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# Fundamental Properties of Source-Excited Field at the Interface of a 2D EBG Material

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## INTRODUCTION

Periodic electromagnetic band gap (EBG) materials have been used recently to modify the radiation pattern and other characteristics of sources located near or within them [1]. The present paper is dedicated to the analysis of some of the fundamental properties pertaining to the field excited by a line source at the interface between an air superstrate and a two-dimensional EBG (electromagnetic band-gap) periodic material. An efficient algorithm based on a moment-method formulation is presented for the evaluation of the field produced by a line source at the interface. The formulation provides physical insight into the nature of the fields via path deformation in the complex wavenumber plane. The three main outcomes of this study are a) an efficient numerical scheme to evaluate the field produced by a localized source, b) a mathematical characterization of the source-excitation problem that can be used to extract the fundamental physics via branch-point singularities in the complex wavenumber plane, and c) a derivation of the field behavior along the interface between the artificial material and the air region. From an asymptotic analysis in the complex wavenumber plane it is found that the spatial wave produced by a line source consists of an infinite number of space harmonics that decay algebraically as  $x^{-3/2}$ . These results are relevant to situations involving antennas or other components mounted in the presence of EBG or other periodic artificial surfaces.

For simplicity, a 2D problem is considered (Fig. 1a) which is invariant along the  $y$  dimension. The periodic artificial material consists of a periodic (along  $x$ ) structure made of layers of conducting cylinders. A finite number of layers is stacked along  $z$ , forming an artificial material slab with a finite thickness. An electric line source in the  $y$  direction (parallel to the periodic elements) is in proximity of the interface of the artificial material, at  $(x_0, z_0)$ . It is shown that the same properties are obtained for the field in proximity of the artificially soft surface [2] shown in Fig. 1b.

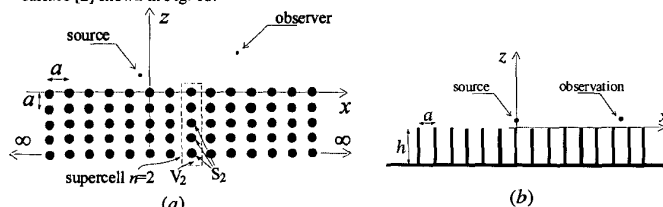


Fig. 1. Geometry of periodic materials and their source excitations. In all cases,  $a$  denotes the periodicity along  $x$ . (a) The EBG material is an infinite periodic structure along  $x$  with period  $a$ , and is truncated in the negative  $z$  direction after a finite number of layers. In the figure, the periodic supercell  $n = 2$  is shown. The source is located in the  $n = 0$  supercell.  $S_n$  denotes the surface of the conductors in the  $n$ th supercell. The  $n$ th supercell region is denoted by  $V_n$ . (b) An artificially soft surface with  $h$  denoting the thickness of the conducting teeth.

### FIELD PRODUCED BY A LINE SOURCE ABOVE A PERIODIC MATERIAL

The electric field in Fig. 1 is polarized along the  $y$  direction. For simplicity we consider here only metallic scatterers such as those shown in Fig. 1. We denote by  $J(\mathbf{r})$  and  $E(\mathbf{r})$  the surface current in the  $y$  direction on the metallic conductors and the electric field directed along  $y$  at any point, respectively. The current  $J_{S, \text{post}}^m$  on the surface of the conductors within the  $n = 0$  supercell due to a phased array of line sources (with a phasing wavenumber  $k_x$ ) is found by solving the EFIE  $\int_{S_0} J_{S, \text{post}}^m(\mathbf{r}', \mathbf{r}_0, k_x) G^m(\mathbf{r}, \mathbf{r}', k_x) d\mathbf{r}' = -G^m(\mathbf{r}, \mathbf{r}_0, k_x)$  for  $\mathbf{r} = S_0$ , where the periodic Green's function  $G^m(\mathbf{r}, \mathbf{r}_0, k_x)$  for the magnetic vector potential is accelerated using the 2D Ewald method [3], [4]. The periodic electric field scattered by the periodic EBG structure from the phased array of line sources is determined by integrating over the post currents  $J_{S, \text{post}}^m$  as

$$E_{sc}^m(\mathbf{r}, \mathbf{r}_0, k_x) = -j\omega\mu \int_{S_0} J_{S, \text{post}}^m(\mathbf{r}', \mathbf{r}_0, k_x) G^m(\mathbf{r}, \mathbf{r}', k_x) d\mathbf{r}' \quad (1)$$

with the integral performed over the post currents within the unit supercell  $S_0$  by using the periodic Green's function  $G^m(\mathbf{r}, \mathbf{r}', k_x)$ . The scattered field with the  $n$ th supercell from the single line source is then found from the field within the 0th supercell in the phased array problem using the array scanning method (ASM) as [5]

$$E_{sc}(\mathbf{r} + na\hat{\mathbf{x}}, \mathbf{r}_0) = \frac{a}{2\pi} \int_{-\pi/a}^{\pi/a} E_{sc}^m(\mathbf{r}, \mathbf{r}_0, k_x) e^{-jk_x na} dk_x, \quad (2)$$

where  $\mathbf{r} \in V_0$ . The total field  $E_{EBG} = E_{sc} + E_{inc}$  is obtained by adding the scattered field (2) to the incident field produced by the line source. It has been observed that the integrand  $E_{sc}^m$  in (2) has a branch point singular behavior at  $k_x = \pm k$  that may cause errors in the numerical integration (2). To overcome this difficulty, the total electric field  $E_{EBG}$  could alternatively be obtained by representing the incident electric field in terms of its spectral representation. The total electric field in (2) is thus expressed as

$$E_{EBG}(\mathbf{r} + na\hat{\mathbf{x}}, \mathbf{r}_0) = \frac{a}{2\pi} \int_{-\pi/a}^{\pi/a} E_{EBG}^-(\mathbf{r}, \mathbf{r}_0, k_x) e^{-jk_x na} dk_x, \quad (3)$$

where  $E_{EBG}^-(\mathbf{r}, \mathbf{r}_0, k_x) = E_{sc}^-(\mathbf{r}, \mathbf{r}_0, k_x) + E_{inc}^-(\mathbf{r}, \mathbf{r}_0, k_x)$ , with  $E_{inc}^-(\mathbf{r}, \mathbf{r}_0, k_x) = -j\omega\mu G^-(\mathbf{r}, \mathbf{r}_0, k_x)$  the incident field produced by the array of line sources. While the integrand  $E_{sc}^m$  in (2) possesses a singularity behavior of the type  $1/\sqrt{k^2 - k_x^2}$ , the integrand  $E_{EBG}^-$  in (3) instead contains weaker branch point singularities of the type  $\sqrt{k^2 - k_x^2}$ . In other terms, the integrand in (3) is smoother than that of (2), thus requiring fewer integration points near the branch point singularities at  $k_x = \pm k$ .

#### Unfolding the Integration path

The integrand in (2) is a periodic function of  $k_x$  with period  $2\pi/a$ . Indeed,  $E_{sc}^m(\mathbf{r}, \mathbf{r}_0, k_x)$  is periodic because  $J_{S, \text{post}}^m(\mathbf{r}', k_x)$  is produced from a periodic (in  $k_x$ ) phased-array source. The field (1) is first inserted into (2), and then the spectral sum for the Green's function  $G^m(\mathbf{r}, \mathbf{r}_0, k_x)$  is used. Since the term  $J_{S, \text{post}}^m(\mathbf{r}', k_x)$  is periodic in  $k_x$ , applying the shift of variable  $k_x + 2\pi p/a \rightarrow k_x$  for every  $p$  term of the spectral sum leads to

$$E_{zca}(\mathbf{r} + na\hat{\mathbf{x}}, \mathbf{r}_0) = \frac{-j\omega\mu}{4\pi j} \int_{S_0} \int_{-\infty}^{\infty} \frac{e^{-j[(x+na-x)k_x + (z-z')k_z]}}{k_z} J_{S, \text{poles}}^{\infty}(\mathbf{r}', k_x) dk_x d\mathbf{r}', \quad (4)$$

which eliminates the sum and expresses the scattered field as a continuous integration over the entire  $k_x$  axis, physically corresponding to a continuous-spectrum plane-wave expansion of the scattered field whose singularities are now analyzed.

#### THE COMPLEX $k_x$ PLANE

In addition to the branch point singularity introduced by the  $k_z$  term in (4), the periodic function  $J_{S, \text{poles}}^{\infty}(\mathbf{r}', k_x)$  introduces a periodic set of branch-point singularities. Furthermore, this function may also exhibit a periodic set of poles, each one representing modal propagation along  $x$ . The branch point singularities at  $\pm k - 2\pi p/a$  of the spectral function  $J_{S, \text{poles}}^{\infty}(\mathbf{r}', k_x)$  arise from the periodic Green's function  $G^{\infty}(\mathbf{r}, \mathbf{r}_0, k_x)$  and are shown in Fig. 2. In Fig. 2, the

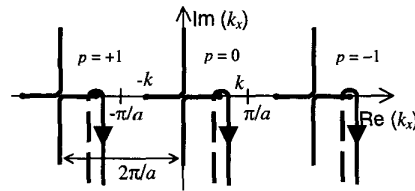


Fig. 2. Spectral  $k_x$  plane. The poles and branch points are periodic in the  $k_x$  plane, with period  $2\pi/a$ . The path deformation around the periodically spaced branch points is shown.

original integration path on the real axis in (4) can be deformed around the spectral singular points to highlight the space-wave and modal contributions. Modes arise from the residue evaluations at the periodic pole locations (not shown in the figure), with the residue at each of the locations determining the amplitude of the corresponding Floquet mode of the guided leaky or surface mode. The space-wave arises from the evaluation of the continuous spectra around each branch point. When  $|z - z'| = 0$ , the vertical paths shown in the figure are the steepest descent paths. From an asymptotic analysis one can infer that the space wave arises from all of the branch points, and consists of an infinite number of space harmonics with spreading factor  $1/(x)^{3/2}$ . The remaining spatial integral in (4) determines the weight of each decaying spatial harmonic.

#### NUMERICAL EXAMPLES

The field decay along the interface for the two structures in Fig. 1 excited by a line source is shown in Fig. 3. In the first case of Fig. 3(a), the electric line source is placed over an EBG slab consisting of three layers of periodic conducting cylinders with normalized radius  $r/a = 0.2$ . The axes of the cylinders in the first row are located at  $z = 0$ . The source is located at  $(x_0, z_0) = (0, 0.5)a$ . The operating frequency corresponds to  $a/\lambda_0 = 0.3$  and is thus in the 0th band gap ( $0 < a/\lambda_0 < 0.48$ ) of the infinite EBG material. The total field  $E_{EBG}$  is plotted versus the distance  $na$  from the line source parallel to the EBG interface at points  $\mathbf{r}_{A,n} = (0.5 + n)a\hat{\mathbf{x}} + 0.5a\hat{\mathbf{z}}$  and  $\mathbf{r}_{B,n} = na\hat{\mathbf{x}} + a\hat{\mathbf{z}}$ , with  $n$  denoting the supercell index. The total field is obtained by adding the scattered field in (2) to the incident field. In Fig. 3(a), it is seen that the total field is dominated by the space wave, and exhibits the expected algebraic decay  $1/(na)^{3/2}$  of the space wave at both observer locations. This indicates the absence of guided modes for this particular structure. In Fig. 3(b) the total field is evaluated along the interface of an artificially soft surface shown in Fig. 1(b) with  $h = 0.5$  cm,  $a = 0.4$  cm, at a

frequency  $f = 10$  GHz. The source is located at  $(x,z) = (0, 0.2)$  cm and the field is observed along the interface at locations (in cm)  $\mathbf{r}_{A,n} = (0.2+na)\hat{x} + 0.2\hat{z}$ . For this geometry the space wave once again exhibits the expected algebraic decay  $1/(na)^{3/2}$ .

### CONCLUSIONS

An efficient numerical scheme has been introduced for calculating the field at the interface of a 2D EBG or periodic artificial material. An analysis of the branch point singularities in the complex wavenumber plane allows for a determination of the physical properties of the field. The asymptotic field behavior  $1/(na)^{3/2}$  that is predicted has been confirmed by calculations.

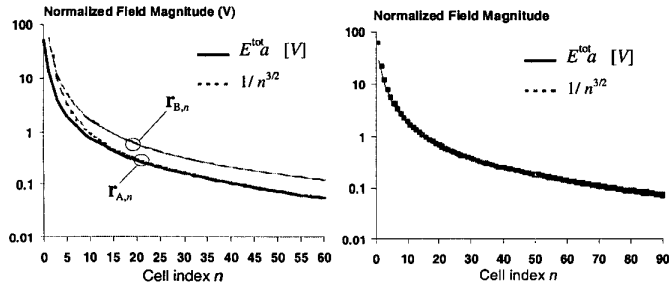


Fig. 3 Spatial decay of the total field produced by a line source over the two types of periodic structures in Fig. 1. (a) Case of line source above the periodic EBG material of Fig. 1(a) made with 3 layers of periodic conducting cylinders. The field is evaluated at points  $\mathbf{r}_{A,n}$  and  $\mathbf{r}_{B,n}$  (defined in the text) where  $n$  denotes the supercell index. (b) Case of line source above the artificially soft surface in Fig. 1(b). In both cases the fields match well with a simple  $1/n^{3/2}$  factor (normalized to the exact fields for large  $n$ ).

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