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Journal

Plasma Physics and Controlled Fusion, 36(7)

ISSN

0741-3335

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Publication Date

1994-07-01

DOI

10.1088/0741-3335/36/7/008

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Doppler shift of the TAE mode frequency in DIII-D

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Received 25 January 1994

Abstract. The Doppler shift caused by toroidal plasma rotation in DIII-D complicates a comparison of the frequency of toroidicity-induced Alfvén eigenmodes (TAE) to theoretical predictions, but also separates toroidal modes in frequency space and provides information about the radial location of the modes. Two independent techniques for estimating the mode frequency and Doppler shift are presented: one based upon the spectrum of multiple toroidal modes and one that utilizes the measured toroidal rotation of the plasma. Both methods indicate the presence of multiple TAE modes located between the $q = 1$ and $q = \frac{3}{2}$ surfaces. The frequencies determined by the two methods agree within 20%, consistent with their estimated uncertainties.

1. Introduction

Toroidicity-induced Alfvén eigenmodes (TAE) have been observed in experiments in the DIII-D (Heidbrink *et al* 1991, Strait *et al* 1993, Turnbull *et al* 1993) and Tokamak fusion test reactor (TFTR) (Wong *et al* 1991, 1992, Durst *et al* 1992) tokamaks. The TAE mode is a discrete mode which was predicted to exist in toroidal plasmas, with a frequency which lies in the toroidicity-induced gap in the shear Alfvén-wave spectrum (Cheng *et al* 1985, Cheng and Chance 1986). This mode is of concern for fusion reactors because of the possibility of resonant interaction with the fusion alpha particles, leading to rapid loss of the alpha particles and failure of thermonuclear ignition.

The instability can be studied in present tokamaks by driving the TAE mode with fast ions produced by neutral-beam injection (Heidbrink *et al* 1991, Wong *et al* 1991) or ion cyclotron heating (Wilson *et al* 1993). The TAE mode is observed to cause a large loss of both resonant (Duong *et al* 1993) and non-resonant (Duong and Heidbrink 1993) fast-ion species. Experimental efforts are in progress to understand the conditions under which this mode becomes destabilized (Strait *et al* 1993).

An important characteristic of TAE modes is that they propagate in the plasma, with a frequency proportional to the Alfvén speed. Simple analytic theory for a TAE mode formed by the coupling of Alfvén modes with toroidal mode number n and poloidal mode numbers m and $m + 1$ gives the frequency as (Fu and Van Dam 1989)

$$f_T = \frac{1}{2\pi} \frac{v_A}{2qR} \quad (1)$$

where

$$q = \frac{2m + 1}{2n} \quad (2)$$

and R is the major radius and $v_A = B/(\mu_0\rho_m)^{1/2}$ is the Alfvén speed, with B the magnetic field and ρ_m the mass density of the plasma. (Throughout most of this paper frequencies will be expressed as f rather than angular frequency ω , for ease of comparison with experimentally measured values.)

The real part of the mode frequency provides a well defined comparison of experiment and theory. The real frequency can be measured much more easily than the details of the mode structure, and it can be predicted theoretically with much greater confidence than the imaginary part of the frequency (i.e. the growth rate).

However, the comparison of measured and predicted frequencies is complicated by the Doppler shift caused by toroidal rotation of the bulk plasma. In DIII-D, all of the neutral beams are co-injected relative to the standard direction for the plasma current, and the plasma usually acquires a toroidal rotation with a frequency of 10 to 20 kHz. Magnetohydrodynamic (MHD) theory predicts the TAE frequency in the fluid rest frame, but plasma rotation can shift the frequency measured in the laboratory frame by an amount comparable to the original frequency in the plasma frame. Therefore, the accuracy of the comparison of measured and theoretical frequencies depends crucially on the accuracy of the correction for this Doppler shift.

The existence of a significant Doppler shift, on the other hand, can be advantageous for some types of data analysis. Determination of toroidal mode numbers is simplified because the Doppler shift separates different toroidal modes in frequency space. Comparison of the Doppler shift to the measured toroidal-rotation profile can provide information about the radial location of the mode. A complementary situation may exist in other devices such as TFTR, where the plasma rotation rate using balanced neutral injection (Wong *et al* 1991) or ICRF heating (Wilson *et al* 1993) is small. Then the absence of a large Doppler shift would allow the frequency to be easily measured, but the possible superposition of several toroidal modes at the same frequency could make a clear determination of the toroidal mode number more difficult.

In previous studies of TAE modes in DIII-D, the Doppler shift was inferred from the frequency spectrum of the magnetic signals. After applying this correction to the data, the measured frequencies agree (to within $\sim 20\%$) with the theoretical frequency, and vary as expected with electron density (Heidbrink *et al* 1991) and toroidal field (Turnbull *et al* 1992a). The fine details of the TAE spectrum also agree with theoretical calculations when this Doppler-shift correction is employed (Turnbull *et al* 1993). These points of agreement are the strongest evidence that the observed instability is, in fact, a TAE mode. This Doppler-shift correction was also employed in recent studies of beta-induced Alfvén eigenmodes (Heidbrink *et al* 1993), along with a second method described below.

The purpose of this paper is to assess the validity and accuracy of these analysis techniques. The Doppler shift inferred from spectral analysis (called method 1 here) is compared with the Doppler shift inferred from measurements of the toroidal rotation of impurity ions (called method 2) both in steady-state plasmas and in plasmas where the rotation speed is rapidly evolving. The good agreement ($\sim 20\%$) between the two techniques confirms the validity of the previous analysis.

Following a brief summary of the experimental techniques in section 2, the paper considers in section 3 some theoretical aspects of the Doppler shift and its relation to the measured plasma rotation. Sections 4 and 5 describe the two methods used to infer the Doppler shift and compare the results. Some inferences about nonlinear coupling of modes are drawn in section 6. Finally, the conclusions are given in section 7.

2. Experimental techniques

The TAE mode has been observed over a wide range of operating conditions in DIII-D. Figure 1 shows the time evolution of a typical L-mode discharge with TAE activity,

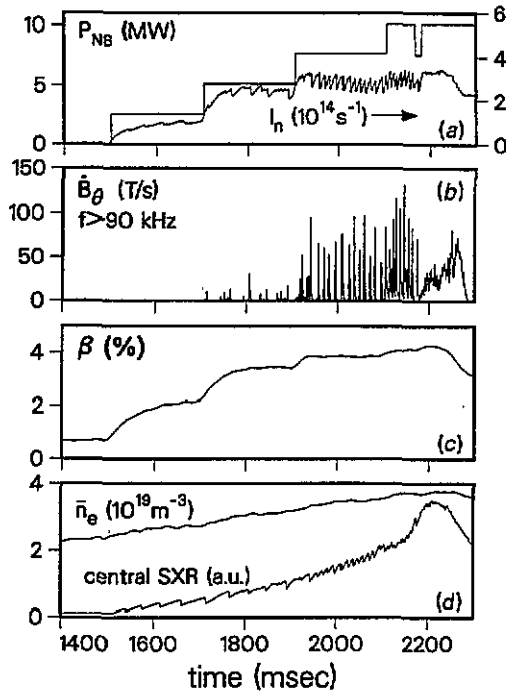


Figure 1. Time traces for a discharge with TAE activity. (a) Neutral-beam power P_{NB} and neutron emission I_n , (b) TAE activity (high-pass filtered magnetic-fluctuation amplitude, $\langle dB_\theta/dt \rangle$, $90 < f < 250$ kHz). (c) β_i from MHD equilibrium fits, (d) line-average density \bar{n}_e and central soft x-ray emission (discharge 71 524: $B_t = 0.8$ T, $I_p = 0.6$ MA).

discharge 71 524. The temperature, density, and safety-factor profiles are given in Strait *et al* (1993) as well as a detailed discussion of the stability properties of this discharge. TAE activity appears as the neutral-beam power reaches 5 MW. The experimentally measured fast-ion pressure, inferred from the 2.5 MeV neutron emission (Duong *et al* 1993), saturates at the onset of the high frequency MHD activity, while an estimate based on classical slowing down predicts that the fast ion pressure should continue to increase almost linearly as the beam power increases. This saturation indicates a large loss of fast ions caused by the instability.

The primary diagnostic for detection of TAE modes is an extensive array of magnetic probes inside the DIII-D vacuum vessel. The location of these probes and other diagnostics is given in Strait *et al* (1993). The Fourier spectrum of the magnetic fluctuations is obtained from a standard cross-spectrum calculation (Jenkins and Watts 1968) for a pair of magnetic probes. The complex cross-spectrum is given by

$$C_{12}(f) = \langle \tilde{B}_{\theta 1}(f) \tilde{B}_{\theta 2}^*(f) \rangle \quad (3)$$

where $\tilde{B}_\theta(f)$ is the complex Fourier transform of a magnetic probe signal (usually a dB_θ/dt signal), and $\langle \rangle$ denotes averaging in the frequency domain over a small interval centred about f . The use of a pair of probe signals and frequency-domain averaging increases the signal-to-noise ratio and improves the estimate of the spectrum in the statistical sense, without sacrificing the bandwidth of the original signal. The cross-power spectrum

$$P_{12}(f) = |C_{12}(f)| \quad (4)$$

for two similar probes then represents the fluctuation power density as a function of frequency; the integral of $P_{12}(f)$ under a spectral peak equals the mean-square amplitude of the mode associated with that peak. The phase spectrum

$$\Theta_{12}(f) = \tan^{-1} \frac{\text{Im}[C_{12}(f)]}{\text{Re}[C_{12}(f)]} \quad (5)$$

gives the phase difference between the probes as a function of frequency.

The cross-power spectrum from a magnetic probe pair located near the outboard midplane, integrated over a short time interval during discharge 71 524, is shown in figure 2. In addition to the usual low-frequency MHD activity at $f < 50$ kHz, a prominent set of peaks is visible between 50 and 150 kHz, the appropriate range for TAE modes as predicted by (1). For these experiments the magnetic probe signals were digitized at a sampling rate of 500 kHz, allowing resolution of frequencies up to 250 kHz which is comparable to the bandwidth of the probe coils.

The toroidal mode numbers associated with the spectral peaks, also shown in figure 2, are obtained from the best fit to the phase differences of a toroidal array of eight probes, all with the same poloidal location. The probes are irregularly spaced with a minimum toroidal spacing of 6 degrees, in principle allowing mode numbers $-30 < n \leq 30$ to be resolved. The toroidal symmetry of the plasma equilibrium allows the toroidal mode number to be obtained quite reliably (Heidbrink *et al* 1991).

The measured spectrum of the TAE modes can be compared to the predictions of two numerical codes. The CONT code (Chu *et al* 1992) calculates the frequency of the Alfvén continuum, with the toroidicity-induced gap in which the TAE frequency is expected to lie. The low- n ideal stability code GATO (Bernard *et al* 1981) is used to search for eigenmodes

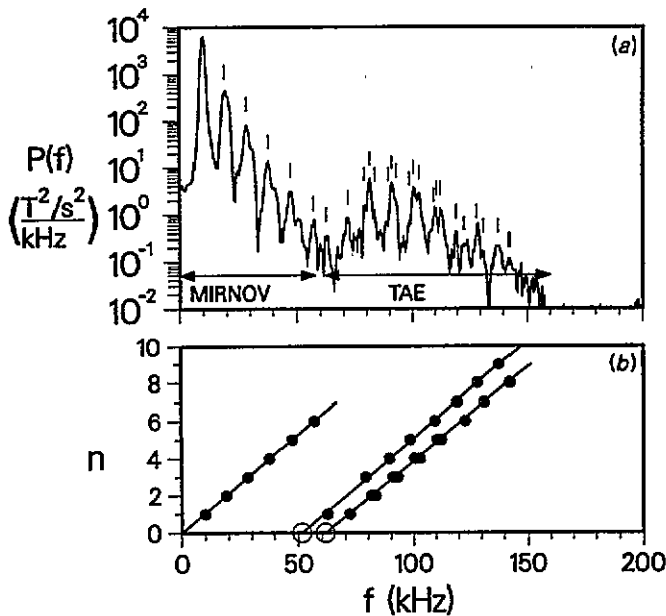


Figure 2. (a) Power spectrum and (b) toroidal mode number spectrum for magnetic fluctuations in a discharge with typical TAE activity, obtained from magnetic-probes near the outboard midplane. Open circles in part (b) indicate the inferred TAE mode frequency in the plasma rest frame (discharge 71 524 at 1874 to 1876 msec).

within the continuum gap (Turnbull *et al* 1992b). The input to these calculations consists of an equilibrium reconstruction for a DIII-D discharge, made with the equilibrium code EFIT (Lao *et al* 1985) using magnetic data as well as measured density and temperature profiles, as described in more detail in Lao *et al* (1990).

The toroidal rotation velocity of the plasma, which is used to determine the Doppler shift contribution to the measured MHD frequency, is obtained from Doppler shifts of spectroscopic lines using the charge exchange recombination diagnostic (Gohil *et al* 1990), typically using He II radiation from the core of the discharge and C IV radiation from the edge. The measured toroidal rotation speed profile is shown in figure 3. Also shown is a spline fit to the data, which will be used to represent this profile in subsequent analysis.

3. Doppler shift of TAE modes: theoretical considerations

In the analysis presented in this paper it is assumed that the Doppler shift arises from a purely toroidal rotation of the plasma. In this section we examine the justification for this assumption, and estimate possible differences between the measured plasma rotation frequency and the Doppler shift of the TAE mode.

Neoclassical theory predicts that poloidal rotation is strongly damped in a tokamak plasma (Rosenbluth *et al* 1973, Stix 1973). Poloidal rotation causes dissipation owing to the motion of the plasma through a spatially-varying magnetic field. In the 'banana' collisionality regime which is relevant to these experiments, the rotation is damped on an ion-ion collision timescale by ion viscosity. The poloidal rotation of the ion fluid can arise from the drift v_E due to a radial electric field, $v_E = E \times B/B^2$, and from the diamagnetic drift v_{*i} associated with the ion pressure gradient, $v_{*i} = -\nabla p_i \times B/en_i Z_i B^2$. The poloidal

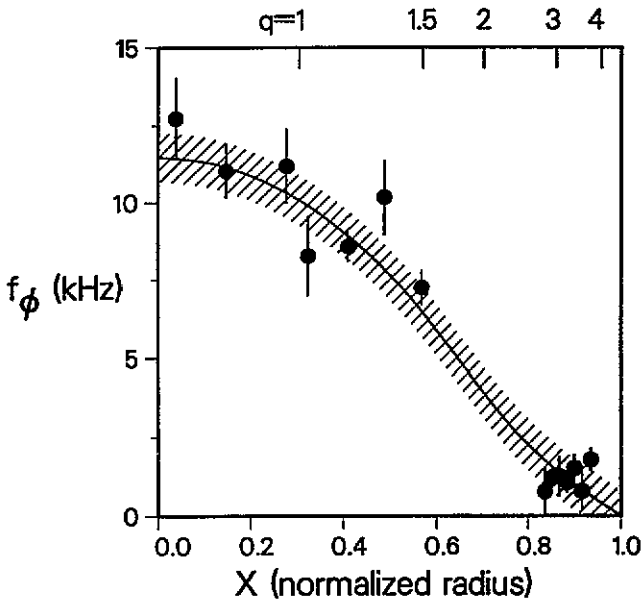


Figure 3. Toroidal rotation frequency f_ϕ profile: charge-exchange recombination data (circles) and cubic spline fit to the data (full curve), with the estimated one-sigma error indicated by the cross-hatching. Abscissa is $X = V_N^{1/2}$, where V_N is the volume inside a flux surface, normalized to the volume inside the last closed flux surface (discharge 71 524 at 1875 msec).

components of these perpendicular drifts are cancelled by the poloidal component of a parallel flow with velocity v_{\parallel} . Then, the net poloidal and toroidal flows are given by

$$v_{\theta} = (v_E + v_{*i}) \frac{B_{\phi}}{B} + v_{\parallel} \frac{B_{\theta}}{B} = 0 \quad (6)$$

$$v_{\phi} = -(v_E + v_{*i}) \frac{B_{\theta}}{B} + v_{\parallel} \frac{B_{\phi}}{B} = -(v_E + v_{*i}) \frac{B}{B_{\theta}} \quad (7)$$

where subscripts θ refer to the poloidal direction and ϕ to the toroidal direction. Assuming that the pressure p and electrostatic potential Φ are constant on a flux surface, we can substitute $\mathbf{E} = -(\nabla\psi) d\Phi/d\psi$ and $\nabla p_i = (\nabla\psi) dp_i/d\psi$. Making use of $|\nabla\psi| = RB_{\theta}$, we find that the toroidal rotation frequency of the ion fluid is

$$\omega_i = \frac{v_{\phi}}{R} = \frac{d\Phi}{d\psi} + \frac{1}{en_i Z_i} \frac{dp_i}{d\psi}. \quad (8)$$

The individual terms $\omega_E = d\Phi/d\psi$ and $\omega_{*i} = (en_i Z_i)^{-1} dp_i/d\psi$ are flux surface constants, so the angular velocity of toroidal rotation of the ions is constant on a flux surface. That is, damping of the poloidal rotation leads to the more general result that the entire flux surface rotates in the toroidal direction as a rigid body.

Now consider a TAE mode with toroidal mode number n and coupled poloidal modes m and $m+1$, localized near the $q = (2m+1)/2n$ surface. Let us assume for the moment that the TAE mode is convected with the ion fluid, and that the poloidal velocity of the ion fluid due to $\mathbf{E} \times \mathbf{B}$ and diamagnetic drifts is perfectly cancelled by the poloidal component of a parallel flow as described above. Then the Doppler shift in the laboratory frame is $\omega_{\text{Dopp}} = \mathbf{k} \cdot \mathbf{v} = k_{\phi} v_{\phi} = n\omega_i$, where \mathbf{k} is the wavevector of the mode.

The physical mechanisms responsible for the Doppler shift are most clearly seen by decomposing the fluid velocity and wavevector into components parallel and perpendicular to the field line. Such a decomposition shows that the primary contribution to the Doppler shift of the TAE mode comes from the perpendicular drift $v_{\perp} = v_E + v_{*i}$. In the case of a tearing mode where $k_{\parallel} = 0$ at the mode rational surface, the Doppler shift is caused by the perpendicular drift only, and is independent of the parallel flow. This also holds approximately for a TAE mode. Although k_{\parallel} at the $q = (2m+1)/2n$ surface is non-zero for a TAE mode, it remains small, so that $k_{\parallel} v_{\parallel} / k_{\perp} v_{\perp} \approx 1/2m$. Thus, for poloidal mode numbers $m \gg 1$ the Doppler shift is primarily related to the perpendicular drifts. However, because the experimental diagnostics measure the ion velocity and mode wavevector in the poloidal and toroidal directions, it is often more convenient to discuss the Doppler shift in terms of poloidal and toroidal components.

The experimentally measured toroidal rotation velocity must be corrected for any differences between the primary ion species and the ion species whose velocity is measured. In DIII-D, the rotation velocity is obtained from the spectroscopic Doppler shifts of impurity radiation. If the ions and impurities are collisionless, neoclassical theory predicts that the ion rotation frequency ω_i exceeds the impurity rotation frequency ω_I (Kim *et al* 1991b), by

$$\omega_i - \omega_I \approx \frac{3}{2} K \frac{1}{en_i Z_i} \frac{dp_i}{d\psi} = \frac{3}{2} K \omega_{*i} \quad (9)$$

where $K \approx \epsilon^{1/2}$ with ϵ the inverse aspect ratio. A parallel electric field may increase the toroidal-rotation difference between main ions and impurities (Kim *et al* 1991b), which would further increase the estimated Doppler shift relative to the measured impurity rotation

frequency. This contribution is small at typical values of the parallel electric field in neutral-beam-heated discharges, and will be neglected here.

The Doppler shift must also be corrected if the mode is not convected exactly with the velocity of the ion fluid. For example, electrostatic fluctuations are convected with the $\mathbf{E} \times \mathbf{B}$ drift only, because of a cancellation in the diamagnetic terms (Kim *et al* 1991a). In the absence of gradients, a shear Alfvén wave must be Doppler shifted by an $\mathbf{E} \times \mathbf{B}$ drift and by a uniform parallel flow, since these amount to changes from one inertial coordinate system to another. However, in the presence of a pressure gradient, the shear Alfvén wave becomes an electromagnetic drift wave with the following dispersion relation (Krall and Trivelpiece 1973):

$$\omega = \frac{k_{\perp} v_{\perp}}{2} \pm \left(\frac{k_{\perp}^2 v_{\perp}^2}{4} + k_{\parallel}^2 v_A^2 \right)^{1/2}. \quad (10)$$

That is, when $k_{\perp} v_{\perp} < k_{\parallel} v_A$ as in the present experiments, the diamagnetic contribution to the Doppler shift is only $k_{\perp} v_{\perp}/2$. If this conclusion is also applicable to the TAE mode, then the Doppler shift estimated from the ion fluid velocity must be reduced by approximately half of the diamagnetic contribution.

Because of the uncertainties in applying some of these correction terms, we have chosen the following expression to estimate the Doppler shift of a TAE mode from the measured impurity rotation velocity:

$$\frac{\omega_{\text{Dopp}}}{n} = 2\pi f_{\phi} = \omega_{\text{T}} + \frac{3}{2} \epsilon^{1/2} \omega_{*i} \quad (11)$$

$$\frac{\delta\omega_{\text{Dopp}}}{n} = 2\pi \delta f_{\phi} = \frac{1}{2} \omega_{*i} \quad (12)$$

where n is the toroidal mode number and $f_{\phi} = \omega_i/2\pi$ is the toroidal rotation frequency of the ions. The expression for the Doppler shift ω_{Dopp} includes the neoclassical difference in rotation between the primary ion species and impurity ions as given in (9), while the uncertainty $\delta\omega_{\text{Dopp}}$ represents an estimate of the range into which other correction terms might fall. For the case shown in figure 3, the estimated difference in rotation frequency between the main deuterium ions and the impurities is 1 to 2 kHz over most of the radius, and the uncertainty is of comparable size.

4. Doppler shift correction: method 1 (spectral analysis)

A simple method for estimating the Doppler-shift correction can be used when more than one TAE mode is present in the Fourier spectrum. Assuming that the poloidal rotation is strongly damped, the frequency in the laboratory frame of a TAE mode with toroidal mode number n will be

$$f_{n,\text{lab}} = f_{\text{T}} + n f_{\phi} \quad (13)$$

where f_{ϕ} is the frequency of the plasma's toroidal rotation as given in section 3. It is important to note that the Doppler shift $n f_{\phi}$ (including the correction terms discussed in section 3) is proportional to the mode number, while the TAE frequency f_{T} does not explicitly depend on mode number (see equation (1)).

The observed spectrum is consistent with the hypothesis that several TAE modes with different toroidal mode numbers are driven at the same radial location in the plasma (Heidbrink *et al* 1991). The mode spectrum shown in figure 2 illustrates the typical features. Several large and small peaks appear in the appropriate frequency range for TAE modes (60 to 150 kHz in this case). The plot of toroidal mode number versus frequency shows clearly that the peaks fall into two families of regularly spaced frequencies, each with consecutive toroidal mode numbers increasing with frequency. (The higher-frequency family may actually consist of two very closely spaced families.) Given such a spectrum, where the observed frequency depends on the toroidal mode number as

$$f_{n,\text{lab}} = f_0 + n\Delta f \quad (14)$$

we hypothesize that f_0 represents the common, n -independent TAE frequency $f_{\text{T}} = f_0$ for several TAE modes at the same rational q surface, and that $n\Delta f$ represents the Doppler shift caused by a toroidal rotation $f_{\phi} = \Delta f$ at that rational q surface. Then from the spectrum of figure 2, we can infer two TAE-mode frequencies in the plasma frame of 62 kHz for the larger peaks and 52 kHz for the smaller peaks, with similar toroidal rotation frequencies for both families of about 9.5 kHz.

Using an equilibrium reconstruction of this discharge incorporating measured temperature and density profiles and available current profile data, the GATO stability code predicts three TAE frequencies, 52, 60, and 74 kHz, for toroidal mode number $n = 3$. The two lower frequencies from the GATO calculation are in good agreement with the two experimental frequencies of 52 and 62 kHz found by method 1, while the higher frequency mode at 74 kHz is expected to be strongly damped (Turnbull *et al* 1993).

The Doppler shift allows an estimate of the radial location of the TAE modes. From the measured radial profile of the toroidal rotation frequency of impurity ions shown in figure 3, and the neoclassical correction of (9) and (11), the inferred Doppler shift corresponds to a normalized radius of $X = 0.5 \pm 0.1$, or a safety factor of $q = 1.25 \pm 0.2$. This radial location is consistent with the location of the Alfvén continuum gap calculated by CONT and the eigenmode structure calculated by GATO, which are shown in Turnbull *et al* (1993). This radial location is also consistent with theoretical estimates of the net growth rate (difference between driving and damping rates) as a function of minor radius, which show a broad maximum at $q \lesssim 1.5$ (Strait *et al* 1993). In a different discharge, soft x-ray measurements also show a maximum TAE fluctuation amplitude near the $q = 1.5$ surface (Heidbrink *et al* 1991).

Method 1 depends on the assumption that each family of spectral peaks corresponds to a family of TAE modes at the same radial location, so that they have the same frequency in the plasma frame and Doppler shifts generated by the same plasma rotation frequency. However, this assumption can only be approximately true. Let us assume that there is a particular radial location in the plasma where the net growth rate (difference between the fast-ion driving rate and total damping rate) is greatest, and that independent TAE modes with various toroidal mode numbers are destabilized close to this location. According to the simple analytic TAE theory expressed in (1) and (2), modes with toroidal mode number n are associated with rational q surfaces spaced $\Delta q = 1/n$ apart. That is, for an arbitrary value of q associated with the location of largest TAE growth rate, there must be a TAE rational surface within $\pm \delta q$, where $\delta q = 1/2n$. With the $\rho_m^{-1/2}$ -dependence of the Alfvén speed on the mass density and the slow radial variation of density typical of the central region of DIII-D discharges, we can assume that the spatial variation of f_{T} is mainly due to the q profile. Relative to the initial assumption of a single spatial location for all toroidal

mode numbers, this spread in spatial locations leads to a spread $\delta f_{n,\text{lab}}$ in the measured frequency:

$$\delta f_{n,\text{lab}} = \frac{df_{\text{T}}}{dq} \delta q + n \frac{df_{\phi}}{dq} \delta q \lesssim \frac{f_{n,\text{lab}}}{2n} + \frac{1}{2} \frac{df_{\phi}}{dq}. \quad (15)$$

For a typical f_{ϕ} profile such as that of figure 3, the first term (due to spread of the TAE frequency) dominates over the second term (due to spread of the Doppler shift) for $n \lesssim 10$, which includes the observed range of mode numbers. Then the uncertainty in the measured frequency (as a representation of independent modes driven in the neighbourhood of a single radial location) is

$$\delta f_{n,\text{lab}} \approx \frac{f_{n,\text{lab}}}{2n}. \quad (16)$$

It is evident from figure 2 that the measured frequency depends linearly on the mode number to a much better degree of accuracy than the uncertainties estimated here would imply. Specifically, the reduced χ^2 values for the fitted lines shown in the figure are on the order of 10^{-2} with (16) for the uncertainties δf_n , indicating that the uncertainties are overestimated by at least an order of magnitude. The good correlation of the frequencies suggests that the different toroidal modes are nonlinearly coupled (see section 6 below).

5. Doppler shift correction: method 2 (measured plasma rotation)

The Doppler shift can also be obtained from direct measurement of the plasma rotation frequency f_{ϕ} , using spectroscopic measurements as described previously. However, the radial location at which the rotation frequency should be evaluated is not known *a priori*. Therefore we evaluate

$$f_n(r) = f_{n,\text{lab}} - n f_{\phi}(r) \quad (17)$$

as a function of radius, where $f_{n,\text{lab}}$ is the observed frequency associated with a toroidal mode number n . Now, $f_n(r)$ represents the frequency of mode number n in a frame of reference rotating with the plasma at radius r . If at a particular radius r_0 , the frequency $f_n(r_0)$ coincides with the theoretically predicted TAE frequency $f_{\text{T}}(r_0)$, then we can say that the observed frequency is consistent with a TAE mode localized at r_0 . Note that method 2 requires the use of a specific theoretical expression for the TAE frequency, unlike method 1. On the other hand, method 2 allows the observed frequencies to originate from arbitrary locations in the plasma, while method 1 assumes several toroidal modes at the same radial location.

The results of analysis using method 2 indicate that the multiple observed frequencies are indeed consistent with several TAE modes at virtually the same radial position. Figure 4 shows the results for the strongest family of peaks in the spectrum of figure 2. The Doppler-corrected frequency $f_n(r)$ is calculated using the observed frequency $f_{n,\text{lab}}$ and corresponding toroidal mode number n obtained from the Fourier analysis as discussed previously, and the measured plasma rotation frequency $f_{\phi}(r)$ from the profile of figure 3 and the neoclassical correction of (11). The intersection of the $f_n(r)$ curves with the simple theoretical expression for $f_{\text{T}}(r)$ given in (1) yields TAE frequencies in the plasma frame of 63 to 65 kHz, and rotation frequencies at the mode location of about 8.8 to 9.3 kHz,

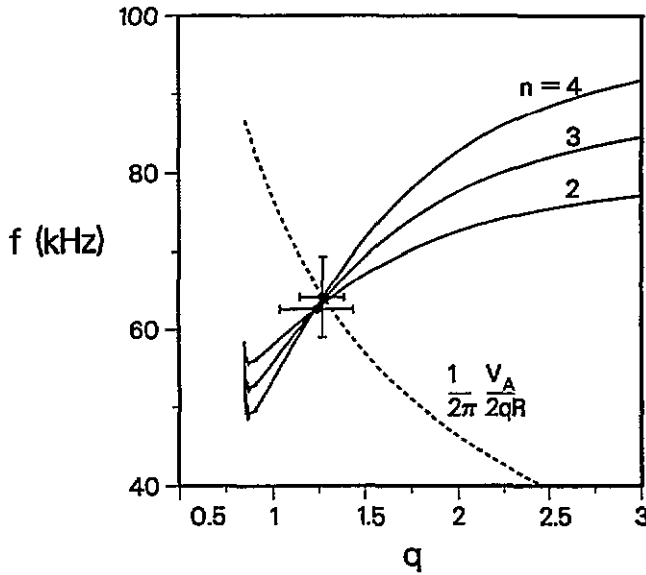


Figure 4. Profiles of observed frequencies shifted to the plasma frame as in (17) for a family of observed spectral peaks (full curves, labelled with toroidal mode numbers), and the TAE frequency from simple theory (broken curve). Abscissa is the safety factor q . Circles with error bars indicate the inferred frequency and radial location for the $n = 4$ mode, using the two methods described in the text (discharge 71 524 at 1875 msec).

in excellent agreement with the values of 62 kHz and 9.5 kHz inferred from method 1. (The agreement for the weaker (52 kHz) set of peaks is similar.) Error bars for the inferred frequency and location of the $n = 4$ mode are shown, based on the uncertainties in the spline fit to the measured rotation data and in the correction term described in section 3.

It can also be seen in figure 4 that the family of $f_n(r)$ curves meets at a common point, at slightly smaller frequency and minor radius than where they cross the $f_T(r)$ curve. The frequency of this common meeting point is equivalent to the prediction of method 1. The radial position of the common meeting point gives the radial location of the TAE modes, under the assumption that the modes have identical frequencies in the plasma frame. An error bar for the inferred location of this meeting point is also shown in figure 4, based on the uncertainties in the toroidal rotation frequency of the ions. The radial location inferred from the two analysis methods agree to within a few centimetres.

As a further test of the validity of the Doppler-shift correction, we study the time evolution of the frequency in a plasma where the Doppler shift undergoes a large change. Here discharge 71 519 is analysed, which is identical to discharge 71 524 but with magnetic-fluctuation data available later in the evolution of the discharge. Late in these high-beta discharges, an $m = 2/n = 1$ mode grows and stops rotating, creating a magnetic drag which slows the rotation of the plasma (Scoville *et al* 1991, La Haye *et al* 1993), but with little effect on the theoretical TAE frequency. The time-dependent rotation profile is shown in figure 5. The time dependence of the TAE spectrum is shown in figure 6. As the Doppler shift decreases, both the frequency and the separation of the TAE peaks decrease. For example, over the time interval 2175 to 2275 msec, the frequency of the $n = 5$ mode changes from about 120 to 75 kHz. Similar behaviour is also seen in other discharges where the $m = 2/n = 1$ mode 'unlocks', the plasma rotation increases, and both the frequency and separation of the TAE peaks increase.

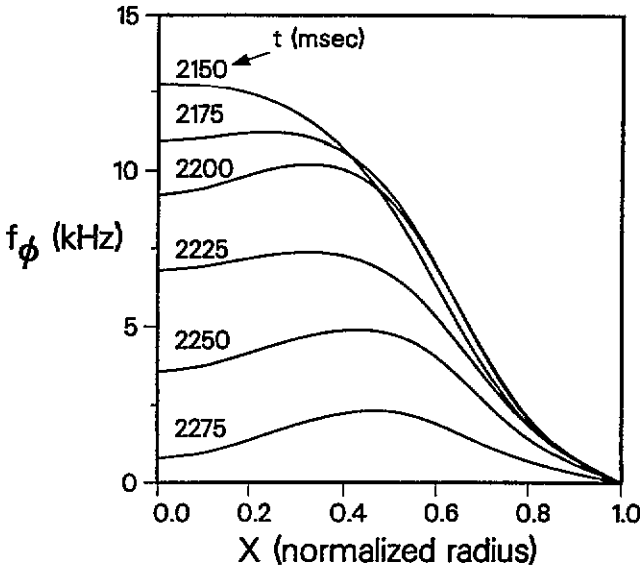


Figure 5. Profiles of toroidal rotation frequency f_ϕ at several times during $n = 1$ mode locking. For clarity, only the spline fits to the data are shown (discharge 71 519: same parameters as discharge 71 524).

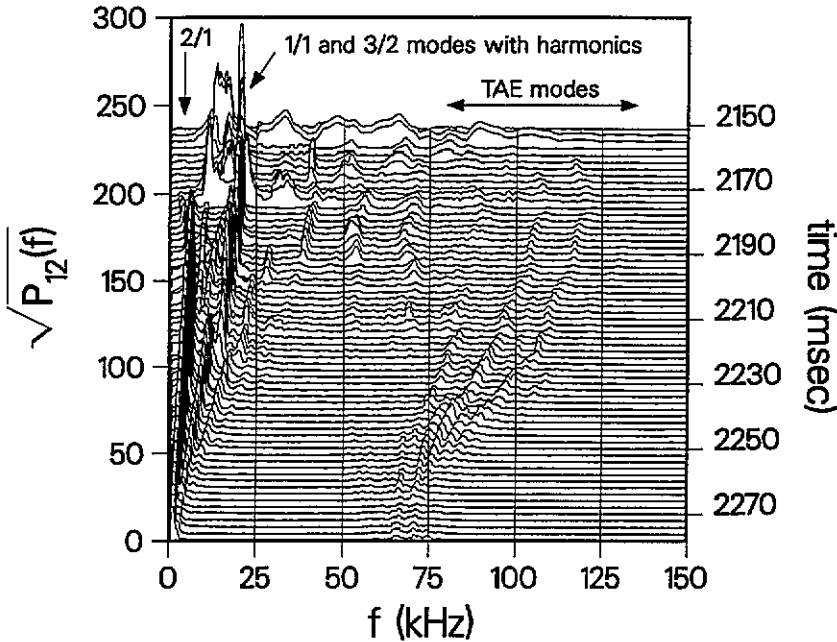


Figure 6. Time evolution of the magnetic fluctuation spectrum during $n = 1$ mode locking (discharge 71 519).

Method 2 was applied to the $n = 5$ frequency from figure 6, using the time-dependent rotation-speed profile. Despite the large change in observed frequency, the

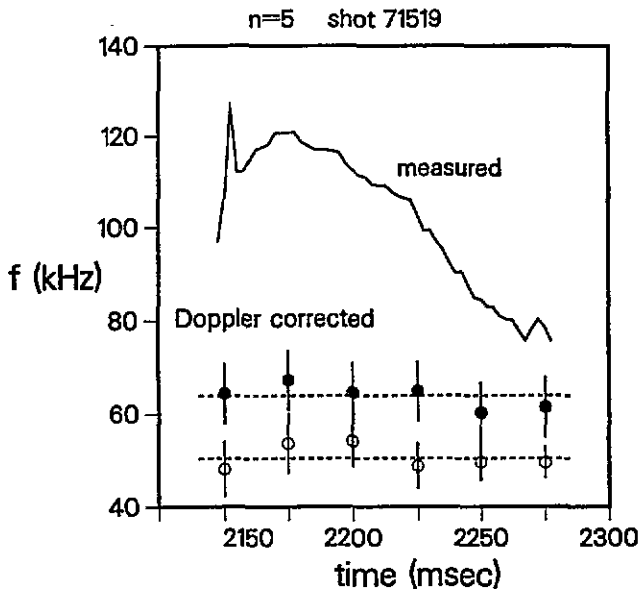


Figure 7. Time evolution of the frequency of the $n = 5$ TAE mode in the laboratory frame (full curve), and in the plasma frame with Doppler shift correction from the measured plasma rotation of method 2 (full circles) and from the simple peak spacing analysis of method 1 (open circles) (discharge 71519).

inferred frequency in the plasma frame remains constant within the estimated uncertainties, as expected (figure 7). The average value of the frequency from method 2 is about $f = 62 \pm 6$ kHz, corresponding to an inferred radial location of $X = 0.56 \pm 0.07$ and $q = 1.27 \pm 0.15$. Method 1 also gives a constant but somewhat smaller mode frequency in the plasma frame, of about $f = 51 \pm 5$ kHz (also shown in figure 7). Because the uncertainties here are dominated by the possible systematic uncertainties in the two methods of analysis, they are not reduced much by averaging several measurements.

The discrepancy between the two methods is partly caused by a downward shift in the TAE frequency in this high-beta phase of the discharge. Method 2 employs the nominal TAE frequency (1), although both theory (Fu and Cheng 1990, Spong *et al* 1990, Villard and Fu 1992) and experiment (Heidbrink *et al* 1993) indicate that the actual TAE frequency falls below this value as the plasma beta increases. If we use the bottom of the TAE gap (calculated by the CONT code) rather than (1), method 2 implies an average frequency of 59 ± 6 kHz, in somewhat better agreement with the frequency implied by method 1.

Good agreement is found in a comparison of the two methods for several different discharges (figure 8). Method 2 tends to yield a Doppler-corrected TAE frequency which is slightly higher than the result of method 1; the average ratio for the cases plotted in figure 8 is 1.05 ± 0.15 . In this figure, the error bars on the plotted points represent random variations in the experimental measurements, dominated by uncertainty in the toroidal-rotation measurement for method 2 and in the Fourier-analysed mode frequencies for method 1. Systematic errors in the Doppler-shift corrections may arise from uncertainties in the underlying models. For the cases in figure 8, equations (12) and (16) estimate a possible systematic error of about 10% for each method, so that the two methods should agree within 20%. This uncertainty in the theoretical expressions is represented by the two broken lines in figure 8 with slopes of 0.8 and 1.2. Most of the experimental points lie within this range,

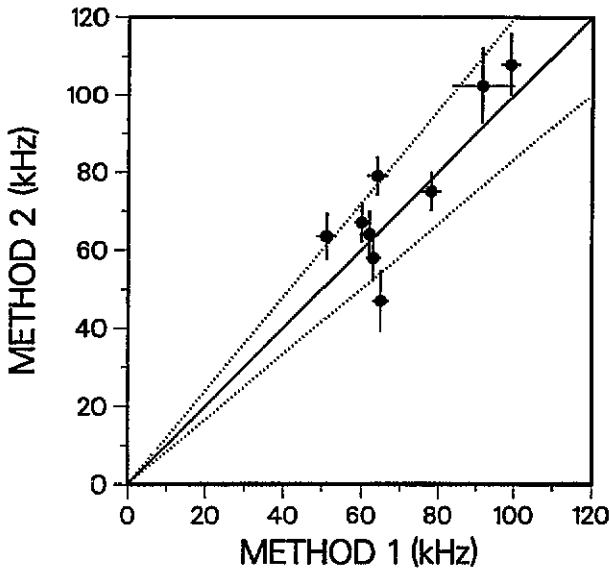


Figure 8. Comparison of the Doppler-corrected frequency from the two methods discussed in the text, for several different discharges. ($B_T = 0.8$ to 1.4 T; $I_p = 0.6$ to 0.8 MA; $P_{NB} = 5$ to 15 MW; $\bar{n}_e = 2.2$ to 4.6×10^{13} cm $^{-3}$; both L-mode and H-mode confinement.) The error bars represent uncertainties in experimental measurements. The dotted lines represent the expected range of agreement between the two methods, from estimates of possible systematic errors in the models.

indicating that the two methods agree within the experimental and theoretical uncertainties.

6. Nonlinear mode coupling

So far we have assumed the presence of multiple-unstable TAE modes whose frequencies in the plasma frame are similar but not necessarily identical. The very good linear relationship between frequency and mode number n which was noted in section 3 suggests that this spectrum may be the result of nonlinear coupling between modes with different n . Although the observed spectrum often resembles what might be expected from coupling to a low-frequency MHD mode, there are also counter-examples which show that in some cases, at least, the observed spectrum cannot arise from this mechanism.

The observed mode spectrum often suggests the possibility of nonlinear coupling between a single unstable TAE mode and a low-frequency MHD mode. The low frequency end of the spectrum (0 to 50 kHz) in figure 2 has a dependence $f'_n = n\Delta f'$, consistent with modes having near-zero frequency in the plasma frame. In this case, the large $n = 1$ peak represents an $m = 1/n = 1$ mode, and the other peaks are harmonics related to a non-sinusoidal structure of the mode. The Doppler shift $\Delta f'$ for this set of peaks is about 9.5 kHz, consistent with modes located at the $q = 1$ surface and also similar to the frequency spacing of the TAE modes. Suppose a single TAE mode with mode number n with a frequency f_n in the laboratory frame is driven unstable. Then nonlinear mode coupling with the large $m = 1/n = 1$ mode could generate sidebands with toroidal mode numbers $n \pm 1$ and frequencies $f_{n\pm 1} = f_n \pm \Delta f'$. Further coupling to the $m = 1/n = 1$ or its harmonics could generate the rest of the train of equally spaced peaks seen in figure 2.

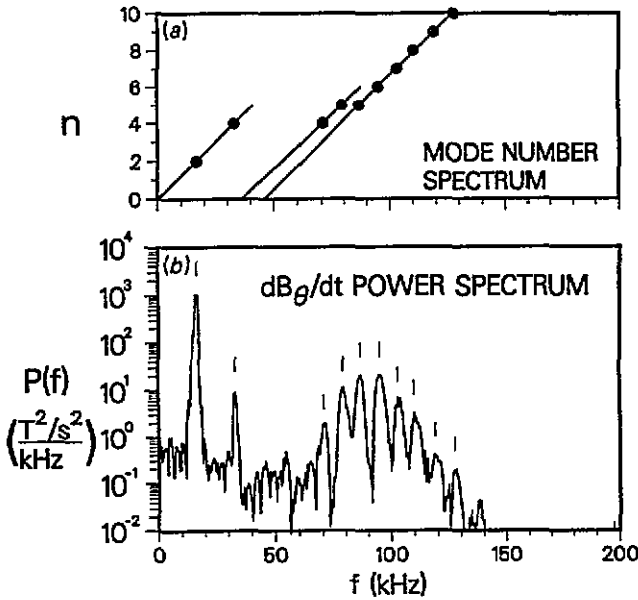


Figure 9. (a) Toroidal mode number spectrum and (b) power spectrum for magnetic fluctuations in a discharge with no $n = 1$ mode activity. (Discharge 71481 at 2052 to 2054 msec: $B_t = 1.0$ T, $I_p = 0.6$ MA, $\bar{n}_e = 3.6 \times 10^{19} \text{ m}^{-3}$, $P_{\text{NB}} = 10$ MW, $\beta_t = 3.0\%$.)

However, while the spectrum of figure 2 appears consistent with coupling to a low frequency $n = 1$ mode, counter-examples to this picture can be found in other cases. For example, figure 9 shows the spectrum from a discharge where no low-frequency $n = 1$ mode is present, but only a large $n = 2$ mode. Nonlinear coupling of a single TAE mode with the $n = 2$ mode should result in toroidal mode numbers with differences of 2 ($n, n \pm 2, \dots$). Yet high-frequency peaks with consecutive mode numbers ($n, n \pm 1, n \pm 2, \dots$) are observed. In other cases, the peak spacing differs slightly from the frequency of the $m = 1/n = 1$ mode.

Therefore, the observed spectrum is consistent with the hypothesis of multiple unstable TAE modes at the same radial location, but not always consistent with the hypothesis of nonlinear interaction with a low-frequency MHD mode. In cases such as figure 2, the apparent equality of the TAE peak spacing and the frequency of the $m = 1/n = 1$ mode may be coincidental. Because the TAE modes are destabilized by the large fast-ion pressure gradient inside the $q = 1.5$ surface, the Doppler shift will necessarily be near the $m = 1/n = 1$ rotation frequency.

Other forms of mode coupling can be imagined. For example, if two TAE modes with mode numbers n and $n + 1$ and frequencies f_n and f_{n+1} are destabilized at nearby radial locations, then nonlinear coupling between these modes could produce an $n = 1$ perturbation with a frequency $\Delta f = f_{n+1} - f_n$. Further coupling could then generate the train of equally spaced peaks seen in figure 2. In this case, the $n = 1$ perturbation could be a localized magnetic perturbation not easily detected by the magnetic probes, or even an $n = 1$ bunching of the fast ions which drive the modes.

The simplest explanation for the observed spectra may be the following. If several TAE modes with different toroidal mode numbers are destabilized in a small region near the radius of maximum fast-ion pressure gradient, then their frequencies in the plasma frame would be close to each other, though not necessarily identical. Nonlinear coupling between

the unstable TAE modes could then lock them to a common frequency in the plasma frame. The linear dependence of the Doppler shift on the toroidal mode number would then produce the observed spectra.

If TAE modes with different toroidal mode numbers are nonlinearly coupled, then method 1 is likely to be the more accurate of the two methods. The possible systematic errors estimated by (12) and (16) typically have similar magnitudes for the two methods. However, in the presence of nonlinear coupling, the systematic error for method 1 could be much smaller than indicated by (16), which assumed independent toroidal modes. In addition, the experimental uncertainties for method 1 are generally smaller than for method 2.

7. Conclusions

There is very good agreement between two essentially independent methods for estimating the frequency of TAE modes in the toroidally rotating plasma frame. These results allow the radial location of the mode to be estimated from the magnitude of the Doppler shift. The inferred radial location is consistent with the predicted TAE mode structure and the profile of fast-ion pressure which drives the modes, and remains constant with time even during large changes in the Doppler shift. The observations of multiple toroidal modes are consistent with multiple unstable TAE modes at approximately the same radial location. Although the observed Fourier spectra cannot always be explained by the hypothesis of nonlinear coupling to low-frequency MHD modes, they do suggest that the multiple unstable TAE modes are nonlinearly coupled to each other. If this is so, then method 1 should be the more accurate of the two. These techniques therefore offer a simple and reliable means of estimating the frequency in the plasma rest frame and the radial location of TAE modes.

Acknowledgments

We gratefully acknowledge helpful discussions with Dr Y-B Kim and thank the CER group for supplying the rotation data. This is a report of work sponsored by the US Department of Energy under Contract No DE-AC03-89ER51114.

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