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K. A. Brueckner

February 3, 1950

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ABSTRACT

The general features of the process of photo-meson production are discussed on a semi-classical basis and also on the basis of perturbation theory in the weak coupling approximation applied to scalar, pseudoscalar, vector, and pseudovector meson theory. These calculations are carried out with the Feynman-Dyson methods to give covariant expressions for the cross sections. The corrections of next higher order in the meson-nucleon coupling are calculated for pseudoscalar mesons and shown to give large contributions. Comparison of the conclusions of the theory with the experimental results seems to indicate that the meson is of spin zero and closely bound to the nucleons as is characteristic of pseudoscalar theory.

THE PRODUCTION OF MESONS BY PHOTONS

K. A. Brueckner

February 3, 1950

The production of mesons by photons has recently been accomplished experimentally,[1] and at present extensive experimental work is being done on the problem.[2] It has been found that the present experimental techniques permit the study of the production of mesons by photons with more accuracy and in more detail than is possible for production by nucleon-nucleon collision. The theoretical analysis of such information is also considerably simplified by the nature of the interaction. It will be shown that the results depend very markedly on the nature of the coupling of the mesons to the electromagnetic field. Since processes involving such coupling can be handled very well for non-relativistic energies by treating the interactions as weak, the usual methods of perturbation theory can be applied with some confidence to this aspect of the production process. The uncertainties of the nature of the coupling of mesons to nucleons, which, as is well known, lead to incorrect prediction of scattering phenomena, do not strongly affect this process. In fact, the very characteristic differences between the behaviour of the photon-ejected spin zero and spin one mesons will be shown to be due almost entirely to the nature of the meson coupling to the electromagnetic field.

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[1] McMillan, Peterson, and White; Science 110, 579 (1949)

[2] Cook, Steinberger, McMillan, Peterson, White; private communications

The theory of the production of photo-mesons has been studied by a number of people,[3] with the most complete work being the recent contribution of Feshbach and Lax. The methods used have been those of perturbation theory in the weak coupling approximation. Effects of the recoil of the nucleons have been neglected. The results obtained differ markedly for the various theories. It is not immediately apparent how the characteristic differences are related to the detailed features of the theories. It is therefore of interest to attempt to understand by classical argument, without reference to perturbation theory, some details of the process. It has further been thought worthwhile to carry out the calculations using the new covariant formalism, eliminating unnecessary approximations and exhibiting the simplicity of the methods. Finally, the higher order corrections in the meson-nucleon interaction have been calculated for the pseudoscalar theory, using the new subtraction techniques, and found to give finite and unambiguous results.

I General Description

A. Ratio of Cross Section for Production of Negative and Positive Mesons

One of the most striking of the early experimental observations on the production of mesons by photons was the excess of negative over positive mesons.[1] At present this is still the best established experimental fact. It was pointed out by Brueckner and Goldberger[4] that very simple classical

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[3] W. Heitler, Proc. Roy. Soc. 166, 529 (1938)
M. Kobayashi and T. Okayama, Proc. Phys. Math. Soc. Japan 21, 1 (1939)
H. S. W. Massey and H. C. Corben, Proc. Camb. Soc. 35, 84 (1939) and 35, 463 (1939)
L. Nordheim and G. Nordheim, Phys. Rev. 54, 254 (1938)
H. Feshbach and M. Lax, Phys. Rev. 76, 134 (1949)
L. Foldy, Phys. Rev. 76, 372 (1949)

[4] K. Brueckner and M. Goldberger, Phys. Rev. 76, 1725 (1949)

arguments could give an explanation of this result. These arguments simply pointed out that there is an essential asymmetry in the production of negative and positive mesons. When the former are produced from neutrons, the charge-carrying nucleon is the final proton with large recoil velocity. When positive mesons are produced, the proton is initially at rest and does not interact through its charge. Using the interaction

$$e \frac{\vec{v} \cdot \vec{A}}{1 - v/c \cos\Theta} \tag{1}$$

which differs from the non-relativistic $e\,\vec{v} \cdot \vec{A}$ because of retardation effects in the interaction of charge with the electromagnetic field, one obtains for the ratio of the interactions leading to the production of positive and negative mesons

$$\frac{I\ (\text{positives})}{I\ (\text{negatives})} = \frac{e \dfrac{\vec{v} \cdot \vec{A}}{1 - v/c \cos\Theta}\ (\text{meson})}{-e \dfrac{\vec{v} \cdot \vec{A}}{1 - v/c \cos\Theta}\ (\text{meson}) + e \dfrac{\vec{v} \cdot \vec{A}}{1 - v/c \cos\Theta}\ (\text{recoil proton})} \tag{2}$$

The ratio of the cross sections then is the square of the ratio of the interactions. Using over-all energy and momentum conservation, this ratio can be written

$$\frac{\sigma\ (\text{positives})}{\sigma\ (\text{negatives})} = \left[1 - \frac{q_o}{Mc^2} (1 - v/c \cos\Theta) \right]^2 \tag{3}$$

where

q_o = meson energy including rest energy

M = nucleon mass

v = meson velocity

θ = angle between meson and photon

The dependence on meson energy and angle of this function is given explicitly in Figure 1.

In this very simple argument, the effects of the magnetic moments of the particles have been ignored. It is of interest, therefore, to look in more detail at the nature of the interaction, including such effects. This will indicate what information can be given by a careful measurement of the negative-to-positive ratio. One can formulate the argument in the following manner: the interaction with the electromagnetic field leading to the ejection of mesons is of the form

$$I = A_\mu \int j_\mu (\vec{r}', t)\, e^{i(\vec{K} \cdot \vec{r}' - K_o t)}\, d\vec{r}' dt \tag{4}$$

where j_μ is the total current carried by the interacting particles. The current can be separated into a curl-free and a divergence-free part, the former corresponding to a linear motion of charge and the latter to circulating currents.

$$j_\mu = q v_\mu + \frac{\partial M_{\mu\nu}}{\partial x_\nu} \tag{5}$$

where v_μ is the relativistic velocity with spacial components $\vec{v}/(1-\beta^2)^{1/2}$, q the charge, and $M_{\mu\nu}$ is an anti-symmetric tensor. The interaction then is

$$\begin{aligned} I &= A_\mu \int \left(q v_\mu + \frac{\partial M_{\mu\nu}}{\partial x_\nu} \right) e^{i(\vec{K} \cdot \vec{r}' - K_o t)}\, d\vec{r}' dt \\ &= A_\mu \int (q v_\mu - i M_{\mu\nu} K_\nu)\, e^{i(\vec{K} \cdot \vec{r}' - K_o t)}\, d\vec{r}' dt \end{aligned} \tag{6}$$

We can easily evaluate this integral, considering each interacting particle separately. If the wave-length of the radiation is large compared with the region over which the charges and currents are distributed, or if one assumes a delta-function of position for the spacial distribution, the dependence on $\vec{r}'$ can be given by

$$\begin{aligned} v_\mu(\vec{r}', t) &= \vec{v}(t)\, \delta\left[\vec{r}' - \vec{r}(t)\right] \\ &= v_\mu(t)\, \delta\left[\vec{r}' - \vec{r}(t)\right] (1-\beta^2)^{1/2} \\ M_{\mu\nu}(\vec{r}', t) &= M_{\mu\nu}(t)\, \delta\left[\vec{r}' - \vec{r}(t)\right] (1-\beta^2)^{1/2} \end{aligned} \tag{7}$$

The integral for each particle then is

$$I = A_\mu \int \left[q v_\mu(t) - i M_{\mu\nu}(t) K_\nu \right] (1-\beta^2)^{1/2}\, e^{i\left[\vec{K} \cdot \vec{r}(t) - K_o t\right]}\, dt \tag{8}$$

Changing variable and partially integrating gives

$$I = A_\mu \int \frac{d}{ds} \left[\frac{v_\mu(s) - iM_{\mu\nu}(s)K_\nu}{K_o(1-\beta\cos\theta)} \right] e^{is} (1-\beta^2)^{1/2} \tag{9}$$

If the interaction takes place over a time much less than the period of the radiation, the variation of the exponential term can be ignored. This gives

$$I = -\Delta \left[\frac{A_\mu P_\mu + \frac{m}{2} M_{\mu\nu} F_{\mu\nu}}{\underset{\sim}{K \circ P}} \right] \tag{10}$$

where the four-vector product is

$$\underset{\sim}{K \circ P} = -K_o P_o + \vec{K} \circ \vec{P}$$

and $\Delta[\quad]$ denotes the change during the interaction.

In this expression for the interaction of the electromagnetic field with the charges and currents of the meson-nucleon system, we have not included the dependence of the meson emission on such factors as the strength of the coupling of the mesons to the nucleons and the spins of the nucleons. We shall assume that a simple multiplicative factor U gives the spin dependence of the forces and the strength of the coupling. When we consider the application of field theory to this process, we shall see that this actually is so for spin-zero mesons. The quantum nature of the process also has not been considered. We actually need the matrix element between the initial and final nucleon states of the operator which we have derived. The interaction therefore can be written for each particle

$$\begin{aligned} I &= \psi_F^\dagger \left\{ U \Delta \left[\frac{q\vec{A} \circ \vec{P} + \frac{m}{2} F_{\mu\nu} M_{\mu\nu}}{\underset{\sim}{K \circ P}} \right] \right\} \psi_I \\ &\equiv \overline{U} \Delta \left(q \frac{\vec{A} \circ \vec{P}}{\underset{\sim}{K \circ P}} \right) + \frac{m}{2} F_{\mu\nu} \Delta (\overline{U M_{\mu\nu}}) \end{aligned} \tag{11}$$

We can use this expression to evaluate the ratio of the cross sections for negative and positive mesons. For simplicity we will take the mesons to have spin zero, i.e., no magnetic moment, and first assume that the nucleons interact only in the proton state. When a positive meson is produced, the proton is the initial nucleon at rest and so interacts only through its magnetic moment with the transverse photon.

$$I(+) = -e \frac{\vec{A} \cdot \vec{q}\, \overline{U}}{K \cdot q} + \frac{m}{2} \frac{\overline{U M_{\mu\nu}}}{K \cdot I} F_{\mu\nu} \qquad (12)$$

q = meson 4-momentum

I = initial nucleon 4-momentum

When a negative meson is produced, the proton is the final recoil nucleon carrying current $e \vec{F}/m$

$$I(-) = e \left(\frac{\vec{A} \cdot \vec{q}}{K \cdot q} - \frac{\vec{A} \cdot \vec{F}}{K \cdot F} \right) \overline{U} - \frac{m}{2} \frac{\overline{UM_{\mu\nu}}}{K \cdot F} F_{\mu\nu} \qquad (13)$$

F = final nucleon 4-momentum

If we now use over-all 4-momentum conservation, we find that

$$\vec{A} \cdot \left(\frac{\vec{q}}{K \cdot q} - \frac{\vec{F}}{K \cdot F} \right) = \frac{\vec{A} \cdot \vec{q}}{K \cdot q} \; \frac{K \cdot I}{K \cdot F}$$

and

$$I(-) = - \frac{K \cdot I}{K \cdot F} \left[e \frac{\vec{A} \cdot \vec{q}}{K \cdot q} \overline{U} + \frac{m}{2} \frac{\overline{UM_{\mu\nu}}}{K \cdot I} F_{\mu\nu} \right] \qquad (14)$$

$$= - \frac{K \cdot I}{K \cdot F} I(+)$$

Therefore, under these assumptions, the plus-to-minus ratio would be

$$\frac{\sigma(+)}{\sigma(-)} = \left(\frac{K \cdot F}{K \cdot I} \right)^2 \qquad (15)$$

$$= \left[1 - q_o/Mc^2 (1 - v/c \cos \theta) \right]^2$$

This is the same result obtained when magnetic moment terms were ignored (Equation 3). It is interesting to observe that the ratio of the linear current interactions is the same as the ratio of the magnetic-moment interactions, so that the plus-to-minus ratio is independent of the values of

$$\overline{U} = \psi_F^{\dagger}\, U\, \psi_I$$

$$\overline{UM_{\mu\nu}} = \psi_F^{\dagger}\, UM_{\mu\nu}\, \psi_I$$

If the nucleons do not interact only in the proton state, but instead with anomalous moments, we can write

$$M_{\mu\nu}\ (\text{proton}) = \gamma_P M_{\mu\nu}$$
$$M_{\mu\nu}\ (\text{neutron}) = -\gamma_N M_{\mu\nu} \qquad (16)$$

where γ = magnitude of magnetic moment of nucleon in Bohr magnetons

$$\gamma_P = 2.87$$
$$\gamma_N = 1.91 \qquad (17)$$

This gives for the ratio of the interactions

$$\frac{I(+)}{I(-)} = \frac{-e\,\frac{\vec{A}\cdot\vec{q}}{K\cdot q}\,\overline{U} - \frac{m}{2}F_{\mu\nu}\left(\frac{\gamma_P}{K\cdot I} + \frac{\gamma_N}{K\cdot F}\right)\overline{M_{\mu\nu}\,U}}{e\,\frac{\vec{A}\cdot\vec{q}}{K\cdot q}\,\overline{U}\,\frac{K\cdot I}{K\cdot F} + \frac{m}{2}F_{\mu\nu}\left(\frac{\gamma_P}{K\cdot F} + \frac{\gamma_N}{K\cdot I}\right)\overline{M_{\mu\nu}U}} \qquad (18)$$

If $\overline{M_{\mu\nu}\,U} = \overline{M_{\mu\nu}}\,\overline{U}$

then the factor $\overline{U}$ cancels. The ratio of the cross section averaged over photon polarization and nucleon-moment orientation gives

$$\frac{\sigma(+)}{\sigma(-)} = \left(\frac{K\cdot F}{K\cdot I}\right)^2 \left[1 + 0\left(\frac{\mu^2}{M^2}\right)\right]$$

Therefore, if the interaction is of this type, the ratio of the cross sections is nearly unchanged by the anomalous moments, since $\frac{\mu^2}{M^2} \sim 2$ percent. This is a result of the predominance of the coupling of the electromagnetic field to the linear motion of charge, i.e., to the electric dipole formed by the meson-nucleon charges.

If $\overline{M_{\mu\nu}U} >> \overline{M_{\mu\nu}}\,\overline{U}$, or what is equivalent, the magnetic moment terms are predominant in the interaction,

$$\frac{\sigma(+)}{\sigma(-)} = \left[1 - \frac{\gamma_P - \gamma_N}{\gamma_P + \gamma_N}\,\frac{q_0}{Mc^2}\left(1 - \frac{v}{c}\cos\theta\right)\right]^2$$
$$= \left[1 - .20\,\frac{q_0}{Mc^2}\left(1 - \frac{v}{c}\cos\theta\right)\right]^2 \qquad (19)$$

This function is given graphically in Figure 1. It is apparent that, if nucleon moment interactions are important, the introduction of a magnetic moment for the neutron removes much of the asymmetry in the process leading to the production of positive and negative mesons, leading to a plus-to-minus ratio which is nearly one.

We remark here that we have assumed that, with photons at energies of about the meson rest energy, the nucleons interact with the anomalous moments observed in a static field. This will be true only if the circulating currents which give rise to the anomalous moments are confined to a region small compared with the wave-length of the radiation. If this is not true, the anomalous moments will show energy dependence and the values of the static moments cannot be used.

We have ignored the interaction of the meson magnetic moments. These could contribute only if the meson were a vector particle, i.e., with spin one. It is apparent that such interactions are symmetrical for the production of either negative or positive mesons. Therefore strong meson-moment interactions would give a negative-to-positive ratio close to one.

One can conclude that a verification of the results (3) would indicate that the meson does not interact strongly through a magnetic moment and that the neutron anomalous magnetic moment does not play an important part in the process. We have seen that these conclusions do not depend on the nature of the coupling of the mesons to the nucleons.

B. Angular Distribution

If the photon is absorbed by the ejected meson at photon energies for example of 200-300 Mev, $\beta = v/c$ for the meson will not be small compared with one. The angular distribution will show large asymmetry about 90 degrees due to the presence in the differential cross section of the denominator

$$\frac{1}{(1 - \beta \cos\theta)^2} \tag{20}$$

which appears because of the retardation effects in the interaction of charge with the electromagnetic field. However, if the absorption of the photon is through its coupling with the nucleon, $v/c << 1$ for the nucleon and the angular distribution will be nearly symmetric about 90 degrees. One would therefore expect that the degree of symmetry would indicate which particle interacts most strongly with the photon.

If the interactions are principally of the form $e\,\vec{v} \cdot \vec{A}$ then, since the photon field is transverse, the angular spectra must fall off to zero at 0 or 180 degrees. Spectra which do not exhibit this behaviour must be due to magnetic moment-like interactions.

II Principal Features of Various Theoretical Results

The calculations, unless otherwise specified, are for the first non-vanishing order in which the process can take place. The nucleons are treated as Dirac particles and the effects of their recoil are fully taken into account. The scalar, pseudoscalar, vector, and pseudovector theories are considered, using the couplings with the nucleon field which do not involve derivatives of the meson field.

The spectra shown in Figures 2, 3, 4 and 5 are for mesons produced by a dK/K photon spectrum. This approximates the bremsstrahlung energy distribution at high energies which is used in the laboratory to produce mesons. The relation between the meson and the photon energy, as a function of angle, is given in Figure 9.

A. Scalar Meson

The scalar meson characteristically shows a dipole angular distribution (Figure 2) at low energies which is strongly distorted forward at energies above a few Mev by the denominator (Equation 20). This is due to the predominance of the photon-meson interaction and the absence of a magnetic moment for the meson. This simple result is quite analogous to the low-energy photo-electric effect in an atom, since the mesons are spherically symmetric in distribution about the nucleon, and the photon interacts relatively weakly with the nucleon. This is in accordance with the fact that the meson cloud about the nucleon extends to distances of the order $\hbar/\mu c$, while the circulating currents associated with the magnetic moment of the nucleon are distributed over a region of order $\hbar/Mc$ and therefore give contributions which are smaller in the ratio $\left(\frac{\mu}{M}\right)^2$. The plus-to-minus ratio from the lowest order calculations is the same as that obtained by the classical argument, as would be expected since the nucleons are treated as Dirac particles and the meson has no magnetic moment interaction.

B. Pseudoscalar Meson

The pseudoscalar theory shows a roughly isotropic angular distribution (Figure 3) indicating the predominance of the coupling of the photon to the magnetic moment of the nucleon. The unimportance of the electric-dipole terms in the interaction, i.e., the coupling to the linear motion of charge, is due to the close binding of the meson cloud to the nucleon. The probability of finding a meson at a distance $\hbar/\mu c$, which is the wave-length of the photon, is relatively small. However, the direct coupling of the photon to the magnetic moment of the nucleon, preceded or followed by the emission of the final meson, is relatively probable. Examination of the matrix elements involved in these transitions indicates that this behaviour is due to the existence of intermediate states in which the nucleon undergoes transitions to negative energy states. For such processes the matrix elements of the couplings to the electromagnetic and meson fields are of the order one instead of v/c for the nucleons. The ratio of negative to positive mesons given by the lowest order calculation is the same as that for the scalar theory.

Preliminary experimental results indicate that the spectrum of mesons observed is roughly isotropic in angular distribution as predicted by the pseudoscalar theory. Therefore this theory was selected for examination of the corrections of higher order in the nucleon-meson coupling constant g. This involves another emission and reabsorption of a virtual meson, as indicated in Figure 7. It was hoped that the investigation of the corrections would answer questions concerning the contribution of the anomalous magnetic moments of the nucleons and the possible large effects of corrections to processes in which the photon is coupled directly to the meson or its associated Dirac vacuum field. Examination of the effects of higher order processes would also indicate the probability that the expansion in powers of $g^2/4\pi$ would actually lead to a convergent series.

The calculation of the next higher order terms shows that they give contributions about as large as the first order terms, therefore casting grave doubts on the convergence of the expansion and the validity of the application of perturbation methods of this kind.

The largest contribution to the corrections comes from the anomalous magnetic moments of the nucleons, Figures 7-B1, 7-B2 and 7-B3. The anomalous moments differ considerably from those which the nucleons exhibit in a static field, as calculated by Case.[5] Non-static effects occur in the interaction which decreases the magnitude of the moments and also change the sign of the proton moment. This result indicates that it may not be correct to assume that the nucleons interact, with their static

[5] Phys. Rev. 76, 1725 (1949)

moments, in high energy processes. The actual magnitude of these corrections, however, probably can be believed only in a very qualitative way.

Corrections in which the meson creates virtual nucleon pairs (Figure 7-A_1), one of which may interact directly with the photon (Figure 7-A_2), give negligible contributions and so do not affect the predominance of the coupling of the photon to the nucleon through the Dirac and anomalous moments.

C. Vector and Pseudovector

The vector theory shows a strongly asymmetric angular distribution (Figure 4), indicating the predominance of the photon-meson interaction. This distribution is also peaked markedly forward showing that the interaction is due at least in part to the magnetic moment of the meson. The pseudovector theory shows a nearly isotropic angular spectrum (Figure 5). This, however, is not due to the largeness of the nucleon photon coupling but to the large magnetic moment interaction of the meson. The negative to positive ratio (Figure 6) also reflects the large effects of the magnetic moment of the spin-one mesons, differing considerably from the result obtained by ignoring the meson moment. Also very characteristic of vector and pseudovector meson is the rapid increase of the cross section with energy (Figures 4 and 5). This is due both to the strong energy dependence of the electromagnetic field coupling with longitudinally polarized mesons and to the magnetic moment interactions.

Higher order corrections probably would not particularly affect these results, since processes involving production of virtual nucleon pairs by the mesons seem to give negligible contributions. Higher order processes involving the coupling of the photon to the nucleon and its associated meson field would give corrections to terms which are already relatively unimportant in the process.

III Method of Calculation, Lowest Order

The calculation of matrix elements can be greatly simplified by use of the Feynman-Dyson methods.[6] The necessary operators can be derived by a technique due to Feynman, the correctness of which can also be demonstrated by the Dyson methods. The calculations can also be carried out by the older methods of perturbation theory to give exactly the results derived here. The meson couplings to the nucleon field that are used are those which do not involve derivatives of the meson field, since these introduce non-renormalizable singularities in higher order processes.

In the following, the notation used has $\hbar = c = M = 1$, where M is the nucleon mass. Therefore, all energies and momenta will be measured in units of the nucleon mass. All products of the form $\underset{\sim}{A}\cdot B$ will be understood to be 4-vector products, with $\underset{\sim}{A}\cdot B = -A_0 B_0 + \vec{A}\cdot\vec{B}$. We also use the notation $\underset{\sim}{U} = U_\mu \gamma_\mu$ with the Dirac matrixes $\gamma_i = i\,\alpha_i\,\beta$ $(i = 1,2,3)$, $\gamma_4 = \beta$. The adjoint operator $\psi^\dagger$ is related to the complex conjugate by $\psi^\dagger = i\psi^* \gamma_4$.

The equation of motion for the fields are

A. Free Particles

$$(\underset{\sim}{P} - i)\psi = 0 \qquad \text{Dirac} \qquad (21)$$

$$(\Box - \kappa^2)\phi = 0 \qquad \text{spin zero} \qquad (22)$$

$$\frac{\partial}{\partial x_\mu}\left(\frac{\partial \phi_\nu}{\partial x_\mu} - \frac{\partial \phi_\mu}{\partial x_\nu}\right) - \kappa^2 \phi_\nu = 0 \qquad \text{spin one} \qquad (23)$$

The last equation can also be written

$$(\Box - \kappa^2)\phi_\nu = 0 \qquad \text{spin one}$$

with the divergence condition

$$\frac{\partial \phi_\mu}{\partial x_\mu} = 0 \qquad (24)$$

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[6] R. P. Feynman, Phys. Rev. 76, 769 (1949)
F. J. Dyson, Phys. Rev. 75, 1736 (1949)

B. Interaction Of Mesons With Dirac Field

$$(\Box - \kappa^2)\phi = g\psi^\dagger U \psi \qquad U = (-1)^{1/2} \quad \text{scalar} \tag{25}$$

$$= \gamma_5 \quad \text{pseudoscalar}$$

$$-\frac{\partial}{\partial x_\mu}\left(\frac{\partial \phi_\nu}{\partial x_\mu} - \frac{\partial \phi_\mu}{\partial x_\nu}\right) - \kappa^2 \phi_\nu = g\psi^\dagger U_\nu \psi \tag{26}$$

$$U_\nu = \gamma_\nu \quad \text{vector}$$

$$= \gamma_5 \gamma_\nu \quad \text{pseudovector}$$

The last equation can be written

$$(\Box - \kappa^2)\phi_\nu = \left(\delta_{\mu\nu} - \frac{\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial x_\nu}}{\kappa^2}\right) g\psi^\dagger U_\mu \psi \tag{26'}$$

$$(\underset{\approx}{P} - i)\psi = g\phi U \psi \quad \text{spin zero} \tag{27}$$

$$= g\phi_\nu U_\nu \psi \quad \text{spin one}$$

C. Interaction With Electromagnetic Field

$$(\underset{\approx}{P} + e\underset{\approx}{A} - i)\psi = 0 \quad \text{Dirac} \tag{28}$$

$$\left[\left(\frac{\partial}{\partial x_\mu} + ieA_\mu\right)\left(\frac{\partial}{\partial x_\mu} + ieA_\mu\right) - \kappa^2\right]\phi = 0 \quad \text{spin zero} \tag{29}$$

$$\left(\frac{\partial}{\partial x_\mu} + ieA_\mu\right)\left[\left(\frac{\partial}{\partial x_\mu} + ieA_\mu\right)\phi_\nu - \left(\frac{\partial}{\partial x_\nu} + ieA_\nu\right)\phi_\mu\right] - \kappa^2\phi_\nu = 0 \quad \text{spin one} \tag{30}$$

To order e, these can be written

$$(\underset{\approx}{P} - i)\psi = -e\underset{\approx}{A}\psi \quad \text{Dirac} \tag{28'}$$

$$(\Box - \kappa^2)\phi = -2ceA_\mu \frac{\partial \phi}{\partial x_\mu} \quad \text{spin zero} \tag{29'}$$

$$(\Box - \kappa^2)\phi_\nu = -ie\left(2A_\lambda \frac{\partial}{\partial x_\lambda}\delta_{\mu\nu} - A_\nu \frac{\partial}{\partial x_\mu} - \frac{\partial A_\nu}{\partial x_\mu} - A_\mu \frac{\partial}{\partial x_\nu}\right)\phi_\mu \quad \text{spin one} \tag{30'}$$

If we consider the direct solution of these equations of motion, following Feynman's general arguments, we find the following expressions to be inserted into the Feynman-Dyson diagrams:

B'. Emission Of a Meson Of Momentum P_μ — gU — spin zero (31)

$$g\left(\delta_{\mu\nu} + \frac{P_\mu P_\nu}{\kappa^2}\right)U_\mu \qquad \text{spin one, polarization } \nu$$

C'. Absorption Of a Photon	$-e\underline{A}$	Dirac
	$2e\underline{A}\cdot\underline{P}'$	spin zero, momentum P'_μ
$e\left[2\underline{A}\cdot\underline{P}'\delta_{\mu\nu} - A_\nu P'_\mu - K_\mu A_\nu - A_\mu q_\nu\right]$		spin one, momentum P'_μ, initial polarization μ, final polarization ν

D. Propagation of an Intermediate Particle With Momentum P'_μ

$$\frac{1}{\underline{P}' - i} \qquad \text{Dirac} \tag{33}$$

$$\frac{-1}{P'^2 + \kappa^2} \qquad \text{boson, mass } \kappa$$

Exponential factors of the form $e^{i\underline{P}\cdot\underline{x}}$ have been omitted since after spacial integrations have been carried out, they simply give 4-momentum conservation at each point of the diagram and for the over-all process.

Using these expressions we can now write down matrix elements directly. A virtual meson emitted by a nucleon can absorb a photon and go into a free meson. This is represented by the diagram A, Figure 7.

$$g e \psi_F^\dagger \left[2\underline{A}\cdot q \quad \frac{-1}{(q-K)^2 + \kappa^2} U\right] \psi_I \phi \qquad \text{spin zero} \tag{34}$$

$$g e \psi_F^\dagger \left(\delta_{\mu\nu} + \frac{q'_\mu q'_\nu}{\kappa^2}\right) U_\mu \left(\frac{-2\underline{A}\cdot q \delta_{\lambda\nu} + A_\lambda q'_\nu + K_\nu A_\lambda + A_\nu q_\lambda'}{(q-K)^2 + \kappa^2}\right) \psi_I \phi_\lambda \tag{35}$$

spin one

The nucleon can emit a real meson, going into a virtual intermediate state, and then absorb the photon. The photon absorption can also come first, followed by the meson emission. This process is represented by the diagram B, Figure 7.

$$-ge\,\psi_F^{\dagger}\left(\underset{\approx}{A}\,\frac{1}{\underset{\approx}{F}-\underset{\approx}{K}-i}\,U\right)\psi_I\,\phi \qquad \text{spin zero} \tag{36}$$

$$-ge\,\psi_F^{\dagger}\left[\underset{\approx}{A}\,\frac{1}{\underset{\approx}{F}-\underset{\approx}{K}-i}\left(\delta_{\mu\nu}+\frac{q_\mu q_\nu}{\kappa^2}\right)U_\mu\right]\psi_I\,\phi_\nu \qquad \text{spin one} \tag{37}$$

If the nucleons are treated as Dirac particles, the photon can be absorbed only by a proton. Therefore, this diagram represents the production of a negative meson. For production of a positive meson, simply replace I by F and invert the order of the operators.

Combining these two contributions from diagrams A and B, we obtain for the lowest order matrix element for the transition

$$M_2 = ge\,\psi_F^{\dagger}U\left(\frac{A\cdot q}{K\cdot q}-\frac{A\cdot I}{K\cdot I}-\frac{\underset{\approx}{K}\underset{\approx}{A}}{2I\cdot K}\right)\psi_I\,\phi \qquad \text{spin zero} \tag{38}$$

For negative mesons replace I by F in the bracket. This is of the form (Equation 11) derived above, with the moment tensor for the Dirac field

$$M_{\mu\nu} = \frac{e}{2}\,\frac{\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu}{2i} = \frac{e}{2}\,\sigma_{\mu\nu}$$

$$M_2 = ge\,\psi_F U_\nu\left[\frac{2A\cdot q\,\phi_\nu + A_\nu\,\phi\cdot K - K_\nu\,\phi\cdot A}{2\,q\cdot K} - \phi_\nu\,\frac{A\cdot I}{K\cdot I}\right. \tag{39}$$

$$\left. -\phi_\nu\,\frac{\underset{\approx}{K}\underset{\approx}{A}}{2I\cdot K} - \frac{F_\nu - I_\nu}{\kappa^2}\left(\frac{q\cdot K\,\phi\cdot A - A\cdot q\,\phi\cdot K}{2q\cdot K}\right)\right]\psi_I \qquad \text{spin one}$$

For negative mesons replace I by F in bracket and

$$U_\nu\underset{\approx}{K}\underset{\approx}{A} \quad \text{by} \quad -\underset{\approx}{A}\underset{\approx}{K}\,U_\nu \tag{40}$$

In these expressions we can demonstrate gauge invariance by substituting

$$A_\mu = A_\mu^0 + \frac{\partial\Lambda}{\partial x_\mu}$$

$$= A_\mu^0 + iK_\mu\Lambda \tag{41}$$

which should leave the matrix element for the transition M_2 unchanged. This is equivalent to showing that replacing A_μ by K_μ reduces M_2 to zero. That this is so can be seen by inspection, using

$$\underset{\approx}{K}\,\underset{\approx}{K} = K_\mu K_\mu = 0$$

If we now specialize to the transverse vector potential, $A \cdot I$ is zero since the momentum I_μ is along the direction of the photon momentum. We then have for the spin-zero mesons

$$M_2 \text{ (positive)} = g e \psi_F^\dagger \phi U \frac{\vec{A} \cdot \vec{q}}{K \cdot q} - \frac{KA}{2 I \cdot K} \tag{42}$$

$$M_2 \text{ (negative)} = -g e \psi_F^\dagger \phi U \left[\vec{A} \cdot \left(\frac{\vec{q}}{K \cdot q} - \frac{\vec{F}}{F \cdot K} \right) - \frac{KA}{2 F \cdot K} \right] \psi_I$$

$$= \frac{K \cdot I}{K \cdot F} M_2 \text{ (positives)}$$

This is the same result as that obtained above (Equation 7).

The differential cross section in the laboratory system then can be obtained from

$$d\sigma = 2\pi \left| M_2 \right|^2 P_F \tag{43}$$

where M_2 is to be summed over meson and final nucleon spins and averaged over initial nucleon spin and photon polarization. The sum over meson spin can be made easily. If ϵ_μ is the 4-vector representing the direction of polarization of the meson which satisfies

$$\epsilon_\mu q_\mu = 0 \qquad \text{divergence condition} \tag{44}$$

$$\epsilon_i^2 + \epsilon_4^2 = \frac{1}{2 q_o} \qquad \text{normalization condition} \tag{45}$$

then

$$\sum_{\text{spin}} \epsilon \cdot A \, \epsilon \cdot B = \frac{1}{2 q_o} \left[A \cdot B - \frac{A \cdot q \; B \cdot q}{q \cdot q} \right] \tag{46}$$

Carrying out the indicated sums and averages gives, after simplification of resulting expressions in the laboratory system, for positive mesons

$$\left[\frac{g^2}{4\pi} \frac{e^2}{4\pi} \frac{\pi}{2} \frac{1}{K_o^2} \right]^{-1} \frac{d\sigma}{dq_o} = \frac{q^2 \sin^2\theta \, (2 - \kappa^2/2)}{(q \cdot K)^2} + 1 - \frac{q \cdot K}{I \cdot K} \qquad \text{scalar} \tag{47}$$

$$\frac{q^2 \sin^2\theta}{(q \cdot K)^2} (-\kappa^2/2) + 1 - \frac{q \cdot K}{I \cdot K} \qquad \text{pseudoscalar} \tag{48}$$

$$\frac{1}{(\underset{\sim}{q}\cdot K)^2}\left[(\underset{\sim}{I\cdot K})^2 - 3\ \underset{\sim}{I\cdot K}\ \underset{\sim}{q\cdot K} + 15/4\ (\underset{\sim}{q\cdot K})^2 + 1/2\ (\underset{\sim}{q\cdot K})^3 \quad \left(\frac{-4}{\underset{\sim}{K\cdot I}} - \frac{1}{\kappa^2}\right)\right. \tag{49}$$

$$\left. + \frac{q^2 \sin^2\theta}{2\kappa^2}\left[(\underset{\sim}{I\cdot K})^2 - 2\kappa^4 - 4\kappa^2\right]\right] \qquad \text{Vector}$$

For pseudovector, add to vector

$$\frac{1}{(\underset{\sim}{q\cdot K})^2}\left[-\frac{2\ (\underset{\sim}{q\cdot K})^2}{\kappa^2} - \frac{2\ (\underset{\sim}{q\cdot K})^3}{\kappa^4} + 6\ q^2 \sin^2\theta\right] \quad . \tag{50}$$

Finally, if we wish to apply these expressions to the calculation of the spectrum of mesons produced by a photon beam which is the result of the bremsstrahlung of energy electrons, we can represent the distribution of energies in the photon beam by

$$\phi\ (K) \quad \frac{dK}{K} \tag{51}$$

where ϕ (K) is a factor, nearly one over the part of the spectrum of interest, which indicates the degree of departure from the simple $\frac{dK}{K}$ distribution.[7] At a given meson energy, the distribution in photon energies leads to a distribution in meson angles, with the energies and angle related by conservation of energy and momentum

$$K = \frac{q_o - \mu^2/2}{1 - q_o + q\cos\Theta} \tag{52}$$

If

$$\frac{d\sigma}{dq_o} = f\ (\Theta,\ q_o) \tag{53}$$

then

$$\frac{d\sigma}{dq_o d\Omega} = f\ (\Theta,\ q_o)\frac{1}{K}\ \frac{dK}{d\Omega}\ \phi(K)$$

$$= \frac{Kq}{q_o - \kappa^2/2}\ \frac{\phi(K)}{2\pi}\ f\ (\theta,\ q_o)$$

This spectrum is that shown in Figures 4, 5, 6 and 7, with ϕ (K) set equal to one.

[7] W. Heitler, The Quantum Theory of Radiation, page 170

IV Higher Order Corrections

The perturbation calculations carried out in II are for the lowest non-vanishing order in the coupling constants g and e. The corrections of order e^3 and higher are considered negligible, which is probably justified for non-relativistic energies because of the smallness of the expansion parameter $\frac{e^2}{4\pi} = \frac{1}{137}$. Therefore, only terms of order eg^3 are calculated. The pseudoscalar theory is considered because it gives predictions in qualitative agreement with the experimental results of McMillan, et al.,[1] and because of the simplicity of the theory. Pseudoscalar coupling is used, which is equivalent to pseudovector coupling in lowest order but not in the higher order processes. The calculations made are for energies near threshold where the momenta of the particles can be ignored relative to the rest energies.

The possible diagrams for the corrections to the lowest order result for the production of positive mesons are given in Figure 7. The diagrams B_3, C, C_1 are forbidden since the nucleons indicated as interacting with the photon are in the neutron state. Diagrams involving neutral mesons are also omitted. Following Dyson,[6] we will first evaluate the corrections to the basic operators which are inserted into the matrix element representing an irreducible diagram, and then specialize to the diagrams listed in Figure 7. Corrections of the following types appear (Figure 8):

I. A nucleon, propagating as a virtual particle, can emit and reabsorb a virtual meson. This corresponds to Dyson's vacuum polarization of the second kind. The diagram includes nucleon mass and mesonic charge renormalization effects.

II. A meson, propagating as a virtual particle, can produce a nucleon pair, which then annihilate to give the meson again. The diagram includes meson mass and mesonic charge renormalization effects.

III. A meson, interacting with the electromagnetic field, can instead produce a nucleon pair one of which interacts with the field.

IV., V. A nucleon, interacting with the electromagnetic field, can emit a virtual meson. Either particle can then interact with the field. This corresponds to the nucleon anomalous moment terms. Diagrams III, IV, and V include electronic charge renormalization effects.

For diagram I, the correction leads to the replacement of

$$\frac{1}{\underset{\approx}{P} - i}$$

by

$$\frac{1}{\underset{\approx}{P} - i}\left\{\frac{g^2}{(2\pi)^4}\int d^4\ell\;\gamma_5\frac{1}{\underset{\approx}{P} - \underset{\approx}{\ell} - i}\gamma_5\frac{1}{\ell^2 + \kappa^2}\right\}\frac{1}{\underset{\approx}{P} - i} \tag{55}$$

the integral can be evaluated, keeping convergence by integrating to a fixed upper limit which will finally be allowed to become infinite, to give

$$J(P) = \pi^2\int_0^1 dx\left[-\underset{\approx}{P}(1-x) + i\right]\left[\ln\frac{\lambda^2}{\phi(P^2)} - 1\right] \tag{56}$$

where

$$\phi(P^2) = x\,(P^2 + 1) + (1-x)\kappa^2 - x^2P^2 \tag{57}$$

Following Dyson we now make the expansion

$$J(P) = U(0) + P_\mu\left(\frac{\partial J}{\partial P_\mu}\right)_{P=0} + R_c \tag{58}$$

where R_c is finite as $\lambda^2 \to \infty$. R_c is now separated into

$$R_c = A + B(\underset{\approx}{P} - i) + (\underset{\approx}{P} - i)\,R_c' \tag{59}$$

where A and B are independent of P and $R_c' \to 0$ as $P^2 \to -1$ and $\underset{\approx}{P} \to i$.

The renormalized J(P) then is

$$(\underset{\approx}{P} - i)\,\pi^2\int_0^1 dx\left\{\frac{ix\,(\underset{\approx}{P} - i) - 2x - (P^2 + 1)\,(1 - x)}{P^2 + 1}\ln\frac{\phi(-1)}{\phi(P^2)} - \frac{2x^2\,(1-x)}{\phi\,(-1)}\right\} \tag{60}$$

Application to diagram B_1, Figure 7 gives

$$M_4(B_1) = ge\,\psi_F^{\dagger}\gamma_5\,\frac{g^2}{16\pi^2}\int_0^1 dx\left\{\quad\right\}\frac{1}{\underset{\approx}{P} - 1}\underset{\approx}{A}\,\psi_I\,\phi \tag{61}$$

where the curly bracket is the same as above. Now if we use the properties of γ_5 to give

$$\psi_F^\dagger \gamma_5 (\underset{\approx}{P} - i) \equiv \psi_F^\dagger \gamma_5 (\underset{\approx}{F} + \underset{\sim}{q} - i)$$
$$\cong -i(2 + \kappa) \psi_F^\dagger \gamma_5$$

we find

$$M_4(B_1) = M_2(B) \frac{g^2}{16\pi^2} \int_0^1 dx \left\{ (-1 + x/2) \ln \frac{\phi(-1)}{\phi(P^2)} \quad \frac{-2 x^2(1 - x)}{\phi(-1)} \right\} \tag{62}$$

This integral can be evaluated numerically to give

$$M_4(B_1) = -1.26 \frac{g^2}{16\pi^2} M_2(B) \tag{63}$$

The correction from this process could also be interpreted as a non-static correction to the magnetic moment of the proton.

For diagram II, Figure 8, the correction leads to the replacement of

$$\frac{-1}{P^2 + \kappa^2}$$

by

$$\frac{-1}{(P^2 + \kappa^2)^2} \frac{g^2}{(2\pi)^4} \int sp \left\{ \gamma_5 \frac{1}{-\underset{\approx}{P} + \underset{\approx}{\ell} - i} \gamma_5 \frac{1}{\underset{\approx}{\ell} - i} \right\} d^4 \ell \tag{64}$$

Carrying out the indicated spur sum and integration gives for the integral

$$J(P^2) = -4 \pi^2 \int_0^1 dx \left\{ - \lambda^2 + \phi \ln \frac{\lambda^2}{\phi} + \phi \right\} \tag{65}$$

where

$$\phi (P^2) = x(1 - x) P^2 + 1 \tag{66}$$

Expansion about $P^2 = -\kappa^2$ gives

$$J(P^2) = J(-\kappa^2) + J' (-\kappa^2) (P^2 + \kappa^2) + R_c \tag{67}$$

where

$$R_c = 4\pi^2 \int_0^1 dx \left\{ \frac{1/2 x^2 (1 - x)^2 (P^2 + \kappa^2)^2}{1 - x (1 - x) \kappa^2} \left[1 + 0 (P^2 + \kappa^2) \right] \right\} \tag{68}$$

Application to diagram A_1, Figure 7, gives

$$M_4(A_1) \cong ge \phi \psi_F^\dagger \gamma_5 \psi_I \frac{-2 \underset{\sim}{A} \cdot \underset{\sim}{q}}{(q - K)^2 + \kappa^2} \frac{q^2}{16\pi^2} \left[\frac{(q - K)^2 + \kappa^2}{15} \right] \tag{69}$$

Near threshold $(q - K)^2 + \kappa^2 = 2\kappa^2$

Using this

$$M_4(A_1) = M_2(A) \frac{g^2}{16\pi^2} \frac{2\kappa^2}{15} \tag{70}$$

Since $\kappa^2 \sim 1/40$, the contribution from this higher order process is less than 1 percent.

For diagram III, replace $2\,e\,\underset{\sim}{A}\cdot\underset{\sim}{P}$

by

$$\frac{g^2 e}{(2\pi)^4} \int d\,4\ell \;\; sp\left\{\gamma_5 \frac{1}{\underset{\sim}{P'} + \underset{\sim}{\ell} - i} \underset{\approx}{A} \frac{1}{\underset{\sim}{P} + \underset{\sim}{\ell} - i} \gamma_5 \frac{1}{\underset{\sim}{\ell} - i}\right\} \tag{71}$$

The integral, after taking indicated spur sum, is

$$J(K) = -8\pi^2 \,\underset{\sim}{A\cdot P} \int_0^1 dx \int_0^x dy \tag{72}$$

$$\left\{ \frac{(1 - x)\phi(K) + 2x + 1}{2\phi(K)} + (2 - 3x/2) \left[\ln \frac{\phi(K)}{\kappa^2} + 3/2\right]\right\}$$

where

$$\phi(K) = 1 - x(1 - x)\kappa^2 - 2y(1 - x)\,\underset{\sim}{K\cdot P}$$

and the final momentum $P = K + P'$ has been taken to satisfy the relation for a free particle

$$P^2 + \kappa^2 = 0$$

Subtraction of the charge renormalization term J(0) leaves the finite expression

$$J(K) - J(0) = -8\pi^2 \,\underset{\sim}{A\cdot P} \int_0^1 dx \int_0^x dy \left\{\frac{2x + 1}{2}\left(\frac{1}{\phi(K)} - \frac{1}{\phi(0)}\right) + (2 - 3x/2) \ln \frac{\phi(K)}{\phi(0)}\right\} \tag{73}$$

$$\cong 4/3\,\pi^2 \,\underset{\sim}{A\cdot P}\,\kappa^4$$

Application to diagram A_2, Figure 7, gives

$$M_4(A_2) = e'/e \; M_2(A) \tag{74}$$

where

$$e'/e = g^2/16\pi^2 \; 4/3 \; \kappa^4$$

Since $\kappa^2 \sim 1/40$, the contribution from this higher order process is again less than 1 percent.

Diagram IV, Figure 8, also gives the anomalous magnetic moment terms for a static field. The expression to be inserted for such a vertex graph, with an initial proton, is

$$\frac{-g^2 e}{(2\pi)^4} \int d^4 \ell \, \gamma_5 \, \frac{1}{\underline{\ell} - i} \, \gamma_5 \, \frac{2A \circ (I - \ell)}{\left[(I - \ell)^2 + \kappa^2\right]\left[(F - \ell)^2 + \kappa^2\right]} \tag{75}$$

For an initial neutron, the interacting meson has a negative charge and this expression changes sign. The integral is

$$\pi^2 \int_0^1 dx \int_0^x \left\{ \underline{A} \left[\ln \frac{\lambda^2}{\phi(K)} - 3/2 \right] + 2y \left[\frac{-(x-y)\underline{I} - (1-x)\underline{F}}{\phi(K)} + i \right] \underline{A \circ I} \right\} \tag{76}$$

where

$$\phi(K) = (1-y)\kappa^2 + y\left[1 + (1-y)I^2 + 2\,\underline{I \circ K}\,(1-x)\right]$$

If one now uses the identity

$$\underline{A \circ I} = \frac{\underline{AK}}{2} + \underline{A}i + \frac{(\underline{F} - i)}{2}\underline{A} + \underline{A}\frac{(\underline{I} - i)}{2} \tag{77}$$

and subtracts the term for which K is zero and the nucleons satisfy the Dirac equation for a free particle, i.e.

$$\psi_F^{\dagger}(\underline{F} - i) = (\underline{I} - i)\psi_I = 0$$

$$I^2 = F^2 = -1$$

one obtains the finite expression

$$\pi^2 \int_0^1 dx \int_0^x dy \left\{ \underline{A} \ln \frac{\phi(0)}{\phi(K)} + 2y \left[\frac{-(x-y)\underline{I} - (1-x)\underline{F} + i}{\phi(K)} \right] \underline{A \circ I} + \frac{2y^2 \underline{A}}{\phi(0)} \right\} \tag{78}$$

Application to diagram B_2, Figure 7, with $\underline{A \circ I} = 0$ for the initial nucleon at rest and a transverse photon, gives

$$M_4(B_2) = e'/e \; M_2(B) \tag{79}$$

where

$$e'/e = g^2/16\pi^2 \int_0^1 dx \int_0^x dy \left[\frac{2y^2}{y^2 + (1-y)\kappa^2} + \ln \frac{\phi_0}{\phi} \right]$$

$$= .88\ g^2/16\pi^2$$

Diagram V, Figure 8, also gives an anomalous magnetic moment term. The expression to be inserted for the vertex graph is

$$\frac{-g^2 e}{(2\pi)^4} \int d\,4\ell\, \gamma_5 \frac{1}{\underset{\approx}{F} - \underset{\approx}{\ell} - i} \underset{\approx}{A} \frac{1}{\underset{\approx}{I} - \underset{\approx}{\ell} - i} \gamma_5 \frac{1}{\ell^2 + \kappa^2} \qquad (80)$$

The integral is

$$2\pi^2 \int_0^1 dx \int_0^x dy \left\{ \frac{\left[-\underset{\approx}{F} + i + y\underset{\approx}{F} + (x-y)\underset{\approx}{I}\right] \underset{\approx}{A} \left[\underset{\approx}{I} - i - y\underset{\approx}{F} - (x-y)\underset{\approx}{I}\right]}{2\phi} + \underset{\approx}{A}/2\,(\ln \lambda^2/\phi - 3/2) \right\} \qquad (81)$$

where

$$\phi\,(K) = x + (1-x)\kappa^2 - 2(1-x)\,(x-y)\,\underset{\sim}{F \cdot K} + x(1-x)\,F^2$$

Subtraction of the renormalization term, using

$$\psi_F\,(\underset{\approx}{F} - i) = (\underset{\approx}{I} - i)\,\psi_I = 0$$

and setting $K = 0$ leaves the finite expression

$$\pi^2 \int_0^1 dx \int_0^x dy \left\{ \frac{\left[-\underset{\approx}{F} + i + y\underset{\approx}{F} + (x-y)\underset{\approx}{I}\right] \underset{\approx}{A} \left[\underset{\approx}{I} - i - y\underset{\approx}{F} - (x-y)\underset{\approx}{I}\right]}{\phi\,(K)} - \frac{x^2 \underset{\approx}{A}}{\phi(0)} + \underset{\approx}{A} \ln \frac{\phi(0)}{\phi(K)} \right\} \qquad (82)$$

Application to the diagram C_3, Figure 7, taking $\underset{\sim}{A \cdot F} = 0$, gives

$$M_4(C_3) = e'/e\ M_2(C) \qquad (83)$$

where

$$e'/e = .32\ g^2/16\pi^2$$

and

$$M_2(C) = ge\,\psi_F^{\dagger} \frac{\underset{\approx}{KA}}{2\ \underset{\sim}{I \cdot K}}\ \psi_I\ \phi$$

This is equivalent to a magnetic moment interaction for the neutron.

If we examine these matrix elements, it is apparent that the corrections from diagrams I, IV, and V are equivalent to an interaction by the nucleons with an anomalous moment in Bohr magnetons

$$\text{proton} \qquad g^2/16\pi^2\ (.88 - 1.26) = -.38$$

$$\text{neutron} \qquad g^2/16\pi^2\ (.88 - .32) = -.56$$

These results are to be contrasted with those of Case,[5] who also calculated the anomalous moments for a static field, using pseudoscalar meson theory. He found that for charge symmetric theory, the anomalous moments were

$$\text{proton} \qquad .65\ g^2/16\pi^2$$

$$\text{neutron} \qquad -1.61\ g^2/16\pi^2$$

where the notation has been adjusted to agree with that used here.

We find therefore that the correction of order eg^3 is the sum of the contributions from the graphs A_1, A_2, B_1, B_2, C_2, C_3. The first two give corrections which are less than 1 percent and can be neglected relative to the others. The remaining correction is

$$M_4 = g^2/16\pi^2 \left[.88 - 1.26\right] M_2(B) - g^2/16\pi^2 \left[-.88 + .32\right] M_2(C)$$
$$= 0.18\ g^2/16\pi^2\ M_2(B)$$

Since $g^2/4\pi$ is about 4π, we see that the contribution from the fourth order terms is about one-sixth as large as that from the lowest order. The smallness of the total effect, however, is due to near cancellation of the large contributions from the individual effects. This cancellation is probably fortuitous and cannot be expected to be repeated for the next order processes. We also see that the corrections do not affect the predominance of nucleon moment interactions, characteristic of the pseudoscalar theory, and that this is probably true if effects of even higher order are included.

V Conclusions

We have seen that certain features of the process of production of mesons by photons are nearly independent of the nature of the coupling of mesons to nucleons. In particular one can expect through the measurement of the distribution in energy and angle of photo-mesons at energies above a few Mev:

1. To determine with some confidence the spin of the meson, through the characteristic behaviour of particles with a magnetic moment in the electromagnetic field. The strong energy dependence of the coupling of spin-one mesons to photons has been used previously by Christy and Kusaka[9] to demonstrate that cosmic ray mesons cannot be of this type.

2. To examine the distribution of mesons about the nucleon, since if the distribution extends to distances of the order $\hbar/\mu c$, one would expect that the electric-dipole terms would be predominant in the coupling to the electromagnetic field. Only if the meson distribution is singular, i.e., closely bound to the nucleons, can one expect the interaction with the circulating currents of the magnetic moments to become relatively important.

3. Through a measurement of the plus-to-minus ratio to determine the effects of a meson magnetic moment and the nature of the anomalous moments for the nucleons. The anomalous moments will be nearly independent of the frequency of the electro-magnetic field only if they are confined to regions small compared with the wave-length of the radiation.

The further details of the calculation made with perturbation theory can probably not be accepted quantitatively since higher order corrections are not negligible. For example, explicit calculation of the higher order effects for the pseudoscalar theory predicts large anomalous magnetic moment interactions for the nucleons including corrections, as large as the lowest order terms, corresponding to polarization of the vacuum by the nucleons. The anomalous moments are considerably

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[9] Phys. Rev. 59, 414 (1941)

smaller than those which the nucleons exhibit in a static field and different in sign for the proton. The importance of the corrections for the pseudoscalar theory appears to be the result of the close binding of the mesons to the nucleons, which in turn leads to the characteristic predominance of the nucleon magnetic moment interactions in the production process. Higher order effects, however, do not change the general features of the lowest order results. There, therefore, seems to be ground for hope that the lowest order calculations for all of the theories may give qualitatively correct results.

The experimental results obtained by the Berkeley workers,[2] although still preliminary, indicate that mesons of 40-100 Mev produced by photons on hydrogen are nearly isotropic in angular distribution from 45-135 degrees in the laboratory system. The cross section for mesons produced in this energy and angular range from hydrogen appears to decrease slowly with increasing meson energy. The magnitude of the total cross section, about 10^{-28} cm^2, can be fitted for the theories giving roughly isotropic angular spectra at low energies (Figures 3 and 5) with a value of the coupling constant $\frac{g^2}{4\pi\hbar c}$ of about 40 for pseudoscalar mesons and 0.4 for pseudovector mesons. The mesons produced from carbon show an excess of negative over positive mesons in the ratio of 1.7 ± 0.2 in the energy range of 30-100 Mev observed at 90 degrees to the photon beam direction. This is in agreement with the ratio given in Figure 1, with the assumption that the neutron does not interact with the electromagnetic field. However, it is not clear that the complications of the binding of the nucleons in the carbon nucleus are unimportant. A more detailed study of the dependence on energy and angle of the negative-to-positive ratio is being carried on at present.

These results all seem to indicate that the general features of the pseudoscalar meson theory are correct, i.e., the zero spin of the meson and the close binding of the meson cloud to the nucleons. It is hoped that the degree of validity of these conclusions will be indicated as the experimental work is completed.

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Titles of Figures

1. The ratio of the cross sections for the production of negative and positive mesons. The solid curves are calculated with the assumption that the neutron does not interact with the electromagnetic field, the dotted curves with the assumption that the neutron interacts with the anomalous moment observed in a static field. The energies indicated in this and the following figures are the meson energies, which are related to the photon energy as shown in Figure 9.

2. Angular distribution of scalar mesons from dK/K photon spectrum.

3. Angular distribution of pseudoscalar mesons from dK/K photon spectrum.

4. Angular distribution of vector mesons from dK/K photon spectrum.

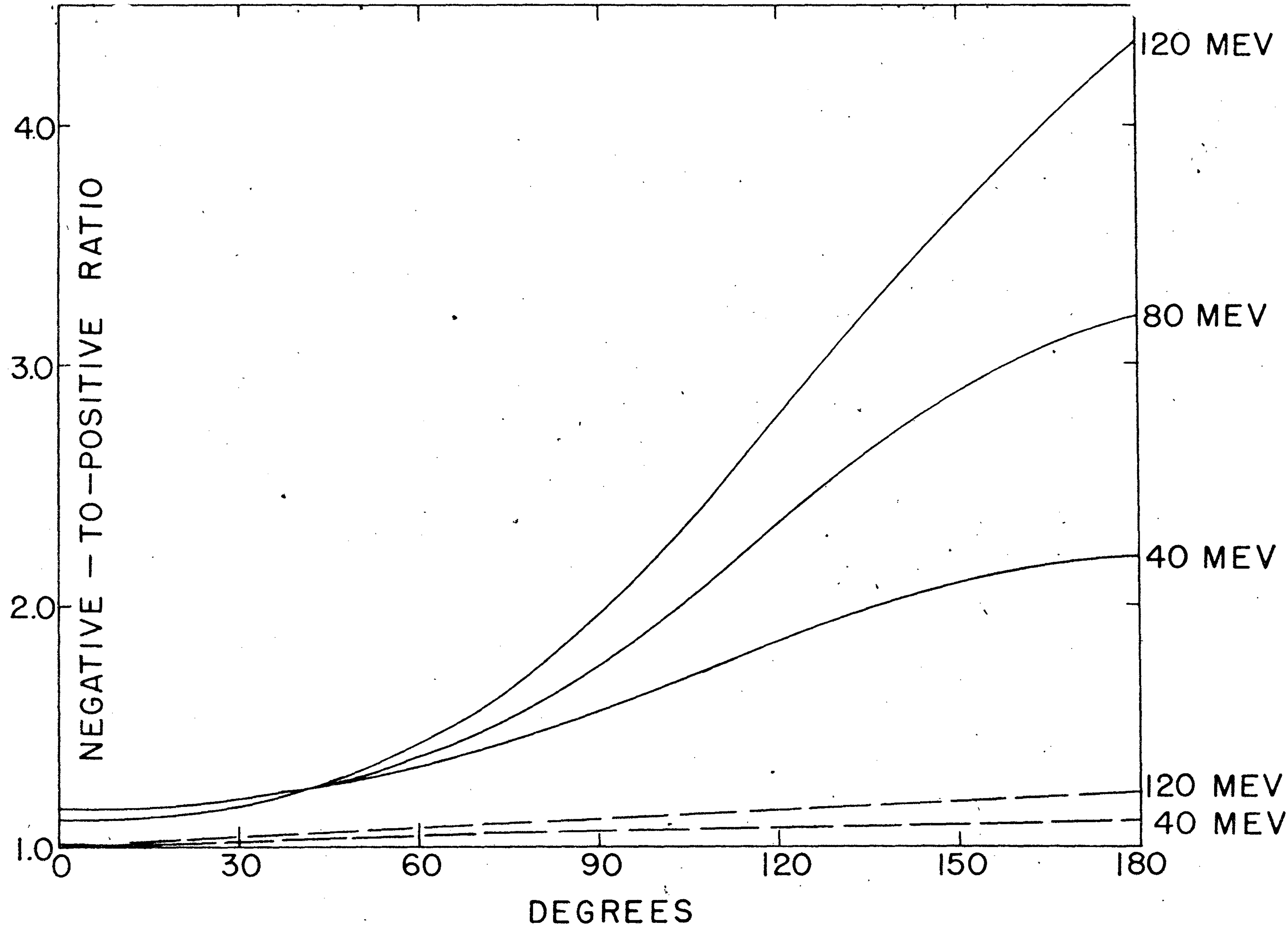
5. Angular distribution of pseudovector meson from dK/K photon spectrum.

6. Ratio of cross sections for production of positive and negative mesons, vector and pseudovector theory.

7. Diagrams for photo-meson production. Diagrams A, B, C are for production in lowest order ge; diagrams with subscripts are for production in order g^3e.

8. Details of higher order corrections.

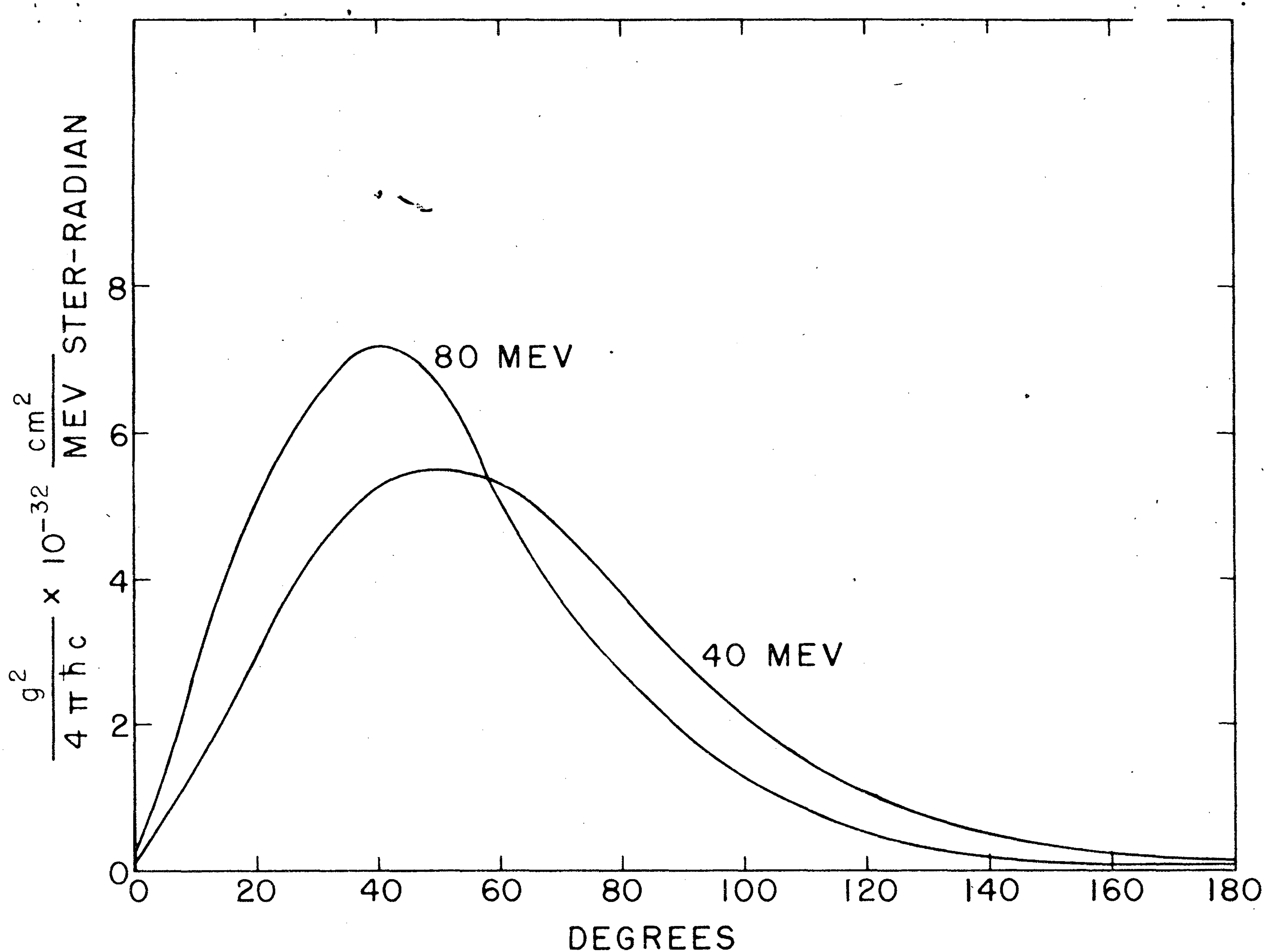
9. Relation between meson and photon energy as a function of the angle of the meson with the photon beam direction.



NEGATIVE – TO–POSITIVE RATIO
1.0
2.0
3.0
4.0
0
30
60
90
120
150
180
DEGREES
120 MEV
80 MEV
40 MEV
120 MEV
40 MEV

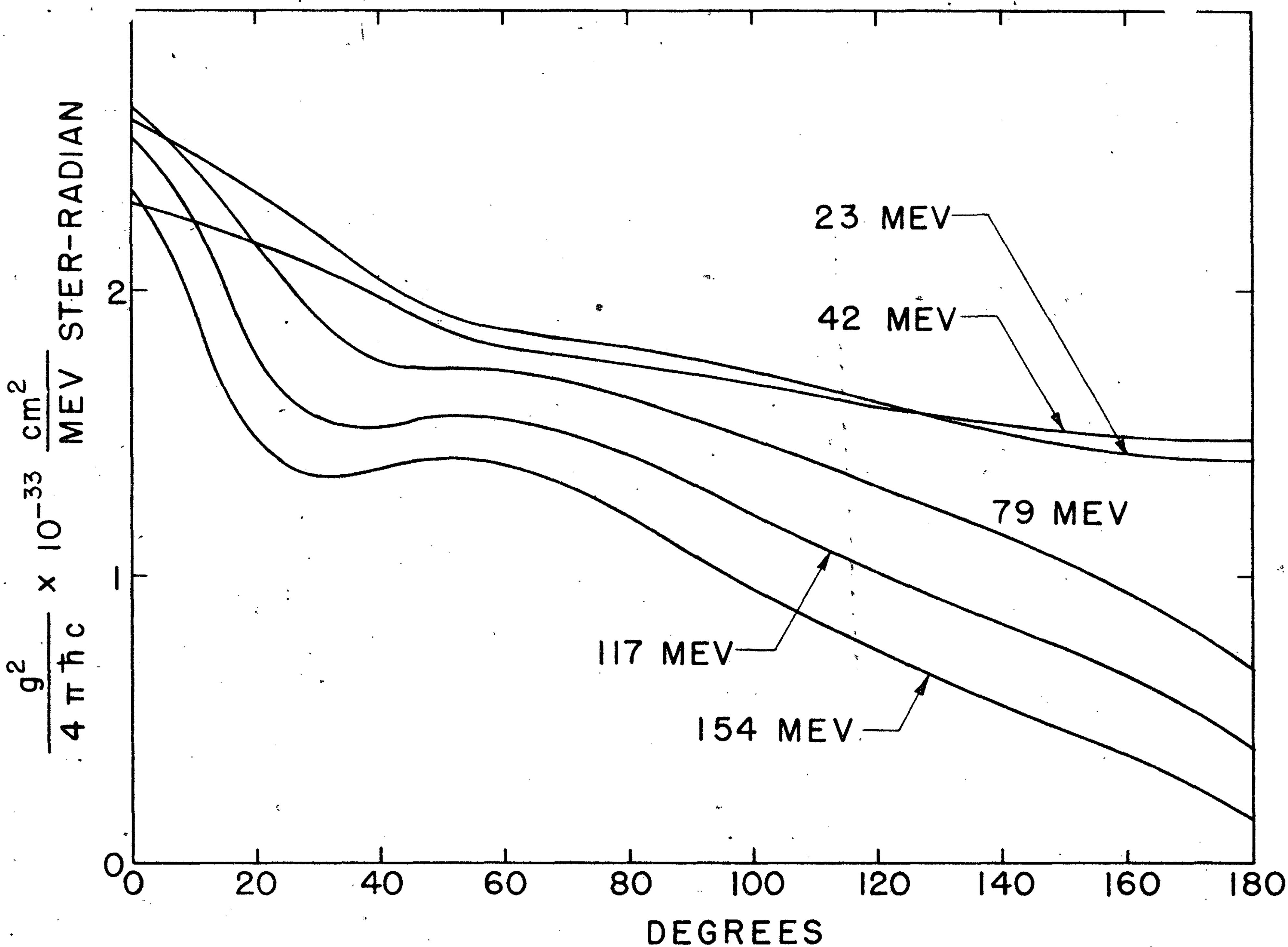
FIG. 1

14558



$\frac{g^2}{4\pi\hbar c} \times 10^{-32} \frac{cm^2}{MEV\ STER\text{-}RADIAN}$
8
6
4
2
0
80 MEV
40 MEV
0
20
40
60
80
100
120
140
160
180
DEGREES

FIG. 2



$\frac{g^2}{4\pi\hbar c} \times 10^{-33}$ $\frac{cm^2}{MEV\ STER\text{-}RADIAN}$
2
1
0
0
20
40
60
80
100
120
140
160
180
DEGREES
23 MEV
42 MEV
79 MEV
117 MEV
154 MEV

FIG. 3

14560-1

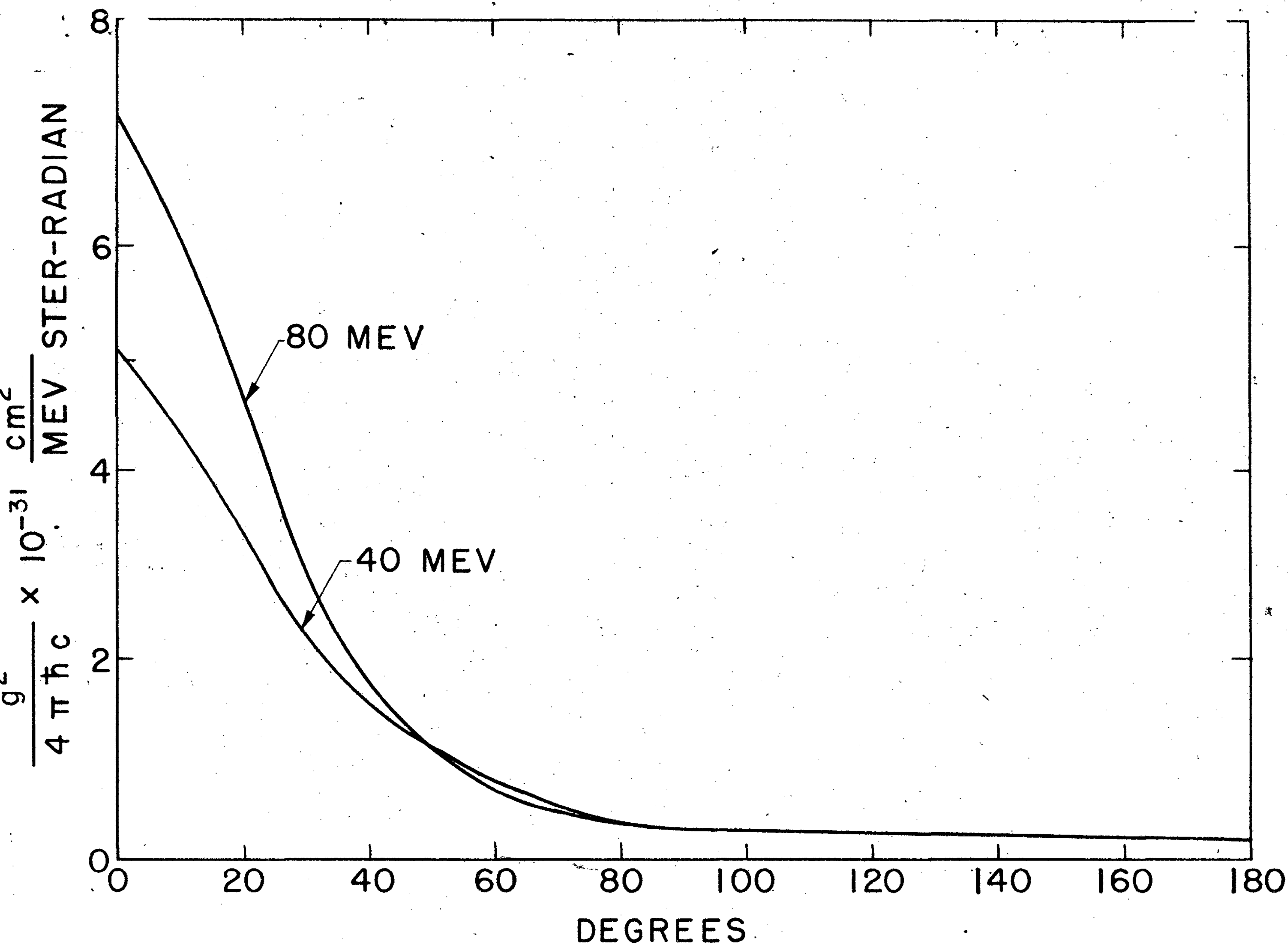
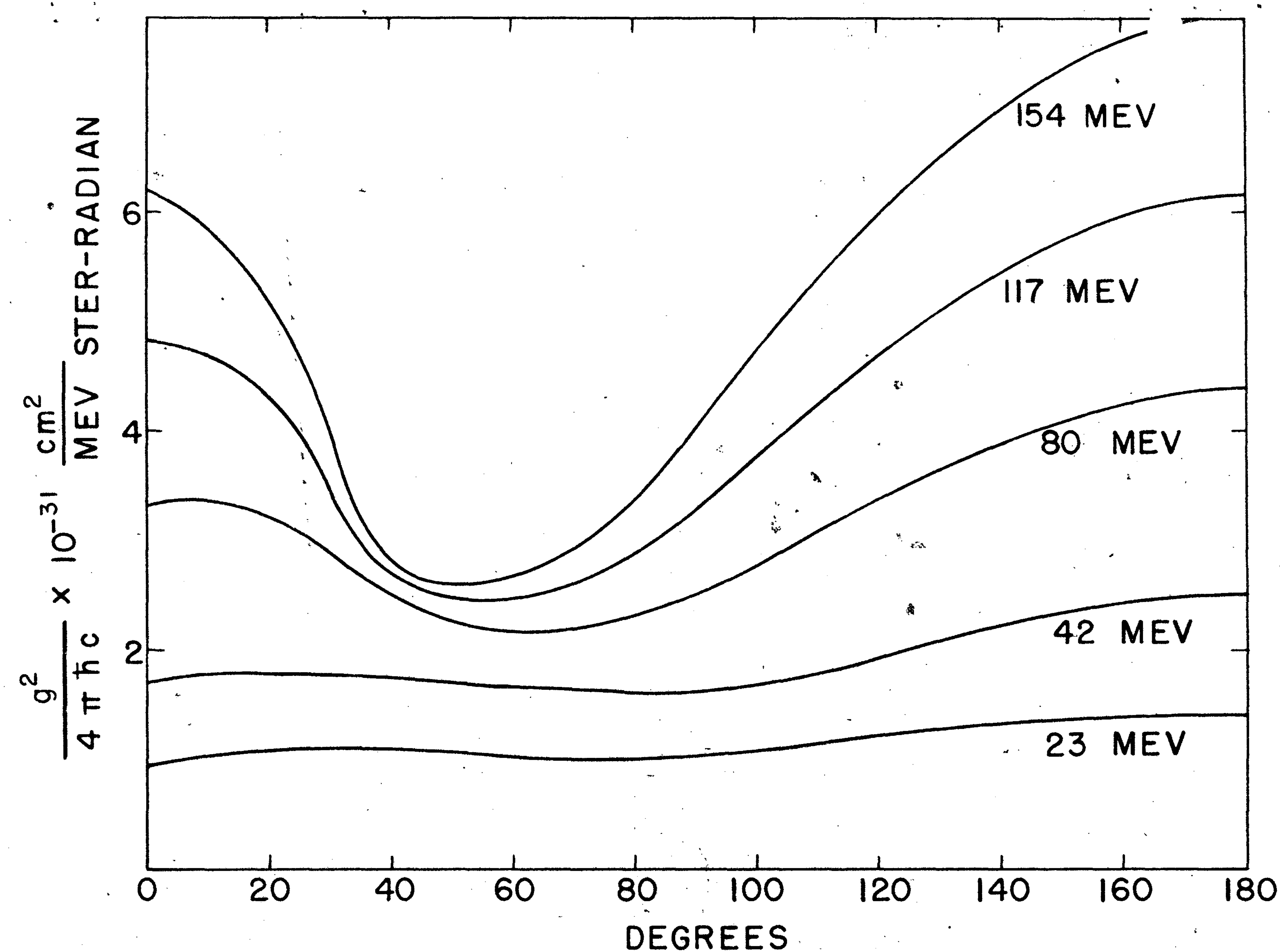


FIG. 4

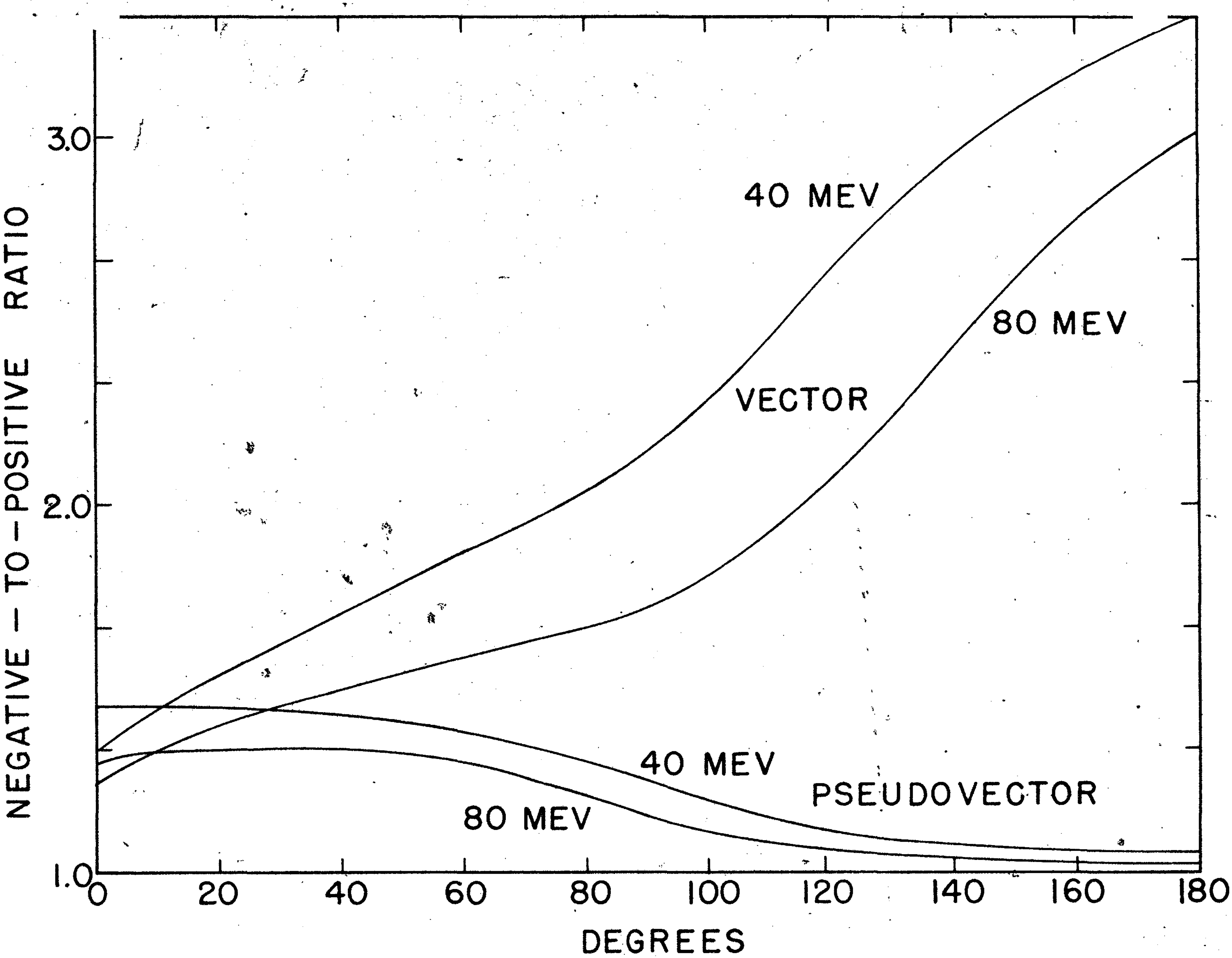
14561



$\frac{g^2}{4\pi\hbar c} \times 10^{-31} \frac{cm^2}{MEV}$ STER-RADIAN
6
4
2
154 MEV
117 MEV
80 MEV
42 MEV
23 MEV
0
20
40
60
80
100
120
140
160
180
DEGREES

FIG. 5

14562-1



3.0
2.0
1.0
NEGATIVE – TO – POSITIVE RATIO
40 MEV
80 MEV
VECTOR
40 MEV
PSEUDOVECTOR
80 MEV
0
20
40
60
80
100
120
140
160
180
DEGREES

FIG. 6

14559-1

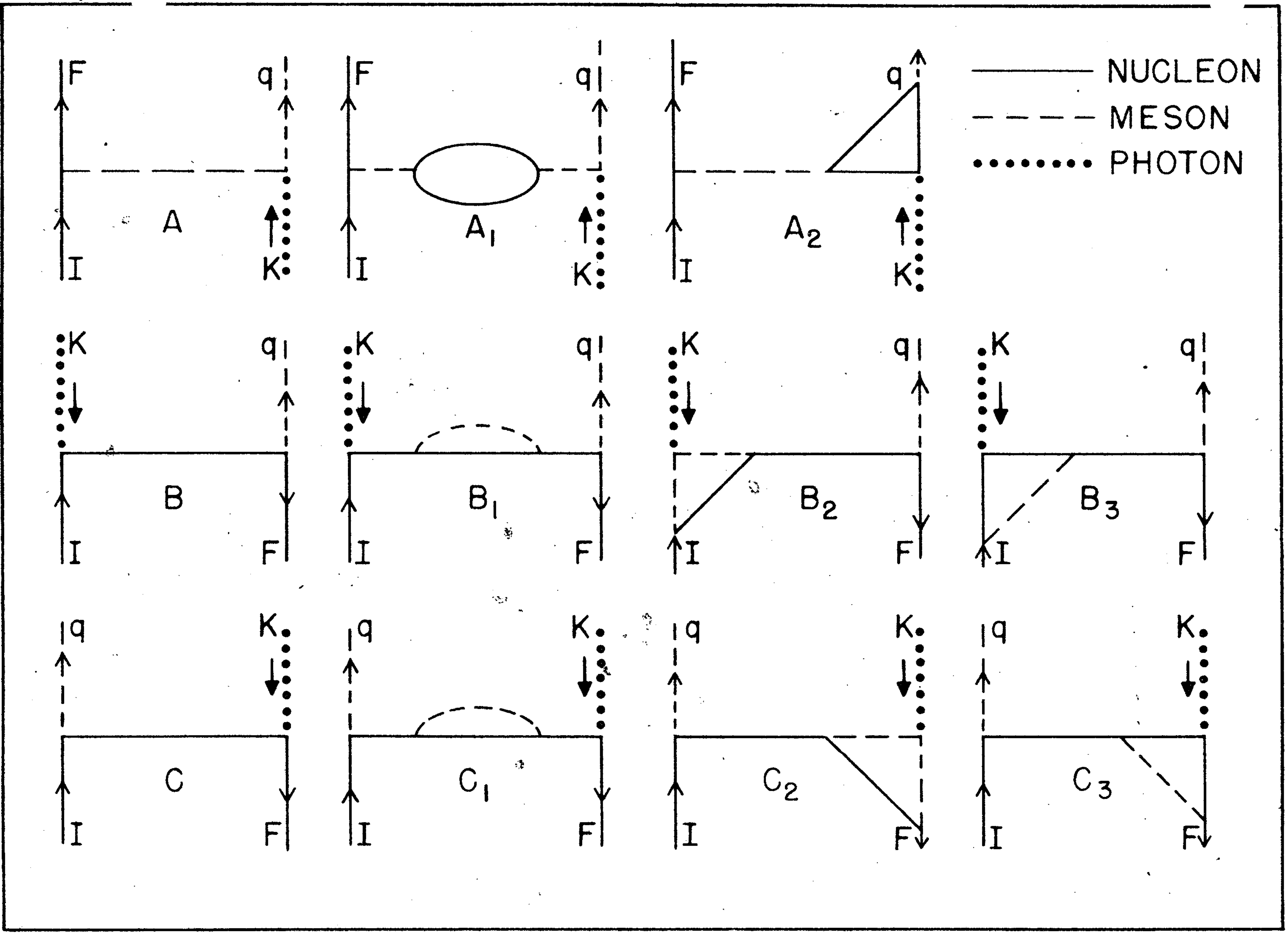
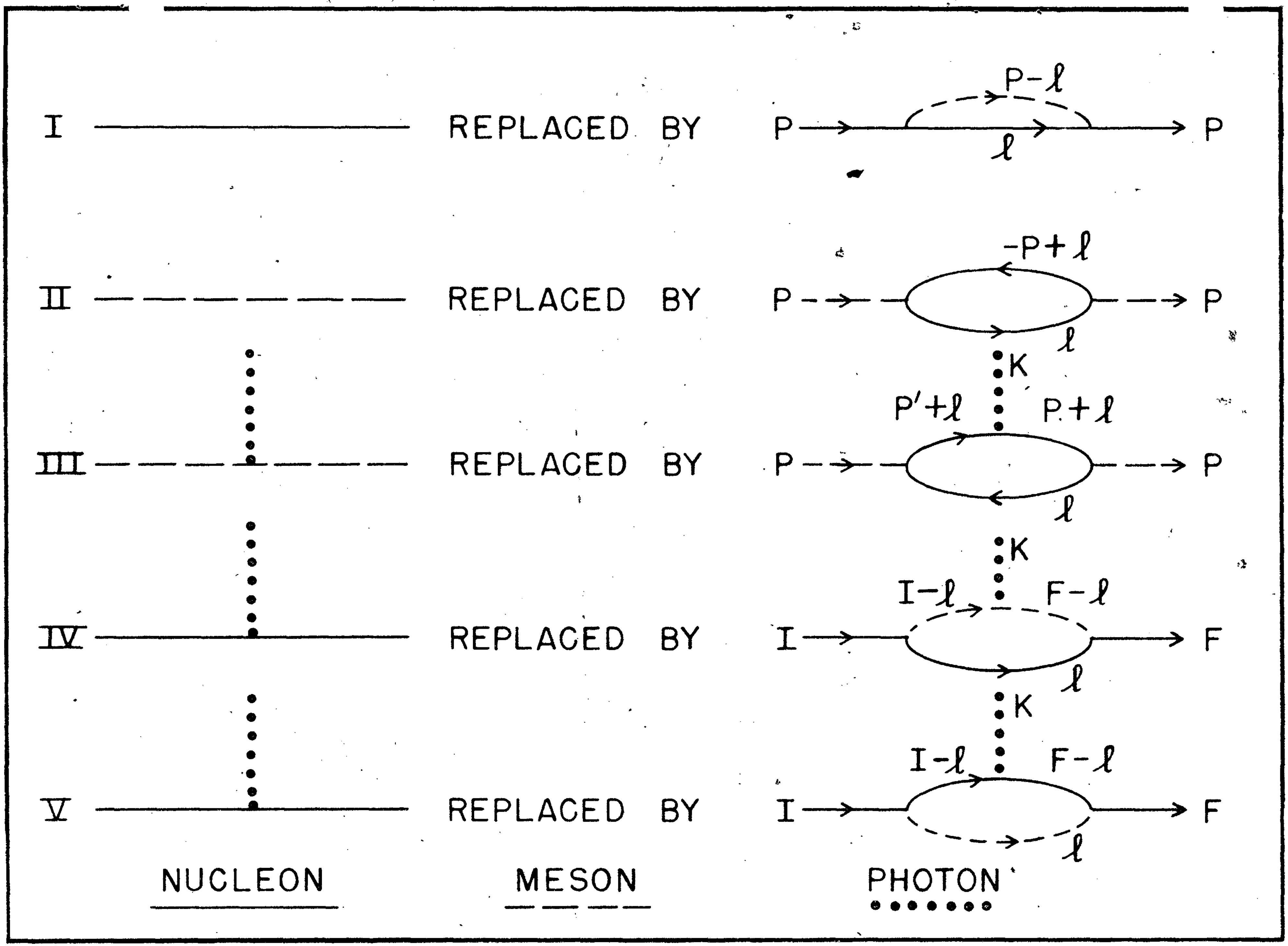


FIG. 7

14565-1



I
REPLACED BY
P
P−ℓ
ℓ
P
II
REPLACED BY
P
−P+ℓ
ℓ
P
K
III
REPLACED BY
P
P′+ℓ
P+ℓ
ℓ
P
K
IV
REPLACED BY
I
I−ℓ
F−ℓ
ℓ
F
K
V
REPLACED BY
I
I−ℓ
F−ℓ
ℓ
F
NUCLEON
MESON
PHOTON

FIG. 8

14566-1

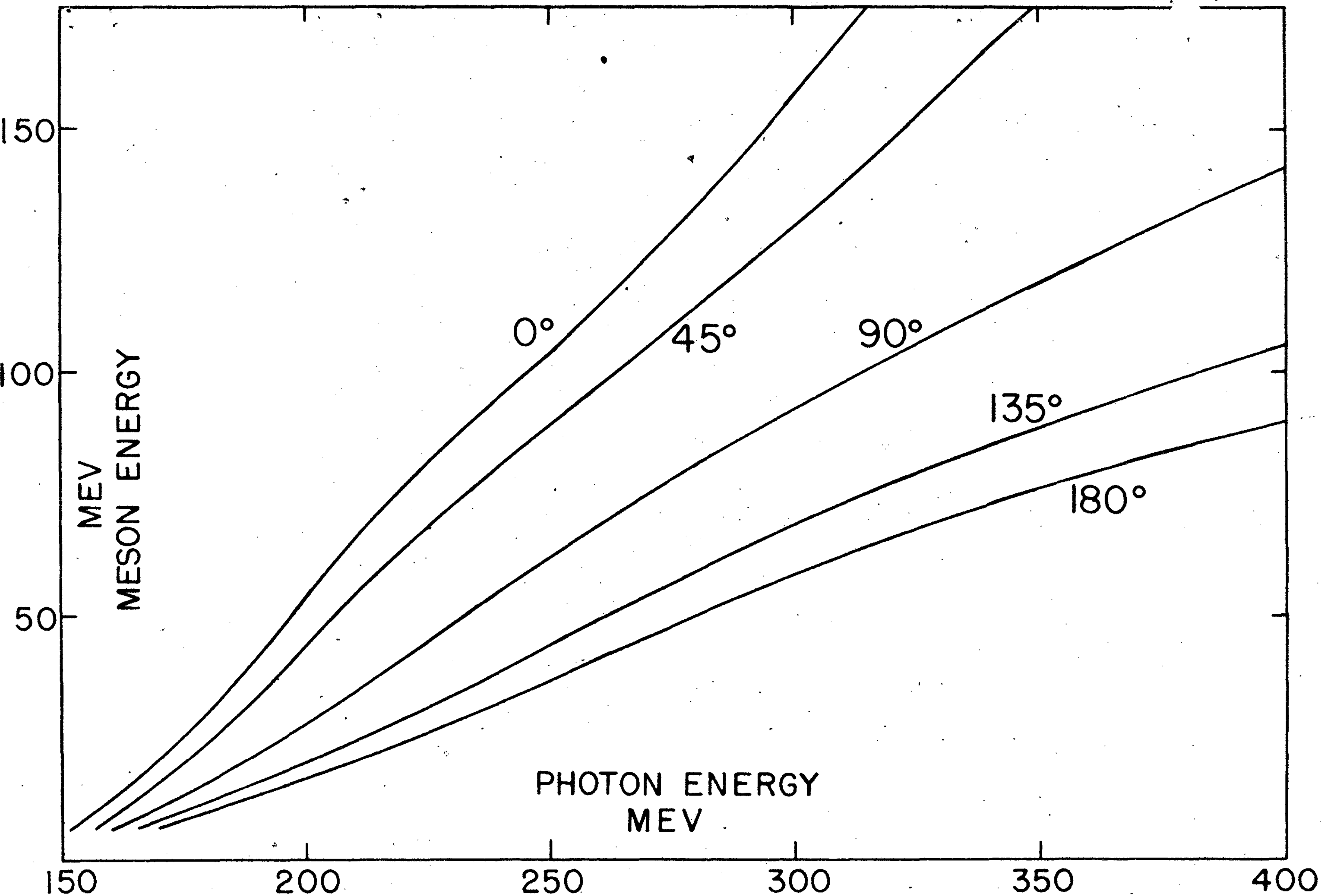

150
100
50
MEV
MESON ENERGY
0°
45°
90°
135°
180°
PHOTON ENERGY
MEV
150
200
250
300
350
400

FIG. 9

14579-1