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# GUT Breaking on the Brane\*

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## Abstract

We present a five-dimensional supersymmetric SU(5) theory in which the gauge symmetry is broken maximally (*i.e.* at the 5D Planck scale  $M_*$ ) on the same 4D brane where chiral matter is localized. Masses of the lightest Kaluza-Klein modes for the colored Higgs and  $X$  and  $Y$  gauge fields are determined by the compactification scale of the fifth dimension,  $M_C \sim 10^{15}$  GeV, rather than by  $M_*$ . These fields' wave functions are repelled from the GUT-breaking brane, so that proton decay rates are suppressed below experimental limits. Above the compactification scale, the *differences* between the standard model gauge couplings evolve logarithmically, so that ordinary logarithmic gauge coupling unification is preserved. The maximal breaking of the grand unified group can also lead to other effects, such as  $O(1)$  deviations from SU(5) predictions of Yukawa couplings, even in models utilizing the Froggatt-Nielsen mechanism.

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# 1 Introduction

The standard model of particle physics, described by the gauge group  $SU(3)\otimes SU(2)\otimes U(1)$  is one of the most successful physical theories ever. Nonetheless, the standard model is theoretically unsatisfying for a number of reasons. For instance, the instability of the weak scale against radiative corrections has motivated the study of numerous theories, including technicolor, supersymmetry and theories with additional dimensions.

While the gauge hierarchy may be the most compelling motivation for new physics, there are additional reasons to consider theories with further structure. It was long ago realized that the gauge group of the standard model could be embedded into a simple group [1]. The appeal of this idea was substantiated by measurements of electroweak observables which suggested that the values of the standard model gauge couplings unify at a scale  $M_{\text{GUT}} \sim 10^{15}$  GeV [2].

As data became increasingly precise, it became apparent that the simplest grand unified theory (GUT), minimal  $SU(5)$ , predicted a value for  $\sin^2 \theta_W$  that was incompatible with observation. However, the combination of supersymmetry and grand unification [3] has met with great success in predicting  $\sin^2 \theta_W$  [4]. For these supersymmetric theories, the GUT symmetry must be broken at a somewhat higher scale,  $M_{\text{GUT}} \sim 2 \times 10^{16}$  GeV.

One smoking gun signal of grand unification is proton decay, which arises from dimension six operators generated by exchange of the  $X$  and  $Y$  gauge bosons, and, in supersymmetric theories, from dimension five operators generated by colored Higgsino exchange [5]. In supersymmetric GUTs, the dimension six operators typically do not lead to observable proton decay, but the dimension five operators do, and place strong limits on these theories [6]. Recently Ref. [7] studied the dimension five proton decay and found that even large masses for the first-two generation sparticles cannot save minimal supersymmetric  $SU(5)$ . They found that successful gauge coupling unification requires that the Higgs triplet mass fall in the range  $1.1 \times 10^{14}$  GeV  $\leq M_{H_C} \leq 9.3 \times 10^{15}$  GeV. However, even with decoupling of the superpartners, the limit from proton decay was shown to require  $M_{H_C} \geq 9.4 \times 10^{16}$  GeV.

While additional field content may generate threshold effects to rectify this, in this paper we propose an alternative possibility. We will embed a unified gauge theory into five dimensions in which gauge and Higgs fields propagate in the additional dimension. We will show that if the GUT is broken on a brane at a high scale (*i.e.*,  $M_*$ , the Planck scale of the five-dimensional theory), the wave functions of the gauge fields and Higgs triplets can develop an approximate node on the brane, suppressing proton decay operators arising from their exchange. We will show that the running above the compactification scale is not only consistent with, but is in fact equivalent to ordinary four dimensional logarithmic unification with the compactification scale identified as the mass of the Higgs triplet.

In doing so, we reformulate the question of the  $M_{\text{GUT}}/M_{\text{Pl}}$  hierarchy because the breaking occurs at the fundamental scale of the theory (which, in general, is still larger than  $10^{16}$  GeV). In its place we require a somewhat large hierarchy ( $O(100)$ ) between the Planck and compactification scales. The traditional GUT scale,  $2 \times 10^{16}$  GeV, is a derived

scale in this framework: there is no new physics there. Because of the  $O(1)$  breaking of the grand unified group on the brane, we will also easily understand deviations from  $SU(5)$  predicted relationships among Yukawa couplings.

## 2 Framework

We consider a 5D supersymmetric  $SU(5)$  theory with the fifth dimension compactified on an  $S_1/Z_2$  orbifold (in section 6 we will consider the possibility of larger gauge groups). The bulk exhibits  $N=1$  supersymmetry in five dimensions, which translates into an  $N=2$  supersymmetry in four dimensions. The fields that propagate in the bulk are contained in a gauge supermultiplet and two Higgs hypermultiplets transforming as  $\mathbf{5}$  and  $\bar{\mathbf{5}}$  under  $SU(5)$ . Under the 4D  $N=1$  supersymmetry preserved after the orbifold compactification, the gauge supermultiplet decomposes into a vector superfield  $V$  and an associated chiral adjoint  $\Phi$ . The two hypermultiplets yield four chiral multiplets  $H_{\mathbf{5}}, H_{\mathbf{5}}^c, H_{\bar{\mathbf{5}}}$  and  $H_{\bar{\mathbf{5}}}^c$ . ( $H_{\mathbf{5}}$  and  $H_{\bar{\mathbf{5}}}^c$  transform as  $\mathbf{5}$  under the  $SU(5)$ , while  $H_{\mathbf{5}}^c$  and  $H_{\bar{\mathbf{5}}}$  as  $\bar{\mathbf{5}}$ .) Under the orbifold  $Z_2$ , which flips the sign of the fifth coordinate ( $y \rightarrow -y$ ), the various superfields transform as

$$\begin{aligned} H_{\mathbf{5},\bar{\mathbf{5}}} &\longrightarrow H_{\mathbf{5},\bar{\mathbf{5}}} & H_{\mathbf{5},\bar{\mathbf{5}}}^c &\longrightarrow -H_{\mathbf{5},\bar{\mathbf{5}}}^c \\ V &\longrightarrow V & \Phi &\longrightarrow -\Phi, \end{aligned} \quad (1)$$

which leaves zero modes only for the MSSM fields and their GUT counterparts.

The  $S_1/Z_2$  orbifold has fixed points at  $y = 0$  and  $y = \pi R$ . We take chiral matter to be localized to the  $y = \pi R$  fixed point, along with an adjoint chiral superfield  $\Sigma$ . The  $SU(5)$  symmetry is broken by the vacuum expectation value (vev) of  $\Sigma$ , which we assume to have the form

$$\langle \Sigma_{ij} \rangle = \begin{pmatrix} 2/5 & 0 & 0 & 0 & 0 \\ 0 & 2/5 & 0 & 0 & 0 \\ 0 & 0 & 2/5 & 0 & 0 \\ 0 & 0 & 0 & -3/5 & 0 \\ 0 & 0 & 0 & 0 & -3/5 \end{pmatrix} \frac{\langle \Sigma \rangle}{\sqrt{2}}. \quad (2)$$

We imagine that this breaking occurs at or near the five-dimensional Planck scale  $M_*$ , which we assume to be much larger than the conventional GUT scale of  $2 \times 10^{16}$  GeV. We will refer to the  $y = \pi R$  fixed point as the GUT-breaking brane.

Bulk fields even under the orbifold  $Z_2$  can couple directly to the fields on the GUT-breaking brane. For instance, the Higgs fields can couple to the chiral matter fields  $T_{\mathbf{10}}$  and  $F_{\bar{\mathbf{5}}}$  as  $(\lambda_u T_{\mathbf{10}} T_{\mathbf{10}} H_{\mathbf{5}} + \lambda_d T_{\mathbf{10}} F_{\bar{\mathbf{5}}} H_{\bar{\mathbf{5}}}) \delta(y - \pi R)$  in the superpotential. We also take there to be a bare Higgs mass term,  $\alpha H_{\mathbf{5}} H_{\bar{\mathbf{5}}} \delta(y - \pi R)$ , as well as Higgs coupling to the GUT breaking,  $(\beta/M_*) H_{\mathbf{5}} \Sigma H_{\bar{\mathbf{5}}} \delta(y - \pi R)$ , in the superpotential. While it would be interesting to study standard solutions to the doublet-triplet splitting problem in this framework, we will not endeavor to do so here. Rather, we will simply tune these two contributions against one another so that the  $SU(2)$  doublets  $H_u$  ( $\subset H_{\mathbf{5}}$ ) and  $H_d$  ( $\subset H_{\bar{\mathbf{5}}}$ ) are nearly

massless, while the colored triplets  $H_C$  ( $\subset H_5$ ) and  $H_{\overline{C}}$  ( $\subset H_{\overline{5}}$ ) end up with a large mass term  $\kappa H_C H_{\overline{C}} \delta(y - \pi R)$ . Note that  $\kappa$  is a dimensionless parameter since  $H_C$  and  $H_{\overline{C}}$  are bulk fields and have dimension  $3/2$ . Here,  $\kappa$  arises as a sum of terms (with higher order terms potentially relevant in the strongly coupled theory). In addition to the bare mass term, the  $H_5 \Sigma H_{\overline{5}}$  term contributes. The natural size of this operator is restricted only by perturbativity, with an upper bound  $\sim 6\pi^2$  suggested by naive dimensional analysis in higher dimensions [8].

## 2.1 Spectrum: Gauge fields

Once the gauge symmetry is broken, the  $X$  and  $Y$  bosons acquire brane-localized masses through the interaction

$$\mathcal{L} \supset \delta(y - \pi R) g_5^2 \langle \Sigma \rangle^2 A_{\mu, \hat{a}} A_{\hat{a}}^\mu \quad (3)$$

where  $\hat{a}$  indexes the broken generators of the group and  $g_5^2$  is the 5D gauge coupling, with mass dimension  $-1$ . If we were to decompose each  $X$  and  $Y$  bulk vector field in a naive Kaluza-Klein (KK) basis, we would estimate that the zero modes have mass  $M_V = g_5 \langle \Sigma \rangle / \sqrt{2\pi R} = g_4 \langle \Sigma \rangle$ . Such an estimate incorrectly assumes that the gauge boson wave functions do not change appreciably in the presence of the large localized mass term. To find the correct spectrum we must solve the differential equation

$$-\partial_y^2 A_\mu + \delta(y - \pi R) g_5^2 \langle \Sigma \rangle^2 A_\mu = m^2 A_\mu, \quad (4)$$

which leads to KK masses  $M_n^G$  given by [8, 9]

$$M_n^G \tan(M_n^G \pi R) = \frac{g_5^2 \langle \Sigma \rangle^2}{2}, \quad (5)$$

where  $n = 0, 1, 2, \dots$ . For  $g_5^2 \langle \Sigma \rangle^2 R \gg 1$  this equation gives a spectrum whose low-lying levels are approximately  $M_n^G = (n + 1/2)M_C$  (with  $M_C \equiv 1/R$ ), whereas the usual KK spectrum is  $m_n = nM_C$ . We see that the KK tower has been shifted up one half unit.

This is easy to understand intuitively. The gauge field picks up a mass of  $g_4 \langle \Sigma \rangle \gg 1/R$  if it does not avoid the brane. If it avoids the brane entirely, it does not “see” the mass term, and picks up a smaller mass of  $1/(2R) = M_C/2$  from a nontrivial profile of the wave function in the extra dimension. The localized mass term acts merely to dynamically assign the boundary condition that the wave functions should vanish at the GUT-breaking brane. The boundary condition is not absolute, of course, so that the spectrum is not precisely spaced according to  $(n + 1/2)M_C$ . The value of the wave function at the GUT-breaking brane goes as  $\cos(M_n^G \pi R) = (2n + 1)M_C / (g_5^2 \langle \Sigma \rangle^2)$ , and there are corrections to the mass which go like  $-(2n + 1)M_C^2 / (\pi g_5^2 \langle \Sigma \rangle^2)$ . Modes with masses near or above the scale  $g_5^2 \langle \Sigma \rangle^2$  become essentially degenerate with the ordinary KK tower,  $m_n = nM_C$ .

We have not broken supersymmetry, and thus the gauginos corresponding to the broken  $SU(5)$  generators have an identical spectrum. The KK towers for the gauge fields are illustrated in Fig. 1a. For each unbroken  $SU(3) \otimes SU(2) \otimes U(1)$  (3-2-1) generator there is

a massless vector multiplet and massive vector multiplets at  $M_C$ ,  $2M_C$ ,  $3M_C$ , and so on. For each broken  $(X, Y)$  generator there is a tower of massive vector multiplets whose low levels are offset from those of the 3-2-1 tower by  $M_C/2$ , but whose higher levels relax into alignment with the 3-2-1 tower. Although in Fig. 1a this transition to degeneracy has already been achieved at  $5M_C$ , for the parameters of interest in our model, the relaxation will actually occur much more gradually, over the span of  $\sim g_5^2 \langle \Sigma \rangle^2 / M_C \sim O(10 - 100)$  KK levels. Note that for each  $X, Y$  massive vector multiplet but one there is a corresponding 3-2-1 massive vector multiplet. The remaining one is paired with the *massless* 3-2-1 vector multiplet, indicating a larger total number of states in the  $X, Y$  tower relative to the 3-2-1 tower. These excess states are the eaten Goldstone degrees of freedom from the  $\Sigma$  multiplet. This means that above the scale  $\sim g_5^2 \langle \Sigma \rangle^2$  the KK towers of gauge fields contributing to the renormalization group (RG) running are completely SU(5) symmetric except that one chiral adjoint for each 3-2-1 vector multiplet is missing. These missing degrees of freedom are provided by the physical  $\Sigma$  fields which remain after the Goldstone components of  $\Sigma$  are eaten by the  $X, Y$  gauge multiplets. Therefore, above the scale  $g_5^2 \langle \Sigma \rangle^2$  and  $m_\Sigma$  (the mass of physical  $\Sigma$  fields), the spectrum contributing to the running is completely SU(5) symmetric.

## 2.2 Spectrum: Higgs fields

The spectrum analysis for the Higgs triplet fields  $H_C$  and  $H_{\overline{C}}$  is somewhat different, because the brane coupling  $\kappa$  is dimensionless, and hence the naive mass that the zero modes would pick up by not avoiding the brane is just  $\kappa M_C$  which is comparable to  $M_C$  for order one  $\kappa$ . For our purposes, we will take  $\kappa$  to be a large parameter, roughly  $O(10)$ , so that the Higgs wave functions will in fact be strongly repelled from the brane. This size for  $\kappa$  is quite consistent with the general features of the theory, which must be somewhat strongly coupled in order to achieve order one top Yukawa and gauge couplings in spite of the large radius.

The equations that determine the colored Higgsino spectrum are

$$-i\bar{\sigma}^\mu \partial_\mu H_C - \partial_y \overline{H}_C^c - \kappa \delta(y - \pi R) \overline{H}_{\overline{C}} = 0, \quad (6)$$

$$-i\bar{\sigma}^\mu \partial_\mu H_{\overline{C}} - \partial_y \overline{H}_{\overline{C}}^c - \kappa \delta(y - \pi R) \overline{H}_C = 0, \quad (7)$$

$$-i\bar{\sigma}^\mu \partial_\mu H_C^c + \partial_y \overline{H}_C = 0, \quad (8)$$

$$-i\bar{\sigma}^\mu \partial_\mu H_{\overline{C}}^c + \partial_y \overline{H}_{\overline{C}} = 0. \quad (9)$$

We can analyze these equations by writing the solutions as

$$H_C^{(c)} = \sum_n \eta_{C,n}^{(c)}(x) g_{C,n}^{(c)}(y), \quad (10)$$

$$H_{\overline{C}}^{(c)} = \sum_n \eta_{\overline{C},n}^{(c)}(x) g_{\overline{C},n}^{(c)}(y), \quad (11)$$

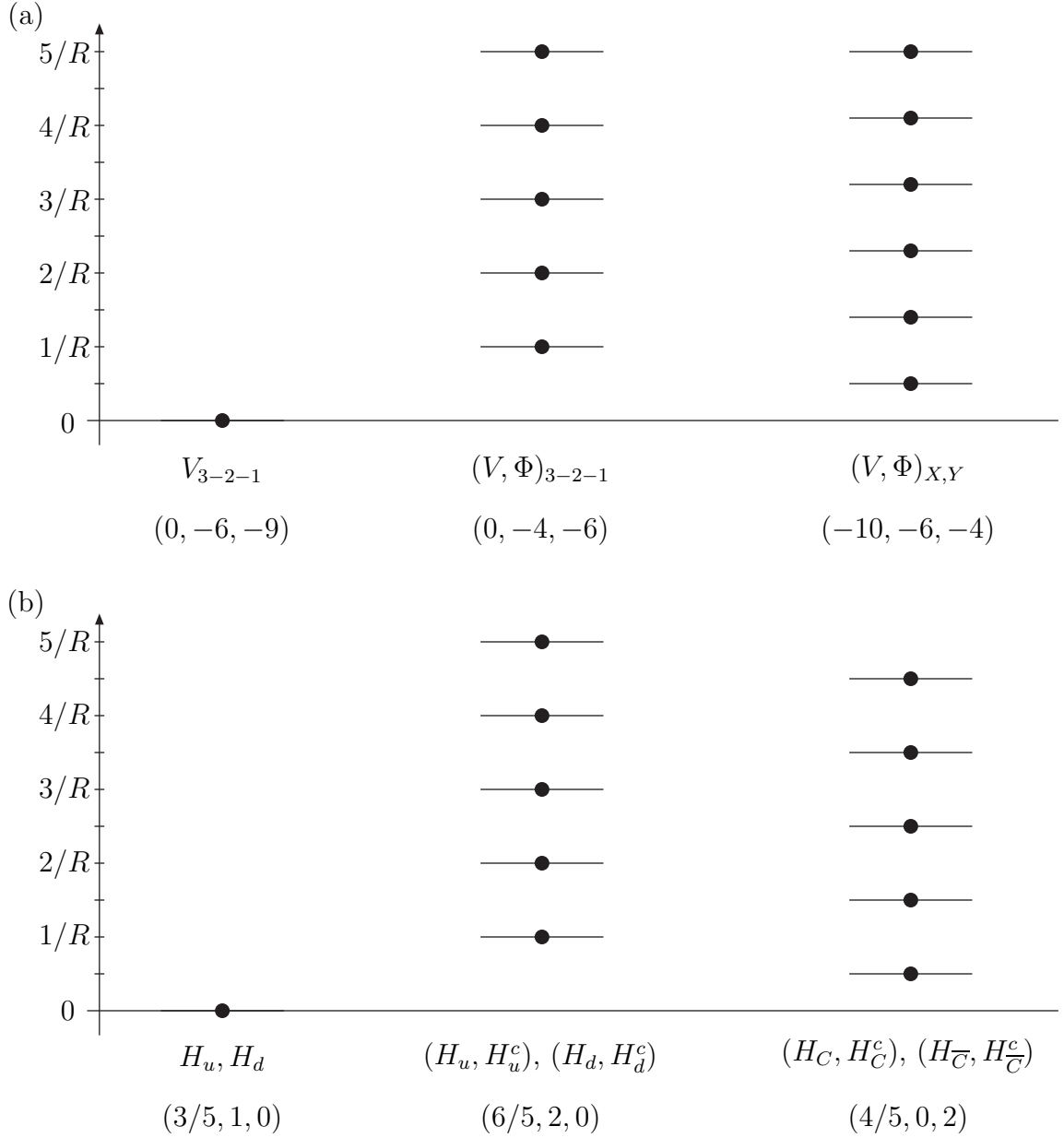


Figure 1: Mass spectrum for the lowest KK modes of the gauge fields (a), and Higgs fields (b) in our model. As explained in the text, the transition to degeneracy between the  $X, Y$  and 3-2-1 towers occurs much more gradually than shown in (a). In (b), the limit of very large  $\kappa$  is taken, so that the slight non-degeneracy between the colored hypermultiplet pairs at each level is not resolved. In both (a) and (b), the triplet of numbers below each tower corresponds to the beta function contribution  $(b_1, b_2, b_3)$  that comes from each level in that tower (for the colored Higgs, the contributions from the nearly-degenerate hypermultiplet pairs are combined).

where  $g_{C,n}$  and  $g_{\overline{C},n}$  are even under  $y \rightarrow -y$ , while  $g_{C,n}^c$  and  $g_{\overline{C},n}^c$  are odd. If we integrate the first two of these equations in a region  $(\pi R - \epsilon, \pi R + \epsilon)$  where  $\epsilon \rightarrow 0$ , we obtain two constraints,

$$\eta_{C,n}^c(x) = \frac{\kappa g_{\overline{C},n}(\pi R)}{2g_{C,n}^c(\pi R - \epsilon)} \eta_{\overline{C},n}(x), \quad (12)$$

$$\eta_{\overline{C},n}^c(x) = \frac{\kappa g_{C,n}(\pi R)}{2g_{\overline{C},n}^c(\pi R - \epsilon)} \eta_{C,n}(x). \quad (13)$$

Here, two constraints must be satisfied mode by mode. Let us define

$$\gamma_{(I,J),n} = \frac{\kappa g_{I,n}(\pi R)}{2g_{J,n}^c(\pi R - \epsilon)}, \quad (14)$$

where  $I, J$  take  $C, \overline{C}$ , and identify both  $-i\overline{\sigma}^\mu \partial_\mu \eta_{C,n}(x) = M_n^H \eta_{\overline{C},n}(x)$  and  $-i\overline{\sigma}^\mu \partial_\mu \eta_{\overline{C},n}(x) = M_n^H \eta_{C,n}(x)$ . This allows us to rewrite Eqs. (6) – (9) as (neglecting singular terms)

$$M_n^H g_{C,n}(y) - \gamma_{(\overline{C},C),n} \partial_y g_{C,n}^c(y) = 0, \quad (15)$$

$$M_n^H g_{\overline{C},n}(y) - \gamma_{(C,\overline{C}),n} \partial_y g_{\overline{C},n}^c(y) = 0, \quad (16)$$

$$M_n^H \gamma_{(C,\overline{C}),n} g_{C,n}^c(y) + \partial_y g_{C,n}(y) = 0, \quad (17)$$

$$M_n^H \gamma_{(\overline{C},C),n} g_{\overline{C},n}^c(y) + \partial_y g_{\overline{C},n}(y) = 0, \quad (18)$$

which give solutions

$$g_{C,\overline{C},n}(y) = N_n \cos(M_n^H y), \quad g_{\overline{C},C,n}^c(y) = N_n \sin(M_n^H y), \quad (19)$$

with  $M_n^H$  defined by

$$\tan^2(M_n^H \pi R) = \frac{\kappa^2}{4}. \quad (20)$$

Thus, the KK masses for the colored Higgs are given by

$$M_n^H = \frac{1}{R} \left( n + \frac{\arctan(\kappa/2)}{\pi} \right), \quad (21)$$

where  $n$  runs from negative infinity to positive infinity (the physical masses are given by the absolute value of  $M_n^H$ ). For large  $\kappa$ ,  $\arctan(\kappa/2) \simeq \pi/2$ , so  $M_n^H \simeq (n + 1/2)M_C$ , and the masses fall into nearly degenerate pairs,  $(M_n^H, M_{-n-1}^H)$ . Note that, in contrast to the case of the gauge fields, even for high values of  $n$ , the triplet Higgsinos do not become degenerate with the doublet Higgsinos.

Supersymmetry fixes the masses and couplings of the Higgs scalars to be the same as for the Higgsinos. The KK towers for the Higgs fields are shown in Fig. 1b. The doublet spectrum consists of a pair of massless  $N = 1$  chiral multiplets, and pairs of exactly degenerate hypermultiplets at  $M_C, 2M_C, 3M_C$ , and so on. For large  $\kappa$ , the triplet spectrum consists of nearly degenerate pairs of hypermultiplets at  $M_C/2, 3M_C/2, 5M_C/2$ , and so on. In Fig. 1b, the non-degeneracy between these pairs is not resolved.



### 3 Differential running

A crucial question in our model is how the gauge couplings evolve above the compactification scale. To address this question, it is useful to focus on the “differential running”, *i.e.*, the non-uniform evolution of the gauge couplings. Above  $M_C/2$  a whole tower of modes contributes to the evolution of  $g_1, g_2$  and  $g_3$ . However, we should emphasize that *this scenario does not employ power law unification*. In contrast with Ref. [10], the overwhelming power-law contribution to the running above  $M_C/2$  is SU(5) universal, so it is useful to focus only on the quantities which distinguish the couplings and lead to non-uniform evolution above  $M_C/2$ . For instance, the matter fields fall into complete SU(5) multiplets so their effects only change the coupling value at unification, but do not influence whether and at what scale the couplings unify. In contrast, the presence of the Higgs doublets leads to non-uniform running. We will see shortly that above  $M_C/2$ , the differential running arises from a sum of a large number of threshold effects.<sup>1</sup>

Let us define the scale-dependent quantities

$$\delta_i(\mu) \equiv \alpha_i^{-1}(\mu) - \alpha_1^{-1}(\mu), \quad (22)$$

with  $\delta_1$  vanishing trivially. The evolution of  $\delta_i(\mu)$  above  $M_C/2$  (but below  $m_\Sigma$ ) takes the form

$$\delta_i(\mu) = \delta_i(M_C/2) - \frac{1}{2\pi}(R_i^H(\mu) + R_i^G(\mu)), \quad (23)$$

where  $R_i^H(\mu)$  represents contributions arising from Higgs loops and  $R_i^G(\mu)$  represents contributions arising from gauge loops. Contributions from matter loops vanish because they contribute universally to the running. Using the known spectra for the various KK towers and their beta function contributions (shown in Fig. 1), we can calculate these quantities. For the Higgs contributions we find

$$R_2^H(\mu) = \frac{2}{5} \log\left(\frac{\mu}{M_C/2}\right) + \frac{4}{5} \sum_{0 < nM_C < \mu} \log\left(\frac{\mu}{nM_C}\right) - \frac{2}{5} \sum_{|M_n^H| < \mu} \log\left(\frac{\mu}{|M_n^H|}\right), \quad (24)$$

$$R_3^H(\mu) = -\frac{3}{5} \log\left(\frac{\mu}{M_C/2}\right) - \frac{6}{5} \sum_{0 < nM_C < \mu} \log\left(\frac{\mu}{nM_C}\right) + \frac{3}{5} \sum_{|M_n^H| < \mu} \log\left(\frac{\mu}{|M_n^H|}\right). \quad (25)$$

Here the  $M_n^H$  are the colored Higgs masses given in Eq. (21). Recall that there are actually two slightly non-degenerate  $|M_n^H|$  near each explicitly shown colored Higgs level in Fig. 1b.

For the gauge contributions we have

$$R_2^G(\mu) = -6 \log\left(\frac{\mu}{M_C/2}\right) - 4 \sum_{0 < nM_C < \mu} \log\left(\frac{\mu}{nM_C}\right) + 4 \sum_{M_n^G < \mu} \log\left(\frac{\mu}{M_n^G}\right), \quad (26)$$

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<sup>1</sup> The threshold correction from KK towers are also discussed in the context of power-law unification [11] and orbifold breaking of the unified gauge symmetry [12].

$$R_3^G(\mu) = -9 \log\left(\frac{\mu}{M_C/2}\right) - 6 \sum_{0 < nM_C < \mu} \log\left(\frac{\mu}{nM_C}\right) + 6 \sum_{M_n^G < \mu} \log\left(\frac{\mu}{M_n^G}\right), \quad (27)$$

where  $M_n^G$  are the  $X, Y$  vector multiplet masses, given by the solutions to Eq. (5).

### 3.1 Higgs contributions

The running of  $\delta_i(\mu)$  is given by

$$\frac{d\delta_i}{d\mu} = -\frac{1}{2\pi} \left( \frac{dR_i^H(\mu)}{d\mu} + \frac{dR_i^G(\mu)}{d\mu} \right). \quad (28)$$

Let us consider the Higgs and gauge contributions to this running separately, starting with the Higgs loops first. We have

$$\frac{dR_2^H}{d\mu} = \frac{1}{5\mu} \left[ (\# \text{ of doublets with mass } < \mu) - (\# \text{ of triplets with mass } < \mu) \right], \quad (29)$$

and

$$\frac{dR_3^H}{d\mu} = -\frac{3}{10\mu} \left[ (\# \text{ of doublets with mass } < \mu) - (\# \text{ of triplets with mass } < \mu) \right]. \quad (30)$$

Here we mean the number of doublet and triplet chiral superfields: there are two massless doublets, four triplets with mass  $\simeq M_C/2$ , four doublets with mass  $M_C$ , and so on.

Both of the above expressions have the expected  $1/\mu$  characteristic of logarithmic running, but have coefficients which *average out to zero*. Below  $M_C/2$  only the massless  $H_u$  and  $H_d$  doublets contribute, so  $R_2^H$  ( $R_3^H$ ) runs in the positive (negative) direction. At  $M_C/2$  we encounter four triplet chiral superfields and  $R_2^H$  ( $R_3^H$ ) now runs in the negative (positive) direction. The directions reverse again when we gain four doublets at  $M_C$ , and again at  $3M_C/2$  when we gain another four triplets. These threshold effects continue, but become increasingly negligible. Taking the triplet masses to be  $(n + 1/2 \pm \delta)M_C$ , the threshold effects from the states between  $nM_C$  and  $(n + 1)M_C$  are proportional to

$$\begin{aligned} \log\left(\frac{\Lambda}{nM_C}\right) + \log\left(\frac{\Lambda}{(n+1)M_C}\right) - \log\left(\frac{\Lambda}{(n+1/2+\delta)M_C}\right) - \log\left(\frac{\Lambda}{(n+1/2-\delta)M_C}\right) \\ = \log\left(\frac{(n+1/2+\delta)(n+1/2-\delta)}{n(n+1)}\right) \end{aligned} \quad (31)$$

which vanishes as  $(1/4 - \delta^2)/n^2$  for large  $n$ .<sup>2</sup>

We conclude that the differential running due to the Higgs fields dies off quickly above the compactification scale. Thus the total Higgs contribution to the running between

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<sup>2</sup>Here we use the step-function approximation for RG running. However, had we used a Jacobi  $\vartheta$  function as in Ref. [10], we would still find the high mass threshold effects to be negligible.

$M_C/2$  and a high scale such as  $m_\Sigma$  or  $g_5^2\langle\Sigma\rangle^2$  is essentially independent of the high scale. Taking the pairs of triplet hypermultiplets to be exactly degenerate, we find that for high scales,

$$\begin{aligned} R_2^H &= -\frac{2}{5} \left( \log \left( \frac{M_C}{M_C/2} \right) - \log \left( \frac{3M_C/2}{M_C} \right) + \log \left( \frac{2M_C}{3M_C/2} \right) - \log \left( \frac{5M_C/2}{2M_C} \right) + \dots \right) \\ &= -\frac{2}{5} \log \left( \frac{\pi}{2} \right), \end{aligned} \quad (32)$$

and similarly,

$$R_3^H = \frac{3}{5} \log \left( \frac{\pi}{2} \right). \quad (33)$$

Note that the ratio  $R_2^H/R_3^H = -2/3$  is the same as one would have calculated by including the contributions from the massless doublets alone. This fact will be important when we compare with the running in 4D minimal SU(5) in section 3.3.

## 3.2 Gauge contributions

In contrast to the Higgs threshold effects, those arising from gauge field loops do not quickly die off. In fact, we will see that these effects add up to an effective logarithmic running all the way up to  $m_\Sigma$  or  $g_5^2\langle\Sigma\rangle^2$ .

The definitions we have used for  $R_i^G$  are very convenient for making connection to ordinary, four-dimensional running. In particular,  $\alpha_1$  receives no gauge contribution below the GUT scale in four-dimensional theories, and likewise here  $R_1^G$  will not receive contributions to differential running.  $R_2^G$  and  $R_3^G$ , on the other hand, *will* receive corrections above the compactification scale.

Referring to Eqs. (26) and (27), we see that the quantities  $R_i^G$  are proportional to the quadratic Casimir coefficients  $C_2(G)$  of the gauge groups. Thus, we can write the gauge contributions as

$$R_i^G(\mu) = -C_2(G_i)\Delta(\mu), \quad (34)$$

where  $C_2(G)$  is 2 for SU(2) and 3 for SU(3). Furthermore, we have

$$\frac{d\Delta(\mu)}{d\mu} = \frac{1}{\mu} \left[ 3 + 2 \left( \# \text{ of } (V, \Phi)_{3-2-1} \text{ levels below } \mu \right) - 2 \left( \# \text{ of } (V, \Phi)_{X,Y} \text{ levels below } \mu \right) \right]. \quad (35)$$

The notation here is as in Fig. 1a. Note that in contrast to the case for the Higgs multiplets, here the coefficient multiplying  $1/\mu$  does not average to zero. Rather, for low modes (such that  $M_n^G \simeq (n+1/2)M_C$ ), the coefficient is 3 up to  $M_C/2$ , 1 between  $M_C/2$  and  $M_C$ , then 3 again until  $3M_C/2$ , and so on. For the higher mass modes, such that the  $X, Y$  and 3-2-1 multiplets are nearly degenerate, the coefficient becomes fixed at 1.

If the running were coming entirely from the massless 3-2-1 vector multiplet, the coefficient multiplying  $1/\mu$  would be 3, so we see that above the compactification scale, the differential running due to the gauge loops is slowed somewhat relative to the ordinary

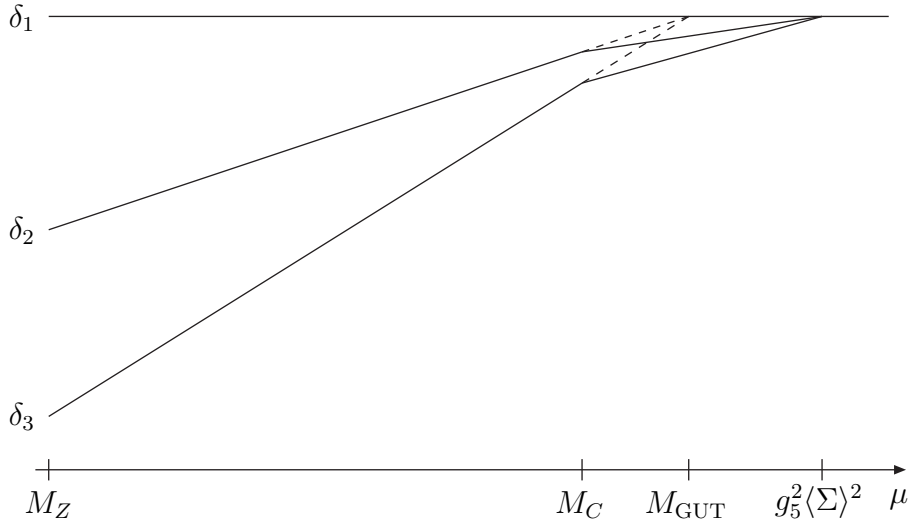


Figure 2: The qualitative picture for gauge coupling unification in our 5D model. We define  $\delta_i(\mu) \equiv \alpha_i^{-1}(\mu) - \alpha_1^{-1}(\mu)$ . The conventional unification scale  $M_{\text{GUT}} \sim 2 \times 10^{16}$  GeV is a derived scale rather than a physical one. Here we assume  $m_\Sigma < g_5^2 \langle \Sigma \rangle^2$ , so that unification is achieved near  $g_5^2 \langle \Sigma \rangle^2$ .

logarithmic evolution in 4D. Note that even once the  $X, Y$  and 3-2-1 multiplets become degenerate, the differential running continues. In this regime, the running is due entirely to the eaten Goldstone states contained in the  $X, Y$  KK tower. The differential running stops completely only once we reach the scale  $m_\Sigma$ , when the rest of the  $\Sigma$  degrees of freedom begin to propagate in the loops.<sup>3</sup>

The qualitative picture for gauge coupling unification in our model is depicted in Fig. 2. Above the compactification scale, the non-uniform evolution of the gauge couplings slows, so that unification occurs at a larger scale than usual. The unification scale will essentially be the larger of  $m_\Sigma$  and  $g_5^2 \langle \Sigma \rangle^2$ . Next we will verify that this picture is in fact correct, and we will make more precise the connection to unification in ordinary minimal SU(5) in 4D.

### 3.3 Unification and minimal SU(5)

Up to this point we have discussed the differential evolution of the couplings above the compactification scale, but have not explicitly demonstrated that the couplings in our model unify in a manner consistent with electroweak scale measurements of the 3-2-1 couplings. Here we will demonstrate that, at one loop, the successful prediction of  $\sin^2 \theta_W$

<sup>3</sup>Or, if  $m_\Sigma$  happens to be below the scale at which the  $X, Y$  and 3-2-1 towers become degenerate (determined by  $g_5^2 \langle \Sigma \rangle^2$ ), the differential running due to the Goldstone degrees of freedom ceases above  $m_\Sigma$ , but the total differential running does not die off until a scale  $\sim g_5^2 \langle \Sigma \rangle^2$ .

of minimal SU(5) with  $M_{H_C} < M_{\text{GUT}}$  [13] is equivalent to the successful prediction of  $\sin^2 \theta_W$  in this model with  $M_C \sim M_{H_C}$ . That is, the fact that minimal SU(5) with a triplet Higgs below the GUT scale can give  $\sin^2 \theta_W$  correctly *guarantees* that this model can do as well.

To see this we must reexamine the running of gauge couplings in four-dimensional, minimal SU(5). Above the Higgs triplet mass, the gauge couplings evolve as

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_{H_C}^{(4d)}) - \frac{1}{2\pi} (7 - 3C_2(G_i)) \log \left( \frac{\mu}{M_{H_C}^{(4d)}} \right). \quad (36)$$

To make connection with our discussion of differential running in the five-dimensional theory, we subtract off a universal piece  $\alpha_1^{-1}$  from each gauge coupling, and we find

$$\delta_i^{4d}(M_{\text{GUT}}) - \delta_i^{4d}(M_{H_C}^{(4d)}) = \frac{3}{2\pi} C_2(G_i) \log \left( \frac{M_{\text{GUT}}}{M_{H_C}^{(4d)}} \right), \quad (37)$$

where  $M_{\text{GUT}}$  is the conventional GUT scale  $\sim 2 \times 10^{16}$  GeV. Below the scale  $M_{H_C}^{(4d)}$ , the running of the gauge couplings in the 5D theory is the same as for the 4D theory. Thus, if we can demonstrate that  $\delta_i^{5d}(\mu) - \delta_i^{5d}(M_{H_C}^{(4d)})$  has the same form as Eq. (37), we will have shown that successful unification in ordinary minimal SU(5) implies successful unification in our model. Here the superscript in  $M_{H_C}^{(4d)}$  is to emphasize that it is the colored Higgs mass that gives successful unification in the 4D theory that is meant, rather than the lightest KK mode of the colored Higgs in the 5D theory.

The Higgs contributions to  $\delta_i^{5d}(\mu) - \delta_i^{5d}(M_{H_C}^{(4d)})$  are

$$- \frac{1}{2\pi} c_i \log \left( \frac{M_C/2}{M_{H_C}^{(4d)}} \right) + \frac{1}{2\pi} c_i \log \left( \frac{\pi}{2} \right), \quad (38)$$

where  $(c_2, c_3) = (2/5, -3/5)$ . The first term comes from the ordinary running due to  $H_u$  and  $H_d$  below  $M_C/2$ , and the second term is the finite running above  $M_C/2$  found in Eqs. (32) and (33). Since the two terms are both proportional to  $c_i$ , we see that the Higgs contribution to the differential running in 4D (which takes place entirely below  $M_{H_C}^{(4d)}$ ) is emulated in 5D with the proper choice of compactification scale. Had the differential running due to the Higgs multiplets turned off right at  $M_C/2$ , we would simply identify  $M_C/2 = M_{H_C}^{(4d)}$ . Because we find a small amount of differential running above  $M_C/2$ , we instead identify  $M_C/\pi = M_{H_C}^{(4d)}$ . We expect small additional corrections to this quantity from further finite one-loop effects, but the precise value  $M_C/\pi$  is not important: what matters is that, given any value of  $M_{H_C}^{(4d)}$  that leads to a successful prediction of  $\sin^2 \theta_W$  in the 4D theory, there is a value for  $M_C$  ( $\sim \pi M_{H_C}^{(4d)}$ ) such that the total differential running due to Higgs loops in the 5D theory will be the same as in the 4D theory.

Having chosen this value for  $M_C$ ,  $\delta_i^{5d}(\mu) - \delta_i^{5d}(M_{H_C}^{(4d)})$  arises entirely from loops of gauge and physical  $\Sigma$  fields:

$$\delta_i^{5d}(\mu) - \delta_i^{5d}(M_{H_C}^{(4d)}) = \frac{3}{2\pi} C_2(G_i) \log\left(\frac{M_C/2}{M_{H_C}^{(4d)}}\right) + \frac{1}{2\pi} C_2(G_i) \left\{ \Delta(\mu) - \theta(\mu - m_\Sigma) \log\left(\frac{\mu}{m_\Sigma}\right) \right\}, \quad (39)$$

where the first term comes from the ordinary 4D running below  $M_C/2$ , and two terms in the curly bracket represent the contributions from gauge KK towers and physical  $\Sigma$  fields, respectively (for the parameterization of the gauge contribution, see Eq. (34)). We find that this has the same form as Eq. (37), with the required identification

$$\left\{ \Delta(\mu) - \log\left(\frac{\mu}{m_\Sigma}\right) \right\}_{\mu \rightarrow \infty} = 3 \log\left(\frac{M_{\text{GUT}}}{M_C/2}\right). \quad (40)$$

Note that  $\Delta(\mu) \rightarrow \log(\mu/M)$  under  $\mu \rightarrow \infty$ , where  $M$  is some mass scale which is a function of  $M_C$  and  $g_5^2 \langle \Sigma \rangle^2$ , so that the quantity in the left-hand side has a well-defined value which is a function of  $M_C$ ,  $g_5^2 \langle \Sigma \rangle^2$  and  $m_\Sigma$ . ( $g_5^2 \langle \Sigma \rangle^2$  is the scale at which the differential running almost ceases in the 5D theory.)

We see that *the form of differential running above the Higgs triplet mass in minimal SU(5) is precisely what arises from threshold effects in the case of the five dimensional theory.* This holds irrespective of whether the step-function approximation of fields in the RG running is very good or not. For instance, had we used the full one-loop calculation as in Ref. [10], all gauge contributions would still be proportional to  $C_2(G)$ . At one loop, the structure of unification is equivalent to that of minimal SU(5). That this model can successfully yield  $\sin^2 \theta_W$  is no more nor less remarkable than in four-dimensional unified theories. The remarkable feature is that the grand unified group is broken “maximally”, at a scale well above the masses of the lightest  $X$  and  $Y$  gauge bosons. This feature changes the predictions of the theory, as we will see in sections 4 and 5.

Let us make one final comment regarding the differential running of the couplings. In a sense, its logarithmic (as opposed to power law) behavior is natural. The power law evolution is characteristic of the bulk (*i.e.*, five dimensional) nature of the gauge coupling. In contrast, the SU(5) breaking occurs strictly on a brane. Simply by Lorentz invariance, the brane breaking cannot contribute to the bulk coupling, but instead should just generate a brane contribution to the gauge kinetic term. Consequently, we would not *a priori* have expected the effects of this SU(5) breaking to exhibit a power law behavior.

### 3.4 Scales

Now that we have shown the direct correspondence between the differential running in our model to that of ordinary minimal SU(5), we can determine the scale at which successful unification occurs. In other words, given a compactification scale  $M_C$ , we can find the values of  $m_\Sigma$  and  $\langle \Sigma \rangle$  that give  $\sin^2 \theta_W$  correctly.

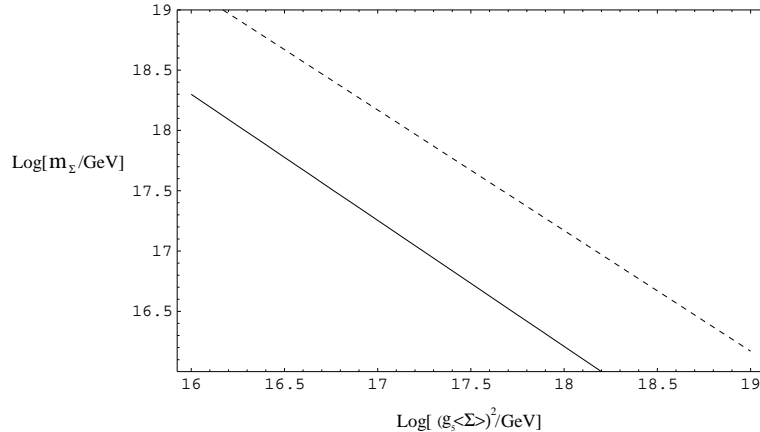


Figure 3: Values of  $M_\Sigma$  and  $g_5^2\langle\Sigma\rangle^2$  which generate the same threshold effect above  $M_C$  as is generated in minimal supersymmetric SU(5) between  $M_{H_C}$  and  $M_{\text{GUT}}$ . The dashed line corresponds to  $M_C = 6 \times 10^{14}$  GeV, while the solid line corresponds to  $M_C = 6 \times 10^{15}$  GeV. To calculate these lines we have used first-order solutions in  $(g_5^2\langle\Sigma\rangle^2 R)^{-1}$  for the  $X$  and  $Y$  masses.

The allowed compactification scales are already known: these are related to the allowed values of  $M_{H_C}$  in minimal SU(5) by  $M_C \sim \pi M_{H_C}$ . Then we need only to find the values of  $m_\Sigma$  and  $\langle\Sigma\rangle$  for which Eq. (40) holds. These values are indicated in Fig. 3 for compactification scales between  $6 \times 10^{14}$  GeV and  $6 \times 10^{15}$  GeV (corresponding to  $M_{H_C} \sim 2 \times 10^{14}$  GeV and  $2 \times 10^{15}$  GeV). For these compactification scales, the 5D Planck scale  $M_*$  is  $(1 \sim 3) \times 10^{17}$  GeV, and Fig. 3 shows that the GUT-breaking scale can be at or near this scale and the unification of three gauge couplings can be attained below  $M_*$  depending on the parameters of the model.<sup>4</sup> In the parameter region where the field theoretic unification works, the ratio of the cutoff to the compactification scales is  $O(10 - 100)$ , and no gauge or Yukawa couplings become nonperturbative below the cutoff scale. Thus, our one-loop treatment of the running is well justified. We also find that  $\langle\Sigma\rangle$  must be somewhat smaller than  $M_*$  in this parameter region. This is consistent with the observation that the theory is more or less strongly coupled at the scale  $M_*$  to have  $O(1)$  Yukawa and gauge couplings in 4D; if the theory is strongly coupled at  $M_*$ ,  $\langle\Sigma\rangle$  is naturally  $\langle\Sigma\rangle \simeq M_*/(4\pi)$ , since the superpotential giving the vev of  $\Sigma$  scales like  $M_*\Sigma^2 + 4\pi\Sigma^3$ . Even then, however, the  $\Sigma$  couples to the other fields in the combination of  $4\pi\Sigma/M_*$ , so that various operators feel order one GUT breaking, since  $4\pi\langle\Sigma\rangle/M_* \sim 1$ . Therefore, we treat as if  $\Sigma$  has a vev of order  $M_*$  in the following discussions, although all the arguments also apply in the strongly coupled case,  $\langle\Sigma\rangle \simeq M_*/(4\pi)$ .

<sup>4</sup> If  $m_\Sigma$  or  $g_5^2\langle\Sigma\rangle^2$  is above the 5D Planck scale, the gauge unification is not completed in a field theoretic regime. In this case, our field theoretic treatment is not fully trustable above  $M_*$  and would have some uncertainties coming from the cutoff scale physics.

### 3.5 Brane operators

We finally comment on effects from brane operators like  $\int d^2\theta (\Sigma/M_*)^m \mathcal{W}^\alpha \mathcal{W}_\alpha \delta(y - \pi R)$  where  $m = 1, 2, \dots$  and  $\mathcal{W}_\alpha$  is the field strength superfield. Although it is possible that these operators are somehow suppressed at the cut-off scale, from effective field theory point of view it is generically expected that they are present with order one coefficients. They give tree-level splitting of three gauge couplings for SU(3), SU(2) and U(1), and could affect the previous analysis of the gauge coupling unification. In particular, since we are considering  $\langle \Sigma \rangle \approx M_*$ , one might think that they give  $O(1)$  correction to  $\sin^2 \theta_W$ . However, we can expect that the effect of these operators are actually smaller by making the following observations.

As an example, let us first consider the extreme case where all the interactions are strongly coupled at the scale  $M_*$ . Then, the operators involving the field strength superfield scale as

$$\mathcal{L}_5 = \int d^2\theta \left[ \frac{M_*}{24\pi^3} \mathcal{W}^\alpha \mathcal{W}_\alpha + \delta(y - \pi R) \frac{1}{16\pi^2} \frac{4\pi \langle \Sigma \rangle}{M_*} \mathcal{W}^\alpha \mathcal{W}_\alpha + \dots \right], \quad (41)$$

where  $4\pi \langle \Sigma \rangle / M_*$  is an  $O(1)$  quantity. (Strictly speaking,  $4\pi \langle \Sigma \rangle / M_*$  must be somewhat smaller than 1 so that all higher dimensional operators involving  $(4\pi \langle \Sigma \rangle / M_*)^m$  do not equally contribute and make the theory unpredictable.) On integrating over  $y$ , the zero-mode 4D gauge couplings  $g_0$  are given as

$$\frac{1}{g_0^2} \sim \frac{M_* R}{12\pi^2} + \frac{1}{16\pi^2}. \quad (42)$$

Here, the first and second terms are SU(5)-preserving and SU(5)-violating contributions, respectively. Since we know that  $g_0 \sim 1$ , we have to take  $M_* R \sim 12\pi^2$  in this strongly coupled case. This shows that the SU(5)-violating contribution coming from brane operators is suppressed by a factor of  $1/(16\pi^2)$  in this case.

In fact, the theory is not truly strongly coupled at the scale  $M_*$  in the realistic case discussed in previous sections so that the one-loop treatment of gauge coupling evolutions is reliable. Nevertheless, the above argument applies more generically; the SU(5)-violating brane contribution is small relative to the SU(5)-preserving bulk contribution due to the large volume factor  $2\pi R M_*$ . Thus, the correction to  $\sin^2 \theta_W$  is expected to be small. We will not discuss possible effects of the brane operators further, and assume that they are negligible in the subsequent discussions.

## 4 Proton decay

One of the key signals of grand unification is proton decay.  $X$  and  $Y$  gauge boson exchange generates dimension six proton decay operators in the low energy theory, and Higgsino triplet exchange generates dimension five operators. One might expect that since the



particles appear at a scale  $\sim M_C$ , this will be the suppression scale of the proton decay operators. One interesting possibility offered by this framework is that *the rate of proton decay is not constrained by gauge invariance to be related to the strength of gauge and Yukawa interactions in the usual way.*

We have already noted that the dangerous particles have wave functions at the  $y = \pi R$  fixed point that are small compared with those of their 3-2-1 or doublet counterparts. On the other hand, there is a tower of states that can mediate proton decay, and we must sum each contribution. In this section, we investigate proton decay operators generated by exchanges of  $X, Y$  gauge bosons and colored Higgsinos, and show that the present model can satisfy the constraints coming from experimental lower bounds on the proton lifetime.

## 4.1 Dimension six operators

Dimension six proton decay operators arise from the exchange of  $X$  and  $Y$  gauge bosons. The coupling of these bosons to fields on the brane is suppressed by a factor  $\cos(M_n^G \pi R)$ . Comparison with four-dimensional theories can be made by replacing

$$\frac{1}{M_{X,Y}^2} \implies 2 \sum_{n=0}^{\infty} \frac{\cos^2(M_n^G \pi R)}{M_n^{G^2}}, \quad (43)$$

where  $M_n^G$  are the masses of the  $X$  and  $Y$  gauge boson KK modes. These masses satisfy Eq. (5), which allows us to approximate the sum as

$$\sum_{n=0}^{\infty} \frac{4}{g_5^4 \langle \Sigma \rangle^4 + M_C^2 (2n+1)^2} = \frac{\pi \tanh(\pi g_5^2 \langle \Sigma \rangle^2 / 2M_C)}{g_5^2 \langle \Sigma \rangle^2 M_C}. \quad (44)$$

Thus, the “effective mass” of the  $X$  and  $Y$  bosons is  $\approx g_5 \langle \Sigma \rangle M_C^{1/2} / (2\pi)^{1/2} = g_4 \langle \Sigma \rangle$ . For  $\langle \Sigma \rangle$  at or near the five dimensional Planck scale, this will typically be larger than  $M_{\text{GUT}}$ , although it is not required to satisfy experimental constraints. In any case, despite the fact that the lightest  $X$  and  $Y$  gauge bosons have masses of only  $\sim 10^{15}$  GeV, it is easy to satisfy proton decay constraints coming from dimension six operators in this model.

## 4.2 Dimension five operators

Dimension five operators come from integrating out the Higgsino triplets. Again, we make connection with the four dimensional theory by finding an effective triplet Higgsino mass by taking the whole sum over KK modes. Here we make the replacement

$$\frac{1}{M_{H_C}} \implies \sum_{n=-\infty}^{\infty} \frac{\cos^2(M_n^H \pi R)}{M_n^H}, \quad (45)$$

where  $M_n^H$  are the masses of the colored Higgs KK modes, given in Eq. (21). Using this spectrum, we obtain the sum

$$\sum_{n=-\infty}^{\infty} \left( \frac{4}{\kappa^2 + 4} \right) \frac{1}{nM_C + M_C \arctan(\kappa/2)/\pi} = \frac{8\pi}{\kappa(\kappa^2 + 4)M_C} \quad (46)$$

Thus, proton decay from dimension five operators is suppressed by a factor  $\sim 8\pi/\kappa^3$  compared to the compactification scale. In ordinary minimal SU(5), proton decay limits require  $M_{H_C} > 9.4 \times 10^{16}$  GeV, so for a compactification scale of  $M_C = 2 \times 10^{15}$  GeV, we would need a  $\kappa$  parameter of ten to adequately suppress proton decay operators. This leaves open the possibility that proton decay could be observed in future experiments. However, for larger  $\kappa$  detection becomes increasingly unlikely.

### 4.3 Derivative operators

In addition to the ordinary Yukawa couplings between the Higgs and matter fields on the brane, there can be derivative couplings of the conjugate Higgs fields to the matter fields as well.<sup>5</sup> These operators such as  $(\zeta_u T_{\mathbf{10}} T_{\mathbf{10}} \partial_y H_{\mathbf{5}}^c + \zeta_d T_{\mathbf{10}} F_{\mathbf{5}} \partial_y H_{\mathbf{5}}^c) \delta(y - \pi R)$  can lead to proton decay. Of course, coefficients  $\zeta_u$  and  $\zeta_d$  of these operators are not related to the usual Yukawa couplings,  $\lambda_u$  and  $\lambda_d$ , by SU(5), so that their actual significance is unknown. Furthermore, there is an ambiguity in what we mean by the derivative of the conjugate field, which is not differentiable at the point  $y = \pi R$ . However, using Eqs. (6) and (7), we can rewrite the coupling as

$$\delta(y - \pi R) \partial_y H_C^c = \delta(y - \pi R) \sum_n \left( M_n^H g_{C,n}(\pi R) - \frac{\kappa}{2\pi R} \sum_m g_{\bar{C},n}(\pi R) \right) \eta_{\bar{C},n}(x), \quad (47)$$

where the summation over  $m$  arises from rewriting the  $\delta$ -function as a sum of the KK mode.

While we can compute proton decay diagrams involving these vertices, it is now apparent that such diagrams have a strong dependence on how we cut off the sum of the KK mode. If the cutoff is done near the fundamental scale, these diagrams can, at least in principle, give comparable contribution to those involving the usual Yukawa couplings. However, it is not entirely clear what the cutoff for these diagrams should be. In the case where the brane is dynamical, the summation in the above operators are cut off at the scale of the brane tension [14]. For couplings at the orbifold fixed point, which is not dynamical and cannot fluctuate, it is unclear whether the cutoff is the fundamental scale or a lower scale, such as the radion mass.

For our purposes here, we will not address these issues further. Since we cannot *a priori* know the size of the couplings of these operators and their flavor structure, estimating the resulting proton decay rate is already very uncertain. However, it is possible that such

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<sup>5</sup> We thank M. Graesser for bringing this to our attention.

operators may provide an opportunity for detectable dimension five proton decay in the near future.

## 5 Yukawa couplings

So far, we have a framework that looks like ordinary SU(5) except with suppressed proton decay. While the breaking occurs at a scale near the 5D Planck scale  $M_*$ , the  $X, Y$  gauge bosons are much lighter. Still, in a very real sense, the breaking of SU(5) is “maximal”, which can manifest itself in deviations from SU(5) expectations. We here consider the effects of such maximal GUT breaking on the fermion Yukawa couplings.

In the minimal SU(5), one important prediction is the unification of the Yukawa couplings. A successful prediction of the theory is the unification of the bottom and  $\tau$  Yukawa couplings at the GUT scale [15]. However, it is well known that the SU(5) relations fail in the lighter first-two generations. For instance, an SU(5) relation  $m_e/m_\mu = m_d/m_s$  fails by a factor of ten.<sup>6</sup>

In ordinary 4D SU(5) GUT, it has been suggested that the operators involving the  $\Sigma$  field can correct this discrepancy [17]. However, this mechanism does not work in most theories where the fermion mass hierarchy is explained by the Froggatt-Nielsen mechanism [18]. In this mechanism, the matter fields carry generation dependent flavor U(1) charges and the various Yukawa couplings are generated through the U(1) breaking spurion. This is an attractive mechanism in that it not only suppresses the first-two generation Yukawa couplings but also suppresses dangerous tree-level dimension five proton decay operators. However, having employed this mechanism, the GUT-breaking operators involving  $\langle \Sigma \rangle$  can modify SU(5) mass relations only by an amount suppressed by a factors of  $M_{\text{GUT}}/M_{\text{Pl}}$ , which is too small to accommodate order one deviations suggested by the observed quark and lepton masses.<sup>7</sup>

In the present model, on the other hand, there is no suppression of higher dimensional operators involving GUT-breaking effects, since  $\langle \Sigma \rangle$  is near the cut-off scale  $M_*$ . In this sense, fields living on the GUT-breaking brane see the GUT broken maximally, and their Yukawa couplings need not respect the SU(5) symmetry. Thus, the failure of SU(5) to describe  $(m_e/m_\mu)/(m_d/m_s)$  seems quite natural. In this framework, however, the success of the  $\lambda_b$ - $\lambda_\tau$  unification must be viewed as an accident, unless there is a reason why the coupling of  $\Sigma$  to the third generation is somewhat suppressed.

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<sup>6</sup>One may be able to correct this prediction by means of contributions from supersymmetry-breaking  $A$  terms to the Yukawa couplings [16].

<sup>7</sup>One possible way to evade this conclusion is to assign a non-vanishing Froggatt-Nielsen charge to the  $\Sigma$  field [19].

## 6 Larger gauge groups

Up to this point, we have restricted our attention to a scenario with  $SU(5)$  gauge group, but it is interesting to consider larger gauge groups. One possible difference comes from additional matter fields required by larger gauge groups. For instance, in  $SO(10)$  there is an additional state, right-handed neutrino. If this additional field acquires a mass around the cut-off scale  $M_*$  from  $SO(10)$  breaking, then there will be a running effect between  $M_C$  and  $M_*$  which is not  $SO(10)$  universal.

It is, however, important that this does not affect our previous analyses; the 3-2-1 unification still works as in section 3 even in the case of  $SO(10)$ . The additional Higgs states do not contribute to differential running above  $M_C$ , and the threshold effects are proportional to 3-2-1  $\beta$ -functions. This means that we can have larger gauge groups broken all the way to 3-2-1 around the cut-off scale. In the  $SO(10)$  case, this may result in too large right-handed neutrino masses to give an appropriate mass scale for atmospheric oscillations [20] through see-saw mechanism [21]. Then, we may need somewhat small coefficient in front of the operator which gives right-handed neutrino masses or have to resort to other ways to generate neutrino masses within supersymmetric models, for instance, though  $R$ -parity violation [22] or supersymmetry breaking [23].

## 7 Conclusions

The possibility that the standard model gauge group is merely a subgroup of a larger, simple group is an attractive one. Unfortunately, the simplest version of supersymmetric  $SU(5)$  predicts proton decay at rates incompatible with experiment.

We have demonstrated that the incorporation of just one new ingredient — an additional dimension in which gauge and Higgs fields propagate — brings about crucial changes relative to ordinary GUTs. The couplings of the lightest  $X$  and  $Y$  bosons are no longer directly linked through gauge symmetry to those of the lightest 3-2-1 bosons, and the couplings of the lightest Higgs triplets are no longer directly linked to those of the Higgs doublets. The strong breaking of the GUT symmetry “pushes away” the wave functions of these states, suppressing the generated proton decay below experimental limits, all while retaining *ordinary, logarithmic unification*. Such features seem special to a five dimensional theory, incapable of reproduction in simple four dimensional theories.

The presence of extra dimensions of this size has been motivated previously as a means to resolve the supersymmetric flavor problem [24]. Our GUT-breaking picture can nicely fit into this framework of supersymmetry-breaking mediation. For instance, if supersymmetry is broken at  $y = 0$  fixed point by  $F$ -term vev of singlet or non-singlet field, it naturally realizes the scenarios of Refs. [25] and [26], respectively. The incorporation of our GUT picture within these scenarios may give interesting signatures, since we now have one more piece of information about parameters of the model; the compactification radius is determined by the gauge coupling unification. We leave an investigation of detailed phenomenology for future work.

To summarize, the framework we have described is very simple, but can describe various observed features such as gauge coupling unification, lack of unification in Yukawa couplings, the absence of proton decay, and so on. It will be interesting to add non-minimal structure to the model as a means of deriving experimental signatures.

## Acknowledgement

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