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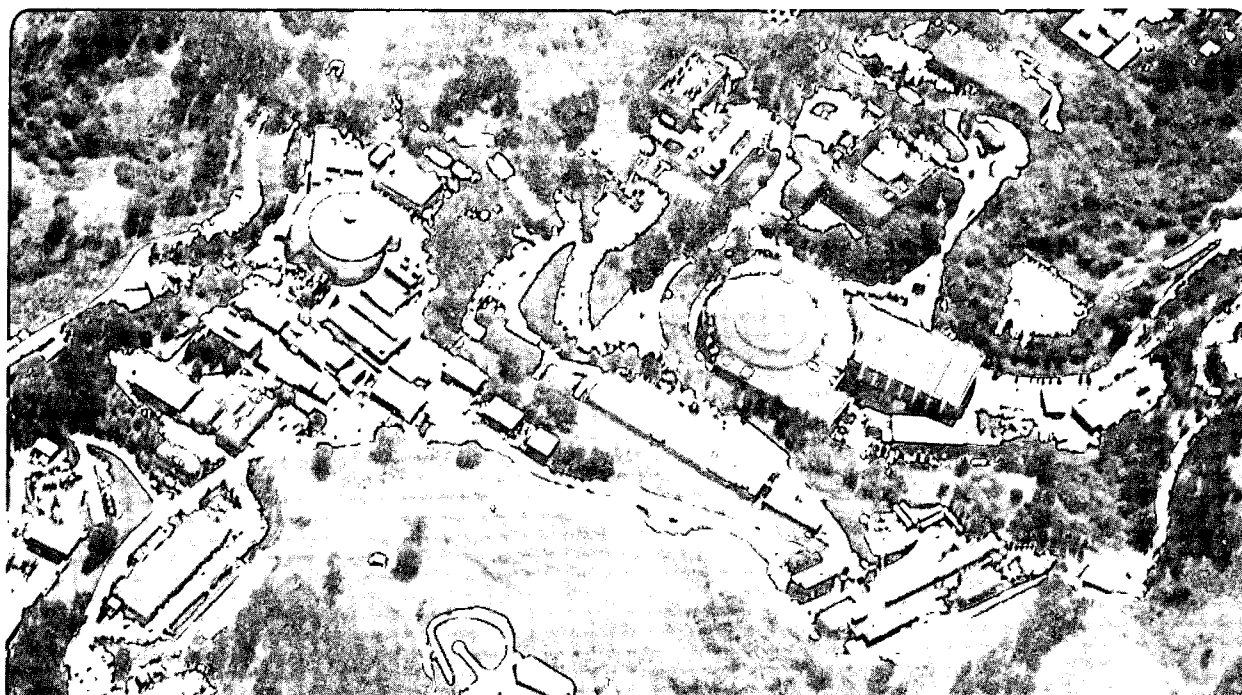## Physics Division

Second Order QCD Corrections to Scalar and Pseudoscalar Higgs Decays into Massive Bottom Quarks

K.G. Chetyrkin and A. Kwiatkowski

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Second Order QCD Corrections to Scalar and Pseudoscalar Higgs Decays into Massive Bottom Quarks[†]

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Abstract

Quark mass effects in $\mathcal{O}(\alpha_s^2)$ QCD corrections to the decay rates of intermediate Higgs bosons are studied. The total hadronic rate and the partial decay rate into bottom quarks are analyzed for the Standard (scalar) Higgs boson as well as for pseudoscalar Higgs bosons. The calculations of three different contributions are presented. First, the flavour singlet diagrams containing two closed fermion loops are calculated for a nonvanishing bottom mass in the heavy top limit. Their leading contribution, which is of the same order as the quasi-massless nonsinglet corrections, and the subleading terms are found. Large logarithms arise due to the separation of the pure gluon final state from the bottom final states. Second, quadratic bottom mass corrections originating from nonsinglet diagrams are presented. Third, nonsinglet corrections induced by virtual heavy top quarks are calculated in leading and subleading orders. It is demonstrated that, in order α_s^2, the first contribution numerically dominates over the second and the third ones.

1 Introduction

In the last years the Standard Model (SM) has faced a remarkable lot of experimental tests, which were performed at LEP and SLC with very high precision. Due to the overwhelming agreement between theory and experiments the description of the world of elementary particles through a $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory has emerged as a profoundly tested and firmly established theoretical framework. Despite of these achievements the detailed nature of electroweak symmetry breaking is still waiting for experimental confirmation. The search for a physical Higgs boson and the study of its properties will be the main subject of future collider experiments.

Standard Model properties of the Higgs boson have been discussed in many reviews (see for example [1, 2]). In the minimal SM one physical scalar Higgs boson is present as a remnant of the mechanism of mass generation. Particularly interesting for the observation of the Higgs boson with an intermediate mass $M_H < 2M_W$ is the dominant decay channel into a bottom pair $H \rightarrow b\bar{b}$. The partial width $\Gamma(H \rightarrow b\bar{b})$ is significantly affected by QCD radiative corrections. First order $\mathcal{O}(\alpha_s)$ corrections including the full m_b dependence were studied by several groups [3, 4, 5, 6, 7, 8]. Second order corrections were calculated in the limit $m_b^2 \ll M_H^2$. Apart from the trivial overall factor m_b^2 due to the Yukawa coupling, corrections were obtained for otherwise massless quarks by [9, 10] and for a nonvanishing mass of the virtual top quark by [11]. Subleading quadratic mass corrections in the m_b^2/M_H^2 expansion were found in [12]. In this work we complete the discussion of quark mass effects in second order QCD corrections. Our calculations provide the so far missing contributions and in part also serve as a crosscheck for existing results.

Theories beyond the SM are usually characterized by an enlarged Higgs sector and may allow for different quantum numbers of the Higgs bosons. For example, one of the most appealing extensions of the SM, the Minimal Supersymmetric Standard Model (MSSM), contains two complex isodoubletts with opposite hypercharge (see e.g. [1]), resulting in five mass eigenstates of the Higgs fields: two scalar (CP-even) neutral H^0, h^0, one pseudoscalar (CP-odd) neutral A and two charged $H^\pm$ physical Higgs bosons. As a consequence QCD corrections to the fermionic decays of a pseudoscalar Higgs have been studied in the past in many works [13, 14, 7, 15, 16]. This was motivation enough to carry out our analysis of quark mass effects in decay rates of pseudoscalar Higgs bosons as well. Our formulae are tailored in such a way that they are immediately applicable to the MSSM. However, the commitment towards this specific choice of model is limited by the fact that supersymmetric QCD with the exchange of gluinos is is not covered in this work.

Let us start our analysis of QCD corrections to the decay rates of the neutral Higgs bosons into bottom quarks with the two point correlators

$$\Pi^{S/P}(q) = i \int dx e^{iqx} \langle 0| \, T \, J^{S/P}(x) J^{S/P\dagger}(0) \, |0\rangle \tag{1}$$

of the scalar current $J^S = \bar{\Psi}_f \Psi_f$ and the pseudoscalar current $J^P = \bar{\Psi}_f i\gamma_5 \Psi_f$ for quarks with flavour f and mass m_f, which are coupled to the scalar Higgs bosons (we

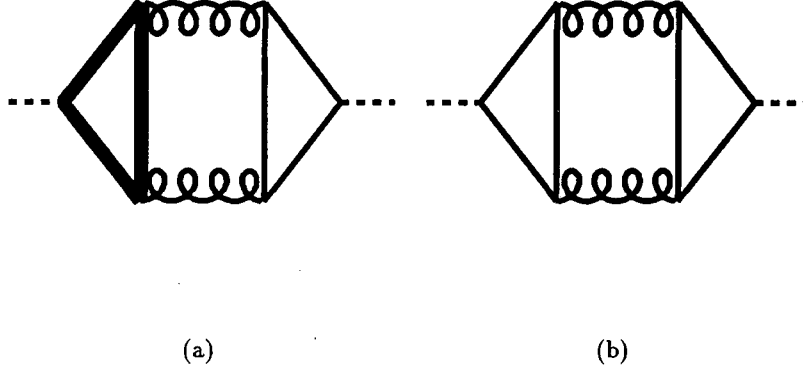<div align="center">(a)   (b)</div>

Figure 1: Diagrams of the $\mathcal{O}(\alpha_s^2)$ singlet contribution. (Thick lines: top, thin lines: bottom, curly lines: gluon, dashed lines: Higgs)

use the generic notation H for H^0 and h^0) and the pseudoscalar one (A) respectively.

The analysis of quark mass effects in second order QCD corrections is the main concern of this work. However, the calculation of $\mathcal{O}(\alpha_s^2)$ corrections to the correlators keeping the exact quark mass dependence would be an enormous task. Fortunately only the first few terms of the expansion in the small parameter m_b^2/s represent already a very good approximation for processes at high energies. The partial decay rates of the Higgs bosons into bottom quarks can therefore be written in the form

$$\Gamma_{b\bar{b}} = \frac{3G_F}{4\sqrt{2}\pi} M_H \bar{m}_b^2 C_{bb}^{S/P} R^{S/P} \tag{2}$$

where all information is contained in the absorptive part of the corresponding current correlator:

$$R(s) = \frac{8\pi}{3s} \mathrm{Im}\Pi(s+i\epsilon)$$

$$= 1 + \Delta\Gamma_1 \left(\frac{\alpha_s}{\pi}\right) + \left(\frac{\alpha_s}{\pi}\right)^2 \left[\Delta\Gamma_2 + \frac{s}{m_t^2}\tilde{\Delta\Gamma}_2\right]$$

$$+ \frac{\bar{m}_b^2}{s}\left(\Delta\Gamma_0^{(m)} + \Delta\Gamma_1^{(m)}\left(\frac{\alpha_s}{\pi}\right) + \left(\frac{\alpha_s}{\pi}\right)^2 \left[\Delta\Gamma_2^{(m)} + \frac{s}{m_t^2}\tilde{\Delta\Gamma}_2^{(m)}\right]\right)$$

$$+ \mathcal{O}\left(\frac{\bar{m}_b^4}{s^2}\right) + \mathcal{O}\left(\frac{s^2}{m_t^4}\right) + \mathcal{O}\left(\frac{\bar{m}_b^2 s}{m_t^4}\right). \tag{3}$$

The coefficients $C_{ff'}^{S/P} = g_f^{S/P} g_{f'}^{S/P}$ are the respective weights of the various couplings between Higgs bosons and fermions. For the MSSM they are given by $g_{up}^{H^0} = \sin\alpha/\sin\beta$, $g_{down}^{H^0} = \cos\alpha/\cos\beta$, $g_{up}^{h^0} = \cos\alpha/\sin\beta$, $g_{down}^{h^0} = -\sin\alpha/\cos\beta$, $g_{up}^{A} = $
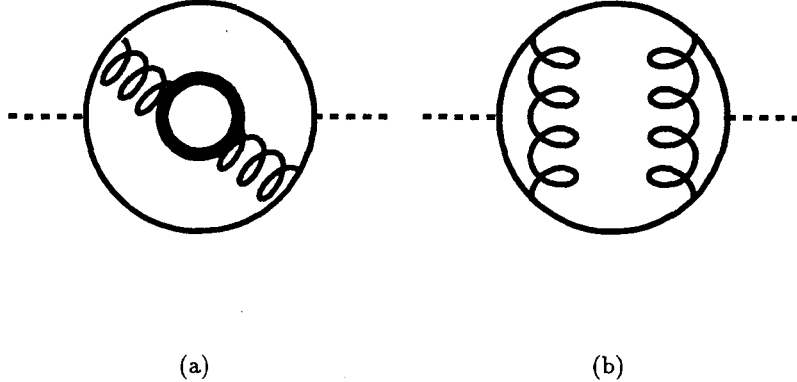
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<div align="center">(a)   (b)</div>

Figure 2: Examples of $\mathcal{O}(\alpha_s^2)$ nonsinglet diagrams.

$\cot\beta$ and $g_{down}^A = \tan\beta$ with α,β denoting the usual mixing angles (see e.g. [1]). All formulae in this work are applicable for the Higgs decay in the mimimal SM with $g_f^S = 1, g_f^P = 0$.

The outline of this work is as follows. The decay rate of the Higgs boson A involves traces over fermion loops which are separately coupled to a pseudoscalar current. For this case some technical details about the treatment of γ_5 in $D = 4 - 2\epsilon$ dimensions are given in Section 2. Flavour singlet contributions to the partial decay rates are discussed in Section 3. The heavy triangle diagram of Figure 1a contains two closed fermion loops composed of a top quark and a bottom quark respectively. It gives rise to a contribution to $\Delta\Gamma_2$, which is of the same order as the otherwise massless nonsinglet corrections. For the scalar Higgs boson it was presented in [17] some time ago. Subleading terms $\tilde{\Delta\Gamma}_2$ are also calculated in Section 3. The light triangle graph of Figure 1b consisting only of bottom loops is suppressed by m_b^2/s. Its contribution to $\Delta\Gamma_2^{(m)}$, obtained by [12] for the scalar Higgs, is only part of the complete answer, since bottom mass corrections of the same order originate also from the heavy triangle diagram and have to be added. This is done for the scalar and the pseudoscalar case in Section 3. Again subleading terms $\tilde{\Delta\Gamma}_2^{(m)}$ are given.

In Section 4 nonsinglet contributions (see Figure 2) to the partial Higgs decay rates are discussed. Quadratic mass corrections to $R^{S/P}$ in second order are presented in this section. Their contributions to $\Delta\Gamma_2^{(m)}$ are obtained from a recent calculation of the corresponding current correlators [18] and agree with [12, 15]. Furthermore the calculation of the double bubble diagram of Figure 2a containing a virtual top quark loop is performed in the heavy top limit. For vanishing bottom mass the exact top dependence of this contribution was obtained in [11]. Our result for $\tilde{\Delta\Gamma}_2$ agrees with the leading (power-suppressed) term in the M_H^2/m_t^2 expansion of [11]. In addition we

present the correction for non-vanishing bottom masses, thus completing the second order corrections $\Delta\Gamma_2, \tilde{\Delta\Gamma}_2, \Delta\Gamma_2^{(m)}, \tilde{\Delta\Gamma}_2^{(m)}$.

In Section 5 the results obtained in the full six flavour theory are expressed in the framework of an effective theory with five active quark flavours.

The numerical size of the corrections and conclusions are given in Section 6.

2   The Treatment of γ_5

The calculations of the two-point correlators are performed with dimensional regularization in the $\overline{MS}$-scheme. For the pseudoscalar correlator we employ the definition of γ_5 in $D \neq 4$ dimensions as suggested by 't Hooft and Veltman [19].

Following [20] we avoid an explicit separation of Lorentz indices into 4 and $(D$-4) dimensions and work, in $D = 4 - 2\epsilon$ dimensions, with the generalized current

$$P^{[\rho\mu\nu\lambda]} = \bar{\Psi} i \gamma^{[\rho\mu\nu\lambda]} \Psi \tag{4}$$

where $\gamma^{[\rho\mu\nu\lambda]} = (\gamma^\rho\gamma^\mu\gamma^\nu\gamma^\lambda + \gamma^\lambda\gamma^\nu\gamma^\mu\gamma^\rho - \gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\rho - \gamma^\rho\gamma^\lambda\gamma^\nu\gamma^\mu)/4$. Taking the limit $D \to 4$ at the end of the calculation, when the result is finite, the pseudoscalar current is recovered by

$$j^P = \frac{i}{4!} \epsilon_{\rho\mu\nu\lambda} P^{[\rho\mu\nu\lambda]}. \tag{5}$$

The corresponding generalized current correlator has the following form

$$
\begin{aligned}
\Pi^{[\rho\mu\nu\lambda]}_{[\rho'\mu'\nu'\lambda']}(q^2) &= i \int dx e^{iqx} \langle 0| \, T \, P^{[\rho\mu\nu\lambda]}(x) P^{[\rho'\mu'\nu'\lambda']}(0) \, |0\rangle \\
&= {}^{[\rho\mu\nu\lambda]}_{[\rho'\mu'\nu'\lambda']} \Pi_1(q^2)
\end{aligned}
\tag{6}
$$

where ${}^{[\rho\mu\nu\lambda]}_{[\rho'\mu'\nu'\lambda']} = (1/4!)\det(g_{\alpha\alpha'})$ with $\alpha = \rho, \mu, \nu, \lambda$ and $\alpha' = \rho', \mu', \nu', \lambda'$. It is convenient to work with the contracted tensor $\Pi^{[\rho\mu\nu\lambda]}_{[\rho\mu\nu\lambda]}$, which is related by

$$\Pi^P = \frac{1}{4!}\left(1 + 8\epsilon + 52\epsilon^2 + \frac{976}{3}\epsilon^2\right)\Pi^{[\rho\mu\nu\lambda]}_{[\rho\mu\nu\lambda]} \xrightarrow{D\to 4} \frac{1}{24}\Pi^{[\rho\mu\nu\lambda]}_{[\rho\mu\nu\lambda]} \tag{7}$$

to the pseudoscalar current correlator Π^P.

3   Singlet Contributions

The flavour singlet diagrams are characterized by two closed quark loops which are connected by gluons only. The heavy triangle graph of Figure 1a containing a top as well as a bottom loop is calculated in the heavy top mass limit. In leading order it contributes to the quasi-massless corrections $\Delta\Gamma_2$: The Yukawa couplings of the fermions yield a factor $m_b m_t$. In addition each fermion trace produces a factor m_b and m_t respectively. Together with a power suppression of $1/m_t^2$ on dimensional grounds

4

all mass factors are combined to m_t^2 and are thus of the same order as the otherwise massless nonsinglet corrections.

This leading and the subleading power suppressed s/m_t^2 contributions are obtained by employing the hard mass procedure [21, 22, 23, 24], which effectively is an expansion in the inverse heavy mass $1/m_t^2$ and has sucessfully been used in previous works [25, 26, 27] for the calculation of singlet diagrams for the decay rate of the Z boson. All possible "hard" subgraphs containing the heavy particle are selected and expanded with respect to the external momenta and the small masses. This formal Taylor expansion is inserted as an effective vertex.

Having integrated out the heavy top $m_t^2 \gg M_H^2, m_b^2$ the remaining diagram still contains two different scales $M_H^2 \gg m_b^2$, where the Higgs mass comes into play through the external momentum of the propagator integral. In analogy to the hard mass procedure the expansion in m_b^2/q^2 is obtained through the hard momentum procedure, where all possible subgraphs, through which the heavy momentum q may be routed, are reduced to a dot and expanded with respect to m_b and eventual small momenta as compared to q. In this way we have calculated the leading and next-to-leading order of the m_b^2/s series. As a result of the expansions the computation is simplified due to a factorization of the integrals. The three loop diagram decomposes into the product of massive tadpole integrals and massless propagator integrals. The latter are computed with the help of the multiloop program MINCER [28] written in the symbolic manipulation language FORM [29].

For the light triangle diagram of Figure 2b containing two bottom loops we also perform an expansion in m_b^2/s. As already stated in [12], the first nonvanishing contribution is suppressed by m_b^2/s and reproduced by our calculation. It has to be combined with the corrections of the same order originating from the heavy triangle diagram. Additionally we find that the corresponding contribution from the pseudoscalar light triangle graph vanishes identically.

The absorptive parts of the singlet diagrams (Figures 1a, 1b) are given by

$$
\begin{aligned}
\Delta\Gamma^{triangle}(H \to b\bar{b}, \text{gluons}) &= \frac{3G_F}{4\sqrt{2}\pi} M_H \bar{m}_b^2 \left(\frac{\alpha_s}{\pi}\right)^2 \\
&\cdot \Bigg\{ C_{tb}^S \Bigg(\frac{28}{9} - \frac{2}{3} \ln \frac{M_H^2}{m_t^2} \\
&\quad + \frac{M_H^2}{m_t^2} \left[\frac{2011}{24300} - \frac{41}{1620} \ln \frac{M_H^2}{m_t^2} \right] \\
&\quad + \frac{\bar{m}_b^2}{M_H^2} \left[-10 + 4\ln \frac{M_H^2}{m_t^2} + \frac{4}{3} \ln \frac{\bar{m}_b^2}{M_H^2} \right] \\
&\quad + \frac{\bar{m}_b^2}{m_t^2} \left[\frac{713}{2700} - \frac{7}{270} \ln \frac{M_H^2}{m_t^2} - \frac{1}{54} \ln \frac{\bar{m}_b^2}{M_H^2} \right] \Bigg) \\
&\quad + C_{bb}^S \frac{\bar{m}_b^2}{M_H^2} 8 \Bigg\}
\end{aligned}
\tag{8}
$$

$$\Delta\Gamma^{triangle}(A \to b\bar{b}, \text{gluons}) = \frac{3G_F}{4\sqrt{2}\pi} M_A \bar{m}_b^2 C_{tb}^P \left(\frac{\alpha_s}{\pi}\right)^2$$

$$\cdot \left\{ \; 4 - \ln \frac{M_A^2}{m_t^2} \right.$$

$$+ \frac{M_A^2}{m_t^2} \left[\frac{61}{324} - \frac{7}{108} \ln \frac{M_A^2}{m_t^2} \right] \tag{9}$$

$$+ \frac{\bar{m}_b^2}{M_A^2} \left[-5 + 2\ln \frac{M_A^2}{m_t^2} - 2\ln \frac{\bar{m}_b^2}{M_A^2} \right]$$

$$\left. + \frac{\bar{m}_b^2}{m_t^2} \left[\frac{19}{108} - \frac{1}{18} \ln \frac{M_A^2}{m_t^2} - \frac{5}{18} \ln \frac{\bar{m}_b^2}{M_A^2} \right] \right\}$$

They are constituted by the sum of all possible cuts of the diagrams and represent the decays into two $(b\bar{b}, gg)$, three $(b\bar{b}g)$ and four particle $(b\bar{b}b\bar{b})$ final states.

Note that the expressions (8) and (9) contain no explicit μ dependence. This is a direct consequence of the fact that the singlet diagrams contain no divergent subgraphs. This, in turn, means that their absorptive parts and, thus, (8), (9) have no dependence on the choice of the renormalization scheme.

The contributions of additional "ultralight" singlet diagrams containing a bottom and another light $(f = u, d, s, c)$ quark loop deserve some extra discussion with respect to their assignment to partial decay rates into specific quark species. The absorptive parts of these diagrams comprise again two $(b\bar{b}, f\bar{f}, gg)$, three $(b\bar{b}g, f\bar{f}g)$ and four particle $(b\bar{b}f\bar{f})$ final states. The cuts with two fermion final states $(b\bar{b}(g), f\bar{f}(g))$ should be calculated separately and could uniquely be assigned to the partial rates $\Gamma_{b\bar{b}}$ and $\Gamma_{f\bar{f}}$ respectively. The situation is different for the four fermion final state $(b\bar{b}f\bar{f})$. An unambiguous assignment to a specific partial rate is not possible in this case. The question to which partial rate this piece should most reasonably be counted must be decided according to the peculiarities of the experimental setup and identification methods. However, since the ultralight singlet contributions are proportional to $m_f^2 m_b^2$, they are much smaller than the already small m_b^4 contribution from the double bottom triangle graph. We therefore have adopted a pragmatic point of view and neglected the ultralight terms in eq. (8). (For the decay of the pseudoscalar Higgs they vanish identically.) No complications of this kind arise for the total Higgs decay into hadrons. We agree to the result in [12] for the light and ultralight singlet contributions to Γ_{had}. However, in view of the above discussion we find that in this work the assignment of the triangle graphs to the partial rates $\Gamma_{f\bar{f}}$ is not well motivated. The complete formula for Γ_{had} will be given in Section 5.

In order to complete our calculation and to arrive at the singlet result for the decay rate $\Gamma_{b\bar{b}}$ into bottom quarks one still needs to subtract from eqs. (8),(9) the pure gluon final state contribution. The decays $H/A \to gg$ through a fermion loop can be found in the literature [30, 31, 1, 32, 33].

Using the notation

$$\beta_f = 4m_f^2/M_H^2$$

$$x_t = \arctan \frac{1}{\sqrt{\beta_t - 1}}$$

$$x_b = \frac{1}{2}\left(\pi + i\ln\frac{1 + \sqrt{1 - \beta_b}}{1 - \sqrt{1 - \beta_b}}\right) \tag{10}$$

$$\mathcal{M}_f^S = -2\beta_f\left[(1 - \beta_f)x_f^2 + 1\right]$$

$$\mathcal{M}_f^P = \beta_f x_f^2$$

one has

$$\Delta\Gamma(H \to gg) = \frac{G_F}{4\sqrt{2}\pi}M_H^3\left(\frac{\alpha_s}{4\pi}\right)^2\left\{2C_{tb}^S\mathrm{Re}\mathcal{M}_t^S\mathcal{M}_b^{S*} + C_{bb}^S|\mathcal{M}_b^S|^2\right\}$$

$$= \frac{3G_F}{4\sqrt{2}\pi}M_H\bar{m}_b^2\left(\frac{\alpha_s}{\pi}\right)^2$$

$$\cdot\left\{C_{tb}^S\left(\ \frac{4}{9} + \frac{\pi^2}{9} - \frac{1}{9}\ln^2\frac{\bar{m}_b^2}{M_H^2}\right.\right.$$

$$+ \frac{M_H^2}{m_t^2}\left[\frac{7}{270} + \frac{7}{1080}\pi^2 - \frac{7}{1080}\ln^2\frac{\bar{m}_b^2}{M_H^2}\right]$$

$$+ \frac{\bar{m}_b^2}{M_H^2}\left[-\frac{4}{9}\pi^2 - \frac{4}{9}\ln\frac{\bar{m}_b^2}{M_H^2} + \frac{4}{9}\ln^2\frac{\bar{m}_b^2}{M_H^2}\right] \tag{11}$$

$$\left.+ \frac{\bar{m}_b^2}{m_t^2}\left[-\frac{7}{270}\pi^2 - \frac{7}{270}\ln\frac{\bar{m}_b^2}{M_H^2} + \frac{7}{270}\ln^2\frac{\bar{m}_b^2}{M_H^2}\right]\right)$$

$$+ C_{bb}^S\left(\ \frac{\bar{m}_b^2}{M_H^2}\left[\frac{4}{3} + \frac{2}{3}\pi^2 + \frac{1}{12}\pi^4\right.\right.$$

$$\left.\left.\left.- \left(\frac{2}{3} - \frac{\pi^2}{6}\right)\ln^2\frac{\bar{m}^2}{M_H^2} + \frac{1}{12}\ln^4\frac{\bar{m}^2}{M_H^2}\right]\right)\right\}$$

$$\Delta\Gamma(A \to gg) = \frac{G_F}{\sqrt{2}\pi} M_A^3 \left(\frac{\alpha_s}{4\pi}\right)^2 \left\{2C_{tb}^P \mathrm{Re}\mathcal{M}_t^P \mathcal{M}_b^{P*} + C_{bb}^P |\mathcal{M}_b^P|^2\right\}$$

$$= \frac{3G_F}{4\sqrt{2}\pi} M_A \bar{m}_b^2 \left(\frac{\alpha_s}{\pi}\right)^2$$

$$\cdot \left\{ C_{tb}^P \left(\frac{1}{6} - \frac{1}{6}\ln^2\frac{\bar{m}_b^2}{M_A^2} \right.\right.$$

$$+ \frac{M_A^2}{m_t^2}\left[\frac{1}{72}\pi^2 - \frac{1}{72}\ln^2\frac{\bar{m}_b^2}{M_A^2}\right]$$

$$+ \frac{\bar{m}_b^2}{M_A^2}\left[-\frac{2}{3}\ln\frac{\bar{m}_b^2}{M_A^2}\right]$$

$$\left.+ \frac{\bar{m}_b^2}{m_t^2}\left[-\frac{1}{18}\ln\frac{\bar{m}_b^2}{M_A^2}\right]\right)$$

$$\left.+ C_{bb}^P\left(\frac{\bar{m}_b^2}{M_A^2}\left[\frac{4}{3}\pi^4 + \frac{8}{3}\pi^2\ln^2\frac{\bar{m}^2}{M_A^2} + \frac{4}{3}\ln^4\frac{\bar{m}^2}{M_A^2}\right]\right)\right\} \tag{12}$$

The singlet contribution to the partial width of the Higgs bosons into bottom quarks is therefore given by

$$\Delta\Gamma^{\mathrm{singlet}}(H \to b\bar{b}) = \frac{3G_F}{4\sqrt{2}\pi} M_H \bar{m}_b^2 \left(\frac{\alpha_s}{\pi}\right)^2$$

$$\cdot \left\{ C_{tb}^S \left(\frac{8}{3} - \frac{\pi^2}{9} - \frac{2}{3}\ln\frac{M_H^2}{m_t^2} + \frac{1}{9}\ln^2\frac{\bar{m}_b^2}{M_H^2} \right.\right.$$

$$+ \frac{M_H^2}{m_t^2}\left[\frac{1381}{24300} - \frac{7}{1080}\pi^2\right.$$

$$\left.- \frac{41}{1620}\ln\frac{M_H^2}{m_t^2} + \frac{7}{1080}\ln^2\frac{\bar{m}_b^2}{M_H^2}\right]$$

$$+ \frac{\bar{m}_b^2}{M_H^2}\left[-10 + \frac{4}{9}\pi^2 + 4\ln\frac{M_H^2}{m_t^2}\right.$$

$$\left.+ \frac{16}{9}\ln\frac{\bar{m}_b^2}{M_H^2} - \frac{4}{9}\ln^2\frac{\bar{m}_b^2}{M_H^2}\right]$$

$$+ \frac{\bar{m}_b^2}{m_t^2}\left[-\frac{713}{2700} + \frac{7}{270}\pi^2 - \frac{7}{270}\ln\frac{M_H^2}{m_t^2}\right.$$

$$\left.\left.+ \frac{1}{135}\ln\frac{\bar{m}_b^2}{M_H^2} - \frac{7}{270}\ln^2\frac{\bar{m}_b^2}{M_H^2}\right]\right)$$

$$+ C_{bb}^S \frac{\bar{m}_b^2}{M_H^2}\left[\frac{20}{3} - \frac{2}{3}\pi^2 - \frac{1}{12}\pi^4\right.$$

$$\left.\left.+ \left(\frac{2}{3} - \frac{\pi^2}{6}\right)\ln^2\frac{\bar{m}_b^2}{M_H^2} - \frac{1}{12}\ln^4\frac{\bar{m}_b^2}{M_H^2}\right]\right\} \tag{13}$$

$$\Delta\Gamma^{\text{singlet}}(A \to b\bar{b}) = \frac{3G_F}{4\sqrt{2}\pi} M_A \bar{m}_b^2 \left(\frac{\alpha_s}{\pi}\right)^2$$

$$\cdot \left\{ C_{tb}^P \left(\frac{23}{6} - \ln\frac{M_A^2}{m_t^2} + \frac{1}{6}\ln^2\frac{\bar{m}_b^2}{M_A^2} \right.\right.$$

$$+ \frac{M_A^2}{m_t^2} \left[\frac{61}{324} - \frac{1}{72}\pi^2 \right.$$

$$\left. - \frac{7}{108}\ln\frac{M_A^2}{m_t^2} + \frac{1}{72}\ln^2\frac{\bar{m}_b^2}{M_A^2} \right]$$

$$+ \frac{\bar{m}_b^2}{M_A^2} \left[-5 + 2\ln\frac{M_A^2}{m_t^2} - \frac{4}{3}\ln\frac{\bar{m}_b^2}{M_A^2} \right]$$

$$\left. + \frac{\bar{m}_b^2}{m_t^2} \left[\frac{19}{108} - \frac{1}{18}\ln\frac{M_A^2}{m_t^2} - \frac{2}{9}\ln\frac{\bar{m}_b^2}{M_A^2} \right] \right)$$

$$\left. + C_{bb}^P \frac{\bar{m}_b^2}{M_A^2} \left[-\frac{4}{3}\pi^2 - \frac{8}{3}\pi^2\ln^2\frac{\bar{m}_b^2}{M_A^2} - \frac{4}{3}\ln^4\frac{\bar{m}_b^2}{M_A^2} \right] \right\} \tag{14}$$

The distinction between bottom and pure gluon final states introduces large logarithms $\ln^2(m_b^2/M_H^2)$ and $m_b^2 \ln^4(m_b^2/M_H^2)$ in the partial decay width $\Gamma_{b\bar{b}}$. Being due to the subtracted decay into two gluons, their physical origin might be traced back to kinematical configurations which correspond to collinear gluons with respect to the virtual bottom quark. This interesting feature deserves further analysis in the future.

4 Nonsinglet Contributions

Second order QCD corrections to the scalar and the pseudoscalar current correlators were calculated recently by one of the authors for the case when the external momentum is much larger than all relevant masses. The general case of nondiagonal currents with quarks of different masses was considered. A detailed description of this work will be published in a separate paper [18]. The results can be applied to the Higgs decay rate in the special case of diagonal currents.

Besides the three loop diagrams of Figure 2b, induced contribution from lower order nonsinglet graphs need to be taken into account, since they lead to second order corrections after renormalization of the coupling constant and the quark mass:

$$\left.\frac{\alpha_s}{\pi}\right|_{bare} = \frac{\alpha_s}{\pi} \left(1 + \frac{\alpha_s}{\pi}\frac{1}{\epsilon}\left[-\frac{11}{4} + \frac{1}{6}n_f \right] \right)$$

$$m_{bare} = m \left(1 - \frac{\alpha_s}{\pi}\frac{1}{\epsilon} \right. \tag{15}$$

$$\left. + \left(\frac{\alpha_s}{\pi}\right)^2 \left[\frac{1}{\epsilon^2}\left(\frac{15}{8} - \frac{1}{12}n_f \right) + \frac{1}{\epsilon}\left(-\frac{101}{48} + \frac{5}{72}n_f \right) \right] \right)$$

For the renormalization of the pseudoscalar currents the two renormalization constants

Z_5^P and Z_{MS}^P are introduced [20]:

$$Z_5^P = 1 - \frac{8}{3}\frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2 \left[\frac{1}{18} + \frac{1}{27}n_f\right]$$

$$Z_{MS}^P = 1 - \frac{\alpha_s}{\pi}\frac{1}{\epsilon} \tag{16}$$

$$+ \left(\frac{\alpha_s}{\pi}\right)^2 \left[\frac{1}{\epsilon^2}\left(\frac{15}{8} - \frac{1}{12}n_f\right) + \frac{1}{\epsilon}\left(+\frac{25}{16} - \frac{11}{72}n_f\right)\right]$$

Ordering the absorptive parts of the corresponding correlators in massless and massive contributions

$$R_{S/P} = R^{(0)} + R_{S/P}^{(m)} \tag{17}$$

one obtains

$$\begin{aligned}
R^{(0)} = \ &1 + \left(\frac{\alpha_s}{\pi}\right)\left[\frac{17}{3} - 2\ell\right] \\
&+ \left(\frac{\alpha_s}{\pi}\right)^2 \left[\frac{10801}{144} - \frac{39}{2}\zeta(3) + \left(-\frac{65}{24} + \frac{2}{3}\zeta(3)\right)n_f' + \pi^2\left(-\frac{19}{12} + \frac{1}{18}n_f'\right)\right. \\
&\left. + \ell\left(-\frac{106}{3} + \frac{11}{9}n_f'\right) + \ell^2\left(\frac{19}{4} - \frac{1}{6}n_f'\right)\right]
\end{aligned} \tag{18}$$

$$\begin{aligned}
R_S^{(m)} = \ &-\frac{6\bar{m}_b^2}{M_H^2}\left(1 + \frac{\alpha_s}{\pi}\left[\frac{20}{3} - 4\ell\right]\right. \\
&+ \left(\frac{\alpha_s}{\pi}\right)^2\left\{\frac{2383}{24} - \frac{83}{3}\zeta(3) + \left(-\frac{313}{108} + \frac{2}{3}\zeta(3)\right)n_f'\right. \\
&+ \pi^2\left(-\frac{9}{2} + \frac{1}{9}n_f'\right) + \ell\left(-\frac{371}{6} + \frac{5}{3}n_f'\right) \\
&\left.\left. + \ell^2\left(\frac{27}{2} - \frac{1}{3}n_f'\right)\right\}\right) \\
&+ 4\left(\frac{\alpha_s}{\pi}\right)^2 \sum_{f=u,d,s,c,b} \frac{m_f^2}{M_H^2}
\end{aligned} \tag{19}$$

$$\begin{aligned}
R_P^{(m)} = \ &-\frac{2\bar{m}_b^2}{M_A^2}\left(1 + \frac{\alpha_s}{\pi}\left[\frac{4}{3} - 4\ell\right]\right. \\
&+ \left(\frac{\alpha_s}{\pi}\right)^2\left\{\frac{1429}{72} - \frac{83}{3}\zeta(3) + \left(-\frac{3}{4} + \frac{2}{3}\zeta(3)\right)n_f'\right. \\
&+ \pi^2\left(-\frac{9}{2} + \frac{1}{9}n_f'\right) + \ell\left(-\frac{155}{6} + \frac{7}{9}n_f'\right) \\
&\left.\left. + \ell^2\left(\frac{27}{2} - \frac{1}{3}n_f'\right)\right\}\right) \\
&+ 4\left(\frac{\alpha_s}{\pi}\right)^2 \sum_{f=u,d,s,c,b} \frac{m_f^2}{M_A^2}
\end{aligned} \tag{20}$$

where $\ell = \ln(M_{H/A}^2/\mu^2)$. We find agreement with [12, 15].

Note that in (17) and (18) n'_f stands for $n_f - 1$. This is because only diagrams without virtual top lines are included there. It also implies that the renormalization of these contributions is to be done by means of (15-16) with n_f substituted by n'_f.

It should be stressed that we are still working in the full QCD including the top quark and $n_f = 6$. The strong coupling constant and light quark masses are, thus, defined with respect to this theory. A transition to effective parameters relevant to the topless QCD with five active quark flavours will be performed in the next section.

The last terms in eqs. (19),(20) arise from double bubble diagrams, where a light quark is running around a virtual fermion loop. Although for these diagrams a four fermion final state $(b\bar{b}f\bar{f})$ is possible, the assignment to the partial decay rate into the primarily produced quark flavour $\Gamma_{b\bar{b}}$ seems to be the natural choice.

It remains to compute the contribution of the double bubble diagram $\Delta\Gamma^{\mathrm{DB}}$ with a virtual top quark loop (see Figure 2a). Again the hard mass procedure can be used and leads to the result:

$$
\begin{aligned}
\Delta\Gamma^{\mathrm{DB}}(H \to b\bar{b}) = {} & \frac{3G_F}{4\sqrt{2}\pi} M_H \bar{m}_b^2 C_{bb}^S \\
& \cdot \Bigg\{ \frac{89}{216} + \frac{1}{3}\ln\frac{M_H^2}{m_t^2}\ln\frac{\mu^2}{m_t^2} - \frac{11}{9}\ln\frac{\mu^2}{m_t^2} - \frac{1}{6}\ln^2\frac{\mu^2}{m_t^2} \\
& + \frac{M_H^2}{m_t^2}\left[\frac{107}{675} - \frac{2}{45}\ln\frac{M_H^2}{m_t^2}\right] \\
& + \frac{\bar{m}_b^2}{M_H^2}\left[-\frac{89}{18} - 4\ln\frac{M_H^2}{m_t^2}\ln\frac{\mu^2}{m_t^2} + 10\ln\frac{\mu^2}{m_t^2} + 2\ln^2\frac{\mu^2}{m_t^2}\right] \\
& + \frac{\bar{m}_b^2}{m_t^2}\left[-\frac{116}{75} + \frac{8}{15}\ln\frac{M_H^2}{m_t^2}\right] \Bigg\}
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
\Delta\Gamma^{\mathrm{DB}}(A \to b\bar{b}) = {} & \frac{3G_F}{4\sqrt{2}\pi} M_A \bar{m}_b^2 C_{bb}^P \\
& \cdot \Bigg\{ \frac{89}{216} + \frac{1}{3}\ln\frac{M_A^2}{m_t^2}\ln\frac{\mu^2}{m_t^2} - \frac{11}{9}\ln\frac{\mu^2}{m_t^2} - \frac{1}{6}\ln^2\frac{\mu^2}{m_t^2} \\
& + \frac{M_A^2}{m_t^2}\left[\frac{107}{675} - \frac{2}{45}\ln\frac{M_A^2}{m_t^2}\right] \\
& + \frac{\bar{m}_b^2}{M_A^2}\left[-\frac{89}{54} - \frac{4}{3}\ln\frac{M_A^2}{m_t^2}\ln\frac{\mu^2}{m_t^2} + \frac{14}{9}\ln\frac{\mu^2}{m_t^2} + \frac{2}{3}\ln^2\frac{\mu^2}{m_t^2}\right] \\
& + \frac{\bar{m}_b^2}{m_t^2}\left[-\frac{16}{25} + \frac{4}{15}\ln\frac{M_A^2}{m_t^2}\right] \Bigg\}
\end{aligned}
\tag{22}
$$

5 Transition to the Effective Parameters

The results obtained above do not allow a naive decoupling of the top quark as they contain the $\log \mu^2/m_t^2$ terms not suppressed by any inverse power of m_t. From a practical

point of view such terms can hardly be considered as potentially large as μ is eventually set to be equal to the Higgs boson mass. Moreover, they may be summed up by using the methods of the effective field theory as it has been recently demonstrated on the example of top mass effects in the hadronic decay rate of the Z boson [34].

Still it seems to be reasonable to express all our results in terms of the effective $\alpha_s^{(5)}$ and $m_b^{(5)}$ appearing in the topless QCD with five quark flavours. The matching equations read [35, 36, 37]:

$$\frac{\alpha_s^{(6)}(\mu^2)}{\pi} = \frac{\alpha_s^{(5)}(\mu^2)}{\pi} + \left(\frac{\alpha_s^{(5)}(\mu^2)}{\pi}\right)^2 \frac{1}{6} \ln \frac{\mu^2}{m_t^2} + \mathcal{O}(\alpha_s^3) \tag{23}$$

$$\bar{m}_b^{(6)}(\mu^2) = \bar{m}_b^{(5)}(\mu^2) \left\{ 1 - \left(\frac{\alpha_s}{\pi}\right)^2 \left[\frac{89}{432} - \frac{5}{36} \ln \frac{\mu^2}{m_t^2} + \frac{1}{12} \ln^2 \frac{\mu^2}{m_t^2}\right] \right\}, \tag{24}$$

The corresponding changes in our results can be summarized as follows:

- As the singlet contributions start from order α_s^2, they do not change its functional form within our accuracy in α_s.

- Nonsinglet contributions (19) and (20) are conveniently arranged not to change their functional form if new $(\alpha_s^{(5)})^2$ terms (that are induced by terms of order $(\alpha_s^{(6)})^0$ and $\alpha_s^{(6)}$ after the replacements (15) and (16) are done) are assigned to the diagrams with a virtual top loop.

- The resulting expressions for the double bubble contributions read:

$$\Delta\Gamma^{\mathrm{DB}}(H \to b\bar{b}) = \frac{3G_F}{4\sqrt{2}\pi} M_H (\bar{m}_b^{(5)})^2 C_{bb}^S \left(\frac{\alpha_s^{(5)}}{\pi}\right)^2$$
$$\cdot \left\{ \frac{M_H^2}{m_t^2} \left[\frac{107}{675} - \frac{2}{45} \ln \frac{M_H^2}{m_t^2}\right] \right. \tag{25}$$
$$\left. + \frac{(\bar{m}_b^{(5)})^2}{m_t^2} \left[-\frac{116}{75} + \frac{8}{15} \ln \frac{M_H^2}{m_t^2}\right] \right\}$$

$$\Delta\Gamma^{\mathrm{DB}}(A \to b\bar{b}) = \frac{3G_F}{4\sqrt{2}\pi} M_A (\bar{m}_b^{(5)})^2 C_{bb}^P \left(\frac{\alpha_s^{(5)}}{\pi}\right)^2$$
$$\cdot \left\{ \frac{M_A^2}{m_t^2} \left[\frac{107}{675} - \frac{2}{45} \ln \frac{M_A^2}{m_t^2}\right] \right. \tag{26}$$
$$\left. + \frac{(\bar{m}_b^{(5)})^2}{m_t^2} \left[-\frac{16}{25} + \frac{4}{15} \ln \frac{M_A^2}{m_t^2}\right] \right\}$$

The decoupling of the top quark holds true for leading as well as subleading bottom mass corrections. The leading (massless) order agrees with the expansion of [11].

6   Discussion

In this section we combine the QCD corrections discussed above. The result is valid for intermediate Higgs masses (in particular $M_H < 2m_t$), since the singlet contribution was calculated in the heavy top limit. Everywhere below the notation $\alpha_s = \alpha_s^{(5)}(\mu)$, $\bar{m}_f = \bar{m}_f^{(5)}(\mu)$ is understrood with $f = u, d, s, c, b$ and $\mu = M_H$. Expressing the decay rate in the following form

$$
\begin{aligned}
\Gamma(H/A \to b\bar{b}) \quad &= \frac{3G_F}{4\sqrt{2}\pi} M_{H/A} \bar{m}_b^2 C_{bb}^{S/P} \\
&\left\{ 1 + \Delta\Gamma_1 \left(\frac{\alpha_s}{\pi}\right) + \left(\frac{\alpha_s}{\pi}\right)^2 \left[\Delta\Gamma_2 + \frac{M_{H/A}^2}{m_t^2}\tilde{\Delta\Gamma}_2\right] \right. \\
&+ \frac{\bar{m}_b^2}{M_{H/A}^2}\left(\Delta\Gamma_0^{(m)} + \Delta\Gamma_1^{(m)}\left(\frac{\alpha_s}{\pi}\right) + \left(\frac{\alpha_s}{\pi}\right)^2 \left[\Delta\Gamma_2^{(m)} + \frac{M_{H/A}^2}{m_t^2}\tilde{\Delta\Gamma}_2^{(m)}\right]\right) \\
&\left. + \mathcal{O}\left(\frac{\bar{m}_b^4}{M_{H/A}^4}\right) \right\}
\end{aligned}
\tag{27}
$$

the coefficients for the scalar Higgs partial width $\Gamma(H \to b\bar{b})$ read

$$
\begin{aligned}
\Delta\Gamma_1 \Big|_{H \to b\bar{b}} &= \frac{17}{3} \\
\\
\Delta\Gamma_2 \Big|_{H \to b\bar{b}} &= 29.147 \\
&+ \frac{C_{tb}^S}{C_{bb}^S}\left[1.57 - \frac{2}{3}\ln\frac{M_H^2}{m_t^2} + \frac{1}{9}\ln^2\frac{\bar{m}_b^2}{M_H^2}\right] \\
\tilde{\Delta\Gamma}_2 \Big|_{H \to b\bar{b}} &= \frac{107}{675} - \frac{2}{45}\ln\frac{M_H^2}{m_t^2} \\
&+ \frac{C_{tb}^S}{C_{bb}^S}\left[-0.007 - \frac{41}{1620}\ln\frac{M_H^2}{m_t^2} + \frac{7}{1080}\ln^2\frac{\bar{m}_b^2}{M_H^2}\right]
\end{aligned}
\tag{28}
$$

13

$$\Delta\Gamma_0^{(m)}\Big|_{H\to b\bar{b}} = -6$$

$$\Delta\Gamma_1^{(m)}\Big|_{H\to b\bar{b}} = -40$$

$$\Delta\Gamma_2^{(m)}\Big|_{H\to b\bar{b}} = -107.755$$

$$-0.98\ln^2\frac{\bar{m}_b^2}{M_H^2} - \frac{1}{12}\ln^4\frac{\bar{m}_b^2}{M_H^2} + 4\sum_{f=u,d,s,c,b}\frac{\bar{m}_f^2}{\bar{m}_b^2} \qquad (29)$$

$$+\frac{C_{tb}^S}{C_{bb}^S}\left[-5.61 + 4\ln\frac{M_H^2}{m_t^2} + \frac{16}{9}\ln\frac{\bar{m}_b^2}{M_H^2} - \frac{4}{9}\ln^2\frac{\bar{m}_b^2}{M_H^2}\right]$$

$$\tilde{\Delta\Gamma}_2^{(m)}\Big|_{H\to b\bar{b}} = -\frac{116}{75} + \frac{8}{45}\ln\frac{M_H^2}{m_t^2}$$

$$+\frac{C_{tb}^S}{C_{bb}^S}\left[0.52 - \frac{7}{270}\ln\frac{M_H^2}{m_t^2} + \frac{1}{135}\ln\frac{\bar{m}_b^2}{M_H^2} - \frac{7}{270}\ln^2\frac{\bar{m}_b^2}{M_H^2}\right]$$

For the decay rate $\Gamma(A\to b\bar{b})$ of the pseudoscalar Higgs boson one has ($\mu^2 = M_A^2$)

$$\Delta\Gamma_1\Big|_{A\to b\bar{b}} = \frac{17}{3}$$

$$\Delta\Gamma_2\Big|_{A\to b\bar{b}} = 29.147$$

$$+\frac{C_{tb}^P}{C_{bb}^P}\left[\frac{23}{6} - \ln\frac{M_A^2}{m_t^2} + \frac{1}{6}\ln^2\frac{\bar{m}_b^2}{M_A^2}\right] \qquad (30)$$

$$\tilde{\Delta\Gamma}_2\Big|_{A\to b\bar{b}} = \frac{107}{675} - \frac{2}{45}\ln\frac{M_A^2}{m_t^2}$$

$$+\frac{C_{tb}^P}{C_{bb}^P}\left[0.051 - \frac{7}{108}\ln\frac{M_A^2}{m_t^2} + \frac{1}{72}\ln^2\frac{\bar{m}_b^2}{M_A^2}\right]$$

$$\Delta\Gamma_0^{(m)}\Big|_{A\to b\bar{b}} = -2$$

$$\Delta\Gamma_1^{(m)}\Big|_{A\to b\bar{b}} = -\frac{8}{3}$$

$$\Delta\Gamma_2^{(m)}\Big|_{A\to b\bar{b}} = 91.006$$

$$-26.32\ln^2\frac{\bar{m}_b^2}{M_A^2} - \frac{4}{3}\ln^4\frac{\bar{m}_b^2}{M_A^2} + 4\sum_{f=u,d,s,c,b}\frac{\bar{m}_f^2}{\bar{m}_b^2} \tag{31}$$

$$+\frac{C_{tb}^P}{C_{bb}^P}\left[-5 + 2\ln\frac{M_A^2}{m_t^2} - \frac{4}{3}\ln\frac{\bar{m}_b^2}{M_A^2}\right]$$

$$\tilde{\Delta\Gamma}_2^{(m)}\Big|_{A\to b\bar{b}} = -\frac{16}{25} + \frac{4}{15}\ln\frac{M_A^2}{m_t^2}$$

$$+\frac{C_{tb}^P}{C_{bb}^P}\left[\frac{19}{108} - \frac{1}{18}\ln\frac{M_A^2}{m_t^2} - \frac{2}{9}\ln\frac{\bar{m}_b^2}{M_A^2}\right]$$

The formula for the decay rate of the standard scalar Higgs boson in the minimal SM may be obtained by setting $C_{bb}^S = C_{bb}^S = 1$. From eqs. (28),(29) one derives

$$\Delta\Gamma_1\Big|_{H\to b\bar{b}}^{SM} = \frac{17}{3}$$

$$\Delta\Gamma_2\Big|_{H\to b\bar{b}}^{SM} = 30.717 - \frac{2}{3}\ln\frac{M_H^2}{m_t^2} + \frac{1}{9}\ln^2\frac{\bar{m}_b^2}{M_H^2} \tag{32}$$

$$\tilde{\Delta\Gamma}_2\Big|_{H\to b\bar{b}}^{SM} = 0.15 - \frac{113}{1620}\ln\frac{M_H^2}{m_t^2} + \frac{7}{1080}\ln^2\frac{\bar{m}_b^2}{M_H^2}$$

$$\Delta\Gamma_0^{(m)}\Big|_{H\to b\bar{b}}^{SM} = -6$$

$$\Delta\Gamma_1^{(m)}\Big|_{H\to b\bar{b}}^{SM} = -40$$

$$\Delta\Gamma_2^{(m)}\Big|_{H\to b\bar{b}}^{SM} = -113.369 + 4\ln\frac{M_H^2}{m_t^2}$$

$$+\frac{16}{9}\ln\frac{\bar{m}_b^2}{M_H^2} - 1.42\ln^2\frac{\bar{m}_b^2}{M_H^2} - \frac{1}{12}\ln^4\frac{\bar{m}_b^2}{M_H^2} \tag{33}$$

$$+4\sum_{f=u,d,s,c,b}\frac{\bar{m}_f^2}{\bar{m}_b^2}$$

$$\tilde{\Delta\Gamma}_2^{(m)}\Big|_{H\to b\bar{b}}^{SM} = -1.027 + \frac{137}{270}\ln\frac{M_H^2}{m_t^2} + \frac{1}{135}\ln\frac{\bar{m}_b^2}{M_H^2} - \frac{7}{270}\ln^2\frac{\bar{m}_b^2}{M_H^2}$$

For our numerical discussion we use as input parameters a top mass of $m_t = 176$ GeV and a bottom pole mass of $m_b = 4.7$ GeV. The latter translates into the running mass $\bar{m}_b(M_H^2) = 2.84/2.75/2.69$ GeV for Higgs masses of $M_H = 70/100/130$ GeV. All other quarks are assumed to be massless. Based on $\Lambda_{QCD} = 233$ MeV one arrives at the following values for the strong coupling constant: $\alpha_s(M_H^2) = 0.125/0.118/0.114$

corresponding to the three differerent values of M_H. The influence of the top quark induced contribution on the second order coefficient is significant. $\Delta\Gamma_2$ is shifted to $(36.60/37.39/38.12)(\alpha_s/\pi)^2$ and is mainly due to the double triangle contribution with its mass logarithms. The quadratic bottom mass corrections prove to be rather small. They add a contribution of $(-0.54/-0.32/-0.22)(\alpha_s/\pi)^2$ to the second order result. Finally we reproduce the expression for the total hadronic width of the Higgs boson in the minimal SM:

$$
\begin{aligned}
\Gamma^{\mathrm{SM}}(H \to \text{hadrons}) \ = \ & \frac{3G_F}{4\sqrt{2}\pi} M_H \sum_{f=u,d,s,c,b} \bar{m}_f^2 \\
& \left\{ 1 + \frac{17}{3}\left(\frac{\alpha_s}{\pi}\right) \right. \\
& + \left(\frac{\alpha_s}{\pi}\right)^2 \left[32.26 - \frac{2}{3}\ln\frac{M_H^2}{m_t^2} \right] \\
& \qquad + \frac{M_H^2}{m_t^2}\left[\frac{5863}{24300} - \frac{113}{1620}\ln\frac{M_H^2}{m_t^2} \right] \\
& + \frac{\bar{m}_f^2}{M_H^2}\left(-6 - 40\left(\frac{\alpha_s}{\pi}\right) \right. \\
& \qquad + \left(\frac{\alpha_s}{\pi}\right)^2\left[-109.72 + 4\ln\frac{M_H^2}{m_t^2} + \frac{4}{3}\ln\frac{\bar{m}_f^2}{M_H^2} \right] \\
& \qquad\quad \left. + \frac{M_H^2}{m_t^2}\left[-1.28 + \frac{137}{270}\ln\frac{M_H^2}{m_t^2} - \frac{1}{54}\ln\frac{\bar{m}_f^2}{M_H^2} \right] \right) \\
& \left. + \sum_{f'=u,d,s,c,b} 12\frac{\bar{m}_{f'}^2}{M_H^2}\left(\frac{\alpha_s}{\pi}\right)^2 \right\}
\end{aligned}
$$

(34)

Contrary to the partial rate into bottom quarks the hadronic width is practically not affected by large logarithms $\ln(\bar{m}_f^2/M_H^2)$, since they vanish in the sum of bottom and gluon final states. Their absence is reflected in the numerical numbers for the second order coefficients. For the massless corrections they read $(33.55/33.12/32.82)/(\alpha_s/\pi)^2$. The mass corrections are again small. They amount to $(-0.19/-0.085/-0.048)(\alpha_s^2/\pi)^2$.

Acknowledgments

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