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Publication Date

1958-12-01

UCRL 8438
cy B2

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Radiation Laboratory

BERKELEY, CALIFORNIA

UCRL-8438

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UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Berkeley, California
Contract No. W-7405-eng-48

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Printed for the U.S. Atomic Energy Commission

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ABSTRACT

Laboratory measurements involving gravitational interaction are limited by the small value of the gravitational coupling constant. The range of possible measurements can be extended by use of a resonant device and by specialized electronics techniques described in this paper. A method for determining the propagation velocity of gravitational interaction v_g is presented. The value of v_g is obtained by measuring the phase difference between electrical signals derived from a pair of electromechanical transducers. Appropriate variations in the experimental arrangement can be made so as to yield v_g in different media. The techniques presented are applicable to other problems in the laboratory; however, this paper deals specifically with the propagation-velocity experiment.

PROPOSED LABORATORY MEASUREMENT OF THE PROPAGATION VELOCITY OF GRAVITATIONAL INTERACTION*

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I. INTRODUCTION

Newton's law of universal gravitation gives the attractive force F between masses M_a and M_b a distance r apart:

$$F = G \frac{M_a M_b}{r^2} ,$$

where

$$G = 6.67 \times 10^{-8} \text{ cgs units.}$$

The small value for the gravitational coupling constant limits laboratory measurements involving gravitational interaction. In spite of this difficulty, the value of G was determined by Cavendish in 1798. Later investigators proposed the phenomenon of resonance as a means of increasing the deflection resulting from gravitational forces. Joly suggested a pendulum arrangement,† and von Eötvös described a torsion instrument, the "Gravitationmultiplicator."** These devices are essentially alike in that r varies periodically, so that the receiving mass M_b, which is part of a mechanical resonator, experiences oscillating forces. By adjusting the period of oscillation of the transmitting mass M_a, one can make the receiving mass M_b oscillate at resonance. Under this condition, the energy transfer between M_a and M_b is maximized, thus facilitating measurement of the deflection amplitude of M_b. Measuring this amplitude, together with knowing the internal friction of the receiver, leads to a value for G.

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*Work performed in part under the auspices of the U.S. Atomic Energy Commission.

†J. Joly, Nature 41, 256 (1890).

**R. von Eötvös, Wied. Ann. 59, 354-400 (1896).

This early work suggests that other experiments involving gravitational interaction might utilize a resonant device. As will be shown, the determination of the propagation velocity of gravitational interaction, v_g, is such an experiment. The method proposed is as follows: A "transmitter," consisting of a pair of connected masses M_T and M_T' rotates about the z axis (Fig. 1). Two "receiving" masses M_1 and M_2 are located a distance X apart on a radius from the z axis. As the transmitting masses M_T and M_T' revolve, the distances separating them from the receiving masses M_1 and M_2 vary periodically, so that M_1 and M_2 experience oscillating forces. The frequency of the oscillating forces is adjusted to the resonant frequency of the receivers. Owing to the distance X separating M_1 and M_2, and to the finite value of v_g, the phase of oscillation of M_2 will be slightly retarded with respect to that of M_1. By measuring this phase difference as a function of the distance X, one can obtain a value for v_g. A series of runs could be made starting with the minimum value of X geometrically possible, and continuing to the largest value of X at which a usable signal can be obtained from the receiver R_2.

The central problem of the experiment is the efficient utilization of the minute gravitational energy available. The method chosen is to generate electrical signals by means of appropriate transducers (described in the following section), amplify these signals, and subsequently compare the phases. What must be shown is that the signal energy is sufficiently above the noise energy that a phase measurement can be made in a reasonable time. To aid in showing this, some specific dimensions and parameters for the apparatus are given. These are not necessarily optimum, and many modifications will undoubtedly suggest themselves.

II. RECEIVERS

1. Geometry

Each receiver (R_1 and R_2 of Fig. 1) is highly evacuated and sealed off (perhaps including a getter), with electrical connections fed out through vacuumtight seals. A small adjustment of resonant frequency is possible in the sealed-off unit because the upper stem (B) vibrates slightly and its mass is adjustable externally. The mass center of the upper stem is relatively far from M_T so that the upper stem will not experience accelerations of the magnitude of those on M_1. As the transmitter rotates with the appropriate period, M_1 is set into motion relative to the walls of its enclosing quartz tube. A suggested resonant frequency is 50 cps for the receiver, in which case the transmitter should rotate at 25 rps.

2. Deflection Amplitude

For an assumed transmitter mass M_T of 20 kg, a receiver mass M_1 of 10 g, and a separation of 20 cm, the peak force F_{peak} exerted on M_1 is (in cgs units)

$$F_{peak} = \frac{6.67 \times 10^{-8} \times 20 \times 10^{3} \times 10}{20^{2}} \approx 34 \text{ microdynes.}$$

(The additional contribution from M_T' is very small at this time since it is at its most remote point.) Because the maximum value of r is large compared to its minimum value, the peak-to-peak value of F is nearly the value above of 34 microdynes, but the sinusoidal component is smaller. Assume a sinusoidal component F_{sin} of 7 microdynes peak at 50 cps. This force acting on a free mass of 10 g gives a peak deflection of

$$x_{peak} = \frac{F_{sin}}{M_1 \omega_r^{2}}$$

$$= \frac{7 \times 10^{-6}}{10\ (2\pi 50)^{2}}$$

$$x_{peak} \approx 7 \times 10^{-12} \text{ cm ,}$$

where ω_r is the angular frequency of the alternating force on the receivers.

M_1 does not, however, act as a free mass. Owing to resonance, the value of x_{peak} is multiplied by the factor Q, where

$$Q = \frac{\text{stored energy in resonator}}{\text{energy loss/radian}} .$$

If it is assumed that a value of $Q = 5000$ can be obtained at a low absolute temperature,* then the maximum deflection x_{max} is

$$x_{max} = 5000\,(x_{peak})$$

$$= 5000\,(7 \times 10^{-12})$$

$$x_{max} = 3.5 \text{ angstroms.}$$

This calculated value (3.5 A) for the maximum deflection is for a particular minimum spacing (20 cm) between M_T and R_1, and indicates the general magnitude of deflection expected in a receiver. Closer spacing may be possible for R_1, while for R_2 greater spacing will be used in the course of the experiment, out to a spacing where the amplitude falls too low to be usable. A deflection of 3.5 A, then, is to be compared to the deflection arising from thermal fluctuations in the receiver, an effect to be discussed in a later section.

3. Capacitor Plates

The relative motion of M_1 can be converted into an electrical signal by making it a part of some electrical circuit, such as the electrostatic

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*A value of $Q > 3000$ was measured for a simple Pyrex resonator operating at room temperature and at a resonant frequency of 87 cps. Because the viscosity of fused quartz is small [H. V. Neher, The Use of Fused Silica, in Procedures in Experimental Physics, ed. by J. Strong (Prentice-Hall, New York, 1938), Ch. V], a high Q is anticipated for the quartz stem (B) at room temperature. A higher Q is expected if we operate the receiver at low temperature. D. L. White, J. Appl. Phys. 29, 856-857 (1958), found that at liquid helium temperature the Q for natural quartz is more than ten times that at room temperature. For a particular vibrational mode he obtained a Q of 55 million. In his investigations the quartz was in a high precision AT crystal unit vibrating on overtones of the thickness shear at about 6.3 and 8.8 Mc.

system described here. The tungsten block M_1 is ground and polished optically flat on two parallel faces, which see corresponding flat faces on the inner wall of the enclosing quartz tube (Fig. 1). The faces (E), when metallized, form the outer plates of a capacitor of which M_1 is the center (moving) member. The quiescent position of M_1 is midway between the outer capacitor plates, with a suggested gap of 0.001 in.

One outer plate is connected to a fixed direct-voltage source of plus 100 v, and the other to minus 100 v. The block M_1, via a metallized line on the inner quartz stem, is connected to a high-impedance amplifier. The electric field between stationary and moving members is 100 v/0.001 in., or 400 μv/angstrom. This is the output signal obtained under motion if the amplifier input impedance is essentially infinite. The expected electrical signal is thus 400 μv/angstrom × 3.5 angstroms= 1.4 mv at the grid of the electrometer tube, discussed in the next section.

4. Amplifiers

An electrometer tube is chosen to observe the 1.4-mv signal. The miniature tube CK5886 (Raytheon) is a possible choice. Suppose M_1 is a tungsten disc 0.25-cm thick. The faces of M_1 will have an area of 2 cm^2 for a 0.5-cm^3 volume, if M_1 weighs approximately 10 g. The capacitance C_f of each face to a stationary plate is (in mks units)

$$C_f = \frac{K_0 A}{d} = \frac{8.85\times 10^{-12}\times 2\times 10^{-4}}{2.54\times 10^{-5}} \approx 70\times 10^{-12} \text{ farad},$$

where K_0 is the permittivity of free space, A is area in m^2, and d is the gap between faces in m. The total capacitance presented to the amplifier input circuit is 2 C_f, or 140 μ μ f. It will be advantageous to adjust the average potential of M_1 to such a value that it is under zero electrostatic force (the possible electrostatic force with unbalance greatly exceeds the oscillating gravitational forces). Therefore we choose a suitable grid resistor, based on considerations of noise and loading. The electrical signal is developed by a generator having an internal impedance equal to that of a 140- μ μ f capacitor; the open-circuit resistor noise is shunted by this capacitance. In this case, a high value of resistance R does not impose the thermal-noise penalty of the open-circuit resistor, for which the mean-square noise emf $\overline{V}^2$ is:

$$\overline{V}^2 = 4(kTR)df ,$$

where df is the bandwidth, T is the absolute temperature, and k is Boltzmann's constant. Instead, the noise for increasingly large R is asymptotic to a value depending upon C_f.* One expects also noise contributions from the tube, from the flicker effect and from shot noise. In general, flicker noise is greater at low frequencies f and may follow a 1/f law, but it should be measured for a particular tube.†

5. Synchronous Frequency-Conversion

Referring to Fig. 2, one sees that some reduction in noise would appear at a frequency above 50 cps. There is a useful method of avoiding the excess noise at low frequency without raising the transmitter frequency. That is to replace the direct-current signal on the fixed plates of the receivers R_1 and R_2 by an audiofrequency signal of say 5 kc. The input signal to the electrometer tube then consists of a pair of sidebands of frequencies 5 kc ± 50 cps, with no accompanying carrier. The modulation process is of the form:

$$\sin \omega_r t \ \sin \omega_a t = \tfrac{1}{2} \cos (\omega_r t - \omega_a t) - \tfrac{1}{2} \cos (\omega_r t + \omega_a t),$$

where ω_a is the audiofrequency and ω_r is the angular frequency of the alternating force on the receivers.

The phase is preserved, so that measuring the phase relation between a pair of similar sidebands is equivalent to a phase measurement at the frequency ω_r, provided that ω_a comes to both receivers from a common source. Frequency conversion of this nature is incorporated in the circuit of Fig. 3 where desirable.

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*A. Von der Ziel, Noise (Prentice-Hall, New York, 1954), p. 13.

†Figure 2 shows this noise measurement as a function of frequency for the CK5886. A 140-μμf capacitor shunted by a 10^{10}-ohm resistor comprises the input circuit (which simulates a receiver for this noise test).

The upper and lower sidebands in this example are sufficiently well separated (100 cps) so that either may be selected with a filter. (Special forms of phase detectors may use both sidebands.)

The electrometer-circuit input impedance must be kept very high to avoid loading the high-Q mechanical resonator; reactive loading will shift the resonant frequency (and hence the phase); resistive loading will reduce the Q.* The input impedance, using an appropriate feedback signal to the base of the 10^{10}-ohm resistor, can be kept suitably high in the frequency range chosen.

6. Signal-to-Noise Ratio of the Receiver

The measured noise is about 0.1 μv for a bandwidth of 1 cps. The proposed receiver (given Q = 5000 and the resonant frequency $f_0 = 50$ cps) has a bandwidth $\Delta f = f_0/Q = 0.01$ cps, with a corresponding time constant $\tau = 2Q/\omega \approx 33$ sec. (This is the time for equilibrium to be re-established in the oscillating system following a temperature change or a change in $F_{\sin}$.)

In order to reduce noise of thermal origin in the receiver itself, we will operate the system at low absolute temperature; liquid helium temperature is suggested. At 4.2°K the energy in the thermal fluctuations of M_1 is of the order $kT = (1.4 \times 10^{-16})\, 4.2 \approx 6 \times 10^{-16}$ erg. We now compare this to the signal energy.

Starting from zero initial signal energy, we find that the circulating signal energy builds up after a time τ to approximately $M_1 v^2$, where v is the velocity ($v = \omega Q x_{\max}$) of M_1. In terms of force, the signal energy E_s is

$$E_s \doteq F_{\sin}(x_{\max})\, Q^2,$$

which gives a signal-to-noise ratio S/N of

$$S/N = \frac{F_{\sin}(x_{\max})\, Q^2}{kT}$$

$$= \frac{5 \times 10^{-6} \times 7 \times 10^{-12} \times 25 \times 10^{6}}{6 \times 10^{-16}}$$

$$S/N \approx 10^{6},$$

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[*] H. F. Olson, Dynamical Analogies (Van Nostrand, New York, 1943), pp. 138-141.

where we use the values $F_{\sin} = 5 \times 10^{-6}$ dyne, $x_{\max} = 7 \times 10^{-12}$ cm, and $Q = 5000$.

7. Receiver Mounting

It is proposed to use a pair of identical receivers R_1 and R_2 and associated amplifiers. The amplifier is physically small; it can be close to the receiver and be powered by batteries. The amplifier output signal could be fed out through the suspension, or be electromagnetically coupled to the external phase-measuring circuit. As a result, the problem of isolating the receiver unit from spurious vibration is reduced to the problem of a single suspension, which could be accomplished magnetically.*

8. Receiver Response

The receiver properties are fundamental to the experiment and must be measured carefully. A way to select a pair of similar receivers is to excite them from a common source of light or microwave radiation chopped at the desired frequency. A radiation pressure of ≈ 5 microdynes rms (5×10^{-11} newton) during the "on" part of the cycle should be exerted on the mass M_1. The required power beamed toward M_1 is $\approx 5 \times 10^{-11}\,c = 15$ mw (where c is the velocity of light). The Q-curves of the receivers can be plotted. Aging effects on the resonant frequency can be observed over a period of time on several receivers (1 part in 10^9 days is anticipated). The relative phase angle of different pairs can be determined with the device described in Sec. IV. If a significant phase difference appears when the source-to-receiver distances are equal, a relative adjustment of receiver

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*With a magnetic suspension, position detectors would electronically control the field. The instantaneous equilibrium is not required to be stable; if the system responds in 1 μ sec, the receiver may fall for part of this time and then be picked up. The resulting displacement S is insignificant:

$$S = \tfrac{1}{2} g t^2 \approx \tfrac{1}{2} \times 10^3 \times 10^{-12} = 0.05 \text{ angstrom},$$

where g is the acceleration of gravity. The restoring forces may be quite non-Hookian if desired, and such a suspension may be given excellent properties as a mechanical filter.

resonant frequency is required. Near the resonant frequency f_0 there will be a phase shift $\Delta\phi \approx (Q\,\pi/2)\,(\Delta f/f_0)$, where Δf is the difference between the driving frequency and f_0. In the above fashion, a pair of receivers having similar Q, similar drift characteristics, and closely matched resonant frequency may be obtained.

9. Receiver Shielding

The receivers must be shielded from spurious fields. Spurious gravitational fields varying at the resonant frequency of the receiver are of course very unlikely, but electromagnetic fields may be anticipated. The effectiveness of proposed shielding for such fields can be tested by placing a receiver in a field much stronger than the measured spurious fields of the transmitter and noting receiver response. Because of the high Q and the coherent detection scheme, only frequencies in a very narrow band near the operating frequency will cause significant disturbance.

III. THE TRANSMITTER

1. Geometry

The transmitter might consist of a pair of 20-kg masses (M_T and M_T' of Fig. 1) connected by a cable or rod 2 meters long and about 3 cm in diameter. The assembly would be rotated at a frequency of 25 rps to excite the receivers R_1 and R_2 (tuned to 50 cps. There are no appreciable alternating stresses in the material, and the problem of decoupling the transmitter and receiver for spurious mechanical signals is simplified, since the fundamental frequency of the transmitter does not excite the receiver. Some properties of the "mass-dipole" transmitter are listed in Table I. Quadrupole and other higher-order configurations have been suggested. The connecting rod or cable might be tapered to achieve some gain in the mass-to-strength ratio of the assembly.

2. Frequency Control

The transmitter is servo-controlled to rotate at a frequency determined by one of the receivers, say R_1. Since the transmitter will be operated in vacuum, the necessary correcting torques will be those required to overcome bearing friction and compensate for slow creep in cable length under tension. Temperature can be controlled to eliminate thermal effects as a cause of frequency drift.

The transmitter is inherently stable over short periods of time. Over longer periods, it will be locked to the frequency of the receivers. It could in fact be locked to an atomic clock, but it is more reasonable to use a receiver (or a mean of both receivers) as the standard, since only relative frequency drift is of great importance.

3. Spurious Radiation

Stray electric and magnetic fields from the transmitter can be detected with sensitive instrumentation, and suitable shields designed and tested. The vibration coupled from the transmitter through its suspension can be monitored continuously by a pickup device similar to the receiver, but directly attached to the suspension rather than isolated. A magnetic suspension for the transmitter would allow it to choose its own spin axis with the restrictions that the vertical force of gravity be just overcome; a couple ($\approx$ 1 in. -lb) should be applied to cause precession of the correct magnitude to follow the earth's rotation.

Table I

Dipole Transmitter Properties

General		Proposed values
mass	M_T	$M_T \approx 20$ kg
cable length	2ρ	$\rho \approx 1$ meter
cable tension	$T_c = M_1\omega^2\rho$	$T_c \approx 50$ tons
angular frequency	ω	$\omega \approx 50\pi$ radians/sec
angular momentum	$2M_1\rho^2\omega$	$I\omega \approx 10^4$ kg meter^2/sec (I is the total moment of inertia of the transmitter)
kinetic energy	$T_c\rho$	$T_c\rho \approx 1/2$ megajoule
peripheral speed	$\omega\rho$	$\omega\rho \approx$ Mach 1/2

4. Optical Reference System

The x, y, and z axes (Fig. 1) may be maintained by an optically-controlled servo system, as follows: Receiver R_2 is arranged to be placed on a line through R_1 and the center of mass of the transmitter. Small flats on the extremities of the transmitter masses contain fiducial marks. The instantaneous position of each mark is observed as it crosses near the x axis by a pulsed light of nanosecond duration,* a phototube, and an electronic time discriminator. Such a system provides the control information to the servo which maintains the angular velocity and the position of the transmitter relative to R_1 and R_2.

Even with such a system, very slight misalignments which may vary with time are inevitable. To compensate for the slight inaccuracies in phase measurements resulting from this situation, additional pairs of receivers could be used. One pair could be located on the (-)x axis diametrically opposite R_1 and R_2. Two other pairs could be located in a similar manner on the y and (-)y axes. The signals from the inner receivers could be averaged to give $\vec{S}_1$, and the signals from the outer receivers averaged to give $\vec{S}_2$.

5. Excitation of Receiver Modes

The receivers can be arranged to respond to either the x or y component of the force. By suitably choosing the cross section of the quartz stem supporting M_1 in the receiver, the x and y vibration modes can be given frequencies differing by more than $1/Q$ and hence be excited separately. The x and y forces as a function of time have the form shown in Fig. 4.

IV. MEASURING ULTRASMALL PHASE ANGLES

The receivers R_1 and R_2 do not operate in the wave zone, but in the induction zone, much less than a wavelength away. The phase detector must therefore be capable of measuring a very small phase difference. If we suppose that a 5-kc carrier is applied to the stationary plates of a

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*Q. A. Kerns, I. R. E. Trans. on Nuclear Sci. NS-3, No. 4, 115 (1956).

receiver, then we will have to make a phase measurement between signals of 5050 cps, choosing the upper sidebands from each receiver. If the receivers provide usable signals for a range of separations X of 30 cm, the phase angle θ will vary by

$$\Delta\theta = 2\pi f_0 \Delta\tau = 2\pi \times 5 \times 10^3 \times 1 \times 10^{-9} \approx \pi \times 10^{-5} \text{ radian,}$$

assuming that $v_g = c$, the velocity of light. Finally, if we agree that a measurement of v_g to an order of magnitude is worthwhile, the phase-measuring device should read to $\pi \times 10^{-6}$ radian, i.e. have no more than 10% error in measuring angles of $\pi \times 10^{-5}$ radian.

In order to make the idea of measuring such small phase angles plausible, the circuit of Fig. 3 was developed and the essential features tested. The explanation follows.

1. A Phase-Measuring Device

The signals $\vec{S}_1$ and $\vec{S}_2$ appear at the output of amplifiers whose inputs are the signals from the receivers R_1 and R_2 (Fig. 3a).* $\vec{S}_1$ and $\vec{S}_2$ are sine-wave signals having the same frequency (or at least the same average frequency), but which are slightly different in phase and amplitude (Fig. 3c). We can install filters to restrict the bandwidth of the amplifiers in the phase-measuring system to a value comparable to the bandwidth of the original signals generated by R_1 and R_2.

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*It is evident that $\vec{S}_1$ and $\vec{S}_2$ are of equal importance. Thus, to get the utmost stability, another complete chain such as that shown in Fig. 3b could be built in which the roles of $\vec{S}_1$ and $\vec{S}_2$ are interchanged. The results obtained from the two chains are then averaged in order to cancel some drift. The entire system could also be "chopper stabilized" by periodically feeding $\vec{S}_1$ and $\vec{S}_2$ from a common source for a time and then setting the output to zero, before returning to the actual signals. These operations would probably be done by automatic circuits.

2. Procedure

By adjustment of individual amplifier gains, we can make $\vec{S}_1$ and $\vec{S}_2$ equal. However, since we wish to compare $\vec{S}_1$ and $\vec{S}_2$ in a precise way, it may be expected that the amplitudes will in general tend to drift apart after an initial setting, sufficiently to obscure the small difference signal due to a phase displacement. Therefore, some automatic means of amplitude regulation may be helpful. The method chosen is to amplitude-modulate, without phase shift, one of the signals, say $\vec{S}_1$, thus producing a signal $k\vec{S}_1$. The subtraction $(k\vec{S}_1 - \vec{S}_2) = \vec{D}$ is then performed (by the shielded transformer shown in Fig. 5) to give a signal $\vec{D}$ whose components D_x and D_y are respectively proportional to the amplitude difference $(|k\vec{S}_1| - |\vec{S}_2|)$ and the very small phase angle θ. The component D_x gives no information regarding θ and, in fact, will have no effect on the over-all performance of the phase detectors provided its magnitude is small. But, as indicated above, its magnitude may drift. To counteract this tendency, the value of k is automatically adjusted so as to keep D_x small. This is accomplished by sufficient loop gain in the feedback system involving the amplitude modulator.

An amplitude modulator which appears satisfactory consists of a "potentiometer" formed of a pair of straight-filament tungsten lamps which electrically terminate the signal cable coming from the amplifier for R_1. (See Fig. 6). The lamps are heated by radiofrequency power from separate sources RF-1 and RF-2, and as a result have resistance above the room-temperature value. By simultaneously increasing the temperature of Lamp 1 and decreasing the temperature of Lamp 2, we can maintain the total resistance (and inductance) at a constant value, which may equal the impedance of the input cable and hence terminate the cable. The value of k, however, has decreased in the process. In this fashion, k may be altered. In a particular experiment the signal $\vec{S}_1$ could be modulated ± 10% with an accompanying phase shift of less than 1 part in 10^8 of 2π radians, which is the limit of the currently available measuring equipment.

While Fig. 3b does not show a second amplitude modulator, for symmetry, a similar amplitude modulator may operate in the opposite sense on the signal $\vec{S}_2$. It may be argued that any drift of amplitude may be accompanied by a drift of phase angle and therefore affect the result.

However, experience with precision amplifiers suggests that gain changes of sufficient magnitude to require correction can occur without accompanying phase-angle changes.

Since the phase angle θ is very small, it is obtained to a good approximation by dividing the amplified signal nD_y (where n is the amplification factor) by $|\vec{S}_2|$. We thus have

$$n\theta = \frac{nD_y}{|\vec{S}_2|}$$

$$\theta = \frac{D_y}{|\vec{S}_2|} \quad .$$

The amplifier gain n will need to be of the order of 10^6 or more.

Differences of 10^{-11} volt between $\vec{S}_1$ and $\vec{S}_2$ are readily detectable in a circuit such as described above. Ordinarily, one would include a "line-stretcher" in one of the signal paths, say S_1, so that θ could be adjusted to zero for each run at a particular receiver spacing. The series of required line-stretcher positions may then be plotted with respect to the series of receiver spacings. In this way, the time-of-flight of the gravitational interaction is directly compared to the "time-of-flight" of an electromagnetic signal through the line stretcher.

3. Testing the Phase-Measuring Device

An audiofrequency oscillator supplies signals to simulate S_1 and S_2. By feeding the oscillator signal through different lengths of coaxial cable to the inputs for S_1 and S_2 (which contain resistive terminations) a pair of signals having a known phase relation is obtained. Separate noise generators inject noise voltages into the two signals; the noise generators must be independent in order to provide noise voltages which are not coherent.

The foregoing test has been performed with a simple version of the phase-measuring device. The experimental result is that time differences of 1×10^{-11} sec between audiofrequency signals can be detected in the presence of noise.

V. CONCLUSIONS

In order to obtain 10^{-10}-second accuracy in our measurements, we require a phase accuracy of approximately 3×10^{-6} radian. If a phase detector can be made to read angles to 1 part in 10^6 with the suggested signal-to-noise ratio of 10^6 and give the result in a reasonable time, the experiment can succeed.

A single phase-measurement can be made in about the time constant (~ 33 sec) of the receivers. One might anticipate measuring to an accuracy of $(1/\omega_r) \times 10^{-6}$ sec in a single measurement. A succession of phase measurements, at intervals of ~ 33 seconds, would yield an average phase measurement, with an improvement in accuracy of $\sqrt{n}$ (where n is the number of measurements) over that of a single measurement.

Reference to the signal-to-noise ratio shows that the receiver-Q appears squared in the ratio S/N. If a higher Q were obtainable and usable, the time constant would be proportionally longer and the accuracy of a single phase measurement would be proportionally better.

Interesting variations in this experiment might be possible in which the space between receivers is filled with some material. The measurement of v_g in free space, however, is of first importance in the experimental science of gravity.

ACKNOWLEDGMENT

I wish to express my gratitude to the Gravity Research Foundation for encouraging the writing and publication of this paper.

LEGENDS

Fig. 1. Experimental arrangement: $M_T M_T'$, transmitter; R_1 and R_2, receivers: A, nodal suspension; B, massive upper stem; C, enclosing quartz tube; D, vibrating quartz stem (hollow); E, metallized faces; M_1 and M_2, tungsten discs. (Not to scale.)

Fig. 2. Measured input-noise as a function of frequency for a type-CK5886 tube connected as shown. The bandwidth for this measurement was 1 cps.

Fig. 3. General circuit schematic: a, R_1 and associated amplifier to generate $\overrightarrow{S_1}$ (R_2 would be treated similarly); b, block diagram of the phase-measuring device; c, vector representation of signals.

Fig. 4. Form of x and y components of the force as a function of transmitter angle θ.

Fig. 5. Electrostatically shielded transformer for measuring very small difference signals.

Fig. 6. Schematic of amplitude modulator.

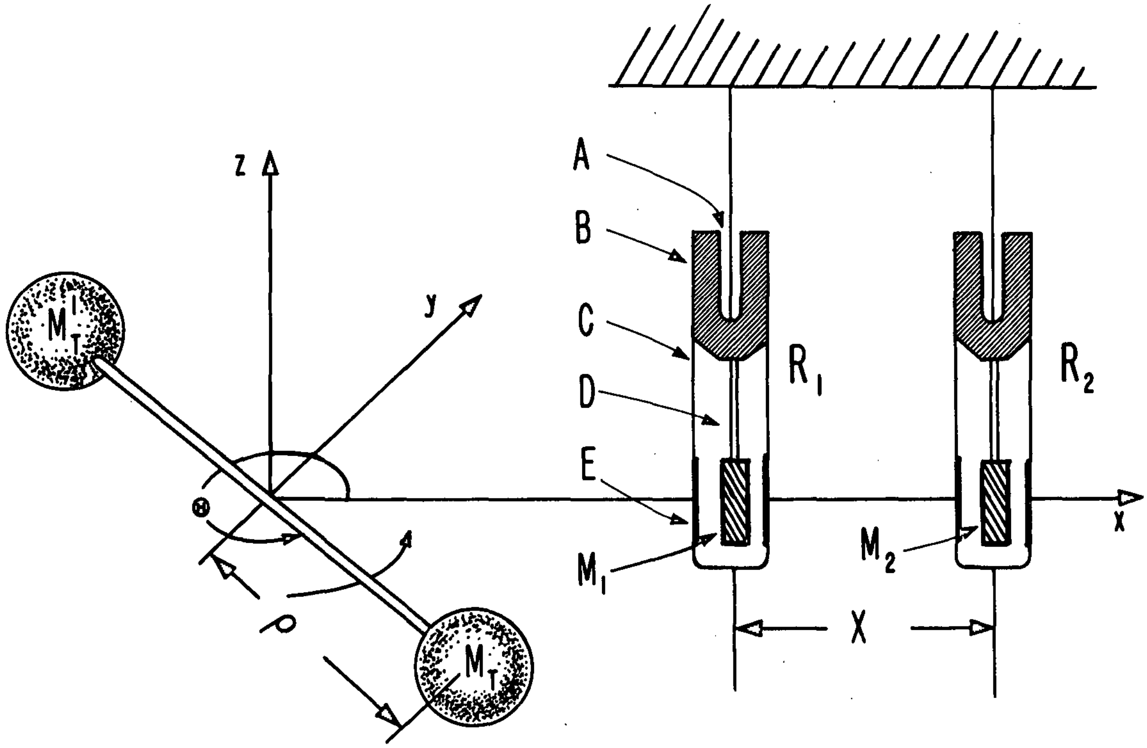

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Fig. 1.

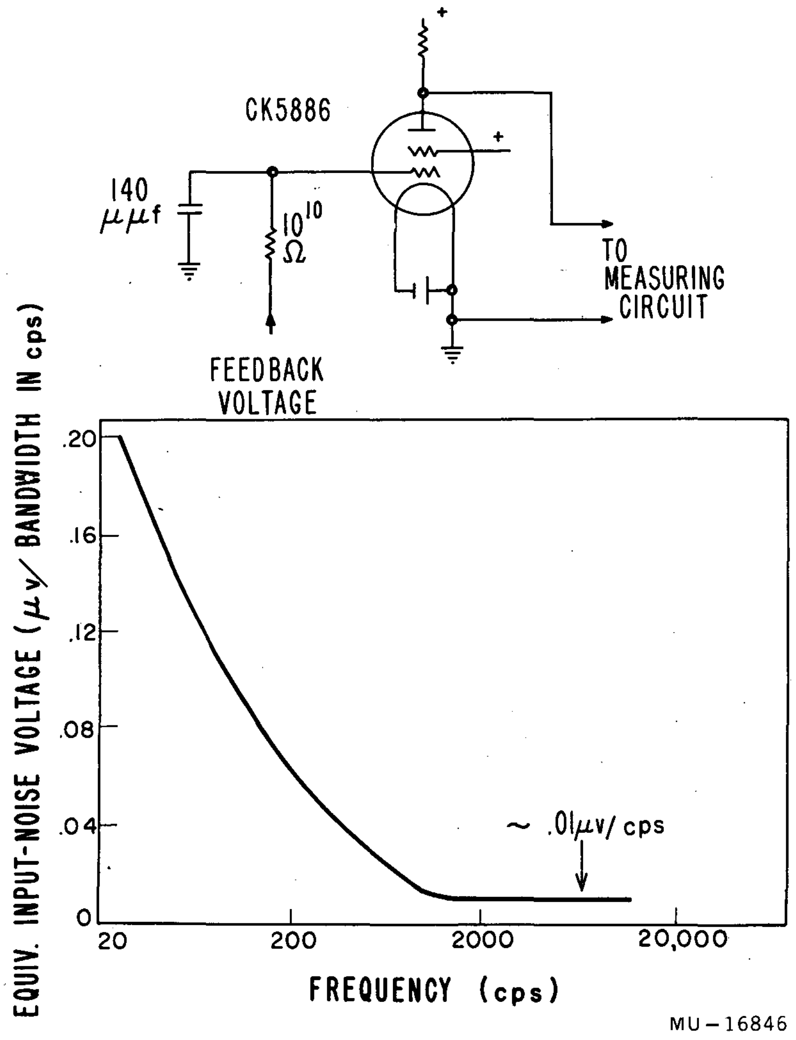

+
CK5886
+
140
μμf
10^{10}
Ω
TO
MEASURING
CIRCUIT
FEEDBACK
VOLTAGE
EQUIV. INPUT-NOISE VOLTAGE (μv/ BANDWIDTH IN cps)
.20
.16
.12
.08
.04
0
~ .01μv/ cps
20
200
2000
20,000
FREQUENCY (cps)
MU-16846

Fig. 2.

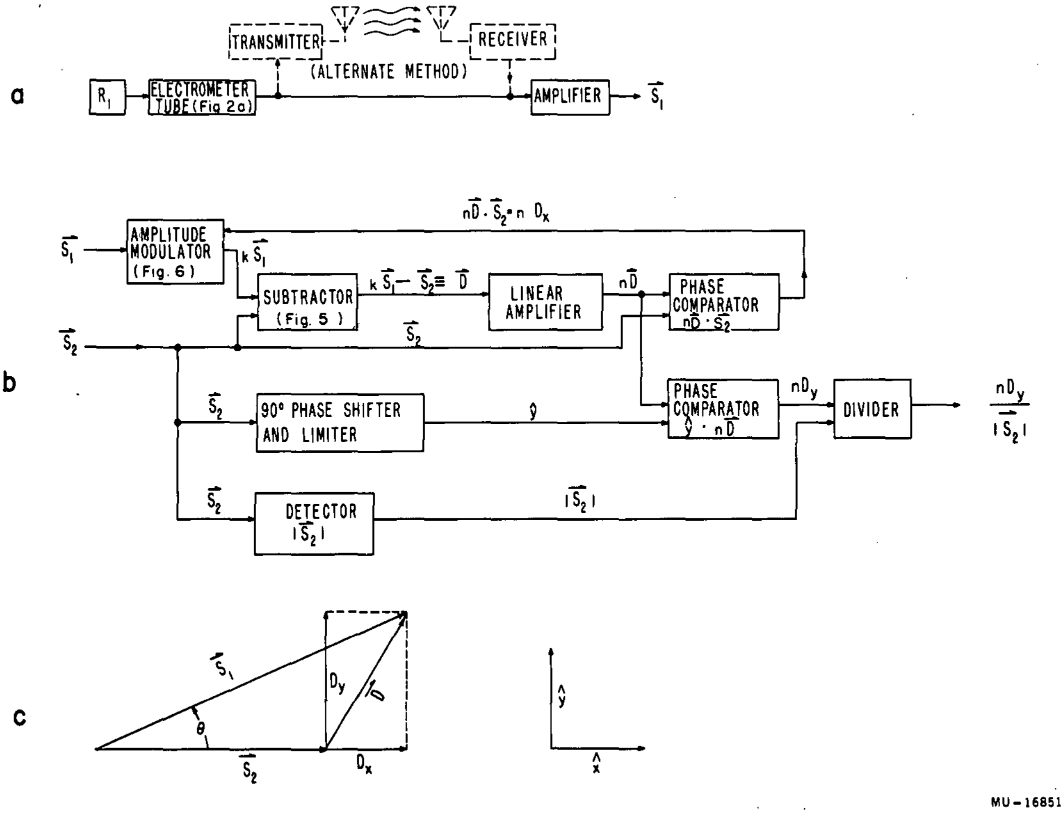
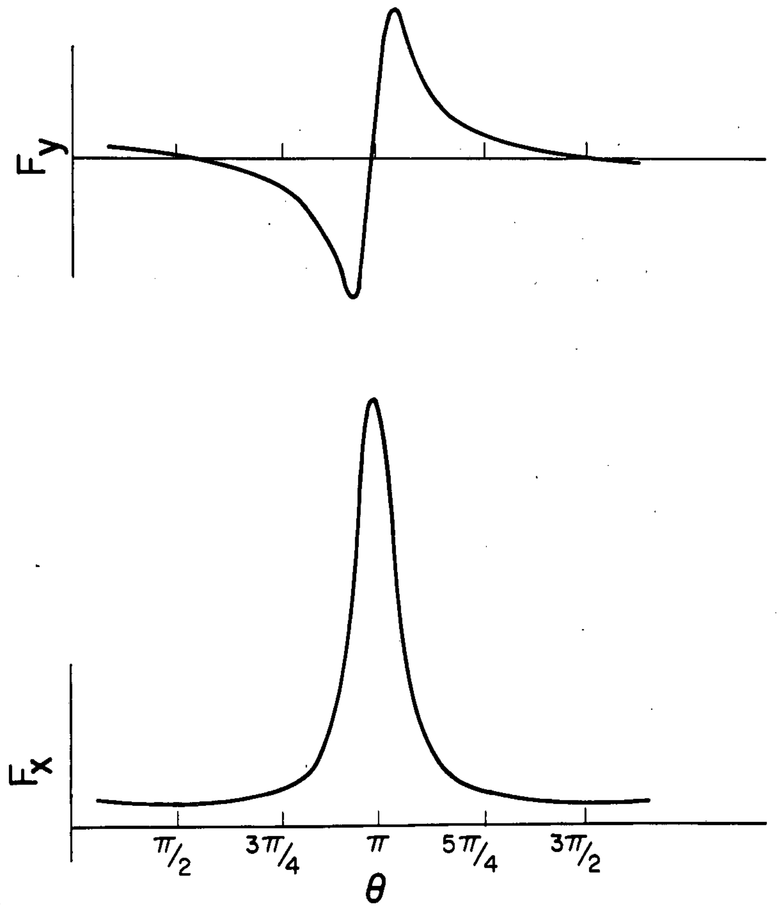

a
TRANSMITTER
RECEIVER
(ALTERNATE METHOD)
R_1
ELECTROMETER TUBE (Fig 2a)
AMPLIFIER
$\vec{S}_1$
b
$n\vec{D}\cdot\vec{S}_2 = n\,D_x$
$\vec{S}_1$
AMPLITUDE MODULATOR (Fig. 6)
$k\vec{S}_1$
SUBTRACTOR (Fig. 5)
$k\vec{S}_1 - \vec{S}_2 \equiv \vec{D}$
LINEAR AMPLIFIER
$n\vec{D}$
PHASE COMPARATOR $n\vec{D}\cdot\vec{S}_2$
$\vec{S}_2$
$\vec{S}_2$
$\vec{S}_2$
90° PHASE SHIFTER AND LIMITER
$\hat{y}$
PHASE COMPARATOR $\hat{y}\cdot n\vec{D}$
nD_y
DIVIDER
$\frac{nD_y}{|\vec{S}_2|}$
$\vec{S}_2$
DETECTOR $|\vec{S}_2|$
$|\vec{S}_2|$
c
$\vec{S}_1$
θ
$\vec{S}_2$
D_y
$\vec{D}$
D_x
$\hat{y}$
$\hat{x}$
MU-16851

Fig. 3.



MU-16848

Fig. 4.

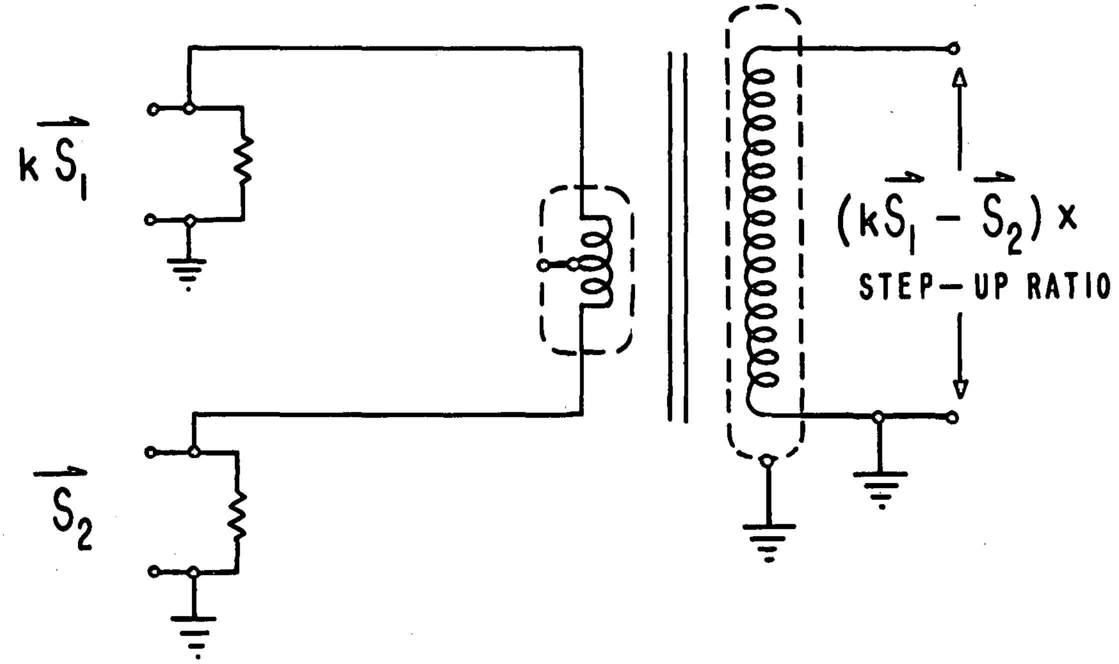


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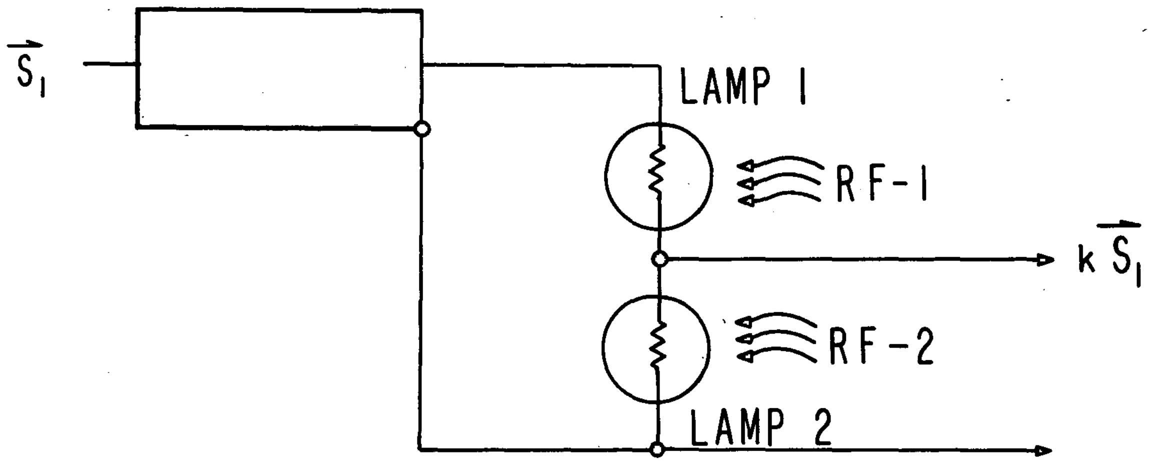
Fig. 5.



MU-16847

Fig. 6.