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REGGE POLE ANALYSIS OF $pn \to np$ AND $p\bar{p} \to n\bar{n}$ SCATTERING*

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ABSTRACT

The differential cross sections for the reactions $pn \to np$ and $p\bar{p} \to n\bar{n}$ have been investigated. It is found that besides the ρ and $R(A_2)$ trajectories, the π and B trajectories must be included. A variety of schemes suggested by four-dimensional symmetry have been investigated. The existence of various daughter trajectories does not suffice to explain the data, though the data can be fitted with a parity doublet, of which the pion may or may not be a member. In the former case some structure must be introduced into the pion residue function.

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INTRODUCTION

We have investigated the differential cross sections for the two charge-exchange processes (I) $pn \to np$, and (II) $p\bar{p} \to n\bar{n}$ within the framework of Regge pole phenomenology.[1] In the absence of cuts these reactions are presumed controlled by the exchange of I-spin = 1, B = 0, Y = 0 trajectories. The main features of the data which must be explained are: (a) the exceptionally sharp peak in the differential cross section of process I with a width of about $0.01(\text{GeV})^2$, (b) the fact that this sharp peak persists to very low energies and the width is almost energy independent, (c) the large difference in the magnitudes of the cross sections for processes I and II at the same value of energy and momentum transfer (for $|t| > 0.02(\text{GeV})^2$), and (d) the energy dependence of $p\bar{p} \to n\bar{n}$ data. Feature (c) can be explained only by the existence of both positive and negative G-parity trajectories which interfere with opposite signs in the two processes.

It has been known for some time that the data cannot be satisfactorily explained with only ρ and $R(A_2)$ trajectories. Even if rapidly varying residue functions are chosen so that the sharp peak of process I is fitted (and this can be done), the difference of magnitude of the two cross sections I and II cannot be explained, since the ρ and R trajectories are roughly equal over the region of interest, and having opposite signature, yield little interference. Moreover, small residues for ρ and R amplitudes are suggested by

the total cross section differences[2] ($\sigma_{\bar{p}p} - \sigma_{\bar{p}n}$ and $\sigma_{pp} - \sigma_{pn}$) which (while possessing large experimental errors) are consistent with zero in the high energy region under consideration. Since only t-channel sense-sense triplet amplitudes which do not vanish at $t = 0$ can contribute to s-channel total cross sections, in this analysis only the ρ and R contribute to these differences. It is therefore to be expected that lower-lying $I = 1$ trajectories which have not been considered in the usual analysis of data up to the present time will play a prominent role here.

Qualitatively one might expect the pion trajectory to be an important factor in determining the sharp peak of the $pn \to np$ cross section, due to the proximity of the pion pole to the forward direction. Extrapolation of the pion residue to the known pion-nucleon coupling constant indicates in fact that the pion contribution must be large near the forward direction (whether or not the pion amplitude vanishes at $t = 0$), and thus should be included in the analysis. Until recently it was assumed that the amplitude to which the pion contributes must vanish at $t = 0$ and thus it was difficult to see how the pion could give rise to a sharp peak. The recent developments in the understanding of daughter trajectories and the idea of conspiracy[3,4] have opened the possibility of at least two types of mechanisms through which the pion could cause a sharp peak in the differential cross section. The first mechanism assumes that the pion contribution does not vanish at the forward direction, in which case one has to assume the existence of a $\tau^P = (\text{even})^+$ trajectory, the other member of the pion

parity doublet. The second possibility is that the pion residue does vanish at $t = 0$, but there exists another pair of trajectories with nonvanishing residues at the forward direction, one of which will interfere with the pion to give rise to the sharp peak of the pn charge-exchange cross section. In this paper we study both of these cases in detail. In Section I we give a brief account of the formalism of $N - N$ scattering processes. In Sections II and III we discuss the above two mechanisms and present the best fits to the data under consideration. However, we would like to close this section by emphasizing the following point: Since our attempts at using the above mechanisms in the simplest and least artificial way did not succeed in fitting the data well, we proceeded to investigate successively more complicated combinations of Regge poles, or residue functions with more structure, in order to see at what degree of complication the data could be satisfactorily fitted. Our final fits turn out to contain enough artificial features so that we do not feel we have completely solved the problems of pn and $\bar{p}p$ charge-exchange scattering, but we nevertheless hope our analysis has shed some light on the problems involved in a Regge pole description of these processes.

I. FORMULAS AND PARAMETERIZATION

A. Formalism of NN Scattering[5]

We define our s and t channels as:

$$s: \quad N_1 + N_2 \rightarrow N_3 + N_4 \ ;$$

$$t: \quad N_1 + \bar{N}_3 \rightarrow \bar{N}_2 + N_4 \ .$$

To order $\frac{1}{z_t}$, the Reggeized t-channel amplitudes are given by:

$$\phi_1 = \sum_i \frac{1}{1-t/4m^2} (1 + \alpha_i) \frac{1 \pm e^{-i\pi\alpha_i}}{\sin \pi\alpha_i} (\gamma_{11}{}^i + \gamma_0{}^i) \left(\frac{s}{s_0}\right)^{\alpha_i} ,$$

$$\phi_2 = \sum_i \frac{1}{1-t/4m^2} (1 + \alpha_i) \frac{1 \pm e^{-i\pi\alpha_i}}{\sin \pi\alpha_i} (\gamma_{11}{}^i - \gamma_0{}^i) \left(\frac{s}{s_0}\right)^{\alpha_i} ,$$

$$\phi_3 = \sum_i \frac{1}{1-t/4m^2} (1 + \alpha_i) \frac{1 \pm e^{-i\pi\alpha_i}}{\sin \pi\alpha_i} \left\{\alpha_i + \frac{1}{z_t}\right\} (\gamma_1{}^i + \gamma_{22}{}^i) \left(\frac{s}{s_0}\right)^{\alpha_i} ,$$

$$\phi_4 = \sum_i \frac{1}{1-t/4m^2} (1 + \alpha_i) \frac{1 \pm e^{-i\pi\alpha_i}}{\sin \pi\alpha_i} \left\{-\alpha_i + \frac{1}{z_t}\right\} (-\gamma_1{}^i + \gamma_{22}{}^i) \left(\frac{s}{s_0}\right)^{\alpha_i} ,$$

$$\phi_5 = \sum_i \frac{2m^2}{s_0} \sqrt{t} \sin \theta_t (1 + \alpha_i) \frac{1 \pm e^{-i\pi\alpha_i}}{\sin \pi\alpha_i} \sqrt{\alpha_i} \gamma_{12}{}^i \left(\frac{s}{s_0}\right)^{\alpha_i - 1} \quad (1)$$

where $z_t = -[1 + 2s/(t - 4m^2)]$, s_0 is a normalization factor which we choose to be 1 $(\text{GeV})^2$, and m is the mass of the nucleon. (In fitting the data, we actually used the more exact form $(s + t/2 - 2m^2)^{\alpha}$ instead of s^{α}). The γ^i are reduced residue functions, but they

may contain zeros at $\alpha_i = 0$ or $t = 0$ depending on the choice of different ghost killing mechanisms[6] or different coupling schemes at the forward direction. When we take out appropriate factors of α_i and t (denoted by $G(\alpha)$ and $\eta(t)$ respectively), we denote the remaining functions as $b(t)$, which we parameterize by exponentials (in a few cases multiplied by a linear polynomial in t). In a few instances when the trajectories went near $\alpha \approx -2$, $G(\alpha)$ included a factor $(\alpha + 2)$. In general, then, we write

$$\gamma^i(t) = G^i(\alpha_i)\, \eta^i(t)\, b^i(t) \qquad (2)$$

where the factors $G^i(\alpha_i)$ and $\eta^i(t)$ will be defined later for each case.

Factorization puts the following constraint on the triplet amplitudes,

$$\gamma_{11}{}^i\, \gamma_{22}{}^i = t(\gamma_{12}{}^i)^2 , \qquad (3)$$

so that either $\gamma_{11}{}^i$ or $\gamma_{22}{}^i$ is proportional to t (i.e., either $\eta_{11}{}^i = t$ or $\eta_{22}{}^i = t$). The ratio $\gamma_{12}{}^i/\gamma_{11}{}^i$ is determined from meson-nucleon scattering.[6]

At $t = 0$ the following additional equation must be satisfied

$$\phi_1(t = 0) - \phi_2(t = 0) = \phi_3(t = 0) - \phi_4(t = 0) . \qquad (4)$$

Since the crossing matrix is orthogonal, the s-channel cross section is given by

$$\frac{d\sigma}{dt} = \frac{1}{2\pi\, s(s - 4m^2)} \left(\sum_{i=1}^{4} |\phi_i|^2 + 4\,|\phi_5|^2 \right). \qquad (5)$$

Equation (1) shows that poles in γ_{11} and γ_0 do not interfere at all, and that there is no interference between poles in γ_1 and γ_{22} to leading order in s. Therefore it is convenient to define a set of amplitudes in the asymptotic region which show this effect explicitly. Let

$$K(s,t) = \frac{\sqrt{2}}{1 - t/4m^2}\, [2\pi\, s(s - 4m^2)]^{-1/2}. \qquad (6)$$

Define a set of amplitudes, g_0, g_1, g_{11}, g_{12}, and g_{22} by

$$g_0 = \sum_i K(s,t)\,(\alpha_i + 1)\, \frac{1 \pm e^{-i\pi\alpha_i}}{\sin \pi\alpha_i}\, \gamma_0{}^i \left(\frac{s}{s_0}\right)^{\alpha_i}$$

$$g_1 = \sum_i K(s,t)\,\alpha_i(\alpha_i + 1)\, \frac{1 \pm e^{-i\pi\alpha_i}}{\sin \pi\alpha_i}\, \gamma_1{}^i \left(\frac{s}{s_0}\right)^{\alpha_i}$$

$$g_{11} = \sum_i K(s,t)\,(\alpha_i + 1)\, \frac{1 \pm e^{-i\pi\alpha_i}}{\sin \pi\alpha_i}\, \gamma_{11}{}^i \left(\frac{s}{s_0}\right)^{\alpha_i} \qquad (7)$$

$$g_{22} = \sum_i K(s,t)\,\alpha_i\,(\alpha_i + 1)\, \frac{1 \pm e^{-i\pi\alpha_i}}{\sin \pi\alpha_i}\, \gamma_{22}{}^i \left(\frac{s}{s_0}\right)^{\alpha_i}$$

$$g_{12} = \sum_i K(s,t)\, \frac{4m^2 - t}{\sqrt{2}\, s_0} \sqrt{t}\, \sin\theta_t\, (\alpha_i + 1)\sqrt{\alpha_i}\, \frac{1 \pm e^{-i\pi\alpha_i}}{\sin \pi\alpha_i}\, \gamma_{12}{}^i \left(\frac{s}{s_0}\right)^{\alpha_i - 1}$$

The cross section is simply given by

$$\frac{d\sigma}{dt} = |g_0|^2 + |g_1|^2 + |g_{11}|^2 + |g_{22}|^2 + |g_{12}|^2 . \qquad (8)$$

The quantum numbers of poles contributing to each amplitude are given in Table 1. With this set of amplitudes Eq. (4) reads

$$g_0(t = 0) = \frac{2m^2}{s - 2m^2} \frac{1}{\alpha_1(0)} g_1(t = 0) + g_{22}(t = 0) \qquad (9)$$

where α_1 refers to the trajectory contributing to g_1. The equation as it stands can be satisfied in many different ways but the recent studies of four-dimensional symmetry restrict us to essentially two conspiracy schemes.

The clearest discussion of the significance of Eq. (9) has been given by M. Toller.[4] Let τ = signature, $\sigma = \tau P$ (P = parity), and c = charge conjugation. Without considerations of four-dimensional symmetry, each trajectory is classified by the three numbers τ, σ, and c. In addition to this, however, one can classify families of trajectories that couple to the $N\bar{N}$ system at $t = 0$ by the Lorentz quantum number M. Let us also introduce the quantity n ($n = 0, 1, 2, \cdots$) which denotes the position of the trajectory in the daughter sequence. There are three ways that sequences of trajectories can couple to the $N\bar{N}$ system at the point $t = 0$. These schemes are illustrated in Fig. 1.

(a) $M = 0, \quad \sigma = 1, \quad \tau = c$

This corresponds to a family of trajectories contributing only to g_{11} at $t = 0$, and therefore not involved in the constraint of Eq. (9). Note that only the even members ($n = 0, 2, \cdots$) of such a family contribute and the odd members ($\sigma = 1, \tau = -c$) do not couple to the $N\bar{N}$ system at all. The ρ and A_2 trajectories presumably define such sequences.

(b) $M = 0, \quad \sigma = -1, \quad \tau = (-1)^{n+1} c$

This corresponds to a family of trajectories, the even members of which ($\tau = -c$) contribute to g_1, with the odd members contributing to g_0. If we denote the parent by A and the first daughter by d, and also label the contribution of each pole to a given amplitude by a superscript, then to the highest order in s, Eq. (9) becomes

$$g_0^{d}(t = 0) = \frac{2m^2}{s\ \alpha_A(0)}\ g_1^{A}(t = 0), \tag{10}$$

with similar equations for other values of n. In terms of the trajectories and residue functions we have

$$\alpha_d(0) = \alpha_A(0) - 1 \tag{11a}$$

and

$$\alpha_A(0)\ \gamma_0^{d}(0) = -\frac{2m^2}{s_0}\ \left(\alpha_A(0) + 1\right) \gamma_1^{A}(0). \tag{11b}$$

Toller shows these to be automatic consequences of Lorentz symmetry.

(c) $M = 1$, $\tau = (-1)^n c$, $\sigma = \pm 1$

This case corresponds to a sequence of trajectories which occur in pairs (parity doublets). The even members have $\tau = c$ and $\sigma = \pm 1$, and contribute to g_0 ($\sigma = -1$) and g_{22} ($\sigma = +1$). When n is odd ($\tau = -c$), however, the $\sigma = +1$ trajectories do not couple to the $N\bar{N}$ system and we have trajectories with $\tau = -c$ and $\sigma = -1$ contributing to g_1. Clearly Eq. (9) is satisfied by groups of trajectories such that the pair contributing to g_0 and g_{22} have the same intercept and the trajectory contributing to g_1 lies one unit higher. Especially, if we denote the first pair ($n = 0$) by d and d' we have

$$g_0^{\,d}(t = 0) = g_{22}^{\,d'}(t = 0) . \tag{12}$$

In terms of the trajectories and residues we have

$$\alpha_d(0) = \alpha_{d'}(0) \tag{13a}$$

and

$$\gamma_0^{\,d}(0) = \alpha_d(0)\, \gamma_{22}^{\,d'}(0). \tag{13b}$$

B. The data and parametrization

The available data for the process $pn \to np$ are at 8 GeV/c for $|t| < 0.5(\text{GeV})^2$ and at lower energies. To make sure that our models are capable of producing the sharp peak for relatively low energies, we have included a set of data at 3 GeV/c in our analysis.

The available data for $p\bar{p} \to n\bar{n}$ are at 5, 6, 7, and 9 GeV/c and $|t| \lesssim 1.3$ $(\text{GeV})^2$. Since this last set of data (especially that at 9 GeV/c) seems to show some structure for $|t| > 0.5$ $(\text{GeV})^2$ which would be difficult to fit with out parameterization, and for the sake of consistency with the pn data, we have only included the data up to $|t| = 0.5$ $(\text{GeV})^2$ in most of our analysis. In general we found it difficult to fit the magnitudes of the data at different energies or for different experiments exactly. However, the data have systematic normalization errors of 30-45% for the 8 GeV/c pn data and 15% for the $p\bar{p}$ data, which are presumably independent of energy and momentum transfer for a given experiment and are not included in the errors used in a χ^2 analysis. Therefore, we have accepted fits which disagree with experiment by overall normalization factors not greater than 25% for 8 GeV/c pn data or 15% for the $p\bar{p}$ cross sections at different energies. We have assumed straight-line trajectories constrained to go through the masses of the corresponding particles when such particles are known. For ρ and R we use the trajectory functions found in previous meson-nucleon fits.[6] Furthermore, the ratios $\gamma_{12}^{i}/\gamma_{11}^{i}$ for ρ and R are given by these fits. However, none of our fits are very sensitive to these ratios, so we do not feel that these constraints provide a good test of factorization. Nucleon-nucleon factorization Eq. (3) is always satisfied by our triplet amplitudes. The magnitude of the pion residue is constrained by Eq. (14)

$$\frac{g^2}{4\pi} = \frac{4m^2 \gamma_0^{\pi}(t = m_\pi^2)}{(4m^2 - m_\pi^2)\pi^2 m_\pi^2 \alpha'_\pi} \approx 14. \qquad (14)$$

We did not accept fits which predicted a value of $g^2/4\pi$ less than 11. Collectively, the constraints mentioned in this section reduce the number of free parameters of all the models discussed here to 17 or fewer.

II. PION PARITY-DOUBLET FITS

In this section we consider fits with the pion amplitude not vanishing at $t = 0$, and thus associated with a parity doublet partner, denoted by π', in the coupled triplet amplitudes. Such a fit was attempted by Frazer and Phillips[7] who encountered the difficulty of obtaining consistency with the known value of $\frac{g^2}{4\pi}$ and a slowly varying residue function. We have found that this difficulty can be overcome by using a parameterization $\gamma_0^{\pi}(t) = (1 - \frac{t}{t_0})\, c\, e^{dt}$. The factor $(1 - \frac{t}{t_0})$ is varied to obtain the correct coupling constant.

The existence of the zero in the pion residue can perhaps be made plausible by the following heuristic argument. If the pion Regge pole (along with the parity doublet partner π') is derived from a pion Lorentz pole (denoted π^L) in the Laplace transform of the nucleon-nucleon scattering amplitude, the Lorentz M-quantum number of π^L is $M = 1$. Now the physical pion π has very small mass. If it had zero mass (i.e., if the pion trajectory were somehow perturbed to pass through the origin, forming a new trajectory π_0), the resulting Lorentz pole π_0^L would have to be classified as $M = 0$ if it were classified at all, since the pion has spin zero. The classification $M = 0$, however, implies the existence of a $J^{PG} = 1^{+-}$ meson with $\alpha(0) = 1$, which is not observed. Hence the π_0 pion is assumed to decouple from the $N\bar{N}$ amplitude (i.e., its residue is proportional to t), the Laplace transform has no pole, and there is therefore no pion classification. This picture can be made consistent

in a natural way if the residue for the physical pion trajectory has a zero which moves to t=0 as the π trajectory is perturbed to pass through the origin, thus decoupling π_0^L from the $N\bar{N}$ channel. We need to assume also that the pion π_0^L decouples from all channels at $t = 0$ so that it is never classified as $M = 0$. Then the π^L pole will be classified as $M = 1$, but the zero-mass trajectory π_0^L will not couple to any channel and so in particular will not be classified with the inconsistent value $M = 0$.[8]

Since the zero of the actual π trajectory differs in position from that of the hypothetical π_0 trajectory by a displacement $\Delta t = m_\pi^2$, it is plausible that the position of the zero in the π residue function may be displaced from $t = 0$ by an amount of the order of m_π^2. If in fact $|\beta^\pi(t) - \beta^{\pi_0}(t)| < |\beta^{\pi_0}(t)|$ on some circle (say for example at $t = m_\pi^2$) where β^{π_0} and β^π are the residues associated with the π_0 and π trajectories respectively, then since β^{π_0} is proportional to t, Rouche's Theorem[9] guarantees the existence of a zero in $\beta^\pi(t)$ somewhere inside the circle (assuming β^{π_0} and β^π are analytic within the circle).

A. Parameterization and Description of the Fit

We now return to the discussion of the fit. Besides the ρ and R, we also have included the B trajectory, assuming $J^{PG} = 1^{++}$ for the B meson. The B amplitude changes sign in processes I and II and thus helps to account for the difference in np and $p\bar{p}$ cross-section magnitudes. The necessary interference therefore is

provided by interference between ρ, R, and π' in the amplitude ϕ_5, and to a lesser extent between π and B. The other amplitudes did not interfere as much due to the fact that the ρ and R cannot conspire (so $\eta_{22}^{\rho} = \eta_{22}^{R} = t$ and $\eta_{11}^{\rho} = \eta_{11}^{R} = 1$) while the π' conspires (so $\eta_{11}^{\pi'} = t$ and $\eta_{22}^{\pi'} = 1$). Thus, while $g_{22}^{\pi'}$ was large, g_{22}^{ρ} and g_{22}^{R} were small, providing little interference. (The ρ and R do not conspire, since experimentally both are known to contribute to total meson-nucleon cross sections). We further assume that the B trajectory is uncoupled at $t = 0$ (so $\eta_0^B(t) = t$). Thus five trajectories were used in this analysis with fourteen free parameters. The additional freedom introduced by the choice of various ghost-killing mechanisms was also investigated; the results are discussed in the Appendix.

The best fit was obtained with $\chi^2 = 93$ for 74 points (without the 5 GeV/c $p\bar{p}$ data). In this fit, the π' was taken to choose nonsense at $\alpha_{\pi'} = 0$, and thus not associated with any 0^+ particle. The Gell-Mann mechanism was chosen for R and the Chew mechanism for ρ. As mentioned before, we cannot fit the different normalizations exactly, so that the curves presented in Figs. 2 and 3 are the calculated curves multiplied by factors 1.0 and 0.75 for the 3 and 8 GeV/c pn data respectively, and by a factor 1.15 for the 6, 7 and 9 GeV/c $p\bar{p}$ data.[10] The 5 GeV/c data needed a different normalization factor (≈ 0.9) which indicated that while we fit the shape of the data at this energy, our model cannot fit the magnitude of the 5 GeV/c data to better than 25% if the normalization uncertainties

are indeed energy independent. The results of the fit are shown in Fig. 2 and 3, and the parameters in Table II. These parameters, however, should not be taken too seriously. The B-trajectory intercept is not well determined, though all fits of this section indicated that $-1.0 \leq \alpha_B(0) \leq -0.1$. Further, fits with the slope of the π' ranging from 0.03 to 1 could be obtained. The exponent for the pion residue turned out to be rather large ($\approx 11\ \mathrm{GeV}^{-2}$). We do not consider this an essential deficiency of this fit, however, since all exponents could be lowered to $\leq 5(\mathrm{GeV})^{-2}$ (consistent with $\frac{g^2}{4\pi} \approx 11$), with an increase of about 10% in χ^2. Better fits could be obtained with higher exponentials, however. The data were relatively insensitive to the precise position of t_0. In fact fits could be obtained by removing the zero entirely while holding the exponentials fixed (thus inconsistent with $\frac{g^2}{4\pi}$). However, t_0 could not become too close to 0 without spoiling the fit for larger values of t, since $(1 - \frac{t}{t_0})$ for small t_0 and moderate t is large. In fact, the high exponential is needed partially to damp this factor for moderate t.

Finally, we remark on the predicted structure near the forward direction of the $p\bar{p}$ data. The data have not been measured close enough to the forward direction to test this feature of our models (the model discussed in Section III also predicts such a structure). This structure, which is not predicted for the 8-GeV/c pn cross section by our models, is due to the different interference of the Regge pole terms in processes I and II and the fact that all the amplitudes which are responsible for the difference of magnitudes of pn and $p\bar{p}$ data vanish at the forward direction. However, from a group theoretical point of view there is no reason for the B-meson to vanish at $t = 0$, and in fact it might be more natural to assume that it is a member of a parity doublet or the daughter of another trajectory. It is clear that by assuming this we could improve our fit, but since the fit is already statistically good and we would have to conjecture another trajectory without experimental support, we feel that any further improvement of this fit along these lines is at the present time meaningless. However, the existence of such a trajectory will be indicated if experiments demonstrate that the large difference in magnitude of pn and $p\bar{p}$ cross sections persists to very small t, since without it the models predict near equality of the pn and $p\bar{p}$ cross sections at $t = 0$. It is interesting to note that if the residue of the B trajectory does not vanish at $t = 0$ and B belongs to a parity doublet, the other member of this doublet will have the quantum numbers of a trajectory like the ρ', the existence of which has already been suggested by many authors.

III. CONSPIRACY PLUS INTERFERENCE

In this section we will assume that the pion contribution vanishes at the forward direction and seek to explain the sharp peak of the $pn \to np$ cross section by the interference of another trajectory in g_0 with the pion. Since we will have to conjecture the existence of some trajectories which have not been established experimentally, and since it is clear that by conjecturing a sufficient number of them we can fit the data, we need some a priori rule as to the number of Regge trajectories we will use in this analysis. Besides the ρ, R, and the pion, we need two other trajectories, one in g_0 to interfere with the pion and the other in g_1 or g_{22} in order to satisfy Eq. (9). Since in general the interference of ρ with other trajectories is not large enough to account for the difference of magnitude between cross sections I and II, we will need another positive G-parity trajectory, and as mentioned in Section II, the B meson (1^{+-}) seems to be a good candidate. Thus at least six trajectories are needed for the analysis of this section.

There are essentially two conspiracy schemes to be considered: (a) The trajectory in g_0 (denoted by d) which interferes with the pion is the daughter of a trajectory in g_1 (denoted by A). The two possible quantum numbers for A are $(\text{signature})^{PG} = (\text{odd})^{+-}$ or $(\text{even})^{-+}$ with the two corresponding daughters $(\text{even})^{--}$ and $(\text{odd})^{++}$. A possible candidate for the first set is the A_1 and its daughter. The daughter trajectory lies one unit below the parent at $t = 0$ and therefore Eq. (10), which has a factor of s in the denominator of the right-hand side, can be satisfied.

It is clear from the outset that if g_0^d interferes with the pion contribution g_0^π to give a sharp peak, it should be large and constitute a considerable portion of the cross section at $t = 0$. Therefore we see that $|g_1^A(0)|^2$ cannot be much larger than $|g_0^d(0)|^2$ since at the forward direction the cross section is equal to $|g_1^A(0)|^2 + |g_0^d(0)|^2$ plus other positive terms. Assuming that $|g_1^A| = |g_0^d|$, we immediately obtain an upper bound for the absolute value of $\alpha_A(0)$. For the incident laboratory momentum of about 10 GeV/c Eq.(10) yields

$$|\alpha_A(0)| \lesssim 0.1.$$

It would then seem that d will lie about one unit below the pion and the interference could not be as energy independent as indicated by the data. Our numerical analysis showed that in fact this was not a serious difficulty, but the scheme failed to fit the data for reasons which can be described in the following way. We can write the ratio of the imaginary to the real part of g_0^d as

$$\frac{|\mathrm{Im}\, g_0^d(0)|}{|\mathrm{Re}\, g_0^d(0)|} = \left| \frac{\sin \pi\alpha_d(0)}{1 \mp \cos \pi\alpha_d(0)} \right| = \left| \frac{\sin \pi_A(0)}{1 \mp \cos \pi\alpha_A(0)} \right|$$

where in the last expression $-$ or $+$ refers to the signature of A being odd or even. For an odd-signature parent (like A_1) with $|\alpha_A(0)| \lesssim 0.1$ this ratio is large and g_0^d is almost imaginary. The pion, however, is almost real near the forward direction and interference

is impossible. For even-signature parents, the ratio is small so that we can get large enough interference. However, the daughter trajectory now has positive G parity and its contribution changes sign as we go from $pn \to np$ to $p\bar{p} \to n\bar{n}$, so that if in one case it interferes with the pion to give a sharp peak, it causes a large enhancement close to the forward direction in the other process. Thus, this scheme could not fit the two sets of data simultaneously. The only way to overcome all of these difficulties was to assume a rapidly varying residue function ($\approx e^{20t}$) for the pion so as to make its contribution very small (with $g^2/4\pi$ fixed). By choosing another rapidly varying residue for g_1^A, it was then possible to get a satisfactory fit to the data. However, we consider this a highly artificial fit (since it is the high A exponential and not the pion which is producing the sharp pn peak) and we do not present it in this paper.

(b) The second possible scheme is to assume the existence of a parity doublet (in addition to the π, which has no partner), which we will denote by d and d', contributing to g_0 and g_{22} respectively. The two possible sets of quantum numbers are

1) $d = (\text{even})^{--}$, $d' = (\text{even})^{+-}$ and 2) $d = (\text{odd})^{++}$, $d' = (\text{odd})^{-+}$.

The second set possesses the same difficulty discussed in the previous part due to the fact that d has positive G parity, but the first set has none of the problems discussed up to this point and we have proceeded to use it in fitting the experimental data. Of course since we have had to assume the existence of a pair of trajectories

with no experimental evidence we cannot claim this to be the correct explanation of the data. We present the fit here as an indication of how this mechanism of conspiracy plus interference may explain the data under consideration.

The best fit corresponded to the parameters presented in Table III. The calculated curves are very similar to those presented in Figs. 2 and 3. In this fit it was assumed that at $\alpha = 0$ the ρ and R both choose nonsense (i.e., the Gell-Mann mechanism), π and B choose sense, and the parity doublet d and d' also choose nonsense. Thus we assume the absence of the 0^- and 0^+ particles corresponding to the d and d' trajectories; their first particles therefore presumably appear at $J = 2$. The total number of free parameters in the fit was 17, and the χ^2 value obtained was 90 for 74 points. This χ^2 corresponds to normalization factors of 0.8 for the 8 GeV/c pn data and no normalization factors for the rest of the data included. Again, the normalization of 5 GeV/c data could not be fitted to better than 25% and these data were not included in the 74 points under consideration. The remarks made in Section II concerning the reliability of the parameters and the structure of the curves also hold with this model.

It should be noted, however, that this model does not involve residue functions with structure and in this sense it may have some advantages over the model described in Section II.

SUMMARY AND CONCLUSION

We have presented certain fits to the pn and $p\bar{p}$ charge-exchange cross sections at different energies. We have argued that the energies involved are not high enough for the leading trajectories to dominate and that the known value of the pion-nucleon coupling constant forces us to include the pion in this analysis, since there are no obvious reasons which would allow us to neglect its contribution. We have then argued that with the pion intercept near zero, other trajectories lying lower than the usual set included in Regge pole phenomenology could play an important role in this energy region, and we have shown how they could be responsible for the special features of the data under consideration. Our results are also consistent with the difference of total cross sections $(\sigma_{\bar{p}p} - \sigma_{\bar{p}n}, \sigma_{pp} - \sigma_{pn})$, but we have not shown any total cross section fits in this paper.

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We wish to thank Professor Geoffrey Chew for his helpful advice and constant encouragement.

APPENDIX: GHOST-KILLING MECHANISMS

A. The ρ and R trajectories

In this appendix we describe fits possessing different ghost-killing mechanisms. All these fits assume the existence of the pion parity doublet described in Section II. The actual details of the mechanisms are described in part C below. These were investigated in the hope of obtaining better nucleon-nucleon fits as well as of distinguishing between the various mechanisms experimentally. Earlier meson-nucleon fits[6] were ambiguous and were unable to resolve these alternatives. We find that many of these same ambiguities persist in the nucleon-nucleon case. In particular, fits were obtained choosing the Chew mechanism for the R trajectory. Two cases were distinguished in the meson-nucleon fits corresponding to a flat R trajectory $(\alpha_R(t) = 0.44 + 0.5t)$ and a steeper R trajectory $(\alpha_R(t) = 0.5 + 0.85t)$, where zeros had to be placed in the non-spin-flip ρ and R residues in the latter case to fit the meson-nucleon data.[6] Thus, for the steep-R fit, double zeros were placed in γ_{11}^{R} and γ_{11}^{ρ}. Fits with $\chi^2 = 88$ and $\chi^2 = 89$ were obtained for 74 data points for the two case respectively. These fits assumed the Chew mechanism for the ρ.

The final case (Gell-Mann mechanism for both ρ and R) was not investigated here due to the background term required by the $\pi^- p \to \pi^0 n$ data. This mechanism was used in the fits described in Section III, however. Questions of the third double spectral function and fixed poles producing a pole in the nonsense-nonsense ρ residue at $\alpha_\rho = 0$ also were not investigated.

B. The pion parity doublet partner π'

The fits described above were all obtained with the π' choosing nonsense at $\alpha_{\pi'} = 0$ (so $G_{11}^{\pi'} = \alpha_{\pi'}$, $G_{22}^{\pi'} = 1$). We here describe the reasoning ruling out sense coupling for π'. We must assume, for sense coupling, that the slope of the π' trajectory is very flat, since no low-mass $J^P = 0^+$ particles are observed. We may then associate the π' trajectory with the recently observed $I = 1, J^{PG} = 0^{+-}$ particle at 1 BeV if $\alpha'_{\pi'} \approx 0.03$. A fit to leading order in z_t was in fact obtained with $\alpha'_{\pi'} = 0.03$. However, the $\frac{1}{z_t}$ term in the amplitudes ϕ_3 and ϕ_4 [which are proportional to $\left(\pm \alpha + \frac{1}{z_t} + 0\left(\frac{1}{z_t^2}\right)\right)$] actually dominates the "leading term" at these energies, in the forward direction (the next term $0\left(\frac{1}{z_t^2}\right)$ is smaller than either). When this $\frac{1}{z_t}$ term was included, the contribution of the π' nonsense-nonsense amplitude increased to the point where Eq. (9) and the $\frac{g^2}{4\pi}$ constraint were no longer compatible. An attempt was made to utilize the second-order interference between the uncoupled triplet amplitude and this $\frac{1}{z_t}$ term by assigning

the B trajectory to the possible B-meson quantum numbers $J^P = 2^-$, assuming that this amplitude is nonvanishing at $t = 0$, but this device failed to remove the difficulty.

C. Details of the mechanisms

The various ghost-killing mechanisms for coupled triplet amplitudes result from the existence of different ways of satisfying analyticity in ℓ and factorization. The d_{10}^{α} function appearing in the sense-nonsense amplitude ϕ_5 leads to a factor $(\alpha)^{\frac{1}{2}}$ in that amplitude. Hence the reduced residue function γ_{12} must contain a factor $(\alpha)^{\frac{1}{2}}$ so that ϕ_5 will not have a branch point at $\alpha = 0$ (γ_{12} may of course have additional factors of α as well). By factorization γ_{12} may be written $(t)^{\frac{1}{2}}\gamma_{12} = \xi_S \xi_N$, so that either $\xi_S \propto (\alpha)^{\frac{1}{2}}$ or $\xi_N \propto (\alpha)^{\frac{1}{2}}$. For odd signatured trajectories the first case yields $\gamma_{11} = \xi_S^2 \propto \alpha$ and $\gamma_{22} = \xi_N^2 \propto 1$ (the Gell-Mann mechanism or nonsense coupling) whereas in the second case $\gamma_{11} \propto 1$ and $\gamma_{22} \propto \alpha$ (the Chew mechanism). For even-signatured trajectories with the Gell-Mann mechanism, the higher-order terms in $\frac{1}{z_t}$ in the 22-coupled triplet amplitude are singular at t_1, where $\alpha(t_1) = 0$. The cancellation of these terms is effected by a trajectory (the "compensating trajectory") in the uncoupled triplet amplitude, having $\alpha(t_1) = -1$.[11] (These trajectories were never included in the analyses, except insofar as the higher-order terms in $\frac{1}{z_t}$ were omitted. However (for example) it should be noticed that, with a slope of 1, the R-compensating trajectory intercept is $\alpha(0) = -0.4$, which is

comparable to intercepts of other trajectories used in the analyses. The compensating trajectory π'_c for the π' has an intercept below -1, so it is safe to neglect the π'_c trajectory).

For even-signatured trajectories with the Chew mechanism, the singularities at $\alpha = 0$ are cancelled by taking $\xi_S \propto (\alpha)^{\frac{1}{2}}$ and $\xi_N \propto \alpha$

If an even-signatured trajectory crosses the $\alpha = 0$ axis at $t_1 > 0$, we have two possibilities: either the trajectory is associated with a spin 0 particle (and so has "sense coupling") or else all five helicity amplitudes are nonsingular at $\alpha = 0$. The latter case can be associated with either the Chew or Gell-Mann mechanism for even signature described above. (In these fits the Gell-Mann mechanism for the π' and the d' trajectories was assumed). If, on the other hand, the trajectory is associated with a spin 0 particle, the sense-sense amplitude must have a pole at $\alpha = 0$, whereas the other amplitudes (sense-nonsense, nonsense-nonsense) are nonsingular. This is accomplished by setting $\xi_S \propto 1$ and $\xi_N \propto (\alpha)^{\frac{1}{2}}$.

TABLE I. Quantum numbers of isospin = 1 trajectories contributing to different amplitudes

Amplitudes	$(\text{signature})^{PG}$	Examples
g_{11}, g_{12}, g_{22}	$(\text{odd})^{-+}$	ρ
	$(\text{even})^{+-}$	A_2, 0^{+-}(?)
g_0	$(\text{odd})^{++}$	B (?)
	$(\text{even})^{--}$	π, A_1 daughter(?)
g_1	$(\text{odd})^{+-}$	A_1 (?)
	$(\text{even})^{-+}$	2^{-+}(?)

TABLE II. Parameters for the fit described in Sec. II. The signs of the residues correspond to the process $p\bar{p} \to n\bar{n}$.

$\alpha_\rho = 0.58 + 1.11t$ (a)	$\alpha_R = 0.5 + 0.86t$ (a)
$\alpha_\pi = -0.025 + 1.25t$	$\alpha_{\pi'} = -0.025 + 1.01t$
$\alpha_B = -0.9 + 1.25t$	
$\frac{b_{12}^{\rho}}{b_{11}^{\rho}} = 8.8\, e^{0.4t}$ (a)	$\frac{b_{12}^{R}}{b_{11}^{R}} = 3.5\, e^{-0.11t}$ (a)
$\gamma_{11}^{\rho} = 0.5\, e^{-3.7t}$	$\gamma_{11}^{R} = 1.8\, e^{10.5t}$ (b)
$\gamma_0^{B} = -955\, e^{2.2t}\, t(\alpha_B + 2)$	$\gamma_0^{\pi} = 0.934\,(1+t/0.013)e^{11t}$ (c)
$\gamma_{22}^{\pi'} = \frac{b_0^{\pi}(0)}{\alpha_\pi(0)}\, e^{5.5t}$	$\gamma_{12}^{\pi'} = -(\alpha_{\pi'})^{\frac{1}{2}}\, 60\, e^{2.2t}$

a. Parameters fixed from meson-nucleon fits.

b. Lowering all exponentials to $\leq 5(\text{GeV})^{-2}$ raised the χ^2 by 10%.

c. Corresponds to $g^2/4\pi = 12.1$.

TABLE III. Parameters for the fit described in Sec. III. The signs of the residues correspond to the process $p\bar{p} \to n\bar{n}$.

$\alpha_\rho = 0.58 + 0.9t$ (a)	$\alpha_R = 0.48 + 0.9t$ (a)
$\alpha_\pi = -0.022 + 1.1t$	$\alpha_B = 0.2 + 0.54t$
$\alpha_d = -0.18 + 1.2t$	$\alpha_{d'} = -0.18 + 1.4t$
$b_{12}^{\rho}/b_{11}^{\rho} \approx -5$ (a)	$b_{12}^{R}/b_{11}^{R} \approx -3$ (a)
$\gamma_{11}^{\rho} = 0.8\,\alpha_\rho\, e^{5t}$	$\gamma_{11}^{R} = -0.26\,\alpha_R\, e^{-3t}$
$\gamma_0^{\pi} = 116.5\,t\, e^{6.2t}$ (b)	$\gamma_0^{B} = 83\,t\, e^{-3t}$
$\gamma_0^{d} = -36.5\,\alpha_d(\alpha_d+2)\, e^{4t}$	$\gamma_{22}^{d'} = -36.5\,(\alpha_{d'}+2)\, e^{2.5t}$
	$\gamma_{12}^{d'} = -341\,(\alpha_{d'})^{\frac{1}{2}}\, e^{4t}$

a. Parameters fixed from meson-nucleon fits.

b. Corresponding to a $g^2/4\pi \approx 12$.

FOOTNOTES AND REFERENCES

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completion of this paper, a preprint from R. J. N. Phillips was received in which conclusions similar to ours were reached.

8. We thank Dr. Jerome Finkelstein for a discussion on this point. There exist, however, heuristic arguments using PCAC, against an $M = 1$ pion assignment. For example see Freedman and Wang (preprint).

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10. Changing the $p\bar{p}$ normalization factors to 1.0 increased the χ^2 by about 10%.

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FIGURE CAPTIONS

1. Different coupling schemes at $t = 0$.
2. $p\bar{p} \to n\bar{n}$ pion parity doublet fits for 6, 7, and 9 GeV/c, and predictions for 5 GeV/c. The calculations have been multiplied by 1.15 and 1.0, respectively. Dotted lines near $t = 0$ indicate predictions of the model.
3. $pn \to np$ pion parity doublet fits for 3 and 8 GeV/c. The calculations have been multiplied by 1.0 and 0.75, respectively.

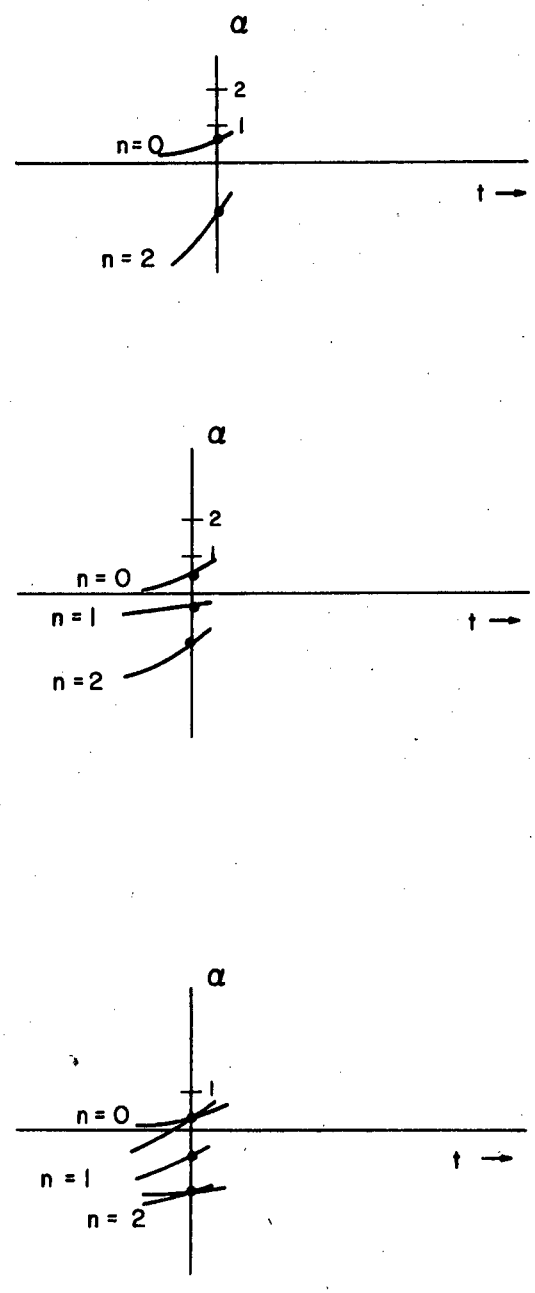

Fig. 1

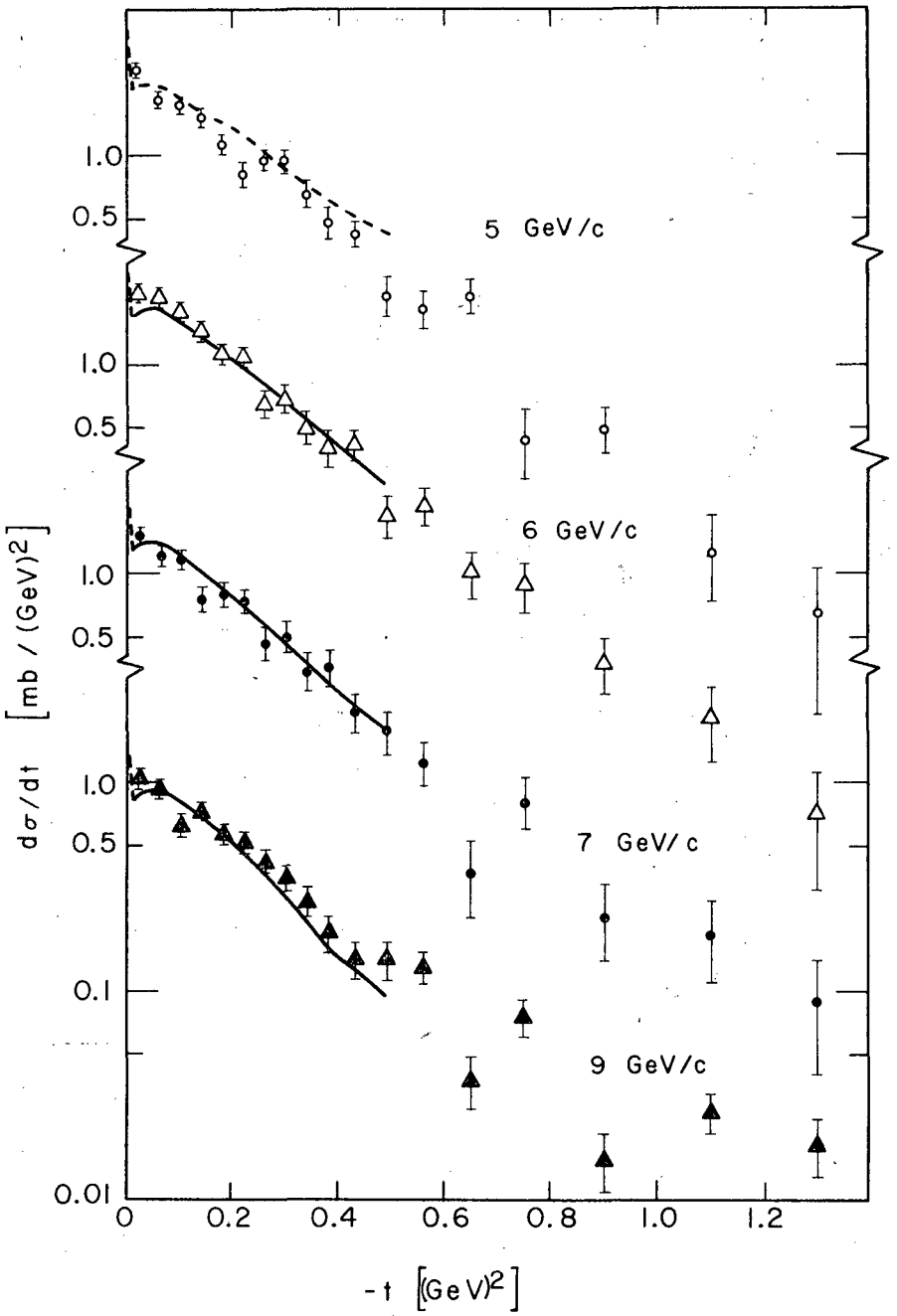
5 GeV/c
6 GeV/c
7 GeV/c
9 GeV/c
dσ/dt [mb / (GeV)²]
-t [(GeV)²]
1.0
0.5
0.1
0.01
0
0.2
0.4
0.6
0.8
1.0
1.2
XBL676-3240

Fig. 2

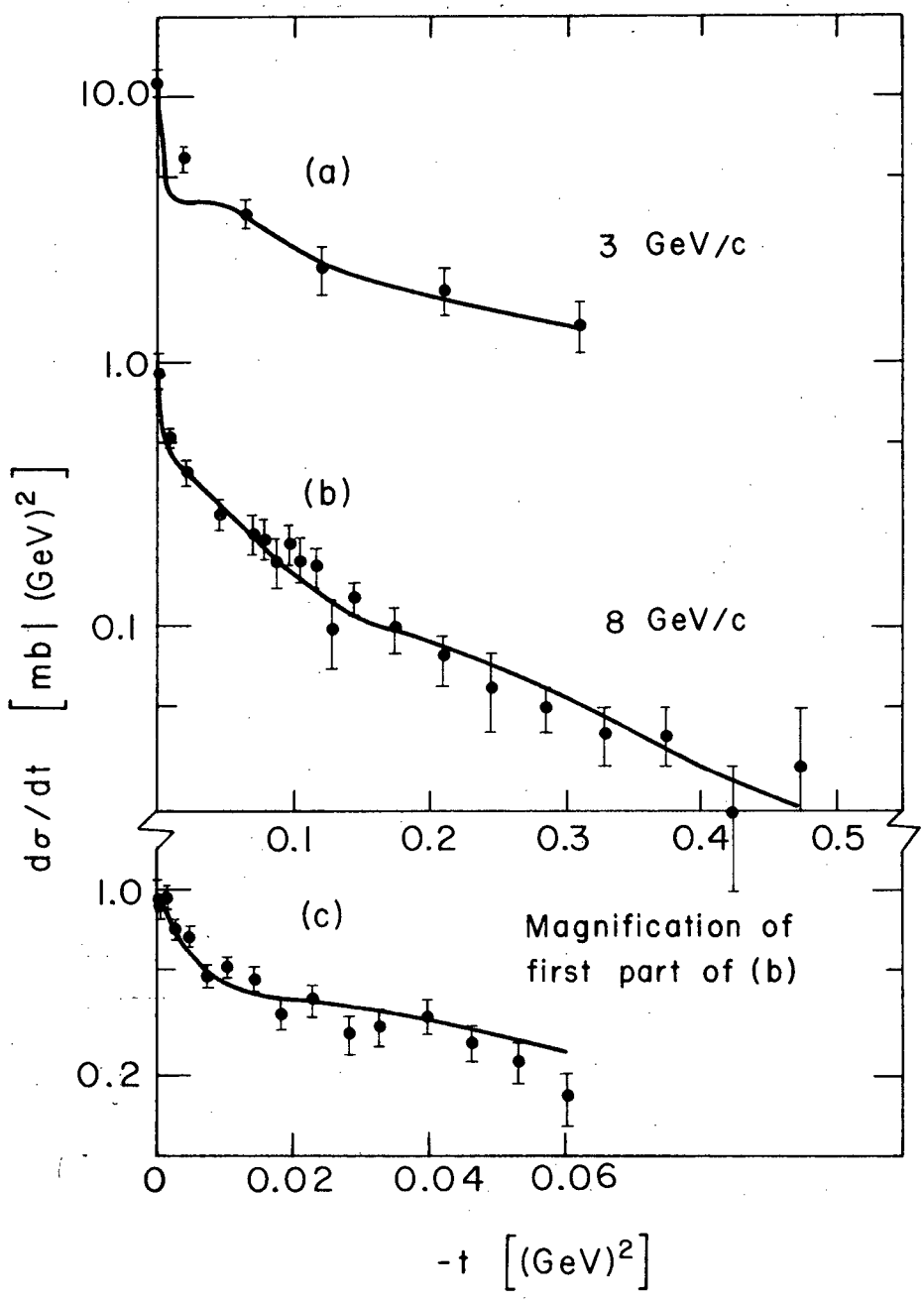
10.0
1.0
0.1
(a)
3 GeV/c
(b)
8 GeV/c
0.1
0.2
0.3
0.4
0.5
1.0
0.2
(c)
Magnification of
first part of (b)
0
0.02
0.04
0.06
dσ/dt [mb] (GeV)²]
-t [(GeV)²]
XBL676-3241

Fig. 3