On the Evaluation of *If p then q* Conditionals

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Abstract

We propose that when evaluating conditionals, people construct an imaginary world that contains the antecedent, and then evaluate the plausibility of the consequent being true in the same world. Thus, when asked for an estimate of the probability of the conditional, people should produce the conditional probability of its consequent given its antecedent. We contrast this view with a view based on the theory of mental models, in which the judged probability of a conditional is derived from the proportion of models in which the premises are true. Study 1 examined this hypothesis by comparing probability estimates for (i) category-based conditional arguments (e.g. *If robins have ulnar arteries then sparrows have ulnar arteries*), (ii) corresponding conditional probabilities in the form of suppositions (e.g. *Suppose you knew that robins have ulnar arteries. How likely would you think it was that sparrows have ulnar arteries?*) and (iii) the argument strength of corresponding inductive arguments (e.g. *Fact: Robins have ulnar arteries. Therefore: Sparrows have ulnar arteries. How convincing do you find this argument?*) All three estimates were highly correlated, a finding that supports our hypothesis. The similarity between the two categories (e.g. robins and sparrows) was also manipulated. Similarity affected all three estimates equally, similar items being given higher estimates than dissimilar items. This finding indicates that similarity is one basis for the plausibility judgements. Study 2 tested our hypothesis using conditional statements with known probabilities. The results favoured our hypothesis. We discuss these results in terms of philosophical and psychological views of conditionals, and suggest that they bring together kinds of reasoning that are traditionally studied separately, such as conditional reasoning, induction, and judgements of probability.

Introduction

Psychological research on inductive and deductive reasoning has traditionally examined reasoning based on premises classified as true. Such research ignores most everyday reasoning, which is based on uncertain premises. Premise uncertainty, in turn, rightly influences the degree of certainty in the conclusion of an inference (e.g. Stevenson & Over, 1995). Understanding everyday reasoning, therefore, involves understanding subjective premise uncertainty, and the way in which such uncertainty gets translated into uncertainty about the conclusion of an inference. The present article investigates subjective uncertainty about conditional premises of the form *If p then q*.

The article focuses on the way in which people evaluate conditional arguments and how they arrive at judgements of the probability of a conditional. We propose that people evaluate conditionals with reference to imaginary situations that they mentally construct; in particular, people evaluate the plausibility of the consequent in an imaginary situation in which the antecedent is true. For example, on encountering the conditional *If you study hard then you will pass the exam,* we propose that reasoners mentally construct an imaginary situation in which they study hard and then pass the exam. This imagined situation may be judged more plausible than one in which they study hard and do not pass the exam. Such a judgement might be mediated by causal schemas, e.g. one's intuitive theories about studying and success or failure (Collins & Michalski, 1989). In other situations, similarity (Osherson et al, 1991) or a judgmental heuristic, such as representativeness or availability (Kahneman et al, 1982) might be used. If people do indeed evaluate conditionals in the above manner, then when asked to estimate the probability of the conditional, they should state the probability of the consequent given that the antecedent is true; that is, they should give the conditional probability.

The view presented above is similar to Ramsey' s (1931) notion of how a conditional is evaluated, and also has some similarities to proposals made by Adams (1975) and Edgington (1995). Ramsey' s idea was that when we evaluate a conditional, we add the antecedent to our stock of beliefs, leaving everything else as undisturbed as possible, and then examine whether our new stock of beliefs contains the consequent. Our proposal that people construct an imaginary world that contains the antecedent is comparable to updating one' s knowledge base by adding the antecedent and making the minimal changes resulting from the presence of the antecedent. People then assess the likelihood that the consequent also holds, using either heuristics or sometimes beliefs about relative frequencies. The psychological validity of this "imaginary worlds" view of conditionals has not yet been tested, although conditionals have been linked to conditional probabilities in other psychological work (Stevenson & Over, 1995; Oaksford, Chater, & Larkin, 2000). The present experiments test this view.

The mental models theory provides a contrasting view of how individuals untrained in logic evaluate the probability of conditional statements. Johnson-Laird et al. (1999) propose that such individuals infer the probability of events by reasoning extensionally. They construct mental models representing true possibilities (the *principle of truth*) and estimate the sum of the probabilities of the models in which the event occurs. *If p then q* conditionals are understood by representing up to the following three explicit mental models (¬ stands for the negation of a premise):

p	q
¬ p	q
¬ p	¬ q

"p ¬q" is not represented because it is a false possibility, though it can be inferred as the complement of the fully explicit models, although this rarely happens (see Barres & Johnson-Laird, 1997). However, consideration of the false possibility is critical for the conditional probability interpretation of conditional statements; the conditional probability of q/p depends on the relative ratio of *pq* to *p¬q* possibilities, i.e. Pr(q/p) = Pr(pq)/ [Pr(pq)+Pr(p¬q)]. Therefore, evidence for a conditional probability interpretation of conditional statements would challenge the mental models theory.

Study 1: Subjective probabilities

In Study 1 we compare the imaginary world hypothesis with the mental models hypothesis by obtaining judgements of (1) the probabilities of conditional arguments, (2) conditional probabilities, and (3) judgments of the convincingness of inductive arguments, that is, judgements of argument strength. Examples of the materials are shown in Table 1.

We propose that when asked to estimate the probability of the conditional shown on the top panel of Table 1, participants will evaluate the conditional in the same way as they evaluate the conclusion of the conditional probability statement shown in the middle panel of Table 1. That is, the reasoning in both cases will be based on the same representation, an imaginary world in which horses have stenozoidal cells and in which the plausibility of cows having stenozoidal cells is assessed.

We also propose that participants evaluate inductive arguments, like the one in the last panel of Table 1, in a similar way. Inductive argument tasks ask participants to assume that p is a fact. Our hypothesis, that when judging the probability of conditional, people imagine a world in which p is true and make judgements about that world, predicts that they should give the same judgement as they give when explicitly told that p is in fact true (i.e., when making argument strength judgements).

Because an imaginary world in which both horses and cows share a property is more representative of the real situation than a world in which horses have the property but cows don't, we expect all three types of judgments to be relatively high. Furthermore, we expect the probability of the conditional to be highly correlated with the conditional probability judgements on the one hand and judgements of argument strength on the other, since we argue that they all measure the same process. Note that an association between argument strength and conditional probability judgements has been presupposed in psychological research (e.g. by Sloman, 1998). By contrast, the mental models view of conditionals does not consider the case in which horses have the relevant property but cows do not. Consequently it would not predict that judgements of the probability of the conditional would be highly related to either conditional probability judgements or argument strength judgements.

Table 1. Study 1: An example of materials used in Study 1. [Note: The example is from the similar condition. Half of the materials were in the dissimilar condition.]

<u>Probability of conditional condition</u>

Peter said the following: If horses have stenozoidal cells, then cows will have stenozoidal cells. How likely do you think it is that what Peter said is true?

0 1 2 3 4 5 6 7 8 9 10
not at all likely very likely

<u>Conditional probability condition</u>

Suppose you knew that horses have stenozoidal cells. How likely would you think it was that cows have stenozoidal cells?

0 1 2 3 4 5 6 7 8 9 10
not at all likely very likely

<u>Inductive argument condition</u>

Fact: Horses have stenozoidal cells

Conclusion: Cows have stenozoidal cells

0 1 2 3 4 5 6 7 8 9 10
not at all convincing very convincing

Study 1 also examined the hypothesis that the similarity between the two categories mediates the judgments in all three conditions. The more similar the categories, the more structure (features and dependencies) their representations share. Thus people are likely to infer that the more known structure two categories share, the more novel structure they are likely to share. We expect, for instance, an imaginary situation, in which similar categories (e.g. cows and horses) share a novel property, will be judged more plausible than an imaginary situation, in which dissimilar categories (e.g. cows and mice) share a novel property. (See Osherson et al, 1991, for a model of how conditional probabilities can be derived from similarity judgements.) Consistent with our view, research on category-based inductive arguments (like the one in the last panel of Table 1) has shown a robust effect of similarity (see e.g. Rips, 1975; Sloman, 1993). Moreover, to the extent that such similarity-based reasoning is non-extensional (see Johnson-Laird et al., 1999), it falls outside the scope of mental models theory, which only considers extensional reasoning.

Method

Participants. Forty-one first-year psychology students volunteered to participate in this study.

Design. Type of Measure (probability of conditional vs. conditional probability vs. argument strength) was crossed with Similarity (similar vs. dissimilar category pairs) in a mixed design with repeated measures on the last factor.

Procedure. Participants were presented with booklets containing 18 examples in one Type of Measure condition. Half of the examples in each condition contained similar and half dissimilar mammal pairs. The assignment of category pairs to similarity conditions was controlled by an independent group of twelve participants who were asked to rate the biological similarity of the 16 mammal pairs in a 0-10 scale, were 0 was labeled as "highly dissimilar" and 10 as "highly similar." The mean ratings for the similar and dissimilar items were, respectively, 7.39 (min=5.92, max=8.92) and 1.74 (min=.92, max=2.33). The results therefore justify the assignment of items to the similar or dissimilar conditions.

Participants in the probability of the conditional condition (N=16) were told that they would be presented with statements uttered by a person. Their task was to say how likely they thought it was that what the person said was true on a 0-10 scale, where 0 was labeled as "not at all likely" and 10 as "very likely." We used this task to obtain judgements of the probability of the conditional to ensure that our instructions did not encourage participants to give conditional probability judgements for superficial reasons[1]. If participants were simply asked "How likely do you think it is that *If p then q?*" they might interpret the question as asking the question "If p, what is the probability that q?" That is, as a direct request for the conditional probability of the consequent given the antecedent. This problem arises because a conditional consists of a main (the consequent) and a subordinate (the antecedent) clause, and it has been shown that, when processing sentences containing main and subordinate clauses, people often assume that the subordinate clause is true (Baker & Wagner, 1987). Our instructions were designed, therefore, to avoid responses based on this kind of linguistic paraphrase and to ensure instead that they were based on a conceptual understanding of the conditional.

Participants in the conditional probability condition (N=10) were told that they would be presented with examples asking them to suppose that a statement is

[1] We thank Phil Johnson-Laird and Vittorio Girotto for suggesting these instructions for the framing of the conditional probability condition.

true. Based upon this supposition, they had to judge the likelihood that a second statement is true. The same scale was used as for the Probability of the conditional participants. Participants in the argument strength condition (N=15) were told that they would be presented with a series of arguments, each containing a fact (which should be taken as true) separated from a conclusion by a line. Their task was to describe how convincing they found each argument on a 0-10 scale, where 0 was labeled as "not at all convincing" and 10 as "very convincing." Participants in all conditions worked through examples similar to the test items before starting the experiment.

Results and Discussion

Correlation statistics Table 2 presents the mean correlation coefficients relating the three types of measures across items. As predicted by the imaginary worlds view, the three measures were significantly correlated (beyond the .001 level).

Table 2. Mean correlation coefficients by items for each of the three conditions. CP=conditional probability. PC=probability of conditional. AS=argument strength.

	PC	*CP*	*AS*
PC	*1.0*	*.99*	*.94*
CP		*1.0*	*.96*
AS			*1.0*

Similarity Table 3 presents mean Type of measure by Similarity estimates. In each Type of measure condition, ratings for similar items were higher than ratings for dissimilar items.

Table 3. Mean Type of Measure by Similarity estimates CP=conditional probability. PC=probability of conditional. AS=argument strength.

	Similar Items	Dissimilar Items
PC	5.78	2.76
CP	6.66	2.83
AS	5.40	2.30

The data from each measure were analyzed by pairwise t-tests for participants, and independent t-tests for items. Pairwise tests were used in preference to a single ANOVA because we cannot assume that the three measures are comparable. For each type of measure, both across participants and items, similarity had a significant effect (beyond the .005 level). These results suggest that the plausibility judgements underlying the imaginary worlds view can be influenced by similarity. The mental models view, however, cannot account for either the correlational results or the effect of similarity.

Study 2: Objective probabilities

Study 2 also investigated how people evaluate conditional statements but with conditionals of known conditional probabilities. The use of known probabilities provides a direct test of our two competing hypotheses, because it allows judgements about the probability of the conditional to be directly compared with the objective conditional probability.

Participants were given three different versions of a text describing a probability problem. For example, a third of the participants read the following text and were then asked to estimate the probability that what Peter said was true:

> In an effort to boost its image, Waterstones bookstore organised lotteries in several Primary schools in Durham. In each school, only the 10 best students participated in the lottery. The name of each participant was written on a piece of paper and was put in a hat. A blindfolded teacher drew a piece of paper from the hat. The student whose name was written on that paper won an autographed storybook. In Durham Gilesgate Primary School the participants were 8 boys and 2 girls. A piece of paper was drawn from the hat. Peter, the father of one of the participants, cannot see the winner's name but says: "If a boy has won the lottery, then my son won it."

According to the imaginary world hypothesis, participants should construct an imaginary situation in which a boy wins the lottery and then consider how likely it is that the boy is Peter's son. Since there are 8 boys all together, this conditional probability is 1/8.

According to the theory of mental models, the correct answer depends upon considering the fully explicit models of the proposition and finding the proportion of models in which the proposition is true. These explicit models, which represent the true possibilities, are shown below, with tags indicating their relative frequencies. (*Boy* stands for the antecedent; *Son* stands for the consequent).

Boy	Son	1/10
¬Boy	Son	0[2]
¬Boy	¬Son	2/10

The proportion of models, therefore, in which the proposition is true is 3/10. We call this the material implication (MI) evaluation of the conditional.

[2] No doubt participants will rule out the possibility of ¬*Boy and Son* on pragmatic grounds. However, for the above problem, this does not affect the predicted probability judgement.

The probability estimates that agreed with one or other of these two evaluations (the conditional probability or material implication) were coded as supportive of either the imaginary world view or the mental models view respectively.

Method

Participants Forty-eight first-year undergraduate volunteers participated in Study 2. The sample included the same forty-one students that participated in Study 1.

Procedure Each participant was presented with a booklet containing one version of the problem given above. In one version the sample of children consisted of 2 boys and 8 girls (the 2b-8g version), in a second of 5 boys and 5 girls (the 5b-5g version), and in a third of 8 boys and 2 girls (the 8b-2g version). Table 4 lists the predictions for the imaginary world and the mental models view for each of the three versions of the problem.

Table 4. Conditional probability and material implication predictions for each of the 3 versions of the problem.

	Imaginary world view (Conditional probability)	Mental models view (Material implication)
2b-8g version	1/2	9/10
5b-5g version	1/5	6/10
8b-2g version	1/8	3/10

Results

Table 5 presents the number of participants in each version of the problem whose response agrees with one of the two evaluation modes for each version. Out of the 48 participants, 44 gave numerical answers.

Table 5. Number of participants in each version whose response falls in one of the two evaluation modes. N=number of participants who gave a numerical response for each version.

	N	Imaginary world View	Mental models view
2b-8g version	13	9	0
5b-5g version	14	5	0
8b-2g version	17	9	1
Total	*44*	*23*	*1*

Out of those 44, 24 gave a response that could be classified in one of the two response modes. The results reported in Table 5, therefore, account for 55% of those responses. The results of all except one of these 24 participants agreed with the conditional probability evaluation ($X^2 = 20.17$). These data clearly favor the imaginary worlds view of conditionals over the mental models view.

The main numeric responses made by the remaining 20 participants were "1/10" or its arithmetic equivalent (N=6), "1/2" or its arithmetic equivalent (N=7), "2/10" or arithmetic equivalent (N=5)[3]. The "1/10" responses are consistent with the mental models view that participants represent an explicit model of the premise and ignore other possibilities. The remaining two responses may reflect failures to understand the conditional.

General Discussion

These results with both subjective and objective probabilities support the imaginary worlds view. Judgements of the probability of a conditional correlated highly with conditional probability judgements and argument strength judgements in Study 1, and they matched the objective conditional probabilities in Study 2. Mental models theory cannot explain these results because it only considers true possibilities. Even if we grant that mental models theory allows that the false possibility may be inferred, the theory still cannot explain our results because it has no mechanism for calculating conditional probabilities when presented with a conditional (see Johnson-Laird et al, 1999).

Furthermore, mental models theory fails to explain the similarity effect found in Study 1. By contrast, this effect follows from the imaginary worlds view, which claims that reasoners evaluate conditionals by representing the antecedent and consequent in an imaginary world and then evaluating the plausibility of this world. Since similarity is a key component of plausible reasoning (Osherson et al, 1991), it follows that similarity should also be a key component in judgements of the plausibility of the consequent being true in a world in which the antecedent is true.

The results of Study 2 also suggest that explicitly presenting the negated antecedent is, in itself, insufficient to promote its inclusion in a mental representation. People might represent such possibilities when background knowledge makes them salient. For example, "If John is in Paris then he is in France" might make people represent the possibility "If John is not in Paris then he is not in France." But as far as the basic evaluation of a conditional is concerned, our results

[3] Some of the responses in the 2b-8g version that are classified as "conditional probability" responses may in fact be "fifty/fifty" responses. However, even if the 2b-8g version is omitted from the analysis, the results still clearly favor the conditional probability view.

suggest that people construct an imaginary world in which the antecedent holds and then consider the likelihood that the consequent holds in the same world.

Our notion of imaginary worlds could be seen as an example of mental models. However, our results suggest that mental models are represented and deployed in ways other than those proposed by Johnson-Laird (e.g. Johnson-Laird et al, 1999) when evaluating the probability of conditionals. For example, the principle of truth cannot apply to uncertain conditionals, since the "false possibility" (p ¬q), must be at least implicitly considered to arrive at the conditional probability. Furthermore, the role of similarity in evaluating an uncertain conditional needs to be included in such a theory.

Our proposal has something in common with possible worlds analyses of ordinary conditionals (Stalnaker, 1968; Lewis, 1973). However, if these conditionals are analyzed in this way in formal semantics, then there are technical reasons why the probability of a conditional cannot be absolutely identified with the corresponding conditional probability. (See Jackson, 1991, for the main technical papers on this issue.) But the technical issue not withstanding, there is reason to hold that the assertion and evaluation of most ordinary conditionals will make them closely related to the corresponding conditional probabilities (Stevenson & Over, 1995; Edgington, 1995). This is all we need for our psychological claims here. Our view is that the judged probability of an ordinary conditional will usually be estimated by assessing the plausibility of the consequent being present in a model that contains the antecedent. Finally, since our views derive from philosophical accounts of conditionals, the present studies also provide a bridge between philosophical and psychological accounts of If p the q conditionals. They also bring together components of reasoning that have been traditionally studied separately such as conditional reasoning, induction, and judgements of probability.

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