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The Case for Open-Market Purchases in a Liquidity Trap

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C04-135 The Case for Open-market Purchase in a Liquidity Trap

Abstract

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The Case for Open-Market Purchases in a Liquidity Trap

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Abstract

Prevalent thinking about liquidity traps suggests that the perfect substitutability of money and bonds at a zero short-term nominal interest rate renders open-market operations ineffective for achieving macroeconomic stabilization goals. We show that even were this the case, there remains a powerful argument for large-scale open market operations as a fiscal policy tool. As we also demonstrate, however, this same reasoning implies that open-market operations *will* be beneficial for stabilization as well, even when the economy is expected to remain mired in a liquidity trap for some time. Thus, the microeconomic fiscal benefits of open-market operations in a liquidity trap go hand in hand with standard macroeconomic objectives. Motivated by Japan's recent economic experience, we use a dynamic general-equilibrium model to assess the welfare impact of open-market operations for an economy in Japan's predicament. We argue Japan can achieve a substantial welfare improvement through large open-market purchases of domestic government debt.

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I. Introduction

Japan's efforts to stimulate its economy over the past decade have led to apparent macroeconomic policy paralysis, with short-term nominal interest rates at their floor of zero and fiscal expansion immobilized by fears of augmenting an already-huge public debt.

Were short nominal interest rates positive, unanticipated open-market purchases of government debt would have the dual benefits of offsetting deflation and reducing the real value of yen-denominated public obligations. Prevalent thinking about liquidity traps, however, suggests that the perfect substitutability of money and bonds at a zero short-term nominal interest rate renders open-market operations ineffective as a stabilization tool.[1]

Even in this circumstance, there remains a powerful argument for large-scale open market operations as a fiscal policy tool. To the extent that long-term interest rates are positive now or short-term interest rates are expected to be positive at some point in the future, trading money for interest-bearing public debt reduces future debt-service requirements and hence the distortions of the requisite taxes. Thus, particularly for an economy in Japan's weakening fiscal position, large-scale open-market operations are an attractive policy, even if these operations are perceived to be totally ineffective at influencing current prices or output.

Yet, our analysis shows that this same reasoning implies that credibly permanent open-market operations *will* be beneficial as a stabilization tool as well, even when the economy is expected to remain mired in a liquidity trap for some time. That is, under the same conditions on interest rates that make open-market operations attractive for fiscal purposes, a monetary

[1] Goodfriend (2000) and King (1999) discuss other channels through which monetary policy might affect the economy even when short-term bond rates are zero. The mechanism we emphasize below is distinct from those that those authors review. In addition, our mechanism does not rely on Bernanke's (2000) "arbitrage" argument for monetary policy effectiveness.

expansion that markets perceive to be permanent will affect prices and, in the absence of fully flexible prices, output as well. Thus, the microeconomic fiscal benefits of open-market operations in a liquidity trap go hand in hand with standard macroeconomic objectives.[2]

Our analysis shows that beneficial macroeconomic policies need not accelerate an economy's escape from a liquidity trap. Indeed, zero nominal interest rates *per se* need not be a problem for policy. Problems can stem, however, from the shocks that drive nominal interest rates to zero, and it is to those shocks that policymakers may wish to respond.

In this paper we use a dynamic general-equilibrium model to assess the welfare impact of open-market operations for an economy in Japan's predicament. We argue that a country like Japan can achieve a substantial welfare improvement through large open-market purchases of debt. The Bank of Japan has indeed been carrying out such operations since March 2001 through its policy of "quantitative easing." Our analysis suggests that Japanese policymakers should underscore the permanence of past operations, perhaps through an announced inflation target range including positive rates, and may need to increase the monetary base even more.[3]

In a flexible-price model with monopolistic competition and distorting taxes, we show that even though Japan currently has zero short-term interest rates, an open-market purchase can counteract deflationary price tendencies. In this setting with flexible prices, the policy will improve welfare by reducing the real value of public debt and hence the excess burden of future taxes. Two preconditions must hold for these effects to be possible. First, long-term nominal

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[2] There is now a large modern literature analyzing alternative strategies for achieving monetary stimulus in a liquidity trap. See Eggertsson and Woodford (2003), McCallum (2003), Orphanides and Wieland (2000), and Svensson (2001, 2003) for discussion and references. Clouse et al. (2003) state that when future nominal interest rates are positive, a permanent monetary expansion has current effects even if current short-term interest rates are zero.

[3] See Shirakawa (2002) for a discussion of quantitative easing in Japan. Bank of Japan operations that purchase private equities, aside from the political problems involved, yield much more uncertain fiscal benefits (corresponding to the riskier return on the equities). Our analysis provides formal support for Bernanke's (2003) proposal for fiscal-monetary cooperation in Japan.

interest rates must be positive at *some* horizon (a condition that does hold in Japan today). Second, the central bank must be able to carry out credibly permanent increases in the level (not necessarily the growth rate) of the money supply, increases that can, but need not, be effected immediately. In Krugman's (1998) account, monetary policy is powerless precisely because of an assumption that the central bank cannot commit itself not to reverse one-off increases in the money supply. Future expected money supply levels are constant because the central bank is assumed unwilling to tolerate any permanent rise in the price level.[4] We argue that the credibility problem Krugman assumes is implausible as a total brake on policy effectiveness.

We also analyze a model with staggered nominal price setting in which anticipated deflation has negative welfare effects. In this setting too, an unanticipated open-market purchase is expansionary. While the open-market purchase again has the advantage of devaluing government debt, it has an additional positive welfare effect by causing a Keynesian temporary output increase. There are further welfare impacts, moreover, due to the effects of unexpected and expected inflation on relative price dispersion, but these are unlikely to offset the primary gains.

The final goal of the paper is to simulate numerically the benefits of open-market expansion. We find that, for an economy with Japan's tax rates and public debt to GDP ratio, open-market purchases of government debt yield large welfare benefits. Sizable benefits can be reaped, as we have noted, even when the accompanying inflationary impact is small.

II. The Term Structure of Interest Rates in Japan

A key assumption in the model we develop below is that short-term nominal interest rates, despite being zero today, are expected to be positive at some date (and in some state of nature) in

[4] Eggertsson and Woodford (2003) restate this result in a more detailed dynamic model.

the future. In other words, market participants see at least some chance that the economy will eventually escape from the liquidity trap. This assumption does not seem overly strong; on the contrary, the assumption of a fully permanent liquidity trap is, on its face, quite implausible. We nonetheless offer a more formal argument, based on the current term structure of interest rates in Japan, to show that our assumption about nominal interest-rate expectations is satisfied for that economy.

Figure 1 shows the evolution of Japan's term structure of interest rates since 1997. Short-term nominal rates are effectively at zero (very slightly positive, but just by enough to cover transaction costs). On the other hand, further out in the term structure—at maturities greater than a year—yields to maturity are higher, with that on the 20-year government bond still around 1 percent per annum.

A simple expectations theory of the term structure would, of course, imply some market expectation of positive future short-term interest rates: otherwise, the entire term structure would be flat at zero rather than upwardly sloped. The expectations theory, however, is highly questionable both on theoretical and empirical grounds. Fortunately we do not need to rely on it. There are other reasons for concluding that the term structure in Figure 1 is inconsistent with the hypothesis of a permanent (with probability 1) liquidity trap. None of the standard explanations for an upward-sloping term structure is plausible in the absence of positive expected future short-term nominal interest rates.

Consider first the possibility of conventional risk premia due to investor risk aversion. A major source of uncertainty in bonds' returns, however, is the future behavior of short-term interest rates. If those rates are at zero, they cannot fall. If investors simultaneously cannot envision an eventuality in which short-term rates might rise, then investors no longer consider short-term rates to be random at all. Under that circumstance, it would be impossible to generate risk premia that might justify the term differentials shown in Figure 1. The relative price of present and all future money payments is fixed at unity, so that money is a perfect substitute for bonds of any maturity.[5]

Since the yields in Figure 1 are government bond yields, what about the possibility of government default as an explanation for relatively high long-term interest rates? That possibility might seem especially compelling in view of Japan's current high debt-GDP ratio, the likely fiscal costs of financial-sector restructuring, and the alarming forecasts for budgetary developments down the road as the population ages.[6] A moment's reflection shows that this is not a plausible explanation for positive long-term rates in a world where short-term rates will never rise above zero. The reason is that, in the latter world, the government can trivially finance all its obligations by printing money. Money creation of such a magnitude could eventually ignite inflation, of course; but in that case, the hypothesis of short-term nominal interest rates frozen at zero would be contradicted.

Finally, consider liquidity effects. With short-term nominal interest rates pegged at zero, marketable debt instruments of different maturities all become equivalent to money, as we have noted. So again, one cannot rationalize term premia of the sort shown in Figure 1 purely as a liquidity effect.

We conclude that the only plausible explanation for the term structure shown in Figure 1 is that investors attach *some* positive probability to Japan's *some day* having positive nominal

[5] Keynes argued that risk premia would keep long-term rates positive even when short-term rates were zero because at low interest rates, bond-prices are volatile and hence bonds must yield a higher excess return in equilibrium (King 1999, p. 39). Keynes's argument presupposes that markets expect a possible future exit from the liquidity trap.

[6] See, for example, Kashyap (2002), who places a lower bound of 24 percent of GDP on the financial cleanup cost, and Dekle (2003).

short-term interest rates. That circumstance, as we now show, is enough to give monetary policy considerable power to enhance economic welfare.

III. Setup of the Model

We consider a model in which the representative household maximizes lifetime utility of consumption (C_t) and labor (L_t) over dates t starting at $t = 0$,

$$U_0 = \sum_{t=0}^{\infty}\left(\prod_{s=0}^{t}\beta_s\right) U(C_t, L_t) = \beta_0 U(C_0, L_0) + \beta_0\beta_1 U(C_1, L_1) + ... + \beta_0\beta_1 \cdots \beta_t U(C_t, L_t) + ... \ ,$$

where β_s is the pure discount factor between the end of period $s-1$ and the end of period s. (We assume throughout that $\beta < 1$ in the long run, so that the product $\prod_{s=0}^{t}\beta_s$ converges to 0 as $t \to \infty$.) We use the dating conventions that consumption and labor occur at the end of the period, assets are indexed by their values at the beginning of the period, and prices are indexed by the end of the period. Throughout most of the analysis, and where not otherwise specified, we use a simplified version of the utility function,

$$U(C_t, L_t) = \log(C_t) - k_t L_t . \tag{1}$$

Formally, the parameter k represents the disutility of labor, so that changes in k from one period to the next are taste shocks. However, as we will model production as a function of labor alone, variations in k will have the same impact as productivity shocks, altering the social cost of transforming forgone leisure into consumption.

There is no capital in the model, so the household holds its financial wealth exclusively in the form of money and interest-bearing government bonds. The household's real wealth at the beginning of period t (before payment of interest) is the sum of its holdings of debt (B) and money (M), divided by the contemporaneous price level (P):

$$V_t = \frac{B_t + M_t}{P_{t-1}}.$$

For most of the paper, we will assume that all debt is of one-period duration and that money is issued directly by the central bank. We consider long-term debt and financial intermediaries as extensions below.

Wealth at the beginning of period t+1 is

$$V_{t+1} = \frac{B_t(1+i_t)}{P_t} + \frac{M_t}{P_t} + \frac{\Omega_t}{P_t} + \frac{w_t L_t}{P_t} - T_t - C_t,$$

where i_t is the nominal interest rate between periods t−1 and t, and w_t, Ω_t, T_t, and C_t are, respectively, the nominal and wage rate, nominal profits, real taxes and real consumption in period t. Combining these two equations and defining the real interest rate by $r_t \equiv (1 + i_t)/(P_t/P_{t-1}) - 1$ yields an expression for the evolution of household wealth:

$$V_{t+1} = V_t(1+r_t) - \frac{i_t M_t}{P_t} + \frac{\Omega_t}{P_t} + \frac{w_t L_t}{P_t} - T_t - C_t.$$

To model money demand, we assume that households face a cash-in-advance constraint, needing to hold money in period t that is sufficient to purchase goods at the end of period t. Taxes are collected in the form of consumption taxes, and households are also required to hold cash in order to pay the taxes on their consumption purchases. If τ_t is the consumption tax rate at date t, the cash-in-advance constraint,

$$M_t \geq (1+\tau_t)P_tC_t, \tag{2}$$

is binding unless the nominal interest rate is zero, so that it is always the case that $i_tM_t=i_t(1+\tau_t)P_tC_t$. Using this fact, we can rewrite the evolution of wealth as

$$V_{t+1} = V_t(1+r_t) + \frac{\Omega_t}{P_t} + \frac{w_tL_t}{P_t} - T_t - (1+i_t+\tau_ti_t)C_t .$$

Because tax collections are given by $T_t = \tau_tC_t$, household wealth evolves as

$$V_{t+1} = V_t(1+r_t) + \frac{\Omega_t}{P_t} + \frac{w_tL_t}{P_t} - (1+\tau_t)(1+i_t)C_t .$$

Solved forward with the transversality condition imposed, this difference equation yields the lifetime budget constraint of the household:

$$V_0 \geq \sum_{t=0}^{\infty}\left(\prod_{s=0}^{t}(1+r_s)^{-1}\right)\left[(1+\tau_t)(1+i_t)C_t - \frac{\Omega_t}{P_t} - \frac{w_tL_t}{P_t}\right].$$

Maximizing utility subject to this budget constraint results in first-order conditions for consumption and labor at each date. Combining the conditions for consumption and labor at date t yields a solution for household consumption at date t,

$$C_t = \frac{w_t}{k_t(1+i_t)(1+\tau_t)P_t}, \tag{3}$$

in which an increase in the labor-disutility parameter, k, has the same impact as a decline in the real wage, discouraging work, consumption, and hence output.

Combining conditions for consumption at successive dates t and t+1 yields the Euler equation,

$$\frac{C_{t+1}}{C_t} = \beta_{t+1}(1+i_{t+1})\frac{P_t(1+\tau_t)(1+i_t)}{P_{t+1}(1+\tau_{t+1})(1+i_{t+1})} = \beta_{t+1}(1+i_t)\frac{P_t(1+\tau_t)}{P_{t+1}(1+\tau_{t+1})}. \tag{4}$$

Note that the cash-in-advance constraint has the effect of replacing the interest rate between dates t and t+1 with the preceding period's interest rate.

We assume that consumption at each date is a composite good. A continuum of producers supplies the individual consumption goods under conditions of market power. We model nominal price stickiness by postulating that each producer must set a nominal price that is maintained over two periods. That is, a posted price is good for the period in which it is set and the following period, with all market demand supplied at that price (as long as price exceeds marginal cost). Price setting is staggered across the two classes of goods. Half of the goods, class 1, have their prices set in odd-numbered periods, while the other half, class 2, have their prices set in even periods. Goods within each type enter the utility function symmetrically, and all goods are produced subject to the simple production function $Y = L$. Letting $c_{ti}(z)$ be the type-z good in class i at date t, the relationship between the composite consumption good and underlying individual commodity consumption is:

$$C_t = 2C_{t1}^{1/2}C_{t2}^{1/2} \qquad \text{where} \qquad C_{ti} = \left[\int_0^1 c_{ti}(z)^{\frac{\rho-1}{\rho}}\,dz\right]^{\frac{\rho}{\rho-1}},\ \rho > 1,\ i = 1,2.$$

That is, goods enter utility via a Cobb-Douglas function of the two class composites, each of which is a CES function of individual types of consumption. The corresponding producer price index is

$$P_t = P_{t1}^{1/2} P_{t2}^{1/2} \qquad \text{where} \qquad P_{ti} = \left[\int_0^1 p_{ti}(z)^{1-\rho} dz \right]^{\frac{1}{1-\rho}} .$$

IV. Effects of Monetary Policy when Short-Term Nominal Interest Rates are Zero: The Flexible-Price Case

It is clearest to start by analyzing open-market operations under the temporary assumption that nominal goods prices are perfectly flexible, that is, are set for one period only. In that case, even though there are two sectors of the economy, monetary shocks have symmetric effects on the sectors' outputs, employments, and prices, and do not drive relative intersectoral prices away from unity.

Initially the cash-in-advance constraint (2) does not bind and the nominal interest rate i = 0, which can occur because expected money growth rates are low (and perhaps even negative) relative to the subjective discount rates reflected in the preference parameters β_s. As per our discussion above, however, we assume that some long-term interest rate is positive (as is currently true in Japan), so that on at least one date T in the future, $i_T > 0$. (Perhaps on that date, consumers become more impatient or the rate of money-supply growth rises.) We show that under flexible prices, monetary policy can affect the price level before date T, notwithstanding the economy's zero nominal interest rate. That is, any prospect of future nominal interest rates above zero, no matter how remote, implies that the economy cannot be in a monetary-policy trap beforehand. Indeed it suffices, once can show, that there be some future state of nature, occurring with any positive probability, in which $i_T > 0$.

Start with the Euler equation (4) for date t, expressed in terms of nominal wages rather than prices based on the conditions (3) for the consumption-labor decisions at dates t and t+1. The result has the very simple form of an "inverse wage Euler equation,"

$$\frac{w_{t+1}}{k_{t+1}} = (1 + i_{t+1})\beta_{t+1} \frac{w_t}{k_t}. \tag{5}$$

We have assumed, however, that the economy is in a liquidity trap and the interest rate is zero from the present date, 0, through period T–1. At date T, the interest rate is positive and the cash-in-advance constraint is binding. To be concrete we will also assume that short-term the nominal interest rate also remains positive for all dates after T, but only some inessential details of our argument change if that is not the case. Under these assumptions, from (5), the wage rate for each date $t < T$–1 obeys the expression

$$\frac{w_t}{k_t} = \frac{1}{\beta_{t+1}} \frac{w_{t+1}}{k_{t+1}}. \tag{6}$$

From (6), we observe that the wages (and prices) rise, fall or remain constant during the zero-interest regime according to whether the term $\beta_{t+1} \frac{k_{t+1}}{k_t}$ is greater than, less than or equal to 1.

For each period $t > T$–1, we have, from the original Euler equation (4) and the cash-in-advance constraint, (2),

$$\frac{M_{t+1}}{M_t} = \beta_{t+1}(1 + i_t) \Rightarrow (1 + i_t) = \frac{M_{t+1}}{M_t \beta_{t+1}}. \tag{7}$$

From (5) and (6), the wage evolves from date T onward according to

$$\frac{w_t}{k_t} = \frac{\beta_t}{\beta_{t+1}} \frac{M_{t+1}}{M_t} \frac{w_{t-1}}{k_{t-1}}. \tag{8}$$

Thus, once we have an expression for $w_{T\text{-}1}$, (7) and (8) provide us with the entire path of wage rates both before date T–1 and after T–1.

To solve for $w_{T\text{-}1}$, write the Euler equation (3) for dates T–1 and T, substituting the cash-in-advance constraint (2) at T and the labor-consumption condition (3) at T–1, to obtain $\frac{M_T}{w_{T-1}/k_{T-1}} = \beta_T$, and so, $\frac{w_{T-1}}{k_{T-1}} = \frac{M_T}{\beta_T}$. As a result, the general solution for the path of wages is:

$$\frac{w_t}{k_t} = \left(\prod_{s=t+1}^{T-1} \beta_s^{-1}\right) \frac{M_T}{\beta_T}, \qquad t \leq T-1, \tag{9a}$$

$$\frac{w_t}{k_t} = \frac{M_{t+1}}{\beta_{t+1}}, \qquad t \geq T-1. \tag{9b}$$

(Note that these two expressions are the same for $t = T$–1.)

We can now see how a permanent step increase ΔM_0 in the money supply's level on date 0 will affect the economy. Let's assume that the increase in the money supply on date 0 does not change any future monetary *growth rates* M_{t+1}/M_t, $t \geq 0$. In that case, M_T rises by the factor $1 + \Delta M_0/M_0$ (as do all subsequent money-supply levels).[7]

[7] As is discussed below, this policy does not change the date at which the nominal interest rate becomes positive.

Equation (9) shows that, notwithstanding zero nominal interest rates prior to date T, all future nominal wage rates, including those for dates 0 through $T-1$, will be scaled by the factor $1+\Delta M_0/M_0$. As can be shown from the more general analysis of price-setting behavior given below, with perfectly flexible product prices, monopolists charge a fixed percentage markup over the wage, so that $P_t = \rho w_t/(\rho - 1)$. Therefore, the current and all future price levels rise by the same percentage as do wages. During a period of zero short-term nominal interest rates, the price level's path is governed by the money stock expected for the first date on which interest rates turn positive.

The intuition is disarmingly simple. On the first date the short-term interest rate turns positive, the money stock determines the price level in the conventional way. As long as the interest rate is zero, however, prices move toward that conventionally determined value at a rate governed by consumers' rate of time preference. By raising the money stock permanently today, the monetary authority can shift upward the date-T terminal condition on the price level. That action necessarily also shifts upward the entire time profile of prices prior to date T.

The dependence of nominal prices on expectations of money supplies on future positive-interest-rate dates, rather than on the current money supply, is the essence of the liquidity trap. However, the central bank can affect today's equilibrium by changing expectations of *future* money supplies. In our analysis, the central bank changes those expectations simply by changing the money stock's level immediately and allowing "base drift." It is not correct, however, to conclude that the current price level is independent of the current money supply in a liquidity trap, given money supplies after the liquidity trap ends. As we shall show in greater detail when we simulate our model, a sufficiently big reduction in current money supplies, given future money, could end the liquidity trap by forcing the short-term nominal interest rate up from zero.

Our infinite-horizon model is in essence just a dynamic extension of Krugman's (1998). Krugman, however, assumes a liquidity trap in the initial period of his model, with an exit from the liquidity trap in the second period and all relevant economic variables stationary thereafter. In our model, however, the liquidity trap can be long-lasting, even indefinitely so in some states of the world. If markets expect the possibility of positive interest rates at any point in the future, however, an immediate action—increasing the date-0 money supply with a credible commitment not to undo the action on some date $t \leq T$—will immediately lift prices.

Of course, one reason that markets might expect positive short-term interest rates in the future would be a government commitment to higher monetary growth rates then. Krugman (1998) and others have argued that such a commitment might be problematic. There are other mechanisms, however, that could produce zero interest rates now, coupled with expectations of future positive interest rates: predictable shifts in productivity growth, predictable shifts in time preference, or (outside the scope of the present model) demographic changes. To consider two concrete examples that might apply to Japan's current circumstances, an aging baby boom cohort with life-cycle savings behavior could induce a very high short-run saving rate that, in the context of our representative-agent model, would translate into a very low discount rate and a very high value for the discount factor, β. This would be particularly true if government promises of old-age pensions were viewed with some skepticism. A similar effect would occur if substantial pessimism or higher risk perceptions induced a rise in precautionary saving.

In general, it is difficult to believe that economic actors would not attach some positive probability to the event of positive interest rates on some, perhaps distant, future date. Nor is it plausible (as we argue in greater detail below) that they would necessarily expect any monetary expansion to be reversed with probability 1. These conditions would be enough to render current monetary expansion effective, as a stochastic extension of our model shows.

It is worth noting that, although we have modeled a closed economy, extension to the open case is easy when prices are flexible. Under a zero domestic nominal interest rate, and with a positive nominal interest rate abroad, the currency would appreciate over time so as to preserve uncovered interest parity. Since that appreciation would exactly offset the difference between domestic and foreign price inflation, relative international prices would not change.[8]

V. Welfare Analysis of an Open-Market Purchase of Government Bonds under Flexible Prices

Consider the impact on welfare of an open-market operation, as measured by a policy index ξ. Given the expression for household utility, we have the general expression

$$\frac{dU_0}{d\xi} = \sum_{t=0}^{\infty}\left(\prod_{s=0}^{t}\beta_s\right)(1-k_tC_t)\frac{1}{C_t}\frac{dC_t}{d\xi} = \sum_{t=0}^{\infty}\left(\prod_{s=0}^{t}\beta_s\right)\left(1-\frac{1}{(1+i_t)(1+\tau_t)(P_t/w_t)}\right)\frac{1}{C_t}\frac{dC_t}{d\xi}\ , \qquad (10)$$

which makes use of the fact that, with flexible prices, the aggregate consumption index C_t equals the aggregate labor input, L_t.[9] Note that this derivative will vanish if there is no seigniorage ($i = 0$), no taxes ($\tau = 0$), and no producer mark-ups ($P/w = 1$): with no distortions present, the allocation is Pareto optimal and any perturbation has no first-order welfare impact. More

[8] The Japanese yen's longstanding secular appreciation process seems to have stopped despite the zero short-term interest rate and the consequent apparent excess return on non-yen currencies. This pattern is difficult to rationalize in a non-stochastic model. Goyal and McKinnon (2003) argue that Japanese investors attach a substantial risk premium to dollar assets, and that this explains the current international configuration of exchange rates and nominal interest rates.

[9] When producer prices are equal for all goods z in sector i (= 1, 2), as is true throughout our analysis, the labor input for sector i equals that sector's sub-index, e.g., $L_{ti} = C_{ti}$. Thus, aggregate labor input $L_t = L_{t1} + L_{t2} = C_{t1} + C_{t2}$, while the aggregate consumption index is $C_t = 2(C_{t1}C_{t2})^{1/2}$. With flexible prices, $C_{t1} = C_{t2}$ and hence $C_t = L_t$.

generally, all of these distortions will be positive and hence cumulative, and any policy that increases consumption will increase welfare.

Let us assume for simplicity that the tax rate τ is always set so as to be constant over time. If we continue to assume that the experiment ξ holds M_{t+1}/M_t fixed in the future, then all of the effects on utility will occur through the tax rate τ. Using expression (3), we obtain[10]:

$$\frac{dU_0}{d\xi} = (1+\tau)\frac{d\left(\frac{1}{1+\tau}\right)}{d\xi}\sum_{t=0}^{\infty}\left(\prod_{s=0}^{t}\beta_s\right)\left(1-\frac{(\rho-1)}{\rho(1+i_t)(1+\tau)}\right) = -\frac{d\ln(1+\tau)}{d\xi}\sum_{t=0}^{\infty}\left(\prod_{s=0}^{t}\beta_s\right)\left(1-\frac{\rho-1}{\rho(1+i_t)(1+\tau)}\right).$$

We can also express the change in utility in dollars (or yen). Starting again with condition (10), use the first-order condition for consumption at each date t,

$$\left(\prod_{s=0}^{t}\beta_s\right)\frac{1}{C_t} = \lambda\left(\prod_{s=0}^{t}(1+r_s)^{-1}\right)(1+\tau_t)(1+i_t),$$

(where λ is the multiplier on the household lifetime budget constraint) to substitute and get

$$\begin{aligned}\frac{dU_0}{d\xi} &= \lambda\sum_{t=0}^{\infty}\left(\prod_{s=0}^{t}(1+r_s)^{-1}\right)\left((1+\tau_t)(1+i_t)-\frac{1}{(P_t/w_t)}\right)\frac{dC_t}{d\xi}\\ &= \frac{\lambda}{P_{-1}}\sum_{t=0}^{\infty}\left(\prod_{s=0}^{t}(1+i_s)^{-1}\right)\left((1+\tau_t)(1+i_t)P_t-w_t\right)\frac{dC_t}{d\xi}.\end{aligned}$$

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[10] This expression can be simplified further if we assume that the discount factor is constant at some value, say β_0, through period T, and constant at some different value, β_1, from period T+1 onward, and the net money growth rate is constant from T onward at some rate μ - 1. For $\beta_0 < 1$ (a comparable expression applies for $\beta_0 > 1$), the simplified expression is:

$$\frac{dU_0}{d\xi} = -\frac{d\ln(1+\tau)}{d\xi}\left\{\left(\frac{\beta_0-\beta_0^{T+1}}{1-\beta_0}\right)\left(1-\frac{\rho-1}{\rho(1+\tau)}\right)+\left(\frac{\beta_0^{T+1}}{1-\beta_1}\right)\left(1-\frac{(\rho-1)\beta_1}{\rho\mu(1+\tau)}\right)\right\}.$$

Because λ is the marginal utility of real wealth at date 0, λ/P_{-1} is the marginal utility of nominal wealth at date 0 (remember that real wealth at date 0 is obtained by deflating nominal wealth by P_{-1}). Thus we can divide both sides by λ/P_{-1} to get the dollar value, at date 0, of the open market operation, say Δ,

$$\Delta = \sum_{t=0}^{\infty}\left(\prod_{s=0}^{t}(1+i_s)^{-1}\right)\left((1+\tau_t)(1+i_t)P_t - w_t\right)\frac{dC_t}{d\xi}, \tag{11}$$

which is a standard result that marginal deadweight loss equals the product of the wedge (due to taxes, seigniorage and non-competitive producer mark ups) times the changes in the distorted quantity (i.e., in vector notation, **t′ΔX**).

For the case we have considered thus far, of flexible prices, M_{t+1}/M_t fixed in the future, and a constant tax rate τ, we have $\frac{dC_t}{d\xi} = -C_t\,\frac{d\ln(1+\tau)}{d\xi}$, so

$$\Delta = -\frac{d\ln(1+\tau)}{d\xi}\sum_{t=0}^{\infty}\left(\prod_{s=0}^{t}(1+i_s)^{-1}\right)\left((1+\tau)(1+i_t)P_t - w_t\right)C_t. \tag{12}$$

The summation in (12) equals the present value of revenue, from taxes and seigniorage, plus monopoly profits, so the dollar value of the welfare effect equals that total times minus the percentage change in $(1+\tau)$. This expression can be further evaluated using the expression for τ derived in Equation (A1) in the Appendix.[11]

[11] For the special case considered in footnote 10, the expression for τ simplifies to:

$$\tau = \frac{\left(\frac{M_0+B_0}{M_T}\right)\beta_0^{T+1} - \left(1-\frac{\beta_1}{\mu}\right)\frac{\beta_0^{T+1}}{1-\beta_1}}{\left(1-\beta_0^{T+1}\right)\frac{\beta_0}{1-\beta_0} + \beta_0^{T+1}\frac{\beta_1}{1-\beta_1} - \left(\frac{M_0+B_0}{M_T}\right)\beta_0^{T+1}}.$$

Expressions (11) and (12) provide a convenient framework for considering the impact of alternative preference assumptions on the magnitude of estimated welfare gains. One can generalize the preferences assumed thus far, based on expression (1) for within-period utility, to

$$U(C_t, L_t) = \left(1 - \frac{1}{\gamma}\right)^{-1} C_t^{1-\frac{1}{\gamma}} - k_t L_t, \tag{1′}$$

in which the parameter γ, equal to the intertemporal elasticity of substitution of consumption, may take on values other than 1 (the logarithmic case). For this more general specification of utility, the analysis thus far goes through with minor modifications, notably that the relationship between consumption and labor at date t now follows the more general rule

$$C_t = \left(\frac{w_t}{k_t(1+i_t)(1+\tau_t)P_t}\right)^{\gamma}. \tag{3′}$$

Expression (11) is not affected by this change in assumption, but using (3′) rather than (3) to simplify leads to a modified version of expression (12),

$$\Delta = -\gamma \frac{d\ln(1+\tau)}{d\xi} \sum_{t=0}^{\infty} \left(\prod_{s=0}^{t} (1+i_s)^{-1} \right) \big((1+\tau)(1+i_t)P_t - w_t\big) C_t. \tag{12′}$$

We see from (12′) that the welfare change is scaled by the intertemporal consumption elasticity, so that, as one would expect, less sensitive preferences would lead to lower estimates of the welfare change.[12] This result should be kept in mind below as we discuss the magnitude of potential welfare gains from policy changes.

[12] A full analysis of the impact of changes in γ would also need to account for differences in the induced change in τ. We present numerical simulations for alternative values of γ in Auerbach and Obstfeld (2004).

VI. The Sticky-Price Case

Continuing to assume that wages are flexible, let us drop the temporary assumption that prices are flexible and consider staggered two-period setting of nominal product prices. From the assumption of profit maximization by producers and the household's Euler equation, we obtain the following expression for the price index for class-i goods, whose price is reset in period t:

$$P_{ti} = \frac{\rho}{\rho - 1}\left[\frac{w_t + \beta_{t+1} w_{t+1} \, {a_t}\!/\!{a_{t+1}}}{1 + \beta_{t+1} \, {a_t}\!/\!{a_{t+1}}}\right], \qquad \text{where } a_t = (1+\tau_t)(1+i_t).$$

Thus, the overall price level in period t is:

$$P_t = \frac{\rho}{\rho - 1}\left[\frac{w_t + \beta_{t+1} w_{t+1} \, {a_t}\!/\!{a_{t+1}}}{1 + \beta_{t+1} \, {a_t}\!/\!{a_{t+1}}}\right]^{1/2}\left[\frac{w_{t-1} + \beta_t w_t \, {a_{t-1}}\!/\!{a_t}}{1 + \beta_t \, {a_{t-1}}\!/\!{a_t}}\right]^{1/2}.$$

Note that this price level expression holds when wages, taxes and interest rates equal the values assumed by producers when prices are set. If we consider an unanticipated policy change at date t, then the prices set in period t-1 will not obey the above expression for P_{ti}, ex post.

If the interest rate, tax rate, wage inflation rate, and discount factor are all constant over time, then the expression for P_t simplifies to

$$P_t = w_t \frac{\rho}{(\rho - 1)(1+\beta)}(1+\beta\mu)^{1/2}\left(\mu^{-1} + \beta\right)^{1/2},$$

where μ is 1 plus the inflation rate. From this expression, it can be shown that the mark-up, P_t/w_t, is a decreasing function of μ for $\mu\beta < 1$ and an increasing function for $\mu\beta > 1$; that is, the mark-up is decreasing with inflation until the rate of inflation equals the pure rate of time preference. Since welfare is inversely related to the markup, this is therefore a model in which even anticipated deflation has welfare costs, and anticipated inflation has some beneficial welfare effects for sufficiently small inflation rates, consistent with the conjecture of Akerlof, Dickens, and Perry (1996).[13]

A major difference now is that it is no longer true that $L = C$. In essence, relative-price distortion between the two price-staggering sectors will lower the consumption index C below the cost of production $L = C_1 + C_2$ whenever the prices charged by the sectors are not equal. So we must calculate aggregate labor supply as the sum of supplies to the two sectors of the economy, which still equal their respective consumption sub-aggregates.

Our goal is to evaluate how lifetime utility

$$U_0 = \sum_{t=0}^{\infty}\left(\prod_{s=0}^{t}\beta_s\right)\left[\log(C_t) - k_t\left(C_{t1} + C_{t2}\right)\right]$$

is affected by an open market purchase of government bonds. As noted above, in the disutility of labor term we can no longer assert that $L = C$ because, with sticky prices, there will generally be asymmetric labor supply to the two sectors of the economy. Since, however,

$$C_{ti} = \frac{1}{2}\left(\frac{P_{ti}}{P_t}\right)^{-1} C_t,$$

we can write utility as

[13] See Wolman (2001) for another model with this property, and Woodford (2003) for a general treatment of inflation and welfare. Schmitt-Grohé and Uribe (2002) and Siu (2004) analyze optimal monetary and fiscal policy in economies with nominal price rigidities.

$$U_0 = \sum_{t=0}^{\infty}\left(\prod_{s=0}^{t}\beta_s\right)\left[\log(C_t) - k_t\left(\frac{P_{t1}+P_{t2}}{2P_{t1}^{1/2}P_{t2}^{1/2}}\right)C_t\right].$$

Note that the ratio of arithmetic to geometric means is always greater than 1 unless the two prices are equal. Thus, the second term in the period utility function incorporates the effects of relative price distortions. Alternatively, let $\delta_t = P_{t1}/P_{t2}$ measure the relative price distortion on date t. Then utility has the form

$$U_0 = \sum_{t=0}^{\infty}\left(\prod_{s=0}^{t}\beta_s\right)\left[\log(C_t) - k_t\left(\frac{1+\delta_t}{2\delta_t^{1/2}}\right)C_t\right].$$

Define the relative-price distortion term as $\phi(\delta) \equiv (1+\delta)/2\delta^{1/2}$. Notice that the derivative $\phi'(\delta) = (\delta^{1/2} - \delta^{-1/2})/4\delta$, which is negative for $\delta < 1$, and that $\phi(\delta) = \phi(1/\delta)$.

For simplicity assume that β is a constant. If we are initially in a position where price-setters' expectations have been realized and the nominal interest rate is 0, then one can show that even with sticky prices $\delta = \beta$ will hold. That is, relative prices (like the price level) are falling at the rate of time preference. In that setting, a small unexpected monetary expansion will raise the prices that are currently set and (for one period) push δ closer to 1, raising welfare through that channel. But a large enough monetary expansion will temporarily exacerbate the relative-price distortion. This gives a rigorous account of the specific costs of unanticipated inflation (as opposed to the better-known costs of anticipated inflation in the optimum quantity of money discussion).[14] Of course, we will have a second unanticipated inflation effect on welfare through the output channel along with a second anticipated inflation effect on the average markup.

[14] Such effects of unanticipated inflation have been assumed on an ad hoc basis (see, e.g., Barro and Gordon 1983) in order to avoid the Calvo (1978) problem of potentially unbounded Nash-equilibrium inflation rates.

These effects are all captured in the expression below:

$$\frac{dU_0}{d\xi} = \sum_{t=0}^{\infty} \beta^{t+1} \left\{ [1 - k_t \phi(\delta_t) C_t] \frac{1}{C_t} \frac{dC_t}{d\xi} - k_t \phi'(\delta_t) C_t \frac{d\delta_t}{d\xi} \right\}.$$

Alternatively, based on expression (3), which continues to hold even under staggered pricing,

$$\frac{dU_0}{d\xi} = \sum_{t=0}^{\infty} \beta^{t+1} \left\{ \left[1 - \frac{\phi(\delta_t)}{(1+i_t)(1+\tau_t)(P_t / w_t)} \right] \frac{1}{C_t} \frac{dC_t}{d\xi} - \frac{\phi'(\delta_t)}{(1+i_t)(1+\tau_t)(P_t / w_t)} \frac{d\delta_t}{d\xi} \right\}.$$

Only in the absence of distortions—that is, with $i = \tau = 0$ and $P/w = \delta = 1$—is this derivative guaranteed to be zero, of course.

The first component of the preceding expression, involving the derivative of total consumption with respect to the policy action ξ, can be calculated just as in the last section. Now, though, the intersectoral relative-price distortion term $\phi(\delta) \geq 1$ reduces the utility value of increments in total consumption. However, with sticky prices the effect of the policy change $d\xi$ on the future path of consumption will differ compared to the flexible-price case. In the present setup, with two-period staggered price-setting, an unexpected monetary expansion on date 0 would raise C_0 above its flexible-price level. This short-run Keynesian effect, associated with a fall in the markup, would reinforce the positive consumption effect due to lower taxes.

The second term above, that involving the derivatives $\phi'(\delta)$, reflects the additional price distortion associated with the policy change. Let us continue to assume that the tax rate is constant and that the open-market purchase at date 0 leaves all future rates of monetary growth unaltered. In that case, only the term $\phi'(\delta_0)$ above is non-zero; $\phi'(\delta_1) = \phi'(\delta_2) = \ldots = 0$ because the only unanticipated shock occurs on date 0, subsequent money-supply growth rates are unchanged, and by date 1 (with two-period overlapping contracts), the pre-shock intersectoral price distribution therefore has been restored.[15]

As we have noted, when the nominal interest rate is zero it is initially the case that $\delta = \beta$ (we assume for simplicity that $t < T-2$). As the tax rate rises immediately to its new constant level as a result of the surprise date-0 monetary expansion, the new relative price on date 0 is

$$\delta_0 = \beta\left(1 + \Delta M_0 / M_0\right).$$

Thus, the additional marginal welfare effect due to the induced date-0 change in intersectoral price dispersion is

$$-\frac{\phi'(\beta)\beta}{(1+\bar{\tau})(\overline{P/w})}\frac{d\left(\Delta M_0 / M_0\right)}{d\xi},$$

where overbars indicate pre-shock levels for the tax rate and markup. As noted above, for a small change the preceding welfare effect is positive, because the initial trend deflation at the rate of time preference is reduced for a period.

The preceding utility effects can be translated into dollar terms using the same transformation we employed in Section V.

VII. Quantitative Estimates of Welfare Gains

How large might the welfare gain from open-market operations be? As just discussed, the effects in the fixed-price case are different only in transitory ways from those of the flexible-price case, so we can get a rough estimate of the magnitude by using expression (12), for the welfare gain

[15] As in the flexible-price case, it remains true that an unexpected monetary expansion on date 0 will not, under our assumptions, change the date on which the nominal interest rate first turns positive.

under flexible prices. That expression says that the welfare effect of an open market operation equals the present value of tax revenues, seigniorage and non-competitive rents, multiplied by minus the percent change in the term $(1+\tau)$, roughly the absolute reduction in the tax rate itself.

To get some idea of the responsiveness of this term to monetary policy, consider the special case in which the discount factor, β, is constant in the zero interest rate regime at some value, say β_0, and constant at some possibly different value, say $\beta_1 < 1$, in the subsequent positive interest rate regime. As discussed above, a temporarily low discount rate could be a contributing cause of a liquidity trap. Under this assumption, an increase in the money stock at date T, M_T, would have the impact on $(1+\tau)$ of $-\dfrac{d\ln(1+\tau)}{d\ln M_T} = \dfrac{1}{X-1}$, where[16]

$$X = \left[\beta_0^{-T} + \beta_0^{-(T-1)} + \ldots + \beta_0^{-1} + 1 + \beta_1 + \beta_1^2 + \ldots\right]\frac{M_T}{M_0 + B_0} \quad . \tag{13}$$

Suppose, for example, that the pure rate of time preference currently in year zero is -0.03 (β_0 = 1/0.97), and that it is expected to increase to +0.02 (β_1 = 1/1.02) in year T = 5. Then the term in brackets in (13) equals $(1-(0.97)^6)/0.03 + 1/0.02 = 55.6$. Hence, for $M_T = M_0$ initially, and a ratio of high-powered money to total government debt (including money) of 0.2,[17] X = 11.1, and hence $-\dfrac{d\ln(1+\tau)}{d\ln M_T} = (1/10.1)$.

To obtain a lower-bound estimate of welfare effects, let's ignore seigniorage and non-competitive rents in expression (12) and consider only the percentage of National Income

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[16] The calculation is based on the formula for the tax rate in footnote 11 and assumes that μ, the gross rate of money-supply growth from date T onward, is 1.

[17] Japan's current monetary base is about 18 percent of GDP and the level of net outstanding general government debt is roughly 70 percent of GDP. (The more widely cited gross government debt figure is about twice as high).

currently raised through taxes. (We present a more complete numerical analysis in the next section.) Our calculation relates to a hypothetical level tax rate consistent with intertemporal budget balance, whereas Japan's general government deficit now exceeds 8 percent of National Income. So we use as our tax rate the Ministry of Finance (2002, sec. II.8) estimate of the overall "potential national burden," which includes hypothetical tax receipts that would suffice to eliminate the current fiscal deficit. That number was 47 percent of National Income for 2002 (and is surely over-optimistic given future social security obligations and financial-sector restructuring costs).[18] From expression (12), we therefore infer a permanent annual welfare gain equal to 0.47/10.1, or nearly 0.05 percent of National Income for each percent increase in the date-T money stock. This is a huge marginal welfare benefit (and, as we have noted, is likely an underestimate of the true benefit that is implied by the model). Extrapolated linearly, it implies that doubling the monetary base would raise welfare permanently by about 5 percent of National Income per year, although linearity breaks down for such a large monetary expansion and the true effect would be smaller. Because the model has some specific simplifying assumptions (including logarithmic consumption preferences), this number should be taken to indicate substantial welfare benefits rather than as a literal best estimate.

VIII. Simulating Various Policy Changes

The preceding section's analysis illustrates the potential welfare benefits of open market operations in a liquidity trap, but with some limits. First, it starts from the premise that a liquidity trap exists and will end at a certain future date, and hence is applicable only to policies

[18] Our model, which abstracts from government spending, implies that $\tau = \{1 - [M_T/(M_0 + B_0)]\}/(X - 1)$ (assuming that $\mu = 1$). While the model is appropriate for capturing quantitatively the seigniorage gains from government bond buybacks and the resultant impact on the "permanent" tax rate, it predicts a tax rate that is much too low and it therefore underestimates the deadweight tax burden. Accordingly we use the actual tax burden to substitute for τ in expression (12).

that do not affect the duration of the liquidity-trap regime. Second, it is accurate only for small open market operations. Third, it does not deal with the effects of sticky prices.

To get a more general idea of the effects of policy changes of different sizes in a variety of environments, including those changes that may affect the dates at which the economy exits the liquidity trap, we turn to numerical simulations. As one cannot determine ex ante whether the economy will be in a liquidity trap in a given period, the key to the methodology is identifying periods in which the economy is constrained. We accomplish this through the following backward solution technique.

Assume first that we know the state of the economy at date t+1, including whether there is a liquidity trap (i.e., whether $i_{t+1} = 0$). Whether or not there is a liquidity trap in period t+1, we can solve for the wage in period t. If the cash-in-advance constraint binds at date t+1, then (from expression (9b)) $\frac{w_t}{k_t} = \frac{M_{t+1}}{\beta_{t+1}}$. If, instead, the cash-in-advance constraint does not bind at date t+1, then $i_{t+1} = 0$ and (from expression (6)) $\frac{w_t}{k_t} = \frac{1}{\beta_{t+1}} \frac{w_{t+1}}{k_{t+1}}$. Now, consider expressions (2) and (3). Together, they imply a notional solution for the period-t nominal interest rate:

$$\tilde{i}_t = \frac{w_t}{k_t M_t} - 1 . \tag{14}$$

If the solution for $\tilde{i}_t$ in (14) is negative, then the cash-in-advance constraint (2) is slack, with the real money stock greater than consumption expenditures. In this case, the interest rate is equal to zero, and the economy is in a liquidity trap, in that the current money supply is irrelevant.[19]

[19] At the borderline where $\tilde{i}$ is exactly equal to zero, the interest rate is zero and the cash-in-advance constraint holds as an equality. It does not bind, though, in the sense that the equilibrium would be unaffected by relaxing the constraint.

Whether or not the economy is in a liquidity trap, we may then solve for the remainder of the date-t variables, and then proceed to a solution for date t-1. Finally, to begin the backward solution process, assume that, for some date in the distant future, we know that the economy has a positive interest rate.

The procedure outlined provides a solution for the entire path of the economy for given paths of the policy variables M and τ. In order to ensure that the government's intertemporal budget constraint is satisfied, we iterate, revising the value of τ (which is assumed to be constant over time) with each iteration to meet the budget constraint. Once the iteration process converges, the value of τ to which behavior responds is consistent with the government's budget constraint, given that behavior.

Having laid out this solution algorithm, we may now demonstrate the claim made above, that a level shift in the money stock at the initial date, with no subsequent change in money growth rates, will not affect the number of periods during which the liquidity trap applies. Consider first dates t from period T-1 onward, where T, once again, is the first date such that $i_T > 0$. For these dates, the current wage is proportional to the next period's money stock. Thus, with no change in the money growth rate between t and t+1, the value of $\tilde{i}_t$ yielded by expression (14) does not change. Now, consider any date t before T-1, for which the wage is proportional to the wage at date T-1, and hence to the money stock at date T. From (14), there will be no impact of the value of $\tilde{i}_t$ at t if the growth of the money stock between periods t and T is unchanged.

It is useful to note that in this model the equilibrium is unique. Benhabib, Schmitt-Grohé, and Uribe (2002) demonstrate that the zero-bound on nominal interest rates can generate multiple equilibria when the monetary authority follows a Taylor-like rule to set the interest rate. Our model escapes this problem because it does not include a feedback policy rule for the nominal interest rate.[20] It is easy to demonstrate, using the solution technique just described, that changing the terminal period has no impact on the equilibrium path. That is, suppose we first assume that the economy is permanently out of a liquidity trap as of date Ψ and solve backward from that date. If the assumption is consistent with this solution, i.e., if the path after Ψ exhibits positive interest rates, then starting the solution from any date $\Psi' > \Psi$ will yield the same equilibrium path for the economy.

We now turn to some numerical simulations. As noted in the previous section, the theoretical model studied here has no government spending other than debt service. To remedy this omission in the simulations, we add a stream of government purchases to the government's intertemporal budget constraint. Purchases at each date t are assumed to be exogenous and, like consumption at that date, proportional to the term $1/k_t$ (see expression (3)). Thus, for a given mark-up, interest rate and tax rate, government purchases will be a constant share of output and consumption over time. We adjust this share so that the tax rate in the initial equilibrium equals the estimate given above for government's share of output in Japan, 47 percent. As before, we also set the initial ratio of high-powered money to high-powered money plus debt at 0.2. For each simulation, we consider the welfare effects of a change in monetary policy in terms of the equivalent variation in resources that would provide the same change in utility.[21]

Because we are interested in studying an economy that is initially in a liquidity trap, we make parameter assumptions consistent with this being the case. Following the assumptions of

[20] Benhabib, Schmitt-Grohé, and Uribe (2002) also investigate conditions under which an anticipated switch from an interest-rate rule to monetary-base targeting can prevent deflationary equilibria from emerging. The model precludes self-fulfilling deflationary paths in which nominal interest rates are zero notwithstanding non-negative money-supply growth. Consumers' desire eventually to spend their rapidly increasing real money balances makes such paths inconsistent with overall equilibrium.

[21] Given the quasi-linear form of the utility function, hypothetical variations in income are absorbed by changes in labor supply, so the calculation amounts to finding the equivalent increase or decrease in labor supply necessary to give the same change in utility.

the previous section, we let the pure rate of time preference initially be negative (-0.03), having it become positive (+0.02) in period 5. In terms of the discount factor itself, β initially is 1/0.97, falling to $\beta_5 = \beta_6 = \ldots = 1/1.02$ in period 5. The low initial rate of time preference pushes the nominal interest rate lower, making a liquidity trap more likely. We also assume that the labor-disutility parameter, k, falls at an annual rate of .05, reaching 1.0 in period 5, when it ceases falling and remains constant thereafter. As discussed, variations in this parameter may be thought of in the same terms as variations in the rate of productivity, with falling k being equivalent to increasing productivity. If productivity is relatively low now but is expected to be higher in the future, then inflation will be lower to make room for the real balances needed to support higher income levels.[22] In line with recent estimates by Nishimura and Shirai (2000), we set the competition parameter, ρ, equal to 10, which induces a modest price-cost ratio of 10/9. Finally, in the initial equilibrium, we set the money stock to 1.0 in period 0 and assume that it grows at a constant annual rate of 2 percent thereafter.

Figure 2 shows the initial equilibrium trajectory of the money stock, inflation rate, and nominal interest rate for the economy just described. Despite a growing money stock, the economy is in a liquidity trap in periods 0 through 4, with a zero nominal interest rate and prices falling at a rate just over 2 percent per year. The period-5 shift in preferences brings the economy out of the liquidity trap, with the interest rate rising to just over 4 percent and deflation ending. (Deflation lessens starting in period 4, due to firms' forward-looking pricing policies.)

Figure 3 shows the impact of one-time increases in the money stock effected by open-market operations in period 0. Recall that, in our model, this family of policies has no impact on

[22] In the context of present-day Japan, one cause of temporarily low productivity might be the banking system's distress, which makes it difficult for firms to finance productivity-enhancing investments. Hayashi and Prescott (2002) attribute Japan's recent output stagnation to low productivity growth, although they argue against the view that financial-system problems have hampered investment.

the timing of the liquidity trap or the interest rate at any date. The figure shows the money stock and inflation trajectories under the baseline equilibrium already discussed, and alternative paths for period-0 increases in the money stock of 1 percent and 10 percent. The welfare gains from these policies are, respectively, 0.09 percent and 0.84 percent of output per year. The first of these gains is somewhat higher than the 0.05 percent derived for a comparable experiment in the previous section, a difference explained by the presence here of the additional distortions of price-cost mark-ups and (after period 4) positive nominal interest rates (see expression (12)). As predicted, the gains are not linear in money stock changes. Still, the gain from a 10-percent increase in the money stock is sizable—nearly 1 percent of output. Such a jump in the money stock does cause some inflation—here nearly 3 percent per year for two years—but even this small temporary surge is an artifact of our assumption that prices adjust over two periods. In this model, a longer period of price adjustment, such as we model in Section XI, would yield a smaller spike and a more prolonged period of inflation within the liquidity trap regime.[23]

Unlike a one-time, unannounced increase in the money stock, an unannounced change in the rate of money growth has the potential to bring the economy out of a liquidity trap immediately. Figure 4 illustrates this, presenting trajectories for the initial equilibrium and for an equilibrium in which the annual growth rate of money growth is raised from 2 percent per year to 4 percent per year through period 5. By the end of this transition period, the money stock is 12.4 percent higher than in the base case with 2 percent money growth.

With faster money growth, the liquidity trap ends immediately. Being out of the liquidity trap, the economy's inflation rate is dictated by this faster money growth rate, and exceeds that

[23] Why does the short-term nominal interest rate not rise in the face of immediate expected inflation? Given our specification of preferences, it is nominal wage growth that determines the interest rate in the $i = 0$ regime. The permanent increase in the money supply leads to a step increase in the entire path of nominal wages, bur to no wage inflation. Essentially, P drops out of the relevant Euler equation, given our consumption function.

of the baseline equilibrium throughout the initial period. This inflation is still modest—ranging from less than 2 percent in period 5 to just below 3 percent in year 1. The welfare gain is 0.66 percent of output per year. Though still significant, this is well below the yearly gain that an immediate 12.4-percent increase in the money stock would deliver—1.01 percent of output. The gradual increase in the money stock weakens the welfare gain by pushing interest rates up immediately, lessening the short-run output surge that accompanies an unexpected burst of inflation.[24] This is illustrated in Figure 5, which shows consumption trajectories for the baseline equilibrium, the equilibrium with faster money growth, and the equilibrium with an immediate jump in the money stock of the same size. Note that the temporary rise in consumption is much smaller under the policy of faster growth, and consumption actually then falls to near the baseline trajectory temporarily due to the higher nominal interest rate.

To conclude this section, we emphasize some general lessons of these simulations. First, the inflationary effects of a step increase in money may well be front-loaded. Even a substantial increase in money could give way to moderate inflation, and possibly a temporary return to deflation, after a transition period. With different parameter settings implying a strong enough underlying deflationary trend, a policy of monetary injections might also succeed only in reducing, but not eliminating, deflation. Nonetheless, substantial welfare gains can be reaped. These observations are relevant in assessing the credibility of permanent monetary expansion, the topic of Section XI.

[24] Our cash-in-advance model's baseline version, which implies a unitary elasticity of consumption with respect to the nominal interest factor, probably exaggerates the welfare cost of deviations from Friedman's optimum quantity of money. The effect is proportionally lower, however, for lower values of the intertemporal substitution elasticity γ. As we have noted, of course, lowering γ will lower other welfare effects as well.

IX. Adding Long-Term Bonds

The analysis so far has assumed that all government debt is of the one-period variety. In reality, of course, the government issues debt with a range of maturities. As discussed above, even when the government is currently in a liquidity trap, sufficiently long-term debt will carry a positive interest rate if the liquidity trap is expected to end at some date with positive probability.

Under our modeling assumption of certainty, long-term and short-term debt would be perfect substitutes from the perspective of investors; there would be no term premium and the long-term interest rate would be a simple function of the time path of short-term interest rates. For example, the value at the beginning of period *t* of a unit coupon payment at a subsequent date *t*+*s* would be:

$$(1+j_{t,s})^{-(s+1)} = [(1+i_t)\,(1+i_{t+1})\,(1+i_{t+2})\cdots(1+i_{t+s})]^{-1} \tag{15}$$

where $j_{t,s}$ is the s+1-period interest rate at date *t* and i_{t+k} is the one-period interest rate from the beginning to the end of period *t*+*k*.

Thus, along any equilibrium path, the composition of government debt is of no consequence. The effects of an unexpected policy change, however, would depend on the maturity structure of debt outstanding at the time of the policy announcement. Whereas one-period bonds' nominal value is unaffected by a change in the path of future short-term rates (because, by assumption, their coupons are reset each period), bonds of longer duration will experience nominal price changes in the opposite direction of changes in future short-term rates. This provides a second channel through which an open market monetary expansion can reduce the real value of government liabilities (money plus debt), in addition to that induced by a price level increase. For our base case considered above—an immediate money supply increase that is sustained permanently, with no change in money growth rates—this new channel will be absent, because short-term rates are unaffected by policy. For this type of policy, the initial maturity structure of debt is irrelevant. For a policy that increases money growth, on the other hand, there will be an increase in short-term interest rates and an immediate decline in the nominal value of outstanding long-term bonds, this reduction reinforcing the policy's welfare enhancing effects. As a consequence, the gap in welfare effects between the two types of monetary policy will be reduced by the presence of long-term bonds.

For example, consider again the numerical simulation of an increase in the money growth rate from 2 percent to 4 percent over the five-year period in which the liquidity trap prevailed in the initial equilibrium. We found that this policy would increase welfare by 0.66 percent of output per year, compared to a welfare increase of 1.01 percent if the ultimate change in the money stock had been effected immediately. Consider now the case in which a certain fraction of the initial debt is long-term, assumed for simplicity to be level-payment consols. For the same policy experiment of increased money growth, the welfare gain would increase to 0.78 percent if consols accounted for half of all initial debt, and 0.91 percent if all debt were long-term. In short, the difference between policies that increase the money stock immediately and those that do so over a few years is overstated by the assumption that all debt is short-term, because this assumption understates the wealth levy that occurs under a policy of gradual money growth.

X. Adding Financial Intermediaries

Until now, we have ignored the existence of financial intermediaries, assuming that the money stock equals the monetary base, and hence is directly controllable by the central bank. With a banking sector, the ability of the central bank to increase the money stock depends on its success at getting the banking sector to translate increases in reserves into increases in the ultimate money stock, through lending. There has been concern in Japan, for example, that the Bank of Japan's increases in the monetary base have failed to have an expansionary impact because these increases have been absorbed by the banking sector in the form of excess reserves.

While such an accumulation of excess reserves would, indeed, cut off the salutary effects of the monetary expansion that we have discussed, one may show by the same logic used above that banks, like other agents in the private sector, should react immediately to a credible increase in the monetary base. Thus, a failure by banks to respond may reflect skepticism the central bank will carry through on its commitment to maintain its increase in the monetary base.

For simplicity of exposition, consider a bare-bones, deterministic model of intermediation in which banks hold the entire monetary base, and hold no excess reserves when the interest rate is positive. When the interest rate is zero, banks are indifferent between holding reserves and making loans, for the two assets yield the same short-term return, namely, zero. Let ν be the ratio of the monetary base to the money stock when there are no excess reserves, and let H_t be the monetary base at date t. Then, at each date t, either $i_t > 0$ and $M_t = H_t/\nu$, or $i_t = 0$ and $M_t \leq H_t/\nu$. In the latter case, M_t is indeterminate, but it will have a lower bound at which further reductions in M_t would induce a positive interest rate, which is inconsistent with $M_t < H_t/\nu$.[25]

With this characterization of bank behavior, it is straightforward to show that the analysis changes little. Consider the backward solution algorithm used above, to determine whether the economy is in a liquidity trap in period t, given a solution for the interest rate in period $t+1$ and hence the wage rate in period t, w_t. There, we used equation (14), repeated here for convenience,

$$\tilde{i}_t = \frac{w_t}{k_t M_t} - 1\,, \tag{14}$$

[25] Krugman (1998) also notes the indeterminacy of money supply in a liquidity trap.

to determine if the economy is in a liquidity trap. With a banking sector, we replace (14) with

$$\tilde{i}_t = \frac{w_t}{k_t(H_t/\nu)} - 1 \tag{14$'$}$$

to solve for the notional interest rate, $\tilde{i}_t$. Given the simplicity of the model and the fact that k is a parameter used for calibration, we can for ease of exposition and without loss of generality set $\nu = 1$, i.e.,

$$\tilde{i}_t = \frac{w_t}{k_t H_t} - 1 \tag{14$''$}$$

If $\tilde{i}_t > 0$, then the economy is not in a liquidity trap, $i_t = \tilde{i}_t$, and hence $M_t = H_t$. If $\tilde{i}_t < 0$, then the economy is in a liquidity trap and the money stock can be less than H_t. But this lower money stock does not disturb the previous solution which assumed that $M_t = H_t$. Consider the next step in the solution process. Once i_t has been determined, we solve for w_{t-1}. The solution for w_{t-1} depends on M_t only if the economy is not in a liquidity trap in period t, in which case $M_t = H_t$. If the economy is in a liquidity trap in period t, then $\frac{w_{t-1}}{k_{t-1}} = \frac{1}{\beta_t}\frac{w_t}{k_t}$, so M_t is irrelevant. Thus, the solution for w_{t-1} is not affected by the fact that M_t can be less than H_t (down to a lower bound of w_t/k_t), and the same logic holds for each successive period in the backward solution process. It follows, then, that the solution for the path of i_t, w_t, and hence prices and output as well will be unaffected by whether banks may hold excess reserves in a liquidity trap. Of particular importance, the change in the equilibrium path of output, prices, wages and interest rates induced by a change in the path of high-powered money will also be unaffected by the ability of banks to hold excess reserves. And, though banks may hold excess reserves while the liquidity trap prevails, the lower bound on the money supply will still increase in response to a permanent increase in the monetary base. For a sufficiently big monetary expansion, the new equilibrium would thus require some immediate expansion in the amount money supplied by the banks, notwithstanding the continuance of zero nominal interest rates.

Consider once again the simulation of an immediate, sustained 10 percent increase in the money stock, now with the added assumption that the government controls H rather than M and that M can be less than H if $i = 0$. Figure 6 shows that path of M and H for this experiment, with these two monetary aggregates no longer forced to be equal by assumption. For concreteness, we show the lower bound for M when the liquidity trap holds and the solution for M is indeterminate. We also show the interest rate, which is the same as in the previous simulation that held $M = H$ and, as before, unaffected by this particular policy experiment. In the simulation, both before and after the policy change, the lower bound on the money stock gradually converges on H as the economy approaches the end of the liquidity trap. But this lower bound shifts upward with the change in policy, along with the high-powered money stock.

Why is the equilibrium unaffected by the introduction of banks? Because banks know that the liquidity trap will end in period T, and that they will cease to have excess reserves at that date, this ties down their behavior in earlier periods. When the money stock is increased permanently, this increases the money stock projected for period T, and the impact of this increase cascades back to the present, following the logic of our previous analysis. Of course, this conclusion requires that banks, as well as other agents in the economy, find the commitment to a sustained policy credible. If the banks believe the expansion of H to be temporary, then that policy will not induce an immediate increase in M.

XI. Credibility of Permanent Money-Supply Changes

As we have noted, Krugman's (1998) dynamic model of the liquidity trap relies on a belief among market actors that the central bank has a rigid future target for the money-supply level. Under that assumption, any announced future increase in the money supply's level lacks credibility and markets expect any current increase to be fully reversed later on.[26] Of course, were Krugman's assumption literally true in our model, the central bank would lose its ability to influence the economy today through open-market operations. How reasonable is this outcome in the context of our model?

A. Credibility and Money-Supply Increases

Krugman's (1998) rationale for assuming that markets believe in a given future money supply (or price level) rests on the credibility problem that inflation-averse central bankers would face in promising future inflation. In Krugman's sticky-price model, the liquidity trap poses a policy problem when full employment requires a fall in the (ex ante) real rate of interest. If the nominal interest rate is not already at zero, the central bank can stabilize by lowering that rate, a current policy action that does not depend on managing market expectations about the future. If the nominal interest rate is at zero, however, the only way to lower the real rate of interest is to convince markets that the price level will be higher *next period* than had previously been expected. Credibility is an issue because the authorities reap the benefit of higher promised prices today, but may be tempted to renege on their promise in the future when it comes time to create the expected inflation. Eggertsson (2003) has neatly formalized this type of credibility problem.

[26] Krugman (1998) models the central bank as targeting a definite historical price level, in the mode of an inter-war central bank returning to pre-1914 gold exchange parity. This behavioral model, fortunately, is no longer plausible.

Our model and simulations, suggest, however, that the preceding characterization of an inflation-averse central bank's credibility problem is unrealistically special and extreme. The associated fiscal benefits affect the credibility of a permanent money-supply increase. There is no welfare gain to reversing a permanent money-stock increase (or, equivalently in our setup, to reneging on a promise to raise the date-T money stock). On the contrary, such a move would have a negative welfare impact in our model compared to non-reversal. This finding is consistent with Eggertsson's (2003) observation that when national policymakers internalize the fiscal benefits of monetary expansion, permanent money-stock increases can become quite credible. Our detailed numerical results suggest, moreover, that for Japan the fiscal benefits are large enough to overwhelm any reasonable fears about inflation, especially starting from a position where prices actually are falling. In other words, the government's net debt is already so large that authorities should perceive very powerful fiscal incentives to end deflation. Following a monetary increase that leaves some public debt outstanding, the authorities' incentive is for more of the same, rather than a reversal.

B. Credible Policy when Central Bankers Are Very Inflation Averse

Eggertsson (2003) and others have argued, however, that an independent central banker's preferences might diverge from those of the general government. In the extreme, the banker might have a lexicographic abhorrence of inflation, and thus be inclined to discount heavily or even ignore the associated benefits from public debt reduction, lower taxes, and higher output. Our model suggests, however, that even in this case, there still can be scope for significant monetary stimulus and debt reduction.

Suppose that the central bank's inflation tolerance is not literally zero; instead, the central bank is willing to tolerate an inflation rate in the range $[-\varepsilon, \pi]$ for some possibly small but positive

ε, π. We also assume that, despite having a lexicographic preference for keeping inflation in its target range, the central bank places some positive weight on at least partially fulfilling its promises by moving the money stock as far as it can in the promised direction without driving inflation our of range. That weight can be made arbitrarily small without affecting the conclusions.[27] A final assumption is that is that along the economy's initial (pre-announcement) path, inflation at date T (i.e., between dates T-1 and T), π_0, is strictly below the central bank's upper limit, π, and the deflation rate at date 0 exceeds the bank's lower limit, ε.

To reduce deflation occurring while the economy is in a liquidity trap, the central bank will wish to increase the money stock immediately and permanently, even if it places no weight at all on the additional benefits that this measure might produce for the economy. One possible equilibrium outcome following such an increase is that markets find the increase credible. As the resulting increase in inflation will play out over a relatively short period, the policy will succeed in reducing deflation around period 0 without increasing inflation around period T. In this equilibrium, the central bank brings the inflation trajectory within its target range, and the action also yields the fiscal and macroeconomic benefits to the economy discussed above.

But another potential outcome is that private agents will not find the announced permanent increase in money credible, in which case prices will not respond immediately, but only when the economy exits the liquidity trap at period T and the higher money stock affects prices directly. At that later date, however, the central bank would be forced to reverse its announced policy if failing to do so leads to an inflation rate in excess of π. But, if inflation in

[27] As will be evident, the argument that follows would go through in essence under alternative preference assumptions. Note that the assumption of central bank indifference within its target range is consistent, in a model with uncertainty, with more aggressive contractionary measures as the top of the range is approached. In a more general setting, the basic argument works even if $\varepsilon = \pi$; the mean of the central bank's inflation target range can be zero provided some range of positive inflation rates is acceptable. Interestingly, Stein (1989) shows how the authorities may be able credibly to announce a target range even when an announced point target is not credible.

period T, π_0, is not already at the central bank's ceiling, π, then this second, Pareto-inferior equilibrium cannot exist. To understand why, consider first the simple case with flexible prices.

Assume that markets do not believe that the central bank will follow through with a permanent deviation from its pre-announcement money-supply path. Then wages and prices do not move prior to date T. On that date, however, the central bank has the option to increase the money supply permanently (relative to the baseline path) by a percentage $x = \pi - \pi_0$ without breaching its upper inflation target. Because the central bank will wish at least partially to fulfill its promise of a higher money supply at T, the private sector must rationally believe that at least an x-percent money supply increase is permanent. That belief implies, however, that the price level (and with it, the entire path of future prices) will rise by x percent *immediately*.

This immediate price increase, though, implies that people would be wrong to expect no *further* increase in the date T price level. Expected inflation between dates T-1 and T is again $\pi_0 < \pi$, so there remains room for some unexpected inflation between dates T-1 and T if people do not believe that a further increase in the initial money stock is permanent. This raises the initial and subsequent price levels further, again making room for a bit more inflation between dates T-1 and T, and so on. By backward induction, the entire announced percent increase in the money stock on date 0 will be viewed as permanent under our central bank preference assumptions. In summary, the equilibrium in which the monetary expansion is non-credible requires, with flexible prices, that inflation at period T already be at least equal to the upper limit of the central bank's target range, π. But, as should be clear, and as we will now illustrate, the same logic applies even with staggered prices, as long as the inflation rate along the baseline path lies below π at date T and for the periods immediately following, during which any price increases induced by a monetary expansion at T might play out.

C. Credible Policy with Sticky Prices: An Example

For this analysis, we extend the numerical simulation model used above to allow for four-year overlapping contracts, as seems to be needed to generate more realistic price persistence.[28] To illustrate concretely the previous theoretical discussion, we assume that the central bank's allowable target range for annual inflation is [-1 percent, 3 percent].

The two solid lines in Figure 7 show the economy's path with and without a permanent, fully credible, 10 percent money-stock increase on date 0. On the initial path, the economy in the liquidity trap experiences deflation outside the central bank's target range, perhaps because of some shock to the economy. Under the credible policy, the bank succeeds in bringing deflation below 1 percent, until the economy exits from the liquidity trap and follows the positive-inflation, baseline path. In this equilibrium the central bank will not wish to reverse the policy, and indeed it will strictly prefer not to, because it places some weight on following its announced policy, which also yields an annual welfare gain of nearly 1 percent of output.

To eliminate the possibility of an equilibrium with policy reversal, however, we must ask what would happen if people believed that the announced money-stock increase was to be reversed on date $T = 5$. The dotted line in Figure 7 shows the dynamics when the public does not believe that the money supply will be above the initial path at $T = 5$, but the central bank nonetheless increases the money supply permanently by 4 percent on date 0. In that case, inflation peaks (on date $T = 8$) at just 3 percent. Given the central-bank preferences assumed above, the public should therefore anticipate that at least 4 percent of the initially announced money-stock increase will indeed be permanent. That anticipation advances the resulting inflation in time, however, making room for further credible permanent monetary expansion on

[28] See Chari, Kehoe, and McGrattan (2000).

date $T = 0$. As argued above, backward induction leads to the solid policy path in Figure 7—a credible 10 percent monetary expansion.

Our argument rests on the assumption that inflation is not already too high on date 5 and after. Otherwise, monetary expansion on date 5, no matter how small, would breach the 3-percent limit. That circumstance would indeed undermine current credibility. Thus, a very stringent upper limit on the central bank's inflation tolerance ultimately reduces its ability to fight *deflation*, for this stance takes away the bank's ability to adopt a credible increase in the money stock while the economy is in a liquidity trap. We return to this issue below.

D. Exchange-Rate Based Policy

A reflationary strategy focused on pegging the exchange rate, such as the one advocated by Svensson (2001, 2003), is in purely economic terms quite similar in its fiscal implications. In principle such a policy could be decided by the government without decisive central bank input, but the approach suffers from credibility problems of its own. A substantial and purposeful exchange rate devaluation by the government of Japan would elicit strong protests from trading partners, and hence pressures for reversal. While the comparable yen depreciation induced by a large one-time money-supply increase would of course put trading partners in the same economic position, the Japanese government would be able to claim that the depreciation was a side effect of domestic policies aimed at enhancing Japanese growth and import demand.[29] Furthermore, if the Japanese government is indeed viewed as being committed to a long-run price-level target, or even if its simply inflation-averse, a depreciated yen currency peg could be

[29] From a political point of view, the incremental market-induced depreciations following *gradual* money-supply expansion would be easier to defend as a side product of domestically necessary policies. As we have shown, however, an anticipated increase in money growth could have a smaller welfare benefit than a commensurate step increase in money. The political difficulties inherent in Svensson's proposal are illustrated by the current U.S. criticism of China for pegging its currency at a competitive level with respect to American goods.

susceptible to a revaluation attack, as were the German and Swiss currencies near the end of the Bretton Woods period.[30]

E. Quantitative Easing in Japan

Starting in March 2001, the Bank of Japan initiated an explicit program of "quantitative easing" that has already expanded the monetary base, but with no discernible impacts on inflation or the nominal exchange rate (Shirakawa 2002). The most dramatic yearly base increase, a log change of 24.3 percent, occurred in the year starting March 2001; in the following year the log base change was 12.8 percent, or about half as big as in the preceding year.[31] Is the Bank of Japan's seeming failure to ignite inflation evidence of deflationary expectations so entrenched that open-market policy cannot be effective? Not necessarily. The quantitative easing was not accompanied by a tax cut, as our analysis would recommend, and the absence of complementary fiscal policy has reduced the total potential expansionary effect. In addition, Japan's price level could well have fallen even more absent the monetary ease—the most recent period of base expansion has seen significant economic shocks, including the slowdown in the United States. There has also been apparent money-demand instability, including a substantial increase in currency demand, perhaps the result of financial-sector distress.

A possible upward shift in the demand function for money cannot be the full explanation for the apparent failure of quantitative easing, however. One notable feature of the recent Japanese experience, similar to what occurred in the United States during the 1930s, is that the evolution of broader monetary aggregates has not come near to matching that of the monetary base; instead, banks have chosen to vastly augment their excess reserves.

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[30] Hungary experienced such an inflow attack on the strong edge of its currency band in January 2003.

[31] Data are reported by the Bank of Japan, http://www.boj.or.jp/en/stat/boj/base0312_f.htm.

Our model suggests two potential explanations for the increase in excess reserves. One is that the bulk of the base expansion is not viewed by markets as permanent, in which case, in a liquidity trap, banks do not lend and the economy is unaffected. Figure 6, however, suggests another possibility. The limited lending out of newly created reserves could simply reflect an expectation that the return of positive interest rates is quite distant. If date T is sufficiently remote in Figure 6, the two cones of indeterminacy can overlap, creating the possibility that an infusion of monetary base stimulates the economy today, while still having little or no effect on the current broad money supply. In that case, the increase in broad money will begin before date T, but with some delay. If this latter explanation rather than the one based on credibility holds, one would have to attribute observed price and exchange rate behavior after March 2001 to strong unobserved deflationary pressures that were partially offset by quantitative easing.

It is also likely that the troubled state of Japan's financial institutions and of many potential borrowers helps to explain the reluctance of banks to lend. Financial-sector distress in Japan must be addressed aggressively and soon, as it is in any case a major impediment to Japanese recovery. Our analysis suggests that market anticipations of a healthy financial sector in the future can enhance the efficacy of current monetary expansion by creating expectations of a higher broadly-defined money stock; this makes financial restructuring all the more urgent.

Given the hesitancy of Bank of Japan policy in general and the retreat from the very strong quantitative easing in the year following March 2001, it is likely that imperfect credibility is part of the explanation for the slow price and exchange rate responses.[32] An important point of our analysis, however, is that in our analytical setting, imperfect credibility need not reside inherently in the objectives of inflation-averse central bankers. In the Japanese case, other forces seem to be at work as well, for example, concerns over the appearance of accommodating public deficits or worries that a return to positive interest rates might injure banks and worsen the public finances. Weakness of financial institutions and of corporate balance sheets no doubt is hampering the effectiveness of monetary transmission as well.[33] Finally, as we have noted, it could well be that Japan's recent deflation would have been harsher absent the Bank of Japan's quantitative easing. The Japanese economy's counterfactual path is not directly observable.

Another possibility suggested by our model is that the public expects very high inflation once short-term interest rates turn positive. As we noted above, that possibility might make it rational for people to expect a future reversal of current quantitative easing. The fact that the Bank of Japan has somehow managed to increase the monetary base by a large amount without seeming effect might make the possibility of high future inflation more plausible. Won't inflation skyrocket if ever the public comes to view past quantitative operations as permanent? High expected future inflation seems an unlikely hypothesis, however, in view of the rather low level of long-term nominal interest rates. In our model, the scope for some credible monetary

[32] As Bernanke (2003, p. 7), puts it, the "obvious reluctance on the part of the BOJ to sail into uncharted waters may have had the effect of muting the psychological impact of the non-standard actions it has taken." In that vein, the concluding remarks of Shirakawa (2002, p. 33), who reviews the quantitative easing policy after one year, could be construed as tentatively declaring the policy a failure and arguing that the failure was to be expected on the basis of "standard theory." Although the Bank of Japan has had goal- as well as instrument-independence since 1998 and has a statutory obligation to maintain "price stability" (see Cargill, Hutchison, and Ito 2000), it has never defined its interpretation of that concept in operationally meaningful quantitative terms. (For a review of Bank of Japan actions and discussions, see Ito 2004.) If the public believes the Bank views price stability narrowly as an inflation rate no greater than zero, then our analysis shows that the Bank indeed will lack the credibility to fight deflation.

[33] The possibility of a direct stimulus that does not rely on the banks' response is an advantage of Svensson's (2001, 2003) exchange-rate based proposal, though, as we have noted, that plan has some disadvantages too.

expansion disappears only when expected future inflation is close to (and on at least one date equal to) the central bank's upper inflation limit.

To the extent that credibility has been a problem, the task of the Japanese authorities may be to persuade markets of the permanence of past monetary expansions more than to carry out further expansions. One way to do so would be the forceful announcement of an explicit allowable inflation target range including *positive* rates of inflation. Even a symmetric range of moderate rates, centered on zero, would aid credibility. Unfortunately, rather than taking an approach that explicitly allows for a period of moderate positive inflation, the majority of the Bank of Japan Policy Board, at its June 25, 2004 meeting, declined to clarify its inflation goals in the alleged interest of preserving monetary-policy "flexibility" and "credibility."[34]

The experience of the United States during the Great Depression provides a perspective on Japan's effort at monetary stimulus. Between 1933 and 1940, the stock of high-powered money in the U.S. nearly tripled.[35] It is noteworthy that after a spike in inflation due to the 1933 devaluation, the U.S. price level rose only gradually until 1941. As is likely in Japan today, monetary expansion had to offset the deflationary pressure caused by an output level well below full employment. Romer (1992) argues persuasively that this monetary expansion was, nonetheless, the main cause of U.S. recovery from the Depression, especially the sharp 1938-1942 increase in output (by 49 percent).

[34] See section III of "Minutes of the Monetary Policy Meeting on June 25, 2004" posted at http://www.boj.or.jp/en/seisaku/04/seisak_f.htm. There are other credibility-enhancing actions available, such as sterilized purchases of foreign exchange (Eggertsson 2003), which have been undertaken, and government withdrawal of short-term government debt and its replacement by long-term debt. The latter action, which apparently has not yet been proposed, would send a credible signal of government intentions to end deflation. Instead, in 2003 Japan's government introduced price-level indexed debt, which could be interpreted by the markets as a bet on a continuing fall in prices.

[35] High-powered money data are taken from Friedman and Schwartz (1963). All other data cited are from the NBER Macro History database.

If the experience of the Great Depression is any guide, Japan may need even more quantitative easing to combat its deflation. However, the Bank of Japan's reluctance to identify a range of moderate positive inflation rates consistent with its view of price stability may be the most important obstacle to ending deflation. The simultaneous opportunity for public debt reduction provides strong additional motivation for this policy.

XII. Summary and Conclusion

We have confirmed the intuition that a substantial monetary expansion undertaken in a liquidity trap should improve welfare by reducing the taxes required in the future to service the national debt. This, in itself, is an important finding, for it suggests a role for monetary policy even if the policy has no immediate impact on prices, output or interest rates. But we have also shown that this policy can effect an immediate expansion in prices and, with less than fully flexible prices, output as well. Thus, monetary policy remains an important policy instrument for an economy mired in a liquidity trap, even if the liquidity trap is severe and expected to last a long time.

Entrenched price expectations surely are a barrier to policy success in Japan. In view of the large economic benefits available, however, sustained policy action coupled with better communication of strategy to the public should be able to modify the deflationary psychology.

Appendix on the Government Budget Constraint

Consider the government's budget constraint. Starting with the expression above for the evolution of household wealth,

$$V_{t+1} = V_t(1+r_t) + \frac{\Omega_t}{P_t} + \frac{w_t L_t}{P_t} - (1+\tau_t)(1+i_t)C_t\,,$$

we impose the national income identity equating purchases to factor incomes and obtain:

$$V_{t+1} = V_t(1+r_t) - [(1+\tau_t)(1+i_t) - 1]C_t .$$

Imposing the transversality condition, we get:

$$V_0 = \sum_{t=0}^{\infty}\left(\prod_{s=0}^{t}(1+r_s)^{-1}\right)[(1+\tau_t)(1+i_t)-1]C_t$$

$$= \sum_{t=0}^{\infty}\left(\prod_{s=0}^{t}(1+i_s)^{-1}\right)\frac{P_t}{P_{-1}}[(1+\tau_t)(1+i_t)-1]C_t$$

$$= \frac{1}{P_{-1}}\sum_{t=0}^{\infty}\left(\prod_{s=0}^{t}(1+i_s)^{-1}\right)[(1+\tau_t)(1+i_t)-1]\frac{w_t}{k_t(1+i_t)(1+\tau_t)}$$

$$= \frac{1}{P_{-1}}\sum_{t=0}^{\infty}\left(\prod_{s=0}^{t}(1+i_s)^{-1}\right)\left[1-\frac{1}{(1+i_t)(1+\tau_t)}\right]\frac{w_t}{k_t}$$

$\Rightarrow$

$$(M_0 + B_0) = \sum_{t=0}^{\infty}\left(\prod_{s=0}^{t}(1+i_s)^{-1}\right)\left[1-\frac{1}{(1+i_t)(1+\tau_t)}\right]\frac{w_t}{k_t} .$$

This expression relates the government's current obligations to its future seigniorage and tax revenue. Note that an open market operation has no impact on the left-hand side of the expression, so one solves for the change in tax rates that keeps the right-hand side constant as the policy changes.

Finally, let us assume that the tax rate τ is constant over all future periods. With this assumption, the path of wages obtained in the text of the paper, and the fact that the interest rate is 0 through date T-1 and determined by the Euler equation based on successive money stocks from T onward, we may use the above expression for the government's budget constraint to solve for the necessary tax rate, τ. We solve for τ to obtain

$$\tau = \frac{\dfrac{M_0 + B_0}{\left(\prod_{t=0}^{T} \beta_t^{-1}\right) M_T} - \sum_{t=T}^{\infty} \left(\prod_{s=0}^{t} \beta_s\right)\left(1 - \beta_{t+1} \dfrac{M_t}{M_{t+1}}\right)}{\sum_{t=0}^{\infty} \left(\prod_{s=0}^{t} \beta_s\right) - \dfrac{M_0 + B_0}{\left(\prod_{t=0}^{T} \beta_t^{-1}\right) M_T}}. \tag{A1}$$

The second term in the numerator reflects the seigniorage collected on household holdings of money after date T-1. The term $\left(\prod_{t=0}^{T} \beta_t^{-1}\right) M_T$ measures the impact of the money stock at date T on the wage rate at date 0. Under the assumption that the cash-in-advance constraint only starts to bind for cash balances held at the beginning of date T, the money stock before T is irrelevant to the determination of wages and prices.

References

Akerlof, George A., William T. Dickens, and George L. Perry. "The Macroeconomics of Low Inflation." *Brookings Papers on Economic Activity* (1:1996): 1-76.

Auerbach, Alan J., and Maurice Obstfeld. "Monetary and Fiscal Remedies for Inflation." Paper presented at the American Economic Association meetings, San Diego (January 2004).

Barro, Robert J., and David B. Gordon. "A Positive Theory of Monetary Policy in a Natural Rate Model." *Journal of Political Economy* 91 (August 1983): 589-610.

Benhabib, Jess, Stephanie Schmitt-Grohé, and Martin Uribe. "Avoiding Liquidity Traps." *Journal of Political Economy* 110 (June 2002): 535-63.

Bernanke, Ben S. "Japanese Monetary Policy: A Case of Self-Induced Paralysis?" In *Japan's Financial Crisis and Its Parallels to U.S. Experience*, ed. Adam S. Posen and Ryoichi Mikitani. Washington, D.C.: Institute for International Economics, 2000.

Bernanke, Ben S. "Some Thoughts on Monetary Policy in Japan." Speech before the Japan Society of Monetary Economics, Tokyo (May 31, 2003).

Calvo, Guillermo A. "On the Time Consistency of Optimal Policy in a Monetary Economy." *Econometrica* 46 (November 1978): 1411-28.

Cargill, Thomas F., Michael M. Hutchison, and Takatoshi Ito. *Financial Policy and Central Banking in Japan* (Cambridge, MA: MIT Press, 2000).

Chari, V. V., Patrick J. Kehoe, and Ellen R. McGrattan. "Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?" *Econometrica* 68 (September 2000): 1151-79.

Clouse, James, Dale Henderson, Athanasios Orphanides, David Small, and Peter Tinsley. "Monetary Policy When the Short-Term Nominal Interest Rate is Zero." *Topics in Macroeconomics* 3 (2003): http://www.bepress.com/bejm/topics/vol3/iss1/art12.

Dekle, Robert. "The Deteriorating Fiscal Situation and an Aging Population." In *Structural Impediments to Growth in Japan*, ed. Magnus Blomström, Jennifer Corbett, Fumio Hayashi, and Anil Kashyap. Chicago: University of Chicago Press, 2003.

Eggertsson, Gauti. "How to Fight Deflation in a Liquidity Trap: Committing to Being Irresponsible." Photocopy: International Monetary Fund (February 2003).

Eggertsson, Gauti, and Michael Woodford. "The Zero Bound on Interest Rates and Optimal Monetary Policy." *Brookings Papers on Economic Activity* (1:2003): 139-233.

Friedman, Milton, and Anna Jacobson Schwartz. *A Monetary History of the United States,1867-1960* (Princeton, NJ: Princeton University Press for the National Bureau of Economic Research, 1963).

Goodfriend, Marvin. "Overcoming the Zero Bound on Interest Rate Policy." *Journal of Money, Credit, and Banking* 32 (November 2000, Part 2): 1007-35.

Goyal, Rishi, and Ronald McKinnon. "Japan's Negative Risk Premium in Interest Rates: The Liquidity Trap and Fall in Bank Lending." *World Economy* 26 (March 2003): 339-63.

Hayashi, Fumio, and Edward C. Prescott. "The 1990s in Japan: A Lost Decade." *Review of Economic Dynamics* 5 (January 2002): 206-35.

Ito, Takatoshi. "Inflation Targeting and Japan: Why Has the Bank of Japan Not Adopted Inflation Targeting?" Working Paper no. 10818, National Bureau of Economic Research (September 2004).

Kashyap, Anil K. "Sorting Out Japan's Financial Crisis." *Federal Reserve Bank of Chicago Economic Perspectives* 26 (Fourth Quarter 2002): 42-55.

King, Mervyn. "Challenges for Monetary Policy: New and Old." In *New Challenges for Monetary Policy* (Kansas City, MO: Federal Reserve Bank of Kansas City, 1999).

Krugman, Paul R. "It's Baaack: Japan's Slump and the Return of the Liquidity Trap." *Brookings Papers on Economic Activity* (2:1998): 137-205.

McCallum, Bennett T. "Japanese Monetary Policy, 1991-2001." *Federal Reserve Bank of Richmond Economic Quarterly* 89 (Winter 2003): 1-31.

Ministry of Finance, Japan. *Current Japanese Fiscal Conditions and Issues to Be Considered*. http://www.mof.go.jp/english/budget/pamphlet/cjfc.htm (July 2002).

Nishimura, Kiyokiko G., and Masato Shirai. "Fixed Costs, Imperfect Competition and Bias in Technology Measurement: Japan and the United States." Working Paper no. F-97, Center for International Research on the Japanese Economy (November 2000).

Orphanides, Athanasios, and Volker Wieland. "Efficient Design of Monetary Policy near Price Stability." *Journal of the Japanese and International Economies* 14 (December 2000): 327-65.

Romer, Christina D. "What Ended the Great Depression?" *Journal of Economic History* 52 (December 1992): 757-84.

Schmitt-Grohé, Stephanie, and Martin Uribe. "Optimal Fiscal and Monetary Policy under Sticky Prices." Working Paper no. 9220, National Bureau of Economic Research (September 2002).

Shirakawa, Masaaki. "One Year under 'Quantitative Easing'." Discussion Paper no. 2002-E-3, Institute of Monetary and Economic Studies, Bank of Japan (April).

Siu, Henry E. "Optimal Fiscal and Monetary Policy with Sticky Prices." *Journal of Monetary Economics* 51 (April 2004): 575-607.

Stein, Jeremy C. "Cheap Talk and the Fed: A Theory of Imprecise Policy Announcements." *American Economic Review* 79 (March 1989): 32-42.

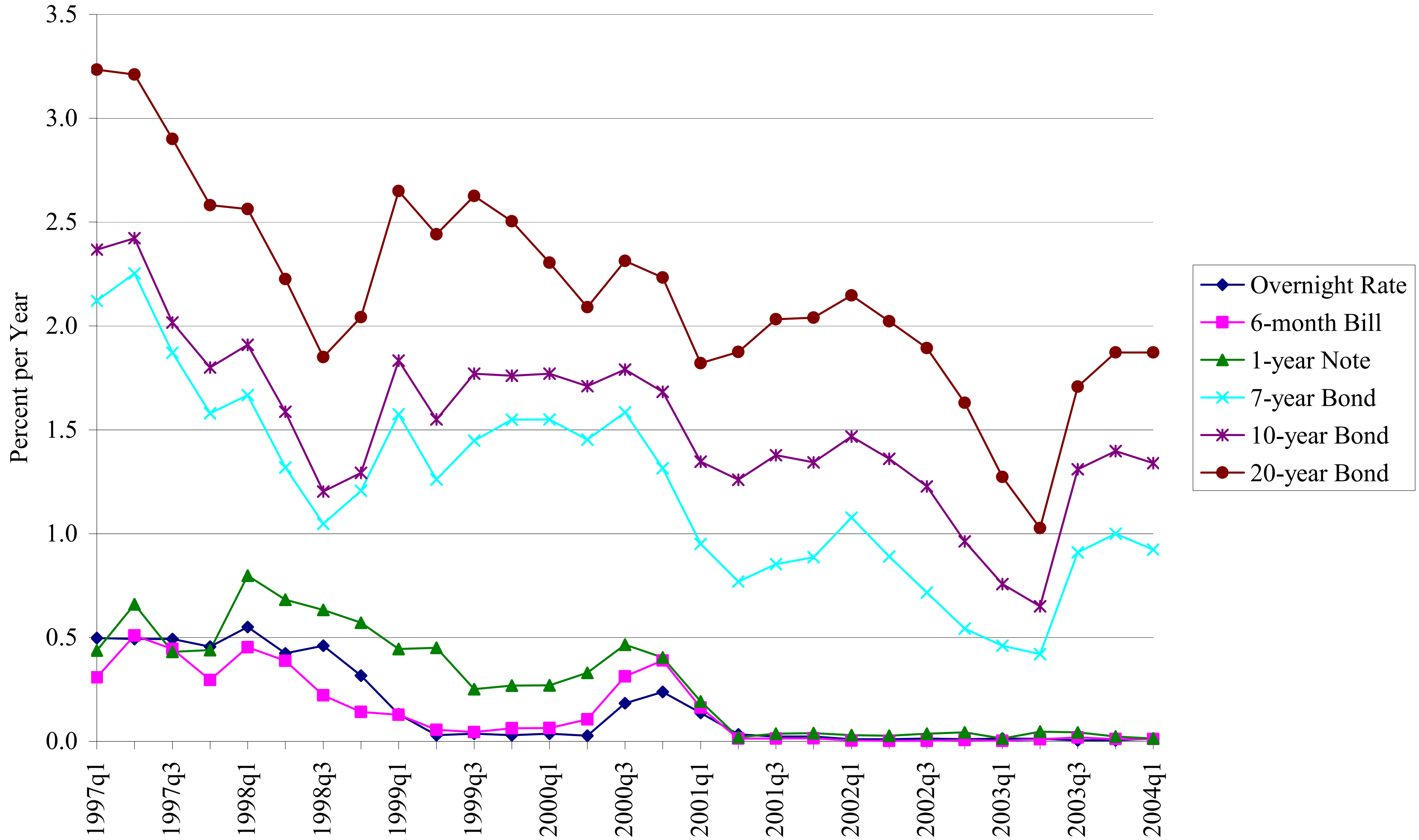
Svensson, Lars E. O. "The Zero Bound in an Open Economy: A Foolproof Way of Escaping from a Liquidity Trap." *Monetary and Economic Studies* 19 (February 2001): 277-312.

Svensson, Lars E. O. "Escaping from a Liquidity Trap and Deflation: The Foolproof Way and Others." *Journal of Economic Perspectives* 17 (Fall 2003): 145-66.

Wolman, Alexander L. "A Primer on Optimal Monetary Policy with Staggered Price Setting." *Federal Reserve Bank of Richmond Economic Quarterly* 87 (Fall 2001): 27-52.

Woodford, Michael. *Interest Rates and Prices: Foundations of a Theory of Monetary Policy* (Princeton, NJ: Princeton University Press, 2003).

Figure 1: Japan's Term Structure



Source: Global Financial Data

Figure 2. Initial Trajectories

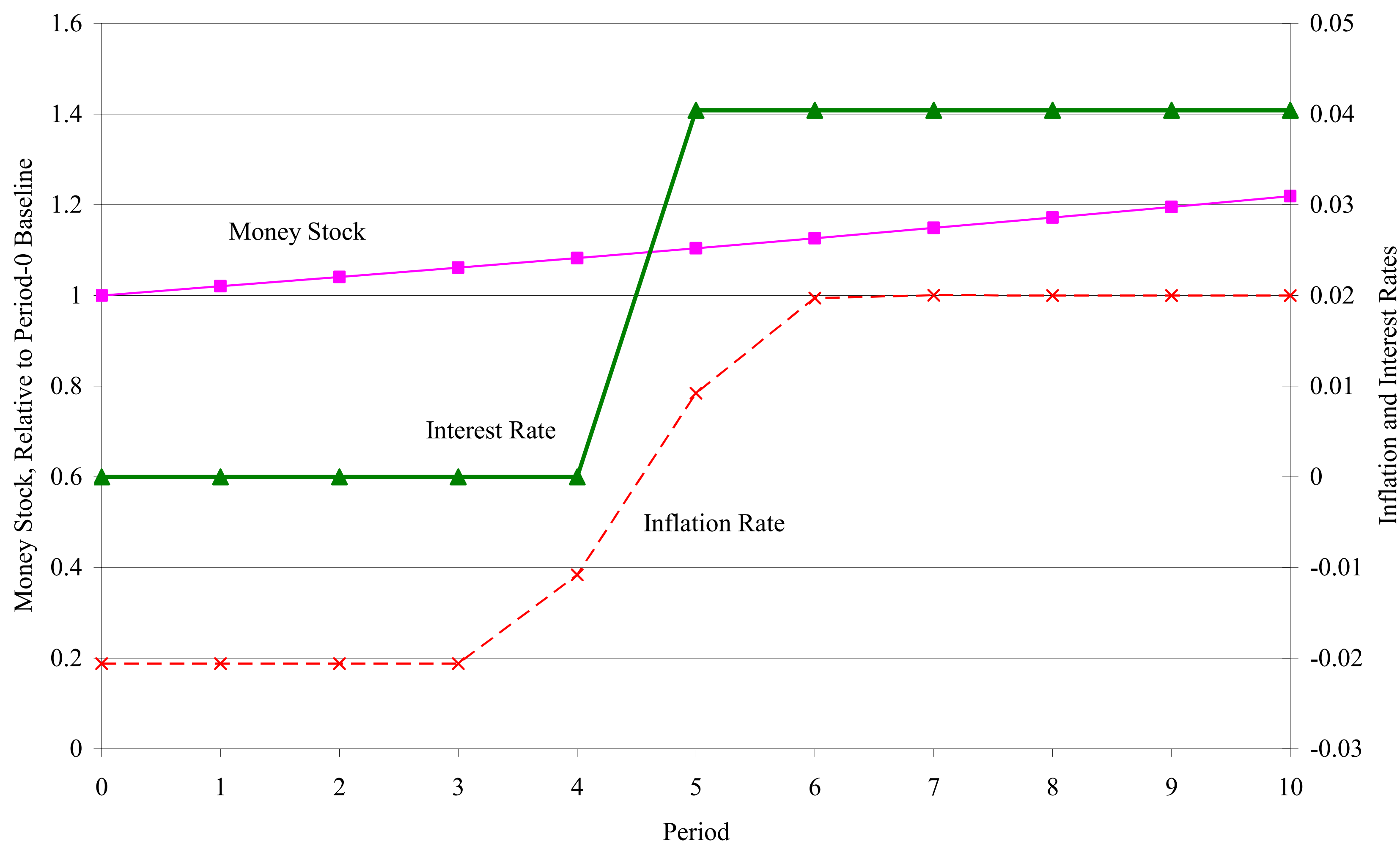


Figure 3. One-Time Increases in the Money Stock

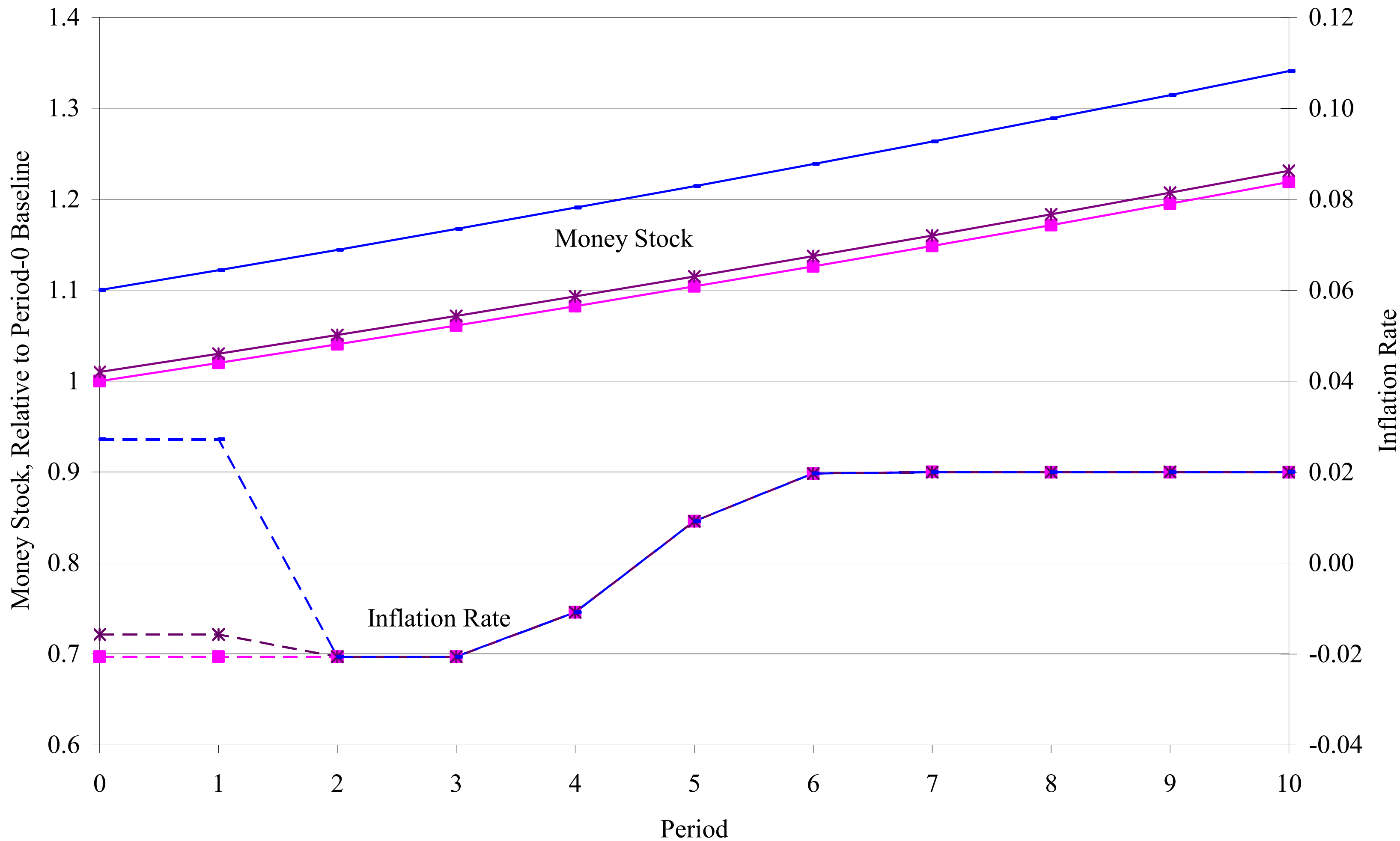


Figure 4. Escaping the Liquidity Trap with Money Growth

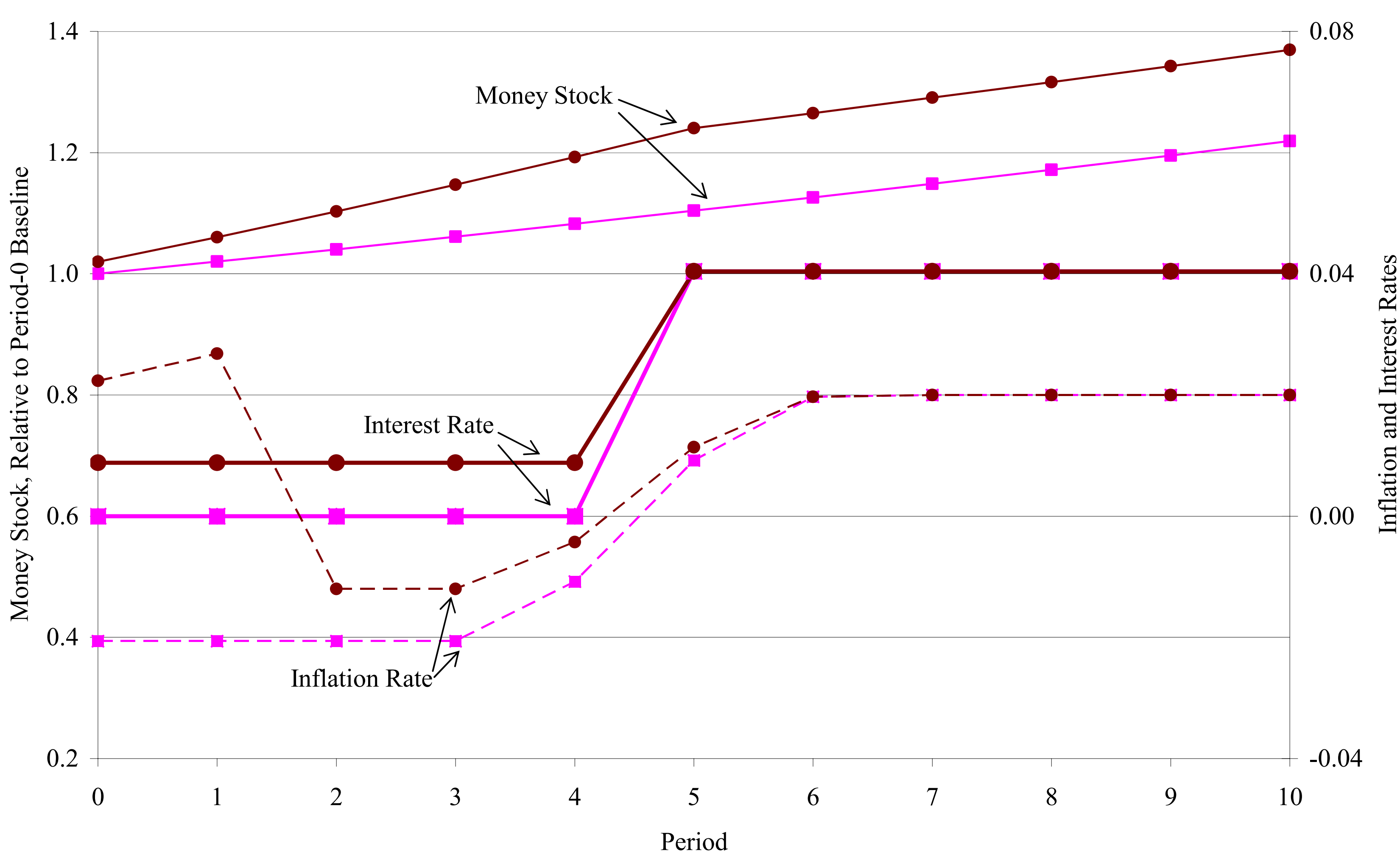


Figure 5. Consumption Trajectories

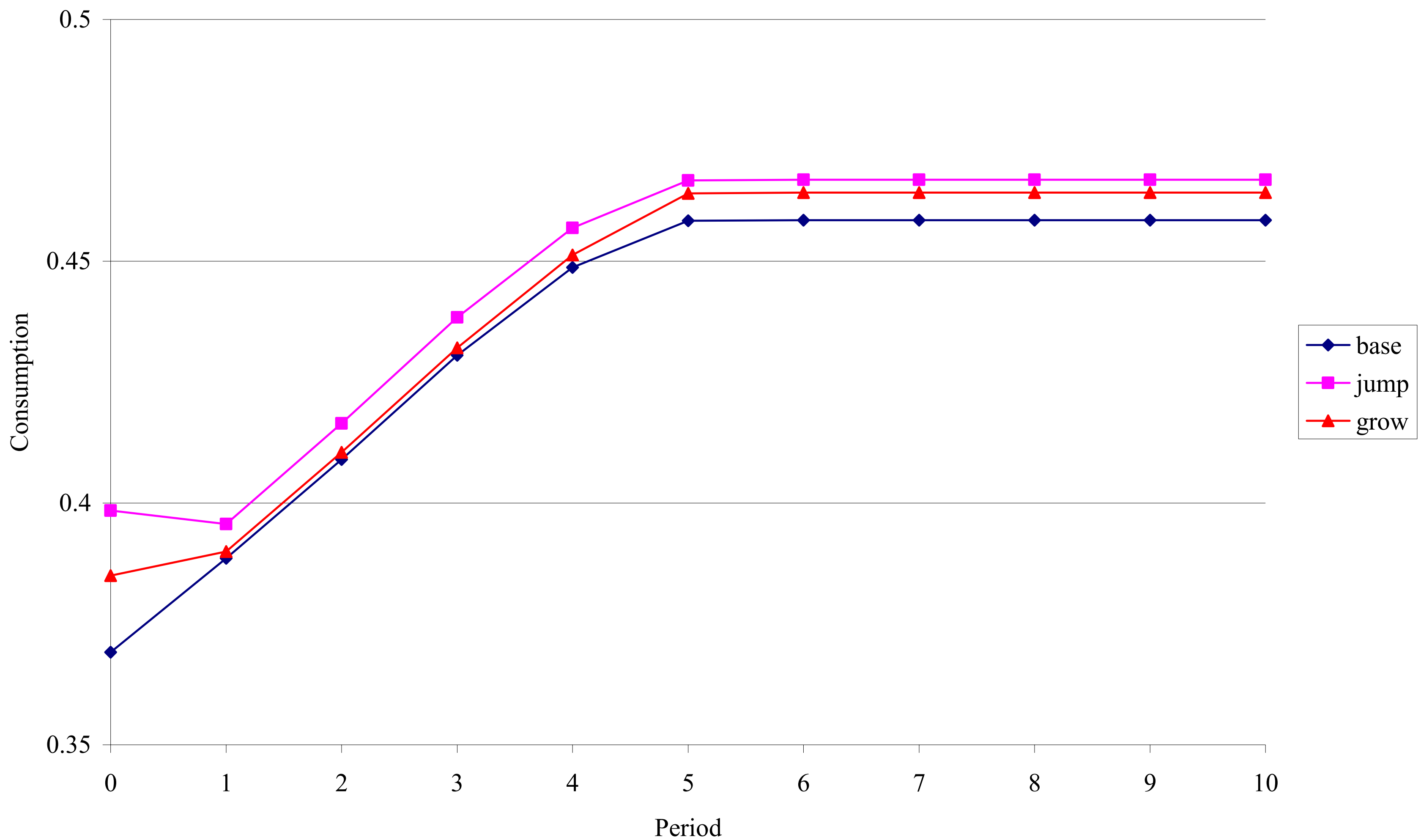


Figure 6. Money Supply and Interest Rates

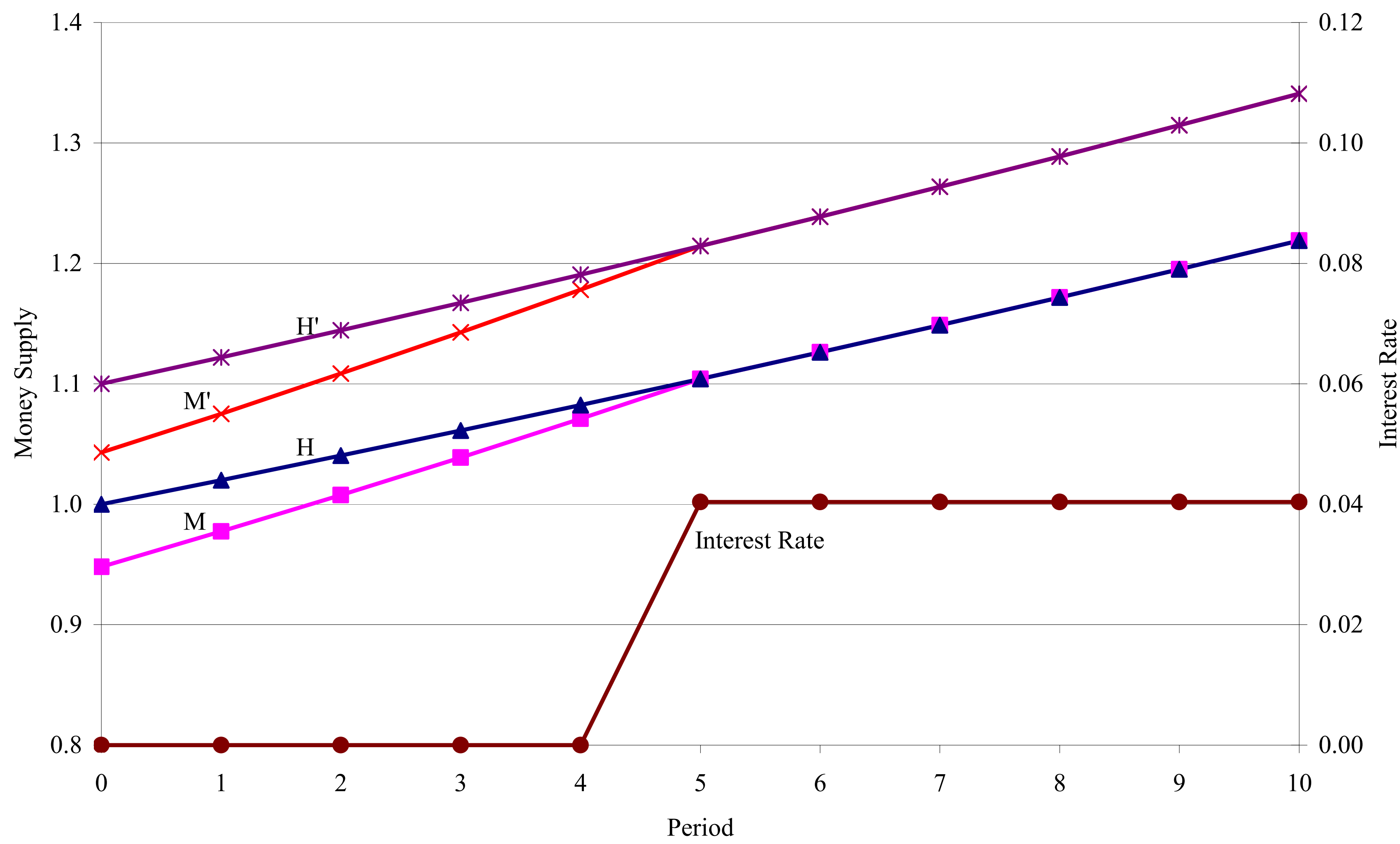


Figure 7. Inflation Under Four-Period Staggered Pricing