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Conceptual Change in a Microcosm: Comparative Analysis of a Learning Event

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Abstract

This article examines a remarkable learning event where a high school class developed, on its own, a stable, normative view of thermal equilibration. The event is also notable because the intuitive ideas that students bootstrapped into their model of equilibration have been thoroughly documented in prior research. Therefore, the process of changing prior conceptions is well delineated. The main point of the article is to review what happened in this microcosm of learning from multiple perspectives to examine how well each perspective can account for the learning that took place. We use three competing views of conceptual change: Knowledge in Pieces, the Theory Theory, and the Ontological View. We argue that Knowledge in Pieces provides a more detailed and more adequate account of the learning that took place, whereas that learning contradicts core commitments of the Theory Theory and of the Ontological View.

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Introduction

The aim of this article is to add a second analysis to a prior fine-grained case study of a learning event [diSessa, 2014a] to examine how different paradigms of conceptual change research can accommodate the general features and details of the original data. Paradigm disputes and attempts to settle them are important in the history of science. However, especially in the social sciences, disputes are often based on incomparable data and general arguments. Here, I use a common data set to interrogate claims of different paradigms.

A Brief History of Conceptual Change and the Context for This Study

Conceptual change names a part of the study of learning that concentrates on the empirically most difficult examples of learning. While part of the conceptual change community also deals with development, these considerations are not relevant here; I focus on learning such as enacted in school. The field of conceptual change arose in the late 1970s and early 1980s in response to a huge burst of studies detailing persistent and non-normative ideas that students seemed to have before instruction [e.g., Pfundt & Duit, 1988], so-called “misconceptions,” which were maintained, shockingly, long into instruction. A brief general history (but more than can be recounted here) appears in diSessa [2014b]. A thorough presentation of the state of the art may be found in Vosniadou [2013].

The essence of this exposition is to contrast three different paradigms within the scholarly literature on conceptual change to see how well they can account for a common data set. First, the original case study was done within the Knowledge in Pieces (KiP) paradigm. Two good general references for KiP are diSessa [1993a] and diSessa, Sherin, and Levin [2016]. A second family of points of view is called the Theory Theory (TT), prominently advanced by Susan Carey, her students, and collaborators. Carey [2009] is the best single reference for her views. The third and final point of view I will consider is the Ontological View (OV). Michelene Chi has been the primary creator and advocate of the OV; Chi [1992; 2005; 2013] serve as core references for her work.

A word about the choice of paradigms to compare is warranted. The field of conceptual change is complex and contested. Dealing with all possible theories and variants in one article would plainly be impossible. So I have cut down the range deliberately, expecting that the insight of a more limited comparison is still worth the effort. It is my impression that range of ideas associated with the TT is wider than the ranges associated with KiP and OV, and it is wide enough—if fully embraced—to incur significant cost in complexity and clarity. Within the TT family I’ve chosen to hew closely to Carey and work most clearly associated with her views in order to avoid such costs.¹

¹ Alternative TT-similar foci might be Vosniadou’s framework theory [Vosniadou & Skopeliti, 2014], Gopnik’s fairly radical version of TT [Gopnik & Wellman, 1994], or some recent TT variants [e.g., Wiser & Smith, 2016]. The latter would particularly

Differences among the chosen paradigms are complex, and I will later need to draw these out in some technical detail. In the meantime, readers unfamiliar with the general claims might benefit from a rough gloss, which is suggested in the paradigms' names. A central issue is the nature of pre-instructional knowledge. The TT view holds that such knowledge is broadly coherent and, indeed, comparable in form to post-instructional ideas, except that it is scientifically incorrect. For example, intuitive ideas of mechanics constitute (with some limitations) a theory comparable to and competitive with Newtonian mechanics. KiP, on the other hand, considers that naïve knowledge is much less coherent, and consists of a great number of ideas ("pieces") that act both as positive resources in learning as well as accounting for students' scientifically incorrect assertions. The OV locates the difference in students' pre-instruction and post-instruction knowledge not so much at the level of content, but in the very set of kinds of things that exist in the world: ontologies. Examples of ontologies are matter, ideas, processes, and basic causal forms.

In addition to being individually respectable, well-established, and well-developed points of view, choosing these three paradigms has two other advantages. First, the perspectives have been in dialog nearly from the start of conceptual change research (see recounting, below), resulting in a reasonably well-developed, public frame for comparison and contrast. Second, all three have treated the topic of thermal equilibration, the instructional topic for the focal learning event. This adds considerable detail and bite to comparisons. Within these paradigms, references for thermal equilibration are as follows: for the OV, Chi [2005; 2013]; for the TT, Wiser and Carey [1983], which Wiser [1995] greatly elaborated with instructional study; for KiP, the original study on which this work is based, diSessa [2014a].

The dialog between KiP, TT, and OV

Even in the early years of conceptual change study, a nascent contest between the TT and KiP perspectives was visible. For example, in the same volume in 1983 [Gentner & Stevens, 1983], work strongly in the TT camp (McCloskey) appeared in contrast to work on KiP (diSessa). In those early years, TT dominated discussion [e.g., Carey, 1985], and KiP was, at best, a minority view [Driver, 1989].

The OV entered the debate in the early 1990s with Chi [1992], just as KiP was beginning to achieve greater visibility. The next year, when a major article appeared on the KiP perspective [diSessa, 1993a], Chi and Slotta [1993] wrote an extended

complex to treat owing to issues such as, for example: (1) whether their theory is similar enough to "classic" TT to be insightfully characterized by the name, (2) whether elements of instruction that are responsible for effects are TT in nature, (3) whether theoretical elements are sufficiently theory-diagnostic to support comparison and contrast.

commentary, countering basic claims of KiP from an OV point of view; diSessa [1993b] wrote a response.

In the years since, the TT, KiP, and the OV have become standard contrasting references by which each paradigm distinguishes itself. For example, Chi [2005, p. 162] contrasts her OV principles and presumptions to KiP. Wiser and Smith [2008] position agreements and disagreements with KiP in comparison to their Theory-Theory-oriented view, and Vosniadou and Skopeliti [2014, pp. 1434-1435] position their variant of the Theory Theory, called the “framework theory,” with respect to KiP and to the OV. I continue to use contrasts with the Theory Theory point of view in explaining KiP [diSessa, 2013]. Indeed, the Wikipedia article on conceptual change [“Conceptual Change,” n.d.] lists the TT (along with the framework theory as a variant), OV, and KiP as basic contemporary approaches to the field. Finally, the TT vs. KiP contrast has spilled over from content-oriented study of conceptual change to students’ epistemic presumptions [Hammer & Elby, 2002].

General compare-and-contrast discussions of the TT, OV, and KiP have been supplemented, although relatively infrequently, with specific data-based arguments. Samarapungavan and Weirs [1997] argued that their data showed substantial consistency and coherence in children’s conceptions, in contrast to core KiP claims (albeit their work was not on physics, but on the topic of origin of species; variations in topic, age of subjects and methodology have dogged resolution of paradigm differences). At least finding common ground on topic, if not on methods or data set, Ioannides and Vosniadou [2002] aimed to support a high degree of coherence in naïve conceptions of force, in contrast to KiP claims. diSessa, Gillespie, and Esterly [2004] countered with dramatically different empirical results on a near-replication of the Ioannides and Vosniadou experiment. Clark, D’Angelo, and Schleigh [2011], in an effort to resolve these differences, used the coding schemes of both diSessa et al. and Ioannides and Vosniadou on a 5-country corpus of data, finding results uniformly consistent with those of diSessa et al., and inconsistent with those of Ioannides and Vosniadou. Gupta, Hammer, and Redish [2010] and Gupta, Elby, and Conlin [2014] take a KiP point of view (broadly construed), using multiple kinds of data and argument to undermine what they see as a key presumption of the OV.²

What does the present work offer in the context of this extended dialog and debate? Why this article, now?

1. A potentially decisive strand of empirical work has blossomed in recent years: microgenetic analysis of student learning. Microgenetic analysis seeks to use all available data *during the process of change* to triangulate on the underlying dynamics [Siegler & Crowley, 1991]. Past work on conceptual

² Gupta and colleagues develop several of the same sorts of arguments that are used here, out of their own data. In particular they point out how the OV seems to marginalize the possibility that naïve knowledge, and even naïve ontologies, can contribute positively to learning science.

change—most obviously in a developmental tradition—has been dominantly within a “snapshot” empirical paradigm, where before and after states are compared. In related educational work, the main empirical strategy has also been to use treatments, short or long, to examine indices of diachronic change.

There is a reasonable body of work employing microgenetic study in education, though little of this literature bears on the comparative focus here. Many educational microgenetic studies, for instance, make no reference at all to theories of conceptual change [e.g., Nemirovsky, 2002]. See also other references in diSessa [2014a, p. 800]. Relevance to the present focus is doubtful,³ and, in any case, it has not been established. A second class of studies makes reference to some relevant themes such as the richness and productivity of pre-instructional knowledge [e.g., Rosebery, Ogonowski, DiSchino, & Warren, 2010], but connection to specifics of theories of conceptual change is weak or non-existent (e.g., Rosebery, et. al. rely on the literary critic and theorist, Bakhtin, whose work was completed before the field of conceptual change began), and comparative work is also missing. A third (and small) class of studies uses microgenetic study to critique theories of conceptual change [e.g., Gupta, Elby, & Conlin, 2014]. But, once again, competitive argumentation is missing. Finally, there has been a fair literature using microgenetic analysis to flesh out details of conceptual change within the KiP framework, but these, too, have minimal reference to other theories of conceptual change [e.g., Kapon & diSessa, 2012; Levrini & diSessa, 2008; Parnafes, 2007; Parnafes & diSessa, 2013].

To sum up, the present study is one of few to employ microgenetic analysis to interrogate, in detail, well-developed theories of conceptual change; to my knowledge, it is unique in engaging those theories in competitive argumentation.

2. The present theoretical landscape is also significantly different from the past; each of these frameworks continues to evolve. Chi [2005] lists major dimensions of change in her claims from earlier work. Chi [2013] continues the evolution. Carey [2009] presents her work as a synthetic whole, emphasizing the processes of change more than in the past. KiP has also continued to evolve and change, with new additions to theory and an increasingly broad portfolio of empirical results and methods [diSessa, Sherin, & Levin, 2016].

Within each of these paradigms, recent changes have made the present comparison more powerful. For example, diSessa [2014a] is one of the few and likely the most detailed studies of learning about thermal equilibration within the KiP perspective.⁴ In addition, that study has a more extensive

³ See the methodological comment at the end of this article.

⁴ A large family of studies of thermal phenomena by Marcia Linn and colleagues [e.g., Linn & His, 2000; Clark, 2006] is close in spirit to KiP, but does not affiliate explicitly with KiP, nor attend to details of KiP theory. Most of those studies are not microgenetic.

analysis of learning mechanisms than in almost all prior work on thermal phenomena, which plays a key role, here. Pairing nicely, Carey's [2009] work gives what I believe is her most elaborate and detailed accounting of mechanisms of change. Chi's [2005; 2013] introduction of the category of *direct causality*⁵ turns out to provide one of the most focused and telling comparisons to the OV in this particular case.

3. This particular case study has some exceptional attributes that recommend it to our attention.
 - a. Fortuitously, many of the knowledge elements identified in the learning event have been extensively studied in prior published research. This means the analysis has greater precision and security than an ad hoc analysis of a single learning event. This specificity concerning knowledge elements allowed most of the analysis of mechanisms, and therefore their comparison with TT and OV. The further fact that the formerly described intuitive knowledge elements were discovered in the domain of mechanics, and not thermodynamics, will also play importantly into comparative analysis.
 - b. Some educationally relevant characteristics of the event are compelling. Students demonstrably learned the target material and showed strong indicators of conceptual change. They showed use of what they learned on several occasions. The learning event also took place over a short period of time (on the order of an hour or two), and without instruction, which, by itself, might challenge some basic assumptions of many views of conceptual change, and also might inspire unusual but effective teaching techniques.
 - c. The students' construction involved specific kinds of thinking that are regarded as "not only incorrect, but misconceived" [Chi, 2013, p. 49], and which adherents of TT and OV perspectives systematically regard as a problematic or impossible basis for learning the subject matter.Each of these points will be elaborated in the analysis below.
4. On a general level, understanding the basic issues of the nature of elements of mind (e.g., concepts, theories, ontologies, or subconceptual elements) that contribute to or block conceptual change, together with their relational structure (e.g., relative coherence) and dynamics during learning (learning mechanisms), remains a compelling challenge for the field to resolve. These topics are central to TT, OV, and KiP debate.

⁵ Chi uses the term *sequential process*. However, she makes clear that this is a causal process, and she uses an alternate phrasing of "direct causal explanations." I prefer the term "direct causality" in part to make broader connections in the literature, specifically literature that holds that learning science involves a succession of more complicated and sophisticated kinds of causality. Chi does not explicitly affiliate with that point of view, or with the description "primitive causality." But she maintains that direct causality is "misconceived" and "incommensurate" with normative science, and the normative equivalent, emergent processes, is initially inaccessible as a form of explanation for learners.

Methodological Preparation

The basic plan for this paper is to develop a systematic framework, a set of dimensions or foci, for comparison between the three relevant perspectives, KiP, TT, and OV, and then to apply that framework to see how each perspective can treat the case of learning in point.

Since the focal learning is based on a single case, one needs to attend to well-known strengths and limitations of case studies. They are very unlikely to settle questions of generality, for example, so one cannot expect this analysis to settle, once and for all, disputes between paradigms. On the other hand, case studies are a better fit for (1) existence proofs, which may concomitantly refute general claims, and (2) discovery. In this case, the original case study uncovered mechanisms of learning that will be relevant to comparative analysis here, although, of course, the generality of such mechanisms requires further study. Concerning existence proofs, one of the core advantages of the case study here is that it uncovers an effective mode of learning, and that mode seems essentially ruled out by some paradigms here compared. I make no claims that that mode is common—in fact, our data strongly suggests it is not. But it shows weaknesses when a paradigm has ruled it out, and strengths when a paradigm can easily accommodate it.

The Original Study

The original study [diSessa, 2014a] involved a systematic search of data from three very similar instantiations of an instructional sequence concerning thermal equilibration for episodes that marked *spontaneous student conceptualizations* (models) of the phenomenon. (A larger corpus of at least six comparable but less similar instructional sequences, involving variously sized classes, with students ranging from sixth grade to early high school provided some additional insight on generality or idiosyncrasy of observations.⁶) In addition, I insisted that the student conceptualizations showed (1) conscious awareness, (2) an ability to support and explain the scheme, (3) significant buy-in from the whole class (“socially shared”), and (4) an ability of nearly all students to demonstrate individual competence with the conceptualization across several contexts of application. Two student constructions meeting these criteria were found and subjected to thorough microgenetic analysis, seeking to use all relevant data from the video to track incremental changes in conceptualization. One of these cases, the one considered here, had the remarkable property of leading to a normative view of thermal equilibration. The other led to a stable, elaborated, but non-normative model. This second study, the development of a stable, elaborated “misconception,” while having its own interesting aspects, is less challenging of core principles of learning, and less educationally relevant: we want students to come to a normative model. It is omitted from consideration here.

⁶ In brief, we saw many similarities in elements across these many cases, although trajectories and macro constructions were quite different.

Microgenetic analysis is a complex, painstaking process with many details and converging lines of argument. However, I can sketch the process as used in this case:

I scanned the whole instructional sequence for the class that developed its own model of thermal equilibration looking for any events that could be seen to be related to the relevant construction. Analysis of any such event focused mainly on (1) the schemata that students used, (2) the comings and goings of schemata, (3) modifications or combinations with other schemata, and (4) the schemata's connection to particular aspects of the real world. In many cases, the discovered schemata had been identified and carefully studied in prior work [diSessa, 1993a], which involved a much larger subject set and many instances of use. Using such prior work, one can sometimes work at "high resolution," matching published descriptions of the schema's typical context of application, and the "slots" of each schema to the features of the world to which students attended and which they fit into the relevant slots. With only one occurrence (e.g., only the event I present below), it would be impossible to determine the typical conditions of activation (hence, to determine whether something unusual has happened in that respect) and similarly extremely difficult to say much in detail about the precise meaning of, for example, schema slots.

When schemata did not match any I could find in the published record, I provided the best ad hoc account that I could.

Fortunately, there is a small core of this complicated analysis of the normative construction, attending to one pivotal construction by one student, which is sufficient for the competitive argumentation presented here. This is, in essence, the microcosm for this paper. I will recount that event and sketch its analysis. A broader view of all relevant events in the full development and use of the model, from the original article, is sketched in the appendix. This sketch includes: (1) listing all events that appeared to relate to the focal event, both before and following it, (2) continuities and discontinuities with what happened before and after the event, and (3) descriptions of other students' reception of the relevant moves.⁷

The next subsection develops the framework for comparing and contrasting KiP, TT, and OV. It consists of dimensions that are diagnostic of core differences in the perspectives. The general positions of each perspective, KiP, TT, OV, are simultaneously described for each theme.

⁷ For a fuller account of the details of microgenetic analysis, the second construction of a non-normative model, more details on the instructional context, and so on, the reader may consult the original analysis [diSessa, 2014a]. I would happily share the original videos with ambitious researchers who wish to do their own analysis from scratch, or to check details of the analysis presented here.

A Framework for Comparing KiP, TT, and OV

The near-universal assumption behind conceptual change research is that the naïve ideas with which individuals enter into learning/instruction provide a challenging background on which to develop new, changed conceptions. Then, one must ask, how should one characterize the naïve ideas that provide the context for difficulties in learning? As anticipated above, this locates a main point of contrast between KiP and other views.

The TT maintains that naïve ideas are coherent, systematically intertwined with one another. Coherence means, essentially, that everything must change in concert; if one concept changes, connected concepts must also change.⁸ This references the famous gestalt switch of Kuhnian scientific revolutions [Kuhn, 1970]. Carey has emphasized Kuhn's ideas, with specific reference to Kuhn's "incommensurability," from her early work [Carey, 1985; 1991] to more recent work [Carey, 2009].

The OV claims that, in the case of deep conceptual change, naïve ideas are ontologically impoverished in the specific sense that, students simply do not have the appropriate scientific ontology; students then may assimilate instructed ideas, mistakenly, to old ontologies, resulting in ideas that are both incorrect and misconceived. Chi's older work [e.g., Chi, 1992] emphasized overuse of the material (matter or physical entities) ontology in learning science, and lack of reasoning based on an expert ontology within the major category of processes, which she described on different occasions as events, acausal interactions, or constraint-based interactions.⁹ The most recent work [Chi, 2013] gives less emphasis to the use of material ontology, and—as anticipated above—emphasizes students' use of the ontology of direct causality (agent/patient causality), categorically distinct (Chi invokes the Kuhnian concept of incommensurable (using the synonym "incommensurate") from the proper scientific ontology of emergent processes, of which students have little or no understanding or awareness.

KiP takes a different view. In this perspective, naïve ideas are neither theories nor sharp contrasts with available ontologies, compared to normative understanding. Instead, the *form* of naïve ideas needs theoretical innovation, compared to constructs such as "theories" or "ontologies." In particular, intuitive ideas are very many, and their contextuality (when they are used and when they are not) is a critical parameter, distinct from what is central to theories or ontologies. Being many and not strongly constrained by ontological dichotomies, intuitive ideas may find useful places in learning, in contrast to their typical characterization as "misconceptions."

⁸ No researcher I know takes "everything must change in concert" literally and categorically to imply reorganization in an instant. But, the idea expresses a fundamental commitment to the forces of coherence that is central and persistent in TT work.

⁹ This development of conceptualization and terminology is described in Chi [2005].

I now develop the elaborated and diagnostic set of dimensions that will be used systematically to compare paradigms. Each paradigm's take in one of these dimensions constitutes a general prediction that the analysis here aims to evaluate in the case in point. Some further elaboration of the themes will follow later in the paper, with important specifics and extended quotations from the relevant authors.

Complexity of the naïve state: KiP holds that naïve knowledge is rich, complex, and diverse. Characterizing it as “a coherent theory,” as TT holds, is misleading, if not categorically wrong. Instead of a few “core concepts,” KiP posits a system of many elements—hundreds or more. These elements have independent developmental histories, so that their level of coherence should be expected to be weak on that basis alone. Similarly, KiP maintains that the OV underestimates the flexibility of naïve knowledge, and overestimates the strength of whatever ontological or causal constraints exist. In the case of heat and temperature phenomena, according to the most recent claims of the OV, the ontology of direct causality is inappropriately used when learning about thermal phenomena; students do not assign heat (thermal equilibration) to the proper scientific ontological category of emergent process because they do not have that category.

Productivity of intuitive knowledge: Naïve theories, according to the TT, are irretrievably wrong when compared to normative science. Naïve ideas are beyond wrong, they are incommensurate¹⁰ with succeeding conceptualizations according to Carey [1991; 2009]. The OV makes a similar assumption: Naïve conceptions—the ones that are difficult to change—are wrong “by definition” [Chi, 2013, p. 50] but also misconceived—incommensurate specifically in the ontological sense. For both TT and OV, the instructed theory simply cannot emerge from the old way of thinking in the relevant domain, so some radical rebuilding using mostly new ideas is necessary. Chi has consistently emphasized the inadvisability or impossibility of building on naïve ideas (naïve ontologies) since her 1992 work. Instead, students must literally be told that their conceptions adhere to a wrong ontology, and the new ontology, in which new scientific concepts must be built, must be directly instructed without reference to the inappropriate, old ontologies. Wiser and Carey also emphasize the lack of continuity between naïve and expert theories, and hence the advisability or impossibility of invoking naïve theories in the learning process. KiP holds that intuitive ideas are essential in scaffolding learning. So, a much more complex and interesting story concerning the relation of old to new conceptions is thereby forecast.

Grain size and structure: The grain-size of conceptual structure of TT analysis is large—theories as a whole, and their constituent (“few, core”) concepts. The description of students' concepts and theories is generally accomplished in a few

¹⁰ The meaning of “incommensurate” is subtle and can be debated. However, for present purposes, a good gist is that “core concepts of a prior theory are not expressible in terms of the core concepts of the successor,” and vice versa.

paragraphs of text. In contrast, KiP targets systems of many sub-conceptual knowledge entities. Extensive description of their individual properties and exactly how they enter into conceptual change is necessary. The OV does not much concern itself with the breadth and detail of naïve ideas. Only the supposed constraint of limited ontologies and limited causal forms is of interest. No specific pathways building on naïve knowledge toward the necessary new ontology or new causality have been put forward; rather, they are generally denied.

It is worth mentioning that KiP does not deny or avoid larger grain-sized analyses. Indeed, the central analysis in this microcosm concerns the development of a stable, widely applicable, and self-consciously held “macro-model” of thermal equilibration. Furthermore, a prominent part of KiP theory is a model of concepts [diSessa & Sherin, 1998], which includes its own definition of and expectations about coherence at that level. At least some TT work acknowledges both large-scale and small-scale mental objects [e.g., Wiser & Smith, 2008], and Chi’s early OV work was explicit about the existence and importance of small-scale level of analysis. So, the fact of finer-grained analyses—and even their necessity—is not an axiomatic distinction between paradigms. However, it is fair to say that neither the TT nor the OV have put forward empirical analyses at a small grain-size, including for the instructional topic here. And, furthermore, it is probably fair to say that the TT and OV at least give the impression that analyses at finer grain sizes are unlikely to be decisive in contests between paradigms. That point will be contested, here.

Learning on a short time scale: As mentioned earlier, KiP can and often does focus on learning and change over small time scales and has increasingly used microgenetic empirical studies that employ process data. In contrast, TT work typically involves cross-sectional, before-and-after analyses using indicators of change. Microgenetic analysis is nearly absent in TT and OV research. So, we can ask: Can TT or OV accounts enfold the phenomena uncovered in the case study?

The following major section describes the case of learning treated here, including the instructional goal and a sketch of specifics concerning student conceptions from prior KiP study that are relevant here. Then, I present the microcosm, itself, and finally enter into the detailed comparisons of TT, OV, and KiP as they apply to this case.

The Case

The case study of learning that constitutes our microcosm comes from a 3-hour instructional unit on thermal equilibration, with a small (6 students), early high school class.

The Instructional Goal

The goal model of thermal equilibration that we wanted to teach is called Newton's law of cooling:¹¹ An object at temperature T equilibrating in an ambience at temperature $T_{ambient}$ follows the differential equation:

$$\frac{dT}{dt} = k (T_{ambient} - T).$$

In words, the time rate of temperature change is proportional to the difference of the temperatures between the environment and the object that is equilibrating. This law leads to a characteristic slowing approach to equilibrium, technically, exponential decay, as in Figure 1.

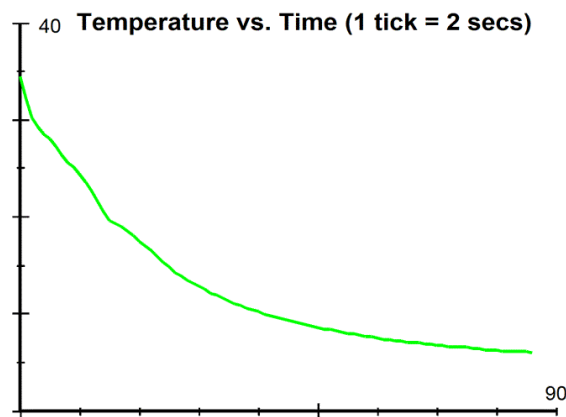


Fig. 1. Student data showing the characteristic “settling in” of equilibration.

Our initial conjecture was that we could scaffold students' seeing the differences of temperatures as a kind of “driving force,” acting on the speed of temperature change. What we found in the core analysis, below, is a very particular learning path to precisely that end.

Sketch of the Instructional Procedure and How the Class Proceeded

The instructional plan was as follows:

1. The students participated in a brief and open full-class discussion of what happens when a glass of water or milk is removed from a refrigerator and placed on a kitchen table.
2. Students were invited to revisit the same phenomenon in terms of graphs: “Show us with a graph how you think the warming happens. Can you explain why your graph happens?”

¹¹ This conventional terminology is somewhat misleading because Newton's law of cooling works just as well for heating. In diSessa [2014a], I used the term “Newton's law of thermal equilibration.”

3. Students were given materials (baths of cold or hot water, test tubes containing hot or cold water, a thermal probe and a computer data-collection program that graphed results) to experiment with heating or cooling.
4. The teacher led a full-class discussion and review of the experimental results and features of what transpired.
5. The teacher scaffolded construction of a computer model that could accommodate the results of the experiment. This is where we intended to introduce the normative model: The change in temperature over a short period of time is proportional to the difference between the current temperature and the ambient temperature.
6. The concept of equilibration was extended by considering a wide range of circumstances where one might observe it.

On the first day of the three-day (one hour per day) instructional treatment, the class went through steps 1-3. Seven minutes into day two, while reviewing the graphs the students had produced (step 4), the critical event analyzed below occurred. There had been no prior attempt to instruct, and reviewing the video data revealed no inadvertent instruction.

The focal event that was analyzed involved one student (called W) who first proposed the model that eventually became consensual. However, microgenetic analysis of the class's thinking both before and after the focal event led to a couple of interesting observations, which we recount later.

Identifying Resources: P-prims as Core Schemata

Before analysis, I need to introduce both the type of intuitive schemata that KiP claims to exist and a number of particular examples that will be relevant here. KiP holds that much of naïve physical intuition consists of a large number of nearly independent, small grain size elements, called p-prims [diSessa, 1993a]. P-prims are similar to physical laws in the sense that they prescribe what happens in situations to which they apply. They are “what just happens, naturally.” However, there are many more p-prims than principles of physics, and, as knowledge elements, p-prims have rather different qualities compared to principles of physics. They are “sub-conceptual” in the sense that they are not, in themselves comparable in complexity to scientific concepts, principles, or theories. However, p-prims do become *part of* the encoding of normative physical concepts and laws. P-prims are only weakly linked to language (there is no conventional lexicon for them), and a lot of their properties flow from their contextuality, exactly when they are invoked or not invoked.

Below is a list of the p-prims from diSessa [1993a] that are needed to understand what our class accomplished. As mentioned, all of these p-prims were discovered in the context of mechanics, force and motion problems, and none involved thermal phenomena. P-prim names are italicized, and bold font marks their first appearance.

Abstract balance is a p-prim that asserts the equality of quantities that “should” balance each other. For example, sides of a pan balance (think of the scales of justice) seem to “want” to level out at the same height.

Abstract balance may be temporarily broken, resulting in **abstract imbalance**. For example, the pan balance may be put “out of equilibrium” by pushing with a finger. However, if the perturbation is removed, the tendency toward balance automatically comes into action; **equilibration** ensues. Two patterns of return to equilibrium are typical. **Slowing equilibration** has the balance re-established in a simple, slowing motion, not incidentally like the graph in Figure 1. **Overshooting equilibration** has balance being reasserted via one or more overshooting motions.¹²

A critical observation is that these equilibration p-prim s are not agentive. Subjects do not search for an agent (e.g., a force, which physicist *require*) that is responsible for equilibration; it just happens. Equilibration p-prim s sit on the natural side of Aristotle’s distinction between natural and violent motions.

In strong contrast, the powerful and central p-prim known as **Ohm’s p-prim** is fundamentally agentive. Roughly, *Ohm’s p-prim* asserts that “more effort begets greater results.” In more detail, some agent¹³ (not necessarily animate—anything that can exert force is agentive in this sense), achieves a result in proportion to its level of agency or activation, and inversely proportional to any resistance involved. Ohm’s p-prim interprets many instances of “effort” and “result,” such as throwing harder to make something go faster or farther, or “working harder” to accomplish some end, such as better grades in school. The OV’s direct causality is closely related to, if not identical to, the agentive causality that one sees in *Ohm’s p-prim*. Chi also uses the language of “agents” to describe what are, in her terminology, the initiators in direct or sequential processes.

In this technical language, I can say now that our instructional intent was to engage *Ohm’s p-prim* in understanding thermal equilibrium, where what I called (informally) “the driving force” is identified with the agent slot in *Ohm’s p-prim*. I can also now anticipate a key difficulty in realizing our instructional intent. Even if students seek to interpret temperature equilibration in terms of equilibration p-prim s—which seems like a good re-use of prior knowledge—one needs somehow to engage agency, also, in their thinking. Since equilibration p-prim s do not ordinarily engage agency, if those p-prim s are activated, something unusual will have to happen to get to an agentive interpretation of thermal equilibration.

Analysis of the Focal Event

I present here an outline of the analysis of the focal event in the microgenetic study where students generated the normative view of thermal equilibration.

¹² The terms “overshooting equilibration” and “slowing equilibration” are not used in diSessa [1993a]. However, these two patterns are described.

¹³ I underscore components (schema slots) of the p-prim.

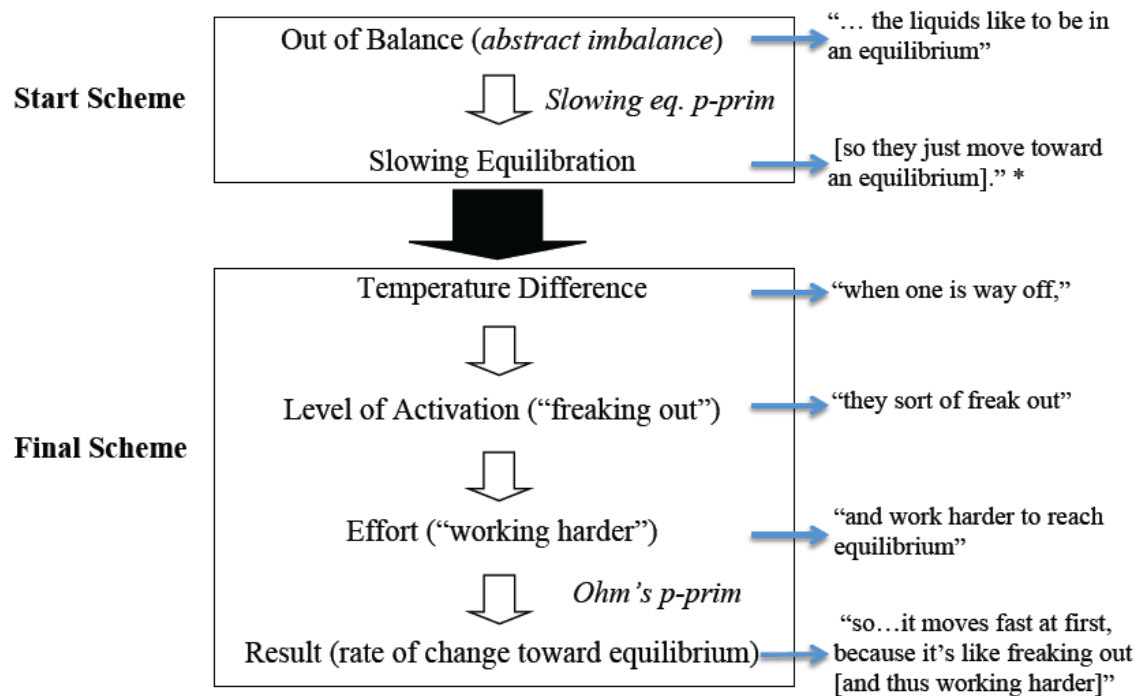
After the class did the experiment on heating and cooling of water in test tubes immersed in baths of warmer or colder water (at the end of day one, in step 3 of the instructional sequence; see Figure 1 for representative data), the class was asked (step 4 of the instruction, at the beginning of day 2) why the temperature changes quickly at first, but slows down later. Just a few minutes into the ensuing discussion, subject W provided the following explanation. Lines are numbered for reference.

1. I think that the liquids like to be in an equilibrium.
2. So, when one is way off, they sort of freak out
3. and work harder to reach equilibrium.
4. And when it's closer to equilibrium, they're more calm,
5. so they sort of drift slowly towards equilibrium.
6. So maybe that's why it moves fast at first, because it's like freaking out.
7. But then it just calms down as it approaches the right temperature.

W had come into this phase with the idea of *slowing equilibration* in place (see appendix, precursor 3). Now, however, he is interpolating that description with a more causal, and definitively agentic interpretation. (This interpolation is the “something unusual,” described above—a clever and unusual link between equilibration *p*-prims and agentic ones.) In line 1, he reiterates his commitment to the underlying reason for equilibration, glossing *abstract balance*: “[L]iquids like to be in an equilibrium.” In line 2, he implicates a rather shocking anthropomorphism, “freaking out,” turning the liquids into agents of the first order. The rest of the class laughed at this, but followed his reasoning and fairly quickly adopted this as the best explanation that they could come up with.

Line 3 “channels” the effort or impetus in the now-agentic liquids into their “working harder,” a prototypical phrase used to implicate *Ohm's p-prim*. At this point, the speed of temperature change is implicit, although it is clear in the context (that is the question being addressed by students). Later in his explanation, rate of temperature change becomes explicit.

Line 4 contrasts the case where the difference of temperatures is less, and, in line 5, one gets a slower rate of equilibration. Lines 6 and 7 reiterate the argument, highlighting the high initial level of agency (“freaking out”), which contrasts with a lower level (“calms down”) later on, corresponding to a slower rate of temperature change.



*Fig. 2. W's "freaking out" explanation. Slowing equilibration (top panel) is replaced by an extended causal chain (bottom panel) featuring Ohm's p-prim as a critical link. Quotes corresponding to elements of the explanation are on the right. Note: Bracketed expressions are inferred from other parts of W's talk. The * marks that W's articulation of slowing equilibration, per se, is clearer elsewhere. See the appendix, precursor 3.*

Figure 2 sketches the whole development implicated in this explanation. W started, essentially, with the p-prim *slowing equilibration* (top panel; precursor 3 in the appendix), and then, in the explanation quoted above (schematized in the bottom panel), he replaces it with a chain of implications that starts with differences in temperature, and ends with the rate of temperature equilibration, just precisely Newton's law of cooling. In later discussion the students spontaneously removed all anthropomorphic language (see mainly developments 4 and 5 in the appendix) and wound up with, essentially, "temperature difference drives rate of temperature change," just exactly our curricular intention.

It is worthwhile pulling one more detail from the full microgenetic analysis into the discussion here. Another student, R, who anticipated some of R's conceptual moves, was, in fact, the first to introduce the idea of "trying harder" to the discussion (appendix precursor 4). Then, she independently upgraded her talk to a dramatic anthropomorphism (the water is "shocked"; appendix precursor 5) during the experiment, out of earshot of W. This seems to emphasize the importance of the

anthropomorphic step in W's proposal. When W produced his explanation, R quickly and emphatically agreed with it (appendix development 1).

Mechanisms of Change

Since a good map of the relevant p-prims exists, one has an unusually clear example of learning with p-prims. Here is what can be seen in this data, in terms of how W produced his "freaking out" model. I describe the relevant changes, the headers below, as "mechanisms."

Causal chaining – W created an explanation of thermal equilibrium by chaining together a set of causal links: Difference in temperature causes "freaking out," "freaking out" causes "working harder," "working harder" characterizes the first slot in *Ohm's p-prim*, which is linked causally to a greater result, a greater speed toward equilibrium.

Causal interpolation – W started with the bare *slowing equilibration* p-prim. But, then, he interpolated a causal chain to explain *slowing equilibration* in an agentive and much more elaborated way.

In our instructional experience, it is very unusual for students to see *slowing equilibration* in heating and cooling contexts. The likely reason is that equilibration p-prims are often supported by spatial symmetry (a pan balance or scales of justice, again, is evocative). In this case, *equilibration* is being applied in an unusual context, in which no evident symmetry can be seen. Unusual invocation, in fact, implicates our next mechanism.

Shifting context – One of p-prims' primary malleabilities is that they may be invoked in circumstances where they are not usually invoked. This constitutes learning to the extent that the new context is felt to be reasonably apt for the p-prim and the new activation is persistent. Here, I already noted that W's starting scheme, *slowing equilibration*, rarely appeared in students' responses to our instruction of thermal equilibration.¹⁴ More centrally, *Ohm's p-prim* gets invoked, it seems, with W's dramatic "freaking out" language, implicating an unusual attribution of agency to the water. However once *Ohm's p-prim* occurred to W, he emphasized its agentive nature in his language: The water is "freaking out."

¹⁴ Another student in this small class used *overshooting equilibration* (appendix, precursor 2), which helps substantiate that equilibration p-prims, generally, may be seen by students to be relevant to this situation, even if relatively rarely. W is not unique in this respect.

Binding¹⁵ – Another of p-prims’ malleabilities is in the features of the world to which they attach. W’s explanation is brilliant (and normative) in connecting to just the right input feature (temperature difference) and output feature (speed of temperature change) in order to reproduce Newton’s law of cooling. P-prims produce commonsensical (and correct) results when bound to certain features of the world, and “misconceptions” when bound to others. Here, for reasons I do not describe (or fully understand), W got his bindings perfectly correct. The student, R, who anticipated W’s dramatic “freaking out” anthropomorphism with her own “shocked” did not on her own produce normative bindings. See the appendix, precursor 5.

Comparative Thematic Analysis

I now undertake to expand this episode of learning into a comparative microcosm on conceptual change, centering on the set of dimensions developed above. The dimensions are slightly elaborated from the original presentation in some cases, and, in order to best match the empirical flow of the microcosm, they are not necessarily mentioned in sequence.

Preliminaries

I begin with a few observations about the status of naïve knowledge—p-prims in particular.

KiP hypothesizes that intuitive causality is distributed in a large set of “loosely connected” p-prims, and, indeed, these p-prims are diverse in their apparent ontological commitments. In this case study, one sees “strange” explanations: *Slowing equilibration* is offered as an autonomous, self-explanatory description, even if it seems to scientists’ ears vaguely articulated as an explanation. However, there is nothing agentic (contrast the OV’s take on the relevant naïve ontology, direct causality) about it.¹⁶ Here, students eventually found an agentic explanation that they preferred, but its quality subsequently changed; anthropomorphism and overt agency disappeared (developments 4 and 5 in the appendix).

Equilibration p-prims seem near form-identical to some abstract principles of physics such as “entropy increases.” Other p-prims and p-prim classes are also devoid of agency and are more geometric than consequentially material in their commitments. See *guiding* (e.g., a train just follows its track; there are no forces), *wobbling*, and figural p-prims (e.g., an orbit around a square planet is, roughly, square) in diSessa [1993a]. Thomas Kuhn [1977] observed that the causality of

¹⁵ The particular real-world entities that are interpreted as relevant examples of the slots in a schema, such as “agents,” and “results” in Ohm’s p-prim, are generally described as the *bindings* of those slots.

¹⁶ I do not think equilibration p-prims express a commitment to material ontology, either, but the argument may be more complex. See continuing discussion just below.

ancients—and, I would add, naïve causality—often seems form-identical to what we see in modern science. It is not at the level of forms of cause or explanation that science advances. I maintain that it is too early in our studies to assert that naïve ideas are strongly limited in terms of their causal forms, compared to professional science. The invocation of agentic causality and its subsequent apparently easy and unproblematic disappearance follow that pattern.

diSessa [2014c] argues that it is a category error to construe p-prims to be true or false. Ecologically, they work well enough in the circumstances in which they are normally evoked: When we work harder, we do, generally, get greater results. So, in general, they work in some circumstances, but not in others. Consider the p-prim that things move in the direction you push them. If a thing is at rest—the condition in which one finds most of the objects around us—that idea works excellently. However, when pushing a moving object, the push combines with existing motion (momentum)—a point that many or even most people do not anticipate [Clement, 1982; diSessa, 1982]. Movement in the direction of push, in that context, is a “misconception.”

If interpreted as universal claims, then p-prims are false. But, there is no basis to interpret them as universal claims. Instead, whether instances of their use lead to valid claims and expectations or invalid ones is established by their invisible and unknown (to subjects) conditions of activation and bindings.

P-prims are often—even usually—used in composition with other knowledge to reach articulate judgments. Then, how can one assess the contribution of one p-prim, in a compound construction, to the truth or falsehood of such a judgment? In general, one cannot.

These characterizations of intuitive knowledge within KiP are strongly differentiating among theories of conceptual change. Naïve theories in the TT perspective, in educational circumstances, are universally considered “false,” and in need of replacement by “correct” knowledge. Carey systematically attributes persistent incorrect answers and failure to learn to naïve theories [Carey, 2009], which are not only non-normative, but also incommensurable with normative ideas. The same is true for OV. Below, Chi aligns herself with Vosniadou, a theory theorist in the broad sense, emphasizing that naïve ideas are assumed to be “incorrect,” and she notes that those ideas must be changed to “correct knowledge.”¹⁷

... [A] student may have already acquired some naïve ideas, either in school or from everyday experiences, that are “in conflict with” the to-be-learned concepts (Vosniadou, 2004). It is customary to assume that the naïve “conflicting” knowledge is incorrect, by some normative

¹⁷ See Smith, diSessa, and Roschelle (1993) for an extended review of the attribution of falsehood to naïve ideas.

standard. Thus, learning [in the case of conceptual change] is not adding missing knowledge or gap filling; rather, *learning is changing naïve conflicting knowledge to correct knowledge.*

[Chi, 2013, p. 49. Emphasis added.]

In another place in that article (p. 50), Chi says that naïve conceptions are false “by definition,” since they are not normative.

Productive Use of Naïve Knowledge

The case study shows unambiguously that naïve knowledge can become involved in students’ coming to understand Newton’s law of cooling. Cases in point include things like *equilibration* p-prims and *Ohm’s p-prim*, and also “primitive” anthropomorphic metaphors (water’s “freaking out”). This fact violates the fundamental and widespread epistemological assumption that naïve ideas or theories are flatly false and for that very reason are in need of dismissal. Below, Chi continues to presume that misconceptions are simply false, and explicitly states that “everyday experiences” and schooling are powerless to help in “confronting” (so as to replace) misconceptions. Within her frame of reference, there is little point to considering whether “everyday experiences” or “misconceptions” (in the guise of p-prims) might actually come to *constitute* part of a normative understanding.

In short, robust misconceptions ... are extremely resistant to change so that everyday experiences encountered during developmental maturation and schooling seem powerless to change them, even when students are confronted with their misconception.

[Chi, 2013, p. 59]

Epistemological presumptions such as these can become hindrances to observation. *Ohm’s p-prim* or *equilibration* p-prims have unusual properties as knowledge: They certainly don’t look like “coherent theories,” and they come and go according to difficult-to-describe contextualities. This is the main reason to consider them to be sub-conceptual or phenomenological, rather than conceptual or theory-like. TT adherents may filter them out of consideration when looking for resources for conceptual change. To them, the appearance of such ideas in data may look like noise in the signal.

Locating Resources: Domain Flexibility

Subsequent to Piaget’s ideas about domain-general development—and regarded as an important corrective to them—it became a persistent assumption that naïve ideas are distinct from domain to domain [Carey, 1991; Hirshfeld & Gelman, 1994; Inagaki & Hatano, 2002; Wellman & Gelman, 1998]. Having identified conceptual development as localized in domains, if conceptual change starts with naïve, coherent and incorrect theories, there is no room for developing more normative ideas from within the domain.

Here, Marianne Wiser explains that her instructional design concerning heat and

temperature works, significantly, by shielding students from using their prior ideas, starting, instead, with a “free-standing” network of ideas, which bootstraps itself. As mentioned earlier, Wiser is particularly relevant to our microcosm because she and Carey claimed to have identified the naïve theory of thermal phenomena.

We believe that our models help students learn the textbook theory because they present the textbook theory as a “free-standing” network of concepts, relations, and explanatory schemata, which constrain each other, and do so transparently enough that *students can construct a new understanding of heat and temperature without “borrowing” and thus interference from the naïve theory.*

[Wiser, 1995, p. 34. Emphasis added.]

The OV has its own version of the necessity of starting from scratch. In building the relevant new ontology, Chi emphasizes that this must be done without “borrowing” anything from the old ontology [Chi, 1992, p. 137]. Chi and Slotta [1993, p. 256] say, “This view of learning physics [the OV] suggests that it is not possible to refine or develop intuitive knowledge to the point that it becomes the veridical physics knowledge....” And Slotta and Chi write:

Teachers should not try to “bridge the gap” between students’ misconceptions and the target instructional material, as there is no tenable pathway between distinct ontological conceptions. ... Indeed, students’ learning may actually be hindered if they are required to relate scientifically normative instruction to their existing conceptualizations.”

[Slotta & Chi, 2006, p. 286]

Consistent with her general theoretical claims, Chi does not identify any particular naïve knowledge resources that might contribute to the construction of the new ontology. Instead, she maintains that relevant new causal forms must be built from scratch, mainly by direct instruction.

The intent to shield students from their own sensibilities toward thermal phenomena may seem suspect to researchers and educators who adhere to constructivist principles, which suggest that learning works by building on and reforming the current state of a student’s knowledge. Nonetheless, the logic is clear. If students have only misconceived ideas concerning a domain, we must avoid or marginalize those ideas, and either build from scratch, or, possibly, draw from elsewhere.

I mentioned, in passing, one critical point concerning the analysis I provided of the microcosm. *All* of the p-prims that fed into the analysis were originally studied using questions about mechanics, the physics of moving, interacting objects. There were no questions about thermal phenomena in the study [diSessa, 1993a].

One can interpret the appearance of mechanical ideas in the domain of thermal phenomena in two ways. First, one could take this to be consistent with the TT claim that there are no positive resources in the relevant domain; one must go outside the domain to find such. This does not let TT off the hook to explain the facts of our microcosm. The TT implicates mechanisms that might use out-of-domain resources (see later discussion of the TT's mechanisms of learning), but, as far as I have read the literature, TT researchers rarely, if ever, identify specific resources, such as the p-prims cited above, on which one might build. In addition, the blanket assumption that naïve resources (theories) are wrong, and therefore have no use, would seem to apply also to out-of-domain resources. And yet, here, the “out-of-domain” resources are also misconceptions in their native domains: *Ohm's p-prim* is non-normative in some circumstances, and so are equilibration p-prims (in circumstances such as a pan balance!) where a physicist demands agentic explanations (equilibration *requires* a force). So we might ask TT researchers, how is it possible that false *and* out-of-domain ideas became a basis for normative understanding? As mentioned, a KiP response to that question is that intuitive resources give correct or incorrect predictions and explanation depending on the context in which they are used, their particular bindings in use, and also on the other ideas recruited to think about particular situations.

Let me elaborate the importance of identifying particular prior resources for learning. The KiP analysis of the case of learning here implicated a particular set of p-prims. There is no basis for claiming any other set exists (although there is no claim that none can exist). For all we know, if those p-prims did not exist, or had different properties, then the use of mechanics as a “source domain” might simply not work. Even more, if one does not recognize sub-conceptual resources, as TT and OV adherents do not, then not only are the particular resources used in this microcosm unrecognizable (they do not fit the theory), but no similar set of resources can exist.

The second way to interpret the fact that apparently out-of-domain ideas strongly support good learning in our microcosm is to question the “out-of-domain” nature of these resources. In my view, this is the best direction to pursue. To pursue it, however, I need a bit more preparation.

Wiser [1995], building on Carey and Wiser [1982], identifies what she claims to be the naïve theory of thermal phenomena, “naïve thermodynamics,” so to speak. The core of this theory, called the source/recipient model, is that hot and cold things act autonomously on other things to change their temperature. “Autonomously” calls out the fact that there is no reciprocal causality. The object that is being heated or cooled does not act on that which heats or cools it, nor does the rate depend on any property of the acted-upon object.¹⁸ This is an example of Chi's direct causality, and

¹⁸ W explicitly remarks that both objects (water in test tube, and water in the bath) freak out. The final scheme produced by this group of students is relational and symmetric: It is the difference in temperatures that drives change, and there is no

she implicates this very way of thinking as a primitive, unscientific causality, incapable of comprehending “emergent” phenomena, such as heating and cooling [Chi, 2013].

Surprisingly, we observed little evidence of the source/recipient model in our microcosm. In the two case studies of students spontaneously developing models of thermal equilibration, only one student (not one from the case developed here) exhibited unambiguous evidence of the core element of the model, the direct causality mentioned just above. Recall that the two case studies in the original study [diSessa, 2014a] occurred before any instruction on the normative model. How is it possible that students did not evoke what, according to Wisser, Carey, and TT principles, are the only resources available to them, their naïve theory of thermodynamics?

Here is a KiP-based interpretation of these facts. First, I do not claim that students do not invoke ideas that Wisser documents as involved in the naïve theory. (I do claim that students are not consistent in use of these ideas, and that the whole set is not nearly so coherent as claimed.) But, students also have *other* ideas—p-prims, phenomenological (sub-conceptual) elements—outside the “theory” (if there is one) that are also available to be used in considering thermal phenomena. That is what is happening here.

The ready availability of other ways of construing thermal equilibration, which showed in extensive, unprovoked expressions of students in our classes, suggests that students do not respect the domain boundaries with which Wisser and Carey align themselves. To the students, it seems that thermal phenomena are not categorically distinct from mechanical ones.

OV adherents are in a slightly different position with respect to the empirical observations of students using “out of domain” knowledge. It is a common assumption in conceptual change research that different domains simply have different explanatory ontologies. So student reliance on an ontologically distinct domain might well be expectable, a cause of misconceptions. But, here, the problem is that students get to the correct model by using ontologically inappropriate ideas.

In general, following constructivist principles, I believe it is important to listen to students, respecting their feelings about what ways of thinking are relevant to which phenomena, and not impose scientists’ conceptions about how the world is carved up into domains, conceptions that are formed by years of scientists’ experience that it is productive, in some ways, to think of mechanics as different from thermodynamics. How else than observing students’ predilection to use

obvious principle for distinguishing source from recipient. Some of our sixth grade students (not in the three closely related editions studied in diSessa [2014a]) explicitly noted that if the equilibrating object were larger than the ambience (the room), then the room would change more than the object.

particular ideas can we develop an empirical and non-question-begging view of domains in students' thinking?

Starting directly with a KiP point of view, the very fact of many, loosely connected elements makes it extremely implausible, if it is possible at all, that all such elements could adhere to one "domain"—could be evoked only in one-domain's specific circumstances. Each element has a range of circumstances in which it will be invoked. While there may be a central tendency of attachment to a certain class of phenomena, unique attachment to one well-defined class, to a domain, is difficult to imagine.

Ohm's p-prim, by itself, is a prototypical example of domain flexibility. I pointed out early on that this p-prim is applied to both mechanical and psychological situations. Indeed, the idea of "more effort" giving rise to "more effect" seems more likely to have originated in personal-psychological phenomena (literally trying harder) than in purely mechanical ones. And yet, *Ohm's p-prim*, in its mature state, easily crosses domain boundaries (e.g., between physics and psychology) that are inviolable, at least in TT versions of conceptual change. Each domain has a distinctive naïve theory. Equilibration p-prims, we have learned in this microcosm, can also travel between mechanical and thermal situations.¹⁹

I close with a methodological point. In the following, Wiser warns researchers that asking students questions that might implicate other domains is dangerous.

... the problems and phenomena used to probe students' knowledge may not belong to the domain of the naïve theory. If students' concepts, like scientists', are legislative and embody hypotheses about the contexts to which they are to be applied, then using those concepts to account for phenomena outside their domain of application is likely to generate inconsistent answers.

[Wiser, 1995, p. 30]

Wiser is ruling out of relevance the research that originally discovered and cataloged much of the knowledge that students in our microcosm used, of their own volition, in coming to understand thermal equilibration. The rationale, as far as I understand it, is that, in asking about "a different domain," one naturally will get different answers, thus "the naïve theory" will appear unstable. But it turns out that

¹⁹ I think it telling that TT adherents have not documented or noticed, as far as I have read, that mechanical intuitions often evoke ideas about equilibration, even as misconceptions. Methodologically, TT researchers might be prematurely settling on the kind of ideas they think students use in a particular domain, those that fit some coherent naïve theory. Thus they would not look for other kinds of ideas, supposedly "out of domain" ideas, or student expression of such ideas might be marginalized as "noise." One of the earliest and best known "naïve theory of mechanics" [McCloskey, 1983] has no elements of equilibration in it.

“instability” means simply that students have a variety of resources that they will use for any given phenomenon, and, here, they willingly imported some that appear to Wiser to be “out of domain.”

Carey and Wiser’s naïve theory seems artificially narrow, recognizing only the (incorrect) resources that define her naïve theory of thermal equilibration, refusing to let students, themselves, speak about domains, what situations invoke what knowledge. I believe it is fair to say that Wiser is imposing what is, at best, a post-conceptual change construction of domains on students.

The implications of domain flexibility are critical for instructional design. We may or must explore widely if we want to build good intuitive bridges to scientific ideas.

To sum up, from a KiP perspective, domains are suspect. The individual elements that make up how students think about a class of phenomena before instruction simply cannot all perfectly respect the domain boundaries to which scientists have grown accustomed.²⁰ However, this “failure” of coherence in naïve thought provides an opening for advancement, as happened here. At least some naïve elements that are freely and spontaneously invoked might be productive. If we use the concept of domain at all for naïve knowledge, we must use it carefully, and respect flexibility and openness if we find it empirically in students’ thinking.

Out of the Shadows Learning

This theme picks out an important thread in the idea of re-use of intuitive knowledge, from whatever domain. Just because students do not usually—or at all—use certain naïve elements in their spontaneous interpretations of a particular set of phenomena does not mean they cannot. In this case, I noted that it was relatively rare that students spontaneously used equilibration ideas in the context of thermal equilibration. Similarly, even when equilibration is invoked, agentive ways of thinking do not come along for free. And yet, this group of students reacted very positively *once these ideas were invoked* by one of their members. In a nutshell, ideas that may be “in the shadows” may be relatively easily brought to prominence. We intended to invoke agentive ideas concerning thermal equilibration; in this case, the students beat us to the punch.

In other KiP-based work, we have seen that, given a choice, students sometimes come to prefer explanations that they, themselves, essentially never produce on their own. I noted here that some of W’s moves were rare in spontaneous consideration of thermal equilibration, but other students found them attractive,

²⁰ Distinctions between domains probably should be regarded as scholastic, rather than scientific. For example, both physics (e.g., how the heart pumps blood), and also chemistry (metabolism) lie at the heart of biology. Modern biology may be difficult to distinguish from biochemistry and biophysics, that is, physics and chemistry specialized to the context of biological systems. The autonomy of chemistry and physics is similarly questionable.

once articulated. Similarly, in the (often) long path toward conceptual change, entire families of p-prims become much more prominent, and, in parallel fashion, other families recede from importance [diSessa, 1993a]. Different instructional choices can even entail different choices of such families [diSessa, 1980].

From the standpoint of KiP theory, these phenomena are not at all surprising. Shifts in activation are common during learning. Naïve conceptions simply do not “know” the range of contexts in which they will find productive use. Contrary to Wiser’s methodological advice, it behooves researchers and curriculum designers to explore widely to see what ideas might be useful—and might even be judged to be useful by students, themselves (such as agency in thermal equilibration)—once evoked.

Beware Characterizing Naïve Causality as “Unproductive” or “Unscientific”

Given the fact that Chi implicates direct, agent/patient causality as insufficient for scientific understanding, most notably in understanding thermal equilibration, our microcosm contains a highly focused and stark irony. These students surpassed their naïve understandings of thermal equilibration precisely because they *did*, quite out of the ordinary, evoke agent/patient causality. Water became an agent, freaking out. A critical component of their learning, *Ohm’s p-prim*, became accessible precisely because they invoked the direct causality implicated by Chi to be responsible for student’s inability to understand thermal equilibration.

As if to heighten the irony, the students went on, spontaneously, to remove all agentive language from their description of thermal equilibration (appendix developments 4 and 5). In the end, their description seemed perfectly mathematical. The rate of change of temperature is merely asserted to be proportional to differences in temperature. Even if agentive causality is “bad,” it seems in this case to be an excellent bootstrapping idea, and is easily removed to leave “more sophisticated” forms of causality. Agency might sometimes be little more than a way of speaking.²¹

Multi-threaded and Stepwise Learning

TT or OV adherents might level the following criticism against the learning that these students accomplished: They do not come to a full appreciation of thermal equilibration. In particular, they do not phrase the idea in terms of both heat and temperature. The general scientific form of thermal equilibration is that heat flows

²¹ Students may be “smarter” than the words they use. These students most certainly knew that the water is not literally an agent, capable of freaking out. They laughed at W’s first formulation and several subsequent mentions. More modest forms of anthropomorphism, “if you bend an object, it *wants to* return to its natural length,” which seem almost conventional, definitively do not imply that students believe inanimate material things have wishes and wants.

proportionally to temperature difference, and, concomitantly, temperature change is, in general, proportional to change in heat.²²

In the following, I comment on a quotation from Wisser [1995, p. 28] in order to make clear that the model our students came to surpasses the source/recipient model, and in what ways, precisely, it did so. I separate the continuous quote into four parts so as most easily to comment on individual points.

[T]hermal equilibrium states the conditions under which heat is exchanged (two objects exchange heat until their temperature is the same); as such it is a principle central to the theory. Since it is based on a concept of heat distinct from temperature, it is not available in the naïve theory.

It turns out that the conditions for the process of equilibration to occur do not need the concept of heat. That equilibration happens as a result of temperature differences is front and center in our students' model. This is, in fact, precisely the normative form.

The limitation is that the students do not yet conceive of the process as heat flow. So far as we saw, our students did not use the concept of heat at all. We did not teach about heat, and they did not learn about it. But, in a little more than an hour's work, without direct instruction, our students most definitely transcended "the naïve theory" with respect to the conditions in which equilibration occurs.²³

But the naïve theory has no need of such a principle because heat exchanges do not have to be explained:

In the students' new model, thermal equilibration *does* have such a principle. The rate of temperature change is driven by a difference in temperatures. Since the naïve theory does not have such a principle, as stipulated by Wisser, these students have cleanly surpassed it. By Wisser and Carey's definition, this is conceptual change, even if it is not the full development that we might like eventually to achieve.

²² The added generality by including the concept of heat is that the proportionality constant between change of heat and change of temperature depends in easily describable ways on such factors as the mass of the sample and its composition. These were not part of our curriculum. We aimed to teach about equilibration, not exclusively about heat and temperature.

²³ It is worth noting that the students also came to a numerically precise specification of temperature change. At stage 5 of our curriculum, they designed a computer program and matched empirical data to it. In this respect, our curriculum is more ambitious than Wisser's, even if it is less ambitious with respect to employing the concept of heat.

Sources emit heat spontaneously; it is in their nature to make other objects warm.

This assumption is gone from the students' model of thermal equilibration, if they ever had it. In their model, equilibration is fundamentally relational. It involves a difference of temperatures between "source" and "recipient." It does not depend solely on the source, per the source/recipient model. From the beginning, W had changes in temperature dependent on both objects (i.e., on difference in temperature), and both objects (both hot and colder water) took consequent actions ("freaked out" and worked harder to get closer to equilibrium).

Some students have their own version of thermal equilibrium, based on a single concept: They know that, eventually, an object placed in contact with a source will reach the temperature of the source but that belief is not central to the naïve theory....

At the end of this statement, in "but that belief is not central to the naïve theory," Wisser denies the possibility of "out of the shadows" learning: Because a naïve idea is not central, it cannot easily become central. TT theorists often list the peripheral becoming central, and vice versa, as an earmark of conceptual change [see Carey, 1991, p. 259]. However, our students have come to know the core, normative idea that temperatures converge as long as they are different. So, converging to the same temperature is not an isolated fact but simple a consequence of the very core of their new "theory." A peripheral idea has moved to the center—once again in about an hour's work.

How much of the full (involving both heat and temperature) conception of thermal equilibration have these students learned? Without any curriculum, focused empirical data, nor ready measures of "amount of learning," this is difficult to say. However, they have come to Newton's formulation, which is historically beyond that of the early scientists (the "Experimenters"), who Wisser and Carey [1983] assert entertained the source/recipient theory.

On a larger plane, KiP opens up a large space of curricular possibilities. We have much more work to do in finding naïve resources that might be productively applied. We also see, here, an excellent intermediate curricular goal, Newton's law of cooling, which seems accessible, once again cleanly transcending "the naïve theory." Describing the naïve theory, even if it exists, does not prepare us in the slightest for the possibility of achieving this intermediate goal. Within KiP, educational design may be decomposed into quasi-independent threads and steps, of which we see a brilliant beginning in our microcosm. And, we can get down to the details of fine-tuning educational interventions, directing students attention and reasoning at fine grain sizes, which every teacher and curriculum developer recognizes is important. Announcing that students have naïve theories does not help with these critical micro-interactive processes. It might well be that a significant problem for TT

approaches to conceptual change is that they set goals globally and too high. The learning territory *between* naïve and scientific theories is not well scouted out. The model developed by these students would be called an “intermediate model” by White [1993]. Our microcosm suggests, as White claimed generally, that some intermediate models are both relatively easy to teach, and on a good path toward true expertise. In this case, the model comes with an exceptional scientific pedigree, as indicated by its name: Newton’s law of cooling.

Let me briefly consider the question of “is this conceptual change?” with respect to the OV. An issue, above, was that our students were not taught and did not learn about heat. This is less of a problem in discussing TT since we can use the details of Wiser and Carey’s account of the naïve theory of thermal equilibrium to argue that, by her own definition, conceptual change (away from the naïve theory) has occurred. The OV case is different. There is, obviously, no basis to claim that our students changed their ontology of heat. So, then, the OV might reject Wiser’s details, and claim that this learning is simply not conceptual change (so it should be relatively easy). There is no basis, within the microcosm, to reject such a claim; it is beyond the scope of this study.

In pursuing these issues beyond our case study, one has at least three options.

Option 1: It is awkward for the OV that, independent of heat and emergent considerations, students did seem to change ontologies in their explanation, arguably twice: *W* started the microcosm with *slowing equilibration*; he then moved to an agentive, directly causal explanation; and, finally, students dropped all agentive framing to settle on an apparently purely mathematical rule, that the rate of temperature change is proportional to temperature difference. In other parts of our data set (mostly not within the microcosm), we saw students proposing mathematical relations as explanatory forms.²⁴ However, I do not pursue the claim that ontological change exists, here, without the concept of heat (and without the emergent ontology) for two reasons. First, as mentioned, arguably the best data regarding students and purely mathematical explanations comes from outside the microcosm. Second, the OV’s attitude toward the ontological commitments of mathematical explanations, and toward those of p-prims like *slowing equilibration*,²⁵ have not been made clear to my knowledge. Discussion would be more productive if a definitive position were in the record.

Option 2: One might demonstrate that the accomplishment of these students is generally difficult, and, therefore, demands explanation even if it does not fit anyone’s theoretical definition of conceptual change. Particular theories of conceptual change lose credence or generality if they cannot account for difficulties

²⁴ An abbreviated documentation and discussion of students’ use of mathematical explanation forms appears in diSessa [2014a, p. 831].

²⁵ In diSessa [1993b], my response to Chi and Slotta’s commentary on diSessa [1993a], I raised the issue of the ontological status of equations and equilibration p-prims. To my knowledge, there has been no public response.

empirically at the level of those investigated in the field of conceptual change. Since there is a lot of literature on thermal equilibration, this might even be pursued by filtering out heat from existing research on student difficulties. Our own experience in a relatively large number of classes is that this accomplishment is, in general, difficult to achieve, arguably on par with many cases that are identified as conceptual change in the literature. The KiP analysis of this learning provides plenty of reasons why it may, in general, be difficult, and none of these are comprehensible within the OV (or TT) perspective. (1) Activation of equilibration p-prims seems unusual, for comprehensible reasons (e.g., lack of typical features of activation, such as spatial symmetry). (2) *Especially* after activation of equilibration ideas, agentic interpretations are unlikely. We noted that in mechanical situations, equilibration seems to rule out agentic (forceful) interpretations [diSessa, 1993a]. (3) Finding the proper bindings to intuitive attributes, which W did masterfully, seems plainly unusual and combinatorially complicated (there are many such bindings, and all must be correct for W's model to be normative). As mentioned earlier, while R followed W's path through activation of equilibration p-prims, to mild agency ("trying"), to dramatic agency ("the water is *shocked*"), she nevertheless failed to find the proper bindings (the equilibrating agents still acted on each other, not on themselves), until W made them. Again, see the appendix, precursors 4 and 5, and comments about continuities and discontinuities in the appendix's account of W's core contribution. Proper activation and bindings are generic difficulties that students need to overcome; they are not ad hoc to this case.

Option 3: One might demonstrate that the accomplishment of this microcosm significantly eases further learning, such as incorporating heat into the picture, which I believe to be the case, but cannot demonstrate. Then, ontological change for heat, even if present, likely could not be the full story. This possibility references White's intermediate models strategy, discussed above.

Mechanisms That Are Adapted to Fine-grained Analysis

A prominent contribution of this work is the development of candidate mechanisms of learning in conceptual change. I compare with some complementary work, mainly from the TT tradition.

Early work on KiP began to consider mechanisms of change. diSessa [1993a] mentions two that appear in the list developed for this microcosm (causal interpolation and shifting context). However, work on discovering mechanisms out of real-time data analysis has accelerated [Izsák, 2000; Parnafes, 2007; Levrini & diSessa, 2008; Kapon & diSessa, 2012]. In contrast, however, TT or OV perspectives rarely or never include observing learning mechanisms in action, comparable to our microcosm.

In my view, this gap in TT and OV work is not accidental. KiP prepares for fine-grained analysis of student thinking, both with respect to time-scale (e.g., an element is activated at a particular time, and it may similarly be observed to cease being used) and with respect to nuance of meaning (all of the listed p-prims are

highly specific). *W* started with a particular monolithic conception of equilibration, *slowing equilibration*. But, then, in the focal event, he activated, connected, and bound slots of an array of different ideas to create a complex model.

In most conceptual change literature, mechanisms of change are not easy to find at any but the coarsest level of description or empirical validation. Chi's description of the emergence of new ontologies is mostly instructional. That is, it mostly prescribes refutation as a way of dealing with naïve ideas, and "direct instruction" to construct new ones. It does not elaborate exactly how refutation or direct instruction works at a learning theoretic level, nor does it empirically track the details of these processes. Below, Chi emphasizes refuting naïve ideas (in contrast to engaging, modifying, and combining naïve ideas, found in our microcosm), and prescribes a particular form of direct instruction. Compare the many specifics in terms of ingredients, changes to ingredients, and compositions, found in our microcosm.

Because [newly constructed ontologies or causal schemes] are different in kind, with mutually exclusive properties, confrontation needs to *reject the [old schema] ...*, and build the alternative [new schema], perhaps through *direct instruction* using contrasting cases.
[Chi, 1992, p. 69. Emphasis added.]

In early work, Chi [1992] asserts that familiar learning mechanisms can enfold this change, although she does not produce specific analyses. In later work [Chi, 2013, p. 61], she allows that the process may be complicated, hence she uses the term "schema" to describe what is constructed. But she does not add detail beyond that description.

Among the most detailed work on mechanisms of learning in the main line of conceptual change literature is that drawn from cognitive-historical studies of conceptual change [Nersessian, 1992; Carey, 2009]. Of course, studying the processes of conceptual change in scientists from historical data does not come down to real-time analyses, a point that Chi also emphasizes. Let us see what is offered, in contrast to the mechanisms described here.

Carey proposes the name "Quinian Bootstrapping" for the process that Wiser described, above, wherein a free-standing, uninterpreted (or minimally interpreted) structure is developed/presented, and then is filled in by various bootstrapping processes. Here is how Carey [2009, p. 418] describes the process.

1. Relations among symbols are learned directly, in terms of each other;
2. Symbols are initially at most only partly interpreted in terms of antecedently available concepts;
3. Symbols serve as placeholders;
4. Modeling processes—analogy, inductive inferences, thought experiment, limiting case analyses, abduction—are used to provide conceptual underpinnings for the placeholders;

5. These modeling processes combine and integrate separate representations from distinct domain-specific conceptual systems; and
6. These processes create explicit representations of knowledge previously embodied in constraints on the computations defined over symbols in one or more of the systems being integrated.

The first three steps describe the initial stage of conceptual change, the free-standing, abstract structure imputed by Wiser. The final three steps describe how bootstrapping proceeds from there.

I find it difficult to find a non-tautological or other-than-common-sense contribution of the description of the schematic initial state. In school, students, of course, are instructed on the formal terms of scientific theory and their relations. But they cannot entirely understand the technical terms (aka “core concepts,” or “symbols”) that are introduced in their science classes, nor their relations, such as $F = ma$. In the history of science, if there is a case where scientists identify some major new concept at first in full-blown form, this would be an interesting and surprising case. (There are books on the historical development of the idea of “force”; quantum mechanics took years to stabilize its primary concepts; Einstein’s general relativity was first developed without tensor equations, and some of Einstein’s early calculations were in error.) So, all the work of Quinian Bootstrapping, as a whole, would seem to be in the listing of particular sub-mechanisms: items 4-6 in Carey’s list. Let us consider them.

What our students did could well be identified as abduction, in Carey’s list of modeling processes (her item 4, above).²⁶ However, this leaves out everything of interest in how they managed to do what they did. Abduction imputes no intuitive schemata, no causal chaining, and no congenial choice of bindings to world attributes. Whether or not the details I offer are complete or correct, there is nothing in the idea of abduction that incorporates *any of the particular things one sees in the data*. It is a “black box” for “good” (in some sense) induction of explanations.

Similarly, one might describe the overt anthropomorphism of “freaking out” as an analogy (item 4, again) or metaphor, drawing from the domain of human experience (psychology) and projecting to the target domain of thermal phenomena. However, this also leaves out critical details of the analysis of *W*’s contribution. *W*’s anthropomorphism serves precisely to invoke agentive *p*-prims, in particular, *Ohm*’s *p*-prim. Without recognizing the power and relevance of *Ohm*’s *p*-prim to this case and how it might lead to an apt causal chain, anthropomorphism, as a general process, may well be (I would judge would likely be) useless.

²⁶ “Abduction” originated in C. S. Peirce’s ideas, and it describes the jump from circumstances to a good (or best) and most economical explanation.

Describing this anthropomorphism as an analogy also leaves out the importance of the particular bindings *W* chose. It is not incidental—in fact it is completely central—that *W* connected degree of agency to temperature difference.

The empirical fact is that students often reject offered instructional analogies [Brown & Clement, 1989; Brown, 1994]. So, using analogies, per se, cannot alone explain success. In this case, a part of the reason for success of anthropomorphism (interpreted, here, as an analogy) came down to students' judgments that particular *p*-prims fit these circumstances. See Kapon and diSessa [2013] for an elaboration of how *p*-prims may systematically explain when students accept or reject instructional analogies.

Continuing through Carey's list, thought experimentation does not appear in our microcosm, nor any mental modeling in the narrower sense of developing a visualizable, runnable mental model of thermal equilibration.²⁷

Mechanisms of learning and conceptual change constitute a complex subject, far beyond exhaustive treatment here. I leave the topic with two observations. I recently undertook to discover any instances of real-time, process analyses of data in the developmental psychological literature concerning conceptual change. This included Carey, her students and collaborators (Wiser and others), and Chi. I found no such studies [diSessa, 2014a]. So, the shift to microgenetic analyses, indeed, might be highly consequential. In addition, I feel it is fair to say that there is simply a very different aesthetic between TT and OV researchers, on one side, and KiP researchers, on the other, concerning the level of detail to which we must attend in order to understand the mechanisms of conceptual change.

Summary and Methodological Reflection

In this article, I attempted to evaluate the fit of three paradigms of conceptual change research to a case study of learning based on systematic differences that distinguish Knowledge in Pieces (KiP), the Theory Theory (TT), and the Ontological View (OV). How do the general and specific prediction for how conceptual change works play out in the details of learning “on the ground,” in a classroom, and with real-time data? I believe the differences are stark and telling.

Basic Epistemological Principles:

KiP finds that the categories of “true” and “false” are largely unilluminating with respect to naïve knowledge in the process of conceptual change. In contrast, TT (and most other approaches to conceptual change) almost always assume that naïve

²⁷ Carey uses “mental modeling” in the broad sense, encompassing all the listed sub-mechanisms in her definition of Quinian Bootstrapping. However, in other places, she emphasized mental modeling in the more specific and traditional sense of developing analog and runnable mental constructions.

ideas are false and misconceived. Similarly, many conceptual change researchers make categorical assumptions about naïve and expert causality, for example, that direct agent/patient causality is useless in understanding scientific ideas such as thermal equilibration. Naïve causality in a KiP view is complex and diverse. There is no particular reason to assume that expert forms are precluded at naïve stages, or that they cannot be relatively quickly developed.

Productivity of Naïve Knowledge:

In our microcosm, naïve ideas such as *Ohm's p-prim* and equilibration ideas—the same ones that are regarded as misconceptions (in certain contexts)—quickly became parts of a conception of thermal equilibration that is virtually indistinguishable from the normative model produced by Newton.

Flexibility of Domain Knowledge:

The common assumption (often directly asserted in TT points of view) that naïve knowledge is locked strictly in particular domains—the domain of a naïve theory—is very hard to support, either theoretically (with KiP principles) or empirically. In our microcosm, p-prims that students apply fluently to mechanical situations quickly and without provocation became centrally involved in their thinking about thermodynamics. Once we let students speak by their use of some ideas, or not, assumptions about domain boundaries that are made by domain professionals—and many conceptual change researchers—may dissolve. Domain flexibility implies that we may need to explore widely to discover naïve knowledge that may be useful for conceptual change.

“Out of the Shadows” Learning:

In TT perspectives, “core concepts” are definitive. Moving from core to periphery, or the reverse, is difficult, and might even be characteristic of intransigent learning targets. In contrast, within KiP, “little” ideas can often become “big” ones, and our microcosm shows that this is not necessarily a difficult transition. In the breadth of our classes, very few students began by thinking about thermal equilibration in terms of either equilibration ideas or in terms of agentive causality. And yet, in our microcosm, our students did both of these things, and they all agreed this was a good idea. I repeat for emphasis, students did this work in a short time period and essentially without instruction. The phenomenon that students may judge some ideas to be good, or even best, while they seldom think of those ideas themselves, is completely comprehensible in the KiP perspective. It is a marginal possibility, if it is possible at all, in TT approaches to conceptual change.

Unfounded Characterization of Naïve Causality as Primitive and Scientifically Useless:

Some approaches to conceptual change (e.g., the OV) assert that students are categorically missing the kind of causality that is needed to understand topics like thermal equilibration. In our microcosm, however, “primitive” direct (agentive) causality (“freaking out” and its consequences) was instrumental in reaching the new, normative conceptualization. Afterward, agentive language was spontaneously

dropped from the students' way of talking, leaving a very professional-sounding version of Newton's law of cooling: Rate of temperature change is driven by the difference of temperatures. Agency facilitated, rather than blocked, learning, and yet it did not remain, as far as we could see, in the students final encoding. It properly served a bootstrapping role.

Multi-threaded and Step-by-step Learning:

TT views mark "the naïve theory" and "the normative theory" as endpoints. KiP, however, understands the naïve state as encompassing rich and diverse resources, and "the normative theory" as something that may be assembled in different threads and stages, using possibly different aspects of the naïve state as resources. While this microcosm does not track a full trajectory to understanding thermal equilibration in terms of both temperature and heat, it shows an apparently stable intermediate stage that cleanly transcends "the naïve theory," as identified by Wiser and Carey. Peripheral ideas, as characterized by Wiser (equilibration approaches a common temperature), become central. The construal of hot objects as intrinsically emitting heat is not present. Instead, a symmetric and relational characterization—the difference of temperatures—is the causal factor in equilibration.

Mechanisms of Learning That Are Adapted to Real-time Learning and Details of Conceptual Structure:

Our study developed candidates for learning mechanisms out of real-time data of students' learning. In contrast, mechanisms in the TT literature are abstracted from accounts of scientists' work that do not jibe with what one sees over the short durations over which one sees students changing their ideas. The general application of KiP mechanisms to student learning seems more fitting to how we might need to direct our students' attention during learning, and how teachers' might evaluate student moves and progress. To a KiP eye, mechanisms such as Quinian Bootstrapping are immensely ambiguous and uncertain in their application and success. In any case, they do not illuminate the learning that students did in our microcosm. I make no pretense to a final judgment about which kind of mechanisms will prove most valuable in understanding conceptual change over the long term. I do think, however, that KiP-styled mechanisms contrast with others with respect to: (1) the means by which mechanisms are abstracted (historical work of scientists, vs. real-world learning of students), (2) their consequent face-value application to students' learning, and (3) the compatibility of these mechanisms with detailed empirical analyses of learning.

Finally, I wish briefly to abstract the characteristics of the case and its analysis that made these conclusions possible. These might establish landmarks or recommendations for those who might want to establish similar results to those here.

1. Much of the knowledge that was involved in the analysis had a published history, and was previously documented and characterized. Descriptions of this knowledge were not ad hoc to the case.

2. These descriptions included specification of when such knowledge is likely to be used. So, for example, one could track innovations in learning that involved the same element, but in unusual circumstances.
3. The data involved was rich enough that innovations could be individually tracked. This is nothing but the central-most principle of microgenetic analysis.
4. The data is sufficiently detailed that it supports discovery and documentation of specific mechanism of learning.
5. Of a slightly different order, but still important, the fact that students constructed a normative idea allowed us to qualify or reject some general statements from TT and OV about how conceptual change must occur. It is probably true that detailed studies of success are much less attended to than studies of failure (“misconceptions”). And yet, they may be differentially more valuable both in terms of theory, but also for practice.

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Appendix: Schematic of episodes in the microgenetic analysis of “freaking out”

This appendix brings to this article, in abridged form, more of the work done in diSessa [2014a] as context for the microcosm.

The class worked mainly as a full-group discussion, moderated by the teacher. The exception was that the class split into two groups to do the lab experiment, with R and W—the two main actors in developing the “freaking out” model—in different groups. Events are labeled by the class session (1, 2, or 3) in which they occurred, by their instructional phase (1-6; see main text section “The Case”; however, this class did not reach phase 6), and by the time of onset of the event, to the nearest minute from the beginning of the relevant class. Hence, Cl 2; IP 4; 7’ means the event occurred during class 2, instructional phase 4, at minute 7 of that class.

Precursor 1 (Cl 1; IP 1; 13’)

Gist: W explains that the cold milk and warm room are battling, and the milk gets “beaten by the room.” “[T]he stronger one affects the weaker one more.”

Continuities: The vibrant anthropomorphism anticipates what W and R will do later. W uses the word “equilibrium,” which, quite unusually compared to other classes, plays a role in this precursor and in later discussion. It may anticipate the use of equilibration p-prims (see precursors 2 and 3).

Discontinuities: In other respects, the description is very different from the “freaking out” description. Instead of each of the equilibrating partners acting on themselves (“trying harder”), they are acting on each other, in a global competition with a winner and a loser.

Social uptake: None apparent.

Precursor 2 (Cl 1; IP 2; 25’)

Gist: C voices, very tentatively, overshooting equilibration. This schema was extremely rare in our instructional experience, but it seemed this class was more attentive to equilibration than others.

Continuities: In the next turn, W, also, used a form of equilibration, slowing equilibration.

Social uptake: None apparent, although W’s next turn (below) might be following suit with a different equilibration form.

Precursor 3 (Cl 1; IP 2; 27’)

Gist: W expresses slowing equilibration, merely describing, not visibly explaining, the pattern of behavior.

Continuities: W is plausibly following C's lead in reverting to a very abstract equilibration schema, which we hypothesize to be the schematic form that was filled in later in the freaking out explanation.

Discontinuities: W abandons his prior vivid anthropomorphism, which will re-emerge in his freaking out explanation.

Social uptake: None apparent.

Precursor 4 (Cl 1; IP 2; 31')

Gist: R offered the same graph as W. When questioned about why that happened, R emphasized the big difference in temperatures, and even talked about "trying harder."

Continuities: R has provided an excellent precursor to the freaking out model. It has (modestly phrased) anthropomorphism, and highlights difference in temperature as a potentially causal element.

Discontinuities: R's explanation is less completely articulated than W's eventual "freaking out": the anthropomorphism is more modest, and it does not bring a symmetry between the two partners in equilibration. It is only the milk, not the room, that is "trying."

Social uptake: None apparent.

Precursor 5 (Cl 1; IP 3; 48')

Gist: R, in her lab with partner Z, further develops her last move, described directly above. She boosts the level of anthropomorphism from modest ("trying") to dramatic (the hot water is "shocked").

Continuities: Anthropomorphism is ratcheted up, matching the level in W's freaking out explanation, still to come.

Discontinuities: However, it is the cold bath that is the agent, and it is cooling the focus of anthropomorphism, the shocked hot water in the test tube. In W's freaking out, each partner in equilibration is agentive, but works on itself.

Social uptake: Z seems in gear with R's explanation (nodding), but the audio of his contributions is uninterpretable.

W's freaking out model (Cl 2; IP 4, 7')

Gist: W proposes his freaking out model.

Continuities: Strong anthropomorphism, similar to his initial try and to R's independent proposal that "the water is shocked." The model may be building a more articulated rationale for slowing equilibration, which constituted W's last turn.

Discontinuities: Many aspects of this explanation seem new. (1) Agency is not "other" directed, but toward oneself: "*The water* is freaking out, so *it* tries harder [and makes itself change faster]," rather than W's earlier "The room beats the milk." (2) There is a relatively long chain of causality: Distance from

equilibrium → freaking out → trying harder → greater result (faster temperature change).

Social uptake: The teacher questions the meaning of “freaking out.” However, uptake is fairly immediate and increasingly broad and clear. See next items. The explanation becomes a reference model for the class and is never challenged.

Development 1 (Cl 2, IP 4, 8')

Gist: R immediately agrees with W, invoking her separate considerations and language. “I agree, and the way I was thinking of it...” She uses her term, “shock” rather than “freaking out,” but recognizes the relation and aligns “shock” and “freaking out”: She uses “stops freaking out” and “calms down,” implicitly parallel to “reduced shock.”

Continuities: R is providing the same explanation as W, recognizing her different language “shock” in place of “freaking out.”

Discontinuities: R does not seem to recognize that the bindings of attributes—in particular, the patient in the agent/patient pair—is different from her previous expression, where cool water acted on warmer water, rather than on itself.

Social uptake: The teacher follows up immediately by putting R’s and W’s expression together. See below.

Development 2 (Cl 2, IP 4, 8')

Gist: The teacher talks about R and W as proposing the same thing: “You guys....” She uses both of their distinctive words: “shock or freaking out or something.” She reviews the contention that shock or freaking out creates faster change when there is more, and slower change when there is less, pointing to relevant parts of the graph. She prompts for agreement, and R and Z (the only students visible in the video at that point) are seen nodding.

Continuity: The teacher repeats, probably for emphasis, what she heard from both R and W, soliciting confirmation from them.

Discontinuity: Minimal.

Social Uptake: The teacher questions the meaning of “freaking out,” but nothing much emerges. She then moves on to other topics.

Development 3 (Cl 2, IP 4, 10')

Gist: The students are discussing whether a particular cooling curve is the mirror image of a heating one, which they determine not to be exactly true. W offers an explanation, that the start of the cooling curve from the hot water (steeper graph) is farther away from room temperature than the cool water hence is freaking out more. W’s model is extended to comparisons between curves, not just segments within one curve. The teacher asks W to clarify “farther away,” which he does with a number line.

Continuity: W's model is used to explain a different observation, comparing different curves.

Discontinuity: This seems to be an advance in generalizing and consolidating the freaking out model.

Social Uptake: The teacher is generally maintaining a neutral position, but comments, "That makes sense." No uptake can be seen or heard on the video from the other students.

Development 4 (Cl 2, IP 5, 45')

Gist: Moving to the "normative model" phase of instruction (IP 5), the teacher helps the students create a computer model by programming. The model has different parameters for temperature and incremental temperature change, and the students used it to match empirical data from heating and cooling curves they had discussed earlier. The teacher notes that the initial temperature change for the hot water curve is larger than that for the cold water and asks for an explanation. With prompting, C offers that it might have to do with "W's number line thing," and, with help, the class develops an explanation aligning with "W's thing."

Continuity: W's model is used in a very different context, that of a computer model and matching curves by changing parameters. Though halting, W's model is brought to bear and matched to these circumstances as an explanation of differing parameters for different curves.

Discontinuity: The computer program context seems clearly different in the student's eyes. So, finding usefulness here probably is a step in confidence in the model and clarity concerning it. In addition, "W's number line thing" is used without mentioning freaking out. Anthropomorphism seems to have been dropped.

Social uptake: The class is working together on the computer model, so uptake of the freaking out model to explain an aspect of the program (with no dissent) seems implicit. In addition, it is C, not W or R, who suggests using "W's thing," reinforcing the contention that the freaking out model has become socially shared.

Development 5 (Cl 3; IP 5; 6')

Gist: Two days later, in a review session, a heating graph and cooling graph were drawn on the board, very much like the data discussed previously. Students noted the gradual decrease in steepness of both, and that the "hot" graph started more steeply. Asked why the differences in starting steepness, C immediately said, "Because the hot one started further away from room temperature [teacher revoicing omitted] so the change was more drastic."

Continuity: The freaking out model is used immediately and fluently. This is its third use without mentioning "freaking out" or any equivalent anthropomorphism.

Discontinuity: None apparent.

Social uptake: At this point, the model seems to be an unproblematic part of all the students' conceptual repertoire.