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ABSTRACT

This device is analyzed as an incompressible viscous fluid. Over-all properties such as resistance, capacitance, and charging time are calculated, and an equivalent circuit is given. One unexpected result is that for a large applied magnetic field, viscous dissipation and joule dissipation contribute equally to the total rate of dissipation.

A THEORY ON A HOMOPOLAR DEVICE

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INTRODUCTION

Recently Baker and his associates published a paper on a hydromagnetic capacitor, usually called a homopolar device.[1] This aroused our attention and interest sufficiently to develop a theory for the operation of this machine. A part of the theory is presented here; the remainder will be published elsewhere later.

The hydromagnetic capacitor as shown in Fig. 1 is an annular tube of rectangular cross section filled with an electrically conducting fluid (assumed incompressible in this analysis). The inner and outer circumferential walls of this device are perfectly conducting electrodes, while the top and bottom horizontal walls are nonconducting. If an electromotive force (emf) is maintained across the electrodes and a uniform magnetic field applied in the axial direction, a Lorentz force develops. This causes the fluid to rotate about the polar axis at a rate such that the viscous force balances the Lorentz force. Kinetic energy is stored in this device, just as electrical energy is stored in a capacitor. In fact, it has been shown experimentally[1] that, in its charging and discharging characteristics, this device behaves very much like a capacitor.

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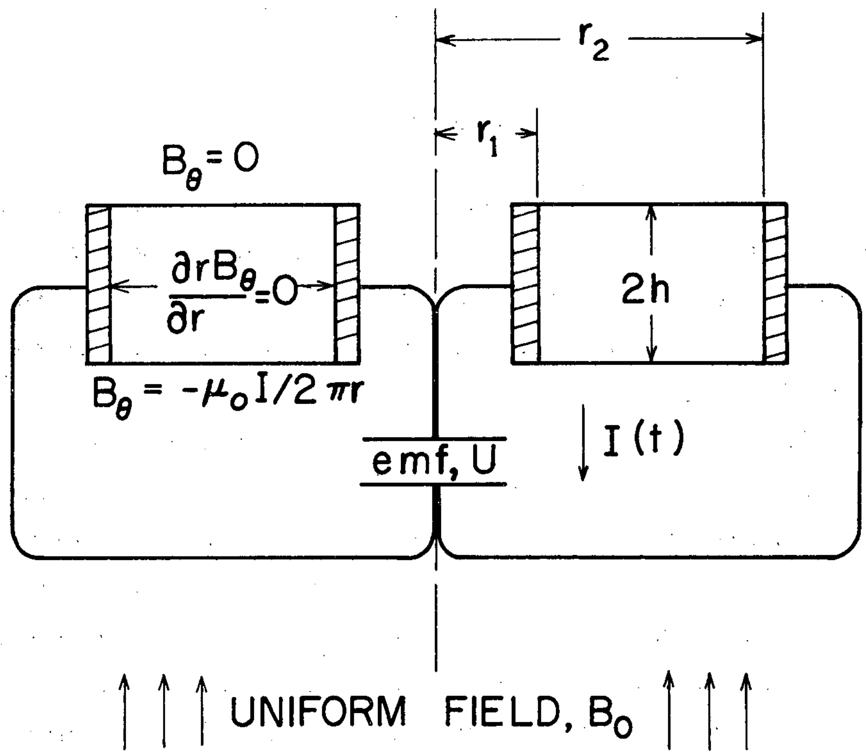

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Fig. 1. Cross section of homopolar device.

DERIVATION OF EQUATIONS

When an electrically conducting incompressible fluid flows in the presence of electric and magnetic fields, the following set of equations (in mks units) express the interaction between the electromagnetic and fluid dynamic effects:[2]

$$\text{curl } \vec{B} = \mu_0 \vec{j} \tag{1}$$

$$\text{div } \vec{B} = 0 \tag{2}$$

$$\text{curl } \vec{E} = -\partial \vec{B}/\partial t \tag{3}$$

$$\vec{j} = \sigma(\vec{E}+\vec{V}\times\vec{B}) \tag{4}$$

$$\rho \; \partial \vec{V}/\partial t = -\nabla p + \vec{j}\times\vec{B} + \rho\nu \nabla^2 \vec{V} \tag{5}$$

$$\text{div } \vec{V} = 0, \tag{6}$$

where B is the magnetic flux density, μ is the permeability, j is the current density, E is the electric field, ρ is the mass density, ν is the coefficient of viscosity, p is pressure, and σ is electrical conductivity. In these equations, free-charge and displacement current have been neglected. In the problem under consideration, the geometry will be assumed axially symmetric (i.e., all physical quantities independent of θ) with the streamlines in circles about the polar axis. That is, in cylindrical (r, θ, z) coordinates, the velocity will have only one component, V_θ, in the θ direction, and this will be independent of θ. In addition, it is assumed that no net current flows in the θ direction, and the applied axial magnetic field B_0 is uniform at infinity. Under these conditions it can be shown that $B_z = B_0$ and $B_r = j_\theta = E_\theta = 0$.[3] There is however, an induced magnetic field B_θ. Equations involving B_θ and V_θ alone are obtained from the θ component of Eq. (5) and the θ component of the curl of Eq. (4). These are

$$\frac{1}{\nu}\frac{\partial V_\theta}{\partial t} = \frac{\partial^2 V_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial V_\theta}{\partial r} - \frac{V_\theta}{r^2} + \frac{\partial^2 V_\theta}{\partial Z^2} + \frac{B_0}{\rho\nu\mu_0}\frac{\partial B_0}{\partial Z}. \tag{7}$$

and

$$\mu_0 \sigma \frac{\partial B_\theta}{\partial t} = \frac{\partial^2 B_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial B_\theta}{\partial r} - \frac{B_\theta}{r^2} + \frac{\partial^2 B_\theta}{\partial Z^2} + \sigma B_0 \mu_0 \frac{\partial V_\theta}{\partial z} . \quad (8)$$

The nonzero components of current and electric field can be calculated in terms of B_θ and V_θ. From Eqs. (1) and (4), we obtain

$$\mu_0 r j_r = - \partial r B_\theta / \partial Z, \quad \mu_0 r j_Z = \partial r B_\theta / \partial r \quad (9)$$

and

$$Er = - \frac{1}{\mu_0 \sigma} \frac{\partial B_\theta}{\partial z} - B_0 V_\theta, \quad E_z = \frac{1}{\mu_0 \sigma} \frac{1}{r} \frac{\partial r B_\theta}{\partial r} . \quad (10)$$

The pressure, p , can be calculated from the r and Z components of Eq. (5), that is,

$$p = - \frac{B_\theta^2}{2 \mu_0} + f(r), \quad \frac{df}{dr} = \frac{\rho V_\theta^2 - B_\theta^2/\mu_0}{r} . \quad (11).$$

A cross section of the system is shown in Fig. 1. In order to preserve axial symmetry, the external circuit is taken here to be a perfectly conducting sheet. If a simple wire were used, peripheral currents would be set up in the circumferential walls and the axial symmetry would be spoiled. At the perfectly conducting circumferential walls, E_z must be zero (continuity of tangential components of $\vec{E}$). From Eq. (10), this implies $\partial r B_\theta / \partial r = 0$ at $r = r_1$ and $r = r_2$. At the horizontal walls, B_θ must be continuous (no surface currents). It remains to determine B_θ in the region exterior to the fluid. Since the current in this region must be zero, Eq. (9) requires that rB_θ be a constant. In the region external to both the fluid and the external circuit, this constant can be taken to be zero, since B_θ must vanish at infinity.

In the loop of the external circuit the constant should be $-\mu_0 I/2\pi$, where I (t) is the total current in the sense indicated in Fig. 1. The boundary conditions are now seen to be $B_\theta = 0$ at the upper wall, and $B_\theta = -\mu_0 I/2\pi r$ at the lower wall.

New variables will be defined by $r = h\xi$, $Z = hn$, $t = T'\tau$, $V_\theta = V'V$, $B_\theta = B'B$, $I = I'i$, and $E_r = E'E$, where ξ, η, τ, V, B, i, and E are dimensionless, h is half the thickness of the capacitor, and the primed symbols are constants which are related by $B' = \mu_0 \sigma h E'$, $V' = h(\sigma/\rho\nu)^{1/2}E'$, $I' = 2\pi\sigma h^2 E'$, and $T' = h^2(\mu_0\sigma/\nu)^{1/2}$. The constant E' which has the dimensions of electric field is left arbitrary for the moment. With this change of variables, Eqs. (7), (8), and (10) take the form

$$\frac{1}{\lambda}\frac{\partial V}{\partial \tau} = \frac{\partial^2 V}{\partial \xi^2} + \frac{1}{\xi}\frac{\partial V}{\partial \xi} - \frac{V}{\xi^2} + \frac{\partial^2 V}{\partial n^2} + M\frac{\partial B}{\partial \eta}, \qquad (12)$$

$$\lambda\frac{\partial B}{\partial \tau} = \frac{\partial^2 B}{\partial \xi^2} + \frac{1}{\xi}\frac{\partial B}{\partial \xi} - \frac{B}{\xi^2} + \frac{\partial^2 B}{\partial \eta^2} + M\frac{\partial V}{\partial \eta}, \qquad (18)$$

and

$$E = -\partial B/\partial\eta - MV. \qquad (14)$$

In these equations, new parameters $\lambda = (\mu_0\sigma\nu)^{1/2}$ and $M = B_0 h(\sigma/\rho\nu)^{1/2}$ have been used. The latter is the Hartmann number. The boundary conditions are as follows: V = 0 on all boundaries, B = 0 at $\eta = 1$, $B = -i/\xi$ at $\eta = -1$ and $\partial\xi B/\partial\xi = 0$ at $\xi = r_1/h$ and at $\xi = r_2/h$. In order to complete the specification of the system, the current i could be specified as a function of time. More generally, the external circuit would be given, and this would afford a relation between the total current and the emf between the electrodes. In other words, the additional relation would be obtained by treating the hydromagnetic capacitor as a circuit element.

The general problem as formulated above is difficult to solve exactly. It will be assumed, therefore, that the device is very thin (thickness much smaller than radial dimensions) and the effects of the circumferential walls will be neglected. To be more precise, the velocity will not be required to be zero at the circumferential walls. It is observed from the boundary conditions that B varies like ξ^{-1} at the lower wall. This is a reflection of the fact that since the total current is conserved, the radial current density must vary in this manner. As a consequence of this, the Lorentz force, which is the product of the applied field and the radial current density, must vary like ξ^{-1}. This motivates the assumptions $V = v(\eta,\tau)/\xi$, $B = b(\eta,\tau)/\xi$, and $E = -e(\eta,\tau)/\xi$. Obviously a velocity of this form cannot be zero at the radial walls. On the other hand, we have $E_z = 0$, so the boundary condition on B at the radial walls is identically satisfied. The equations which v, b, and e must satisfy are

$$\frac{1}{\lambda}\frac{\partial v}{\partial \tau} = \frac{\partial^2 v}{\partial \eta^2} + M\frac{\partial b}{\partial \eta}, \tag{15}$$

$$\lambda\frac{\partial b}{\partial \tau} = \frac{\partial^2 b}{\partial \eta^2} + M\frac{\partial v}{\partial \eta}, \tag{16}$$

and

$$e = \frac{\partial b}{\partial \eta} + Mv. \tag{17}$$

The boundary conditions are $v = 0$ at $\eta = \pm 1$, $b = 0$ at $\eta = 1$, and $b = -i$ at $\eta = -1$.

STEADY-STATE FLOW

The simplest problem which can be solved for the hydromagnetic capacitor, is its steady-state operation with a constant potential difference, U, maintained between the circumferential walls. In order to make the boundary conditions on b antisymmetric, b will be replaced by $b + i/2$. This has no effect on the equations (in this study case) but changes the boundary conditions to $b = \pm i/2$ at $\eta = \pm 1$. With this change, it is apparent that b must be an odd function of η while v must be even. Eqs. (15) and (16) can each be integrated once, yielding

$$\partial v/\partial \eta + Mb = 0 \tag{18}$$

and

$$\partial b/\partial \eta + Mv = e. \tag{19}$$

The constant of integration in Eq. (18) has been made zero since b is odd and v is even, and the constant in Eq. (19) has been identified with e by virtue of Eq. (17). Therefore, e, which is proportional to E_r is independent of η. The reference electric field will be chosen so that $e = 1$. This is accomplished by integrating E_r between the circumferential walls and setting this result equal to the applied emf, U. The result is $E = U/h \ln r_2/r_1$. Now, when b is eleiminated between Eqs. (18) and (19), the resulting equation for v is

$$\partial^2 v/\partial \eta^2 - M^2 v = - M, \tag{20}$$

and the desired solution, vanishing at both walls, is

$$v = 1/M\,(1 - \cosh M\eta/\cosh M). \tag{21}$$

From Eqs. (18) and (21), we obtain

$$b = (1/M) \sinh M\eta/\cosh M. \tag{22}$$

By the use of this result and the condition $b = i/2$ at $\eta = 1$, the dimensionless current, i, can be determined, that is,

$$i = 2 \tanh M/M. \tag{23}$$

From this the total current is found to be

$$I = \frac{4\pi\sigma h}{\ln r_2/r_1} \frac{\tanh M}{M} U \equiv U/R_0 . \tag{24}$$

This expression gives the total resistance, R_0, of the capacitor as

$$R_0 = \frac{\ln r_2/r_1}{4\pi\sigma h} \frac{M}{\tanh M} . \tag{25}$$

Equation (25) is plotted versus M in Fig. 2, where it is seen that for large M the resistance is approximately M times the resistance with no magnetic field. The final expressions for velocity and induced field are

$$V_\theta = \frac{U}{B_0 \ln r_2/r_1} \; \frac{1}{r} \left(1 - \frac{\cosh (MZ/h)}{\cosh M} \right) \tag{26}$$

and

$$B_0 = \frac{U \mu_0 (\sigma \rho \nu)^{1/2}}{B_0 \ln(r_2/r_1)} \; \frac{1}{r} \; \frac{\sinh (MZ/h)}{\cosh M} . \tag{27}$$

When M is very large, these last two relations are approximately

$$V_\theta \sim \frac{U}{B_0 \ln r_2/r_1} \; \frac{1}{r} , \quad B_0 \sim 0,$$

except in a boundary layer with thickness of the order h/M.

Several over-all properties of the hydromagnetic capacitor are of considerable interest. If the device is considered as a circuit element with resistance R_0, the rate of dissipation should be I^2R_0 and, indeed, direct calculation of the total viscous and joule dissipation rates show this to be the case. The local rate of viscous dissipation is $\rho \, \nu \, (\partial V_\theta / \partial Z)^2$ while the rate of joule dissipation is $j_r^{\,2}/\sigma$ or $(\partial B_\theta / \partial Z)^2 / \mu_0^{\,2} \sigma$. The total rates of viscous and joule dissipation are the integrals of these quantities over the whole volume of the device. Using Eqs. (26) and (27) and carrying out the integration, we obtain for the total dissipation rates respectively,

$$\Phi_v = \frac{2\pi\sigma h U^2}{\ln r_2/r_1} \left(\frac{\tanh M}{M} - \frac{1}{\cosh^2 M} \right) \tag{28}$$

and

$$\Phi_j = \frac{2\pi\sigma h U^2}{\ln r_2/r_1} \left(\frac{\tanh M}{M} + \frac{1}{\cosh^2 M} \right) . \tag{29}$$

The sum of these two expressions, the total rate of dissipation, is U^2/R_0 which of course is the sum of I^2R_0. A quantity of greater interest is the ratio,

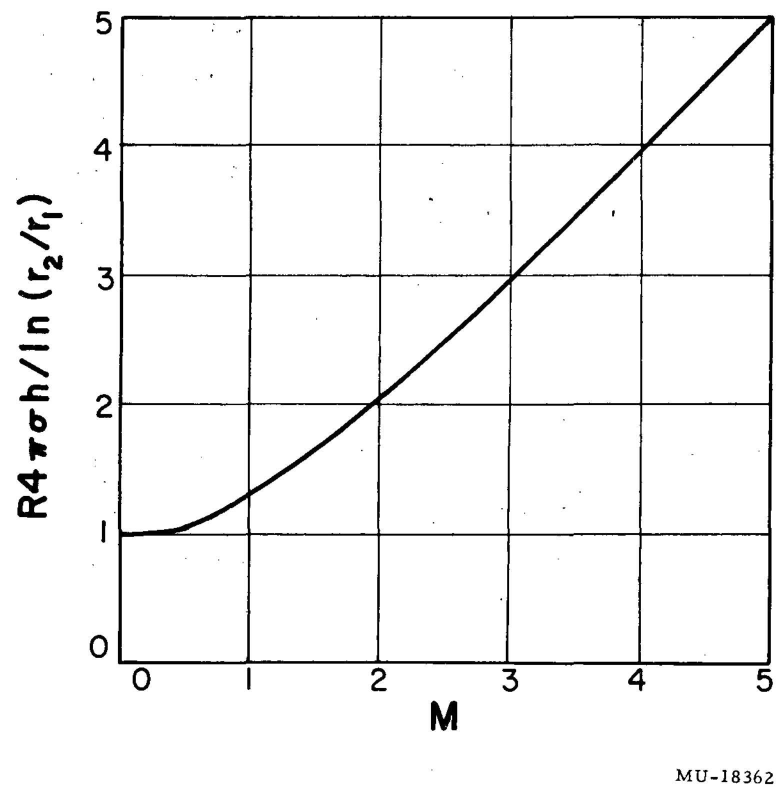

Fig. 2. Steady-state resistance versus the Hartmann number, M.

$$\frac{\phi_v}{\phi_j} = \frac{\sinh M \cosh M - M}{\sinh M \cosh M + M}, \tag{30}$$

which shows how much the different dissipation mechanisms contribute to the total rate of dissipation. This ratio is plotted versus M in Fig. 8. It is clearly seen that the viscous and joule dissipations contribute equally to the total dissipation rate when M is greater than about three.

Another interesting over-all property is the total kinetic energy (K. E.) stored in the device. This is easily calculated from Eq. (26):

$$\text{K. E.} = \frac{2\pi h^3 \sigma U^2}{\nu \ln r_2/r_1} \frac{1}{2M^2} \left[3 \left(1 - \frac{\tanh M}{M}\right) - \tanh^2 M \right]. \tag{31}$$

The total kinetic energy as a function of M is presented in Fig. 4. The notable features of this result are that the maximum kinetic energy occurs at a quite small value of M (about 1.5), and for large M the kinetic energy varies like M^{-2}.

CHARGING CAPACITOR

Although the steady-state problem gives information about the total kinetic energy, it is not as yet clear to what extent the device behaves like a capacitor. This is best decided from an analysis of the transient characteristics of the device. The simplest transient problem will be considered, namely, charging the device from rest by a constant emf source (i.e., an infinite capacitor). Even this problem is of considerable difficulty unless further simplication of the equations is made. In Eqs. (15) and (16) observe that the coefficients of the derivatives with respect to time are λ^{-1} and λ. For most materials λ is a very small number (for mercury $\lambda^2 \simeq 10^{-7}$), therefore the term $\lambda\, \partial b/\partial\tau$ will be neglected in Eq. (16). It will be shown later just how small λ must be for this approximation to be valid. Since this term is neglected, it is again possible to replace b by $b + i/2$, and to integrate Eq. (16) once. The constant of integration which results is again identified with $e(\tau)$ by virtue of Eq. (17). However, $e(\tau)$ must be independent of time, because the applied emf is steady. For this reason it is possible to choose the reference electric field E' so that $e = 1$.

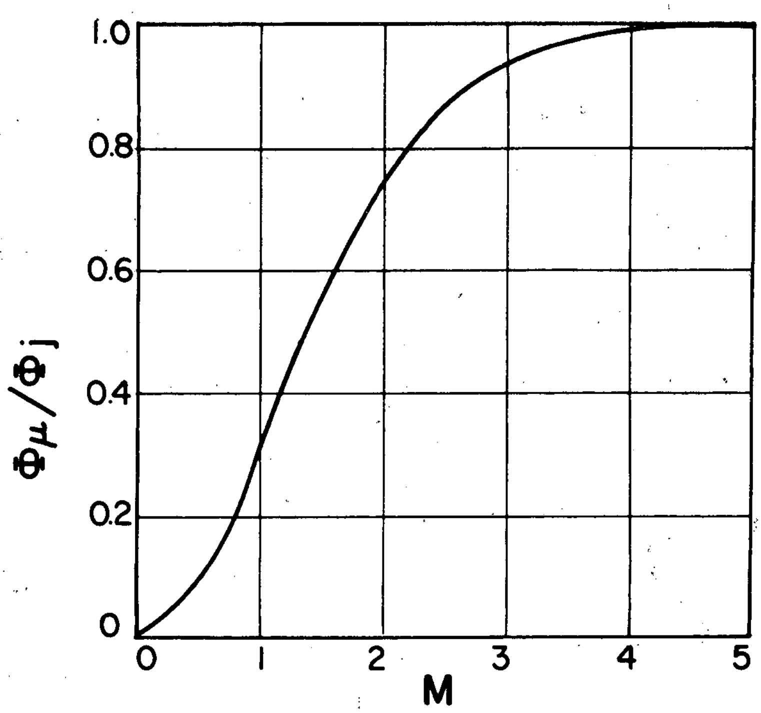

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Fig. 3. Ratio of viscous dissipation rate to joule dissipation rate versus the Hartmann number, M.

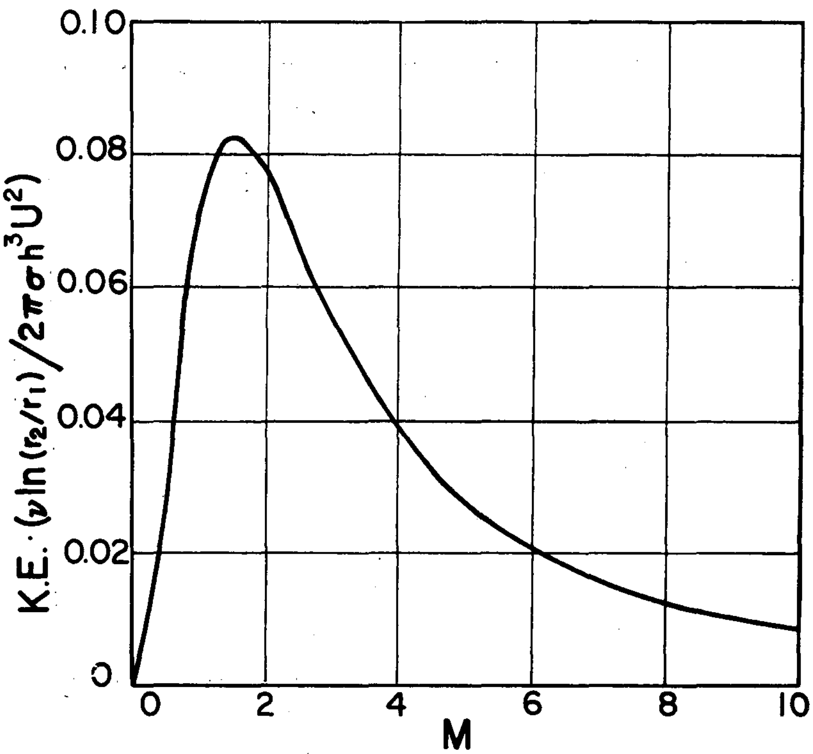

Fig. 4. Total kinetic energy versus the Hartmann number, M.

This is accomplished, as in the steady-state problem, by $E' = U/\hbar \, r_2/r_1$. When $\partial b/\partial \eta$ is eliminated from Eq. (15), the problem reduces to solving

$$\frac{1}{\lambda}\frac{\partial v}{\partial \tau} = \frac{\partial^2 v}{\partial \eta^2} - M^2 v + M \tag{32}$$

and

$$\partial b/\partial \eta + Mv = 1, \tag{33}$$

with v and b equal to zero when $\tau = 0$ and the boundary conditions $v = 0$ at $\eta = \pm 1$ and $b = \pm i/2$ at $\eta = \pm 1$. The solution of this problem is obtained by assuming

$$v = \frac{1}{M}\left(1 - \frac{\cosh M\eta}{\cosh M}\right) + \sum_{n=0}^{\infty} v_n(\tau) \cos \beta_n \eta . \tag{34}$$

and

$$b = \frac{1}{M}\frac{\sinh M\eta}{\cosh M} + \sum_{n=0}^{\infty} b_n(\tau) \sin \beta_n \eta , \tag{35}$$

where $\beta_n = (n + 1/2)\pi$. Substituting Eqs. (34) and (35) into Eqs. (32) and (33) and equating to zero the coefficients of the cosines and sines, we obtain $\lambda^{-1}\, \partial v_n/\partial \tau = -(\beta_n^2 + M^2)v_n$ and $\beta_n b_n + Mv_n = 0$. The first of these integrates simply to $v_n(\tau) = v_n(0)\, e^{-\lambda\left(\beta_n^2 + M^2\right) Z}$, while the second gives $b_n(\tau)$ in terms of $v_n(\tau)$. The initial value $v_n(0)$ is easily found by expanding $M^{-1}(1 - \cosh M\eta/\cosh M)$ in a series of cosines.
This gives

$$v_n(0) = -\frac{2M(-1)^n}{\beta_n(\beta_n^2 + M^2)}$$

The final expressions for v and b are

$$v = \frac{1}{M}\left(1 - \frac{\cosh M\eta}{\cosh M}\right) - \sum_{n=0}^{\infty} \frac{2M(-1)^n \cos\beta_n\eta}{\beta_n(\beta_n^2 + M^2)}\, e^{-\lambda\left(\beta_n^2 + M^2\right)\tau} \tag{36}$$

and

$$b = \frac{1}{M}\frac{\text{Sinh } M\eta}{\cosh M} + \sum_{n=0}^{\infty} \frac{2M^2(-1)^n \text{Sin}\beta_n\eta}{\beta_n^2(\beta_n^2 + M^2)} e^{-\lambda(\beta_n^2+M^2)\tau} . \quad (37)$$

When the condition $b = i/2$ at $\eta - 1$ is used in Eq. (37), an expression for the total current $I = I^{'}i = (M/2 \tanh M)(Ui/R_0)$ results, namely,

$$I(t) = \frac{U}{R_0} + \frac{2M^3}{\tanh M}\frac{U}{R_0}\sum_{n=0}^{\infty}\frac{1}{\beta_n^2(\beta_n^2+M^2)} e^{-\lambda(\beta_n^2+M^2)t/T^{'}} .$$

(38)

Now consider the circuit shown in Fig. 5. If this system is charged from rest by a constant emf, U, the total current is

$$I(t) = \frac{U}{R_0} + \sum_{j=1}^{\infty} \frac{U}{R_j} e^{-t/R_jC_j} \quad (39)$$

Equation (39) is of the same form as Eq. (38); in fact, Fig. 5 gives an equivalent circuit for the hydromagnetic capacitor with

$$R_j = R_0\beta_{j-1}^2 (\beta_{j-1}^2 + M^2) \tanh M /2M^3 \quad (40)$$

and

$$C_j = (T^{'}/R_0)\, 2\, M^3/\lambda\, \beta_{j-1}^2 (\beta_{j-1}^2 + M^2)^2 \tanh M. \quad (41)$$

The characteristic times of the separate capacitance-resistance combinations are

$$t_j = R_j\, C_j = T^{'}/\lambda\,(\beta_{j-1}^2 + M^2) . \quad (42)$$

The largest of these, t_1, can be taken as the characteristic time for the hydromagnetic capacitor:

$$t_1 = T^{'}/\lambda\,(M^2 + \pi^2/4) = h^2/\nu\,(M^2 + \pi^2/4) . \quad (43)$$

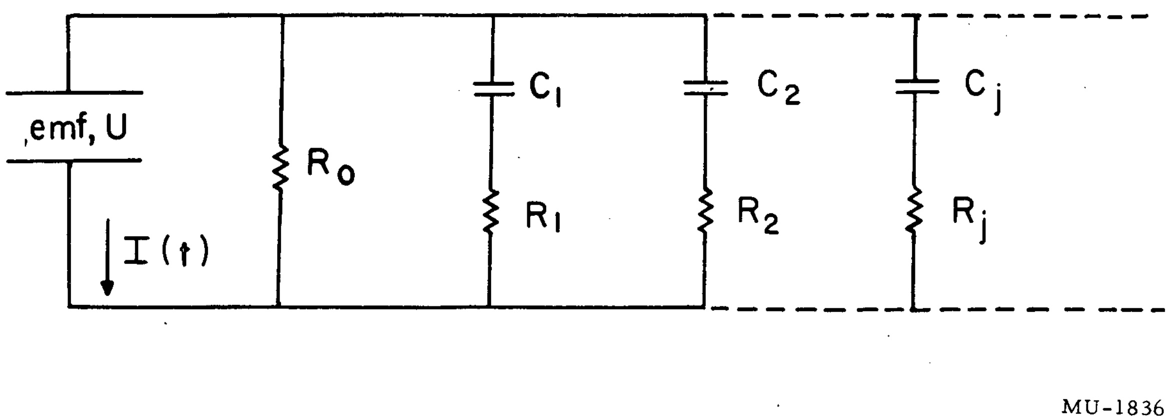

Fig. 5. Equivalent circuit for the hydromagnetic capacitor.

When M is very large -- e.g. greater than 25 -- Eq. (38) can be approximated by

$$I(t) = 1\,(U/R_0) + 2\,M\,(U/R_0)\,e^{-\lambda M^2 t/T'} \sum_{n=0}^{\infty} \beta_n^{-2}$$

$$= (U/R_0) + M\,(U/R_0)\,e^{-\lambda M^2 t/T'}$$

In this approximation, the equivalent circuit takes the much simpler form shown in Fig. 6, with

$$R_1' = R_0/M = (\ln r_2/r_1)\,/\,4\pi\sigma h\,, \tag{44}$$

$$C_1' = T'/\lambda R_1' M^2 = 4\pi\rho h/B_0^2 \ln r_2/r_1\,, \tag{45}$$

$$t_1' = R_1' C_1' = T'/\lambda M^2 = \rho/\sigma\, B_0^2. \tag{46}$$

The total capacitance of the general equivalent circuit (Fig. 5) is

$$C = \sum_{j=1}^{\infty} C_j = \frac{2M^3T'}{\lambda R_0 \tanh M} \sum_{n=0}^{\infty} \frac{1}{\beta_n^2(\beta_n^2 + M^2)^2}\,, \tag{47}$$

From this, one would calculate $CU^2/2$ as the total steady-state energy stored. A direct calculation of the steady-state kinetic energy [Eq. (31) and Fig. 4], by first expanding the steady-state velocity, Eq. (26), to a cosine series and then carrying out the integration, shows K. E. = $CU^2/2$. One might logically ask why the electrical and magnetic energy do not appear in this expression, that is, why isn't the total energy equal to $CU^2/2$. The answer to this is that the electrical energy was lost when the displacement current was neglected and the magnetic energy disappeared when $\lambda\,\partial b/\partial\tau$ was dropped from Eq. (16).

It is apparent from Fig. 5 that when the capacitors C_j are being charged, current must flow through the resistances R_j. Therefore, besides the constant rate of dissipation in the shunt resistance R_0, an additional amount of dissipation is intrinsically associated with the charging process. The rate at which energy is put into the system is

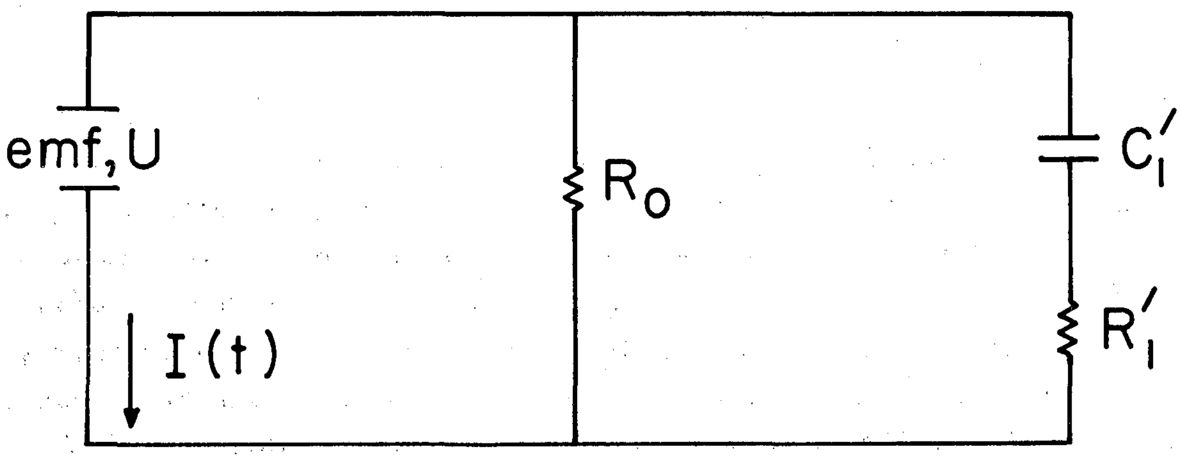

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Fig. 6. Equivalent circuit for large values of the Hartmann number, M.

$$IU = (U^2/R_0) + \sum_{j=1}^{\infty} (U^2/R_j)\, e^{-t/R_jC_j} \tag{48}$$

The term U^2/R_0 is the constant rate of dissipation in R_0. The total energy put into the remainder of the circuit is

$$\int_0^{\infty} \sum_{j=1}^{\infty} (U^2/R_j)\, e^{-t/R_jC_j}\, dt = \sum_{j=1}^{\infty} C_j U^2 , \tag{49}$$

which is just twice the energy stored in the system. This shows that the additional energy dissipated is equal to the total energy stored in the system.

A point which remains to be settled is the validity of neglecting $\lambda\, \partial b/\partial \tau$ in Eq. (16). A criterion which seems adequate is that $\lambda\, \partial b/\partial \tau$ must be much less than $\partial^2 b/\partial \eta^2$. Upon comparing the magnitude of these as calculated from Eq. (37), it is seen that the inequality will be approximately satisfied; if $\lambda^2(M^2+\pi^2/4)$ is much less than $\pi^2/4$. For large M this requires $\lambda^2 M^2 << 2.5$. If M is small, it will be satisfied by $\lambda^2 << 1$. For mercury we have $\sigma \simeq 10^6$, $\nu \simeq 10^{-7}$, and $\rho \simeq 13\times10^3$ in mks units, therefore $\lambda^2 = \mu_0\, \sigma\nu \sim 10^{-7}$. For a magnetic field of 25,000 gauss (2.5 Weber/M^2) and h = 1cm, M approximately equals 675. Therefore, $\lambda^2 M^2$ is about 0.5, which is much less than 2.5. These calculations show that the assumptions made are valid for mercury when the applied field is smaller than 25,000 gauss.

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