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Structure of ^{29}F in the Rotation-aligned Coupling Scheme of the Particle-Rotor Model

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Abstract

Recent results from RIKEN/RIBF on the low-lying level structure of ^{29}F are interpreted within the Particle-Rotor Model. We show that the experimental data can be understood in the Rotation-aligned Coupling Scheme, with the $5/2^+$ ground state as the bandhead of a decoupled band. In this picture, the energy of the observed $1/2^+_1$ state correlates strongly with the rotational energy of the core and provides an estimate of the 2^+ energy in ^{28}O. Our analysis suggest a moderate deformation, $\epsilon_2 \sim 0.17$, and places the 2^+ in ^{28}O at ~ 2.4 MeV.

The structure of exotic neutron-rich nuclei is one of the main science drivers in contemporary nuclear physics research. Our current knowledge of nuclear structure, towards the driplines, has clearly established that the paradigm of magic numbers and doubly magic nuclei as we know it near stability changes across the nuclear landscape [1]. It is expected that these changes in the underlying single-particle structure are intimately related to specific aspects of the effective nuclear force, specifically to its tensor and three-body (or higher) components.

Thus, a detailed mapping of shell evolution and collectivity at the limits of isospin becomes a key element to understand the atomic nucleus and all of its many-body intricacies. The Islands of Inversion at N=8, 20, and 40 provide dramatic examples with the underlying physics mechanism driven by the important role of the neutron-proton force [1–4]. The effect of isospin on the monopole average of the central and tensor components of the force affects the neutron effective single-particle energies (ESPEs) in such a way that expected shell closures are quenched, opening the door for the collective degrees of freedom to become relevant in the low-lying excitation spectra of these systems, where single-particle excitations were anticipated.

The study of odd-A and odd-odd nuclei has traditionally been a tool of choice to disentangle the competition of single-particle and collective motion insofar as they can be regarded, at least a priori, as one or two valence nucleon(s) coupled to a core.

As nicely discussed in Ref. [5], a very appealing region to study is that near ^{28}O, which is today accessible experimentally and also theoretically with state-of-the-art large scale Shell Model calculations. The well known magic numbers, Z=8 and N=20, would anticipate ^{28}O as a doubly magic nucleus, however, the recent study of ^{29}F carried out at RIKEN/RIBF suggests otherwise [5]. As noted, the relatively low transition energy of the $1/2^+_1$ state in ^{29}F (1.08 MeV) largely disagrees with Shell Model predictions restricted to the *sd* model space, ≈ 3.5 MeV. Based on their calculations using the SDPF-M effective interaction, the authors determined that the N= 20 shell gap is quenched for ^{29}F and concluded: *"... extending the Island of Inversion to isotopes with proton number Z= 9."*

In this work we follow-up on this conclusion and ask whether the observed structure in ^{29}F is amenable to a description in terms of a collective picture [6], within the framework of the Particle-Rotor Model (PRM) [7, 8].

Considering ^{28}O as our core, an inspection of a Nilsson diagram [9] suggests that the well bound odd proton will occupy the single-j multiplet originating from the $d_{5/2}$ orbit, namely the levels $\frac{1}{2}$[220], $\frac{3}{2}$[211], and $\frac{5}{2}$[202], with its Fermi energy at the $\Omega = 1/2$.

The Hamiltonian of the system can be written as [7, 8]:

$$H = E_\Omega + \frac{\hbar^2}{2\mathscr{I}} I(I+1) + H_C \tag{1}$$

Here E_Ω relates to the intrinsic level energies, $\mathscr{I}$ is the core moment-of-inertia, and H_C the Coriolis coupling term given by

$$H_C = -\frac{\hbar^2}{2\mathscr{I}}(I_+ j_- + I_- j_+) \tag{2}$$

where $I_\pm$ and $j_\pm$ the ladder operators for the total and single particle angular momenta respectively.

Given the conditions above and for small to moderate deformations ($\epsilon_2 \sim 0.15$) the Coriolis matrix elements[1] ($\sim \hbar^2 I j/\mathscr{I}$) dominate over the intrinsic level spacings, ($\Delta E_{\Omega,\Omega\pm1} \sim \hbar\omega_0\epsilon_2$), and a rotation-aligned coupling limit is anticipated [10, 11]. In this case, the lowest-lying (*yrast*) band has spins $I = j,\ j+2,\ j+4,\ ...,$ and energy spacings equal to that of the core; this type of band is referred to as a decoupled band. Specific to our case, the ground state is then predicted to be $5/2^+$, for which a geometrical picture is shown on the left panel of Fig. 1.

[1] We have not used any explicit attenuation of these matrix elements in the PRM calculations.

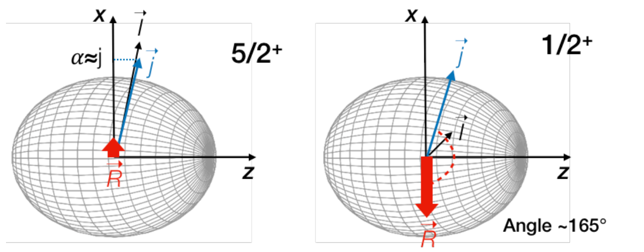

Figure 1: Schematic diagram to illustrate the vector coupling for the lowest states in ^{29}F. The black arrows represent the total momentum, $\vec{I}$, while the blue arrows represent $\vec{j}$ and the red arrows the core rotation $\vec{R}$.

As illustrated on the right panel, the excited $1/2^+_1$ state must have, by necessity, anti-parallel coupling of $\vec{j}$ with the core rotation, $\vec{R}$.[2] Therefore it follows that in the decoupled limit ($\epsilon_2 \rightarrow 0$) the energy of the $1/2^+_1$ state with respect to the ground state is proportional to the rotational energy of the core, $E_{2^+}(core)$, and provides, in the case of ^{29}F, a proxy for the 2^+ energy in ^{28}O. Since the splitting of the Nilsson multiplet is proportional to the quadrupole deformation, $\sim \hbar\omega_0\epsilon_2$, we expect a trade-off between $E_{2^+}(core)$ vs. ϵ_2, and thus a range of possible solutions matching the experimental results.

In Fig. 2 we compare PRM solutions as a function of the deformation, to a calculation of the core energy given by $E_{2^+} = 3\hbar^2/\mathscr{I}$ with the moment of inertia calculated using the Migdal formula [12–14],

$$\mathscr{I} = \frac{\mathscr{I}_{rigid}}{(1 + (\frac{2\Delta}{\hbar\omega_0\epsilon_2})^2)^{3/2}} \quad (3)$$

using an isospin dependent expression of the pairing gap, Δ, from Ref. [15], adjusted to this region of the nuclear chart.

The values E_{2^+} and ϵ_2 where the two curves intersect defines a consistent solution to the problem. To estimate an uncertainty in the adopted solution we take into account an uncertainty of $\sim \pm$ 13% in Δ entering the calculation of $\mathscr{I}$ [16], and obtain the blue shaded band, from which $E_{2^+} \approx 2.4 \pm 0.2$ MeV and $\epsilon_2 \approx 0.17^{+0.15}_{-0.2}$.

A given state, $|I, \alpha\rangle$, of angular momentum I and projection α onto the rotation axis (x-axis) has a wavefunction of the form:

$$|I, \alpha\rangle = \sum_{\Omega=1/2}^{5/2} C^{\alpha}_{I\Omega}|I, \Omega\rangle \quad (4)$$

It can be shown [10, 11], that in the rotation-aligned coupling limit, the amplitudes are given by the Wigner d-function evaluated at $\pi/2$, the angle between the symmetry and rotation axes:

$$C^{\alpha}_{I\Omega} = d^{j}_{\alpha,\Omega}(\pi/2) \quad (5)$$

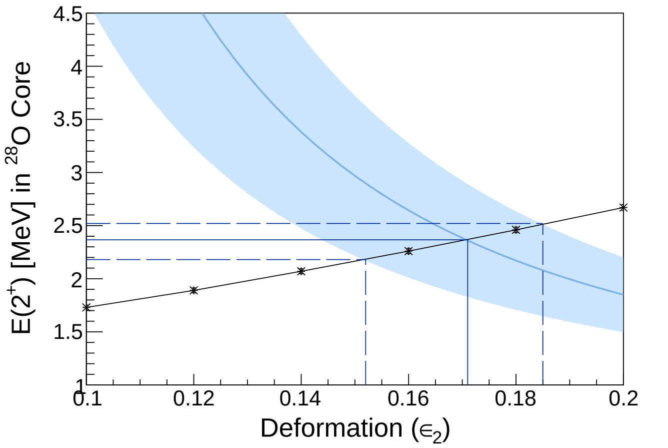

Figure 2: The $E_{2^+}(core)$ required to: 1) reproduce the energy of the $1/2^+_1$ state, 1.08 MeV, as a function of deformation (black line) and 2) calculated with the Migdal formula (blue line) using a pairing gap, Δ=1.5 MeV. The shaded band is an estimate of the theoretical error in the calculation of $\mathscr{I}$.

For our adopted PRM solution, the square amplitudes for the lowest two states, $5/2^+$ and $9/2^+$ of the *yrast* band ($\alpha = j = 5/2$) are given in Fig. 3 where they are compared to the limit above showing that the structure can be interpreted, indeed, as a decoupled band. Some geometrical and spectroscopic properties of the calculated low-lying levels are summarized in Table 1.

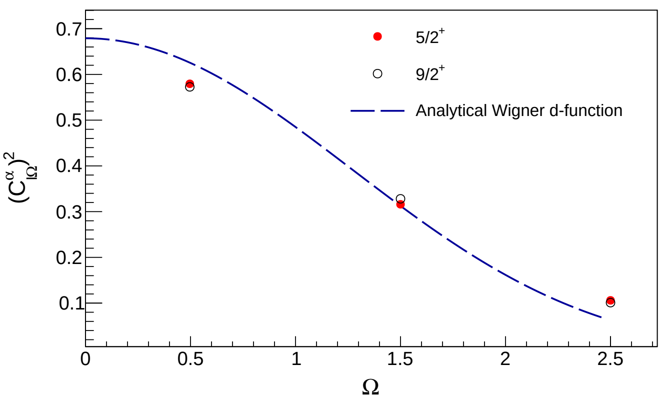

Figure 3: Wavefunctions of the $5/2^+$ (solid circles) and $9/2^+$ (open circles) states compared to the decoupled limit given by the d-function (dashed-line).

If the conditions for a rotational aligned coupling scheme persist also in ^{30}F, the odd-neutron will occupy the $\frac{1}{2}[330]$ level of the $f_{7/2}$ Nilsson multiplet. In complete analogy with the odd-A case, we expect a doubly-decoupled band structure [17] with spins $I = (j_\pi + j_\nu)$, $(j_\pi + j_\nu) + 2$, ..., also following the core spacings. Thus, we predict the ground state of ^{30}F to be 6^-. Of course, it is also possible that the occupation of the deformation driving $\frac{1}{2}[330]$ neutron level may polarize the core to a larger ϵ_2 and a strong coupling scheme be realized, with the $\Omega_\pi + \Omega_\nu$ configuration being favored and the $|\Omega_\pi - \Omega_\nu|$ nearby. In this limit the ground state will be 2^-. With the substantial difference

[2] The angle can be estimated from the semi-classical expression: $cos\ \theta = \frac{1}{2}(I(I+1) - R(R+1) - j(j+1))/\sqrt{R(R+1)j(j+1)}$, giving $\theta \approx 165°$.

Table 1: PRM results for the low-lying levels of ^{29}F. The lowest two states have been observed experimentally. Magnetic moments have been calculated with no-quenching of g_s, and $g_R = Z/A$. (In boldface we indicate the *yrast* band members).

State	Energy [MeV]	$\langle R \rangle$	E_{rot} [MeV]	$\langle I_z \rangle$	$\langle \vec{I} \cdot \vec{j} \rangle / \lvert I \rvert$	$\langle \vec{R} \cdot \vec{j} \rangle / \lvert R \rvert$	Magnetic Moment [μ_N]	Quadrupole Moment [eb]
5/2$^+$	0	0.67	0.43	0.08	2.65	-0.21	4.6	-0.06
$1/2^+$	1.08	1.84	2.00	0.5	1.83	-2.65	2.4	0
$3/2^+$	2.2	2.01	2.32	-1.04	1.58	-2.19	2.5	0.026
9/2$^+$	2.6	2.28	2.9	0.05	2.55	1.67	5.3	-0.1
$7/2^+$	3.2	2.1	2.50	0.48	2.25	0.12	4.1	-0.024

in spin predicted for the ground state in these two coupling schemes, a measurement of the (unbound) ground-state resonance in ^{30}F will be interesting to illuminate our understanding.

In summary, the recent experimental results of Ref. [5] and Shell Model calculations suggest the extension of the $N = 20$ Island of Inversion to the Fluorine isotopes. We have shown that the low-lying excitation spectrum of ^{29}F can be understood in terms of a collective picture, with a level structure corresponding to the rotation-aligned coupling limit of the PRM. The Coriolis coupling effects on the proton $d_{5/2}$ Nilsson multiplet give rise to a (favored) decoupled band. Thus, the $5/2^+$ bandhead naturally emerges as the ground state, and the $1/2^+$ as the first excited state, with its excitation energy depending directly on the $E_{2^+}(core)$. We have found a consistent solution at a deformation of $\epsilon_2 \approx 0.17^{+0.15}_{-0.2}$ that suggests an excitation energy of the 2^+ in ^{28}O at $E_{2^+} \approx 2.4 \pm 0.2$ MeV in line with the conclusions reached in Ref. [5] based on the SDPF-M effective interaction. PRM predictions for some spectroscopic observables were also presented. Similar conditions in ^{30}F would give rise to a $\pi d_{5/2} \otimes \nu f_{7/2}$ double-decoupled structure with a predicted 6^- ground state.

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References

References

[1] O. Sorlin and M. Porquet, Prog. Part. Nucl. Phys. **61**, 602 (2008), and references therein.
[2] A. Poves and J. Retamosa, Phys. Lett. **B 184**, 311 (1987).
[3] T. Otsuka, *et al.*, Phys. Rev. Lett **87**, 082502 (2001).
[4] K. Heyde and J. L. Wood, Rev. Mod. Physics, **83**, 1467 (2011).
[5] P. Doornenbal, *et al.*, Phys. Rev. **C 95**, 041301(R) (2017).
[6] A. Bohr and B. R. Mottelson, *Nuclear Structure Volume II*, (W. A. Benjamin, Inc., Advanced Book Program; Reading, Massachusetts; 1975).
[7] S. E. Larsson, G. Leander and I. Ragnarsson, Nucl. Phys. **A307**, 189 (1978).
[8] I. Ragnarsson and P. B. Semmes, Hyperfine Interactions **43**, 425, (1988).
[9] S. G. Nilsson, Selsk. Mat. Fys. Medd. **16**, 29 (1955).
[10] F. S. Stephens, R. M. Diamond, and S. G. Nilsson, Phys. Lett. **B 44**, 429 (1973).
[11] F. S. Stephens, Rev. Mod. Physics , **47**, 43 (1975).
[12] A. B. Migdal, Nucl. Phys. **13** 655, (1959).
[13] Ibid [6], page 82.
[14] R. Bengtsson, Proceedings of the XVIII Mikołajki Summer School On Nuclear Physics, Poland, page. 13, 1986.
[15] A.S. Jensen, P.G. Hansen, and B. Jonson, Nucl. Phys. **A431** 393 (1984).
[16] A. O. Macchiavelli, *et al.*, To be published.
An analysis of pairing gaps derived from Eq. (3) using known E_{2+} energies and quadrupole deformations across the nuclear chart shows deviations of this order when compared to gaps extracted from mass formulae.
[17] A. J. Kreiner, Z. Physik A **288** 373, (1978), and references therein.