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UNIVERSITY OF CALIFORNIA  
RIVERSIDE

Essays on Estimation of Technical Efficiency and on  
Choice Under Uncertainty

A Dissertation submitted in partial satisfaction  
of the requirements for degree of

Doctor of Philosophy

in

Economics

by

Aditi Bhattacharyya

June 2009

Dissertation Committee:

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2009

The Dissertation of Aditi Bhattacharyya is approved:

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To Baba and Ma

## ABSTRACT OF THE DISSERTATION

Essays on Estimation of Technical Efficiency and on  
Choice Under Uncertainty

by

Aditi Bhattacharyya

Doctor of Philosophy, Graduate Program in Economics  
University of California, Riverside, June 2009  
Professor R. Robert Russell and Professor Aman Ullah, Co-Chairpersons

In the first two essays of this dissertation, I construct a dynamic stochastic production frontier incorporating the sluggish adjustment of inputs, measure the speed of adjustment of output in the short-run, and compare the technical efficiency estimates from such a dynamic model to those from a conventional static model that is based on the assumption that inputs are instantaneously adjustable in a production system. I provide estimation methods for technical efficiency of production units and the speed of adjustment of output for cases when they are time-invariant and when they vary with time. I also apply the methods to a panel dataset on private manufacturing establishments in Egypt.

The dynamic frontiers with time-invariant and time-varying technical efficiency are estimated using the System Generalized Method of Moments estimator and the Generalized Least Squares estimator with instrumental variables, respectively. The



results for the Egyptian private manufacturing sectors show that the speed of adjustment of output is significantly lower than unity, the static model underestimates technical efficiency on average, and the dynamic model captures more variation in the time pattern of technical efficiency. Further, the ranking of production units based on their technical efficiency measures changes when the lagged adjustment process of inputs is taken into account.

In another essay, I characterize a class of rules for decision-making under the type of non-probabilistic uncertainty that was first axiomatically analyzed by Arrow and Hurwicz (1972). In this framework, the agent knows the possible states of the world and the outcome of each of her actions for each state, but does not have any information about the probabilities with which each state occurs. The decision-making rules characterized in this essay focus on the outcome(s) which occupy the middle position(s), when all outcomes of an action under different states of the world are arranged according to the agent's preference ordering defined over the outcomes. The existing literature in the Arrow-Hurwicz framework has mainly considered 'max'-based or 'min'-based rules and their variants, which reflect rather extreme forms of optimism or pessimism on the part of an agent. In contrast, the results of this essay characterize a decision rule that reflects a more 'balanced' attitude of the agent.

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# Chapter 1

## Introduction and Research Objectives

Measuring efficiency of production units is an important aspect of productivity analysis. The concept of stochastic production frontier introduces a plausible approach for estimating technical efficiency of production units under different assumptions about the production model. The standard stochastic production frontier identifies two main sources for the deviation of actual output from the maximum possible output, given the inputs. A part of this deviation is attributed to the symmetric random shocks to a production system that are not under the control of a producer (e.g., uncertainty about the weather, and input market conditions). The other reason for the failure to produce the potential output, given a set of inputs, is the presence of technical inefficiency caused by factors such as managerial error and coordination failures. Accordingly, a firm is said to be technically inefficient if it produces below the production frontier, and the corresponding technical inefficiency is measured by the deviation of the actual output from the frontier, after accounting for the random shocks to the system.

Based on advanced econometric techniques for both cross sectional and panel data models, this literature has grown to analyze technical efficiency of production units under different assumptions for the production model. Most of the existing studies on stochastic frontiers and technical efficiency focus on the static analysis of a producer's behavior. The static production frontier analysis assumes that the inputs are instantaneously adjustable within a production system. Accordingly, any shortfall from

the maximum possible output is attributed to random shocks to the production system and the technical inefficiency of the production unit.

However, an input may take time to adjust within a production system in the short-run and, during the period of adjustment, the actual output may not reach the maximum possible level, even in the absence of any other sources of inefficiency in the production system. The factors of production may take time to adjust because of different contractual bindings as well. Moreover, it may be too costly to change the amount of certain inputs in the short-run, and this may result in lower output than the optimal one. Further, not only the adjustment process of inputs, but also the changes in demand for output and the expectation about future economic conditions may lead to a sub-optimal output production. Therefore, producing at a level lower than the maximum possible level may not always result from inefficiency of a production unit, and it may be a conscious choice of the producer.

By assuming instantaneous adjustment of all inputs, a static production model fails to capture the short-run dynamics of a production process. As a result, the static model is likely to either underestimate or overestimate technical efficiency of production units when actual output is lower than the optimal level due to short-run quasi-fixity of inputs. However, the direction of bias in technical efficiency estimates from a static model is not uniquely justified using a stochastic production frontier approach as this approach only identifies technical efficiency of each production unit relative to the best-practice frontier. Since the short run behavior of a producer during the initial phase of adjustment can be substantially different from those occurring once the long run

equilibrium has been attained, a dynamic production model that allows for the lagged adjustment of inputs is a more suitable one to estimate efficiency of production units in the short-run.

One of the main objectives of this dissertation is to construct a dynamic stochastic production frontier incorporating the sluggish adjustment of inputs and to provide an estimation method to measure the speed of adjustment and technical efficiency of production units from such a dynamic frontier. When the inputs need time to adjust within a production system, or if the costs of adjustment of the quasi-fixed inputs are too high, the change in the actual output between two time periods is likely to be a fraction of the desired change in output. The gap between the actual change in output and the desired change is determined by the rate of adjustment of inputs. Based on this idea, I construct a dynamic production frontier in chapter 2 that shows that output of a production unit in any time period is not only determined by the level of inputs used in that period, but also by the output that was produced in the last period. The proposed dynamic production frontier is then estimated using the System Generalized Method of Moments estimator for dynamic panel data models, and the speed of adjustment of output and producer specific technical efficiency is estimated accordingly.

The dynamic production model as discussed in chapter 2 can further be extended and formulated under less restrictive assumption regarding the production process. In reality, as inputs get more familiar with a production system, their speed of adjustment is likely to improve. As a result, the deviation of actual change in output from the desire change is also likely to vary over time. More specifically, it is likely that the gap between



the actual change and the desired change in output will fall over time. Further, if we study a production behavior for a substantially long time period, and the economic structure is sufficiently competitive, then the inefficiency effect of a production unit is also likely to change over time. Therefore, in chapter 3, I formulate a dynamic production frontier with time-varying speed of adjustment of output and technical efficiency of production units. I use the Generalized Least Squares estimator with instrumental variable to estimate the frontier and extend the existing econometric methods for dynamic panel data model with time-varying individual effects to suit my purpose of efficiency estimation.

Since the basic idea behind incorporating the lagged adjustment of inputs while measuring efficiency of production units is to identify the true efficiency in the short-run that is not recognized by the static models, I compare the efficiency estimates from the proposed dynamic model with time-invariant and time-varying technical efficiency with the corresponding static versions of such models as well. The existing literature in this area does not identify the possible difference in the efficiency estimates from a dynamic and a static production model. Also, estimation of technical efficiency and speed of adjustment, both of which may vary over time, has not been discussed so far in the context of a dynamic production model.

I apply the proposed dynamic model and estimation methods on a panel dataset on private manufacturing establishments in Egypt from the Industrial Production Statistics of the Central Agency for Public Mobilization and Statistics (CAPMAS). Estimation of technical efficiency and ranking of production units according to their efficiency levels are important aspects of productivity analysis, based on which,

producers often take critical decisions about their production plans. The next two chapters of this dissertation provide a more realistic and rigorous approach for capturing the dynamics of a production system, and measuring technical efficiency accordingly. A dynamic production model is particularly suitable for a country like Egypt, where sluggish adjustment of inputs is a very plausible phenomenon, in light of the facts that during the period under consideration Egypt employed unskilled and semi-skilled labor in the manufacturing sectors and also underwent through several structural changes in those sectors.

Another objective of this dissertation, though not directly linked to the previous ones, is to characterize median-based rules for decision making under uncertainty. Thus, in chapter 4, I characterize a class of rules for decision-making under the type of non-probabilistic uncertainty that was first axiomatically analyzed by Arrow and Hurwicz (1972). In this framework, the agent knows the possible states of the world and the outcome of each of her actions for each state, but does not have any information about the probabilities with which each state occurs. The decision-making rules characterized in this chapter focus on the outcome(s) which occupy the middle or median position(s), when all outcomes of an action under different states of the world are arranged according to the agent's preference ordering defined over the outcomes.

The existing literature in the Arrow-Hurwicz framework has mainly considered 'max'-based or 'min'-based rules and their variants that reflect rather extreme forms of optimism or pessimism on the part of an agent. In contrast, I characterize a decision rule that reflects a more 'balanced' attitude of the agent. In light of the agent's usually limited

capacity for processing information, it seems intuitively plausible to assume that an agent, when confronted with the problem of choice under uncertainty, may concentrate on some ‘focal’ outcomes for each action. It is, however, not clear why the agent will necessarily look only at the extreme outcomes, i.e., the best or worst outcomes, of each action. An alternative focal point for each action may be its median outcome(s). Though decision rules based on the median outcome(s) seem to have considerable intuitive plausibility, the structure of these rules in the Arrow-Hurwicz framework has not been explored so far. I aim to fill this gap in the literature by providing an axiomatic characterization of a class of median-based decision rules for choice under non-probabilistic uncertainty of the Arrow-Hurwicz type.

The remainder of this dissertation is organized as follows. Chapter 2 discusses a dynamic production frontier and measures time-invariant technical efficiency and speed of adjustment of output of private manufacturing sectors of Egypt using the proposed dynamic production model. Chapter 3 further extends the dynamic production frontier discussed in chapter 2 to incorporate time-varying speed of adjustment and technical efficiency of Egyptian private manufacturing sectors and provides econometric methods to estimate the model. Chapter 4 characterizes median-based rules for decision making under complete ignorance, and chapter 5 summarizes and concludes the dissertation. Tables and Figures presenting the results from the analyses in chapter 2 and 3 are included in the Appendix A and B at the end of each chapter.

## **Chapter 2**

### **Adjustment of Inputs and Measurement of Time-Invariant**

#### **Technical Efficiency: A Dynamic Panel Data Analysis**

##### **2.1. Introduction**

Production frontier estimation and the measurement of technical efficiency of production systems have been important areas of research for more than half a century. Following the pioneering work of Aigner, Lovell, and Schmidt (1977) and Meeusen and Broeck (1977), who independently proposed the estimation of stochastic production frontier, this field has further grown with important contributions by many researchers (see Schmidt and Lovell (1979), Jondrow et al. (1982)). These studies have posited two main causes for the deviation of actual output from the maximum possible output (potential output), given the inputs. A part of this deviation is attributed to the symmetric random shocks to a production system that are not under the control of a producer (e.g., uncertainty about the weather, and input market conditions). The other reason for the failure to produce the potential output, given a set of inputs, is the presence of technical inefficiency caused by factors such as managerial error and coordination failures. Accordingly, a firm is said to be technically inefficient if it produces below the production frontier, and the corresponding technical inefficiency is measured by the

deviation of the actual output from the frontier, after accounting for the random shocks to the system.

Based on this concept, the literature has expanded to include both time-invariant and time-varying technical efficiency measures (see Cornwell, Schmidt and Sickles (1990); Kumbhakar (1990); Kumbhakar (1991); Battese and Coelli (1992); Lee and Schmidt (1993); Ahn, Lee, and Schmidt (1994); and Kumbhakar, Heshmati, and Hjalmarsson (1997)), as well as cross sectional and panel data models of stochastic frontier estimation (see Schmidt and Sickles (1984)). A general discussion on the measurement of productive efficiency and the related literature can be found in Lovell (1996), Kumbhakar and Lovell (2000), and Coelli et al. (2005).

Most of the existing studies on stochastic frontiers and technical efficiency focus on the static analysis of a producer's behavior and, therefore, fail to capture the dynamic nature of a firm's optimization process. In other words, these studies assume that when a unit of input is introduced into the production system, it immediately contributes to production at its maximum possible level. Accordingly, any shortfall from the potential output is attributed to random shocks and the technical inefficiency of the production unit. However, once introduced to a production system, an input may require some time for adjustment within the system. For example, consider a firm using labor and capital as inputs to produce a single output. If new capital is introduced, then the existing labor force has to be reassigned to the new capital stock, and during this process, output cannot be produced at the maximum level. Similarly, a newly hired labor unit or an employee will take time to get familiar with the production process. The factors of production may

take time to adjust because of different contractual bindings as well. Given such a process of adjustment of inputs in the short-run, it may not be possible for a firm to catch up with the production frontier instantaneously following the introduction of a new input, even in the absence of any other source of inefficiency. A vast literature on the source, structure, size and specification of adjustment costs (Lucas (1967a, 1967b); Treadway (1971); and Hamermesh and Pfann (1996)) has established the importance of such an adjustment process in the theory of production.

Evidently, behind the productivity change of a firm, a dynamic process is likely to be at work in terms of input adjustment. This dynamic adjustment process is a natural phenomenon of any production system and thus, the shortfall in the output that results from the dynamic adjustment of inputs does not really represent inefficiency of the production unit. The adjustment process of inputs is rather an inherent characteristic of any production system that cannot completely be controlled by the producers. Therefore, the shortfall in the actual output that results from the sluggish adjustment of input should not be considered as technical inefficiency of the production unit. In reality, the total deviation of actual output in the short-run can be attributed to three sources, viz. presence of the random shocks, presence of any inefficiency within the production unit, and the lagged adjustment of input that may prohibit a production unit from reaching the maximum possible output level.

The static production frontier model assumes all inputs are instantaneously adjustable, and thus ignores the important impacts of short-run fixity of certain inputs. As a result, a static production model attributes the shortfall in production to the inefficiency

of the production unit and random shocks, even if it is caused by the internal adjustment of inputs. However, depending on the adjustment costs for quasi-fixed inputs, the production units also adjust capacity utilization. Further, when there is any shock to the production system, the short-run behavior of a firm during the initial phase of adjustment can be substantially different from those occurring once the long run equilibrium has been attained. Therefore, the short-run quasi-fixity of inputs, changes in the demand for output, and also the expectation about future economic conditions may lead to a level of capacity utilization that is lower than the long run static equilibrium level (see Berndt and Fuss (1986), Morrison (1986)). Hence, in the presence of lagged adjustment of inputs, a static production frontier model that ignores the effect of input adjustment on output may misspecify the process of output generation. Consequently, technical efficiency measures from such a misspecified model are likely to be biased.

Little work has so far been done to incorporate the dynamic adjustment process of inputs while measuring technical efficiency of a production unit. Among studies that have considered dynamic models for technical efficiency using panel data, Ahn, Good, and Sickles (2000) allow for the lagged adjustment of inputs to explain the autoregressive nature of the technical efficiency component that varies with time. They also measure the speed of sluggish adoption of technological innovations and the associated efficiency loss. However, sluggish adjustment of inputs not only affects the adoption of technological innovations, but can also affect the whole production process by restricting output from reaching its maximum possible level. Further, ranking of the production units are also likely to be altered in the presence of lagged adjustment of inputs. Ayed-Mouelhi

and Goaid (2003) follow Ahn, Good, and Sickles (2000) to measure technical efficiency of Tunisian textile, clothing, and leather industries. Another study by Kumbhakar, Hesmati, and Hjalmarsson (2002) formulates the input requirement frontier for a firm in a dynamic set-up, but it does not shed any light on specifying a stochastic frontier for measuring technical efficiency and the speed of adjustment of inputs, when inputs need time for adjustment. Finally, a recent paper by Asche, Kumbhakar, and Tveteras (2008) formulates the dynamic profit maximization process of a firm and test whether there are adjustment costs associated with the inputs. However, their study does not discuss the effect of such dynamic optimization on the firms' technical efficiency in the short-run.

The objective of this chapter is to measure a firm's technical efficiency in the presence of lagged adjustment of inputs. Specifically, this chapter presents a dynamic, stochastic production frontier incorporating the lagged adjustment of inputs, and also compares the estimates of time-invariant technical efficiency of production units from such a dynamic model with the estimates from a static production model that assumes instantaneous adjustment of all inputs. For this purpose, I use a panel dataset on private manufacturing establishments in Egypt from the Industrial Production Statistics of the Central Agency for Public Mobilization and Statistics (CAPMAS).

The remainder of this chapter is organized as follows. The main theoretical and econometric models are presented in sections 2.2 and 2.3, respectively. Section 2.4 elaborates on the estimation methods. Results from the empirical analysis are described in section 2.5, and finally, section 2.6 presents concluding remarks.



## 2.2. Theoretical Model

Let  $y_{it}^*$  be the maximum possible production level of firm  $i$  that uses a vector of inputs  $X_{it}$  at time  $t$ . However, after introduction of inputs, it is logical to have a time lag before they produce at their maximum possible level. Therefore, because of the underlying adjustment process of inputs, it is likely that a newer input will contribute less to the output than the older ones. Accordingly, the actual output of firm  $i$  at time  $t$ , given by  $y_{it}$ , is determined by the speed of adjustment of inputs and the history of input usage. Thus, the actual output is a function of the current and past input levels, and the speed of adjustment of inputs.

Let  $\lambda$  ( $0 \leq \lambda \leq 1$ ) be the speed of adjustment of inputs. I assume that  $\lambda$  is constant over time and identical for all inputs and for every production unit. Then the actual output of firm  $i$  at time  $t$  is given by-

$$y_{it} = f(\lambda, X_{it}, X_{it-1}, X_{it-2}, \dots) \quad (2.2.1)$$

The change in actual output between any two periods is a combined result of contribution of new inputs, a part of which is adjusted during the period, and contribution of part of the old inputs that adjusts in that period. Therefore, during the adjustment process of inputs, the current output  $y_{it}$  is likely to be higher than  $y_{i(t-1)}$  but lower than  $y_{it}^*$ , when  $y_{it}^*$  is increasing over time, and the actual change in output is likely to be a fraction of the change in output that is needed to catch up with the potential output at any given time period. Let us refer to the change in output that is needed in any period to catch up with the potential output, as the ‘desired change’ in output. Further, the

difference between the actual and the desired change in output is likely to depend on the speed of adjustment of inputs. In other words, the dynamic production process of output generation can be represented by the partial adjustment scheme -

$$y_{it} - y_{i(t-1)} = \lambda(y_{it}^* - y_{i(t-1)}) \quad (2.2.2)$$

If the speed of adjustment is lower than unity, then the change in actual output will be lower than the desired change. Moreover, the higher is the speed of adjustment of inputs, the lower is the deviation of the desired change in output from the actual change, and the desired change in output is exactly similar to the actual change when the speed of adjustment is unity, i.e., inputs are instantaneously adjusted in the production system. To further analyze the production model, let us consider a Cobb-Douglas function for the production of potential output<sup>1</sup>-

$$\ln y_{it}^* = \beta_0 + \sum_{m=1}^M \beta_m \ln x_{mit} + \sum_{t=2}^T \delta_t D_t, \quad (2.2.3)$$

where  $i = 1, \dots, N$  denotes the production unit,  $t = 1, \dots, T$  represents the time periods,  $m = 1, \dots, M$  represents the inputs used in production,  $\beta_m$  is the marginal effect of the  $m$ th input on the potential output, and  $\beta_0$  is the intercept of the potential production frontier. I introduce the time dummy variables  $D_t$  in the production model to incorporate the pure technological change as proposed by Baltagi and Griffin (1988). Thus, no specific structure is imposed on the behavior of the technological change.  $\delta_t$  captures the effect of technological changes on the potential output. The partial adjustment scheme for output generation is then given by-

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<sup>1</sup> The analysis is valid for more general production functions.

$$\ln y_{it} = (1-\lambda) \ln y_{i(t-1)} + \lambda(\beta_0 + \sum_{m=1}^M \beta_m \ln x_{mit} + \sum_{t=1}^T \delta_t D_t) \quad (2.2.4)$$

Using (2.2.4) for output produced in each period, the partial adjustment scheme of output as given in (2.2.2) can further be restated as follows-

$$\ln y_{it} = \lambda \ln y_{it}^* + (1-\lambda)(\lambda \ln y_{i(t-1)}^* + (1-\lambda) \ln y_{i(t-2)})$$

$$\text{or, } \ln y_{it} = \lambda \beta_0 + \lambda(1-\lambda)\beta_0 + \lambda(1-\lambda)^2 \beta_0 + \lambda(1-\lambda)^3 \beta_0 + \dots + \lambda \sum_{m=1}^M \beta_m \ln x_{mit} +$$

$$\lambda(1-\lambda) \sum_{m=1}^M \beta_m \ln x_{mi(t-1)} + \lambda(1-\lambda)^2 \sum_{m=1}^M \beta_m \ln x_{mi(t-2)} + \lambda(1-\lambda)^3 \sum_{m=1}^M \beta_m \ln x_{mi(t-3)} + \dots$$

$$+ \lambda \delta_2 D_2 + \lambda(1-\lambda) \delta_2 D_2 + \lambda(1-\lambda)^2 \delta_2 D_2 + \lambda(1-\lambda)^3 \delta_2 D_2 + \dots$$

$$+ \lambda \delta_3 D_3 + \lambda(1-\lambda) \delta_3 D_3 + \lambda(1-\lambda)^2 \delta_3 D_3 + \lambda(1-\lambda)^3 \delta_3 D_3 + \dots \quad (2.2.5)$$

Therefore, the partial adjustment scheme for actual output at time  $t$  demonstrates that the current output depends on the current and past inputs, and on the speed of adjustment of inputs. A fraction,  $\lambda$ , of an input  $x_{mi(t-k)}$  introduced by firm  $i$  in the period  $t-k$  ( $0 < k < t$ ), contributes to the output in that period. In period  $t-k+1$ ,  $\lambda$  fraction of the remaining  $(1-\lambda)x_{mi(t-k)}$  adds to the output, and again  $\lambda$  fraction of the unadjusted  $(1-\lambda)^2 x_{mi(t-k)}$  contributes to output in  $t-k+2$ . Following this process,  $\lambda$  fraction of  $(1-\lambda)^k x_{mi(t-k)}$  contributes to output at time  $t$ . Therefore, the marginal effects of current inputs on the current output are higher than those for the inputs from previous periods. With a speed of adjustment that is less than unity, these marginal effects are declining in a geometric progression for inputs introduced in previous periods. In other words, the most recent past of input usage receives the greatest weight in determining the current

output, and influence of past inputs will fade out uniformly with the passage of time. Therefore, the distant past receives arbitrarily small weight.

Further, following Kennan (1979), I can draw implications of rational expectation equilibrium from the partial adjustment scheme of output as specified in our framework. Under the rational expectation hypothesis, it can be shown that the solution to a problem that minimizes a quadratic loss function, results in a partial adjustment model for the current output. The total loss in output is generated by the loss due to deviation of current output from the potential output, and the loss due to the lagged adjustment of inputs.

### **2.3. Econometric Model**

The potential output is a hypothetical characterization of the maximum possible output and is not observed in reality. The actual output is generally above or below the potential output because a production system is exposed to random shocks that may positively or negatively affect production plans. Moreover, a production unit is likely to suffer from technical inefficiency that may lower the actual output. The stochastic version of (2.2.2), which is more realistic, considers a composite error term that accounts for the random shocks to a production unit, and the technical inefficiency of that unit. I obtain the stochastic versions of the dynamic output generation process (2.2.2) by considering a composite error term ( $\varepsilon_{it}$ ) consisting of symmetric random shocks  $v_{it}$  to firm  $i$  at time  $t$ , and the producer specific effects,  $u_i$ , that determine the technical efficiency of each production unit and are constant over time.

Therefore, the dynamic stochastic production frontier that incorporates the sluggish adjustments of inputs and time-invariant technical inefficiency, is given by-

$$\ln y_{it} = (1 - \lambda) \ln y_{i(t-1)} + \lambda(\beta_0 + \sum_{m=1}^M \beta_m \ln x_{mit} + \sum_{t=2}^T \delta_t D_t) + \varepsilon_{it}, \quad (2.3.1)$$

where  $i = 1, \dots, N$ ,  $t = 1, \dots, T$ ,  $m = 1, \dots, M$ . In (3.1),  $\varepsilon_{it} = v_{it} - u_i$ , and  $u_i \geq 0$  captures the producer specific, time-invariant, non-negative inefficiency effects for production unit  $i$  with  $E(u_i) = \mu$ , and variance  $\sigma_u^2$ .  $v_{it}$  are the random shocks to the production unit  $i$  at time  $t$ , with zero mean and variance  $\sigma_v^2$ . I further assume that  $\lambda\beta_0 - \mu = \beta_0^*$ ,  $u_i^* = u_i - \mu$  such that  $u_i^* \sim iid(0, \sigma_u^2)$ . The time dummies,  $D_t$ , have value equals unity for year  $t$  and zero otherwise. The standard structure of the error component as discussed in Blundell and Bond (1998) is also maintained as follows-

1.  $u_i^*$  is uncorrelated with  $v_{it}$ , i.e.  $E(v_{it}u_i^*) = 0$  for all  $i = 1, \dots, N$ , and  $t = 1, \dots, T$ .
2.  $v_{it}$  is serially uncorrelated, i.e.  $E(v_{it}v_{is}) = 0$  for all  $i = 1, \dots, N$ , and  $t \neq s$ .
3.  $E(y_{it}v_{it}) = 0$  for  $i = 1, \dots, N$ , and  $t = 2, \dots, T$ .

In the dynamic model (2.3.1), the parameter  $\lambda$ , which is invariant over time, producer, and inputs, reflects the fraction of the desired change in output that is realized in any period. Following Schmidt and Sickles (1984), the most efficient production unit in the sample is assumed to be 100% efficient, and technical efficiency of other units are measured relative to the best-practice frontier -

$$TE_i = \exp\{-(\max_i(-\hat{u}_i^*) - (-\hat{u}_i^*))\} \quad (2.3.2)$$

where a consistent estimator of  $\hat{u}_i^*$  is given by -

$$\hat{u}_i^* = \frac{-1}{T-1} \sum_{t=2}^T \left( \ln y_{it} - (1-\hat{\lambda}) \ln y_{i(t-1)} - \hat{\beta}_0^* - \hat{\lambda} \sum_{m=1}^M \hat{\beta}_m \ln x_{mit} - \hat{\lambda} \sum_{t=2}^T \hat{\delta}_t D_t \right) \quad (2.3.3)$$

The conventional static specification of the stochastic production frontier assumes instantaneous adjustment of inputs while catching up with the potential output and hence  $\lambda = 1$  for the static version of (2.3.1). Formally, the static production frontier is given by -

$$\ln y_{it} = \beta_0 + \sum_{m=1}^M \beta_m \ln x_{mit} + \sum_{t=2}^T \delta_t D_t - \eta_i + v_{it} \quad (2.3.4)$$

Here,  $\eta_i$  represents the non-negative producer specific inefficiency effects. Therefore, the technical efficiency is measured from (2.3.4) as

$$TE_i = \exp\{-(\max_i(-\hat{\eta}_i^*) - (-\hat{\eta}_i^*))\} \quad (2.3.5)$$

where  $\eta_i^* = \eta_i - E(\eta_i)$ ,  $\beta_0^* = \beta_0 - E(\eta_i)$ ,  $\eta_i^* \sim iid(0, \sigma_\eta^2)$ , and  $v_{it} \sim iid(0, \sigma_v^2)$ . If the producer specific effects are fixed, then (2.3.4) is estimated as a fixed effects model and the producer specific effects are consistently estimated as

$$\hat{\eta}_i^* = \frac{-1}{T} \sum_t \left( \ln y_{it} - \hat{\beta}_0^* - \sum_{m=1}^M \hat{\beta}_m \ln x_{mit} - \sum_{t=2}^T \hat{\delta}_t D_t \right) \quad (2.3.6)$$

Alternatively, if the producer specific effects are random, then (2.3.4) is estimated as a random effects model<sup>2</sup> and the estimates of producer specific effects are given by-

$$\hat{\eta}_i^* = \frac{-\sigma_\eta^2}{T\sigma_\eta^2 + \sigma_v^2} \sum_t \left( \ln y_{it} - \hat{\beta}_0^* - \sum_{m=1}^M \hat{\beta}_m \ln x_{mit} - \sum_{t=2}^T \hat{\delta}_t D_t \right) \quad (2.3.7)$$

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<sup>2</sup> A detailed discussion on the model specification and related prediction procedures can be found in Baltagi (1995).

The static model as represented in (2.3.4) omits the lagged adjustment phenomenon of inputs and is likely to provide biased estimates of technical efficiencies of the production units, particularly in the short-run, if the true process of output generation is dynamic. Also, the ranking of firms based on their technical efficiency estimates will be biased if the ranking is obtained from a similarly misspecified static model. Therefore, in the presence of sluggish adjustment of inputs, a static model cannot identify the true process of output generation or the true technical efficiency of a production system. A dynamic model is more suitable for this purpose.

## **2.4. Estimation Methods**

The dynamic model of production as given in (2.3.1) includes the one period lagged dependent variable as a regressor along with other exogenous variables. Both  $y_{it}$  and  $y_{it-1}$  are functions of  $u_i$ , leading to a correlation between one of the regressors and the error term. Thus the OLS estimator is biased and inconsistent even if  $v_{it}$  are not serially correlated. Arellano and Bond (1991) suggested a generalized method of moments (GMM) estimator for the dynamic panel data model that consistently estimates a dynamic panel data model. The basic principle of such estimation is to use a first difference transformation to eliminate the individual specific effects and then to consider the dependent variable with two period lags or more lags as valid instruments. The GMM estimator is more efficient than the Anderson-Hsiao (1982) instrumental variable estimator. Ahn and Schmidt (1995) derived additional non-linear moment restrictions and

the estimation method is further generalized and extended by Arellano and Bover (1995) and Blundell and Bond (1998).

I use the system GMM estimator proposed by Blundell and Bond (1998) which uses a set of moment conditions relating to the first differenced regression equation, and another set of moment conditions for the regression equation in levels. According to Blundell and Bond (1998), the first differences of the two or more period lagged dependent variable are valid instruments for the equation in levels, and two or more period lagged dependent variable in level are relevant instruments for the equation in first differences. In addition, the exogeneity or predeterminedness of some or all of the other explanatory variables ( $x_{mit}$ ) also generates more instrumental variables for estimation<sup>3</sup>. This GMM estimation is consistent for large  $N$  and finite  $T$ , and is more efficient than the estimator proposed by Arellano and Bond (1991).

To estimate (2.3.1), I use  $\ln y_{it-2}$ ,  $\ln x_{mit-1}$ , and  $\ln x_{mit-2}$ ,  $m = 1, \dots, M$ , as instruments for the equation in first difference and  $(\ln y_{it-1} - \ln y_{it-2})$ ,  $(\ln x_{mit-3} - \ln x_{mit-4})$ ,  $m = 1, \dots, M$ , are the instruments used for the equation in levels<sup>4</sup>. A crucial assumption of the validity of GMM estimates is that the instruments are exogenous. I verify joint validity of the instruments with the Sargan test. Further, I use the one step GMM estimator for which

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<sup>3</sup> Inputs  $x_{mit}$  are likely to be correlated with the producer specific effects  $u_i$  for all  $i = 1, \dots, N$ ,  $t = 1, \dots, T$ , and  $m = 1, \dots, M$ .

<sup>4</sup> Though it is possible to have more instrumental variables for our model, considering even deeper lag of the instrumental variables that I am using, I do not use all available instruments, as too many instruments may over fit the endogenous variable and weaken the power of the Hansen test to detect over identification. Given the sample with 28 groups, I choose to use 26 instruments from the recent lags, for which the power of the Sargan test is the largest.



the estimates are consistent<sup>5</sup>. I also employ small-sample corrections to the covariance matrix estimate, and the standard errors, which are robust to heteroskedasticity and arbitrary pattern of autocorrelation within production units. Furthermore, consistency of the GMM estimator relies upon the fact that the idiosyncratic errors are serially uncorrelated. If the differenced error term is second-order serially correlated, then  $\ln y_{it-2}$  is not a valid instrument for the first differenced equation<sup>6</sup>. The Arellano and Bond (1991) test is applied to the residuals in differences to test for second-order autocorrelation.

The static model with time-invariant technical efficiency as given in equation (2.3.4) is estimated as a random effects model<sup>7</sup> and accordingly the technical efficiency is estimated using (2.3.7).

## **2.5. Empirical Analysis**

### **2.5.1. Data**

To illustrate the theoretical model empirically, I use the panel data for nine years (1987/88 – 1995/96) on the private sector manufacturing establishments in Egypt, obtained from the Industrial Production Statistics of the Central Agency for Public

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<sup>5</sup> While the coefficient estimates of two-step GMM estimator are asymptotically more efficient, I do not find any difference in the estimates of coefficients from the one-step and two-step estimation procedure. Since my purpose is to estimate technical efficiency, I use the one-step estimation results only.

<sup>6</sup> By construction, the differenced error term is expected to be first order serially correlated and the evidence of the correlation is uninformative.

<sup>7</sup> Hausman's specification test (1978) for equation (3.4) using the sample suggests random effects specification.

Mobilization and Statistics (CAPMAS). The data is in three-digit ISIC (International Standard Industrial Classification) level and for 28 sectors with the total number of observation being 252. The broader categories of output include food, tobacco, wood, paper, chemicals, non-metallic products, metallic product, engineering products, and other manufacturing products. Table A.1 in the appendix A presents the description of each sector.

This data set is directly taken from a study by Getachew and Sickles (2007) and details about the data can be found in their paper. They use the superlative index number approach to aggregate the data to the three-digit level, such that the establishments in each sector can be viewed as homogeneous in terms of production technology. To get a single aggregate measure of output from heterogeneous and multi-product firms, they consider total revenue from these firms for goods sold, industrial services provided to others, and so on. Finally, they obtain the quantity indices for output and inputs by deflating the total value of output and inputs by the relevant price indices.

Capital, labor, energy, and material are the inputs for the manufacturing sectors' output. As found by Getachew and Sickles (2007) the quantity indices for output and inputs grew over the period under consideration. The summary statistics of the indices are presented in Table A.2 in the appendix A. Getachew and Sickles (2007) use this data set to analyze relative price efficiency of the Egyptian manufacturing sectors, but they do not measure technical efficiency of these sectors.

The private sector has always been important for the economic growth and development in Egypt. However, the Egyptian government adopted rigorous privatization

policies in the early 1990 that were followed by increased growth of the private manufacturing sectors, and as a result, Egypt's manufacturing sector became the highest contributor to the value-added at the national level. Several sub-sectors of the private manufacturing sector (like food and textile) generated good opportunities of employment for unskilled and semi-skilled labors, particularly in a labor abundant country like Egypt. Moreover, during the 1990s, the activities that contributed higher value-added at the national level got more priorities and as a result the input ratios were changing within different sectors. Since frequent or rapid changes in the input ratios and use of unskilled and semi-skilled labor are potential source of sluggish adjustment of inputs, I expect the production process and technical efficiency of the Egyptian private manufacturing sectors to be affected by the adjustment of inputs.

### 2.5.2. Results

I consider a Cobb-Douglas production function for the potential output of the manufacturing sectors<sup>8</sup>. Therefore, the dynamic model corresponding to equation (2.3.1) is given by -

$$\ln y_{it} = (1 - \lambda) \ln y_{i(t-1)} + \lambda(\beta_0 + \sum_{m=1}^4 \beta_m \ln x_{mit} + \sum_{t=2}^9 \delta_t D_t) + \varepsilon_{it} \quad (2.5.1)$$

where,  $\varepsilon_{it} = v_{it} - u_{it}$ ,  $u_{it} \geq 0$ . The inputs are capital, labor, energy and material with  $m = 1$  for capital,  $m = 2$  for labor,  $m = 3$  for energy, and  $m = 4$  for material. The estimation

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<sup>8</sup> I compare a Cobb-Douglas and a more general translog production function for the data. Based on the information criterion (AIC and BIC) from these two models, I select the Cobb-Douglas production function.

results for this model are derived using the Blundell and Bond (1998) system GMM<sup>9</sup> estimator and are given in column (1) of Table A.3. The standard errors of the estimates are robust to heteroskedasticity and arbitrary patterns of autocorrelation within sectors, and I also incorporate the small-sample corrections to the covariance matrix estimate.

From the estimation results I find that the one period lagged output has a significant<sup>10</sup> positive effect on the current output, where output is measured in logarithm. Using the estimated value of  $1 - \hat{\lambda} = 0.16$ , I calculate the fraction of desired change in output that is realized as  $\hat{\lambda} = 0.84$ . Therefore, the actual change in output of a sector in any period is 84% of the change in output that is needed to catch up with the potential output in that period. Further, the  $p$ -value for the estimate of  $(1 - \hat{\lambda})$  is 0.01, which suggests that  $\hat{\lambda}$  is significantly different from unity at the 1% level of significance. Therefore, assuming similar speeds of adjustment for inputs across sectors, this result supports the partial adjustment scheme for output that is generated by the sluggish adjustment of inputs in the production system.

The  $F$  statistic for testing overall fit of the production model, as reported in column (1) of Table A.3, confirms a good fit for the model ( $F(8, 27) = 381.21$ ,  $p$ -value = 0.000). Since the purpose of this chapter is to identify the true technical efficiency of the sectors, the significance levels of the input elasticities are not of much interest. However,

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<sup>9</sup> I use Stata command `xtabond2` developed by David Roodman (2006).

<sup>10</sup> Estimated coefficient of the lagged level of output =  $\hat{\lambda} = 0.16$ , significant at the 10% level.

I find that labor and material have significant input elasticities in the dynamic production model<sup>11</sup>.

Consistency of the system GMM estimator relies upon the fact that the idiosyncratic errors are not serially correlated. The AR(2) test statistic ( $p$ -value = 0.966), as reported in column (1) of Table A.3 corresponds to the test of the null hypothesis that the residuals in the first-differenced regression exhibit no second order serial correlation. Following the test procedure proposed by Arellano and Bond (1991), a negative first order serial correlation in the equation in first differences is expected and the AR(1) test statistic supports that. Thus, the random shocks to the sectors are not serially correlated and the estimation results are consistent. The validity of the GMM estimates also depends on the assumption of exogeneity of the instruments. The Sargan test statistic for testing exogeneity of the instrumental variables, as reported in column (1) of Table A.3, supports validity of the instruments ( $p$ -value = 0.688). The GMM system estimation uses internal instruments for estimation, and thus, there can be several valid instrumental variables. I chose the set of instrumental variables for which the Sargan test of exogeneity was the most powerful.

The static stochastic frontier corresponding to equation (2.3.4) that assumes instantaneous adjustment of all inputs, i.e.,  $\lambda = 1$ , is given as follows-

$$\ln y_{it} = \beta_0 + \sum_{m=1}^4 \beta_m \ln x_{mit} + \sum_{t=2}^9 \delta_t D_t - \eta_i + v_{it} \quad (2.5.2)$$

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<sup>11</sup> Though capital and energy do not have significant input elasticities, I do not drop them from our production model, because they are valid inputs, and have positive elasticities as expected.

I estimate (2.5.2) as a random effects model, following the Housman's specification test (1978) results. The estimation results are also presented in column (2) of Table A.3, that shows that the input elasticities are not drastically different from those estimated using the static model. Finally, the average of the time-invariant technical efficiency estimates from the dynamic (2.5.1) and the static (2.5.2) models are shown in Table A.4. I find that the technical efficiency estimate from the dynamic model is 74.5% for a sector on average, whereas, the estimate from the static model is 70.02% on average. Thus, the technical efficiency estimates from these two models are not similar, and the absolute difference between the estimates is 7.14 percentage points on average, with a maximum of 17.16 percentage points. Further, I find that the static model underestimates the technical efficiency of sectors by 4.48 percentage points on average, and this underestimation can be as high as 17.16 percentage points.

The estimates of technical efficiency from the dynamic and the static specification of production model clearly suggests that by misspecifying the process of output generation in the short-run, the static model generates biased estimates of technical efficiency in the presence of lagged adjustment of inputs. However, due to model misspecification, the technical efficiency estimates from static model can be either higher or lower than the estimates from the dynamic model, for a particular sector. Though the direction of bias may not be uniquely identified for all sectors while comparing results from the static and the dynamic model, I find that the static model underestimates technical efficiency by 4.67% on average. This underestimation may be a result of considering the natural process of input adjustment as a source of inefficiency of

production units. Nevertheless, as the relative efficiency is recovered from the stochastic frontier, the static model may overestimate technical efficiency of a specific production unit. The technical efficiency estimates for each sector from the dynamic and static production model are given in column (1) and (2) of Table A.5, and in Figure A.1.

The significant effect of the one period lagged output on the current output level clearly suggests that the static model is misspecified. Consequently, I find that the ranking of sectors according to the dynamic and static model specifications are also not the same. The ranking of sectors based on their technical efficiency estimates are given in column (3) and (4) of Table A.5. The two model specifications, though uniquely identifying the best sector among all, generate different internal ranking for the other sectors. Since, the dynamic model identifies true efficiency of a sector, the relative ranking of the sectors based on their technical efficiency is more reliable when formulated from such a dynamic model. The Spearman's correlation coefficient for these ranks from the dynamic and static model is found to be 0.59. Though the ranks of sectors as generated from the static and the dynamic model may not be independent, clearly they are different. Since several organizational and production decisions are taken based on the relative efficiency of the sectors, a true ranking as generated by the dynamic model is more reliable.

## 2.6. Conclusion

This chapter outlined a theory for a dynamic stochastic production frontier that described the process of output generation in the presence of lagged adjustment of inputs. The dynamic production model acknowledged the fact that output could be lower in the short run when the inputs are adjusted within a production system, and accordingly measured technical efficiency of production units. The chapter also discussed estimation methods for time-invariant technical efficiency measures of production units, based on such a dynamic model. It further illustrated the methods of estimation using data from the private manufacturing sectors in Egypt, and found that the speed of adjustment of output was significantly lower than unity for the period under consideration. This, in turn, suggests that the conventional static model that assumes instantaneous adjustment of inputs is misspecified, and provides biased estimates of technical efficiency. Comparing the technical efficiency estimates from the dynamic model with those from a static model, I found that the static model underestimates technical efficiency of different sectors by 4.5 percentage points on average that could be as high as 17.16 percentage points. The dynamic production model, as discussed in this chapter, considered the sluggish adjustment of inputs and estimated the true efficiency of the production units in presence of any such adjustment of inputs. Further, I found that the static production model provided an inappropriate ranking of sectors based on these biased estimates of technical efficiency.

Estimation of technical efficiency and ranking of production units according to their efficiency levels are important aspects of productivity analysis. Producers often take



critical decisions about their production plans that are, in part based on such technical efficiency measures. This chapter has provided a more realistic and rigorous approach for capturing the dynamics of a production system, and measuring technical efficiency, particularly for country like Egypt where sluggish adjustment of inputs is a very plausible phenomenon in light of the facts that during the period under consideration, Egypt employed unskilled and semi-skilled labor in the manufacturing sectors and also underwent through several changes in the manufacturing production. In particular, this chapter offers a novel approach towards estimating the speed of adjustment of output and technical efficiency, and thereby, captures the effect of lagged adjustment of inputs on output.

The theoretical and econometric models, as discussed in this chapter, are based on the simplifying assumption that the speed of adjustment of inputs is similar for all inputs, every production unit, and all time periods. However, different production units are likely to have different speeds of adjustment that may change over time along with the technical efficiency as well. Similarly, the adjustment processes of different inputs are also likely to be different. While this chapter does not discuss methods to estimate technical efficiency under less restrictive assumptions, these should be interesting areas of exploration for future research in this field. Moreover, instead of measuring relative efficiency of production units and thus failing to generally specify a direction of bias of efficiency estimates from a misspecified production model, using bootstrapping techniques to compare the efficiency estimates from a static and a dynamic model would be another area to explore in the future.

## Appendix A: Tables and Figures

**Table A.1: Sectors and the Industrial Activities at the three-digit ISIC level**

<b>Sector Number</b>	<b>Industrial activity</b>
1	Food manufacturing
2	Other food manufacturing
3	Beverage and liquor
4	Tobacco
5	Manufacture of textile
6	Manufacture of wearing apparels
7	Manufacture of leather products
8	Manufacture of footwear
9	Manufacture of wood products
10	Manufacture of furniture & fixture
11	Manufacture of paper products
12	Printing and publishing industries
13	Manufacture of industrial chemicals
14	Manufacture of other chemical products
15	Other petroleum and coal
16	Manufacture of rubber products
17	Manufacture of plastic products
18	Manufacture of pottery and china
19	Manufacture of glass and glass products
20	Manufacture of other non metallic products
21	Iron and steel basic industries
22	Non-ferrous basic industries
23	Manufacture of fabricated metal products
24	Manufacture of machinery except electrical
25	Manufacture of electrical machinery
26	Manufacture of transport equipment
27	Manufacture of professional equipment
28	Other manufacture industries

**Table A.2: Variable Descriptions and Summary Statistics**

<b>Variable</b>	<b>Description</b>	<b>Number of Observation</b>	<b>Mean</b>	<b>Standard deviation</b>	<b>Minimum</b>	<b>Maximum</b>
<b>Yearid</b>	id number for 9 years of data for each sector	252	5	2.59	1	9
<b>Sectorid</b>	id numbers for the 28 three digit manufacturing sectors	252	14.5	8.09	1	28
<b>Output</b>	Output quantity index	252	2888.90	3333.39	67	19236
<b>Capital</b>	Capital quantity index	252	288.84	475.29	1	3437
<b>Labor</b>	Labor quantity index	252	273.34	344.06	10.50	1689.2
<b>Energy</b>	Energy quantity index	252	61.97	116.56	0.20	860.1
<b>Material</b>	Material quantity index	252	1823.44	2168.83	44.8	11853.8

Source: Getachew and Sickles (2007).

**Table A.3: Estimation Results from Dynamic and Static Specifications (Time-Invariant Technical Efficiency Model)**

	<b>Dynamic Specification</b>	<b>Static Specification</b>
<b>Coefficients</b>	<b>(1)</b>	<b>(2)</b>
lag_ln(output)	0.16*** [0.06]	-
ln(capital)	0.02 [0.05]	0.014 [0.01]
ln(labor)	0.22** [0.09]	0.123*** [0.04]
ln(energy)	0.04 [0.05]	0.044** [0.02]
ln(material)	0.65*** [0.09]	0.833*** [0.03]
Constant	0.33 [0.29]	0.803*** [0.15]
AR(1)	-2.72***	-
AR(2)	-0.04	-
Sargan test	12.79	-
Observations	140	252
Number of sectors	28	28
Number of instruments	26	-
R-squared	-	0.973
F (10, 27)	381.21***	-

*Note:* Robust standard errors are reported in parentheses; \* significant at 10%; \*\* significant at 5%; \*\*\* significant at 1%; AR(1) and AR(2) represent the Arellano-Bond (1991) test statistics for the first order and second order serial correlation in the first differenced residuals respectively; The null hypothesis for Sargan test is that instruments used are not correlated with the residuals; the instruments used are  $\ln(\text{output})_{it-2}$ ,  $\ln(\text{capital})_{it-1}$ ,  $\ln(\text{capital})_{it-2}$ ,  $\ln(\text{labor})_{it-1}$ ,  $\ln(\text{labor})_{it-2}$ ,  $\ln(\text{energy})_{it-1}$ ,  $\ln(\text{energy})_{it-2}$ ,  $\ln(\text{material})_{it-1}$ , and  $\ln(\text{material})_{it-2}$  for the equation in first differences, and are  $\ln(\text{output})_{it-1} - \ln(\text{output})_{it-2}$ ,  $\ln(\text{capital})_{it-3} - \ln(\text{capital})_{it-4}$ ,  $\ln(\text{labor})_{it-3} - \ln(\text{labor})_{it-4}$ ,  $\ln(\text{energy})_{it-3} - \ln(\text{energy})_{it-4}$ , and  $\ln(\text{material})_{it-3} - \ln(\text{material})_{it-4}$  for the equation in levels; The regressions also include dummy variables for the different time periods that are not reported.

**Table A.4: Difference in the Time-Invariant Technical Efficiency Estimates from Static and Dynamic Specifications**

<b>Variables</b>	<b>Mean</b>	<b>Maximum</b>
Technical Efficiency_Dynamic	74.5	100
Technical Efficiency_Static	70.02	100
Difference in Efficiency Estimates	7.14	17.16
Underestimation by Static Model	4.48	17.16

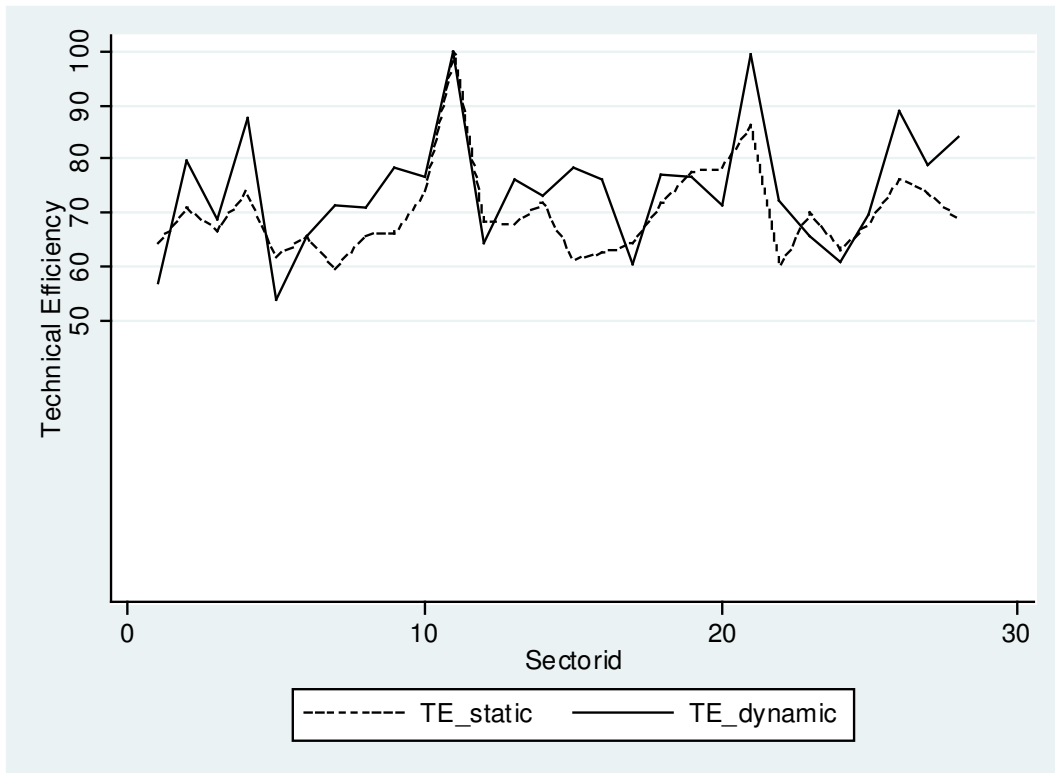
*Note:* The technical efficiency estimates from the dynamic and the static models are presented in percentage terms. These estimates show the efficiency level of a production unit relative to the most efficient unit in the sample. The difference in efficiency estimates is calculated by taking the absolute difference in the technical efficiency estimates from the dynamic and the static model. The difference in efficiency estimates and the underestimation by static model are presented in terms of percentage points.

**Table A.5: Time-Invariant Technical Efficiency Estimates and Ranking of Sectors under Dynamic and Static Specifications**

Sectorid	Technical Efficiency from Dynamic Specification (%) (1)	Technical Efficiency from Static Specification (%) (2)	Rank_Dynamic Specification (3)	Rank_Static Specification (4)
1	57.12	64.35	27	21
2	79.62	70.94	6	11
3	68.91	66.88	21	17
4	87.51	74.28	4	6
5	54.17	61.84	28	25
6	65.81	65.69	23	20
7	71.46	59.56	17	28
8	71.27	65.87	19	19
9	78.46	66.34	9	18
10	76.63	73.58	11	8
11	100.00	100.00	1	1
12	64.58	68.50	24	14
13	76.37	67.90	13	16
14	73.13	71.96	15	9
15	78.58	61.42	8	26
16	76.20	62.66	14	24
17	60.64	64.30	26	22
18	77.11	71.78	10	10
19	76.59	77.72	12	4
20	71.41	78.14	18	3
21	99.67	86.20	2	2
22	72.54	60.19	16	27
23	65.86	70.23	22	12
24	60.81	63.29	25	23
25	69.69	68.10	20	15
26	88.96	76.20	3	5
27	78.76	73.80	7	7
28	84.29	68.81	5	13

*Note:* Technical efficiency of a sector is measured relative to the most efficient sector.

**Figure A.1: Time-Invariant Technical Efficiency Estimates from Dynamic and Static Specifications**



## **Chapter 3**

### **Adjustment of Inputs and Measurement of Time-Varying**

#### **Technical Efficiency: A Dynamic Panel Data Analysis**

##### **3.1. Introduction**

Estimation of technical efficiency of production units using a stochastic frontier approach and panel data has been a popular area of applied research for the last couple of decades. The advantage of using panel data in stochastic production frontier analysis is that it enables one to estimate efficiency of production units without imposing restrictive assumptions on them. Earlier research on measuring time-invariant technical efficiency (Schmidt and Sickles (1984)) has been further developed by Cornwell, Schmidt and Sickles (1990); Kumbhakar (1990); and Battese and Coelli (1992) to incorporate time-variation in technical efficiency of a production unit. They assume the technical efficiency of production units to be a parametric function of time. Lee and Schmidt (1993) capture temporal variation in efficiency in a more flexible fashion. They consider the temporal pattern of efficiency to be the same for all production units without assuming any functional form. According to Lee and Schmidt (1993), the producer specific effect and its time pattern are unknown parameters to be estimated. A recent research by Ahn, Lee, and Schmidt (2007) further extends this idea and discusses



estimation of time-varying technical efficiency from a stochastic frontier model with multiple time-varying individual effects.

All of the existing studies on stochastic frontiers with time-varying technical efficiency focus on the static analysis of a producer's behavior, and therefore, fail to capture the dynamic nature of a firm's optimization process. In the previous chapter, I have discussed a dynamic stochastic production frontier that incorporates the effects of short-run adjustment of inputs on the actual output generation process. The previous chapter shows that in the presence of the short-run quasi-fixity of inputs, changes in the demand for output and expectations about future economic conditions leads to a partial adjustment process for the actual output. The time-invariant technical efficiency from such a dynamic model is found to be different than those obtained from a static model. However, neither chapter 2 nor the above mentioned works discussed a production frontier that allows for the short-run adjustment of inputs and production plans when both the speed of adjustment and technical efficiency are varying with time.

Ahn, Good, and Sickles (2000) allow for the lagged adjustment of inputs to explain the autoregressive nature of the technical efficiency component that varies with time. They also measure the speed of sluggish adoption of technological innovations, but the speed is assumed to be constant over time. In reality, sluggish adjustment of inputs not only affects the adoption of technological innovations, but can also affect the whole production process by restricting output from reaching its maximum possible level. Moreover, with time, as the inputs get more familiar with a production system, their speed of adjustment is likely to improve as well. For example, a worker hired in the past

is likely to learn faster than a newly hired worker. As a result, the deviation of actual change in output from the desired change is also likely to vary over time. More specifically, it is likely that the gap between the actual change and desired change in output falls over time. Further, if a production system is studied for substantially long period of time, and the economic structure is sufficiently competitive, the inefficiency effect of a production unit is also likely to change over time (Kumbhakar and Lovell, 2000).

Therefore, in this chapter, I present a dynamic production model with time-varying speed of adjustment of output and technical efficiency. Measuring efficiency from such a dynamic model is also not straightforward. Particularly, consistent and efficient estimation of dynamic panel data model with time-varying individual effects<sup>12</sup> is an attractive area of research even in the current time. The first and widely known paper in this area is by Holtz-Eakin, Newey, and Rosen (1988), who discuss the estimation method for a dynamic panel data model with time-varying individual effects, but do not discuss estimation of the time-varying individual effects from such a model. By adapting their method, I extend it to suit my purpose of technical efficiency estimation and apply it in this chapter.

The objective of this chapter is thus, to present a dynamic stochastic production frontier that allows for short-run quasi-fixity of inputs and provide estimation methods to measure the speed of adjustment of output and technical efficiency of production units, both of which vary over time. The chapter also compares the estimates of time-varying

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<sup>12</sup> The econometric dynamic panel data model with time-varying individual effects corresponds to the dynamic production model with time-varying technical efficiency.

technical efficiency of production units from such a dynamic model with the estimates from a static production model that assumes instantaneous adjustment of all inputs. For this purpose, I use a panel dataset on private manufacturing establishments in Egypt from the Industrial Production Statistics of the Central Agency for Public Mobilization and Statistics (CAPMAS).

The remainder of this chapter is organized as follows. The model specifications are discussed in section 3.2. Section 3.3 and 3.4 elaborate on the estimation methods and empirical analysis, respectively. Finally, section 3.5 presents concluding remarks.

### 3.2. Model Specification

The dynamic production model showing the relationship between actual change and the desired change in output between two periods is given as -

$$\ln y_{it} - \ln y_{i(t-1)} = \lambda_t (y_{it}^* - \ln y_{i(t-1)}), \quad 0 \leq \lambda_t \leq 1 \quad (3.2.1)$$

where,  $i = 1, \dots, N$  denotes the production unit,  $t = 1, \dots, T$  represents the time periods,  $y_{it}$  is the actual output of producer  $i$  at time  $t$ ,  $y_{it}^*$  denotes the maximum possible output of producer  $i$  at time  $t$ , and  $\lambda_t$  is the fraction of desired change in output that is realized in time  $t$ . I assume the gap between the actual change ( $y_{it} - y_{i(t-1)}$ ) and the desired change ( $y_{it}^* - y_{i(t-1)}$ ) in output is similar for all producers. If the maximum possible output is generated by a Cobb Douglas production function, then (3.2.1) can be represented as

$$\ln y_{it} = (1 - \lambda_t) \ln y_{i(t-1)} + \lambda_t (\beta_0 + \partial t + \sum_{m=1}^M \beta_m \ln x_{mit}) \quad (3.2.2)$$

where  $x_{mit}$  is the  $m$ th input used by producer  $i$  at time  $t$  for  $m = 1, \dots, M$ ,  $\beta_m$  is the elasticity of the  $m$ th input, and  $\beta_0$  is the intercept of the potential production frontier.  $\delta$  captures the effect of technological changes on the potential output<sup>13</sup>.

The stochastic version of the dynamic production model allows for the presence of inefficiency in a production system and also accounts for the random shocks. Thus the stochastic dynamic production frontier corresponding to (3.2.2) is given by

$$\ln y_{it} = (1 - \lambda_t) \ln y_{i(t-1)} + \lambda_t \ln y_{it}^* + e_{it} \quad (3.2.3)$$

$$\text{or, } \ln y_{it} = (1 - \lambda_t) \ln y_{i(t-1)} + \lambda_t (\beta_0 + \delta t + \sum_{m=1}^M \beta_m \ln x_{mit}) + e_{it} \quad (3.2.4)$$

The composed error term  $e_{it}$  can be decomposed into the technical inefficiency term,  $\theta_t f_i$  ( $\theta_t f_i \geq 0$ ), that varies with time and the symmetric random shock,  $\tau_{it}$ , i.e.,  $e_{it} = -\theta_t f_i + \tau_{it}$ , where  $\tau_{it} \sim iid(0, \sigma_\tau^2)$ .  $\theta_t$  captures the time-varying influence of the producer specific inefficiency  $f_i$  on the current output. In this formulation, the temporal pattern of technical inefficiency is the same for all production units. However, as discussed by Lee and Schmidt (1993), this structure is less restricted than the structures proposed by Cornwell, Schmidt, and Sickles (1990) and Kumbhakar (1990).

To measure the time-varying technical efficiency and speed of adjustment of output, I consider a Cobb-Douglas production function with constant elasticity of inputs for the potential output. Since I do not expect the elasticities to vary when the inputs are producing at their maximum possible level, the assumption of constant elasticities of

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<sup>13</sup> Since all the parameters in (2.2) vary with time and the sample size is not very large for this analysis, I consider only time trend instead of time dummies to reduce the number of parameter estimates from (2.2).

inputs for the potential output is reasonable. This assumption also assures a considerable reduction in the number of parameter estimates from a small sample. The estimation method for (3.2.4) is discussed in the next section. However, once the parameters  $\theta_t$ , and the firm specific effect  $f_i$  are estimated, the technical efficiency is measured as –

$$TE_{it} = \exp\{-\max_j(-\hat{\theta}_t \hat{f}_j) - (-\hat{\theta}_t \hat{f}_i)\} \quad (3.2.5)$$

If the speed of adjustment of all inputs is assumed to be unity, as in the static specification of equation (3.2.4), the following represents the static stochastic frontier model -

$$\ln y_{it} = \beta_0 + \rho t + \sum_{m=1}^M \beta_m \ln x_{mit} + \pi_{it} \quad (3.2.6)$$

where  $\pi_{it} = -\rho_t \kappa_i + w_{it}$ ,  $\rho_t \kappa_i \geq 0$  represent the technical inefficiency of producer  $i$  at time  $t$ ,  $\rho_t$  is the time-varying influence of the producer specific effect  $\kappa_i$ , and the symmetric statistical noise  $w_{it} \sim iid(0, \sigma_w^2)$ . The static stochastic frontier (3.2.6) is estimated following the methods suggested by Lee and Schmidt (1993). Since inputs are likely to be correlated with the producer specific effects, (3.2.6) is estimated as fixed effects model and  $\rho_t$  and  $\kappa_i$  are estimated. The estimation procedure is discussed in detail in the next section. Then the technical efficiency from (3.2.6) is calculated as -

$$T\tilde{E}_{it} = \exp\{-\max_j(-\hat{\rho}_t \hat{\kappa}_j) - (-\hat{\rho}_t \hat{\kappa}_i)\} \quad (3.2.7)$$

In the presence of short-run adjustment of inputs, the technical efficiency estimates using the static model (3.2.7) is expected to be biased as compared to those obtained from the dynamic model (3.2.4).

### 3.3. Estimation Methods

To estimate the dynamic panel data model with time-varying technical efficiency, as given in equation (3.2.4), I adapt the method described by Holtz-Eakin, Newey, and Rosen (1988). For identification purposes, I assume that

$$E[\ln y_{is} \tau_{it}] = E[\ln x_{mis} \tau_{it}] = E[f_i \tau_{it}] = 0, \quad (s < t), \quad (m = 1, \dots, M) \quad (3.3.1)$$

The error term in (3.2.4) does not have a mean value zero. Therefore, I transform equation (3.2.4) to eliminate the individual effects in the following way. Let  $r_t = \frac{\theta_t}{\theta_{t-1}}$ . I consider (3.2.4) for period  $(t-1)$ , multiply it by  $r_t$ , and take the difference of the derived equation from (3.2.4) for period  $t$ . This gives us the following quasi-transformed equation-

$$\begin{aligned} \ln y_{it} = & \lambda_t \beta_0 - r_t \lambda_{t-1} \beta_0 + r_t \lambda_{t-1} \partial + (1 + r_t - \lambda_t) \ln y_{it-1} - r_t (1 - \lambda_{t-1}) \ln y_{it-2} + \lambda_t \sum_{m=1}^M \beta_m \ln x_{mit} \\ & + \lambda_t \partial t - r_t \lambda_{t-1} \partial t - r_t \lambda_{t-1} \sum_{m=1}^M \beta_m \ln x_{mit-1} + e_{it} - r_t e_{it-1} \end{aligned} \quad (3.3.2)$$

The regressors in (3.3.2) involve one period lagged dependent variable that is correlated with the error term. However, the orthogonality conditions in (3.3.1) imply that the error term in (3.3.2) satisfies the following conditions -

$$E[\ln y_{is} \varepsilon_{it}] = E[\ln x_{mis} \varepsilon_{it}] = E[f_i \varepsilon_{it}] = 0 \quad \text{for } s < t-1, \quad m = 1, \dots, M$$

where,  $\varepsilon_{it} = e_{it} - r_t e_{it-1}$ . Therefore, the vector of instrumental variables that is available to identify the parameters of (3.3.2) is  $Z_{it} = [\ln y_{it-3}, \dots, \ln y_{i1}, \ln x_{mit-2}, \ln x_{mit-3}, \dots, \ln x_{mi1}]$ .

The vectors of observation on  $i = 1, \dots, N$  for a given time period are given by

$$Y_t = [\ln y_{1t}, \dots, \ln y_{Nt}]'$$

$$X_t = [\ln x_{m1t}, \dots, \ln x_{mNt}]', m = 1, \dots, M$$

The vectors of the right hand side variables, error term, and coefficients of (3.3.2) for a given time period are given by  $W_t, V_t$  and  $B_t$  respectively, where

$$W_t = [e, Y_{t-1}, Y_{t-2}, t, X_{mt}, X_{m-1}] \text{ for } m = 1, \dots, M$$

$$V_t = [v_{1t}, \dots, v_{Nt}]'$$

$$B_t = \begin{bmatrix} (\lambda_t - r_t \lambda_{t-1}) \beta_0 + r_t \lambda_{t-1} \partial \\ 1 + r_t - \lambda_t \\ -r_t (1 - \lambda_{t-1}) \\ (\lambda_t - r_t \lambda_{t-1}) \partial \\ \lambda_t \beta_1 \\ \vdots \\ \lambda_t \beta_M \\ r_t \lambda_{t-1} \beta_1 \\ \vdots \\ r_t \lambda_{t-1} \beta_M \end{bmatrix}$$

Therefore, I can write equation (3.3.2) as

$$Y_t = W_t B_t + V_t \text{ for } t = 4, \dots, T \quad (3.3.3)$$

Further, combining observations for each time period, (3.3) can be written as

$$Y = WB + V \quad (3.3.4)$$

where,

$$Y = [Y'_4, \dots, Y'_T]'$$

$$B = [B'_4, \dots, B'_T]'$$

$$V = [V'_4, \dots, V'_T]'$$

$$W = \text{diag}[W'_4, \dots, W'_T]'$$

and  $\text{diag}[\ ]$  denotes a block diagonal matrix with the given entries along the diagonal.

Thus, the matrix of instrumental variable for period  $t$  is  $Z_t = [Y_{t-3}, \dots, Y_1, X_{mt-2}, \dots, X_{m1}]$  for  $m = 1, \dots, M$ . Consider  $Z = \text{diag}[Z_4, \dots, Z_T]$ .

The covariance matrix  $\Omega$  of the transformed disturbances is  $\Omega = E\{Z'VV'Z\}$ . To estimate  $\Omega$ , I use the two-stage least squares (2SLS) estimator of  $B_t$ , given by  $\tilde{B}_t$ , as the preliminary consistent estimator where

$$\tilde{B}_t = [W'_t Z_t (Z'_t Z_t)^{-1} W'_t Z_t (Z'_t Z_t)^{-1} Z'_t Y_t] \quad (3.3.5)$$

Then, the vector of residuals for period  $t$  is given by -

$$\tilde{V}_t = Y_t - W_t \tilde{B}_t \quad (3.3.6)$$

A consistent estimator of  $(\tilde{\Omega} / N)$  is then formed by -

$$(\tilde{\Omega} / N)_{rs} = \sum_{i=1}^N (v_{ir} v_{is} Z'_{ir} Z_{is}) / N \quad (3.3.7)$$

where  $v_{it}$  ( $t = r, s$ ) is the  $i$ th element of  $V_t$  and  $Z_{it}$  is the  $i$ th row of  $Z_t$ .

For the empirical analysis, (3.3.2) is estimated by the method of GLS (generalized least squares) with  $\ln y_{it-3}$  as the instrumental variable. Since  $N$  is not large (28) for the sample used in this chapter, I do not use all the available instruments, in order to avoid the



problem of too many instruments. Given the choice of instrumental variable, (3.3.2) is estimated for  $t \geq 4$ .

Holtz-Eakin, Newey, and Rosen (1988) do not discuss about estimation of the individual specific effects that vary with time. However, the main objective of this chapter is to estimate the time-varying technical efficiency of a production unit, which is a part of the composite error term. For this purpose, I estimate (3.3.2) following the method discussed above and get estimates for  $(2M + 4)$  parameters, where each of these parameters is a nonlinear function of  $(M + 5)$  distinct parameters given by  $r_t$ ,  $\lambda_t$ ,  $\lambda_{t-1}$ ,  $\beta_0$ ,  $\partial$ , and  $\beta_1, \dots, \beta_M$ . Thus, once (3.3.2) is estimated, I have an over identified system of  $(2M + 4)$  equations to identify  $M+5$  parameters, for  $M \geq 1$ . I denote the vector of  $(M + 5)$  parameters by  $\varphi_t$  and the system of equations by  $g(\varphi_t)$ . The  $(2M + 4)$  estimates from (3.3.2) are given by  $a_t$ ,  $b_t$ ,  $c_t$ ,  $d_t$ ,  $f_{1t}, \dots, f_{Mt}$ , and  $h_{1t}, \dots, h_{Mt}$ , and hence,  $g(\varphi_t)$  is given by

$$g(\varphi_t) = \begin{bmatrix} (\lambda_t - r_t \lambda_{t-1}) \beta_0 + r_t \lambda_{t-1} \partial - a_t \\ 1 + r_t - \lambda_t - b_t \\ -r_t(1 - \lambda_{t-1}) - c_t \\ \lambda_t - r_t \lambda_{t-1} - d_t \\ \lambda_t \beta_1 - f_{1t} \\ \vdots \\ \lambda_t \beta_M - f_{Mt} \\ r_t \lambda_{t-1} \beta_1 - h_{1t} \\ \vdots \\ r_t \lambda_{t-1} \beta_M - h_{Mt} \end{bmatrix}$$

To identify the parameters of the original dynamic production model (3.2.4), I solve the following optimization problem<sup>14</sup> subject to the condition that the speed of adjustment of output in each period  $\lambda_t \in [0,1]$  and the input elasticities ( $\beta_m$ ) are non-negative. Thus, I get unique estimates for the parameters in the original model as given in (3.2.4) and also

for  $r_t = \frac{\theta_t}{\theta_{t-1}}$  by the following -

$$\text{Min}_{\varphi_t} g(\varphi_t)'g(\varphi_t) \text{ subject to } 0 \leq \lambda_t \leq 1, \text{ and } \beta_m \geq 0 \text{ for } m = 1, \dots, M.$$

Further, to identify  $\theta_t$ , I normalize<sup>15</sup>  $\theta_T = 1$  and accordingly identify  $\theta_t$  for the periods for which (3.3.2) is estimated. Finally, I estimate the sector specific effect  $f_i$  by the ordinary least squares method for each sector using the following equation

$$\hat{\phi}_{it} = -\hat{\theta}_t f_i + \tau_{it} \quad (3.3.8)$$

$$\text{where, } \hat{\phi}_{it} = \ln y_{it} - (1 - \hat{\lambda}_t) \ln y_{i(t-1)} - \hat{\lambda}_t (\hat{\beta}_0 + \hat{\delta}t + \sum_{m=1}^M \hat{\beta}_m \ln x_{mit}) \quad (3.3.9)$$

The time-varying technical efficiency is estimated following equation (3.2.5)

To compare the technical efficiency estimates from (3.2.4) with those from the static version of the model that assumes the speed of adjustment is constant and equals unity, I estimate equation (3.2.6), following the method suggested by Lee and Schmidt (1993). Relying on the results of Hausman's specification test (1978), I estimate (3.2.6)

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<sup>14</sup>  $\text{Min}_{\varphi_t} g(\varphi_t)'V(\varphi_t)g(\varphi_t)$ , where  $V(\cdot)$  represents variance, makes no considerable changes in the results.

<sup>15</sup> Lee and Schmidt (1993) suggest the normalization  $\theta_1 = 1$  for the static model with similar time-varying technical efficiency structure. However, our model being a dynamic one, the parameters cannot be estimated for the initial period and we choose the normalization with respect to the last period.

as a fixed effects model such that the producer specific effects are treated as parameters to be estimated. In a general notation the model can be summarized as

$$\ln y_{it} = X'_{it}\beta + \pi_{it} \quad (3.3.10)$$

where,  $\pi_{it} = -\rho_i \kappa_i + w_{it}$ ,  $X_{it}$  is the vector of regressors including a constant term, time trend, and M inputs in logarithmic term.  $\beta$  is the vector of input elasticities, and  $w_{it}$  are assumed to be independently and identically distributed with mean zero and variance  $\sigma_w^2$ .

The  $T$  observations for production unit  $i$  can be written as

$$y_i = X_i\beta + \xi k_i + w_i \quad (3.3.11)$$

where,

$$y_i = (\ln y_{i1}, \dots, \ln y_{iT})', \quad X_i = (X_{i1}, \dots, X_{iT})', \quad w_i = (w_{i1}, \dots, w_{iT})', \quad \text{and } \xi' = (\rho_1, \dots, \rho_{T-1}, 1).$$

The estimator of  $\beta$  is given by

$$\hat{\beta} = \left( \sum_i X'_i \hat{M}_\xi X_i \right)^{-1} \sum_i X'_i \hat{M}_\xi y_i \quad (3.3.12)$$

Where  $\hat{M}_\xi = I_T - \hat{\xi}(\hat{\xi}'\hat{\xi})^{-1}\hat{\xi}'$ , and  $\hat{\xi}$  is the eigenvector of  $\sum_i (y_i - X_i\hat{\beta})(y_i - X_i\hat{\beta})'$

corresponding to the largest eigenvalue<sup>16</sup>. To implement the fixed effects estimator of

Lee and Schmidt (1993), first, I estimate  $\beta$  by the ordinary least squares method (OLS)

as

$$\tilde{\beta} = \left[ \sum_i (X_i - \bar{X})(X_i - \bar{X})' \right]^{-1} \sum_i (X_i - \bar{X})(y_i - \bar{y})$$

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<sup>16</sup>  $M_\xi$  is a  $T \times T$  idempotent matrix such that  $M_\xi \xi = 0$ .

where,  $\bar{X} = \sum_i X_i / N$ , and  $\bar{y} = \sum_i y_i / N$ . Using this initial estimate of  $\beta$ , I iterate the estimation process till it converges. Finally, the producer specific effects are estimates as  $\hat{k}_i = \hat{\xi}'(y_i - X_i \hat{\beta}) / \hat{\xi}' \hat{\xi}$ . Then, the time-varying technical efficiency is estimated from (3.2.7).

### 3.4. Empirical Analysis

#### 3.4.1. Data

The dynamic production frontier and the estimation method, as discussed in section 3.2 and 3.3, respectively, are applied on a panel data set for nine years (1987/88 – 1995/96) on the private sector manufacturing establishments in Egypt. The details about the data are discussed in chapter 2 that shows that the process of output generation for the Egyptian private manufacturing sectors is a dynamic one. Based on this result, we further extend the dynamic stochastic production frontier to incorporate time-varying speed of adjustment and technical efficiency, and apply the model on the same dataset from Egypt. Table B.2 and B.3 reproduces the description of each sector and the summary statistics of the input and output data.

#### 3.4.2. Results

As discussed before, the technical efficiency as well as the speed of adjustment of output may vary over time. More specifically, it is likely that the rate of adjustment of an input improves over time by the process of learning and doing, and as a result, the speed of adjustment of output increases as well. Consequently, the technical efficiency of a

production unit is likely to increase with time. Using a Cobb-Douglas production function to specify production of the potential output of the manufacturing sectors<sup>17</sup>, the dynamic specification as given in equation (3.2.4) is estimated as -

$$\ln y_{it} = (1 - \lambda_t) \ln y_{i(t-1)} + \lambda_t (\beta_0 + \partial t + \sum_{m=1}^4 \beta_m \ln x_{mit}) + e_{it} \quad (3.4.1)$$

where  $e_{it} = -\theta_t f_i + \tau_{it}$ . The inputs are capital, labor, energy and material with  $m = 1$  for capital,  $m = 2$  for labor,  $m = 3$  for energy, and  $m = 4$  for material.

Following the method described in section 3.3, we estimate the speed of adjustment of output for time periods  $t = 4, \dots, 9$ , and the time varying technical efficiency for each sector  $i = 1, \dots, 28$ . I use the two-stage least squares results that are consistent and find that the coefficient of the lagged dependent variable in the transformed equation (3.3.2) is positive and significant for every period, implying that the true process of output generation is dynamic. The coefficient estimates<sup>18</sup> of the lagged dependent variable are given in Table B.3 along with their  $t$ -ratios that use heteroskedasticity corrected standard errors<sup>19</sup>. Finally, I recover the parameter estimates from the original model for each period by minimizing an over identified system of equation.

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<sup>17</sup> I compare a Cobb-Douglas and a more general translog production function for the data. Based on the information criterion (AIC and BIC) from these two models, I select the Cobb-Douglas production function.

<sup>18</sup> The coefficient of the lagged dependent variable in the transformed equation (3.3.2) is given by  $(1 + r_t - \lambda_t)$  for each  $t$ .

<sup>19</sup> The total number of parameter estimates from the quasi-transformed model is 72 and I present only the relevant ones.

The estimation results show that the speed of adjustment of output ranges from 43% to 55% for the sample over the period under consideration (given in Figure B.1), with an average of 49%. Thus, on average, the actual change in output in any period is 49% of the change in output that is needed to catch up with the potential output. Moreover, the gap between the change in actual output and the desired change reduces over time as the inputs get more time to learn and adjust within the production system.

The average time-varying technical efficiency as measured from (3.3.8) is given in Table B.4, which shows that during the period, the private manufacturing sectors of Egypt were approximately 90% technically efficient on average. To compare these results with the estimates from a static stochastic frontier, I also estimate the time-varying technical efficiency from the static model –

$$\ln y_{it} = \beta_0 + \partial t + \sum_{m=1}^4 \beta_m \ln x_{mit} + \pi_{it} \quad (3.4.2)$$

where  $\pi_{it} = -\rho_t \kappa_i + w_{it}$ . The average technical efficiency for a sector during the period under consideration is found to be only 79% when measured from a static model that assumes instantaneous adjustment of all inputs. Thus, in the presence of sluggish adjustment of inputs, a static model misspecifies the production process and underestimates the true technical efficiency of a sector on average which is likely to be the result of attributing the shortfall in output that occurs during the short-run adjustment of inputs to inefficiency of the production unit.

Further, I find that the absolute difference between the efficiency estimates from the static and the dynamic model is 17 percentage points on average, and can be as high

as 54 percentage points for a sector in a period. Since the static model seems to underestimate the technical efficiency, I present the magnitude of this underestimation in Table B.4 as well. I find that the static model underestimates the technical efficiency of production units by 11 percentage points on average, i.e., the static model underestimates technical efficiency of a sector in a period by 12%, on average.

Instead of presenting the technical efficiency for all observations I present the average for each sector in Table B.5. Figure B.2 further illustrates the contents of this table. From column (1) and (2) of Table B.5, that show the average technical efficiency estimates for each sector respectively, it is evident that by ignoring the adjustment process of inputs, the static model underestimates the technical efficiency for most of the sectors on average.

I also find that the ranking of sectors from the dynamic and the static model are markedly different, and the best performing sector is also not the same according to these two production models. The ranking of sectors according to the dynamic and the static production model are given in column (3) and (4) of Table B.5, respectively. Further investigation on the ranks of sectors, as assigned by the dynamic and static production model, reveals that the Spearman's correlation coefficient is 0.34 for them, and I cannot reject the hypothesis that the ranks from the static and dynamic model are independent at the 5% significance level ( $p$ -value for the test statistic is 0.08).

Finally, I look into the pattern of variation of technical efficiency over time for each sector, and compare them as obtained from a dynamic and a static production model. The time-varying technical efficiency estimates from both models are presented in

Figure B.3, separately for each of the 28 sectors in the sample (Figure B.3(i) – B.3(xxviii)). Figure B.3 reveals that the dynamic production model identifies more variation in the time pattern of technical efficiency, for each sector, when compared to the pattern of time variation of technical efficiency as estimated from a static production frontier. Thus, by ignoring the lagged adjustment of inputs, the static model not only provides biased estimates of technical efficiency, but it also fails to capture the temporal variation in the efficiency measures.

A closer look at the economic conditions of Egypt during the period under consideration reveals that the Egyptian government adopted rigorous privatization policies in the early 1990. Since then, there have been substantial changes in the structure of the private manufacturing activities. The new economic policies enhanced competition and opened up possibilities for further privatization through international investment banking. Consequently, it tended to attract investment for high technology and managerial and marketing skills that was likely to foster higher level of productivity and efficiency. From Figure B.3, it is visible that starting with 1991/1992, which is the 5<sup>th</sup> year in Figure, technical efficiency of each sectors improved substantially as shown by the efficiency estimates from the dynamic production model. Every sector followed a upward rising trend in the technical efficiency after 1991/1992, signifying the effects of new economic policies implemented by the Egyptian government in early 1990s. As a result of these new economic policies, production resources were geared more toward the sectors, that were likely to promote growth, and the private manufacturing sectors were the prominent ones among them. Thus the production in the private manufacturing



sectors experienced significant change in the input structure. Moreover, the private manufacturing sector was also a source of employment for the unskilled and semi-skilled labor. Therefore, it is very plausible that the inputs of production exhibited substantial adjustment process during 1990s, supporting a dynamic production model, and efficiency of sectors markedly improved in the 1990s.

However, the pattern of time variation in technical efficiency for each sector as estimated by the static production model fails to capture this phenomenon as shown in the Figure B.3. By assuming instantaneous adjustment of inputs, the static model estimates a steady but slow improvement in efficiency for all the sectors, and thus do not show the marked improvements in efficiency of sectors after implementation of the privatization policies. Therefore, it is clear from the Figure B.3 that the dynamic production model captures more variation in the time pattern of technical efficiency than the static model, by allowing for sluggish adjustment of inputs.

### **3.5. Conclusion**

This chapter discussed estimation methods for the speed of adjustment of output and technical efficiency of production units that vary over time from a dynamic stochastic production frontier, which described the process of output generation in the presence of lagged adjustment of inputs. The dynamic production model acknowledged the fact that output could be lower in the short-run when the inputs were adjusted within a production system, and accordingly measured technical efficiency of production units. The chapter further illustrated the methods of estimation using data from the private

manufacturing sectors in Egypt, and found that the speed of adjustment of output was significantly lower than unity for the period under consideration. The dynamic model also identified that the gap between the actual change in output and the desired change reduced slowly over the period under consideration. This, in turn, suggests that the conventional static model that assumes instantaneous adjustment of inputs is misspecified, and provides biased estimates of technical efficiency. Comparing the technical efficiency estimates from the dynamic model with those from a static model, I found that the static model underestimated technical efficiency of different sectors by 11 percentage points on average that could be as high as 54 percentage points.

Further, I found that the dynamic production model captured more variation in the time-pattern of technical efficiency of a production unit as compared to a static production model, and provided an internal ranking of production units considering their short-run adjustment of production plans. Particularly, for the private manufacturing sectors of Egypt, I found that efficiency of the sectors significantly increased after implementing privatization policies in the early 1990s that was captured by the dynamic production model but not by the static production model.

To conclude, this chapter has provided a more realistic and rigorous approach for capturing the dynamics of a production system, and measuring the speed of adjustment of output and technical efficiency both of which may vary over time. The dynamic production frontier, as discussed in this chapter is particularly suitable for country like Egypt where sluggish adjustment of inputs is a very plausible phenomenon in light of the facts that during the period under consideration, Egypt employed unskilled and semi-

skilled labor in the manufacturing sectors and also underwent through several changes in those sectors. Since producers often take important production decisions based on the efficiency of the units, a dynamic frontier that incorporates the short-run quasi-fixity of inputs is a reasonable one to use for this purpose.

The theoretical and econometric models, as discussed in this chapter, are based on the simplifying assumption that the speed of adjustment of inputs is similar for all inputs, and every production unit. However, different production units and inputs may have different speeds of adjustment. The econometric method for estimating such a dynamic production frontier with time-varying individual effects with large  $N$  (number of production units) and fixed  $T$  (time period under consideration) is an open research area till now. While this chapter does not discuss methods to estimate technical efficiency under less restrictive assumptions, these should be interesting areas of exploration for future research in this field.

## Appendix B: Tables and Figures

**Table B.1: Sectors and the Industrial Activities at the Three-digit ISIC Level**

<b>Sector Number</b>	<b>Industrial activity</b>
1	Food manufacturing
2	Other food manufacturing
3	Beverage and liquor
4	Tobacco
5	Manufacture of textile
6	Manufacture of wearing apparels
7	Manufacture of leather products
8	Manufacture of footwear
9	Manufacture of wood products
10	Manufacture of furniture & fixture
11	Manufacture of paper products
12	Printing and publishing industries
13	Manufacture of industrial chemicals
14	Manufacture of other chemical products
15	Other petroleum and coal
16	Manufacture of rubber products
17	Manufacture of plastic products
18	Manufacture of pottery and china
19	Manufacture of glass and glass products
20	Manufacture of other non metallic products
21	Iron and steel basic industries
22	Non-ferrous basic industries
23	Manufacture of fabricated metal products
24	Manufacture of machinery except electrical
25	Manufacture of electrical machinery
26	Manufacture of transport equipment
27	Manufacture of professional equipment
28	Other manufacture industries

**Table B.2: Variable Descriptions and Summary Statistics**

<b>Variable</b>	<b>Description</b>	<b>Observation</b>	<b>Mean</b>	<b>Standard deviation</b>	<b>Minimum</b>	<b>Maximum</b>
<b>Yearid</b>	id number for 9 years of data for each sector	252	5	2.59	1	9
<b>Sectorid</b>	id numbers for the 28 three digit manufacturing sectors	252	14.5	8.09	1	28
<b>Output</b>	Output quantity index	252	2888.90	3333.39	67	19236
<b>Capital</b>	Capital quantity index	252	288.84	475.29	1	3437
<b>Labor</b>	Labor quantity index	252	273.34	344.06	10.50	1689.2
<b>Energy</b>	Energy quantity index	252	61.97	116.56	0.20	860.1
<b>Material</b>	Material quantity index	252	1823.44	2168.83	44.8	11853.8

Source: Getachew and Sickles (2007).

**Table B.3: Coefficient of the Lagged Dependent Variable in the Time-Varying Technical Efficiency Model**

<b>Year</b>	<b>Coefficient of lagged dependent variable</b>	<b>t-ratio</b>
4	0.069	5.61
5	0.073	8.43
6	0.084	7.83
7	0.095	10.77
8	0.087	12.26
9	0.082	20.41

*Note:* The results are from the two-stage least squares analysis. The standard errors are corrected for heteroskedasticity.

**Table B.4: Difference in the Time-Varying Technical Efficiency Estimates from Dynamic and Static Specifications**

<b>Variables</b>	<b>Mean</b>	<b>Maximum</b>
Technical Efficiency_Dynamic	90.26	100
Technical Efficiency_Static	79.48	100
Difference in Efficiency Estimates	17.12	54.47
Underestimation by the Static Model	10.77	54.47

*Note:* The technical efficiency estimates from the dynamic and the static models are presented in percentage terms. These estimates show the efficiency level of a production unit relative to the most efficient unit in the sample.

The difference in efficiency estimates is calculated by taking the absolute difference in the technical efficiency estimates from the dynamic and the static model. The difference in efficiency estimates and the underestimation by static model are presented in terms of percentage points.

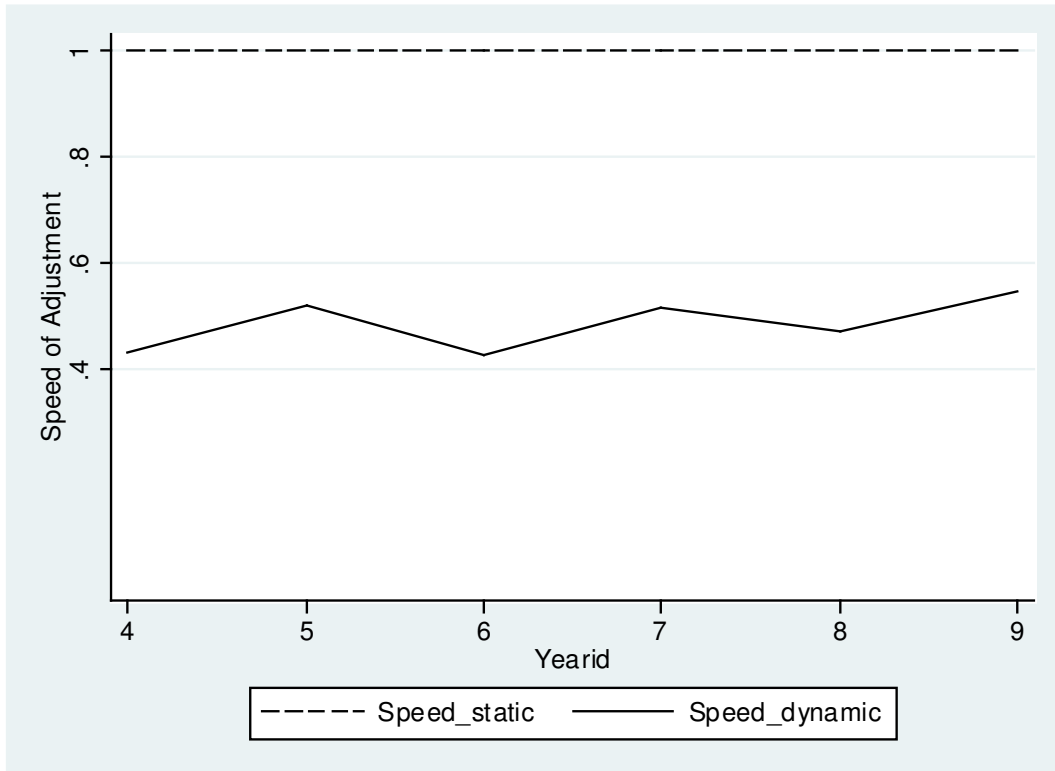
**Table B.5: Average Time-Varying Technical Efficiency Estimates and Ranking of Sectors under Dynamic and Static Specifications**

Sectorid	Technical Efficiency from Dynamic Specification (%) (1)	Technical Efficiency from Static Specification (%) (2)	Rank_Dynamic Specification (3)	Rank_Static Specification (4)
1	90.07	81.19	13	14
2	89.18	67.03	21	25
3	92.19	86.81	3	7
4	89.09	71.25	22	22
5	89.62	84.95	18	8
6	88.45	82.72	28	12
7	90.22	99.55	12	2
8	88.52	88.55	24	6
9	88.91	91.85	23	5
10	89.67	76.13	16	18
11	88.50	47.61	26	28
12	90.36	84.18	10	10
13	89.95	80.61	15	15
14	89.97	69.99	14	23
15	92.06	100.00	4	1
16	94.77	97.04	1	4
17	90.59	82.81	9	11
18	94.30	82.17	2	13
19	90.78	74.78	7	21
20	90.78	69.28	8	24
21	89.47	56.08	19	27
22	90.26	98.02	11	3
23	91.83	75.53	5	20
24	89.63	84.61	17	9
25	91.72	77.94	6	17
26	88.47	58.61	27	26
27	88.52	76.07	25	19
28	89.41	80.23	20	16

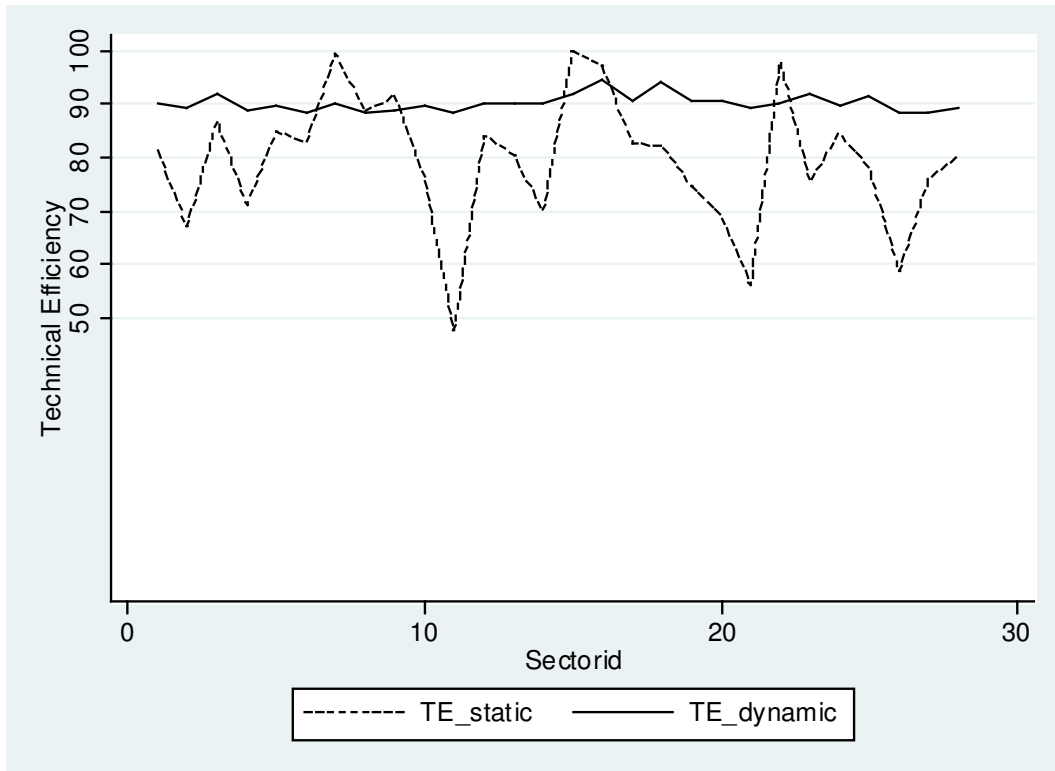
*Note:* Technical efficiency of a sector is measured relative to the most efficient sector.



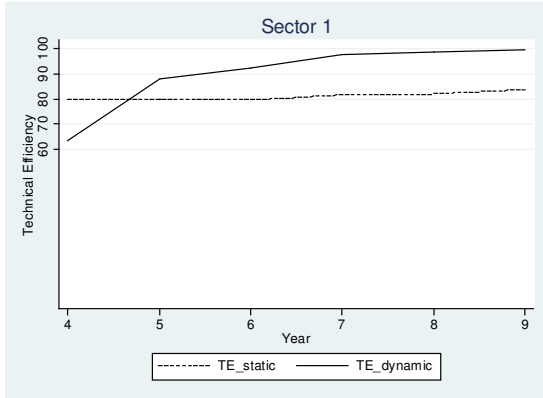
**Figure B.1: Speed of Adjustment from Dynamic and Static Specifications**



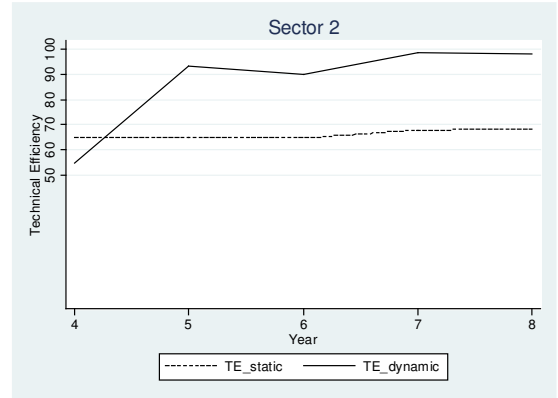
**Figure B.2: Average Time-Varying Technical Efficiency for Sectors from Dynamic and Static Specifications**



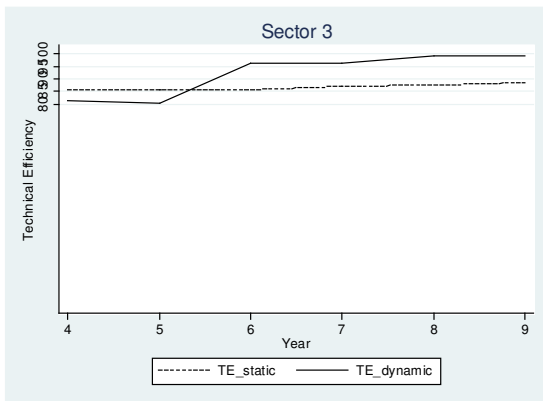
**Figure B.3: Time-Varying Technical Efficiency for Different Sectors**



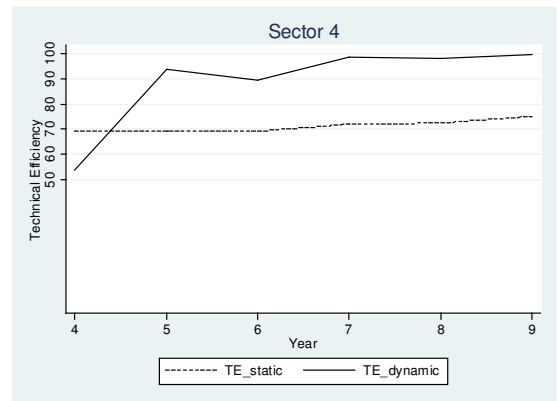
**(Figure B.3(i))**



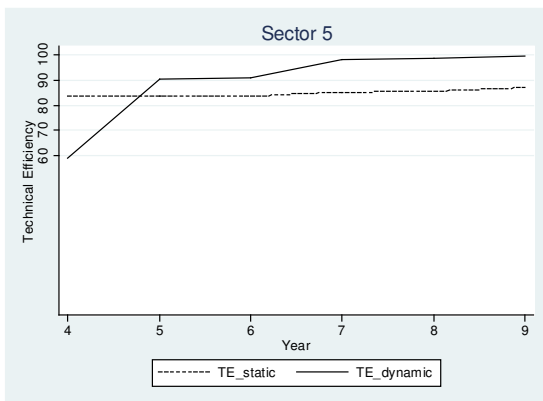
**(Figure B.3(ii))**



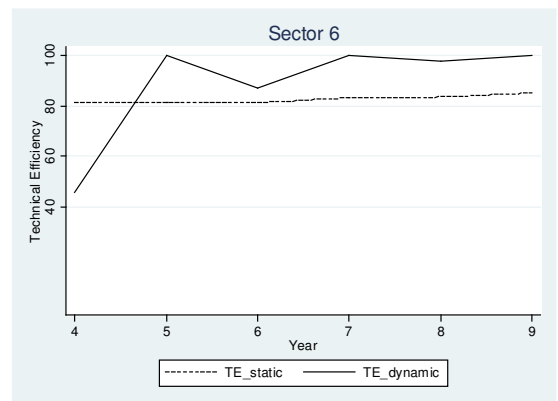
**(Figure B.3(iii))**



**(Figure B.3(iv))**

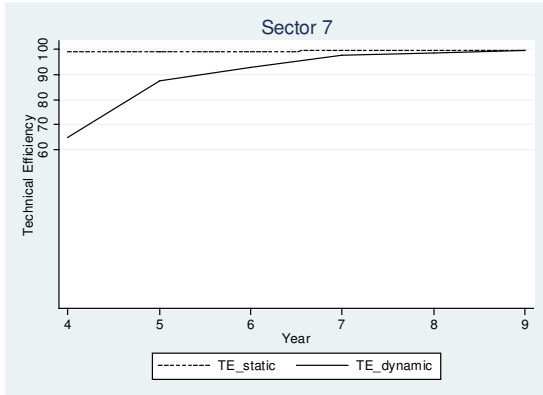


**(Figure B.3(v))**

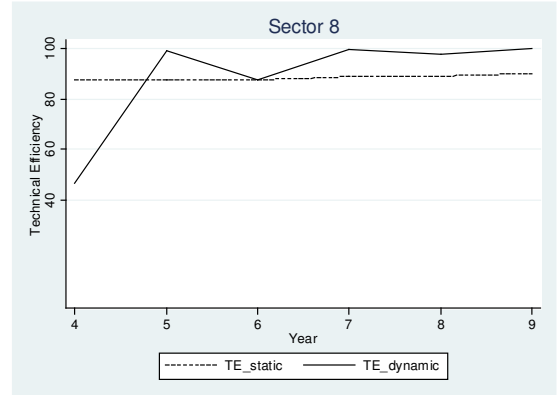


**(Figure B.3(vi))**

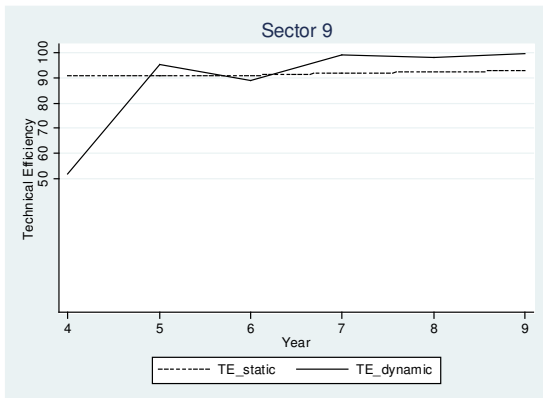
**Figure B.3 continued**



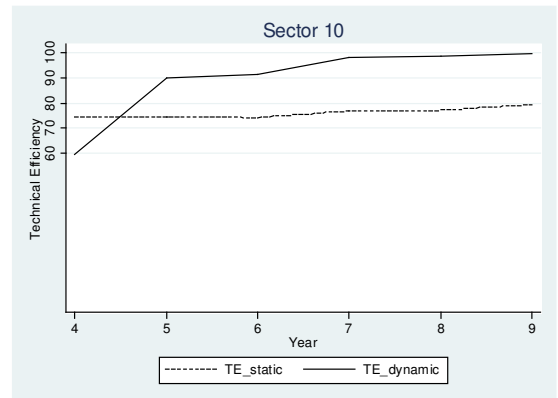
**(Figure B.3(vii))**



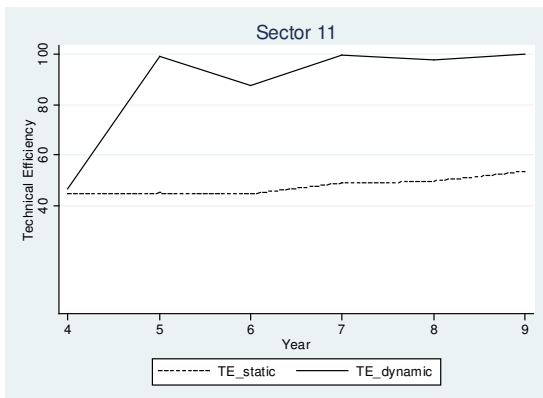
**(Figure B.3(viii))**



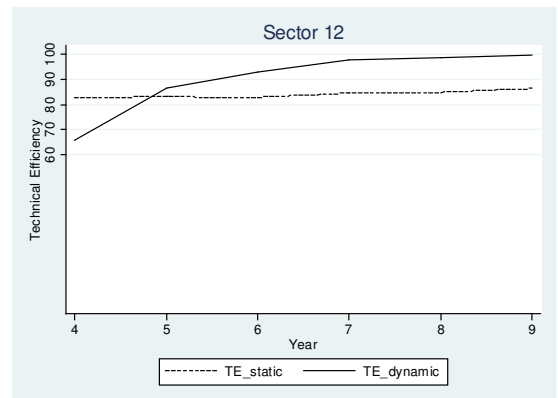
**(Figure B.3(ix))**



**(Figure B.3(x))**

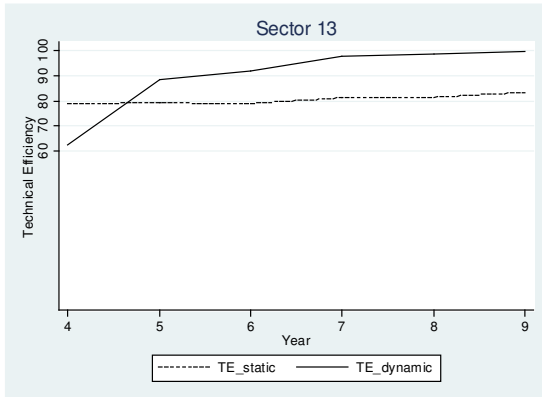


**(Figure B.3(xi))**

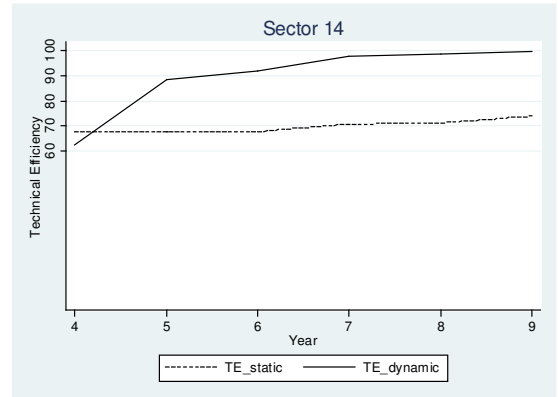


**(Figure B.3(xii))**

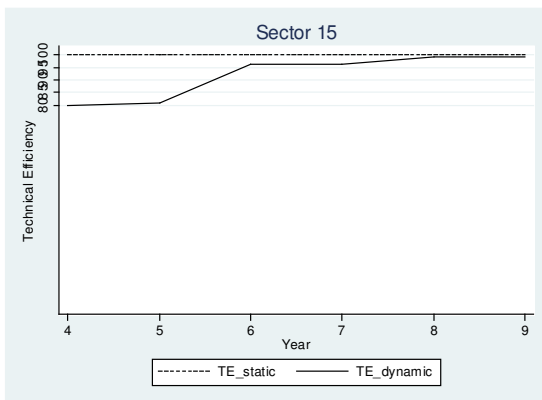
**Figure B.3 continued**



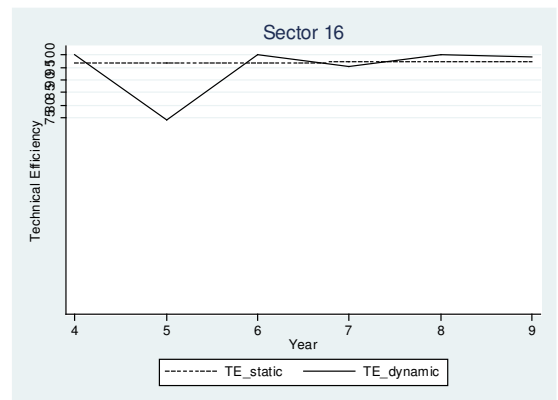
**(Figure B.3(xiii))**



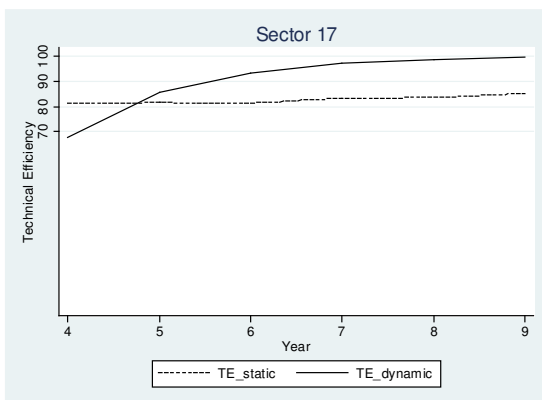
**(Figure B.3(xiv))**



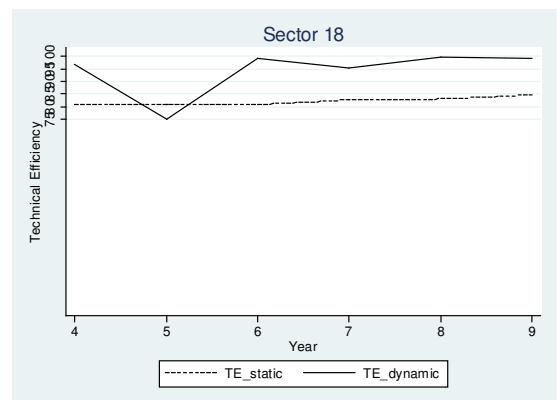
**(Figure B.3(xv))**



**(Figure B.3(xvi))**

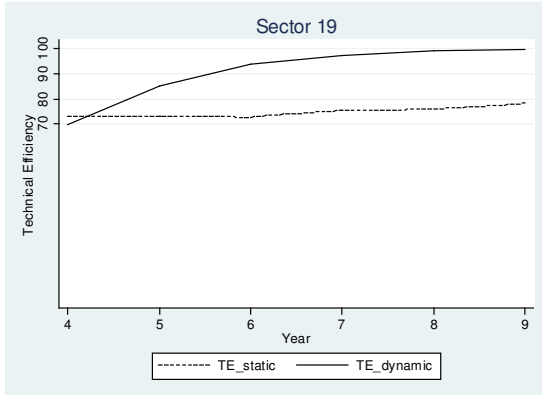


**(Figure B.3(xvii))**

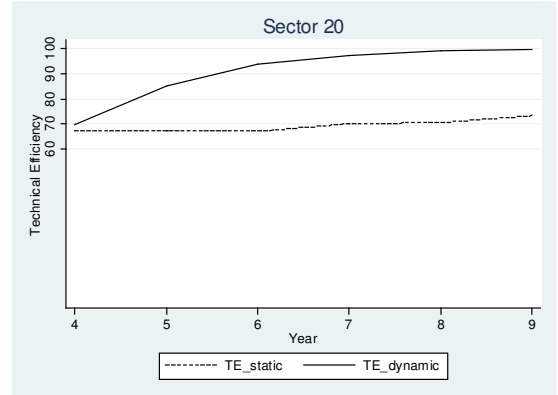


**(Figure B.3(xviii))**

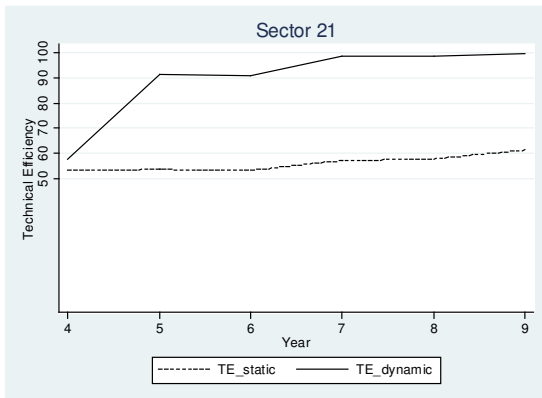
**Figure B.3 continued**



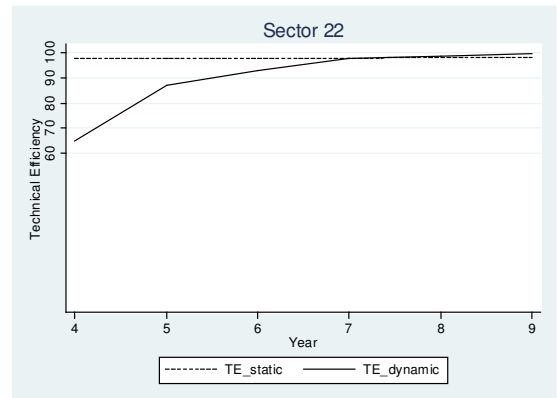
**(Figure B.3(xix))**



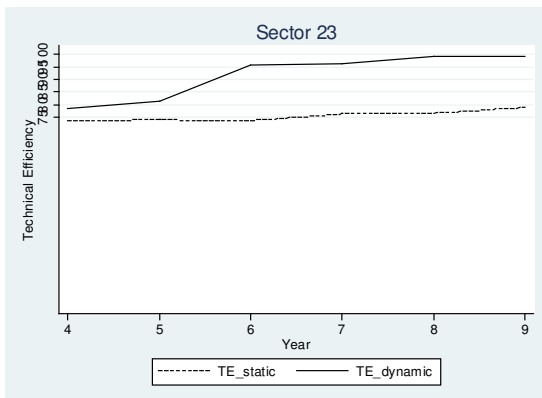
**(Figure B.3(xx))**



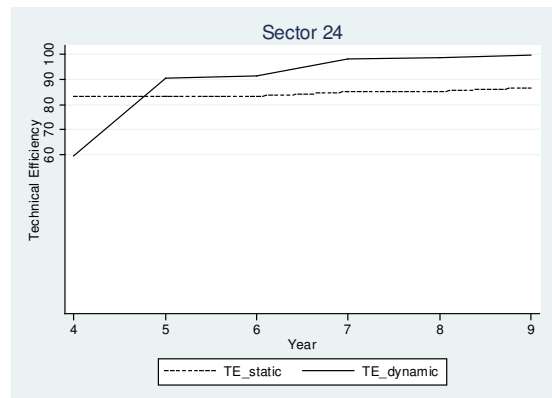
**(Figure B.3(xxi))**



**(Figure B.3(xxii))**

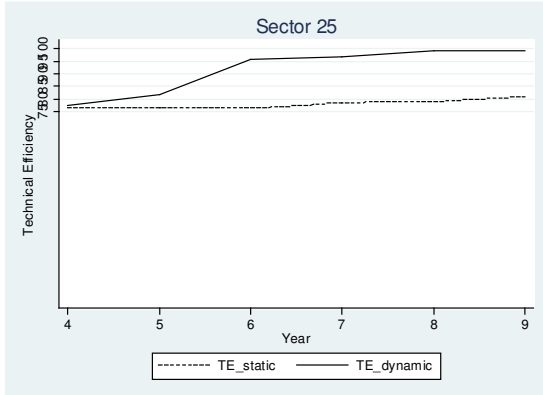


**(Figure B.3(xxiii))**

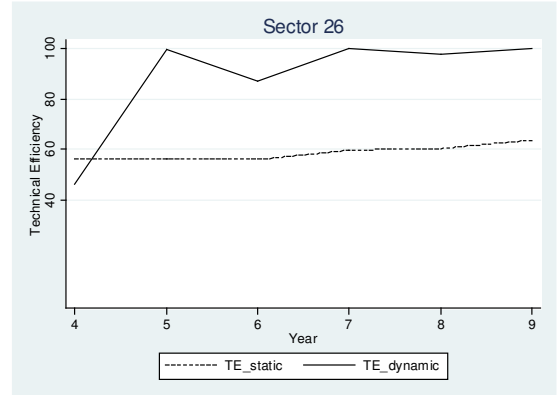


**(Figure B.3(xxiv))**

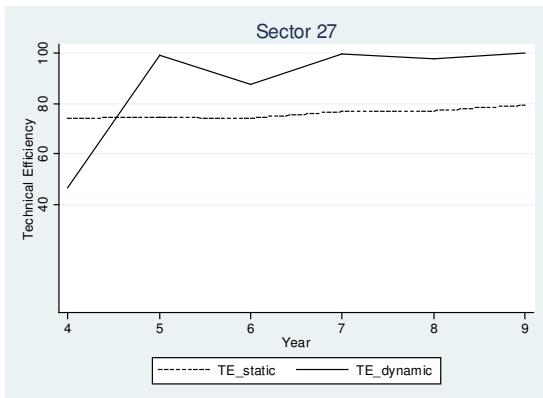
**Figure B.3 continued**



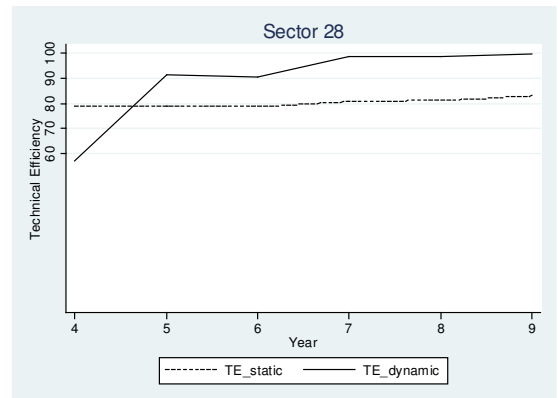
**(Figure B.3(xxv))**



**(Figure B.3(xxvi))**



**(Figure B.3(xxvii))**



**(Figure B.3(xxviii))**

## Chapter 4

### Median-based Rules for Decision-making under Complete Ignorance

#### 4.1. Introduction

This chapter characterizes a class of rules for decision-making under the type of non-probabilistic uncertainty, which were first axiomatically analyzed by Arrow and Hurwicz (1972). Under this type of uncertainty, the agent knows different possible states of the world and the outcome of each of her actions for each state, but does not have any probabilistic information, such as exact probabilities, the likelihood ranking<sup>20</sup>, or probability intervals<sup>21</sup> for these states. Following Arrow and Hurwicz (1972), several writers (see, for example, Maskin (1979), Barrett and Pattanaik (1984), and Barbera and Jackson (1988)), have discussed different rules of decision-making under uncertainty of the Arrow-Hurwicz type. All these contributions, however, focus on ‘max’-based or ‘min’-based rules and variants of such rules. In light of the agent’s usually limited capacity for processing information, it seems intuitively plausible to assume that an agent, when confronted with the problem of choice under uncertainty, may concentrate

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<sup>20</sup> See Kelsey (1986) for a discussion of decision-making when the agent has only the likelihood ranking of the states, but not their exact probabilities.

<sup>21</sup> See Gilboa and Schmeidler (1989) for a model of decision-making where the agent has a probability interval for each state of the world.



on some ‘focal’ outcomes<sup>22</sup> for each action. It is, however, not clear why the agent will necessarily look only at the extreme outcomes, i.e., the best or worst outcomes, of each action. An alternative focal point for each action may be its median outcome(s)<sup>23</sup>. The ranking of actions on the basis of their extreme outcomes involves excessive optimism or pessimism on the part of the agent. In contrast, the focus on the median outcome(s) in ranking alternative actions can be interpreted as a characteristic of more balanced behavior. Though decision rules based on the median outcome(s) seem to have considerable intuitive plausibility, the structure of these rules in the Arrow-Hurwicz framework has not been explored so far. The purpose of this chapter is to fill this gap in the literature by providing an axiomatic characterization of a class of median-based decision rules for choice under non-probabilistic uncertainty of the Arrow-Hurwicz type<sup>24</sup>.

The structure of the chapter is as follows. In section 4.2, we introduce the basic notation and assumptions. Section 3 presents the axioms with illustrative examples. The main result and its proof are given in section 4.4. Section 4.5 contains an example of a median-based rule. Finally, section 4.6 concludes.

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<sup>22</sup> The idea that the agent may consider only some focal outcomes of each available action goes back to Milnor (1954) and Shackle (1954). It may be worth recalling that the paper of Arrow and Hurwicz (1972) was published in a volume in honor of Shackle.

<sup>23</sup> For a precise definition of the median outcome(s) of an action, see Section 2 below.

<sup>24</sup> Nitzan and Pattanaik (1984) characterize a class of median-based decision rules in a framework which was first introduced by Kannai and Peleg (1984), and which is very different from that of Arrow and Hurwicz (1972). Nitzan and Pattanaik, (1984), as well as Kannai and Peleg (1984), do not introduce the states of the world into their model; they assume that the agent knows only the set of outcomes for each action.

## 4.2. Notation and Assumptions

**Assumption 4.2.1.** The universal set of outcomes,  $X$ , is a non-empty and convex subset of  $\mathfrak{R}^n$ , where  $n$  is some fixed positive integer.

**Assumption 4.2.2.** The agent has a convex ordering  $\succsim$  over  $X$ , such that for some  $x, y \in X$ ,  $x \succsim y$  and not  $(y \succsim x)$ .

The asymmetric and symmetric factors of  $\succsim$  are given by  $\succ$  and  $\sim$ , respectively.

Let  $d(x, y)$  denote the Euclidean distance between  $x, y \in X$ .

A decision problem is defined by a (finite) non-empty set of states of the world,  $s$ . Let  $Z$  be the class of all decision problems and let the elements of  $Z$  be denoted by  $S, S', S''$  etc. Given  $S \in Z$ , let  $A(S)$  denote the set of all possible functions  $a: S \rightarrow X$ . The elements of  $A(S)$  are called actions. For a decision problem  $S = \{s_1, \dots, s_m\}$ , where  $m$  is a positive integer, an action specifies exactly one  $n$ -tuple of real numbers for each of the  $m$  states of the world, and hence, can be thought of as an  $mn$  dimensional vector of real numbers. It may be worth noting here that this representation of actions as  $mn$  dimensional vectors of real numbers allows us later to introduce the property of continuity of the agent's ordering over actions for a given decision problem (see Assumption 4.2.3 below).

A typical decision problem with two states of the world  $S = \{s_1, s_2\}$ , two actions  $a, b \in A(S)$ , and outcomes  $a(s_1), a(s_2), b(s_1), b(s_2) \in X$  is described as follows—

$$\begin{array}{ccc}
& & S \\
& & s_1 \quad s_2 \\
a & a(s_1) & a(s_2) \\
b & b(s_1) & b(s_2)
\end{array}$$

An action  $a \in A(S)$  is *trivial*, if for all  $s, s' \in S$ ,  $a(s) = a(s')$ . A trivial action  $a \in A(S)$ , such that for all  $s \in S$ ,  $a(s) = x$ , is denoted by  $\llbracket x \rrbracket$ .

**Assumption 4.2.3.** For all  $S \in Z$ , the agent has a weak preference ordering  $R_S$  defined over  $A(S)$ , such that,

(i)  $R_S$  is continuous<sup>25</sup> over  $A(S)$ ,

and

(ii) for all  $x, y \in X$ ,  $x \succsim y$  iff  $\llbracket x \rrbracket R_S \llbracket y \rrbracket$ .

$I_S$  and  $P_S$  are the symmetric and asymmetric factors, respectively, corresponding to  $R_S$

**Remark 4.2.4.** Given Assumption 4.2.3, the ordering  $\succsim$  over  $X$  is continuous.

For all decision problems  $S \in Z$ , and, for all  $a, b \in A(S)$ , we write  $a * b$  iff the outcomes of action  $a$  corresponding to the different states of the world in  $S$ , constitute a permutation of the outcomes of action  $b$ .

Let  $S = \{s_1, s_2, \dots, s_m\} \in Z$  be a decision problem and let  $a \in A(S)$ . Let the outcomes in the set  $A(S)$  be indexed as  $x_1, x_2, \dots, x_m$  such that for all  $k$  ( $m > k \geq 1$ ),

---

<sup>25</sup> As noted earlier, actions for any given decision problem with  $m$  states of the world can be thought of as  $mn$  dimensional vectors of real numbers. Therefore, continuity of the agent's ordering over  $A(S)$  can be defined in the usual fashion.

$x_k \succ x_{k+1}$ <sup>26</sup>. Then the set of median outcome(s) of action  $a$  is denoted by  $med(a)$  and is

defined to be:  $\left\{x_{\frac{m-1}{2}+1}\right\}$  if  $m$  is odd, and  $\left\{x_{\frac{m}{2}}, x_{\frac{m}{2}+1}\right\}$  if  $m$  is even.

The agent follows a median-based rule iff for all  $S \in Z$  and for all  $a, b \in A(S)$ ,  $aI_S b$  if  $|med(a)| = |med(b)|$ <sup>27</sup> and there exists a one-to-one function  $h$  from  $med(a)$  to  $med(b)$  such that for all  $x \in med(a)$ ,  $x \sim h(x)$ .

For example, consider the following decision problem  $S = \{s_1, s_2, s_3, s_4, s_5, s_6\}$ , actions  $a, b \in A(S)$ , and outcomes  $x_1, x_2, x'_2, x_3, x'_3, x''_3, y_3, y_4, y'_4, y_5, y_6, y'_6 \in X$  such that,  $x'_2 \sim y'_4$ ,  $x_3 \sim y_5$ ,  $x_1 \succ x_2 \sim x'_2 \succ x_3 \sim x'_3 \sim x''_3$ , and  $y_3 \succ y_4 \sim y'_4 \succ y_5 \succ y_6 \sim y'_6$ .

	$S$					
	$s_1$	$s_2$	$s_3$	$s_4$	$s_5$	$s_6$
$a$	$x'_2$	$x_3$	$x'_3$	$x_1$	$x_2$	$x''_3$
$b$	$y_5$	$y_3$	$y_4$	$y_6$	$y'_4$	$y'_6$

We choose the indexing of outcomes, such that,  $x_1 \succ x_2 \succ x'_2 \succ x_3 \succ x'_3 \succ x''_3$  and  $y_3 \succ y_4 \succ y'_4 \succ y_5 \succ y_6 \succ y'_6$ . Then,  $med(a) = \{x'_2, x_3\}$ ,  $med(b) = \{y'_4, y_5\}$ , and a median-based rule will yield  $aI_S b$ .

<sup>26</sup> If there are more than one way of indexing the outcomes in this fashion, we choose one of them and keep it fixed.

<sup>27</sup>  $|med(a)|$  represents cardinality of the set of median outcome(s) from action  $a$ .

Note, that the class of median-based rules is not necessarily a singleton. Consider the following decision problem  $S = \{s_1, s_2\}$  and two actions  $a, b \in A(S)$  such that,  $b(s_1) \succ a(s_1)$  and  $a(s_2) \succ b(s_2)$ . It is easy to see that, both  $aP_S b$  and  $bP_S a$  are consistent with a median-based rule as we have defined it. This shows that a median-based rule need not be unique.

	$S$	
	$s_1$	$s_2$
$a$	$a(s_1)$	$a(s_2)$
$b$	$b(s_1)$	$b(s_2)$

### 4.3. The Axioms

We shall now introduce several plausible properties that the agent may satisfy. The properties are also illustrated with examples. We shall later characterize median-based rules in terms of these properties.

**Axiom 4.3.1.** *Neutrality:* Suppose  $S, S' \in Z$ ,  $|S| = |S'|$ ,  $a, b \in A(S), a', b' \in A(S')$ .

Further, suppose there exists a one-to-one function  $f$  from  $S$  to  $S'$  such that, for all  $s, \hat{s} \in S$ ,  $[a(s) \succ b(\hat{s}) \text{ iff } a'(f(s)) \succ b'(f(\hat{s}))]$ ,  $[b(\hat{s}) \succ a(s) \text{ iff } b'(f(\hat{s})) \succ a'(f(s))]$ ,  $[a(s) \succ a(\hat{s}) \text{ iff } a'(f(s)) \succ a'(f(\hat{s}))]$ ,  $[a(\hat{s}) \succ a(s) \text{ iff } a'(f(\hat{s})) \succ a'(f(s))]$ ,  $[b(s) \succ b(\hat{s}) \text{ iff } b'(f(s)) \succ b'(f(\hat{s}))]$ , and  $[b(\hat{s}) \succ b(s) \text{ iff } b'(f(\hat{s})) \succ b'(f(s))]$ . Then  $[aR_S b \text{ iff } a'R_{S'} b']$  and  $[bR_S a \text{ iff } b'R_{S'} a']$ .

Suppose two decision problems  $S$  and  $S'$  have equal number of the states of the world. Neutrality then requires that, if the ranking of outcomes from two actions  $a$  and  $b$ , in decision problem  $S$  is “analogous” to the ranking of the outcomes of two actions  $a'$  and  $b'$ , in the decision problem  $S'$ , then the ranking of  $a$  and  $b$  will be similar to the ranking of  $a'$  and  $b'$ .

For example, neutrality implies [ $aR_S b$  iff  $a'R_{S'} b'$ ] and [ $bR_S a$  iff  $b'R_{S'} a'$ ] in the following two decision problems  $S$  and  $S'$  where  $s'_1 = f(s_1)$ ,  $s'_2 = f(s_2)$ , and  $p, q, r, x, y, z \in X$  such that  $p \succ q \succ r \succ x \succ y \succ z$ .

$S$			$S'$		
	$s_1$	$s_2$		$s'_1$	$s'_2$
$a$	$r$	$y$	$a'$	$q$	$x$
$b$	$z$	$p$	$b'$	$y$	$p$

Neutrality has several plausible implications that have been discussed in the literature for decision-making under complete uncertainty. First, neutrality implies that the identities of the states of a decision problem do not matter while ranking actions in a decision problem; only order of the outcomes under different states matters. Thus, neutrality is similar to the well-known ‘symmetry’ axiom introduced by Arrow and Hurwicz (1972), but it is stronger than the ‘symmetry’ axiom. The ‘symmetry’ axiom as discussed in Arrow and Hurwicz (1972) requires the image set of the mapping from one decision problem to the other to be identical with the domain set, whereas, the image set can be different than the domain set under neutrality.

Second, neutrality implies that, while ranking two actions, only the ranking of outcomes from these two actions are relevant. The ranking of outcomes, at least one of which does not occur in the two actions under consideration, is of no importance. This may be noted as “Independence of the Irrelevant Outcomes”.

Thus, in the presence of neutrality, only the ordering of the relevant outcomes under the different states is considered. At first sight, this may seem implausible. Consider the following example with two states and two actions where the outcomes are assumed to be monetary magnitudes.

	$S$			$S'$	
	$s_1$	$s_2$		$s'_1$	$s'_2$
$a$	7	1	$a'$	7000	4.99
$b$	5	5	$b'$	5	5

Suppose, an outcome  $x$  is at least as good as  $y$  iff  $x \geq y$ . It is possible for an agent to have  $aI_S b$  and  $a'P_{S'} b'$ , violating neutrality. However, the Arrow-Hurwicz (1972) framework of complete ignorance provides only ordinal information about an agent’s preference over the outcomes. Since the ordering of outcomes from  $a, b \in A(S)$  is the same across states as the ordering of outcomes from  $a', b' \in A(S')$ , neutrality seems to be a plausible axiom in this framework.

**Lemma 4.3.2.** Suppose the agent satisfies neutrality. Then, for every decision problem  $S = \{s_1, s_2, \dots, s_m\} \in Z$  and for all actions  $\bar{a}, a \in A(S)$ , such that,  $a * \bar{a}$  and  $a(s_1) \succ a(s_2) \succ \dots \succ a(s_m)$ , we must have  $aI_S \bar{a}$ .

**Proof:** Let  $S = \{s_1, s_2, \dots, s_m\} \in Z$  and let  $\bar{a}, a \in A(S)$  such that  $a * \bar{a}$  and  $a(s_1) \succ a(s_2) \succ \dots \succ a(s_m)$ . Since  $a * \bar{a}$ , there exists a one-to-one function  $f$  from  $S$  to  $S$  such that for all  $s \in S$ ,  $a(s) = \bar{a}(f(s))$  and hence  $a(s) \sim \bar{a}(f(s))$ . Therefore, by neutrality [ $aR_S \bar{a}$  iff  $\bar{a}R_S a$ ] and [ $\bar{a}R_S a$  iff  $aR_S \bar{a}$ ]. Since, by connectedness of  $R_S$ , we have ( $aR_S \bar{a}$  or  $\bar{a}R_S a$ ), it follows that  $aI_S \bar{a}$ .

**Axiom 4.3.3. Duality:** Suppose  $S, S' \in Z$ ,  $|S| = |S'|$ ,  $a, b \in A(S)$ ,  $a', b' \in A(S')$ , and  $s, \hat{s} \in S$ . Further, suppose there exists a one-to-one function  $g$  from  $S$  to  $S'$  such that, for all  $s, \hat{s} \in S$ , [ $a(s) \succ b(\hat{s})$  iff  $b'(g(\hat{s})) \succ a'(g(s))$ ], [ $b(\hat{s}) \succ a(s)$  iff  $a'(g(s)) \succ b'(g(\hat{s}))$ ], [ $a(s) \succ a(\hat{s})$  iff  $a'(g(\hat{s})) \succ a'(g(s))$ ], [ $a(\hat{s}) \succ a(s)$  iff  $a'(g(s)) \succ a'(g(\hat{s}))$ ], [ $b(s) \succ b(\hat{s})$  iff  $b'(g(\hat{s})) \succ b'(g(s))$ ], and [ $b(\hat{s}) \succ b(s)$  iff  $b'(g(s)) \succ b'(g(\hat{s}))$ ]. Then [ $aR_S b$  iff  $b'R_{S'} a'$ ] and [ $bR_S a$  iff  $a'R_{S'} b'$ ].

Suppose two decision problems  $S$  and  $S'$  have the same number of states of the world. Duality then requires that, if the ranking of outcomes from two actions  $a$  and  $b$  in decision problem  $S$  is the 'reverse' of the ranking of the outcomes of two actions  $a'$  and  $b'$  in the decision problem  $S'$ , then the ranking of  $a$  and  $b$  must be the 'reverse' of the ranking of  $a'$  and  $b'$ .

In the following two decision problems  $S$  and  $S'$  such that  $s'_1 = g(s_1)$ ,  $s'_2 = g(s_2)$ ,  $s'_3 = g(s_3)$ ,  $p, q, r \in X$  and  $p \succ q \succ r$ , duality implies [ $aR_S b$  iff  $b'R_{S'} a'$ ] and [ $bR_S a$  iff  $a'R_{S'} b'$ ]



	S					S'		
	$s_1$	$s_2$	$s_2$		$s'_1$	$s'_2$	$s'_3$	
$a$	$p$	$q$	$r$		$a'$	$r$	$q$	$p$
$b$	$r$	$r$	$p$		$b'$	$p$	$p$	$r$

**Axiom 4.3.4. Weak Dominance:** For all decision problems  $S \in Z$ , and for all  $a, b \in A(S)$ , if  $a(s) \succ b(s)$  for all  $s \in S$ , then  $aR_S b$ .

Thus, if, for every state of the world, an action yields an outcome that is better than the outcome from another action, then the former action is at least as good as the later one. For example, in the following decision problem S, where  $a, b \in A(S)$ ,  $p \succeq q \succ r \in X$ , weak dominance requires  $aR_S b$ .

	S	
	$s_1$	$s_2$
$a$	$p$	$q$
$b$	$r$	$r$

#### 4.4. The Main Result

**Proposition 4.4.1.** Suppose, Assumptions 4.2.1 through 4.2.3 hold. Then the agent follows a median-based rule if she satisfies neutrality, duality, and weak dominance.

We proceed to the proof of Proposition 4.4.1 via a series of lemmas. Throughout the proof, it is to be understood that Assumptions 4.2.1, 4.2.2, and 4.2.3 hold, and the agent satisfies neutrality, duality, and weak dominance.

Let  $S = \{s_1, \dots, s_m\} \in Z$  be any decision problem such that  $\bar{a}, b, a \in A(S)$ ,  $|med(\bar{a})| = |med(b)|$ , there exists a one-to-one function  $h$  from  $med(\bar{a})$  to  $med(b)$  such that for all  $x \in med(\bar{a})$ ,  $x \sim h(x)$ ,  $a * \bar{a}$ , and  $a(s_1) \succ \dots \succ a(s_m)$  (4.4.1)

We assume that (4.1) holds for the rest of our discussion.

**Lemma 4.4.2.** For all  $S \in Z$  such that  $|S| \leq 2$ , and for all  $a, b \in A(S)$  such that  $b(s) \sim v$  for all  $s \in S$ , and  $\{v\} = med(a)$ , we must have  $aI_S b$ .

*Proof:* If  $|S| = 1$ , then,  $aI_S b$  follows immediately by reflexivity of  $R_S$ . If  $|S| = 2$ , then,  $aI_S b$  follows from reflexivity of  $R_S$  and neutrality.

**Lemma 4.4.3.** Let  $S = \{s_1, \dots, s_{2m+1}\} \in Z$  be such that  $m$  is a positive integer. Let  $a, b \in A(S)$  be such that  $a(s_1) \succ a(s_2) \succ \dots \succ a(s_{2m+1})$ , and  $b(s) \sim a(s_{m+1})$  for all  $s \in S$ . Then, we must have  $aI_S b$ .

*Proof:* Consider  $S$  and  $a, b \in A(S)$  as specified in the statement of Lemma 4.4.3. For the sake of convenience, we represent  $a, b, a', b' \in A(S)$  as follows:

$S$					
	$s_1$	$s_2$	.....	$s_{2m}$	$s_{2m+1}$
$a$	$a(s_1)$	$a(s_2)$	.....	$a(s_{2m})$	$a(s_{2m+1})$
$b$	$b(s_1)$	$b(s_2)$	.....	$b(s_{2m})$	$b(s_{2m+1})$
$a'$	$a(s_{2m+1})$	$a(s_{2m})$	.....	$a(s_2)$	$a(s_1)$
$b'$	$b(s_{2m+1})$	$b(s_{2m})$	.....	$b(s_2)$	$b(s_1)$

Recall that  $b(s) \sim a(s_{m+1})$  for all  $s \in S$  and  $a(s_1) \succ a(s_2) \succ \dots \succ a(s_{2m+1})$ . Hence, by neutrality,  $aI_S a'$  and  $bI_S b'$ . By transitivity of  $R_S$ , we then have -

$$(aR_S b \text{ iff } a'R_S b') \text{ and } (bR_S a \text{ iff } b'R_S a') \quad (4.4.2)$$

By duality, we have -

$$(aR_S b \text{ iff } b'R_S a') \text{ and } (bR_S a \text{ iff } a'R_S b') \quad (4.4.3)$$

By connectedness of  $R_S$ , either  $aP_S b$ , or  $bP_S a$ , or  $aI_S b$ . If  $aP_S b$ , then, by (4.4.2) and (4.4.3),  $a'P_S b'$  and  $b'P_S a'$ , which is a contradiction. Similarly  $bP_S a$  yields a contradiction. Thus we must have  $aI_S b$ .

**Lemma 4.4.4.** Let  $S = \{s_1, \dots, s_{2m}\} \in Z$  be such that  $m$  is a positive integer. Let  $a, b \in A(S)$  be such that  $a(s_1) \succ a(s_2) \succ \dots \succ a(s_m) \succ a(s_{m+1}) \succ a(s_{m+2}) \succ \dots \succ a(s_{2m})$ ,  $[b(s_i) \sim a(s_m)]$  for  $i = 1, \dots, m$ , and  $[b(s_i) \sim a(s_{m+1})]$  for  $i = m+1, \dots, 2m$ . Then we must have  $aI_S b$ .

*Proof:* The proof follows exactly similar logic as described in the proof of Lemma 4.4.3.

**Lemma 4.4.5.** Let  $x, y \in X$  be such that  $x \succ y$  and let  $\tilde{\epsilon}$  be such that  $d(x, y) > \tilde{\epsilon} > 0$ .

Then, for every positive integers  $m$ , there exist  $w_1, w_2, \dots, w_m \in ]x, y[$  such that  $x \succ w_1 \succ w_2 \succ \dots \succ w_m \succ y$ , and  $\tilde{\epsilon} > d(w_1, y) > d(w_2, y) > \dots > d(w_m, y)$ .

*Proof:* We first show that,

for all  $q, r \in X$  such that  $q \succ r$ , and all  $\tilde{\epsilon}$  such that  $d(q, r) > \tilde{\epsilon} > 0$ , there exists  $w \in ]q, r[$  such that  $\tilde{\epsilon} > d(w, r) > 0$ , and  $q \succ w \succ r$ . (4.4.4)

Let  $q, r \in X$  be such that,  $q \succ r$ . Let  $\tilde{\epsilon}$  be such that,  $d(q, r) > \tilde{\epsilon} > 0$ .

Since  $q \succ r$ , by convexity of  $\succ$ ,  $w \succ r$  for all  $w \in ]q, r[$ . (4.4.5)

Noting  $q \succ r$ , by the continuity of  $\succ$ , there exists  $w \in ]q, r[$  such that  $\tilde{\epsilon} > d(w, r) > 0$  and  $q \succ w$ . (4.4.6)

(4.4.4) follows from (4.4.5) and (4.4.6).

Now, let  $x, y \in X$  such that  $x \succ y$ , let  $\tilde{\epsilon}$  be such that  $d(x, y) > \tilde{\epsilon} > 0$ , and let  $m$  be any positive integer.

Since  $x, y \in X$ ,  $x \succ y$ , and  $d(x, y) > \tilde{\epsilon} > 0$ , by (4), there exists  $w_1 \in ]x, y[$ , such that  $\tilde{\epsilon} > d(w_1, y) > 0$ , and  $x \succ w_1 \succ y$ . (4.4.7)

Since  $w_1, y \in X$ ,  $w_1 \succ y$ , and  $d(w_1, y) > \epsilon' > 0$ , for some positive  $\epsilon'$ , by (4.4.4) again, there exists  $w_2 \in ]w_1, y[$  such that,  $\epsilon' > d(w_2, y) > 0$ , and  $w_1 \succ w_2 \succ y$ . (4.4.8)

Thus, we have  $w_1, w_2 \in ]x, y[$  such that,  $d(x, y) > \tilde{\epsilon} > d(w_1, y) > \epsilon' > d(w_2, y)$ , and  $x \succ w_1 \succ w_2 \succ y$ . Continuing in this fashion, for all  $x, y \in X$  such that,  $x \succ y$ , all  $\tilde{\epsilon}$  such that  $d(x, y) > \tilde{\epsilon} > 0$ , and every positive integers  $m$ , there exist  $w_1, w_2, \dots, w_m \in ]x, y[$  such that  $x \succ w_1 \succ w_2 \succ \dots \succ w_m \succ y$ , and  $\tilde{\epsilon} > d(w_1, y) > d(w_2, y) > \dots > d(w_m, y)$ .

Lemma 4.4.3 showed that, in a decision problem with an odd number of states of the world, if two actions  $a$  and  $b$  are such that  $b$  always yields outcomes that are indifferent to the median outcome of  $a$  and if  $a$  does not yield indifferent outcomes for any two distinct states of the world, then  $a$  and  $b$  must be indifferent. Our next lemma, Lemma 4.4.6, extends Lemma 4.4.3 by relaxing the requirement that  $a$  does not yield

indifferent outcomes for any two distinct states of the world. Lemma 4.4.7 extends Lemma 4.4.4 in an analogous fashion.

**Lemma 4.4.6.** Let  $S = \{s_1, \dots, s_{2m+1}\} \in Z$  be such that  $m$  ( $m \geq 1$ ) is a positive integer. Let  $a, b \in A(S)$  be such that,  $a(s_1) \succsim a(s_2) \succsim \dots \succsim a(s_{2m+1})$ , and  $b(s) \sim a(s_{m+1})$  for all  $s \in S$ . Then, we must have  $aI_S b$ .

*Proof:* Consider  $S = \{s_1, \dots, s_{2m+1}\} \in Z$ . Let  $a, b \in A(S)$  be such that  $a(s_1) \succsim a(s_2) \succsim \dots \succsim a(s_{2m+1})$ , and  $b(s) \sim a(s_{m+1})$  for all  $s \in S$ . Now, partition  $S$  into  $S_1, S_2, \dots, S_t$  such that, [for all  $j \in \{1, \dots, t\}$ , and all  $s, s' \in S_j$ ,  $a(s) \sim a(s')$ ], and [for all  $j \in \{1, \dots, t-1\}$ , all  $s \in S_j$ , and all  $s' \in S_{j+1}$ ,  $a(s) \succ a(s')$ ]<sup>28</sup>. For all  $j \in \{1, \dots, t\}$ , let  $m(j)$  be the cardinality of  $S_j$ .

By our assumption, there exist  $y_0, y \in X$  such that  $y_0 \succ y$ . Then, by Lemma 4.4.5, there exists  $y_1, \dots, y_t \in ]y_0, y[$  such that,  $y_0 \succ y_1 \succ y_2 \succ \dots \succ y_t \succ y$  and  $d(y_0, y) > d(y_1, y) > d(y_2, y) > \dots > d(y_t, y)$ . Let  $\varepsilon$  be a positive number such that  $\varepsilon = \min(d(y_0, y_1), d(y_1, y_2), \dots, d(y_{t-1}, y_t))$ . Consider  $(\varepsilon/k)$ , where  $k$  ( $k \geq 2$ ) is any positive integer. Then, for every  $j \in \{1, \dots, t\}$ , by Lemma 4.4.5, there exist  $w_{j,1}, w_{j,2}, \dots, w_{j,m(j)}$  such that,  $y_{j-1} \succ w_{j,1} \succ w_{j,2} \succ \dots \succ w_{j,m(j)} \succ y_j$ , and  $(\varepsilon/k) > d(w_{j,1}, y_j) > d(w_{j,2}, y_j) > \dots > d(w_{j,m(j)}, y_j)$ .

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<sup>28</sup> If  $t = 1$ , then  $a, b \in A(S)$  are trivial actions such that, Lemma 4.6 follows immediately, by reflexivity of  $R_S$ .

Let  $a_{\varepsilon/k}$  be an action such that for every  $j \in \{1, \dots, t\}$ , and  $n \in \{1, 2, \dots, m(j)\}$ ,  $a(s_{m(1)+m(2)+\dots+m(j-1)+n}) = w_{j,n}$ . It is clear by lemma 4.5 that for all  $s \in S$ ,  $a_{\varepsilon/k}(s) \succ a(s)$ , and  $(\varepsilon/k) > d(a_{\varepsilon/k}(s), a(s)) > 0$ .

Hence, as  $k \rightarrow \infty$ ,  $a_{\varepsilon/k}$  converges to action  $a$ . (4.4.9)

Further, note that, for every  $k$ ,  $V_k \succ V$ , where  $\{V_k\} = \text{med}(a_{\varepsilon/k})$  and  $\{V\} = \text{med}(a)$ . Then, by Lemma 4.4.2,  $a_{\varepsilon/k} I_S \llbracket V_k \rrbracket$  for every  $k$ <sup>29</sup>. Again, by Assumption 4.2.3,  $\llbracket V_k \rrbracket P_S \llbracket V \rrbracket$ . Hence, by transitivity of  $R_S$ ,  $a_{\varepsilon/k} P_S \llbracket V \rrbracket$  for all  $k$ .

Finally, noting (4.4.9) and  $a_{\varepsilon/k} P_S \llbracket V \rrbracket$ , we have  $a R_S \llbracket V \rrbracket$ , by continuity of  $R_S$ . (4.4.10)

All that remains to be shown is that  $\llbracket \text{not } a P_S \llbracket V \rrbracket \rrbracket$ , which given (4.4.10), will give us  $a I_S \llbracket V \rrbracket$ . Suppose  $a P_S \llbracket V \rrbracket$ . Then, by continuity of  $R_S$ , there exists large enough  $k'$ , such that,  $a P_S \llbracket V_{k'} \rrbracket$ . But, given  $a_{\varepsilon/k} I_S \llbracket V_k \rrbracket$  for every  $k$ , we must have  $a_{\varepsilon/k'} I_S \llbracket V_{k'} \rrbracket$ . Therefore, by transitivity of  $R_S$ , it follows that, for some  $k'$ ,  $a P_S a_{\varepsilon/k'}$ . This contradicts weak dominance, since, as we noted earlier,  $a_{\varepsilon/k}(s) \succ a(s)$  for all  $s \in S$ . This completes the proof of Lemma 4.4.6.

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<sup>29</sup> Note, that as  $k \rightarrow \infty$ ,  $\llbracket V_k \rrbracket$  converges to  $\llbracket V \rrbracket$ .

**Lemma 4.4.7.** Let  $S = \{s_1, \dots, s_{2m}\} \in Z$  be such that  $m$  ( $m > 1$ ) is a positive integer. Let  $a, b \in A(S)$  be such that,  $a(s_1) \succ a(s_2) \succ \dots \succ a(s_m) \succ a(s_{m+1}) \succ a(s_{m+2}) \succ \dots \succ a(s_{2m})$ ,  $[b(s_i) \sim a(s_m)]$  for  $i = 1, \dots, m$ , and  $[b(s_i) \sim a(s_{m+1})]$  for  $i = m+1, \dots, 2m$ . Then, we must have  $aI_S b$ .

**Proof:** Consider  $S = \{s_1, \dots, s_{2m}\} \in Z$  such that  $m$  ( $m > 1$ ) is a positive integer. Let  $a, b \in A(S)$  be such that,  $a(s_1) \succ a(s_2) \succ \dots \succ a(s_m) \succ a(s_{m+1}) \succ a(s_{m+2}) \succ \dots \succ a(s_{2m})$ ,  $[b(s_i) \sim a(s_m)]$  for  $i = 1, \dots, m$ , and  $[b(s_i) \sim a(s_{m+1})]$  for  $i = m+1, \dots, 2m$ . Now, partition  $S$  into  $S_1, S_2, \dots, S_t$  such that, [for all  $j \in \{1, \dots, t\}$ , and all  $s, s' \in S_j$ ,  $a(s) \sim a(s')$ ], and [for all  $j \in \{1, \dots, t-1\}$ , all  $s \in S_j$ , and all  $s' \in S_{j+1}$ ,  $a(s) \succ a(s')$ ]. The rest of the proof is similar to the proof of Lemma 4.4.6.

**Proof of Proposition 4.4.1:** Finally, given (4.4.1) and Lemma 4.3.2, we have  $aI_S \bar{a}$ . Thus, by transitivity of  $R_S$ , and following Lemmas 4.4.2, 4.4.3, 4.4.6, and 4.4.7, we must have  $\bar{a}I_S b$ .

## 4.5. An Example

We have shown that, given Assumptions 4.2.1, 4.2.2, and 4.2.3, if the agent satisfies neutrality, duality, and weak dominance, she must follow a median-based rule. We now give an example where Assumptions 4.2.1, 4.2.2, and 4.2.3 as well as neutrality, duality, and weak dominance are all satisfied.

Let  $X$  be any non-empty and convex subset of  $\mathfrak{R}^n$  and let  $\succsim$  be any convex and continuous ordering over  $X$ , such that, for some  $x, y \in X$ ,  $x \succ y$ . Thus, by our specification, Assumptions 4.2.1, 4.2.2, and 4.2.3 (i) are satisfied. Let  $U$  be a real valued and continuous utility function representing  $\succsim$ . Clearly, such a utility function  $U$  exists<sup>30</sup>. For every decision problem  $S \in Z$ , let  $R_S$  defined over  $A(S)$  be such that, for all  $a, b \in A(S)$ ,  $a R_S b$  iff  $\sum_{x \in med(a)} U(x) \geq \sum_{y \in med(b)} U(y)$ . Clearly, for every  $S \in Z$ ,  $R_S$  is continuous and for all  $x, y \in X$ ,  $[[x]] R_S [[y]]$  iff  $x \succsim y$ , and hence Assumption 4.2.3 (ii) is satisfied. Further, for all  $S \in Z$ , and for all  $a, b \in A(S)$ , if  $|med(a)| = |med(b)|$  and there exists a one-to-one function  $h$  from  $med(a)$  to  $med(b)$  such that for all  $x \in med(a)$ ,  $x \sim h(x)$ , then  $\sum_{x \in med(a)} U(x) = \sum_{y \in med(b)} U(y)$  and hence  $a I_S b$ .

## 4.6. Conclusion

Most of the papers, which discuss non-probabilistic uncertainty of the Arrow-Hurwicz type, focus on what may be called positional decision rules. The positional rules characterized in this literature mainly consider the best or the worst outcomes. Lexicographic variants of such rules have also been discussed. It is, however, surprising

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<sup>30</sup> See Debreu (1959).



that none of the papers in this area have dealt with the case when the agent makes decision on the basis of the median outcome(s) of her actions. In this chapter, we have sought to fill this gap by providing an axiomatic characterization of median-based rules.

## **Chapter 5**

### **Concluding Remarks**

This dissertation outlined a theory for a dynamic stochastic production frontier that described the process of output generation in the presence of lagged adjustment of inputs. Since the estimation methods for time-invariant and time-varying technical efficiency using such a dynamic model is not straightforward, I discussed the relevant estimation methods from the existing literature and extended it to suit the purpose of efficiency estimation. It further illustrated the methods of estimation using data from the private manufacturing sectors in Egypt, and found that the speed of adjustment of output was significantly lower than unity, both for the time-invariant and time varying technical efficiency models as discussed in chapter 2 and 3, respectively. This, in turn, suggests that the conventional static model that assumes instantaneous adjustment of inputs is misspecified, and provides biased estimates of technical efficiency. Chapter 3 also showed that as inputs got more time to adjust within a production system, the gap between the actual change and the desired change in output reduced over time.

By assuming instantaneous adjustment of inputs, a static production model attributes the shortfall in actual output arising from short-run adjustment of inputs to inefficiency of production units. However, the dynamic adjustment of inputs is a natural phenomenon that cannot be completely eliminated by the producers, and hence, the reduced output in the short-run should not be identified as inefficiency of the production

units. Comparing the technical efficiency estimates from the dynamic model with those from a static model, I found in chapter 2 that the static model underestimated time-invariant technical efficiency of Egyptian private manufacturing sectors by 4.5 percentage points on average and this underestimation went up to 17 percentage points for some sectors. Allowing for the technical efficiency to vary over time, in chapter 3, I similarly found that the static model underestimated efficiency of a sector at a time period by 11 percentage points on average and this underestimation could be as high as 54 percentage points for some sectors. Though it is clear that the static production model provides biased estimates of technical efficiency when the true process of output generation is dynamic, the direction of bias in the technical efficiency estimates are not uniquely identified in this analysis. The reason for that is the stochastic frontier approach, as followed in the first two chapters, identifies relative efficiency only and not the absolute level of technical efficiency of production units. The static model is expected to underestimate the absolute technical efficiency of production units by labeling the shortfall in actual production that results from the short-run quasi-fixity of inputs, but the deviation of each unit's efficiency level from the best-practice frontier can be either overestimated or underestimated by the static model.

I also found that the ranking of the sectors based on the efficiency measures from a dynamic model were substantially different when compared with the ranking obtained from a static model. Since production plans are often taken on the basis of efficiency measures, such measures should be derived considering the true production process. Therefore, if inputs require time to adjust within a production system, then a dynamic

production model is a more suitable one to measure efficiency of such a production system in the short-run. Based in this idea, Chapter 2 and 3 of this dissertation provided a more realistic and rigorous approach for capturing the dynamics of a production system, and measuring technical efficiency. In particular, chapter 3 offered a novel approach towards estimating the speed of adjustment of output and technical efficiency that vary over time, and thereby, captures the effect of lagged adjustment of inputs on output.

The theoretical and econometric models, as discussed in chapter 2 and 3, are based on the simplifying assumption that the speed of adjustment of inputs is similar for all inputs and every production unit. However, different production units are likely to have different speeds of adjustment. Similarly, the adjustment processes of different inputs are also likely to be different. While this dissertation does not discuss methods to estimate technical efficiency under less restrictive assumptions, these should be interesting areas of exploration for future research in this field. Further, it would be a novel area of future research to apply econometric techniques like bootstrapping method to compare the difference in the absolute technical efficiency estimates from static and dynamic models to uniquely identify the direction of bias in the estimates from a static model, when the inputs are not instantaneously adjusted.

Finally, chapter 5 characterized decision rules under a particular type of non-probabilistic uncertainty, known as complete ignorance, focusing on the median outcomes of an action taken by an agent. The chapter also provided an example of such median-based rules. Most of the papers, which discuss non-probabilistic uncertainty under complete ignorance, focus on what may be called positional decision rules. The

positional rules characterized in this literature mainly consider the best or the worst outcomes. Lexicographic variants of such rules have also been discussed. It is, however, surprising that none of the papers in this area have dealt with the case when the agent makes decision on the basis of the median outcome(s) of her actions. In chapter 5, I have sought to fill this gap by providing an axiomatic characterization of median-based rules. However, characterizing median-based rules when the agent has lexicographic preference is an interesting area of research yet to be considered.

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