate consumer utility. Cook et al. (2009) analyze epidemiological and economic field data from two sites in Kolkata, India,

[1]The role of subsidies in market coordination has also been studied in other contexts that exhibit network effects. For example, within the context of security, Zhuang (2010) and Zhuang et al. (2007) show that providing fully subsidized security to targeted agents (with heterogeneous time preferences) can reduce the total social cost of security and improve the performance of the system.

to estimate the socially optimal subsidy for a cholera vaccine (disease subject to similar network externalities as influenza).

In most of the prior work, however, vaccine production is either neglected (e.g., Bauch and Earn 2004, Bauch et al. 2003, Geoffard and Philipson 1997) or considered as deterministic and exogenous (e.g., Brito et al. 1991). Recently, Mamani et al. (2012) consider the role of governments in market coordination through subsidies in the presence of multiple vaccine manufacturers, but they do not take into account the yield uncertainty inherent in the production process, which, as shown here, has major implications in the subsidy program. In particular, they find that a constant one-part subsidy coordinates the market in a deterministic setting. In extending their results, one would naturally expect that a constant two-part subsidy should align the incentives of different parties under the presence of production uncertainty. Our results, however, indicate that, when the yield is stochastic, a constant two-part subsidy is just *not* sufficient to align both consumer demand and production quantity with the first-best outcome; rather, a menu of subsidies is necessary. The purpose of this research is to bridge this conspicuous gap in the literature by considering yield uncertainty and network externalities together in order to develop a more comprehensive analysis and to provide a more practical coordination scheme.

The remainder of the paper is organized as follows. We discuss related prior work in Section 2. Section 3 develops the modeling framework and examines the market equilibrium. In Section 4, we employ a total social welfare function to identify the socially optimal outcome. Then, in Section 5, we derive a two-part subsidy scheme that induces this outcome. Section 6 concludes the paper and offers directions for future research.

2 Literature Review

The topic of network externalities and consumers' willingness to pay in the context of public goods in general, and vaccines in particular, is gaining traction in the operations management community. Cho (2010) considers the issue of consumers' willingness to pay and price elasticity to determine the socially optimal policy for selecting strains that are to be included in an influenza vaccine. This threshold policy is obtained based on a trade-off: On one hand, retaining the current composition of influenza strains could lead to an ineffective vaccine if new strains were to emerge. On the other, including new strains in the vaccine composition could increase the production yield uncertainty. Arifoglu et al. (2012) combine consumption-side externalities with the supply uncertainty, and show that the limited availability of vaccines lead to an inflated demand. They quantify the level of ineffectiveness of the decentralized model with partially centralized scenarios. In this paper, we, too, combine negative network effects with yield uncertainty to identify the level of inefficiency in a market-based system. However, unlike Arifoglu et al. (2012), we investigate this inefficiency with the lens of contract theory to design incentive mechanisms that can induce consumers and suppliers to make decisions that are aligned with the social optimum.

Our work is closely related to the recent work by Mamani et al. (2012); similar to theirs, our model also includes consumers' free-riding behavior, the production cost borne by the vaccine

manufacturer, and a central planner's desire to achieve the social optimum using subsidy. In addition, however, we also consider the production yield uncertainty faced by the manufacturer. In our model, the yield is stochastic, and hence the vaccine coverage is uncertain. Consequently, the quantity decision is made by the manufacturer under yield uncertainty, whereas the demand decisions are made by consumers facing both the supply uncertainty and the negative network effect. These two decisions are not necessarily aligned with the social objective, and a central planner has a role in aligning them towards the social optimum. This modeling difference has critical consequences on the equilibrium solution, social optimum, and coordination mechanisms. In particular, in (Mamani et al. 2012), where the yield is deterministic, a one-part subsidy is shown to be sufficient in coordinating the market. In our paper, unsurprisingly, the stochastic yield makes a one-part subsidy ineffectual for market coordination. Perhaps more unexpectedly, even a two-part fixed subsidy scheme cannot coordinate the market. The coordinating subsidy is a two-part *menu* of subsidies that depends on the coverage. This fundamental difference in the equilibrium outcome is essentially a result of the significant differences in the modeling contexts, requiring different analyses.

Recent literature studies contract design issues when considering customers' willingness to pay for public interest goods. For example, Taylor and Yadav (2011) investigate two forms of contracts for public products (e.g., essential medicines): a *purchase subsidy* (to the retailers) and a *sales subsidy* (to the consumers). They show that when consumers are homogeneous in their valuation of the product, the subsidy provider (donor) would prefer a purchase subsidy, whereas a sales subsidy is more preferable for heterogeneous consumers. A key assumption in (Taylor and Yadav 2011) is that the retailer has freedom to set prices after market uncertainties are resolved and the product is acquired. This is a valid assumption for their model as they consider populations in developing countries where the retailer can change prices based on market conditions. In our model, however, the uncertainty exists with respect to the yield (not the demand), and pricing and production decisions are made prior to the realization of the yield. The manufacturer, once committed to a fixed price, cannot change it easily. Ovchinnikov and Raz (2012) study a number of mechanisms to coordinate price and quantity decisions in a market for public goods where these decisions are made simultaneously. They show the effectiveness of subsidies compared to rebates in coordinating either price or quantity decisions. They also show that a joint rebate-subsidy mechanism can coordinate both decisions. However, both the aforementioned papers assume a deterministic supply but a random demand. We, on the other hand, consider a scenario where supply is uncertain and demand is deterministic (e.g., the case of pre-orders for influenza vaccine). Finally, we focus on a specific class of products—vaccines—rather than a generic product. This allows us to derive a demand function that is based not only on economic parameters such as the vaccine price, but also on other epidemiological factors such as the population's perception of the vaccine effectiveness and the dynamics of the infection.

In summary, the major contributions of our paper are two fold: First, we build on a stream of work that combines concepts from three distinct bodies of literature—operations management, economics, and epidemiology—to generate insights about public policy issues related to vaccine

pricing, coverage, and subsidies in the context of potentially imperfect vaccines. Second, our model contributes to the operations management literature by considering contracting issues within a production process with random supply and price-dependent demand. While others have examined single-period models with random and price-dependent demand (Petruzzi and Dada 1999), as well as supply contracts in that setting (Cachon 2003), we show that effects of random supply are different than those of random demand when considering price-sensitive consumers, and thus the resulting coordinating mechanism must differ accordingly. More specifically, we show that a menu of two-level incentive mechanism is needed to coordinate the market, and a simple two-part fixed subsidy is not sufficient to align both the quantity and pricing decisions simultaneously. Our results have important implications in terms of the role of governments and global health organizations in managing vaccine programs and provide directions for appropriate incentive schemes that can work towards a greater good.

3 The Model

In this section, we develop a model of the vaccine market with a single manufacturer supplying a specific vaccine for a fixed population. We consider the following sequence of events in this sequential game. At the beginning of the production season, the manufacturer decides the vaccine price and the production volume, and incurs the corresponding production cost. Next, consumers follow with their choice of whether (or not) to get immunized, which determines the demand for the vaccine. This decision is based on the consumers' expected utility that takes into account their probability of infection, disutility from the infection, and the price they pay for the vaccine. Finally, the total vaccine coverage is determined based on the realized production yield and the demand. Any leftover vaccine is discarded and any unmet demand is lost.

We note that Arifoglu et al. (2012) consider a different model of consumer behavior in which individuals make the decision to search for vaccines based not only on the vaccine price but also the availability of vaccines. There, the demand for vaccines is determined based on the vaccine price and the realized production yield in order to incorporate the "availability effect" in their model. In this paper, we assume that vaccine production quantity or yield does not significantly affect the demand for vaccines. This assumption is justified in the following contexts:

- There are situations where the realized (or targeted) production quantity is not public information, or simply when its effect on consumer behavior is not as significant as the vaccine price.

- In some cases, a significant portion of the demand for vaccines is based on pre-orders received from healthcare providers that are placed well in advance of the production yield realization; for example, see (Fine 2004, CDC 2011a) for the case of influenza vaccines.[2] In this case while consumers do not directly purchase the vaccine from the manufacturer, the total demand from

[2]In case of influenza vaccines, such pre-orders happen between January and March (CDC 2011a). While flu vaccines can be purchased at a later time closer to the flu season, Fine (2004) reports that suppliers' emphasis and preference is on pre-booking.

a healthcare provider for a certain vaccine price should effectively mirror the actual demand from consumers based on demand and price elasticity information from prior periods.

3.1 Characterization of Consumer Behavior

We now provide a description of the consumer model, also borrowed from (Mamani et al. 2012). We consider consumers to be heterogeneous, in the sense that direct and indirect costs of infection varies from one person to another. This is captured by indexing consumers by a parameter u that is assumed to be uniformly distributed over the interval $(0, 1)$; u can be viewed as the relative loss suffered by an individual from an infection. This assumption allows us to model individuals who perceive the vaccine as dangerous and are thus more willing to risk infection rather than be vaccinated (low value of u), as well as individuals like children and the elderly who, for example, are more prone to complications when contracting influenza and would thus experience more disutility from an infection (high value of u). The absolute cost to consumer u from an infection can then be expressed as Lu, where L is a constant.

Because of different infection dynamics for the vaccinated and unvaccinated fractions of a population, the infection probabilities are also different for each group. Following Mamani et al. (2012), by $p(f)$ and $P(f)$, we denote the infection probabilities for the vaccinated and unvaccinated fractions, respectively, for an overall vaccine coverage of f; $P(f)$ must be strictly greater than $p(f)$ for the vaccine to have any value. The infection probability for the entire population, $r(f)$, can also be viewed as the overall fraction of infected individuals in the population:

$$r(f) = fp(f) + (1 - f)P(f).$$

Let the price of the vaccine be W. Since vaccines are not fully effective, an immunized person incurs a vaccine cost of W and may still get infected. Therefore, $W + Lup(f)$ represents the total expected cost of getting vaccinated. On the other hand, a consumer who has not been immunized does not incur the cost of W but has a higher chance, $P(f)$, of becoming infected. The expected cost incurred by a unimmunized individual is thus $LuP(f)$. Therefore, for the indifferent consumer, the average cost of getting vaccinated and the cost of an infection are the same. Let $\bar{u}$ denote such a marginal consumer, then we have: $L\bar{u}P(f) = W + L\bar{u}p(f)$, or $W = L\bar{u}(P(f) - p(f))$. Any consumer with $u \geq \bar{u}$ gets immunized, whereas anyone with $u < \bar{u}$ does not. Therefore, $\bar{u} = 1 - f$, as u is uniformly distributed over $(0, 1)$. Substituting this and using the fact that $P(f) - p(f) = \frac{r(f)-p(f)}{1-f}$, we get:

$$w = \frac{W}{L} = r(f) - p(f) \equiv T(f). \tag{1}$$

Equation (1) tells us that consumers make their vaccination decisions in such a manner that the normalized price, w, of a vaccine represents the marginal reduction it induces in the probability of infection.

As discussed earlier, we do not consider the effect of production uncertainty and possible production shortage on consumers' demand decision. Of course, this modeling framework can be easily adapted to the case where an aggregated pre-order is placed by healthcare providers prior to the

production season, when there is simply no information about the vaccine yield or production shortage. This is because, as illustrated in the next section, the consumer choice model in (1) is equivalent to the one where the total orders placed by consumers (or healthcare provider) follows a piece-wise linear form, which is a standard assumption in many price-demand models.

3.2 Infection Probability

In order to complete the characterization of consumer behavior, we need to find the functional forms of $r(f)$, $p(f)$, and $T(f)$. In a large part of the literature on vaccine programs, it is assumed that an individual receiving a vaccine becomes completely immune from the underlying infection, that is, the vaccine has been assumed to be perfect. This assumption, however, may not be valid for a number of infectious diseases of interest, such as seasonal influenza and HIV.[3] In this regard, we follow Mamani et al. (2012) and assume that vaccines may indeed be imperfect—a vaccinated consumer, albeit less likely, can still get and transmit the infection. There are two ways in which an imperfect vaccine affects the transmission of the disease: First, imperfect vaccines can reduce the probability of becoming infected if exposed to an infectious contact, which is known as the vaccine effect on susceptibility. Second, imperfect vaccines can also affect the probability of disease transmission when a vaccinated individual gets infected; this is known as the vaccine effect on infectiousness (Datta et al. 1998, Hill and Longini 2003). To combine these two effects, we take an approach that is similar to the one adopted by Longini et al. (1978). Let $0 < \phi \leq 1$ be the vaccine efficacy parameter that reflects the combined effect of vaccine on transmission, which includes both susceptibility and infectiousness effects. Note that $\phi = 1$ corresponds to the perfect vaccine case.

Following Mamani et al. (2012), we express $r(f)$ as (see Appendix A in the Electronic Companion to this paper for more details):

$$r(f) = \begin{cases} 0, & \text{if } f > F = \frac{R_0 - 1}{\phi R_0} \\ 1 - \phi f - \frac{1}{R_0}, & \text{otherwise,} \end{cases}$$

where R_0, the *basic reproduction number*, is defined as the total number of secondary infections caused by an infectious individual in an otherwise susceptible population. Therefore, R_0 is a measure that indicates whether the introduction of a single initial case of infection into a susceptible population will result in a nontrivial outbreak. It must be noted that, if $R_0 \leq 1$, the infection probability is zero, and the disease transmission dies out. Hence, for the remainder of the paper, we only consider the interesting case of $R_0 > 1$.

We note that $F = \frac{R_0 - 1}{\phi R_0}$ is the minimum vaccination fraction that drops the overall infection probability, $r(f)$, to zero. In epidemiology, this threshold is called the *critical vaccination fraction*; it represents the minimum level of vaccine coverage necessary for providing the so-called *herd immunity* effect—a situation that arises when immunization level in the population is sufficiently

[3] Seasonal influenza strains constantly mutate over time making a vaccine an imperfect match to the circulating strains in a community (Gross et al. 1995, Hill and Longini 2003, Sullivan 1996). Within the context of potential HIV vaccines, many experts believe that, even if a vaccine were to become available in the future, it would likely provide only a partial protection from the disease (Datta et al. 1998, Hillier et al. 2005, Longini et al. 1996).

high so that the disease is eliminated entirely from the population. It should be noted that F could be greater than one, in which case the disease cannot be completely eradicated by a vaccination program alone. Therefore, we introduce $\bar{F} \equiv \min\{F, 1\}$.

Before we discuss the functional form of $p(f)$, and hence that of $T(f)$, it is necessary to see how the negative network effect manifests itself in this context. The willingness to pay for consumer u is given by $\text{WTP}_u = Lu(P(f) - p(f))$. The fact that WTP_u depends on f, the market coverage, is an evidence of the presence of a network effect. Now, note that, when $\phi = 1$, $p(f)$ becomes zero as the vaccine is perfect; if, in addition $R_0 \to \infty$, then $r(f) \to (1-f)$ and $P(f) \to 1$; this is because, when the reproduction number becomes large, an unvaccinated individual is almost certain to contract the infection. In this case, $P(f) - p(f) = 1$, and WTP_u becomes independent of the coverage level. Therefore, in this limiting case, the actions of other individuals become irrelevant. As a result, the externality effect completely disappears—the only way to enjoy the benefits of the vaccine is to acquire it. In contrast, if $R_0 < \infty$ or $\phi < 1$, the term $P(f) - p(f)$ is lower than 1 and is no longer independent of f. This implies that, for the same consumer, WTP_u is lower than in the limiting case considered above, and its specific value depends on the overall coverage level. Therefore, the desire to purchase depends not only on a consumer's own preference, but also on decisions made by other consumers. This happens because an unvaccinated individual could benefit from the (partial) immunity of the vaccinated population. Put differently, unlike typical products or services, in the vaccine market, a consumer does gain some utility from the vaccine, without having to actually purchase it. Hence, the coverage level has an impact on the purchase decision of an individual.

Finally, in order to find an expression for $p(f)$, we note that it must abide by two properties. First, the probability of infection for vaccinated individuals must be less than that for the general population, i.e., $p(f) < r(f)$, for all f. Second, when the vaccine is fully effective, a vaccinated individual will not contract the disease, implying $p(f) = 0$ if $\phi = 1$. The following functional form abides by these two conditions and provides a good approximation of the exact infection probability function (see Appendix B in the Electronic Companion to this paper for details):

$$p(f) = \eta(1-\phi)r(f),$$

where $0 \le \eta \le \frac{1}{1-\phi}$ is a constant; in this paper, we use this functional form for $p(f)$. Using this approximation, we find that $\text{WTP}_u = \frac{Lu\theta r(f)}{1-f}$, where $\theta = 1 - \eta(1-\phi)$. Therefore, the externality effect, which can be viewed as the strength of the dependence of WTP_u on f, can be written as:

$$\text{EF}_u = \left|\frac{\partial \text{WTP}_u}{\partial f}\right| = \begin{cases} 0, & \text{if } f > F = \frac{R_0 - 1}{\phi R_0}, \\ \frac{Lu\theta}{(1-f)^2}\left|1 - \phi - \frac{1}{R_0}\right|, & \text{otherwise.} \end{cases}$$

It is now easy to see that, when $f < F$, EF_u depends on both R_0 and ϕ; it approaches zero when $R_0 \to \infty$ and $\phi \to 1$, but is non zero otherwise. Finally, $T(f)$ can be written as:

$$T(f) = \theta r(f) = (1 - \eta(1-\phi))\, r(f). \tag{2}$$

3.3 Production Decision

As mentioned earlier, we consider a production process with uncertain yield. This is particularly relevant in the context of influenza vaccines, where uncertainties in the manufacturing process have been shown to play a key role in decisions across the entire vaccine supply chain (Chick et al. 2008); our model reduces nicely for vaccines with deterministic yield (Mamani et al. 2012).

We consider a stochastically proportional yield for the final production output (e.g., Yano and Lee 1995). Specifically, let q be the target production level normalized with respect to the size of the population; then, we assume that the final production output is Uq where $0 < U < \bar{U}$ is a random variable with an average $\mathbb{E}[U] = \mu$, a probability distribution function $J(\cdot)$, and a probability density function $j(\cdot)$, where $j(x)$ is positive only if $x \in (0, \bar{U})$ and zero otherwise; the possibility that $U > 1$ is not excluded in our model and represents a situation in which the yield is higher than the targeted production quantity.

We analyze a market where the vaccine price is determined before the production campaign and the realization of the random yield, such as the influenza vaccine market; CDC (2011b), for example, lists vaccine prices as early as January. An underlying assumption of our model is that, once committed to a price, the manufacturer cannot simply change it based on the production yield or other market conditions. This is a plausible scenario for markets such as the US (CDC 2011b). Taylor and Yadav (2011) study an alternative model where prices are set after the market conditions are realized. This is the case when the supplier has the authority to change prices based on the realized demand and available quantity and is perhaps a better representation of situations in developing countries.

In a market where pricing decisions are made prior to random yield realization, the vaccine price does not necessarily clear the market and the manufacturer can have vaccine shortage or excess. We assume that the manufacturer is subject to a constant (normalized with respect to L) marginal production cost of c per unit. For a production yield U, a normalized vaccine price w, and a production quantity q, the manufacturer's profit is:

$$\Pi(q, w, U) = w \min\{Uq, f\} - cq.$$

3.4 Market Equilibrium

In a monopoly market, the manufacturer chooses his vaccine price and production quantity in order to maximize his expected profit, anticipating the effect of his decision on the final production output and consumers' reaction:

$$\pi(q, w) = \mathbb{E}_U[\Pi(q, w, U)] = \mathbb{E}_U\left[w \min\{Uq, f\} - cq\right] = w \int_0^{\frac{f}{q}} xq dJ(x) + w \int_{\frac{f}{q}}^{\bar{U}} f dJ(x) - cq,$$

where f is given by Equation (1) and can be substituted to rewrite the manufacturer's expected profit, $\pi(q, w)$, as a function of q and f:

$$\pi(q, f) = T(f) \int_0^{\frac{f}{q}} xqdJ(x) + T(f) \int_{\frac{f}{q}}^{\bar{U}} fdJ(x) - cq. \tag{3}$$

To solve the manufacturer's problem, we first find the optimal production quantity for a fixed demand f, denoted by $q_m(f)$, and then we solve for the equilibrium demand, f_m. Let f be fixed such that $f < \bar{F}$. Taking the derivative with respect to q, using Leibnitz's rule, and rearranging terms, we get the following result.

Lemma 1 *Let $\tilde{q}$ be the unique solution of:*

$$\int_0^{\frac{f}{q}} xdJ(x) - \frac{c}{T(f)} = 0.$$

Then, $q_m(f)$ is given by:

$$q_m(f) = \begin{cases} \tilde{q}, & \text{if } T(f) \geq \frac{c}{\mu}, \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

Note that when the production cost exceeds the expected revenue from the vaccine ($\mu T(f) < c$), the manufacturer would simply set $q_m(f) = 0$.

Example 1 *If U follows a uniform distribution on $[a, 2\mu - a]$, then $J(x) = \min\left\{1, \frac{x}{2(\mu-a)}\right\}$. In that case:*

$$q_m(f) = \begin{cases} \sqrt{\frac{f^2 T(f)}{a^2 T(f) + 4c(\mu-a)}}, & \text{if } T(f) \geq \frac{c}{\mu}, \\ 0, & \text{otherwise.} \end{cases}$$

Figure 1 illustrates the behavior of $q_m(f)$ for two different values of ϕ, when U follows a uniform distribution on [0,2] and $R_0 = 2$, $c = 0.1$, and $\eta = 3.3$. This figure shows that $q_m(f)$ equals zero when $T(f) > \frac{c}{\mu}$. This implies that, given a ϕ, there exists a threshold for f—a point of discontinuity always below the critical vaccination fraction, F—beyond which the production quantity is zero. Indeed, beyond this point, the vaccine price becomes so low that production costs outweigh expected revenues and the manufacturer chooses not to enter the market. This threshold occurs at a lower value of f when the vaccine is less perfect because the equilibrium price of a vaccine with a lower ϕ is also lower. We also notice that for a fixed coverage level f, the lower the value of ϕ, the lower is the quantity produced by the manufacturer, since the vaccine price decreases as ϕ decreases. Finally, we note that $q_m(f)$ is not necessarily monotonic in f, even to the left of the threshold; this can be clearly seen in the plot corresponding to $\phi = 1$. There, the quantity first increases in f as a larger demand provides incentives to supply higher quantities. However, beyond a certain value, the price of the vaccine, $w = T(f)$, becomes too low and the supplier, in turn, chooses to reduce the

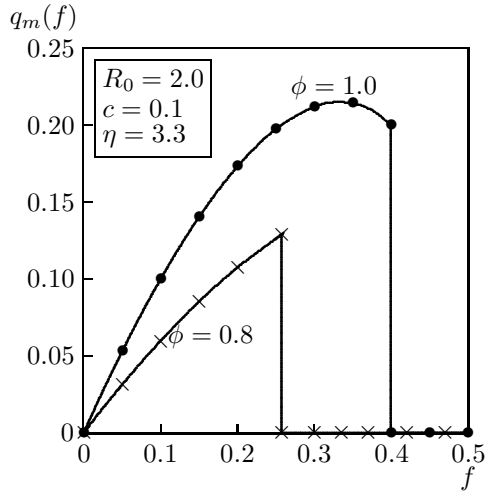


Figure 1: Optimal Monopoly Production Quantity with $U \sim$ Uniform$[0, 2]$

quantity produced. This observation is consistent with observations in traditional markets where a monopolist is known to under-produce.

Using Lemma 1, we can reduce the objective function to a function of a single variable, f, by substituting for $q_m(f)$ from Equation (4) into Equation (3). For the Newsvendor problem with price-dependent and random demand, this method was introduced by Whitin (1955). We follow a similar process when the supply is random. It turns out that the profit function $\pi(q_m(f), f)$ is not concave; however, as indicated by the next theorem, we can still prove the existence and uniqueness of an optimal solution for a large class of random variables, including the increasing failure rate (IFR) distributions for supply uncertainty:

Theorem 1 *Suppose, for all z, the distribution function $J(z)$ satisfies the following condition:*

$$2\rho(z)(1 - J(z)) + \left(\int_0^z x dJ(x) + z \int_z^{\bar{U}} dJ(x) \right) \frac{d\rho(z)}{dz} \geq 0,$$

where $\rho(z) = \frac{j(z)}{1-J(z)}$ is the hazard rate. Then, the equilibrium monopoly price is $w_m = \theta r(f_m)$, and the monopoly production quantity is $q_m(f_m)$ units, where $q_m(f)$ is obtained from Equation (4), and f_m is the unique f in the region $\left(0, \bar{F} - \frac{c}{\mu\phi\theta}\right)$ that satisfies $\frac{d\pi(q_m(f),f)}{df} = 0$ if $\bar{F} - \frac{c}{\mu\phi\theta} \geq 0$, and $f_m = 0$ otherwise.

It is interesting to observe that, despite the similarities between our model and the Newsvendor model with price-dependent demand, the condition of the hazard rate function in Theorem 1 is different than the similar condition for the latter model (e.g., Petruzzi and Dada 1999). In classical Newsvendor models, the price-dependent demand is either (linear) additive or (exponential) multiplicative. In our model, however, the supply uncertainty is multiplicative in nature, and, at the same time, the demand function is nonlinear. Thus, the objective function in our model is affected by the supply uncertainty in a different manner than in those models where demand is uncertain.

Example 2 *Once again, let us assume that U is uniformly distributed over $[a, 2\mu - a]$, i.e., $J(x) = \min\left\{1, \frac{x}{2(\mu-a)}\right\}$. Then, from Equation (3) and Example 1, we get:*

$$\pi(f) = \frac{f\left((2\mu - a)T(f) - \sqrt{(a^2T(f) + 4c(\mu - a))\,T(f)}\right)}{2(\mu - a)},$$

which can be maximized using Theorem 1 to obtain f_m.

Figure 2 represents the equilibrium in the case of a perfect vaccine ($\phi = 1$) for a range of values of c when U follows a uniform distribution on [0,2] and $\eta = 3.3$. In order to see the impact of yield uncertainty, we also plot the value of f_m for the case of deterministic yield (i.e., $U = 1$).

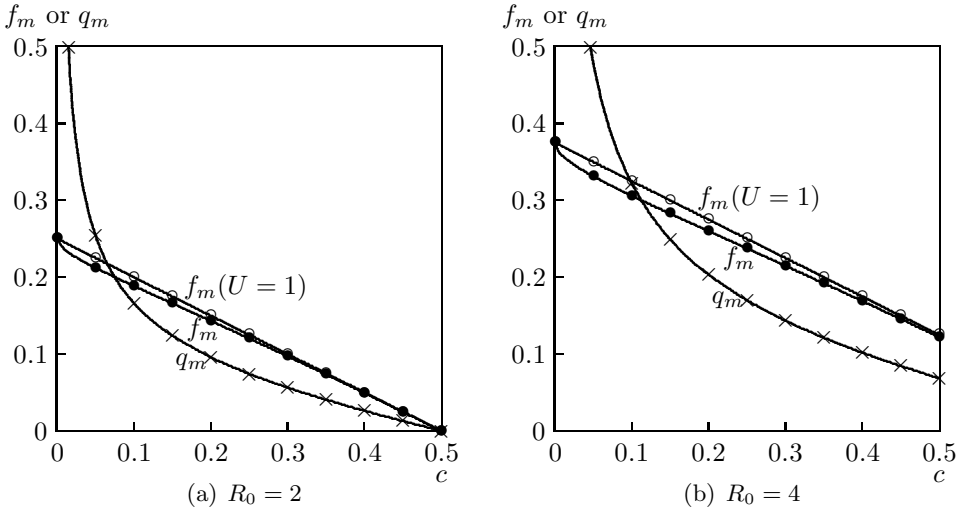
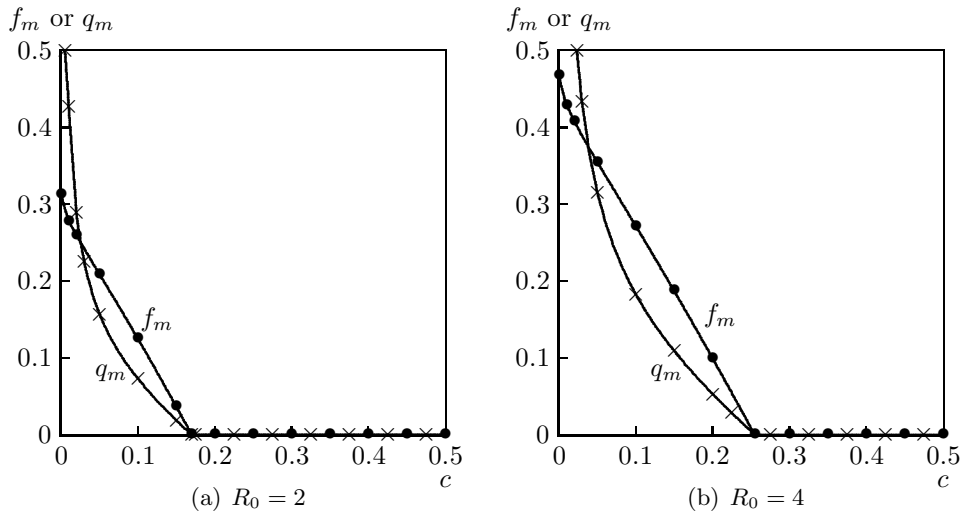


Figure 2: Equilibrium Production and Coverage for a Perfect Vaccine ($U \sim$ Uniform$[0, 2]$)

Comparing the two cases in the figure, it is clear that the presence of yield uncertainty implies a lower vaccine coverage at equilibrium. This finding is in conformity with the results from previous studies (on operational risks born by influenza vaccine suppliers), which conclude that manufacturing uncertainties help explain low vaccine supplies and sometimes even shortages (Fine 2004). Moreover, high externality effects (low R_0) lead to lower coverage as a single vaccination helps protect more individuals against the disease. We also observe that when the per unit cost is low, the manufacturer has an incentive to produce high quantities of the vaccine (compared to the equilibrium coverage); as the cost increases, however, the yield uncertainty makes it too risky to produce extra vaccines, and the quantity drops below the equilibrium demand.

In order to see the impacts of vaccine effectiveness ϕ and the distribution of U, we also plot the equilibrium outcome for $\phi = 0.8$ in Figure 3, for two values of R_0 and two different distributions of U. The comparison of Figure 3 with Figure 2 makes the role of vaccine effectiveness clear. First, for low values of the production cost, the vaccine coverage at equilibrium decreases with vaccine effectiveness, but this trend reverses for high production cost. This illustrates that a more

ineffective vaccine requires more coverage in the population to achieve the same level of protection, which is possible only when the vaccine is cheap enough to produce. When it is more expensive, a lack of effectiveness makes the vaccine less worthwhile to purchase, and coverage drops. Moreover, the equilibrium quantity is lower than when the vaccine is perfect. The impact of R_0 in Figure 3 is similar to the one see in Figure 2.



$\phi = 0.8$; $\eta = 3.3$.

Figure 3: Equilibrium Production and Coverage for an Imperfect Vaccine ($U \sim$ Uniform$[0, 2]$)

4 First Best Solution

In order to get a more complete picture of whether or not the existence of negative network effects and yield uncertainties in the production process leads to market inefficiencies, we need to solve the first best problem, whereby the social planner maximizes the total social welfare, which is the sum of all the surpluses for the manufacturer and the consumers (vaccinated and unvaccinated). In addition, we also account for the societal surplus that may accrue from a lower infection rate.

Manufacturer surplus: The manufacturer surplus is the net profit earned by the manufacturer when the target production quantity is q, and a fraction $g = \min\{Uq, f\}$ of the population is vaccinated.

$$\text{Manufacturer surplus} = \sigma_M = wg - cq.$$

Vaccinated consumers' surplus: When individuals are vaccinated, their probability of infection reduces and the expected cost from infection is lower. The aggregate normalized cost for purchasing the vaccine is wg. If these individuals were not vaccinated, the total vaccinated fraction would be zero and their probability of getting infected would be $r(0)$; if they do get infected, they incur a normalized infection cost $u \in [1 - g, 1)$. The surplus gained from

vaccination for this fraction of the population is then the difference between the costs with and without vaccination.

$$\text{Vaccinated surplus} = \sigma_V = \int_{1-g}^{1} x(r(0) - p(g))dx - wg = \left[\frac{1}{2} - \frac{(1-g)^2}{2}\right](r(0) - p(g)) - wg.$$

Unvaccinated consumers' surplus: One of the key differences between vaccines and most other products is that an individual can enjoy some surplus even if (s)he does not purchase or acquire the product, because of the immunity generated by other vaccinated individuals that leads to the negative network externality effect. The unvaccinated consumers' surplus is determined as the difference in utility between when the remaining consumers get vaccinated and when they do not. Each individual in the unvaccinated fraction, with a normalized infection cost $u \in (0, 1-g)$, has a probability $P(g)$ of becoming infected when the other fraction is vaccinated, and a probability $r(0)$ of becoming infected when the other fraction is not vaccinated. Calculating the difference in total cost between these two situations yields the total surplus gained for this fraction of the population from the presence of the vaccine. Therefore, the unvaccinated fraction $(1-g)$ gets a surplus of:

$$\text{Unvaccinated surplus} = \sigma_N = \int_{0}^{1-g} x\left(r(0) - P(g)\right)dx = \left(r(0) - P(g)\right)\frac{(1-g)^2}{2}.$$

Societal surplus: To obtain the total social welfare, we also consider the indirect cost to the society—we assume that every individual who gets infected poses a normalized loss of λ on the society; this cost could include a societal loss of work time and healthcare costs. Ovchinnikov and Raz (2012) consider a similar term in their social welfare function called the "externality effect" that represents the public interest component of the product. Since, a fraction $r(g)$ is likely to get infected in an expected sense, we write the societal surplus as:

$$\text{Societal surplus} = \sigma_S = -\lambda r(g).$$

Combining all these together, we get the normalized total social welfare:

$$\text{Total Social Welfare} = \sigma_M + \sigma_V + \sigma_N + \sigma_S = \frac{r(0)}{2} - \frac{(1-g)r(g) + gp(g)}{2} - \lambda r(g) - cq,$$

where, as before, $g = \min\{Uq, f\}$ is the actual fraction of the population that get vaccinated. Maximizing this total social welfare then is equivalent to minimizing the following social cost:

$$\begin{aligned} \text{SC}(q, f, U) &= \frac{(2\lambda+1)r(g) - gT(g)}{2} + cq \\ &= \frac{(2\lambda+1)r(\min\{Uq, f\}) - \min\{Uq, f\}T(\min\{Uq, f\})}{2} + cq. \end{aligned}$$

Thus, the central planner's objective would be to minimize this expected social cost function:

$$\begin{aligned}\Gamma(q,f) &= \mathbb{E}_U[\mathrm{SC}(q,f,U)] = \mathbb{E}_U\left[\frac{1}{2}\left((2\lambda+1)r(\min\{Uq,f\}) - \min\{Uq,f\}T(\min\{Uq,f\})\right) + cq\right] \\ &= \int_0^{\frac{f}{q}} \frac{1}{2}\left((2\lambda+1)r(xq) - xqT(xq)\right) dJ(x) + \int_{\frac{f}{q}}^{\bar{U}} \frac{1}{2}\left((2\lambda+1)r(f) - fT(f)\right) dJ(x) + cq.\end{aligned}$$

Taking the partial derivative of Γ with respect to f and q leads to:

$$\frac{\partial \Gamma}{\partial q} = -\int_0^{\frac{f}{q}} \frac{1}{2}\left(\phi(2\lambda+1) + T(xq) + xqT'(xq)\right) xdJ(x) + c, \tag{5}$$

$$\frac{\partial \Gamma}{\partial f} = -\frac{1}{2}\left(\phi(2\lambda+1) + T(f) + fT'(f)\right) \int_{\frac{f}{q}}^{\bar{U}} dJ(x). \tag{6}$$

The next result characterizes the first best solution:

Theorem 2 *Let f^* and q^* be the socially optimal vaccine coverage and production quantity. Furthermore, let $\tilde{q}^*$ be the solution of:*

$$\int_0^{\frac{\bar{F}}{q}} \frac{1}{2}\left(\phi(2\lambda+1) + T(xq) + xqT'(xq)\right) xdJ(x) - c = 0. \tag{7}$$

Then,

$$q^* = \begin{cases} \tilde{q}^*, & \text{if } c \le \mu\left(\phi\lambda + \frac{\phi + T(0)}{2}\right), \\ 0, & \text{otherwise,} \end{cases} \qquad \text{and} \quad f^* = \min\{\bar{F}, \bar{U}q^*\}.$$

Theorem 2 states that, when the production cost (c) is low, or when the infection cost for the society (λ) is high, it is socially optimal to set the demand at its maximum (the critical vaccination fraction or the maximum possible production). However, this does not imply that it is socially optimal to always reach this high vaccine uptake. Having the demand at its maximum ensures that the demand will not be the bottleneck in determining the coverage. The actual coverage is the minimum of the demand and the realized production, and will thus be determined by the yield realization and quantity q^*.

Example 3 *Suppose U is uniformly distributed over $[a, 2\mu - a]$. Let $\tilde{q}^*$ be the unique positive solution of:*

$$\frac{\phi}{4}(1 + 2\lambda + \bar{F}\theta)\left(\left(\frac{\bar{F}}{q}\right)^2 - a^2\right) - \frac{\bar{F}\theta\phi}{3}\left(\left(\frac{\bar{F}}{q}\right)^2 - a^3\frac{q}{\bar{F}}\right) = 2c(\mu - a).$$

Then, from Theorem 2, we get:

$$q^* = \begin{cases} \tilde{q}^*, & \text{if } c < \frac{\phi\mu}{2}\left(1+2\lambda-\frac{\bar{F}\theta}{3}\right) - \frac{a^2\bar{F}\theta\phi}{3(2\mu-a)}, \\ \frac{3(\phi\mu(1+2\lambda+\bar{F}\theta)-2c)}{2\phi\theta((2\mu-a)^2+2a\mu)}, & \text{if } \frac{\phi\mu}{2}\left(1+2\lambda-\frac{\bar{F}\theta}{3}\right) - \frac{a^2\bar{F}\theta\phi}{3(2\mu-a)} \le c \le \frac{\phi\mu}{2}\left(1+2\lambda+\bar{F}\theta\right), \\ 0, & \text{otherwise}, \end{cases} \quad \text{and}$$

$$f^* = \min\{\bar{F}, q^*(2\mu-a)\}.$$

Furthermore, if $a = 0$ and $\mu = 1$, that is, if U follows a uniform distribution on [0,2], then $\tilde{q}^* = \frac{\bar{F}}{2}\sqrt{\frac{\phi(3+6\lambda-\bar{F}\theta)}{6c}}$. For that case, the first two panels in Figure 4 depicts the socially optimal vaccine coverage and production quantity for two different values of each ϕ and R_0, when $\lambda = 1$ and $\eta = 3.3$. Similar to the market equilibrium case, for low values of c, the targeted production

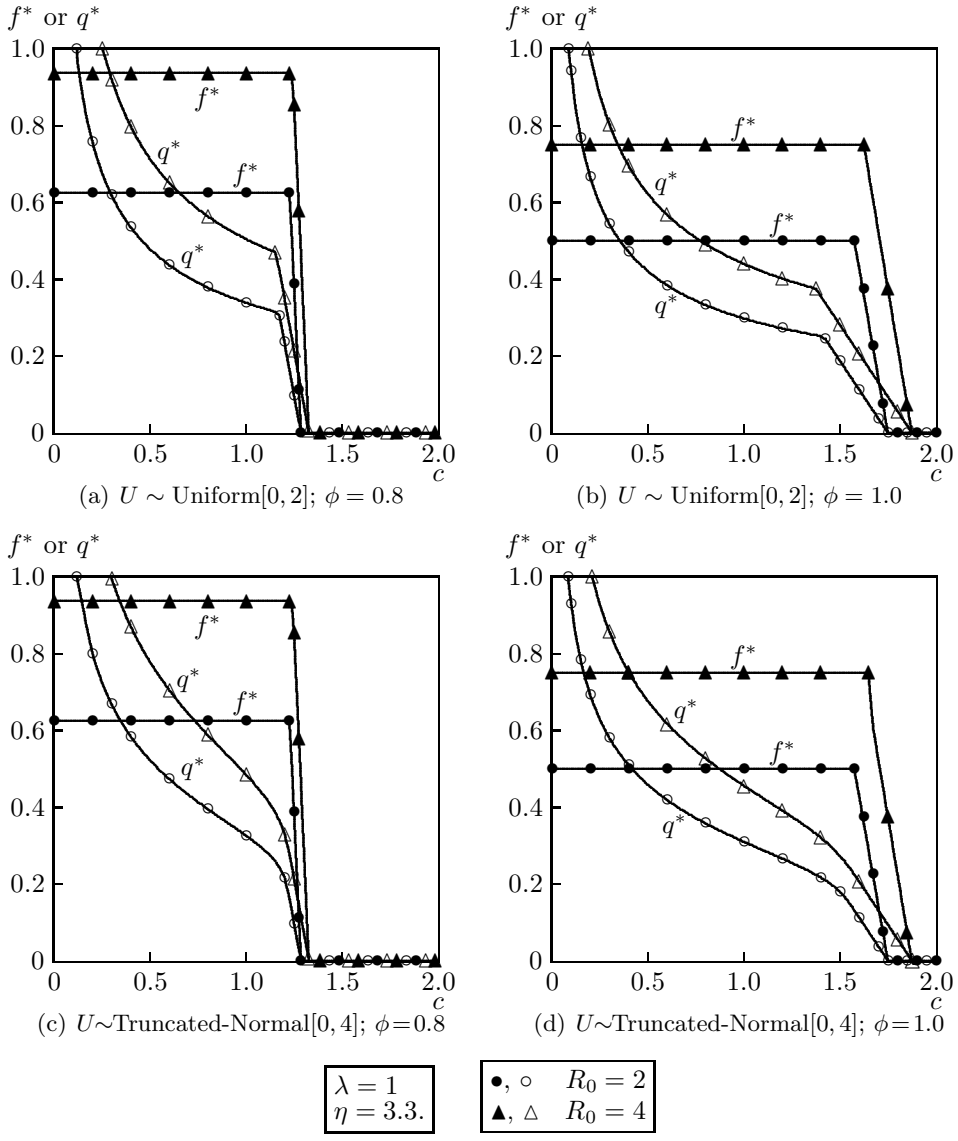


Figure 4: Socially Optimal Production and Coverage

quantity exceeds the demand so as not to risk inventory shortages. However, as the cost parameter, c, increases, the targeted production falls below the demand to avoid over-producing expensive vaccines that are likely to go unsold. The impact of vaccine effectiveness is also clear from Figure 4. For low values of c, the optimal production and coverage levels both increase as the effectiveness, ϕ, decreases. This is to be expected—as the vaccine effectiveness goes down, it becomes necessary to immunize a larger fraction of the population to obtain the same level of immunity and to produce more to meet that higher coverage level. However, at high values of c, maintaining a high level of immunity is no longer desirable. There, the trend reverses. Both optimal production and coverage level decrease quickly and reduce to zero as the vaccine effectiveness decreases. It is not optimal to keep producing an ineffective vaccine at a high production cost. It is also clear from these plots that the socially optimal production and coverage levels increase with R_0. This is expected—as R_0 increases, the higher level of infectiousness makes it necessary to produce more vaccines in order to immunize a larger fraction of the population.

In order to verify the robustness of our results with respect to the distribution of U, we have numerically tested them with other distributions as well. In particular, in the last two panels in Figure 4, we report the results when U follows a truncated normal distribution on $[0, 4]$; for the sake of easy comparison, the mean and the standard deviation of the truncated normal distribution, in this as well as other illustrative plots in this paper, are kept the same as those for the uniform distribution used in the first two panels in Figure 4.[4] The similarity of the last two panels with the first two clearly illustrates the robustness of our results with respect to the distributional assumptions about U.

5 Incentive Mechanism to Coordinate the Market

This section studies the effects of a subsidy program as a coordinating mechanism to align the incentives of different entities in a vaccine market. We introduce and motivate the subsidy scheme used to coordinate the market and show that a menu of two-part subsidies can indeed achieve this goal. We also examine the efficiency of *partial subsidy* programs, where a one-part mechanism is implemented by only subsidizing vaccines to the consumers.

5.1 Subsidy scheme

Comparing the results from Sections 3 and 4—or more specifically comparing Figures 2 and 3 with Figure 4—we can easily see that negative network effect and yield uncertainty can lead to large market inefficiencies. An important thrust of this paper is to understand whether, and how, a social planner could intervene in the vaccine market to align the equilibrium outcome with the social optimum. Of course, as amply demonstrated in Figure 4, achieving the herd immunity may not always be optimal for the society as a whole (if, for example, the vaccine is very expensive to

[4] We set the mean and standard deviation of the underlying normal distribution at 0.858205 and 0.689323, respectively; this way the mean and the standard deviation of the truncated distribution matches those of Uniform[0,2], which are 1 and $\frac{1}{\sqrt{3}}$, respectively.

produce and an infection causes little harm or disutility to consumers). Government intervention should thus aim at not necessarily achieving a vaccine coverage equal to the critical vaccination fraction, but rather to some fraction that optimizes a measure of the societal benefit. In this section, we propose an incentive mechanism that can eliminate the market inefficiencies and achieve the social optimum.

As mentioned earlier, we consider a situation where the vaccine prices are set well before the production yield is realized, sometimes as early as a year before production starts. We focus on subsidies to simultaneously coordinate price and quantity decisions. Furthermore, since both price and quantity decisions are made before resolving the yield uncertainty, a simple one-parameter subsidy is not sufficient to coordinate both decisions at the same time. This is due to the fact that a subsidy that can provide incentives to the individuals to ensure a sufficiently high level of vaccine coverage (coordinate price) does not necessarily align the manufacturer's production decision, under yield uncertainty, with the central planner's objective (coordinate quantity). A similar behavior can be observed in the classical Newsvendor model with price-dependent demand (e.g., Cachon 2003). Therefore, we consider a two-parameter incentive mechanism (s, e), in which a subsidy s is provided to the consumers (to coordinate the vaccine price or equivalently vaccine coverage, f), and, under a cost sharing proposal, the central planner agrees to pay e to the manufacturer for every production unit (to coordinate production quantity, q).[5] The cost sharing component in this contract is analogous to the purchase subsidy, and the subsidy component of our contract is similar to the sales subsidy in (Taylor and Yadav 2011). One way to implement the contract, among several other possible alternatives, would be to offer both incentives as a *package*. The manufacturer could purchase the raw materials for the production directly from the government at a per unit cost of $c - e$. In return, the manufacturer would get a payment of s for each vaccine sold to the consumers, whereas the customers would only pay $w - s$ to the manufacturer at the time of purchase. Thus, the effective production cost to the manufacturer would become $c - e$, and the demand-price relationship can be written as:

$$w = s + T(f).$$

The manufacturer's profit can be written as follows.

$$\pi(q, f) = (T(f) + s) \int_0^{\frac{f}{q}} xq dJ(x) + (T(f) + s) \int_{\frac{f}{q}}^{\bar{U}} f dJ(x) - (c - e)q.$$

In what follows, we first obtain the optimal production quantity for a fixed demand f, denoted by $q_m(f)$, and then find the equilibrium demand, f_m. Let f be fixed such that $f \leq \bar{F}$. Taking the derivative with respect to q, we can determine the optimal production quantity $q_m(f)$.

[5]This is in direct contrast with the case where the vaccine price can be set after the production yield is realized. If that were indeed the case, then one can modify (Taylor and Yadav 2011) using the approach presented in this paper to account for the production uncertainty and show that this market can be coordinated with just a single-parameter subsidy.

Lemma 2 *Let $\tilde{q}$ be the unique solution of:*

$$\int_0^{\frac{f}{q}} xdJ(x) - \frac{c-e}{T(f)+s} = 0.$$

Then, $q_m(f)$ is given by:

$$q_m(f) = \begin{cases} \tilde{q}, & \text{if } T(f) \geq \frac{c-e}{\mu} - s, \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

Example 4 *If U follows a uniform distribution on $[a, 2\mu - a]$, then:*

$$q_m(f) = \begin{cases} \sqrt{\frac{f^2(T(f)+s)}{a^2(T(f)+s)+4(c-e)(\mu-a)}}, & \text{if } T(f) \geq \frac{c-e}{\mu} - s, \\ 0, & \text{otherwise.} \end{cases}$$

Using the first order optimality conditions, we can find the equilibrium for the decentralized vaccine market. More specifically, for a pair (s, e), Equation (8) allows us to reduce the objective to a function of only f by substituting $q_m(f)$ into $\pi(q, f)$. Before we derive the subsidy pair (s, e), it is important to note that it is not possible to coordinate the market with fixed values of s:

Proposition 1 *Let $s > 0$ be a fixed subsidy provided to each vaccinated consumer and e the unit production payment provided to the manufacturer. Then, there is no contract with fixed payments (s, e) that can coordinate the market.*

Proposition 1 tells us that the consumer subsidy must be given as a *menu* of subsidies, that is, s should be a function of the coverage level f. This is in direct contrast with the results in (Mamani et al. 2012), where not only is the subsidy one-part because yield is deterministic, but it is also fixed and does not change with the coverage level. The following result characterizes a subsidy/cost-sharing scheme (s, e) that can coordinate the vaccine market.

Theorem 3 *Let f^* and q^* be the socially optimal demand and production quantity. Let e and $s(f)$ be given by:*

$$e = \begin{cases} c - (T(f^*) + s(f^*)) \int_0^{\frac{f^*}{q^*}} xdJ(x), & q^* > 0, \quad \text{and} \\ 0, & q^* = 0, \end{cases} \tag{9}$$

$$s(f) = \begin{cases} \theta f, & 0 \leq f < f^*, \\ \theta f^*, & f^* \leq f < \bar{F}, \\ 0, & f \geq \bar{F}. \end{cases} \tag{10}$$

Then, the contract $(s(f), e)$, in which a subsidy of $s(f)$ is provided to each vaccinated consumer and a unit production payment of e is provided to the manufacturer, coordinates the market.

Theorem 3 implies that it is indeed possible for a central planner to coordinate the market and induce it to achieve the social optimum, but not via a fixed two-part subsidy program. We illustrate this with the following example:

Example 5 *If U follows a uniform distribution on $[a, 2\mu - a]$, then the optimal contract is $(s(f), e)$, where $s(f)$ is as shown in Equation (10) and e is given by:*

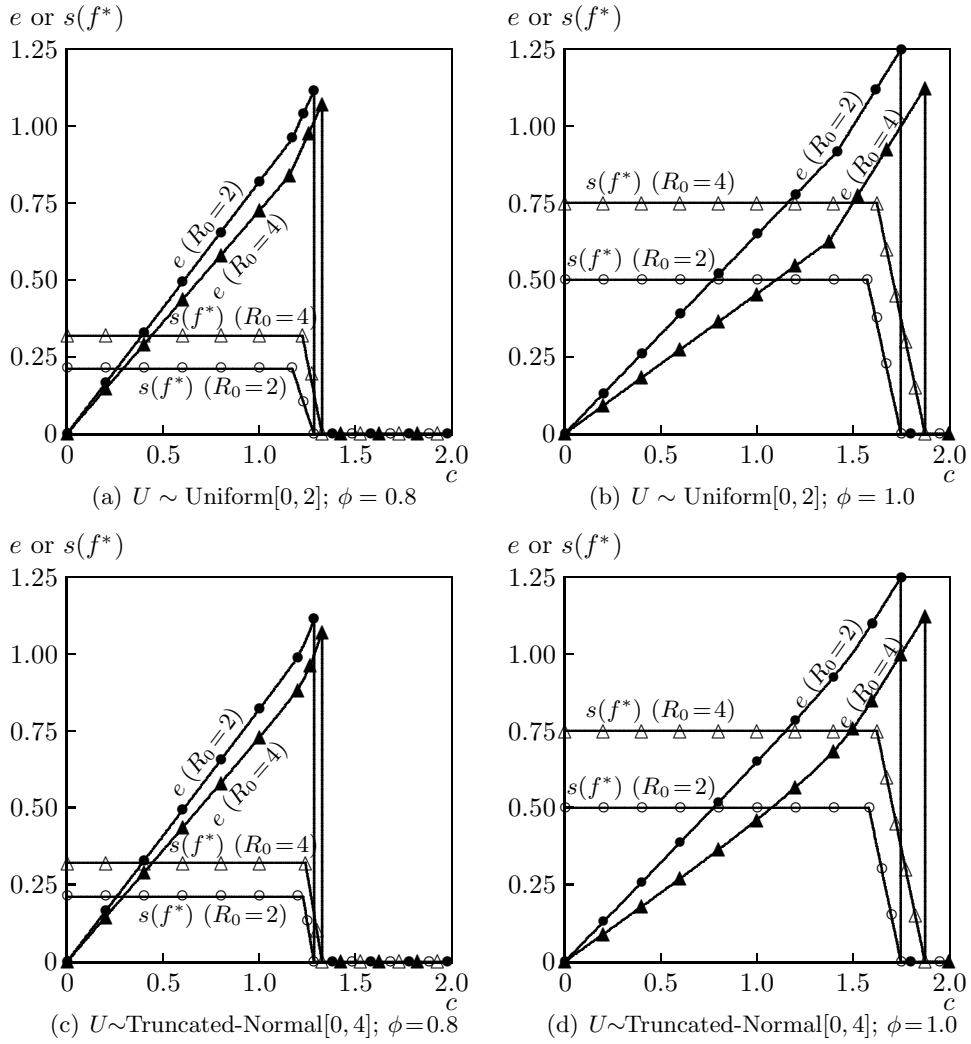
$$e = \begin{cases} c - \frac{\bar{F}\theta}{4(\mu-a)}\left(\left(\frac{\bar{F}}{q^*}\right)^2 - a^2\right), & \text{if } c < \frac{\phi\mu}{2}\left(1 + 2\lambda - \frac{\bar{F}\theta}{3}\right) - \frac{a^2\bar{F}\theta\phi}{3(2\mu-a)}, \\ c - \theta\mu\left(\bar{F}\phi + q^*(2\mu - a)(1-\phi)\right), & \text{if } \frac{\phi\mu}{2}\left(1 + 2\lambda - \frac{\bar{F}\theta}{3}\right) - \frac{a^2\bar{F}\theta\phi}{3(2\mu-a)} \le c \le \frac{\phi\mu}{2}\left(1 + 2\lambda + \bar{F}\theta\right), \\ 0, & \text{otherwise}, \end{cases}$$

where q^* is as given in Example 3.

Figure 5 shows how the optimal subsidy payments change with c, the unit production cost, for two values of ϕ and R_0 each, and two distributions of U; in these plots, $\lambda = 1$ and $\eta = 3.3$.

It is clear from this figure that e, the payment to the manufacturer, first increases with c, as the social planner tries to compensate the manufacturer for the higher cost of production directly, but, after c increases beyond a threshold, where it is not socially optimal to produce the vaccine, this subsidy diminishes in size. As long as f^* is kept at a constant value, the subsidy s also remains constant, and s drops to zero when f^* drops to zero. It is interesting to observe, from Figure 5, that as R_0 increases, i.e., as the negative externality effect lessens, the subsidy to the manufacturer decreases and that to consumers increases. Indeed, a higher value of R_0 implies a higher vaccinated fraction at the social optimum, hence a higher subsidy to consumers is necessary to induce the socially optimal coverage. Some of this subsidy to consumers gets transferred to the manufacturers via a higher demand, and thus the direct subsidy to the manufacturer decreases. Finally, as the vaccine becomes less effective, that is, as ϕ becomes smaller, the subsidy to consumers decreases. Two effects are at play here for the overall impact of vaccine effectiveness on the subsidies. On one hand, a less effective vaccine requires more individuals to be vaccinated to achieve the same level of immunity within the population; hence, for low c, f^* increases as ϕ decreases (see Figure 4). On the other, subsidizing a less effective vaccine represents a more wasteful measure to protect the population as more vaccinated individuals will contract the disease, especially when the vaccine is costly to produce. As a result, for high values of c, f^* is zero for a less effective vaccine, but f^* is positive for a more effective vaccine. The socially optimal subsidy program is a combination of these two effects. We find that the second effect dominates the first and the subsidy to consumers is lower for a less effective vaccine. Finally, a quick comparison of the first two panels in Figure 5 with the last two reveals that our results are quite robust to distributional assumptions about U.

In fact, our sensitivity analysis with respect to the distributional parameters of U, such as its mean and standard deviation, show that, although the results do depend on these parameters, the optimal subsidy is not very sensitive to small changes in these parameter values. In order to illustrate this, in Figure 6, we plot e as a function of the standard deviation of U for two different values of ϕ. As can be clearly seen from this figure, small inaccuracies in the estimation of the



(a) $U \sim$ Uniform$[0,2]$; $\phi = 0.8$ (b) $U \sim$ Uniform$[0,2]$; $\phi = 1.0$

(c) $U\sim$Truncated-Normal$[0,4]$; $\phi=0.8$ (d) $U\sim$Truncated-Normal$[0,4]$; $\phi=1.0$

$\lambda = 1$; $\eta = 3.3$.

Figure 5: Optimal Subsidy Payment

standard deviation of U would not much impact the effectiveness of the proposed subsidy scheme.

We now make two important observations about Theorem 3:

Remark 1. Implementing the subsidy, $s(f)$, based on the vaccine demand, f, may be challenging in practice. However, since f and w are directly related, $w = T(f) + s(f)$, one can rewrite the subsidy based on the vaccine price, w, instead. To see this, we substitute the subsidy from Theorem 3 and solve for f to obtain:

$$f = \frac{1}{1-\phi}\left(\frac{w}{\theta} - \phi F\right).$$

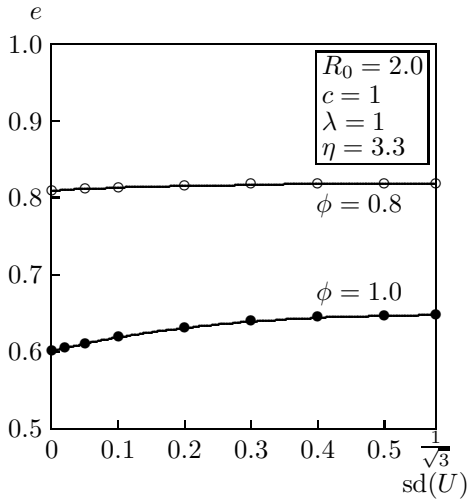


Figure 6: Optimal Subsidy, e, as a Function of Standard Deviation of U ($U \sim$ Uniform$[a, 2-a]$)

Therefore, the subsidy can be implemented based on the vaccine price as follows:

$$s(w) = \begin{cases} \frac{1}{1-\phi}\left(w - \theta\phi F\right), & 0 \le w < \theta\left(\phi F + (1-\phi)f^*\right) \\ \theta f^*, & \theta\left(\phi F + (1-\phi)f^*\right) \le w < \theta\left(\phi F + (1-\phi)\bar{F}\right) \\ 0, & w \ge \theta\left(\phi F + (1-\phi)\bar{F}\right). \end{cases}$$

Remark 2. The piecewise linear subsidy scheme described in Theorem 3 is only one of many mechanisms that can coordinate this market. In general, a subsidy $s(f)$ achieves the first best solution as long as it is a non-decreasing function of f that satisfies the following properties:

- $s'(f) \ge -T'(f)$ for $0 \le f < f^*$,
- $s'(f) \le -T'(f)$ for $f^* < f < \bar{F}$, and
- $s(f) = 0$ for $f \ge \bar{F}$.

5.2 Effect of the Subsidy Scheme on the Social Welfare

We plot the total social welfare as a function of c in Figure 7, for two different values of each R_0 and ϕ with $\lambda = 1$, and $\eta = 3.3$, when U is uniformly distributed over $[0, 2]$; the results are very similar when U follows a truncated normal distribution. In order to see how well the subsidy scheme works, we also plot the social welfare when no subsidy is provided.

It is clear from this figure that the subsidy scheme provides a significant increase in the total social welfare. We can also see that the social welfare increases with ϕ, while it decreases with R_0. This is expected. As ϕ increases, the vaccine becomes more effective, resulting in better control of the disease and higher social welfare. On the other hand, as R_0 increases, the disease becomes more contagious and the social welfare diminishes. Finally, as expected, the total social welfare decreases with c—as the cost of production increases, the optimal production quantity becomes lower and the resulting social welfare goes down.

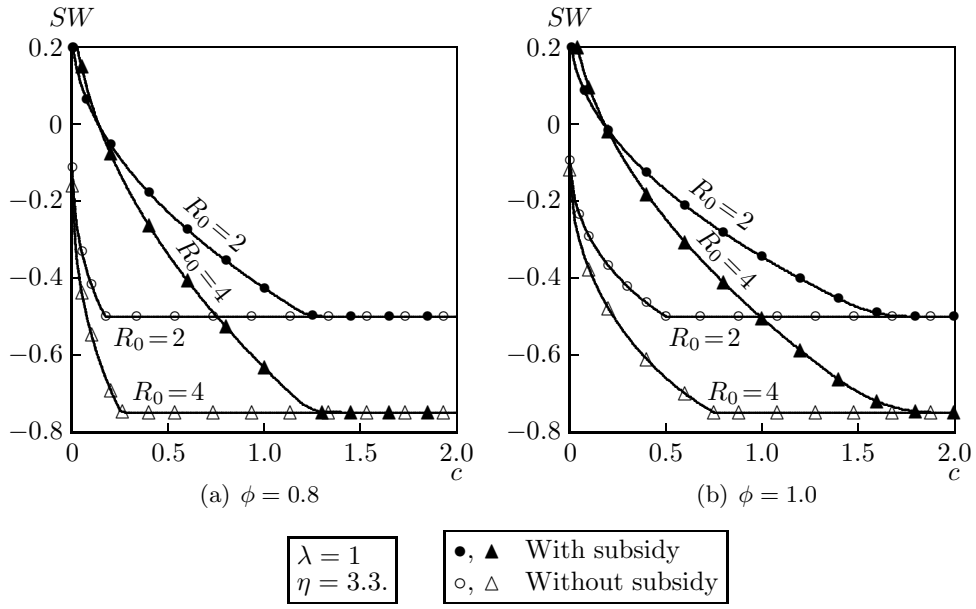


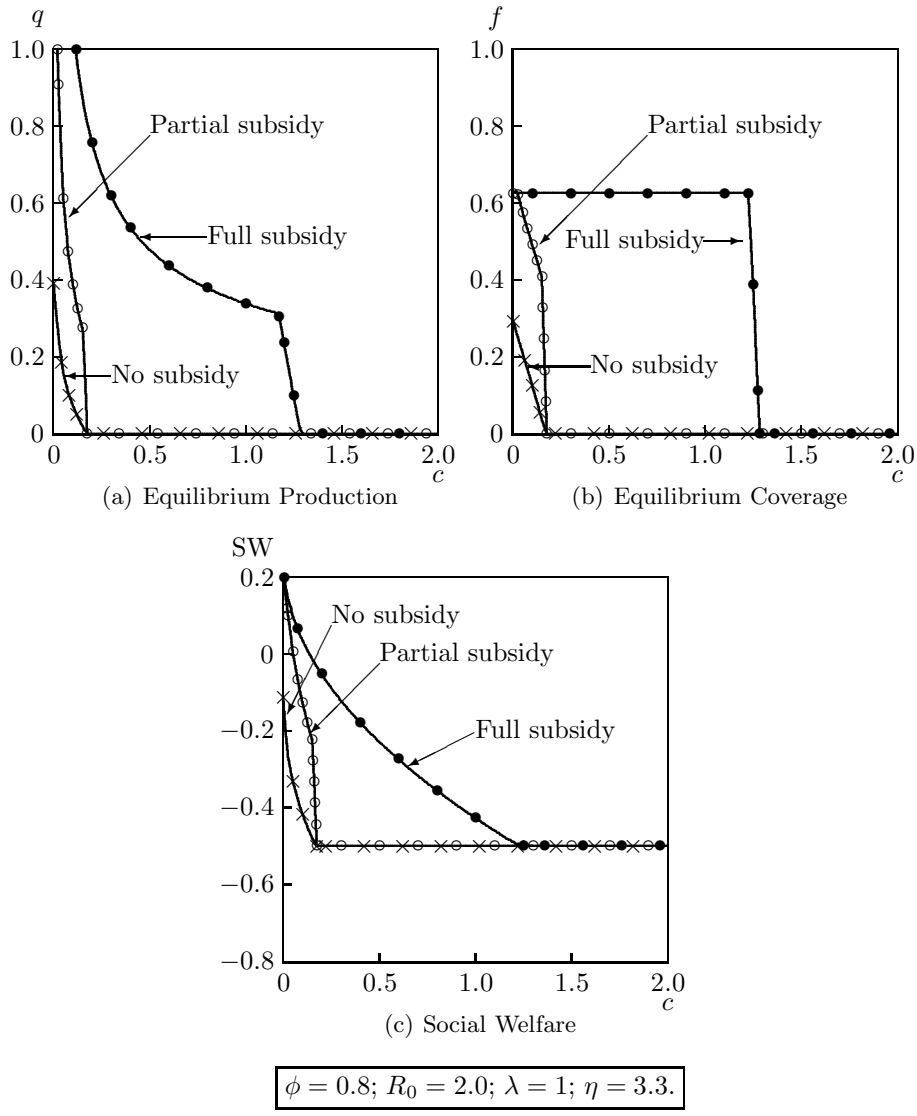
Figure 7: Total Social Welfare With and Without Subsidy ($U \sim$ Uniform$[0, 2]$)

5.3 Partial subsidy

Much of the prior literature from health economics has studied the role of subsidies to consumers only to improve vaccine coverage and consumer surplus, while ignoring the manufacturer surplus and the role of subsidies to the manufacturer, as detailed in Section 2. To further demonstrate the importance of taking into consideration the manufacturer in a subsidy program, we now investigate the effect of a subsidy program that is focused on the consumers only, ignoring the manufacturer, and that only aims at coordinating the consumer demand with that of the first-best solution.

We consider a partial subsidy program, where the manufacturer receives no subsidy, but the consumer subsidy is $s(f)$, as given in Theorem 3, and the coverage level is determined as an equilibrium outcome of this modified game. The resulting equilibrium can be easily computed in a manner similar to Theorem 1, after replacing $T(f)$ with $T(f) + s(f)$. Figure 8 depicts, for U following a uniform distribution on [0,2], the equilibrium solution and total social welfare for (i) no subsidy (i.e. the equilibrium solution found in Section 3), (ii) a partial subsidy to consumers only, and (iii) the full subsidy proposed in Section 5.1; the plots are remarkably similar for other distributions as well. Unsurprisingly, the partial subsidy offers some benefits in terms of social welfare compared with no subsidy at all.

However, the gap in social welfare between the partial subsidy and the full subsidy is still quite large. Evidently, it is critical for a social planner to take into account the manufacturer surplus when designing a subsidy program. Therefore, the need to subsidize the production of vaccines to achieve the best coordination results and social welfare benefits cannot be overlooked, as has often been done in prior literature.



(a) Equilibrium Production (b) Equilibrium Coverage

(c) Social Welfare

$\phi = 0.8$; $R_0 = 2.0$; $\lambda = 1$; $\eta = 3.3$.

Figure 8: Production, Coverage, and Social Welfare Under Different Subsidy Programs

6 Conclusions

Infectious diseases remain an important world-wide public health threat. Vaccines are available for many of these diseases and are generally considered a safe and cost-effective prevention tool. Government intervention via vaccine subsidy programs is an important weapon against the spread of infectious diseases. Without this type of programs, spontaneous market behavior leads to a vaccine coverage that is often too low to achieve herd immunity and is always less than the socially optimal level of vaccine coverage. There are two reasons for this: On the consumer side, there are negative network effects of vaccines—as more individuals get vaccinated, the chance of infection, and hence the willingness to pay for the vaccine, for an unvaccinated individual, decreases. On the supply side, production uncertainties make it too risky for manufacturers to produce sufficiently large quantities. Government intervention may improve public health by providing subsidies as incentives

to reduce these inefficiencies. In this paper, we synthesize concepts from three areas—economics, operations management, and epidemiology—to analyze the effect of the network externalities and yield uncertainties on the outcome of the monopoly market as well as the first best solution. We also derive a coordination scheme via subsidies to align the market outcome with the centralized solution. We show that a fixed one- or two-parameter subsidy structure cannot provide appropriate incentive to the consumers and the vaccine manufacturer to guarantee a socially optimal vaccine coverage that maximizes the total social welfare, but a two-part menu of subsidies can.

We find that our results are quite robust to the distributional assumptions about the production yield. This is amply illustrated in Figures 4 and 5, where we plot the socially optimal outcomes and subsidy payments, respectively, for two different distributions of the yield. In the first two panels in each figure, the yield is uniformly distributed and, in the last two, it follows a truncated normal distribution (with the same mean and variance as the uniform distribution). It can be clearly seen that the results do not seem sensitive to the distribution of the production yield; in fact, two very different types of distributions lead to very similar results for other key metrics such as equilibrium outcome (Figures 2 and 3) and social welfare (Figures 5 and 7) as well. This robustness of the equilibrium outcome, first best solution, coordinating subsidy scheme, and social welfare with respect to the distribution of the yield has important practical implications. Often, it is quite difficult to accurately estimate the distribution of an uncertain parameter such as the production yield. It is common to have good estimates for some of its characteristics, such as the mean and variance, but to expect a full knowledge of the exact distribution is often unrealistic. Our results indicate that this would not be an issue within our research context, as the findings are almost unchanged when the distribution is modified while retaining the mean and variance at their respective values. The material implication is that inaccuracies in estimating the yield distribution would not have much of an impact in the implementation of our approach.

There are several directions in which our work could be extended. For example, in this paper we consider a monopoly market with a single manufacturer. It would be interesting to investigate the impact of competition in this market. We could do this by extending our model to consider two or more competing firms in the same vaccine market. Although the algebra becomes intractable, we can obtain numerical results in that case. Second, in this study, we assume a constant unit cost of production. In reality, there are usually large fixed costs and economies of scale in vaccine production. Both these factors tend to impact the production decision in exactly the opposite manner, when compared to the impacts of negative network effect and yield uncertainty—while the former two favor a higher output, the latter two tend to induce the manufacturer to under-produce. This trade-off is an interesting topic to explore. While analytical solutions are difficult to obtain, once again, we can obtain numerical solutions for this trade-off. Furthermore, we do not address the issue of how to pay for the subsidy. In order to achieve revenue-neutrality, one would need to develop a combined tax-subsidy scheme where the taxes exactly pay for the subsidies. Finally, in practice, a big portion of the total demand for influenza vaccine is realized as pre-orders from healthcare providers, and the manufacturer caters to the current unmet demand only after meeting the pre-orders received prior to vaccine production. In this paper, we assume that the total demand

from a healthcare provider for a certain vaccine price effectively mirrors the actual demand from consumers based on demand and price elasticity information from prior periods; this translates to merging the actual consumers with healthcare providers as the "consumers" in our model. It may be interesting to extend the model to distinguish the pre-order and consumer demands. We are examining all these issues in an ongoing project to develop a more complete picture of the public policy challenges in this area.

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Electronic Companion to:
Operational Issues and Network Effects in Vaccine Markets

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Appendices

A Estimation of $r(f)$

The analysis presented in the paper, relies on an underlying epidemic model to obtain the infection probability $r(f)$. This appendix presents two variants of the textbook SIR epidemic model, one with vital dynamics and one without. We derive a closed form solution for infection probability of an SIR model with vital dynamics which gives rise to the formulas used in the paper. Unfortunately, for an SIR model without vital dynamics, there is no closed-form solution. However, we show that, for diseases of interest and for realistic values of infection model parameters, the same formula can be used as an effective approximation for the true probability function. Using the notation in the paper, a vaccine's effectiveness is denoted by a parameter ϕ, where $0 < \phi \leq 1$.

A.1 An SIR Model with Vital Dynamics (Endemic Model)

In this model, births and deaths are explicitly taken into account. The birth and death rates are equal so that the total size of the population would remain constant. Notice that such models can be used for diseases which can stay in the population for a long time period (e.g., smallpox), but may not be appropriate for diseases that can invade a population for a relatively short period of time (e.g., influenza).

A standard formulation for the SIR epidemic with vital dynamics (endemic model) uses a three-compartment model in which individuals are either susceptible to the disease (S), infectious (I), or recovered (R). The fractions of individuals in these three states at time t are represented as $S(t)$, $I(t)$, and $R(t)$, respectively. When expressed as a set of differential equations, epidemiologists tend to parameterize the endemic model by:

$$\begin{aligned} \frac{dS}{dt} &= \gamma(1-\phi f) - \beta SI - \gamma S, \\ \frac{dI}{dt} &= \beta SI - \frac{I}{\delta} - \gamma I, \text{ and} \\ \frac{dR}{dt} &= \frac{I}{\delta} + \gamma\phi f - \gamma R, \end{aligned}$$

where γ is the birth and death rate, β is the mean transmission rate, and $\delta > 0$ is the mean duration of the infection; clearly, $\gamma\delta$ is then the infectious period as a fraction of mean lifetime

Let $R_0 = \frac{\beta\delta}{\gamma\delta+1}$ be the basic reproduction number and $F = \frac{R_0-1}{\phi R_0}$ be the critical vaccination fraction. Let $\hat{S}$ and $\hat{I}$ be the number of susceptible and infected individuals (vaccinated and unvaccinated) in the population in the equilibrium. One can show that if $f \geq F$, then the system converges to the disease-free state $(\hat{S}, \hat{I}) = (1-\phi f, 0)$, whereas if $f < F$, then the system converges to the following stable endemic state $(\hat{S}, \hat{I}) = \left(1-\phi f, \frac{\phi\gamma\delta}{\gamma\delta+1}(F-f)\right)$ (Bauch and Earn 2004). As a result, the infection probability of a susceptible individual can be expressed as the proportion of susceptible individuals becoming infected versus dying in any unit of time. Therefore, the total fraction of infected individuals in the population is:

$$r(f) = (1-\phi f)\frac{R_0(\gamma\delta+1)\hat{I}}{R_0(\gamma\delta+1)\hat{I}+\gamma\delta} = 1 - \phi f - \frac{1}{R_0},$$

which is the same as the probability function presented in the paper.

A.2 An SIR Model without Vital Dynamics (Epidemic Model)

This is an epidemic model which does not include newborns or regular deaths in the population due to the relatively small window of the infection period. This model has been shown to be consistent with history of many diseases including influenza. As a result, letting $\gamma = 0$ in the above-mentioned system of differential equations leads to:

$$\begin{aligned} \frac{dS}{dt} &= -\beta SI, \\ \frac{dI}{dt} &= \beta SI - \frac{I}{\delta}, \text{ and} \\ \frac{dR}{dt} &= \frac{I}{\delta}. \end{aligned}$$

Timely vaccination followed by the onset of (instantaneous) infections from exogenous sources results in initial conditions $R(0) = f\phi$, $S(0) = (1-f\phi)(1-\chi)$, $I(0) = (1-f\phi)\chi$, where χ is the initial infected fraction of the population. As expected, this number is typically very small ($0 < \chi \ll 1$).

It can be shown that the attack rate in this case does not have an explicit form solution like the previous section. However, it can be approximated as follows (Chick et al. 2008):

$$r(f) = (1-\phi f)\left[1 - (1-\chi)e^{-R_0 r(f)}\right].$$

Moreover, it can be shown that the critical vaccination fraction for this case is the same as the one for the endemic model, i.e., $F = \frac{R_0 - 1}{R_0 \phi}$, where $R_0 = \beta\delta$ is the basic reproduction number. Chick et al. (2008) also argue that, for a small enough value of χ, the overall infected fraction of the population, $r(f)$, can be well estimated by a piecewise linear function. Using the first-order Taylor series approximation, we get:

$$r(f) = \max\left\{1 - \frac{1}{R_0} - f\phi, 0\right\},$$

which is identical to the endemic case.

B Estimation of $p(f)$

In this appendix, we extend the SIR models developed in Appendix A to account for the disease dynamic within the vaccinated and unvaccinated populations. Such compartmental models are mainly discussed in the context of HIV and HPV vaccines in the epidemiology literature (Anderson and Hanson 2005, Hughes et al. 2002, McLean and Blower 1993). In this section, we use a variant of the mentioned models to fit other diseases of interest such as influenza.

B.1 An SIR model with imperfect vaccines

As before, we consider two variants of the SIR epidemic model, one with vital dynamics and one without. Unlike for the total fraction of the infected population, we are not able to find a closed-form analytical solution for the infection probability of vaccinated population. However, we can show that a simple piecewise linear function can provide a fairly effective approximation for $p(f)$ for both scenarios.

In order to differentiate between the vaccinated and unvaccinated populations we define the following five states for the individuals at any time t. Let $S_n(t)$ be the fraction of unvaccinated susceptible individuals, $I_n(t)$ the fraction of unvaccinated infected individuals, $S_v(t)$ the fraction of vaccinated susceptible individuals, $I_v(t)$ the fraction of vaccinated infected individuals, and $R(t)$ the fraction of recovered population. With imperfect vaccination and vital dynamics, assuming that vaccination occurs at birth, the system of differential

equations characterizing the epidemic model can be written as:

$$\begin{aligned}
\frac{dS_n}{dt} &= (1-f)\gamma - \gamma S_n - \beta S_n I_n - \beta S_n I_v, \\
\frac{dS_v}{dt} &= f\gamma - \gamma S_v - \beta(1-\phi) S_v I_n - \beta(1-\phi) S_v I_v, \\
\frac{dI_n}{dt} &= \beta S_n I_n + \beta S_n I_v - \frac{I_n}{\delta} - \gamma I_n, \\
\frac{dI_v}{dt} &= \beta(1-\phi) S_v I_n + \beta(1-\phi) S_v I_v - \frac{I_v}{\delta} - \gamma I_v, \text{ and} \\
\frac{dR}{dt} &= \frac{I_n}{\delta} + \frac{I_v}{\delta} - \gamma R,
\end{aligned}$$

with the initial conditions $R(0) = 0$, $S_v(0) = 0$, $S_n(0) = 1 - \chi$, $I_v(0) = 0$, $I_n(0) = \chi$, where χ is as defined earlier.

Setting $\gamma = 0$ in the set of differential equations above would lead to the infection dynamics for an infection without vital dynamics:

$$\begin{aligned}
\frac{dS_n}{dt} &= -\beta S_n I_n - \beta S_n I_v, \\
\frac{dS_v}{dt} &= -\beta(1-\phi) S_v I_n - \beta(1-\phi) S_v I_v, \\
\frac{dI_n}{dt} &= \beta S_n I_n + \beta S_n I_v - \frac{I_n}{\delta}, \\
\frac{dI_v}{dt} &= \beta(1-\phi) S_v I_n + \beta(1-\phi) S_v I_v - \frac{I_v}{\delta}, \text{ and} \\
\frac{dR}{dt} &= \frac{I_n}{\delta} + \frac{I_v}{\delta}.
\end{aligned}$$

Timely vaccination followed by the onset of (instantaneous) infections from exogenous sources results in initial conditions $R(0) = 0$, $S_v(0) = f$, $S_n(0) = 1 - f - \chi$, $I_v(0) = 0$, $I_n(0) = \chi$, where χ is the initial infected population who initiate the epidemic.

B.2 Further Approximation of $p(f)$

While we are not able to find a closed-form solution for the infection probability of the vaccinated population, $p(f) = \left(\int_0^\infty I_v(t)dt \right) / f$, in either one of the epidemic models, we can estimate this function numerically. Based on the numerical estimates, we find that, for a suitable choice of η, the following is a reasonably close approximation of $p(f)$, both with and without vital dynamics:

$$p(f) = \eta(1-\phi) r(f).$$

In Figure A1, the actual values of $p(f)$—obtained from numerical solutions of the ODEs for the case without vital dynamics with $R_0 = 2$ and $\phi = 0.9$—are plotted as points (represented by $\times$). The approximation of $p(f)$ is drawn as a solid line. In this particular case, the best fit is obtained for $\eta = 3.3$. This figure clearly shows that our approximation is quite close to the reality. Similar plots were obtained for other values of R_0 and ϕ, as well as for the case with vital dynamics.

C Proofs

Proof of Theorem 1

First, we assume that the vaccine is perfect. Then $\phi = 1$ and the demand equation becomes $w = \bar{F} - f$.

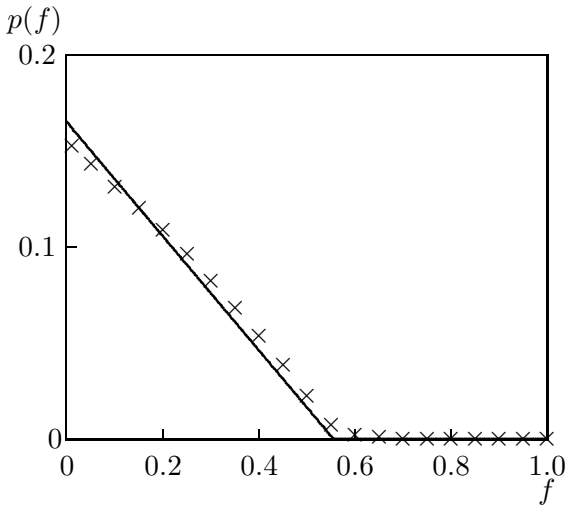

Figure A1: Actual and Approximated Values of $p(f)$ for $R_0 = 2$ and $\phi = 0.9$

Let $t = \bar{F} - f$, and $z = \frac{f}{q}$. Then, $f = \bar{F} - t$, and $q = \frac{\bar{F}-t}{z}$. Thus, we can rewrite the objective as:

$$\pi(t,z) = \frac{\bar{F}-t}{z}\left[t\int_0^z x\,dJ(x) + tz\int_z^{\bar{U}} dJ(x) - c\right].$$

Taking the first derivatives with respect to z and t, we get:

$$\begin{aligned}\frac{\partial\pi(t,z)}{\partial z} &= -\frac{\bar{F}-t}{z^2}\left[t\int_0^z x\,dJ(x) - c\right], \text{ and}\\ \frac{\partial\pi(t,z)}{\partial t} &= \frac{1}{z}\left[(\bar{F}-2t)\int_0^z x\,dJ(x) + (\bar{F}-2t)z\int_z^{\bar{U}} dJ(x) + c\right].\end{aligned}$$

Define functions $R(z)$ and $S(t)$ as follows:

$$R(z) = t\int_0^z x\,dJ(x) - c, \text{ and} \tag{A1}$$

$$S(t) = (\bar{F}-2t)\left[\int_0^z x\,dJ(x) + z\int_z^{\bar{U}} dJ(x)\right] + c. \tag{A2}$$

We will shortly prove that $R(z)$ has at most two roots between zero and its upper limit ($\bar{U}$), when t is chosen such that $S(t) = 0$. Note that the roots of $R(z)$ are the roots for $\frac{\partial\pi(t,z)}{\partial z}$ and vice versa; the same is true for $S(t)$ and $\frac{\partial\pi(t,z)}{\partial t}$. Setting $S(t) = 0$, and defining

$$\Theta(z) = \int_0^z x\,dJ(x) + z\int_z^{\bar{U}} dJ(x)$$

leads to:

$$t = \frac{1}{2}\left[\frac{c}{\Theta(z)} + \bar{F}\right].$$

Now, replace this t into the expression for $R(z)$ in Equation (A1) to obtain:

$$R(z) = \frac{1}{2}\left[\frac{c}{\Theta(z)} + \bar{F}\right] \int_0^z x\, dJ(x) - c.$$

Taking the derivative of $R(z)$, we get:

$$\begin{aligned}
\frac{dR(z)}{dz} &= \frac{1}{2}\left[\left(\frac{c}{\Theta(z)} + \bar{F}\right) z j(z) - \frac{c(1-J(z))}{\Theta^2(z)} \int_0^z x dJ(x)\right] \\
&= \frac{j(z)}{2}\left[\left(\frac{c}{\Theta(z)} + \bar{F}\right) z - \frac{c}{\Theta^2(z)\rho(z)} \int_0^z x dJ(x)\right],
\end{aligned}$$

where $\rho(z) = \frac{j(z)}{\bar{J}(z)}$ is the hazard rate of random variable U with a pdf $j(z)$, a cdf $J(z)$, and a complement cdf of $\bar{J}(z) = 1 - J(z)$. The second derivative of $R(z)$ where its first derivative is zero can, then, be found:

$$\begin{aligned}
\left.\frac{d^2R(z)}{dz^2}\right|_{\frac{dR(z)}{dz}=0} &= \frac{1}{2}\left[\frac{c\Theta(z) - cz\bar{J}(z)}{\Theta^2(z)} + \bar{F} - \frac{czj(z)}{\Theta^2(z)\rho(z)} + \frac{c[2\rho(z)\bar{J}(z) + \Theta(z)\rho'(z)]\int_0^z x dJ(x)}{\Theta^3(z)\rho^2(z)}\right] \\
&= \frac{1}{2}\left[\frac{c\int_0^z x dJ(x)}{\Theta^2(z)} + \underbrace{\bar{F} - \frac{cz\bar{J}(z)}{\Theta^2(z)}}_{\star} + \frac{c[2\rho(z)\bar{J}(z) + \Theta(z)\rho'(z)]\int_0^z x dJ(x)}{\Theta^3(z)\rho^2(z)}\right]. \qquad \text{(A3)}
\end{aligned}$$

As a result, if (A3) is positive then we can conclude that $R(z)$ can have at most two roots which is the desired result. To show this, we first prove that the portion marked as $(\star)$ in (A3) is positive:

$$\begin{aligned}
\bar{F}\Theta^2(z) - cz\bar{J}(z) &= \bar{F}\left(\int_0^z x dJ(x) + z\bar{J}(z)\right)^2 - cz\bar{J}(z) \\
&\geq \bar{F}\left(\int_0^z x\, dJ(x) + z\bar{J}(z)\right) z\bar{J}(z) - cz\bar{J}(z) \\
&= z\bar{J}(z)\left[\bar{F}\left(\int_0^z x\, dJ(x) + z\bar{J}(z)\right) - c\right]. \qquad \text{(A4)}
\end{aligned}$$

On the other hand, setting $S(t)$ to zero in (A2), we get:

$$\begin{aligned}
0 &= S(t) = (\bar{F} - 2t)\left[\int_0^z x dJ(x) + z\int_z^{\bar{U}} dJ(x)\right] + c \\
&\geq -\bar{F}\left[\int_0^z x dJ(x) + z\int_z^{\bar{U}} dJ(x)\right] + c \qquad (\text{setting } t = \bar{F}).
\end{aligned}$$

Thus, (A4) and the portion of (A3) marked as $(\star)$ are both non-negative. As a result, (A3) is non-negative for distributions that satisfy $2\rho(z)\bar{J}(z) + \Theta(z)\rho'(z) \geq 0$. Therefore the derivative of the objective function has at most two roots with the first one being the local maximum and the second one the local minimum. Next we show that the minimum is always outside of the feasible range of vaccination fractions, $f \in (0, \bar{F} - \frac{c}{\mu})$ or

equivalently $t \in (\frac{c}{\mu}, \bar{F})$. To see this, we replace $t = \frac{c}{\mu}$ in (A1):

$$R(z) = \frac{c}{\mu} \int_0^z x\, dJ(x) - c \leq 0.$$

Now, if the vaccine is imperfect, we have:

$$T(f) = \theta r(f) = \begin{cases} 0, & \text{if } f > \bar{F} \\ \theta\left(1 - \phi f - \frac{1}{R_0}\right), & \text{otherwise.} \end{cases}$$

Let $t = \theta\left(1 - \phi f - \frac{1}{R_0}\right)$, and $z = \frac{f}{q}$. Then, $f = \frac{1}{\phi}\left(1 - \frac{1}{R_0} - \frac{t}{\theta}\right)$, and $q = \frac{1}{\phi z}\left(1 - \frac{1}{R_0} - \frac{t}{\theta}\right)$. Thus, we can rewrite the objective as:

$$\pi(t, z) = \frac{1}{\phi z}\left(1 - \frac{1}{R_0} - \frac{t}{\theta}\right)\left[t \int_0^z x\, dJ(x) + tz \int_z^{\bar{U}} dJ(x) - c\right].$$

The rest of the proof proceeds similarly to the perfect case above. ■

Proof of Theorem 2

Let q be the production quantity. We first prove that the derivative of Γ with respect to f, $0 \leq f \leq \bar{F}$, is always non-positive. Using (**??**), for all f, $0 \leq f \leq \bar{F}$, we have:

$$\begin{aligned}
\frac{\partial \Gamma}{\partial f} &= -\frac{1}{2}\left(\phi(2\lambda + 1) + T(f) + f\,T'(f)\right) \int_{\frac{f}{q}}^{\bar{U}} dJ(x) \quad \leq \quad -\frac{1}{2}\left(\phi + T(f) + f\,T'(f)\right) \int_{\frac{f}{q}}^{\bar{U}} dJ(x) \\
&\leq -\frac{1}{2}\left(\phi + T(\bar{F}) + \bar{F}\,T'(\bar{F})\right) \int_{\frac{f}{q}}^{\bar{U}} dJ(x) \qquad = \quad -\frac{1}{2}\left(\phi + \bar{F}\,T'(\bar{F})\right) \int_{\frac{f}{q}}^{\bar{U}} dJ(x) \\
&\leq -\frac{1}{2}\left(\phi - \theta\phi\bar{F}\right) \int_{\frac{f}{q}}^{\bar{U}} dJ(x) \qquad \leq \quad 0.
\end{aligned}$$

Now we consider two cases: (1) $\bar{F} \leq q\bar{U}$ and (2) $\bar{F} > q\bar{U}$. For the first case using the inequality above we find $f^* = \bar{F}$. Under the second case however, for all $q\bar{U} \leq f \leq \bar{F}$, the integral and hence the derivative is zero. Defining the optimal fraction to be the smallest fraction optimizing the objective we get: $f^* = \min\{\bar{F}, \bar{U}q\}$.

Note that even though $f^* = \min\{\bar{F}, \bar{U}q\}$, in order to find the optimal q we can simply replace f by $\bar{F}$ in the $\Gamma(q, f)$ expression and achieve the same objective. Thus one can write the objective function as a function of q, only. Taking the first and second derivatives of the objective with respect to q, leads to:

$$\begin{aligned}
\frac{\partial \Gamma}{\partial q} &= -\int_0^{\frac{\bar{F}}{q}} \frac{1}{2}\left(\phi(2\lambda + 1) + T(xq) + xqT'(xq)\right) x dJ(x) + c, \text{ and} \\
\frac{\partial^2 \Gamma}{\partial q^2} &= -\int_0^{\frac{\bar{F}}{q}} \left(\frac{d}{dx}\left(T(xq) + xqT'(xq)\right)\right) \frac{x}{2} dJ(x) + \frac{1}{2}\left(\phi(2\lambda + 1) + T(\bar{F}) + \bar{F}T'(\bar{F})\right)\left(\frac{\bar{F}^2}{q^3}\right) j\left(\frac{\bar{F}}{q}\right).
\end{aligned}$$

Note that the term $\frac{d}{dx}\left(T(xq) + xqT'(xq)\right)$ is non-positive, as the function $fT(f)$ is concave. Furthermore, $\phi + T(\bar{F}) + \bar{F}T'(\bar{F}) \geq 0$ from above, making the second derivative non-negative. Thus, the first-order

condition could be used to find the optimal q. If

$$-\mu\left(\phi\lambda+\frac{\phi+T(0)}{2}\right)+c=\left.\frac{\partial\Gamma}{\partial q}\right|_{q\to 0}>0,$$

then the objective is increasing over the entire set of feasible production quantities, and hence $q^*=0$. Otherwise, q^* can be found by solving the first order condition:

$$\int_0^{\frac{\bar{F}}{q}}\frac{1}{2}\left(\phi(2\lambda+1)+T(xq)+xqT'(xq)\right)xdJ(x)=c,$$

which completes the proof. ■

Proof of Proposition 1

Given a fixed contract (s,e), we first take the derivative of the manufacturer's cost

$$\pi(q,f)=(T(f)+s)\int_0^{\frac{f}{q}}xqdJ(x)+(T(f)+s)\int_{\frac{f}{q}}^{\bar{U}}fdJ(x)-(c-e)q$$

with respect to f and q:

$$\begin{aligned}\frac{\partial\pi(q,f)}{\partial q} &= (T(f)+s)\int_0^{\frac{f}{q}}xdJ(x)-(c-e),\ \text{and}\\ \frac{\partial\pi(q,f)}{\partial f} &= T'(f)q\int_0^{\frac{f}{q}}xdJ(x)+T'(f)\int_{\frac{f}{q}}^{\bar{U}}fdJ(x)+(T(f)+s)\int_{\frac{f}{q}}^{\bar{U}}dJ(x).\end{aligned}$$

To show that the contract is coordinating, we should demonstrate that $f=f^*$ and $q=q^*$ satisfy the optimality conditions. Showing the optimality of $q=q^*$ for $f=f^*$ leads to:

$$\left.\frac{\partial\pi(q,f^*)}{\partial q}\right|_{q=q^*}=(T(f^*)+s)\int_0^{\frac{f^*}{q^*}}xdJ(x)-(c-e)=0.$$

Next, to show that a fixed two-part subsidy cannot coordinate this market we consider two cases: (i) $\bar{F}\geq\bar{U}q^*$, and (ii) $\bar{F}<\bar{U}q^*$.

(i) Suppose that $f^*=\bar{U}q^*\leq\bar{F}$. Then taking the left derivative of manufacturer's profit at f^* leads to

$$\begin{aligned}\left.\frac{\partial\pi(q^*,f)}{\partial f}\right|_{f=(f^*)^-} &= \left(T'(f)q^*\int_0^{\frac{f}{q^*}}xdJ(x)+T'(f)\int_{\frac{f}{q^*}}^{\bar{U}}fdJ(x)+(T(f)+s)\int_{\frac{f}{q^*}}^{\bar{U}}dJ(x)\right)\Bigg|_{f=(f^*)^-}\\ &= T'(\bar{U}q^*)q^*\int_0^{\bar{U}}xdJ(x).\end{aligned}$$

Therefore, the left derivative is negative and the optimal value of f under the contract will not be the same as $f^*=\bar{U}q^*$.

(ii) Suppose that $f^* = \bar{F} < \bar{U}q^*$. Then taking the right derivative of manufacturer's profit at f^* leads to

$$\begin{aligned}\left.\frac{\partial \pi(q^*, f)}{\partial f}\right|_{f=(f^*)^+} &= \left(T'(f)q^* \int_0^{\frac{f}{q^*}} x dJ(x) + T'(f) \int_{\frac{f}{q^*}}^{\bar{U}} f dJ(x) + (T(f)+s) \int_{\frac{f}{q^*}}^{\bar{U}} dJ(x) \right)\Bigg|_{f=(f^*)^+} \\ &= s \int_{\frac{f^*}{q^*}}^{\bar{U}} dJ(x).\end{aligned}$$

Therefore, for any non-zero subsidy to consumers, the right derivative will be positive and the optimal value of f under the contract will not be the same as $f^* = \bar{F}$. ■

Proof of Theorem 3

We first take the derivative of

$$\pi(q, f) = (T(f) + s(f)) \int_0^{\frac{f}{q}} xq dJ(x) + (T(f) + s(f)) \int_{\frac{f}{q}}^{\bar{U}} f dJ(x) - (c - e)q$$

with respect to f and q:

$$\begin{aligned}\frac{\partial \pi(q, f)}{\partial q} &= (T(f) + s(f)) \int_0^{\frac{f}{q}} x dJ(x) - (c - e), \text{ and} \\ \frac{\partial \pi(q, f)}{\partial f} &= (T'(f) + s'(f))q \int_0^{\frac{f}{q}} x dJ(x) + (T'(f) + s'(f)) \int_{\frac{f}{q}}^{\bar{U}} f dJ(x) + (T(f) + s(f)) \int_{\frac{f}{q}}^{\bar{U}} dJ(x).\end{aligned}$$

We demonstrate that $f = f^*$ and $q = q^*$ in (**??**) satisfy the optimality conditions. Showing the optimality of $q = q^*$ for $f = f^*$ is straightforward as:

$$\left.\frac{\partial \pi(q, f^*)}{\partial q}\right|_{q=q^*} = (T(f^*) + s(f^*)) \int_0^{\frac{f^*}{q^*}} x dJ(x) - (c - e) = 0$$

Next, we show that f^* is the optimal vaccine demand for $q = q^*$. For this purpose, we show that $\left.\frac{\partial \pi(q^*, f)}{\partial f}\right|_{f<f^*} > 0$ and $\left.\frac{\partial \pi(q^*, f)}{\partial f}\right|_{f<f^*} \leq 0$:

$$\begin{aligned}\left.\frac{\partial \pi(q^*, f)}{\partial f}\right|_{f<f^*} &= (T'(f) + s'(f))q^* \int_0^{\frac{f}{q^*}} x dJ(x) + (T'(f) + s'(f)) \int_{\frac{f}{q^*}}^{\bar{U}} f dJ(x) + (T(f) + s(f)) \int_{\frac{f}{q^*}}^{\bar{U}} dJ(x) \\ &= \theta(1 - \phi)q^* \int_0^{\frac{f}{q^*}} x dJ(x) + \theta(1 - \phi) \int_{\frac{f}{q^*}}^{\bar{U}} f dJ(x) + (T(f) + s(f)) \int_{\frac{f}{q^*}}^{\bar{U}} dJ(x) > 0,\end{aligned}$$

as $s(f) = \theta f$ by the definition of subsidy in this region. On the other hand,

$$\left.\frac{\partial \pi(q^*, f)}{\partial f}\right|_{f>f^*} = \begin{cases} -\theta\phi q^*\mu & ; \text{ if } \bar{U}q^* < \bar{F} \\ 0 & ; \text{ if } \bar{U}q^* \geq \bar{F} \end{cases} \leq 0.$$

Note that $f^* = \bar{F}$ in the case of $\bar{U}q^* \geq \bar{F}$. Therefore, $T(f) = T'(f) = s(f) = s'(f) = 0$ for all $f > f^*$. Furthermore, in the case of $f^* = \bar{U}q^* < \bar{F}$, by the definition of the subsidy we have $T'(f) + s'(f) = T'(f) = -\theta\phi$ for $f > f^*$. As a result $f = f^*$ when $q = q^*$ leading to the desired result. ■

D Notational Summary

Table D1: Notational Symbols and Their Meanings

Symbol	Meaning
c	Marginal production cost of a vaccine
e	Production subsidy per vaccine
$J(\cdot)$, $j(\cdot)$	Probability distribution and density functions of U
f	Vaccine coverage level
F, $\bar{F}$	Critical vaccination fraction; $\bar{F} = \min\{F, 1\}$
g	Realized coverage level; $g = \min\{Uq, f\}$
L	Maximum possible cost from an infection to a consumer with $u = 1$
$p(f)$, $P(f)$	Infection probability for the vaccinated and unvaccinated populations
q	Normalized target production level
$r(f)$	Infection probability for the entire population
R_0	Basic reproduction number
s	Subsidy given to each vaccinated consumer
SC	Realized total social cost
SW	Expected total social welfare
$T(f)$	$T(f) = r(f) - p(f)$
u	Index of consumers' relative cost from an infection
$\bar{u}$	Index of a consumer indifferent between getting or not getting the vaccine
U	Fractional yield; $0 < U \leq \bar{U}, \mathbb{E}[U] = \mu$
W, w	(Normalized) price of a vaccine; $w = \frac{W}{L}$

Symbol	Meaning
Γ	Expected total social cost
η	A constant; $0 \leq \eta \leq \frac{1}{1-\phi}$
θ	$\theta = 1 - \eta(1 - \phi)$
λ	Normalized loss to the society per infection
μ	$\mu = \mathbb{E}[U]$
Π , π	Realized and expected profit of the manufacturer
$\rho(\cdot)$	Hazard rate; $\rho(z) = \frac{j(z)}{1-J(z)}$
σ_M	Manufacturer's expected surplus
σ_V, σ_N	Expected surplus for the vaccinated and unvaccinated consumers
σ_S	Expected societal surplus
ϕ	Effectiveness of a vaccine

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