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ELASTIC STRAIN ENERGIES OF UNDISSOCIATED JOGS

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INTRODUCTION

Kroupa and Brown[1] recently calculated the strain and interaction energies of jogs based on Yoffe's method[2] of combining solutions for energies of simple dislocation line configurations. These calculations are repeated here employing the more basic method of analysis due to Kröner[3] and expounded by deWit.[4] Whereas this method gives the same interaction energies as those obtained by Brown and Kroupa, small differences from Brown and Kroupa's results are obtained for the elastic self energies of jogs. These differences can be interpreted in terms of the differences in the cut-off of the dislocation cores.

ELASTIC STRAIN ENERGIES

Elastic strain energies were determined for the double jog configuration shown in Fig. 1. As described by deWit the energy, E, is given by

$$E = 1/2\, b_i b_j M_{ij} \tag{1}$$

where b_i is the component of the Burgers vector in the i direction and M_{ij} is an inductance factor given by

$$M_{ij} = \frac{-G}{8\pi} \oint_C \oint_{C'} \left\{ \left[\frac{1+\nu}{1-\nu} \right] R_{,kk}\, dl_i'\, dl_j \right. $$
$$\left. + \frac{2}{1-\nu} \left(R_{ij} - R_{,ll}\, \delta_{ij} \right) dl_k'\, dl_k \right\} \tag{2}$$

over the realm where Hookes law applies, isotropy is assumed, G is the shear modulus of elasticity, ν is Poisson's ratio, C and C' imply the same closed dislocation loop and dl_i and dl_i' refer to the ith components of the line elements along the dislocation. The symbols refer to common tensor notation where δ_{ij} is the Kronecker delta, $R = (X_i X_i)^{1/2}$, and

$X_i = x_i - x_i'$, x_i being the ith component of the coordinates of the dislocation segment dl, etc. The singularity at the dislocation core was eliminated from the integral (Eq. 2) by avoiding a segment of width $\zeta = b$ on either side of the singularity in a manner similar to that of Lothe.[5] Care must be taken in setting the limits of integration of Eq. 2 so that this condition is <u>always</u> fulfilled.

Whereas Eq. 2, when integrated around a closed loop, gives the total elastic strain energy of the loop our interest centers only on the jog energies and their interaction energy. Therefore the jogged dislocation segment of the loop is taken to extend undeviated to ± R where $R \gg a_1 \simeq a_2$, $R \gg r$ and the remainder of the loop is very far from the jogs. When the energy of a similar loop containing an equally long unjogged dislocation is subtracted from the energy of the jogged loop, Eq. 2 reveals that the energy so obtained refers only to the energies of the jogs on a long dislocation line where the total energy E^T of the jogs is

$$E^T = E_{j1}^S + E_{j2}^S + E^I \{r\}, \tag{3}$$

the first two terms to the right of the equality sign referring to the self energies of the jogs and the last term referring to the interaction energy of the pair of jogs. The interaction energy neglecting higher order terms in a_1/r and a_2/r is the same as that given by Kroupa and Brown, namely

$$E^I = \frac{-Ga_1 a_2}{8\pi(1-\nu)r} \{[b_1^2 + (1-2\nu)b_2^2 + (1+\nu)b_3^2]\cos\alpha - 2\nu b_1 b_2 \sin\alpha\} \tag{4}$$

The self energies, as shown in the first two rows of the following table, are slightly different from the values obtained by Kroupa and Brown.

Table of Jog Self Energies

Dislocation type	Frank	Edge	Screw
Elastic Energy of Kroupa and Brown*	$\dfrac{Gb_1^2 a}{4\pi(1-\nu)}\left[\ln\dfrac{2a}{\epsilon}-2\right]$	$\dfrac{Gb_2^2 a}{4\pi}\left[\ln\dfrac{2a}{\epsilon}-\dfrac{2-\nu}{1-\nu}\right]$	$\dfrac{Gb_3^2 a}{4\pi(1-\nu)}\left[\ln\dfrac{2a}{\epsilon}-2+\nu\right]$
Elastic Energy obtained by Us*	$\dfrac{Gb_1^2 a}{4\pi(1-\nu)}\left[\ln\dfrac{a}{\zeta}+2\left(\dfrac{\zeta}{a}-1\right)\right]$	$\dfrac{Gb_2^2 a}{4\pi}\left[\ln\dfrac{a}{\zeta}+\dfrac{(2\frac{\zeta}{a}-1)-\nu(\frac{\zeta}{a}-1)}{(1-\nu)}\right]$	$\dfrac{Gb_3^2 a}{4\pi(1-\nu)}\left[\ln\dfrac{a}{\zeta}+(2-\nu)\left(\dfrac{\zeta}{a}-1\right)-1\right]$
Suggested Core Energy Form	$\dfrac{Gb_1^2 a}{4\pi(1-\nu)}\left[\ln\dfrac{\zeta}{b}-2\left(\dfrac{\zeta}{a}-1\right)+\alpha_F\right]$	$\dfrac{Gb_2^2 a}{4\pi}\left[\ln\dfrac{\zeta}{b}-\dfrac{(2\frac{\zeta}{a}-1)-\nu(\frac{\zeta}{a}-1)}{(1-\nu)}+\alpha_E\right]$	$\dfrac{Gb_3^2 a}{4\pi(1-\nu)}\left[\ln\dfrac{\zeta}{b}-(2-\nu)\left(\dfrac{\zeta}{a}-1\right)+1+\alpha_S\right]$
Total Jog Energy	$\dfrac{Gb_1^2 a}{4\pi(1-\nu)}\ln\dfrac{a}{b}e^{\alpha_F}$	$\dfrac{Gb_2^2 a}{4\pi}\ln\dfrac{a}{b}e^{\alpha_E}$	$\dfrac{Gb_3^2 a}{4\pi(1-\nu)}\ln\dfrac{a}{b}e^{\alpha_S}$

*Both ϵ and ζ are of the order of magnitude of b

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This difference very likely arises from a difference in the application of the cut-off of the core energy. Whereas Kroupa and Brown suggest that the equations are invalid for small values of a_1 and a_2 because their energies then become negative, it is our contention that the equations are correct for small jogs, any negative values arising because of the additonal discarded core volume in the jogged dislocation. On this basis, we suggest that the energy be empirically formulated as given in the third row of the Table of Jog Self Energies where the α's have the values appropriate for the cut-off that was employed. Then the corresponding total energy of the jogs are as given in the last row of the table.

ACKNOWLEDGMENT

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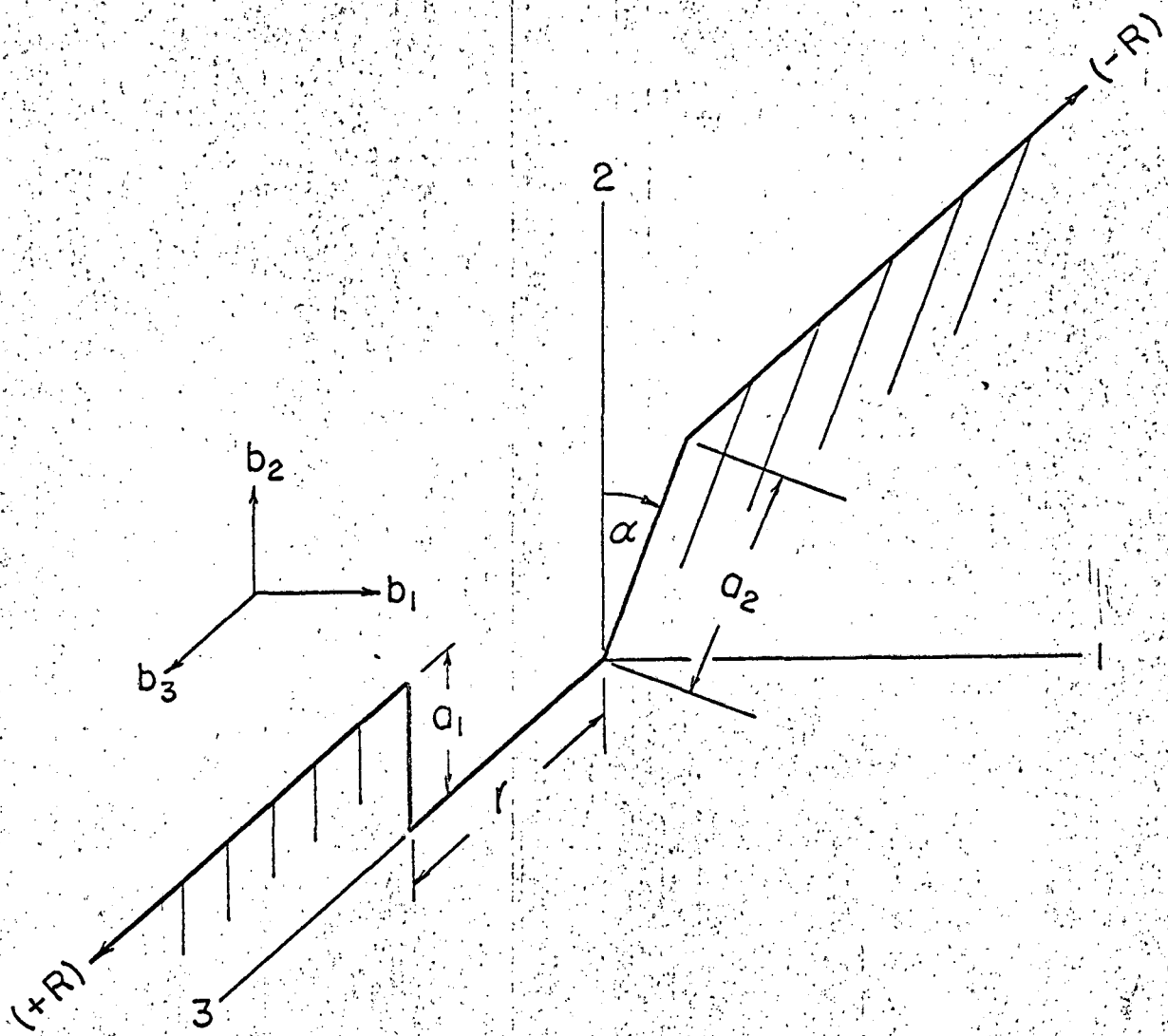
FIG. I. DOUBLE JOG CONFIGURATION.