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UNIVERSITY OF CALIFORNIA, SAN DIEGO

Problem Solving Toward Mathematical Understanding:
Instructional Design for Students with Learning Disabilities

A Thesis submitted in partial satisfaction of the requirements
for the degree Master of Arts

in

Teaching and Learning (Curriculum Design)

by

Renate Ward

Committee in charge:

Bernard Bresser, Chair
Cheryl Forbes
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2012

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Chair

University of California, San Diego

2012

DEDICATION

I dedicate this to my mother who always believed in my ability to achieve all that I set out to accomplish. I also dedicate this to my three children, Daunielle, Alisha, and Tyler who are my inspirations and the reason I strive to be my best.

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ABSTRACT OF THESIS

Problem Solving Toward Mathematical Understanding:
Instructional Design for Students with Learning Disabilities

by

Renate Ward

Master of Arts in Teaching and Learning (Curriculum Design)

University of California, San Diego, 2012

Bernard Bresser, Chair

Over three decades of data continue to show a lack of mathematical achievement for students of color, minority language speakers, students living in poverty, or those who have learning disabilities (LD). *Problem Solving Toward Mathematical Understanding* (PSTMU) is designed to teach LD children multiple ways to represent and solve problems, improve reasoning skills, and persevere. Through the use of higher-order questioning, students develop metacognitive awareness helping them monitor the effectiveness of a strategy and to consider different options. *PSTMU* is designed to develop a deeper understanding of mathematical concepts through scaffolded

instruction, peer-talk, teacher-talk, and group discussions. Students come to a shared understanding of the material and observe multiple approaches to solving problems. The curriculum was implemented, evaluated, and revised over seven weeks with a group of nine middle school students in an urban school setting. The students were of low socio-economic background, diverse ethnicities, and all had one or more learning disabilities. Qualitative and quantitative measures were used to determine the effectiveness of this approach. Students' problem solving skills, ability to reason, provide proof, effectively communicate mathematically, and create and use representations in their work was evaluated through a rubric scored by two raters.

Observations, class work, and audio recordings were used to support the findings. Surveys and questionnaires were used to rate metacognitive awareness and attitude. The data indicated that all students increased their abilities in two or more of the areas evaluated. The attitudes of six out of the nine students improved and overall students became more flexible in their use of strategies to solve problems.

I. Introduction

What is the goal of education? Why do students attend school for more than twelve years in the United States? Is it to receive information from the one who holds the knowledge, the teacher, and regurgitate it when needed? Or is it essentially to develop and master problem solving skills in order to be able to solve the problems we are confronted with during our lives? Independent thinking is necessary in order to be successful in life and problem-solving skills can lead to this. Fleischner and O'Loughlin (1985) indicate that while each academic area infuses problem solving skills into its curriculum, the formal teaching of these skills often occurs in the mathematics classroom. However, data shows that the way we approach mathematical education in the US is failing many of our students.

Data collected for over three decades continues to show dire results for students of color, minority language students, students living in poverty, and students with learning disabilities (LD) (NAEP, 2009). The gap between the achievement of Caucasian middle-class students and the aforementioned groups of students is not closing (NAEP, 2009). Based on my fourteen years of experience in the classroom and current observations, it is clear to see that students struggle with word problems and instruction is not focused on problem solving activities. Little time is spent discussing problems or writing reflectively about the problem solving process. Virtually no time is spent asking children to

ponder and think because it takes time away from teaching procedures to complete mathematical computation. However, focusing on procedural skills has not been a successful approach for all students, particularly those with learning disabilities (NCES, 2009). Cawley (2002) has found that students with disabilities do better by making math meaningful to them. In his article he discusses the difference between “knowing” and “doing” mathematics. When faced with a mathematics problem a student knows mathematics when the basic principles of the problem are comprehended, the student is aware that there is more than one way to explain the problem, and that there may be more than one acceptable answer. Doing mathematics is the ability to apply different mathematical principles and strategies to solve a problem. Cawley believes that overemphasizing the “doing” and neglecting the “knowing” has contributed to the difficulties students with learning disabilities face with understanding mathematics.

I began my career in education as a teacher of deaf and hard-of-hearing students. I spent the first 10 years teaching math to middle school children whose primary mode of communication was American Sign Language (ASL). I also taught a kindergarten class for two years for deaf children with multiple disabilities. In my final two years in this substantially separate public school setting I taught math at the elementary level. It was during these years that my desire to provide access to quality math instruction with the proper supports for students with disabilities took its roots.

As a math teacher in the 1990's, I took many classes taught through Math

Solutions, an educational approach that advocates for problem solving with a strong focus on communication. I also attended workshops that focused on the use of manipulatives to support understanding math concepts. My philosophy was that math concepts should be uncovered not covered. By this I mean that students should explore and discover the concepts of mathematics through engagement in activities rather than a teacher covering the material through lectures. Another phrase I remember that guided my teaching stated that all students have gifts, they just unwrap them at different times. These words of wisdom, though not mine, were posted in my classroom and emulated through my teaching approach. I used manipulatives, visual supports, and language scaffolds because deaf students are visual learners and tend to have delays in language development both in writing and in ASL. Because this was the model that was encouraged at my school and supported through district professional development in mathematics, I was unprepared when I left teaching deaf students to work as a school psychologist in the general education setting.

To my surprise math was taught very differently at the middle school level with general education students with or without learning disabilities. While observing students as a school psychologist I saw math classes that entailed 20 minutes of home work review, followed by 15-20 minutes of teaching computational procedures, with the remaining 10 minutes or so practicing what was learned, which was to be completed for homework. I never observed problem solving activities, I rarely heard children explaining their reasoning, and I certainly never saw children writing in journals or reflecting on their thinking

processes. I saw the same thing in the math support class where students with learning disabilities receive extra math instruction. Not only that, but additional time was lost completing homework assignments that were overdue in some of their other classes. I did not see scaffolding or language supports to help the students access the curriculum to the best of their ability. Many were doing poorly in their general education math class and they were frustrated. Over the years, through interviews, I found that for many students, math was their least favorite class. As a former mathematics teacher, I was frustrated too, so I decided to do something about it. This decision brought me to UCSD to pursue a higher degree in the area of teaching and learning with the focus this past year on curriculum design.

I chose to work specifically with students with learning disabilities because research indicates that traditional methods of instructional support, including the practice and repetition of basic skills are not improving these students' scores on standardized tests, especially when compared to similar aged peers (Bottge, 2001). As a school psychologist I am privy to these academic assessments, which occur every three years for their IEP re-evaluation process. For the majority of LD students there is little if any improvement in their standardized math scores during this three-year period. The way these students are supported through special education needs to change. *Problem Solving Toward Mathematical Understanding: Instructional Design for Students with Learning Disabilities* is an attempt to offer an alternative to how we teach mathematics to students with disabilities.

Research suggests that metacognition, or “thinking about thinking” is important for all areas of academic achievement. Kramarski and Mevarech (2003) conducted a study that examined the effects of both cooperative learning and metacognitive training on mathematical reasoning in the classroom. This study looked at which classroom organization (individual or cooperative learning) with or without metacognitive training, was most effective in enhancing mathematical reasoning. Therefore there were four groups that were explored: cooperative learning combined with metacognitive training, cooperative learning without metacognitive training, as well as individual learning both with and without metacognitive training. The results indicated that the group with cooperative learning combined with metacognitive training outperformed all groups and those with metacognitive training outperformed those without. Children with metacognitive skills do better on problem solving activities and use more strategies such as planning, monitoring their progress, and evaluating the process (Scheid, 1989; Schraw & Dennison, 1994). Many children develop these “executive functioning” skills on their own but some, for whatever reason, do not. For these children, metacognitive instruction can be beneficial (Manning & Payne, 1996; Mevarech, Kramarski, & Arami, 2002). A key question is how do we teach students who have not developed the ability to select cognitive strategies, apply them, monitor their effectiveness, and adjust them as necessary?

A number of cognitive processes are called upon when a student is faced with a problem-solving task. Students with learning disabilities have processing deficits. That is the definition of a learning disability. It is “...a disorder in one or

more of the basic psychological processes involved in understanding or in using language, spoken or written, which may manifest itself in imperfect ability to listen, think, speak, read write, spell, or to do mathematical calculations” (IDEA, 1997). Many LD students require special education supports due to being two or more grade levels behind their non-LD peers. The students in this mathematics classroom were all children with Individual Educational Plans (IEP) and include the following disabilities: Specific Learning Disability, Intellectually Disabled, and Other Health Impairment. Although their handicapping condition affects them in a variety of ways in terms of academic achievement, all of these students are struggling in mathematics such that they require an additional mathematics class outside the general education classroom setting. Each of my students receives their primary mathematics instruction within a general education classroom with non-disabled peers.

Regardless of whether a child has a learning disability or not, I argue that the curriculum I have designed can benefit all students who are struggling in the area of problem solving. The data I collected indicate that scaffolding, or instructional supports designed to facilitate learning, can facilitate the development of metacognitive processes as well as cognitive processes. These specialized supports will enable students to engage in problem solving activities by supporting their ability to read a problem, select a strategy, monitor their progress, evaluate the outcome and adjust if necessary. My curriculum calls for the teacher to model metacognitive processes and support the students’ abilities to problem solve through metacognitive questioning. It also provides for think-

pair-share, and group discussions. In this way, as stated by Manning and Payne (1996), “social contexts provide the foundational core of cognitive and metacognitive development” (p.103). Reflective writing on how a task is approached allows the student the time to further explore his or her thinking and demonstrates to the teacher the development of critical metacognitive skills. Writing is scaffolded through the use of sentence frames and vocabulary banks that foster the use of the mathematical vocabulary necessary for effective writing. Sentence frames are a form of scaffolding that provides a structure to help students find the right words to explain or describe their thinking. Therefore, through this scaffolding, the teacher anticipates and tries to eliminate difficulties in an effort to allow for more efficient learning (Manning & Payne, 1996).

Thus far, I have discussed why special education students continue to struggle and how the curriculum I developed will benefit their abilities to improve in the area of mathematical problem solving. The next section is intended to give the reader an in-depth look at why there is a need to make changes in the way educators teach special education students problem solving in mathematics.

II. Assessment of Need

Mathematics achievement in the United States has always been a matter of contention for educators when it comes to comparing our students' abilities to those of other nations. Literacy in mathematics, meaning the ability to take mathematical skills and use them to solve mathematical problems in real life, is a skill that every child needs in order to become a productive member in our society (Stuart & Dahm, 1999). With the No Child Left Behind Act of 2001 came more accountability and more standardized assessments in U.S. schools that receive federal funding.

No Child Left Behind is a federal act, which mandates that all children achieve at the proficient level in English language arts, mathematics, and science by the year 2014. However, the definition of proficient varies from state to state as each state is allowed to define the proficient cutoff levels. Therefore, these levels may vary across the nation. Nonetheless, we compare our children's achievement at the international, national, state, district, and school levels. When comparing students' achievement in the United States with other nations one finds that we rank below many nations, as can be found in the Program for International Student Assessment, 2006 report released by the US Department of Education (DOE, 2007). This program measures math, reading, and science literacy every three years with emphasis on one subject area in depth. In 2006 science was the area assessed in depth; however, achievement in math was also reported as a minor subject. The data indicated that the average score in

mathematical literacy in the United States was lower than the Organization for Economic Cooperation and Development's (OECD) average with standard scores of 474 and 498 respectively. Thirty-one jurisdictions scored higher and 20 scored lower. The OECD is the sponsoring intergovernmental organization of PISA that consists of 30 member organizations (Baldi, Jin, Skemer, Green, & Herget, 2007).

The United States has been monitoring the achievement of our students for many years through assessments such as the National Assessment of Educational Progress (NAEP, 2009). The NAEP provides us with the Nation's Report Card, which reports on the biennial national testing that is federally mandated at the fourth and eighth grade levels. According to the Institute of Education Sciences, the 2009 statistics for eighth grade mathematics indicate that nationally, California's average score was lower than 45 of the participating states/jurisdictions. California basically tied with four other states and was higher than only two other jurisdictions. Compared to the nation, the average score for eighth grade students in California was lower by 12 points, with scores of 270 and 282 respectively. Nationally the average score has increased from 2007 to 2009 at grade eight but remained the same at grade four. There were significant differences at the $p < .05$ level between California's results in 1990, 1992, 1996, and 2000 when compared to the state score in 2009; however, accommodations were introduced in 2000. This indicates that the probability of this difference occurring by chance alone is less than 5%. When you look at the score gap that existed between to 25th percentile and the 75th percentile in both 1990 and 2009

the gap is not closing. It is, in fact, two points higher. In 2009 the gap was 53 points and in 1990 the gap was 51 points (NAEP, 2009).

Table 1. “Gaps persist despite gains for some student groups” (NAEP, 2009. P.5)

Characteristic	Grade 4		Grade 8	
	Since 1990	Since 2007	Since 1990	Since 2007
Overall	▲	◆	▲	▲
Race/ethnicity				
White	▲	◆	▲	▲
Black	▲	◆	▲	▲
Hispanic	▲	◆	▲	▲
Asian/Pacific Islander	▲	◆	▲	▲
American Indian/ Alaska Native	‡	◆	‡	◆
Type of school				
Public	▲	◆	▲	▲
Private	▲	◆	▲	◆
Gaps				
White-Black	Narrowed	◆	◆	◆
White-Hispanic	◆	◆	◆	◆
Private- Public	Narrowed	◆	◆	◆

▲ Indicates the score was higher in 2009.
◆ Indicates no significant change in the score or the gap in 2009.
‡ Reporting standards not met. Sample size insufficient to permit a reliable estimate.

In terms of the gaps between White-Black, White-Hispanic and Private-Public, scores have continued to show no significant change for fourth and eighth grade mathematics students since 1990 except for some narrowing of the White-Black gap and the Public-Private gaps that existed in 1990 at the fourth grade level. When looking at score gaps what is particularly interesting is the gap that exists for our students with learning disabilities who receive services through California’s special education programs.

Nationally the scores for students with learning disabilities, either with an IEP or a 504 plan, are significantly below those of the general public. In 2005, 2007, and 2009 the average scores of students without disabilities were 283, 285 and 287 respectively. The average score for students with learning disabilities in those same years were 245, 246 and 249 respectively. This indicates that the score gap in 2005 and 2009 remained the same at 38 points each. When breaking down the scores for each average subscale within the eighth grade test it is clear that special education students lag significantly behind their non-special education peers (National Center for Education Statistics, 2009).

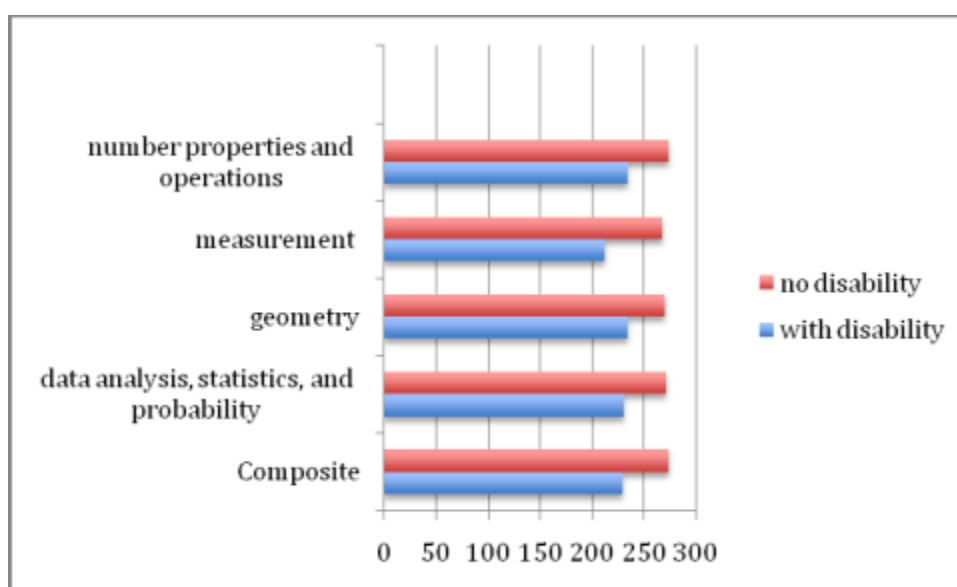


Figure 1. Grade 8 2009 CST math: Average subtest and composite scores by disability status of student

California as a state continues to struggle with raising mathematics achievement. So too does the San Diego School District, where this curriculum design project was conducted. San Diego Unified School District is the second largest school district in California and, according to their website, services

131,541 students in grades pre-K through 12. This includes 118 elementary schools, 24 middle schools, 28 high schools, 13 atypical schools, and 45 charter schools. Of these students, 16,062 receive Special Education services. That is equivalent to approximately 12.2%. It is a diverse school district with 45.7% Hispanic, 23.9% white, 11.8% African-American, 6% Filipino, 5.1% Indo-Chinese, 3.3% Asian, 0.4% Native American, 0.8% Pacific Islander, and 3.1% Multi Racial/Ethnic. There are 30.2% English learners and 59.1% are eligible for free or reduced meals (San Diego Unified School District, 2011).

At the federal level, San Diego Unified School District's (SDUSD) student achievement and teacher accountability is assessed through the NAEP assessment as previously discussed. At the state level the California High School Exit Examination (CHSEE), California English Language Development Test (CELDT), FITNESSGRAM Physical Fitness Test, and Standardized Testing and Reporting (STAR) are all administered annually (SDUSD, 2011). The STAR testing also includes the California Modified Assessment, or the CMA, used with children with learning disabilities, if appropriate. At the district level, students are assessed using the Benchmarks for Literacy, Mathematics, and Science; Developmental Reading Assessment (DRA); End-of-Course Examinations; and Writing and Reading Assessment Profile (WRAP) (SDUSD, n.d.). All of these standardized assessments serve the purpose of measuring student achievement and providing for teacher accountability. The STAR testing done each spring at the district level identifies student achievement in English language arts, mathematics, and science and holds the school accountable to the No Child Left

Behind mandate that all children achieve at the proficient level in these areas by 2014.

My investigation was conducted at Harper Middle School (all the names of people and places used in this thesis are pseudonyms). It houses grades six through eight. According to the School Accountability Report for Harper Middle School, during the year 2009-2010, the ethnic make-up of the student population included include Hispanic (59.4%), White 24%, 4.8% Indochinese, 4% two or more races, 3.5 % African American, Asian (2.1%), Filipino (1.8%) and 0.2 % each for Native American and Pacific Islander. Of these students, 69.7% receive free or reduced meals, 26.2% are English learners, and 17.2 % receive Special Education services. I chose to use the STAR assessment scores when comparing Harper Middle School to the state or district achievement. The following chart, retrieved from <http://www.sandi.net> on October 11, 2011, indicates the percentage of students who scored at or above Proficient on the STAR: Mathematics Assessment.

Table 2. Students who scored at or above proficient on the STAR testing

Grade	School			District			State		
	07-08	08-09	09-10	07-08	08-09	09-10	07-08	08-09	09-10
6	40.0	45.7	56.8	45.9	49.2	54.8	44	49	52
7	41.4	39.6	65.5	44.7	47.1	52.5	41	43	52
8	23.2	34.5	44.8	31.9	36.0	38.9	35	40	43

Although these scores indicate that Harper Middle School is achieving above both the state and district levels for 2009-2010, we must remember that 45 states or jurisdictions scored above California in terms of the average score of students in math achievement. With over 16,000 students receiving Special Education in the San Diego Unified School District, I wondered how well our students with special needs performed at Harper Middle School on the STAR mathematics test. Again, data on the Harper Middle School Accountability Report Card released in the Spring of the 2009-2010 school year indicate that among those students with disabilities in grade six, 37.1% scored at or above the proficient level, compared to 59.2 % of those without disabilities. At grade seven, 56.0% scored at or above proficient while 66.1% of their non-disabled peers reached this achievement. Grade eight students with special needs did not fair as well with only 24.4% achieving at this level, although 46.7 percent of students without disabilities were meeting or exceeding state standards. It is again worth reiterating that compared to other states and nations these scores portray a dismal picture of the success of California students in the area of mathematics. This is even more so for our special student groups which, for this investigation, highlights students with learning disabilities. What then should students be able to do in the realm of mathematics to be considered literate?

In 2000 National Council of Teachers of Mathematics (NCTM) developed principles and standards intended to guide what mathematics educators should teach and what children should learn in order to foster mathematical literacy. No longer is it enough to memorize facts, formulas, or algorithms in an attempt to

pass a mathematics class. Children need to understand the mathematical concepts and processes they are learning in order to function effectively in our technologically advanced society. Mathematics should no longer be considered only for a select few. All students, including those with learning disabilities, have the right to have opportunities and options available to them when they leave high school and not to have doors shut due to lack of mathematical competence (NCTM, 2000). LD students are likely to have many disadvantages such as, poor computational skills, difficulty processing information, as well as locating relevant information, and they are likely to have more difficulty than the average student in reasoning and problem solving skills. However, with the right support, these students can succeed at higher levels than they are currently performing (Gagnon & Maccini, 2001). Low expectations, poor teaching, and curricula that do not focus on important mathematics are no longer acceptable. The vision for mathematics education for which the NCTM (2000) advocates is one where...

Students confidently engage in complex mathematical tasks...They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers...Orally and in writing, students communicate their ideas and results effectively (p. 3).

Given these statistics and the goals proposed by the NCTM standards, how do we move our students forward from these dismal results? Research has

indicated that there is promise in some methods for students with disabilities.

The next section will look at what research has found and what we can learn from the literature in order to support the special education student population.

III. Review of Relevant Research

A large amount of research has been conducted in the area of developing students' mathematical problem solving skills. The research stems from a strong need for effective strategies that teach more than just procedural knowledge but instead offer a balance by also focusing on conceptual knowledge and reasoning skills. The current method of teaching students mathematics in the United States is failing many of children as was documented in the needs assessment of the previous chapter. Not only are schools failing students of color but also children of varying linguistic and cultural backgrounds. Students with learning disabilities (LD) are also, as a group, not making significant gains in the area of mathematics. In 1992 the National Assessment of Educational Progress (NAEP) conducted a mathematical assessment and its findings suggested that students receiving mathematics instruction in the United States are severely deficient in their mathematical problem solving abilities (Jitendra and Xin, 1997). After twenty years the statistics continue to be dismal. Students with learning disabilities often have reading difficulties in combination with mathematical difficulties making word-problem solving an even more challenging task. As a result, for over more than two decades, research has investigated and empirically validated alternative approaches and practices to teaching problem solving skills in mathematics for all students.

Since 1989 the National Council of Teachers of Mathematics (NCTM) has called for teachers to approach mathematical instruction in a new way. The

council revised their math standards and principles in 2000. The document addresses students with learning disabilities stressing the need for opportunities and supports to help them succeed (NCTM, 2000). Over the last several years there has been a joint project between the Council of Chief State School Officers (CCSSO) and the National Governors Association called the Common Core State Standards Initiative. Common K-12 reading and math standards have been developed to better prepare students for college and careers (ASCD, n.d.). So far, these standards have been accepted in 45 states and 3 territories with California adopting the common core standards on August 2, 2010 (Common Core State Standards Initiative, n.d.). These standards do not directly address how the needs of students with learning disabilities or English language learners should be taken into account; however they are designed for all students.

To be successful in mathematical problem solving students must have sufficiently developed several components and be able to integrate them successfully. Studies have shown that students deficient in self-regulation, or the ability to regulate, monitor, and control one's thinking processes have a very difficult time developing problem-solving skills despite interventions. Motivational components are important as well (Kajamies, Vauras, & Kinnunen, 2010). In addition, a solid knowledge base in mathematics, including easy retrieval of facts, algorithms and rules, etc., as well as reading comprehension, are important skills necessary for solving word problems. Verschaffel et al. (1999) pointed out that real-world knowledge about the situation embedded in the word problem is also important for comprehending the problem. What has not worked with students

with learning disabilities is to focus predominantly on basic skills instruction while ignoring problem solving experiences due to lack of automaticity. Instead, a balanced instructional approach that infuses basic remediation practice within meaningful problem solving experiences, along with explicit instruction is seen as more beneficial (Bottge, 2001).

The purpose of this chapter is to review research that is relevant to improving mathematical problem solving skills of students with learning disabilities. I include a review of research in the area of metacognitive instruction to gain an understanding of how this can improve LD students' achievement in mathematics. I also review how attitude and motivation can affect students' performance in mathematics. And finally, I review research on the importance of communicating in the mathematics classroom to develop better reasoning abilities.

Metacognition and Achievement

Metacognitive instruction has been shown to have the potential of increasing achievement. Metacognition is the ability to monitor and mediate one's own thoughts (Schraw, 1994). When confronted with a novel problem in mathematics a student with well-developed metacognitive skills is able to analyze the problem, monitor the solution process (self-regulate), revise the process as necessary, and evaluate the answer for correctness (Kramarski & Mevarech, 2003).

In a review of research Bottge (2001) pointed out that cognitive and metacognitive strategies combined with direct instruction show some promise in

helping students with learning disabilities improve in word problem solving abilities. With this direct instruction word problems can then be used as a medium through which these skills can be integrated into problem solving activities. Thus LD students can learn to use a variety of strategies necessary for handling all problems (Reid & Lienemann, 2006). Cognition strategies include, but are not limited to, such things as comprehension, prediction, planning, calculation, and evaluation (Montague, 1992), whereas metacognition entails knowing how, when, and why to use a strategy and the ability to analyze and shift directions if necessary when solving problems. Schraw and Dennison (1994) conducted a study to assess metacognitive awareness and in their article they indicated that Swanson (1990) found sixth grade students who were metacognitively aware performed better on problems and used more strategies than those that were unaware.

In a study conducted by Mevarech and Kramarski (1997) junior high school students received metacognitive instruction for one academic year using a method called IMPROVE. These students significantly outperformed those in the control group on various mathematics achievement measures (Mevarech, Kramarski, & Arami, 2002). In another study by Kramarski and Mavarech 384 eighth grade students from 4 junior high schools were instructed using one of four methods: cooperative learning with metacognitive training, cooperative learning without metacognitive training, and individualized learning with and without metacognitive training. The metacognitive training used three sets of self-addressed metacognitive questions. These were comprehensive, strategic

and connection questions and were used as prompts for the students. The findings showed that transfer, or the ability to apply skills in a novel situation, were improved from one mathematical topic to another that was not taught when students were exposed to metacognitive training. This was true for both the individual and cooperative groups. The findings also showed higher scores on a questionnaire designed to measure metacognitive knowledge. In terms of mathematical reasoning these two groups did better in terms of achievement on the post-test as well as justifying their reasoning (Kramarski & Mevarech, 2003).

In another study conducted on four classes of fifth graders, researchers designed an experiment where students were taught a series of heuristics embedded in an overall metacognitive strategy. Heuristics include such strategies as making a drawing, looking for patterns, making a chart or table, or guessing and checking. Students were found to have deficiencies in their mathematical knowledge base as well as these valuable strategies. Seven classes were used as a control group and followed the regular mathematics curriculum. After the intervention, improvements were found on different aspects of mathematical modeling and problem solving achievement in the experimental group. Also positively affected were students' persistence and enjoyment as well as their beliefs and attitudes toward problem solving (Verschaffel, et al., 1999).

The research suggests that students with learning disabilities benefit from direct instruction in cognitive and metacognitive processes, as well as problem solving strategies. Direct instruction ensures that students have the skills

necessary to improve their performance when attempting word problems or other novel tasks.

Motivation and Attitude

Academic intrinsic motivation is positively related to achievement in school and, when exhibited in the primary grades, can be predictive of later motivation (Gottfried, Fleming & Gottfried, 1994). According to Gottfried (1985), academic intrinsic motivation relates to persisting with challenging tasks in order to master them and encompasses an enjoyment of learning in the school setting. In the classroom students are often motivated to complete homework assignments, class work, and to participate via rewards. Rewards are often used on a contingent basis to motivate the student to conform to someone else's goals, such as curriculum goals, school goals or society's goals. When this occurs students often focus on getting the reward instead of learning the material. This extrinsic motivation changes the attitude one brings to the activity (Kohn, 1993). Deci and Flaste (1995) contend that we cannot motivate people through rewards and expect to have a lasting effect.

Teachers need to set up an environment that motivates students by tapping into their interests and providing richer experiences so that intrinsic motivation is brought to the surface (Deci & Flaste, 1995). Students are also more likely to be motivated if they feel the learning is relevant to real-life situations (D'Amico & Gallaway, 2008). By offering activities that incorporate students' interests and culture, or by providing motivating modes of learning such as technology and

blogging, the teacher is more likely to tap into the students' intrinsic motivation for learning (Deci & Flaste, 1995).

Attitudes and beliefs can affect how a student performs on the mathematical problem solving activities. Chronic failure can affect a student's beliefs about themselves which in turn can affect their motivation. Fleischer and O'Loughlin (1985) discuss the effect of what they call "affective aptitudes." This is extremely important to consider with LD students who have struggled their entire academic career. These students may exhibit a learned helplessness and require significantly more encouragement than their non-disabled peers.

Beliefs held by students concerning the nature of mathematics may also affect students' motivation. If the student believes that there is only one right answer and only one correct way to solve a mathematics problem they may attempt the problem and give up easily when not successful without considering alternative strategies (Verschaffel, et al., 1999). They may also believe that the ability to do mathematics is innate. Schoenfeld (1992) argues that people in the United States are more likely than those in Japan to believe this and therefore, as a society, this belief becomes reinforced. To counter these beliefs and attitudes Schoenfeld (1992) would suggest that the mathematics teacher should create an environment that allows for risk-taking, discussions of strategies and ideas, and the logical reasoning behind them. A risk-taking classroom culture fosters a socialization process where students interact with their peers, the teacher, and the mathematics in order to develop mathematical thinking (Schoenfeld, 1992). This creates a safe environment for students to listen to

others approaches, strategies, and ideas while, at the same time allows for mistakes or misinformation to be acknowledged and dispelled.

In summary, motivation can be influenced by several factors, which can affect a student's ability to achieve in mathematics. These include the presence of intrinsic motivation within a student and whether a teacher can provide activities that foster such motivation for those who don't. Attitudes and beliefs about oneself and mathematics can stifle motivation as well. A classroom culture that encourages risk-taking can provide a forum where students can begin to dispel self-fulfilling beliefs that one is not capable of achieving in mathematics and allow them to move toward developing higher-level mathematical thinking skills. That being said, the ability to use communication to express mathematical ideas and reasoning is an important skill to foster in order to develop mathematical thinking.

Mathematics and Communication

Considerable research has demonstrated the effectiveness of communication in the mathematics classroom on student achievement. The QUASAR (Quantitative Understanding: Amplifying Student Achievement and Reasoning) project, a large-scale research project was a five-year study that took place in six urban middle schools with culturally and linguistically diverse student populations (Silver & Stein, 1996). All the students involved lived in poverty. The schools populations ranged from 300-1500 students. The project aimed at increasing reasoning and problem solving in mathematics while developing better conceptual understanding. The instructional approach promoted communication

through collaboration, cooperative learning, and explaining reasoning, to a much greater extent than often seen in the mathematic classroom (Silver & Stein, 1996). Silver and Stein (1996) used the Quasar Cognitive Assessment Instrument (QCAI) specifically developed to assess communication, problem solving and reasoning skills and found clear evidence that students improved in all these areas. Conceptual understanding also greatly improved. Students were also given one of two 1992 NAEP tasks following the first year of implementation and did considerably better when compared to NAEP's similar disadvantaged urban groups. Another finding was that more than 40 percent of participating students were eligible for placement in grade nine algebra classes four years after the projects implementation as opposed to only 8 % after the first year of implementation (Silver & Stein, 1996).

Schoenfeld (2002) also indicated the importance of students learning to reason and communicate. In addition, the NCTM (2000) and the Common Core Content Standards for Mathematics both call for this at each grade level. However, communicating mathematically may be difficult for some students, in particular students with learning disabilities that affect reading and/or writing, as well as English language learners (ELL). For these students teachers need to provide supports while they are learning to talk about mathematical ideas (Gutierrez, 2002). Gutierrez analyzed three high school teachers' work with large numbers of Latina/o students who had advanced through their curriculum over a 13-month period of time. Although these students were mostly English dominant the analysis indicated that strategies typically used by elementary and middle

school teachers as well as in bilingual education settings were also successful with these students. His article focused mainly on the use of language in the classroom and the fact that all students, regardless of their primary language, need to learn to communicate using the mathematical register, a language that differs from everyday language (Gutierrez, 2002). Doty, Mercer and Henningsen suggests that encouraging students, within group problem solving situations, to debate, explain and discuss will improve students understanding and self-reliance in mathematics (1999).

Incorporating writing within the mathematics classroom is also beneficial to students' learning. Writing assignments help students make sense of mathematics by organizing and clarifying ideas, as well as monitoring and reflecting on the problem solving process (Burns, 2007). Math has its own vocabulary that is typically not a part of our everyday usage. In order to talk about mathematical ideas students need to have the appropriate vocabulary as well as an understanding of the math it describes. Therefore vocabulary must be explicitly taught. There are mathematical words that exist that have different meanings from their common meanings (Burns, 2007). Burns advocates for instructional strategies that contain the use of word charts, repeated use, students pronouncing the words, encouraging the use of the vocabulary during instruction, and explaining the words while connecting it to the learning experience. Melanese, Chung and Forbes (2010) also suggest identifying the differences in mathematical and common meanings and for finding cognate from ELL students' primary language when applicable. Cognates are words within the

same language that can have meanings that are slightly different or completely different. Clarifying the differences between cognates could be especially helpful for ELL students who also have a disability.

Summary

Research on the subjects of metacognition, motivation, mathematical communication and problem solving indicates that although it is possible to improve LD students' achievement in these areas it is demanding and difficult (Witzel & Riccomini, 2007). That difficulty, however, does not mean teachers should shy away from the challenge. All students need to have the opportunity to learn mathematics in a way that will make them able to function successfully in a society that demands higher-order thinking abilities in mathematics. The data suggest that although basic skills and automaticity are fundamental to high achievement in mathematics, these skills can be taught and reinforced while infusing metacognitive instruction throughout the curriculum. By developing activities that will motivate and challenge students, providing scaffolding and modeling is necessary for success. By encouraging communicating in the mathematics class, teachers not only develop students' high-order thinking skills but challenge their beliefs regarding mathematics. It is a matter of equity in learning for all students. These ideas have been recommended since the mid-1980 especially with the call for reform under the NCTM (1989) Curriculum and Evaluation Standards. Why then do the scores of particular groups of students (i.e., African American, ELL and LD) remain stagnant and the achievement gap between these groups and White students remain so wide? Perhaps the answer

lies within the curriculum that schools are using in the mathematics classrooms of K-12 U.S. students.

IV. Review of Existing Approaches to Learning

Academic standardized testing abounds in this day and age as teachers are under a great deal of pressure to improve students' academic performance, particularly in the areas of mathematics and reading. With the renewal of No Child Left Behind (NCLB) in 2001 all students are expected to perform at the proficient level on statewide, standardized assessments by 2014. This includes students with disabilities. Schools, districts, and states are held accountable and provisions have been put in place for those not making adequate yearly progress. Also, this piece of legislation calls for closing the gap that exists between high performing and low performing students. In particular we see significant gaps among various minority groups, students from low socioeconomic background, limited English proficient students, as well as students with learning disabilities. The 1997 Amendment to the Individuals with Disabilities Education Act (IDEA) protects students with disabilities indicating that these students have the right to access the general education curriculum, receive instruction on the same skills and concepts as those without disabilities, and make adequate progress toward specific goals (IDEA, 1997). Yet these students continue to fail. Schools have attempted to address this issue in many ways, curriculum being only one.

School systems spend a great deal of money on textbooks and middle school mathematics teachers rely heavily on them to ensure they are teaching to the standards in mathematics. Not only that, but the San Diego Unified School

District (SDUSD) requires Benchmark Testing to be done every three weeks to ensure students are grasping and retaining the curriculum (SDUSD, n.d.). However, not all textbooks are well designed or address the standards adequately. Nor do all textbooks address the needs of a diverse student population with various learning levels. In 2000 the National Council of Teachers of Mathematics (NCTM) developed principles and standards that called for a shift in mathematics education from procedural knowledge and rote learning to an approach that provides problem solving opportunities to develop thinking and reasoning skills and a solid conceptual understanding of the mathematical concepts and skills presented (Schoenfeld, 2002). Over the years many publishers have attempted to include in their textbooks more problem solving opportunities and activities that embrace the ideas of the NCTM's Principles and Standards. I will review the NCTM Standards, a seventh grade mathematics textbook (Prentice Hall Mathematics) and a more innovative approach to teaching mathematics (Math Solutions). By examining these we can get a glimpse at what is available to districts and teachers in terms of curricula and pedagogy. In addition, the National Common Core Standards adopted in 2010 provides a glimmer of hope that future textbooks and training of math teachers will focus on what students should understand in mathematics rather than predominantly what they can do procedurally. While no initiative comes with a guarantee of success these standards hold high expectations and uniform standards among the states and territories that have adopted them.

NCTM Standards

The NCTM Principles and Standards is not a curriculum textbook but instead a resource and guide. First I will examine these standards and then use them to evaluate two different curriculum materials. There are ten standards for grades prekindergarten through grade 12. These describe what students should be able to understand, know, and be able to do at each grade level. The Content Standards, which describe what students should learn, include Number and Operations, Algebra, Geometry, Measurement, as well as Data Analysis and Probability. A significant amount of attention is focused on algebra and geometry. This is due to the fact that U.S. students typically perform most poorly in the area of geometry when assessed and compared both domestically and internationally and algebraic thinking is widely held as important in today's workplace (NCTM, 2000). The remaining standards, known as Process Standards, include Problem solving, Reasoning and Proof, Communication, Connections, and Representation. These describe the process of learning and using the content of the curriculum. Each standard has goals that are the same across all grade levels (NCTM, 2000). In looking at the Process Standards one can see the shift from rote learning and procedural knowledge to developing a deep conceptual understanding where students use higher-order thinking skills to solve problems. Table 3 contains the Process Standards mentioned above. The NCTM standards call for monitoring the problem solving process, communicating mathematically, analyzing and evaluating the thinking of others, applying and adapting a variety of strategies, and constructing new mathematical knowledge

through the use of problem solving, just to name a few. However, these are the areas where my curriculum is focused and it is with this in mind that I will evaluate the following two curriculum materials.

Table 3. Process standards for grades 6-8 (NCTM, 2000, p.402)

Problem Solving Standard for Grades 6-8

Instructional programs from prekindergarten through grade 12 should enable all students to—

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof Standard for Grades 6–8

Instructional programs from prekindergarten through grade 12 should enable all students to—

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

Communication Standard for Grades 6–8

Instructional programs from prekindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

Connections Standard for Grades 6–8

Instructional programs from prekindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.

Representation Standard for Grades 6–8

Instructional programs from prekindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

Prentice Hall Mathematics

The textbook series that has been adopted in California by San Diego Unified School District at the middle school level is Prentice Hall Mathematics (Charles, McNemar, & Ramirez, 2009). Charles, McNemar, and Ramirez co-authored the Pre-Algebra textbook, used in grade seven. The vocabulary development consultants include Kate Kinsella whose specialty is Second Language Acquisition and Kevin Feldman, whose specialties include Special Education, Learning Disabilities, and Instructional Design as well Curriculum and Instruction.

The suggested pacing guide for this curriculum consists of 169 days of instruction scheduled from September through mid-June and consists of 12 chapter units. The guide, provided by the school district, gives a time-line to which the teacher is required to adhere. This leaves little time to address areas that are not covered sufficiently in the textbook such as problem solving and writing activities. It also presumes that students learn at the same pace and does not leave room in the schedule to re-teach or extend the lessons. Table 4 shows the content covered in each chapter. As can be seen there is quite a bit of material to be taught over the course of one school year with little time left for problem solving, discussions, group work, or writing about mathematical ideas

Table 4. Pre-algebra contents by chapter (Charles et al., 2009)

Chapter	Content
1	Algebraic Expressions and Integers
2	Solving One-Step Equations and Inequalities
3	Decimals and Equations
4	Factors, Fractions, and Exponents
5	Operations With Fractions
6	Ratios, Proportions, and Percents
7	Solving Equations and Inequalities
8	Linear Functions and Graphing
9	Spatial Thinking
10	Area and Volume
11	Irrational Numbers and Nonlinear Functions
12	Data Analysis

Each chapter includes a Check Your Readiness section, Checkpoint Quizzes 1 and 2, Guided Problem solving, Activity Lab, Vocabulary Builders, Mathematical Reasoning, as well as a Standards Mastery and Assessment section. The latter includes Test-Taking Strategies, Chapter Review, Chapter Test, Multiple Choice Practice and Standards Mastery Cumulative Practice. Students having difficulty can refer to the “Go for Help” points where they can review lessons or access the textbook online. Once online, students can access the Video Tutor Help, Active Math, the Homework Video Tutor, Lesson Quizzes, a Vocabulary Quiz and a Chapter Test. At the beginning of each chapter in the

Teacher's Edition there are suggestions under a section titled Universal Access: Solutions for All Learners. A suggestion is given under the headings of Special Needs, Below Level, Advanced Learners and English Learners.

The curriculum is very focused on the California standards. The Teacher's Edition indicates, within each chapter, the standards that are addressed and the strands to which they pertain. The manual offers a table, which indicates correlations to the California math content standards with the Prentice Hall math lessons. Number Sense has 51 lessons in which the standard is either introduced, developed or mastered; Algebra and Functions, as well as Measurement and Geometry have 55 lessons; Statistics, Data Analysis and Probability has only 9 lessons that address these standards; and Mathematical Reasoning is uncovered in 77 lessons (Charles, et. al., 2009).

Advantages

As indicated above, this textbook is very much tied to the current California standards and the Teacher's Edition provides a great deal of information pertaining to them. It also supplies information in the student's textbook for the students and their parents. It appears to be well organized, with skills being introduced in a logical order where later skills build on earlier skills. The same occurs with mathematical concepts. The program uses both formative and summative assessments, and it also has an online Success Tracker where teachers can access student progress, find students' strength and weaknesses, generate reports and even personalize remediation based on each student's performance (McNemar and Ramirez, 2009). Resources are provided for

students with special needs, English learners, gifted and talented students, as well as advanced learners.

Disadvantages

Math textbooks heavily influence teachers' daily decisions about instruction. In reviewing this curriculum series the textbook does not provide a good match for the learning needs of students with disabilities. In order to provide a match the teacher would have to provide a number of modifications for these students to benefit (Witzel & Riccomini, 2007). Teachers would have to be knowledgeable about assessing where the child is at when entering the mathematics classroom for the first time. They would also have to know how to scaffold to support the development of mathematical knowledge by providing a bridge from the students' current independent level to where the teacher wants them to be. Content difficulty levels can also be controlled by scaffolding the instruction (Sood & Jitendra, 2007). Minor suggestions are included for diverse learners; however, they are very limited. Although there are resources for special populations of students, these may not be available in some schools if these resources are not purchased. For example, an All-in-One Student Workbook, Adapted Version is available for students with learning disabilities as well as Spanish Practice Workbooks, a Multilingual Handbook and Vocabulary Worksheets for English language learners.

Another concern stems from the availability of on-line tutoring and homework help. Disadvantaged students such as those with low-socioeconomic status may not have the resources to purchase a computer and/or pay a monthly

fee for Internet access. Students with learning disabilities often have below grade level reading comprehension and may not have an appropriate reading level to access the online help (King-Sears & Duke, 2010). This may also pose difficulties for English language learners. In terms of access to the written material within the textbook itself, this may also present a problem for these students. The teacher's notes on Universal Access give only one simple suggestion at the beginning of each chapter for diverse learners.

In terms of the Process Standards as developed by NCTM, Prentice Hall Mathematics does offer opportunities to answer questions and write about math. An example taken from the textbook asks: "How can you use integers to describe elevations above and below sea level?" (p. 21). Problem solving activities are found at the end of the chapter and are not embedded into the curriculum. The pacing guide is rigid and the location of these problems is so obscure they are often left out of lesson plans. Moreover, the problems provided do not require higher order thinking skills and the ability to use a variety of strategies to solve the problem. Nor do they encourage reflective practices. You also do not see activities within the textbook that will excite students and call for them to engage in peer or group discussions, make conjectures, explore them and refine them as needed. The word problems appear to be similar to those seen in most textbooks, typically being the type that invites students to practice the skills recently taught. The NCTM standards call for problems that are not readily solved with only procedural knowledge. They call for problems that develop flexibility in problem solving and reasoning skills, an ability to represent problems

in different ways, an ability to reflect on the problem solving process they are engaging in, and the ability to shift to different approaches until a solution is found (NCTM, 2000).

Math Solutions

Math Solutions is another resource for teachers of mathematics. Math Solutions is a company that publishes their own materials. Through the purchasing of their resources they spread a philosophy of teaching and learning in mathematics. Math Solutions calls for teachers to have a deep understanding of the concepts they teach. They advocate for teachers to have insights in how children learn in the area of mathematics. They offer classroom-tested lessons and activities and encourage teachers to adapt them based on their awareness of how their students learn, and the lessons in order to address students' particular needs (Burns, 2007).

Marilyn Burns is the founder of Math Solutions and its mission. Burns first began teaching in 1962 and the third edition of her resource book, *About teaching mathematics: A K-8 resource*, is the culmination of many years of hard work and research since the first edition went to press in 1981. The book is designed to help teachers develop a deeper understanding of the underlying mathematics concepts behind the content they teach, have more clarity in how children learn mathematics, further develop skills in implementing effective instructional strategies, as well as to be better enable teachers to integrate assessment into their instruction and provide a safe and supportive learning environment for children (Burns, 2007).

The book contains five parts that include Raising the Issues; Instruction Activities for the Content Standards; Teaching Arithmetic; Mathematical Discussions, and Questions Teachers Ask. Numerous activities are provided in Part 2 that allow students to explore content in the areas of “measurement; probability and statistics; geometry and spatial sense; logical reasoning; patterns, functions, and algebra; and number and operations” (Burns, 2007, p. xv). These activities are classroom tested, therefore they have been shown to excite students and engage them in mathematical exploration. The book also discusses the NCTM 2000 Principles and Standards, addresses why we cannot effectively teach mathematics until we, as teachers, have a deep understanding. It also provides information on examining children’s arithmetic errors in order to align instruction for corrective measures. Math Solutions also prescribes to the philosophy of not teaching arithmetic skills in isolation but rather infusing these skills throughout the problem solving process and embedding it into contextual situations (Burns, 2007).

Advantages

There are several advantages to this resource source. First, the activities adhere to the standards, which were developed by the NCTM in 2000. They also believe that children can learn mathematical concepts and arithmetic skills through a problem solving approach. Research studies indicate that solving word problems is far more difficult for students than solving computational problems. In her book, Burns indicates that this is due to the fact that children do not really deeply understand the underlying meanings of the operations and

therefore have difficulty making connections when confronted with word problems (2007). The problem solving activities provided involve more than what is typically demanded in the solving of word problems found in many textbooks. They tap into the Process Standards of the NCTM Principles and Standards in that they ignite mathematical reasoning abilities, flexibility in thinking, creativity in approaches, and conjectures that need to be proven and justified. Oral and written communication of mathematics is also emphasized. Problem solving strategies are taught that are useful for analyzing a problem and solving it successfully. These strategies include looking for a pattern, constructing a table, making an organized list, acting the scenario out, representing by drawing a picture or using objects, using the Guess and Check method, working backwards, writing an equation, solving a simpler or similar problem, and making a model (Burns, 2007). Class discussions of how various students solved the problem illustrates that there is not necessarily one approach or strategy. This helps develop students' flexibility in thinking and perseverance when faced with obstacles while looking for a solution.

Another advantage of this approach is that the activities are designed for all children thereby providing the scaffolding, language, etc., needed to address diverse learners.

Disadvantages

In her book, Burns (2007) addresses arguments that arise when teachers are presented with this approach to teaching mathematics. The first argument presented is that there just isn't enough time in the school day to focus on

problem solving when students do not have the basic arithmetic skills needed to pass standardized assessments. Another argument is that parents expect basic arithmetic skills to be taught in schools. However, as indicated above, adequate computational skills do not necessarily produce good problem solvers. The third argument that is addressed in the book is that students have so much difficulty with these basic skills because they are unable to problem solve due to computational errors. The final argument addressed is that not all teachers feel they are strong enough in mathematics to comfortably teach problem solving skills (Burns, 2007). This is likely more of an issue at the elementary level and for special education teachers where a credential in mathematics is not required.

This brings forth another disadvantage of this resource in that it is not a textbook with an outline and a step-by-step process on which teachers, who are not comfortable with their mathematical knowledge, can rely. Teachers may not be able to fully implement the activities without engaging in professional development where they can experience the lessons first hand. Teachers also need to have confidence in their own ability to infuse into the standard textbook curriculum opportunities to do activities that allow students to develop important skills necessary in today's world, such as mathematical reasoning, ability to discuss mathematical ideas clearly and efficiently, and to look at numbers in a different way (Burns, 2007).

A Glimmer of Hope: Common Core Standards

In 1997 California adopted its own mathematics content standards (California Department of Education, 2006). These standards have driven

standardize testing, which in turn has since driven curriculum and instruction. These standards have also failed many of our students in their ability to obtain mathematical literacy because they do not focus on problem solving and communication. With the adoption of the California Common Core State Standards (CCSS) for Mathematics in August 2010, those concerned about mathematical achievement find hope that mathematics curricula and instruction will provide more balance between procedural knowledge and conceptual knowledge as well as the ability to discuss and write about mathematics. The standards call for students to “make sense of problems and persevere in solving them; reason abstractly and quantitatively; construct viable arguments and critique the reasoning of others; model with mathematics; use appropriate tool strategically; attend to precision; look for and make use of structure; and look for and express regularity in repeated reasoning” (California Department of Education, 2010 p.1-2).

Information from the California Department of Education, however, indicates that fully implementing these standards will take several years. During this time new curriculum frameworks, instructional materials, and assessments will be implemented (California Department of Education, 2011). In a PowerPoint presentation developed by Tom Adams, director of Standards, Curriculum Frameworks and Instructional Resource Division, the materials implementation is projected to be between November 2016 and November 2018 (CDOE, 2011). Is it fair that our students must wait this long?

Conclusion

The NCTM Principles and Standards developed in 2000 set off a change in how mathematical teaching and learning was viewed. Schools were asked to adapt and change their view of mathematical teaching. No longer was it acceptable to teach through rote learning and memorization, but instead through mathematical teaching that developed the ability to reason mathematically; communicate numerically; allow for flexibility of thinking; and all those things for which the NCTM has developed standards.

One of the ways in which schools have adapted is through the curriculum materials they use. This review of curricula takes a look at a seventh grade pre-algebra textbook published through Pearson Prentice Hall and a more innovative approach developed by Marilyn Burns and through the organization, Math Solutions. Both are examined using the NCTM, 2000 Principles and Standards.

The California Common Core Standards brings hope that new curriculum materials and instruction will have the structure that is found in the Prentice Hall Mathematics series and the types of activities that call for a problem solving approach as seen in the resource books available through Math Solutions. Unfortunately, it will take several years before these standards are fully implemented. In the meantime, my curriculum shows one way teachers can infuse activities that provide access for all students, develops skills that will improve problem solving abilities (reasoning) and students' ability to reflect on their problem solving process (metacognition), while at the same time improving their attitude toward math and increasing their motivation.

V. Problem Solving Toward Mathematical Understanding

Introduction

The call for reform in mathematics began with the National Council for Teachers of Mathematics' (NCTM) Curriculum and Evaluation Standards that set a precedent for a change in mathematical teaching and learning in 1989. After over a decade of controversy, the NCTM developed the principles and standards that currently exist. Many curriculum series have tried to adapt their programs to be more in line with these standards in order for all students to develop a deep understanding of mathematical concepts, higher-level reasoning and problem solving skills, and the ability to communicate about mathematical ideas. However, the data, retrieved in November 2011, suggest that not all students are succeeding, particularly students of some minority groups, low socioeconomic status, English language learners and students with learning disabilities (San Diego Unified School District, n.d.).

Research on mathematics achievement for children suggests that cognition, metacognition, and motivation are all components necessary to be successful in mathematics. Studies have shown that metacognition, in terms of monitoring the problem solving process, and motivation are important components in order for students to achieve and without these, interventions do not always provide enough support to successfully develop problem-solving skills (Kajamies, Vauras, and Kinnunen, 2010). When considering the additional challenges that students with learning disabilities face learning mathematics,

Bottge indicates that direct instruction with metacognitive strategies have shown promise for this particular group (2001). Not only this, but research supports the supposition that competence in language is also important as limitations can hinder higher-level mathematical achievement (Carnine, 1997).

The curriculum I have designed, *Problem Solving Toward Mathematical Understanding: Instructional Design for Students with Learning Disabilities*, (hereafter referred to as Problem Solving Toward Mathematical Understanding or PSTMU) pulls together the knowledge gained from the research by infusing metacognitive questioning, language scaffolding, as well as oral and written reflection in a problem solving approach as an addition to the curriculum adopted by the school system. Scaffolding of the problem solving activities, by beginning with an easier problem and moving onto more difficult problems, allows students to develop confidence and motivation. At the same time this scaffolding allows students to focus their cognitive energy on the problem solving process rather than on difficult computations, an area in which many students with learning disabilities are particularly weak.

Appropriate Settings for Implementation

Special education students are found in a variety of settings. Some receive their educational instruction in the general education setting, perhaps with accommodations, such as preferred seating or extra time to complete assignments, and/or modifications to the curriculum. Other students may receive their instruction in the general education environment with a co-teaching partnership between a general education teacher and a specialist in special

education. Others receive their instruction in self-contained classes with other special education students and a special education teacher. And still others are in the general education setting with support but also receive supplemental instruction in academic areas such as math or English in a substantially separate class that is not within the general education setting typically referred to as a resource class. In whatever environment a student with a learning disability receives specialized instructional assistance, there is a need to scaffold instruction so that the student can access the curriculum. That being said, how does my curriculum aid in the instruction of mathematics within any and all of these settings?

Problem Solving Toward Mathematical Understanding can be implemented within any of these settings. First and foremost is the idea that students need to be have the opportunity, on a consistent basis, to develop problem solving skills in mathematics that do not focus solely on practicing a computational method or a rote procedure. Children need to practice solving problems that do not present a readily identifiable equation or method that has been studied in a unit, but rather, to struggle with how to solve a problem and to engage in the use of a variety of problem solving strategies. The solutions may or may not include a traditional computational method or formula that was recently covered in the unit of instruction. Children need to uncover ways to solve problems that may be non-traditional (e.g. drawing a picture), in a mathematics teacher's experience, but are still valued and result in a procedure or strategy that provides a resolution to the problem presented. These skills will

be essential to students in order for them to become contributing members of society as 21st century jobs that require higher levels of abstract tasks such as solving novel problems (Hilton, 2008).

How would a teacher implement *Problem Solving Toward Mathematical Understanding* in a general education setting? Typically, this type of setting is comprised of general education students who are of varying levels of mathematical ability and perhaps several students with learning disabilities. The teacher would follow the curriculum chosen by their school but perhaps take one day a week to do a problem solving activity that is aligned with the standard they are currently addressing. An example of this would be the activity I used with my students titled "Guess the Function." The standard addressed is a one-step or multiple-step function in a pre-algebra class. The scaffolding suggested in my curriculum is made available to any student whose abilities may require these supports. These scaffolds may be suitable for English language learners, lower functioning students, or students with learning disabilities. Higher functioning students will likely dismiss the additional scaffolding and approach the task or use it in a way that best suits their abilities. I believe that providing scaffolding is a good practice in order to differentiate instruction within the general education setting.

In a substantially separate class or in the resource class, both of which do not include general education students, the teacher may have students at a variety of mathematical proficiency levels with a range of learning disabilities. This teacher has the flexibility to use the curriculum in whatever way best suits

the children in order to follow the academic goals of their Individual Educational Plans, or IEP. This instructional design can be used to supplement the school's chosen curriculum, as in the general education setting, or as an approach to teaching the standards through a problem solving approach.

Curriculum Overview of Goals and Constructs

Problem Solving Toward Mathematical Understanding is designed to teach children that there are many ways to represent and solve mathematics problems, to improve their mathematical reasoning skills, and to persevere after attempting to solve a problem without initial success. Through the use of metacognitive questions as raised by the teacher, the students begin to develop metacognitive awareness that helps them monitor if a strategy is working and drives them to consider different options when it is not. Metacognitive questions provoke the students to think about what they are doing and evaluate its effectiveness. If it is not effective they reconsider what the problem is asking and how to solve it. Therefore, they engage in a recursive approach to problem solving that promotes perseverance. Perseverance is also fostered by the culture of the classroom where a safe environment encourages risk taking, discussion and sharing of ideas, as well as focusing on multiple ways to solve a problem instead of one, and only one formulaic method. In this way, *Problem Solving Toward Mathematical Understanding* is designed to help children develop a deeper understanding of the concepts embedded in the problem through peer-talk, teacher-talk, and group discussions designed to help students come to a shared understanding of the material presented as well as to observe

that there is not only one correct approach to solving a problem. It is through this socialization process that mathematical knowledge is constructed.

Table 5 provides an overview of how the goals of this curriculum are connected to the constructs and the curriculum features. It also provides information on what was used to assess the students on their progress toward the curriculum goals.

Table 5. Connections between goals, constructs, curriculum features, and the evaluation plan

Goal for Students	Research Constructs	Curriculum Features	Data for assessment of Student Learning and/or evaluation
Make sense of problems and persevere in solving them.	Scaffolding Attitude Motivation Metacognition	-Manipulatives -Discourse (teacher-talk, peer-talk/work, group discussion) -Direct teaching of strategies -Direct teaching of problem solving process -Metacognitive questioning	-Observation -Pre/post -Metacognitive questionnaire -Pre/post attitude survey -Student work -Field notes
Students will reason abstractly and quantitatively.	Scaffolding Metacognition	-Graphic organizers -The Process Sheet -Metacognitive questioning -Journals -Sentence Frames -Reflective Writing	-Journals -Observations -Student work -Field notes -Rubric
Students will construct viable arguments and critique the reasoning of others.	Scaffolding Metacognition Communication	-Group discussion -Peer-talk/work -Metacognitive modeling -Sentence frames	-Observations -Tape recordings -Field notes

The goal of this approach is to develop the skills necessary to problem solve. These skills include the ability to read a problem, analyze its meaning and what it is asking for, plan a strategy, implement the strategy, evaluate the

process and the outcome and, if necessary, to continue this process by returning to areas that need additional attention, until a solution is found (Schoenfeld, 1985). The approach also provides supports for students to develop the ability to represent problems in a variety of ways and to discuss and justify their solution through the use of sentence frames and journal writing on the process.

Metacognitive awareness is called upon in the reflective writing activity that asks the children to think about different aspects of the problem and their ability to find a solution. This awareness was specifically elicited from the students through the teacher's use of questioning that draws upon the students' ability to analyze what they were doing and why they were doing it. The questions this design will attempt to answer include the following:

- ❖ How does a metacognitive and scaffolded approach to mathematical problem solving improve the achievement of students with learning disabilities in the area of mathematical reasoning?
- ❖ How does a metacognitive and scaffolded approach improve the ability of students with learning disabilities to communicate mathematically both orally and in writing about how they approached a problem and their reasoning behind this approach?
- ❖ How does a metacognitive and scaffolded approach improve students' attitudes and motivation allowing them to persevere in solving problems?

Curriculum Features

Problem Solving Toward Mathematical Understanding uses features that research supports as being beneficial to student achievement.

Direct Teaching of Strategies

Students received direct instruction on the use of strategies to analyze and solve problems. These strategies include looking for a pattern, constructing a table, making an organized list, representing by drawing a picture, using objects, guessing and checking, working backwards, writing an equation as well as solving a simpler or similar problem (Burns, 2007). This direct teaching was scaffolded using visual supports that showed the step-by-step process for each strategy with multiple examples. These strategies were then displayed in the classroom for the students to refer to when needed.

Direct Teaching of Problem Solving Process

The overarching structure of *PSTMU* is a recursive approach to the problem solving process. The approach emphasizes identifying the problem, planning the solution, executing the plan, monitoring its effectiveness, and going back to any and/or all of these steps until a solution that makes sense is found. Scaffolding was provided by discussing and modeling each step, again providing multiple, continued exposure and reference to the process. It was displayed visually as it was modeled and was also pasted into the students' journals for easy access when writing about the problem solving process after each activity. Metacognitive questioning focused on what strategy was used and why and it

was continuously connected to the problem solving process and its recursive nature.

Manipulatives

Manipulatives or physical objects, such as colored tiles, are sometimes necessary supports for some problem solving activities in this curriculum. Other times they may not be necessary. These are tools that can support learning by helping students with learning disabilities make abstract ideas more concrete and comprehensible. Clements (1999) indicates that manipulatives can help students construct meaningful ideas in the mathematics classroom if they are used appropriately during activities with teacher guidance. Sowell (1989) states that attitudes of students regarding mathematics can change with the use of manipulatives during instruction. Based on my own teaching experience working with students with disabilities, it is a good practice to have a variety of manipulatives available in the classroom where children who need them can access them easily. Manipulatives are tactile providing a three dimensional tool to aid in the learning of mathematical concepts.

Teacher-Talk

Teacher-talk is the type of communicative instruction you give children that includes modeling self-talk or thinking aloud, modeling mathematical language usage, asking questions that will guide the student without telling them what to do, making connections, and eliciting prior knowledge. This list is by no means all-inclusive and should not be taken as such, but instead, as a sample of the different kinds of talk encompassed within this type of discourse. Increasing

peer-talk and decreasing teacher-talk is a desired outcome of this curriculum. Allowing students more time to engage in discourse gives them the opportunity to verbally make sense of new information and ideas, to construct meaning by listening to the thinking and strategies of their peers, and it give teachers access to students' understanding of the material presented (Burns, 2007). Effective teacher-talk will facilitate learning and communication among the students (Cullen, 1998).

Peer-talk/Group work

Based on my experience as a middle school math teacher, giving students a problem to solve in groups of two or three, and monitoring so that everyone participates, fosters peer-talk, another opportunity to develop mathematical communication. The teacher can then move throughout the classroom listening to the discussions, informally assess students' knowledge or confusions, ask questions and guide students when necessary. Research has shown that there is a positive relationship between peer-talk during collaborative activities in the classroom setting and constructing knowledge (Mercer, Wegerif & Dawes, 1999).

Group Discussion

Through teacher facilitation and guidance, group discussions are used to develop several areas important to mathematical understanding. These include metacognitive awareness, reasoning skills, use of mathematical language, as well as ability to effectively explain the problem solving process. These discussions allow for students to communicate their strategies, describe the steps they used to solve a problem, and explain why it makes sense to them.

Metacognitive Questioning

Metacognitive awareness allows one to examine and act on one's own thoughts. Some children develop these skills on their own while others do not. Through metacognitive questioning, the teacher asks high-level questions that require the students to probe the underlying reasons for their choices and why they make sense. The teacher's intention is for the student to reflect on their thinking process. The teacher also models metacognition through "think aloud". This is when a teacher demonstrates his or her thoughts by verbalizing them while going through the problem solving process. By promoting discussion and reflection, the children move toward an improved ability to self-evaluate their performance and their work.

Graphic Organizers

The graphic organizer used in *PSTMU* serves a unique function. It is a scaffold used to organize the students' writing while they are developing skills to write about mathematics. The organizer provides sentence frames that the students complete. There are two options available in terms of sentences frames. One set is used if the first attempt at solving the problem was successful. The second set is used if the first attempt did not work and the student needed to change the strategy.

The Process Sheet

The *PSTMU* approach provides scaffolding for students in need of support. Students with learning disabilities often have difficulties with written language and can have difficulty deciding what relevant information should be

included in their writing about problem solving. This sheet, which is pasted into their journals for each problem solving activity, prompts the student to show their work, provide a description of what they did, indicate the solution, and explain why it makes mathematical sense to them. The sheet provides the scaffolding needed to develop better writing skills for mathematical communication. It also prompts them to look for clue words that may give them an idea of the operations that may be necessary to solve the problem (Gagnon & Maccini, 2001).

Journals

Problem Solving Toward Mathematical Understanding uses journals to promote the ability to communicate mathematically using mathematical language and to reflect on the problem solving process, thereby developing metacognitive awareness. The journals serve as an assessment tool for the teacher as well.

Reflective Writing

PSTMU also uses more focused reflective writing that draws students' attention to what they found most difficult about the activity, what was easiest about the activity, and to identify something new that they learned. This is an additional piece of the curriculum intended to foster metacognition, as well as to serve as a guide for the teacher to address any areas of weakness for the students.

Group or individual Sharing of Process

Making the thought process explicit is an important aspect of *PSTMU*, therefore a safe learning environment is important to develop. Guidelines for classroom discussion should be established at the beginning of the year and

monitored closely so everyone feels safe to share their work and ideas even if they did not find the solution. From my experience, a safe environment fosters a positive attitude toward mathematics and increases motivation when students enjoy sharing their work. Listening to others share their work and ideas provides opportunities for students to see that there are multiple ways of approaching a mathematical problem and that mathematical ideas can be represented in a variety of ways.

Sentence Frames

Sentence frames or sentence starters are writing scaffolds typically used with English language learners in the classroom setting (Walter, 2004).

However, because students with cognitive processing deficits often have problems with reading and writing, frames or starters are a useful support that enables the students to participate in journal writing and reflective writing. This support also helps students to practice and begin to internalize aspects of academic language in the mathematics classroom.

Activities

Problem Solving Toward Mathematical Understanding is structured so that a variety of problem solving activities can be incorporated into mathematical instruction. These problem-solving activities use novel problems that do not have a readily identifiable solution. The problems are drawn from a variety of sources including books, articles, and the Internet. These problems can be adapted as needed for your particular group of students. Examples of such activities can be found in the Appendix section of this paper.

Summary

The *Problem Solving Toward Mathematical Understanding* approach attempts to help students with learning disabilities improve their problem solving skills. It strives to help these students increase their motivation by improving their ability to make sense of problems and develop perseverance when faced with a challenging task. It focuses on improving students' ability to reason and communicate using mathematical ideas. In doing so, this approach makes an effort to foster in these students a positive attitude regarding mathematics.

VI. Implementation and Revision of Problem Solving Toward Mathematical Understanding

Introduction

Problem Solving Toward Mathematical Understanding was implemented in one school in San Diego County. Harper Middle School had a population of 850 students, and was predominantly Latino. Other populations included White, Indochinese, African American, Asian, Filipino, Native American, Pacific Islander, and student of two or more races. Almost seventy-five percent of the population received free or reduced meals. The school was responsible for providing Free Appropriate Public Education (FAPE) to the 17.2% of their students who have learning disabilities (LD). This is an educational right that the Rehabilitation Act of 1973 and Individuals with Disabilities Education Act (IDEA) guarantees to children with disabilities (U.S. Department of Education, 2010). Most of the eligible students at Harper were educated within the general education setting with Specialized Academic Instructional (SAI) support provided. Others received support through the Resource Support Program (RSP) and a small group of students received instruction in Special Day Classes (SDC) for their core classes, which included math, science, English and reading. SDC students were typically in the general education setting for physical education and social studies or history.

Problem Solving Toward Mathematical Understanding was designed specifically for the RSP population. The RSP program provided students with

additional math support in a small group setting with a special education teacher. It was conducted outside of the general education classroom. I began the implementation of the approach in a mixed sixth and seventh grade level mathematics Resource Support classroom where students were provided with an additional 50-minute period of math support. All of these students were in a general education math class as well. As a guest teacher, during the implementation, I came into the classroom and taught three days a week for seven weeks. Each period of teaching lasted 30-35 minutes. The time limit was due to the classroom teacher's requirement of allowing time for students to work on overdue work or current homework.

The Setting and Participants

The curriculum I designed was conducted with an initial group of 12 special education students. By the end of the first implementation only 7 of the original students remained and two students entered the class while the implementation was in progress. Therefore, these two students were not exposed to the full seven-week curriculum. One of these two students received only two weeks of instruction and was therefore not included in the data collection. He did not receive the same direct instruction of the problem solving process or the strategies. He also did not complete the math attitude survey or the metacognitive questionnaire. Another student, who left after four weeks, was included in the final data because he had received the direct instruction, completed the survey and questionnaire, and participated in the pre-implementation problem solving activity. The final problem solving activity he

participated in was used as his final assessment. Yet another student moved and the remaining had their annual IEP meetings where the IEP Team determined the math support class was no longer necessary.

The group of students I taught was originally a seventh-grade support math class with an educational specialist certificated to work with special education students. There was originally only one sixth-grade student in this class. By the end of the implementation there were three sixth-grade students in the class. The class was taught by a special education teacher, commonly referred to as an educational specialist, and met 5 days a week for 50 minutes per day outside the general education setting, but on the comprehensive middle school campus. The disabilities found within this group included Intellectual Disability or a cognitive deficit (ID), Other Health Impairment (OHI) and Specific Learning Disabilities (SLD).

To clarify these terms for those who are not familiar with special education terminology, Intellectual Disability refers to cognitive functioning or intellectual ability that falls in the deficit range. Cognitive deficits can range from mild to profound. Students who qualify for special education services under the nomenclature of OHI may have medical issues or Attention Deficit Hyperactivity Disorder (ADHA) of which there are three types: attention, hyperactivity, or combined. The students in this class who qualified under this criterion had a diagnosis of ADHD. Specific learning disabilities found among this group of students included auditory processing (AP), visual processing (VP), processing speed (SP), visual-motor integration (VMI), Attentional Processing (ATP), and/or

short-term memory (STM). Auditory and visual processing are the abilities to interpret and store auditory and visual stimuli. This includes short-term memory. Visual-motor integration is the ability to integrate fine motor finger and hand movements with visual perceptual skills. And finally, processing speed is the speed and efficiency in the ability to do automatic cognitive tasks. In addition to the LD risk factors, ten of the original students were also English language learners with proficiencies that ranged from Early Intermediate-to-Intermediate levels. Two students were native English speakers.

Because these students all had Individual Educational Plans (IEP), their mathematics skills were assessed every three years. At Harper Middle School these skills were measured by administering the Woodcock-Johnson Tests of Achievement (WJ-III). A Broad Math Index was generated using Standard Norm Referenced Scores. My students' scores ranged from 73 to 89 (low to low average range) where standard scores between 90-110 are considered to be in the average range. When discussing standard scores the mean score is 100 with a standard deviation of 15. On the problem-solving subtest of the WJ-III their scores ranged from 67 to 85 (very low to low average range). These scores were derived from their most recent triennial IEP re-evaluation assessment, which occurred within the last three years. With their scores being so depressed compared to similar aged peers it made sense to me that providing these students with more problem solving experiences would be beneficial since this was lacking in the general education curriculum.

The students in my class represented some of the children with the highest needs. These scores indicated that the supports, which were supposed to help them succeed in the classroom, were not producing promising results. They were not succeeding when it came to developing the mathematical concepts and reasoning skills necessary to contribute in today's society. Table 6 provides a summary of the characteristics for each student included in the final evaluation of this curriculum.

Table 6. Student characteristics

Student #	Gender	Grade	Disability	Cognitive Functioning Level	English Language Developmental Level (4/2011)
1	M	7	ADHD	Low Average	Intermediate
2	F	6	AP/PS/VP	Average	Native English
3	F	7	STM/ Nonverbal cog. deficit	Nonverbal- Low	Intermediate
4	M	7	ADHD	Average	Native English
5	F	7	AP/STM/ATP	Nonverbal- Average	Early Advanced
6	M	7	AP	Average	Early Advanced
7	F	7	AP	Average	Intermediate
8	F	7	Cog. Deficit	Very Low	Early Intermediate
9	M	6	VMI/AP/PS	Low Average	Native English

The Teacher

The educational specialist of this class had a master's degree in special education. Prior to my taking over the class she typically spent three days a week reviewing the material the students were learning in their general education math classroom. For the seventh-grade students this was pre-algebra and for the sixth-grade students this was 6th-grade-level curriculum. Topics covered in the curriculum include Decimals and Integers, Exponents, Factors, and Fractions, Operations with Fractions, Equations, Ratios, Rates and Proportions, Percents, Geometry, Measurement, Patterns and Rules, Displaying and Analyzing Data, and Using Probability. Two days a week the students worked on Learning Upgrade. This was a computer software program designed to remediate and provide practice with math computational skills from grade three through algebra. Time was allotted Monday through Thursday for homework, lasting approximately 20 minutes.

I became the primary teacher three days a week during the implementation of this curriculum. The regular classroom teacher was rarely involved in the classroom instruction, instead using the time to catch up on her other special education responsibilities such as IEP paperwork. There was, however, an instructional aide in the classroom that assisted as needed while the students worked on the problem solving activities.

Determining Baseline Abilities: Pre-implementation

Prior to implementation of this curriculum I gathered some baseline data that I could use to evaluate the effectiveness of this program. The students

completed a questionnaire designed to assess their metacognitive abilities. I created the questionnaire based on the recursive problem solving process I intended to instill in the students. My recursive process was developed by adapting an example of the steps to solving word problems found in Manning and Payne's work (1996). These steps include identifying the problem, planning the solution, solving the problem, and checking the solution. I wanted to assess if the students monitored their efforts and if they returned to prior steps when their approach was unsuccessful.

The students also completed a questionnaire intended to assess their attitude toward mathematics. This survey was adapted from Technical Education Resource Centers (TERC) Mathematics. The original can be found on their website at <http://mathequity.terc.edu>, and contains a scoring rubric. Higher scores suggested a more positive attitude toward mathematics (TERC, 1997).

I also gave the students the Brigance Test (Curriculum Associates' Inc., n.d.) consisting only of the problem solving section of this assessment tool. The Brigance Test is a standardized test often used by special education specialists to assess students' mathematics skills both in computation and word problems. It can assist in identifying students' academic functioning level as well as strengths and weaknesses. Using this assessment tool allowed me to have a gauge for where the children were functioning in regards to word problems prior to implementation.

Finally, I presented Secret Number Puzzles # 1 and 2 using the *Secret Number Puzzles* adapted from Melanese, Chung and Forbes (2011) for the

children to solve. The purpose was to assess the students' skills as to how they approached the problems, persevered, as well as their ability to discuss and write about the processes they used in finding a solution. Secret Number Puzzle #1 was used as a comparison to the final assessment problem given at the end of the implementation.

Implementation of Problem Solving Toward Mathematical Understanding

Problem Solving Toward Mathematical Understanding was implemented over a seven- week period of time from January 2011 through February 2011. Problem solving activities were chosen from a variety of sources based on the flexibility of adaptation, whether or not the concepts had been previously studied in the general education classroom, and within the students' current level of functioning. The result was a variety of problem solving activities that did not revolve around a particular computational skill or formula that the students were currently studying and therefore the solution was not readily identifiable. Table 7 indicates the sequence of events including revisions made that occurred during the implementation.

Table 7. Sequence of activities and revisions

Time Frame	Day One	Day Two	Day Three
Pre-	Secret Number Puzzle #1 (Melanese, Chung, & Forbes, 2011)	Secret Number Puzzle #2	No class
Week 1	Brigance Testing Metacognitive Questionnaire -Math Attitude Survey	-Direct teaching: Problem Solving Process	Direct teaching: Problem Solving Strategies
Week 2	-Revised Metacognitive Questionnaire administered. -Introduced the Process Sheet for journaling -Consecutive Sums Problem (Burns, 2007)	Consecutive Sums Problem continued	Guess the Function Problem #1 (Melanese, Chung, & Forbes, 2011)
Week 3	Guess the Function Problem #2	Guess the Function Problem #3	Create your own Guess the Function Problem
Week 4	Pentominoes Problem (Burns, 2007)	Pentominoes Problem continued	Sense of Nonsense (Burns, 2007)
Week 5	-Cats & Birds: -Mathematical vocabulary and algebraic expressions review (Melanese, Chung, & Forbes, 2011) Introduced Task Organizer	-Cats & Birds Problem Presented 1 clue	-Cats & Birds Problem continued -Problem solving checklist replaced task organizer
Week 6	-Cats & Birds continued Introduced all 6 clues	Cats & Birds continued	Introduced reflective writing: Cats & Birds
Week 7	-Assessment: Farmer Ben Problem (The Singapore Maths Teacher, 2005) -Introduce graphic organizer for the writing process	Farmer Ben Problem continued	Farmer Ben Problem continued -Post Metacognitive Questionnaire -Post Math Attitude Survey

Direct Teaching: Problem Solving Process

Research suggests that direct instruction combined with cognitive and metacognitive strategies has shown some promise in improving mathematical achievement among students with learning disabilities (Bottge, 2001). At the beginning of the implementation of this curriculum the problem solving process, as I developed it for this program, was directly taught to the students. Each part of the process was reviewed and modeled using a simple word problem. The process included Identify the Problem, Plan the Solution, Solve the Problem, and Check the Solution. Each section had identifiable parts for the children to consider during each step; I called this, "Let's think about thinking." Throughout the implementation students were told that they could return to any of the prior steps if what they were doing was not working. This chart was then pasted onto the first page of their journal to refer to as necessary. The original chart was modified after the implementation to show the recursive process (see Figure 2). I did not feel that my initial design conveyed that this was a recursive process; therefore, I added arrows that made it clear that this was part of the process and this version is included in the Appendix.



Let's Think About Thinking

Identify the Problem	Plan the Solution	Solve the Problem	Check the Solution
<ul style="list-style-type: none"> a. Read the word problem aloud to myself twice. b. Do I understand the question? c. I understand or I don't understand. d. If I don't understand I talk it over with my partner or someone else who does understand. 	<ul style="list-style-type: none"> a. Do I know what to do and how to do it? b. What should I do first? c. What do the key words tell me to do? d. This is going to be easy/hard for me. 	<ul style="list-style-type: none"> a. I need to add, subtract, multiply or divide first because _____. b. There is one step. (There is more than one step.) c. I need to follow through with solving my problem. d. I can do this. (I can't do this.) If not, find help. 	<ul style="list-style-type: none"> a. Did I complete everything? b. Does it make sense? c. What do I need to recheck? Do I need to return to prior steps (Identify, Plan, or Solve)? d. I think it's correct? e. It is my best try!
	Return to identifying the problem as necessary.	Return to identifying the problem or planning the solution as necessary.	Return to identifying the problem, planning the solution, or solving the problem as necessary.

Figure 2. The problem solving process revised

Direct Teaching: Problem Solving Strategies

A second part of the direct teaching was directed at building a repertoire of problem solving strategies that the students could call upon to help them solve a problem. I presented the strategies on the Promethean board to the class and then ask them to practice it individually. I also found a helpful website that had many of the strategies at primary 3-6 grade levels that were presented in a slide show modeling the strategy and checking the solution (The Singapore Maths Teacher, 2005). I also provided several more examples for the students to try. After practicing, each strategy was written on chart paper and remained on the easel at the front of the classroom. During the implementation process this information was always referred to when the students began an activity or journal writing. I reminded them to change strategies if the one they were using was not working and to include the strategies used in their journal writing.

Establishing Groups

Throughout the curriculum implementation, students worked individually, in pairs, or in groups of three. Most often they worked in pairs due to the small

number of students in the classroom. During the pre-implementation problem solving activities the students worked independently, thereby allowing me to assess their baseline skills without support from their peers. I used my prior observations of the students to plan groupings for the first problem solving activity. During those observations I noted who appeared to feel comfortable with whom and if any student seemed reluctant to work with the opposite gender, a common occurrence in middle school. After day one of the Consecutive Sums Problem I quickly saw that two boys, in particular, could not work together due to behavioral issues. The special education teacher confirmed this but I felt compelled to allow them to remain together for the two sessions needed to complete the Consecutive Sums Problem because the boys had already contributed their ideas and I wanted to see their completed work. From that point on I used my knowledge, based on standardized testing, observations, and the pre-implementation problem solving activities to pair the academically stronger students with the lower functioning students. This allowed for additional modeling of higher-level problem solving thinking, discourse and questioning. It also avoided problem behaviors by lowering the frustration level that may have occurred if both students were struggling. I also considered their ability to work together due to personality compatibility or incompatibility.

Activity One: Consecutive Sums Problem

As the first week began in earnest I had already lost two of the original group of 12 students and a new student was added. With the remaining eleven students we worked on the Consecutive Sums Problem (Burns, 2007) as the first

problem because I felt they would be familiar with the concept of consecutive numbers and, in terms of computational skills, the problem only involved simple addition. I did not want the students to be overburdened by procedural difficulties, but rather, to focus their cognitive energy on learning the problem solving process and to use mathematical reasoning.

I began the class with the question: What are consecutive numbers? I then asked for an example and a non-example. When I was sure the students understood, I asked them to tell me a way to write the number nine as a sum of consecutive numbers. The students readily generated $4+5$. I asked them if they could find another way. They could not, and I subsequently asked them to consider $2+3+4$. I then presented the problem on the board:

Find all the ways to write the numbers from one to twenty-five as the sum of consecutive numbers. Some may be impossible. Is there a pattern for these?
(Burns, 2007)

I told the students to search for other patterns as well, such as how many different sums there are for different numbers. I asked them to work in groups of three (which I set up) and to put the names of the members of their group on their recording sheet. I instructed them to write statements that described the patterns they found. All of this was through a multisensory approach that included the information and directions on the Promethean board where I could write what the children stated. I directed the students to consider the following questions:

Can you see a pattern to the numbers that are impossible? How could you describe that pattern in a summary statement? What do you notice about all the numbers that had three possible sums? Which numbers had only one possible sum?
(Burns, 2007)

The students had difficulty working with consecutive numbers. By that, I mean they would combine sums that were not consecutive (i.e. $3+6=9$). I often had to ask them if their number sentences had consecutive numbers. I reconvened the group and discussed and reviewed the original instructions using several sums. Upon releasing them back to the task at hand they continued to appear confused. I suggested the use of a number line to some pairs, which offered a visual support that appeared to be beneficial. I then regrouped the class and provided a number line as a visual on the Promethean board for them to access if necessary. In retrospect I would provide more examples of consecutive sums and have included similar examples in the Appendix.

During their investigation I circled the room listening to discussions and asked guiding questions to alleviate confusions or misunderstandings. I recorded comments I heard after the class dismissed. I prompted the students to search for patterns and reiterated that some sums may be impossible to do with consecutive numbers. We had previously discussed and practiced finding patterns as a strategy for problem solving; therefore, the students were familiar with this approach, albeit not yet skilled. For example, Michael (pseudonym), a student who chronically failed math but was one of the more motivated students, quickly stated, "I see a pattern! Even numbers don't work!" I asked him "How do

you know that?” He responded, “2, 4, 6, 8, 10 and 12 don’t work!” I drew his attention to the sum of 6 and encouraged him to use more than two consecutive numbers. He then found $1+2+3$ as a solution thereby nullifying what he misconstrued as the pattern. The other students were not able to find patterns and they also had trouble looking beyond using more than two consecutive numbers for sums until I asked questions that guided them to further explore multiple numbers.

During the following class session the students continued to work with their partners to complete the Consecutive Sums Problem. At times the students became frustrated; a few, however, appeared to enjoy the challenge. I encouraged the students not to give up and think of it as a puzzle. I also confessed that I too had to work out the problem at home to find a solution. I even showed them my messy attempt at searching for patterns and solutions. In doing so, I let them know that I also have to persevere to solve problems and don’t always know the answer immediately.

Finally, I pulled the students together for each group to answer the questions that were presented on the board. Each group came up to the front of the classroom and addressed the questions. Choosing the strongest group first to model the process, I guided them through their presentation. My intention was to have them discuss the process and their reasoning first, and then write about it using the sentence stems in their journals. I was hoping that, after rehearsing the answers, the writing would not be so difficult for the students. This proved to be very helpful to the students. For the most part, they were eager to share their

work. Having peers stand beside them for support and having the cues on the board seemed to provide a sense of security for most of the students. Stacy, a very shy girl, never volunteered information during class discussion. As each student took a turn answering a question, she did not hesitate when her turn arrived. This indicated to me that the classroom environment was a safe place that would foster classroom discussions and sharing.

After the presentations the students were then asked to write in their journals. Protests abounded with students stating that this was not an English class. I discussed with the students the importance of writing in mathematics to support their learning. I also told them it provided me with a glimpse of their understanding or confusions so I can better teach them. I indicated that the writing should not be too difficult because they had just presented to the group what they needed to write in their journals. The students were told that they were not required to use the sentence starters but they were there to guide their writing if needed. It is important to scaffold the writing activity for students with learning disabilities and English language learners alike to provide a structure when writing and to differentiate instruction for the students at varied writing and language levels (Melanese, Chung, & Forbes, 2011).

The writing process was arduous for some and painstakingly slow for others. All the students took more time than I expected to write the brief 3-4 sentences I provided with the sentence frames. These included such starters as “My group divided the work by____” “Our method was/was not a good one because_____” (refer to figure 6.). Time management became an issue between

the time it took for completing the problem and the additional time that was required by the students for the writing task. Initially I had expected to complete one activity with journal writing per class period. When all was said and done, each activity took two to three days and on some occasions, such as with this activity, extended through the full 50-minute period.

Figure 3 shows the visuals and language supports provided during the writing process for the students.

Class Discussion and Journal Writing

How did your group divide up the work?
My group divided the work by _____.

Was your method a good one? Why or why not?
Our method was/was not a good one because _____.

What strategies did your group use?
Guess and check
Work backwards
Make an organized list
Look for a pattern
Others?
The strategies we used were _____. ***First we tried _____.***
Then we tried _____.

How were you sure your group found all the possible ways to write any particular number?
We were sure we found all the possible ways for each number because _____.

Figure 3 Visual and language support for LD students

As the students wrote in their journals I saw the first indication regarding how much work I needed to do to move these kids forward in their ability to problem solve in mathematics. First I needed to break through the resistance to writing they had developed over the years, but also their learned helplessness and the notion that they were incapable of doing challenging work. My initial lesson plan

had been too zealous and could not be completed in one period. It was also at this point that the special education teacher made it clear that she expected me to limit my sessions to 30 minutes to allow for homework time as this session took the full period of 50 minutes. Looking back I would now start this lesson with fewer numbers and perhaps increase to more numbers after they were able to be successful with the more restricted set. I think too, it would have been beneficial to generate more examples from the students of ways to write sums using consecutive numbers until they were confident in their ability to find more independently. In the Appendix I have kept the total sum at twenty-five but recommend adapting the lesson by using a smaller or larger total sum based on your students' abilities. This activity also prompted me to set an alarm on my phone to warn me to start wrapping up the lesson in order to allow writing time. It also prompted me to create the Process Sheet.

Figure 4 shows the resistance one student displayed to group work. The students were expected to work as a group of three but the student on the left initially had difficulty working as a team. Over the course of the implementation she did become more tolerant of group work.



Figure 4. Consecutive sums problem: A student's initial resistance to group work

Figures 5 and 6 show the various levels of abilities within the class. The students were given sentence frames to write about the process they used to divide the work and solve the problem. Some students were able to use the sentence frames to develop a paragraph and included them in their writing while others had difficulty with this. Because the students' levels were so varied I decided that more structure was needed for the journal writing process. I wanted to see their work in addition to telling me what they did and why the answer made mathematical sense.

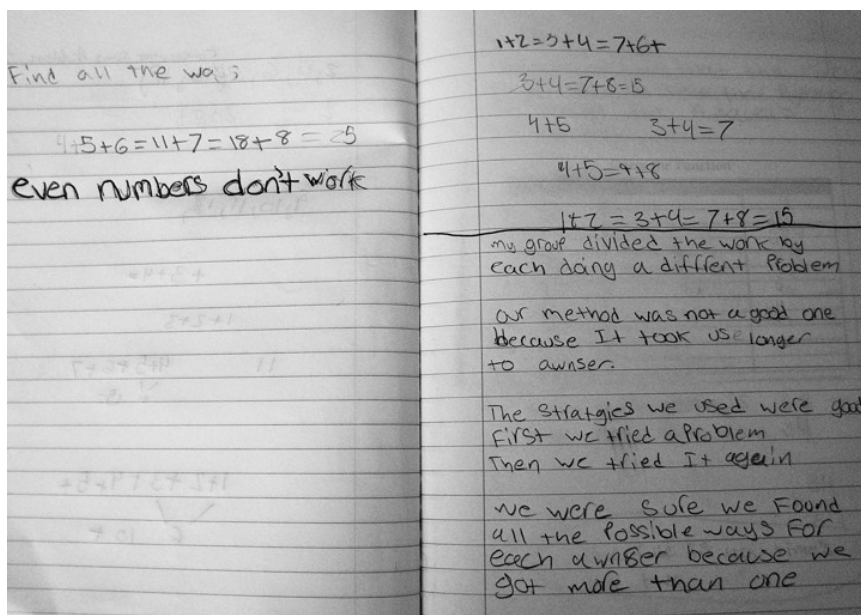


Figure 5. Writing sample: More advanced

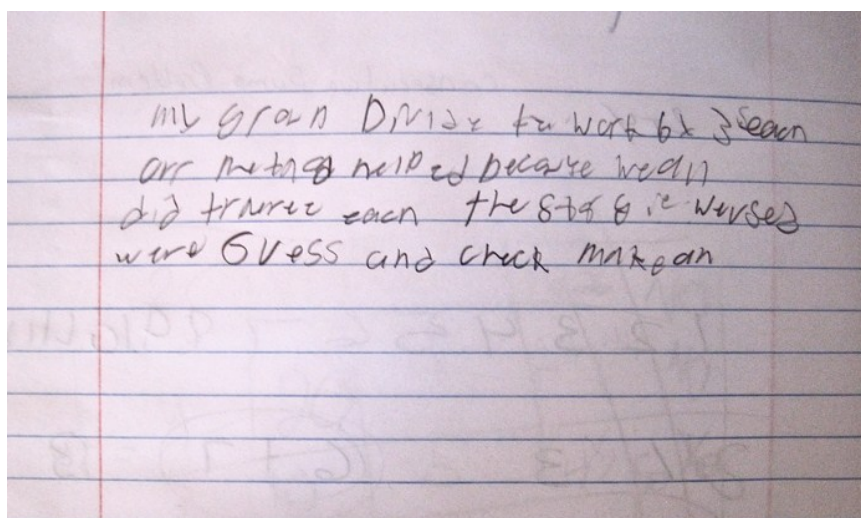


Figure 6. Writing sample: Less advanced

Activity Two: Guess the Function

The second week began with my first revision to the curriculum. After watching the students struggle with the journal writing I devised what I call The Process Sheet and it is included in the Appendix. I explained to the students that

they were to show their work in the top section and do reflective writing in the bottom section indicating what they did, the solution, and why it made mathematical sense to them. Because I introduced a different reflective writing task much later in the curriculum, in the Appendix, I revised the bottom section to read Journal Writing. This proved helpful although I continued to provide sentence starters or sentence frames to support the writing process.

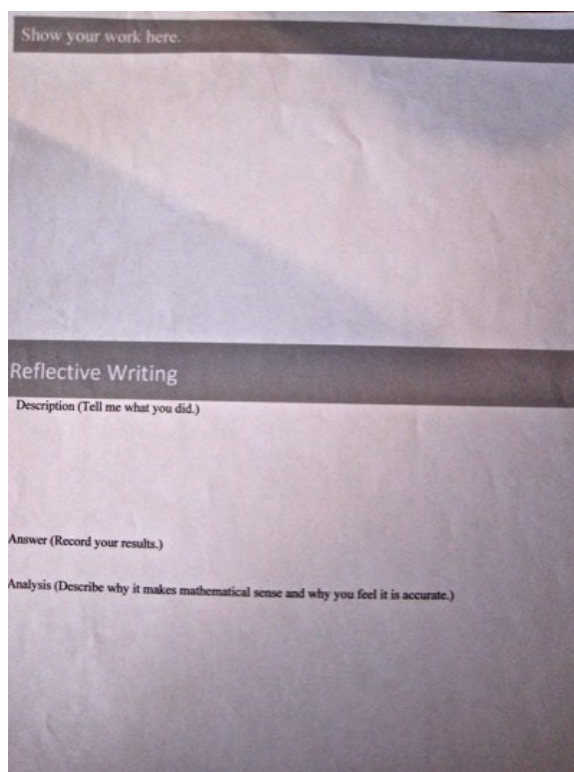


Figure 7. The process sheet

The second problem solving activity was called *Guess the Function* and was adapted from Melanese, Chung, & Forbes (2011). I copied only the function chart from the reproducible master provided in the book but did not include the sentence frames on this sheet. I instead presented the sentence frames on the

Promethean board so I could direct their attention as a group and have a discussion on these.

The first problem was a one step function. A function is a relationship between a set of inputs and a set of outputs. In this case only one computational rule is operated on the input value to yield the output value. I provided them with either the input or the output values and they had to figure out the rule or function. The students, with guidance, were able to find the solution for the first problem; however, when they proceeded to write about it they wrote a sentence instead of a function, therefore the distinction had to be reviewed. I explained to the students that functions have a formal notation and are often expressed as an algorithm or formula such as $3n$, rather than “the rule is you times it by three” as one student wrote. On this day four of the students were absent, therefore missing the introductory lesson of this problem. One student, it turns out, had chronic absences since kindergarten, two others were reported to be sick, and the last student absence was unexplained. Due to this, I had to review the previous day’s lesson on the following day. This became a chronic issue during implementation.

As the week progressed we moved into two-step functions, with which the students had a great deal of difficulty. One student, John, began the process by looking for consecutive numbers indicating his perseveration on the previous activity, sum of consecutive numbers problem. For example, this student took the input of 3, added 4 to get the output of 7. For the next input value of 5 he tried to add 6 to generate the output of 13. Seeing that this did not work he

asked me if he had to use consecutive numbers. All the students needed to be reminded that the function, or rule, must work for all the input/output data. Often, I had to bring them back to this important piece. The students had a tendency to erase their mistakes, and I had to discuss, as a group, the importance of saving all work so they could write about the strategies they tried in their journals. After allowing time for students to work, and noting that some students were able to find the function, I asked them to write about what they did to arrive at their answer. At this point I visually displayed three sentence frames, as seen in Figure 8, to help them in their writing. I modeled the use of these frames while pointing to the corresponding information on the input/output table illustrated on the board to provide them with an auditory and visual association.

<p>If the _____ value is _____, then the _____ value is _____. (Input/output) (Input/output)</p> <p>I know for every input value n, the value of the output is _____.</p> <p>I can conclude that the function is _____, because _____.</p>

Figure 8. Sentence frames

The students had difficulty writing on the process sheet in terms of where to put the actual work and where these sentences fit in the reflection writing section. They also put the wrong information in the blank lines. Because of this, the following day I presented an example of a journal entry for the previous day's function problem. I displayed this as a model, as shown below in figure 9, under the doc-camera and we discussed it.

Solve your work here.

input	output
3	7
5	13
20	58
10	28
9	16
n	$3n-2$

① $3+4=7$ $5+4=9$ X
 ② $3 \cdot 2 = 6$ NOPE
 ③ $3 \cdot 3 = 9 - 2 = 7$ times 3, minus 2
 $5 \cdot 3 = 15 - 2 = 13$
 $20 \cdot 3 = 60 - 2 = 58$
 $10 \cdot 3 = 30 - 2 = 28$
 $9 \cdot 3 = 27 - 2 = 25$
 $n \cdot 3 = \underline{3n-2}$ Function

Reflective Writing

Description (Tell me what you did.)
 First I tried $3+4=7$ but $5+4$ doesn't equal 13. Then I tried multiplying by 2 but that didn't work. Finally I tried 3 times 3 but that was nine, so I had to subtract 2 to get 7. It work for all of them.

Answer (Record your results.) $3n-2$

Analysis (Describe why it makes mathematical sense and why you feel it is accurate.)
 If the input is 3 and I multiply by 3 and subtract 2 the output is 7. The function, using n , is $3n-2$. I works for any number.

Adapted from Burns, 1990, p.112

Figure 9. A model of a journal entry for the students

I also rewrote the sentences frames to provide more guidance as to what to put where.

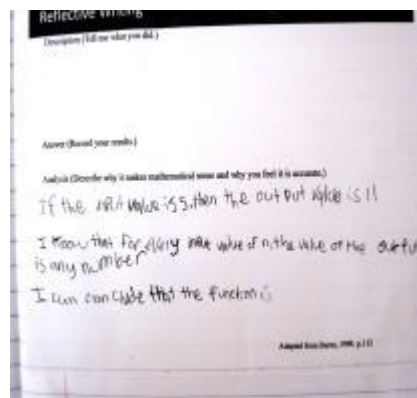
Sentence Frames			
You DO NOT have to use these sentence frames. You CAN create your own sentences.			
Analysis: Why does it make mathematical sense?			
If the _____ value is _____,	then the _____ value is _____.		
input/output	put a number here	put a number here	input/output
I know that for every input value of n , the value of the output is _____.			
put your rule here			
I can conclude that the function is _____ because _____.			
put your function here		your reason	

Figure 10. Revised sentence frames

Because the students had such a difficult time with a two-step function, I extended the lesson to include another one. The students continued to have a difficult time and most could not move beyond searching for a one-step function. Two students did find the solution, however. The students were then asked to write in their journals. Upon completion they were asked to share this under the doc-camera regardless of whether or not they arrived at the correct solution. I felt it was important to do this in order to establish a safe environment where students could take risks and not obtain the correct answer without fear. This would allow for students to feel free to explore new strategies and hopefully improve in the areas addressed as the goals of this curriculum. One student, Anita, who was functioning at the lowest level in this class and had emotional difficulties as well, offered to present her work. She did not hesitate to say that she did not find the solution. Manuel, another student, was able to define the function as $3n-4$ in his journal within the table but was not able to clearly write his conclusion even with the sentence frames. Therefore, when he came forward to present he was unable to articulate the process. Michael, the highest functioning student in the class, had worked with him during this activity and come forward to assist him. He explained how they multiplied each input by three and then subtracted 4 to get the output. He was not able to elaborate the process without prompting from me. Manuel's work can be seen in Figure 11 and 12 below.

Guess the Function

Input	Output	Output
5	$5 \cdot 3 + 4$	11
2	$2 \cdot 3 + 4 =$	2
12	$12 \cdot 3 + 4$	40
3	$3 \cdot 3 + 4$	13
9	$9 \cdot 3 + 4$	31
100	$100 \cdot 3 + 4$	304
6	$6 \cdot 3 + 4$	14
n	$3n + 4$	



Figures 11 & 12. Guess the function: Student work

Finally, the last activity I did during this problem solving lesson was to allow the students to create their own function along with an input/output table with the values. The students then exchanged their product with another student who tried to solve it. This gave me the opportunity to see who felt comfortable with attempting a two-step problem and who could solve them. They continued to need quite a bit of support to create and solve these functions. The students appeared to enjoy creating the functions as well as the table of values, but several became frustrated when attempting to solve others' creations. Part of this frustration appeared to be due to the fact that a couple of students made one or two computational errors, which threw off the student guessing the function. Others had difficulty with two-step functions such as the one in Manuel's work shown above because many of the students had poor number sense and were not fluent with simple computation. Therefore, they relied mainly on guess and check and without that solid number sense their attempts were without direction. In retrospect I realized that it is important for the teacher to give a quick scan for computational errors. Another option is for me to take the papers home and

check them before distributing them for the students to solve the following day. I noted this precaution in the lesson plan found in the Appendix.

If a student had a question during the problem solving activities, I would often direct the question to the whole class and have a discussion. I also used metacognitive questioning to evoke reflection. Examples of metacognitive questions include “How do you know that”, “Does anyone have a different answer/suggestion?” “Why do you think that is/is not true?” At other times I would ask them to talk to their partner. If they continued to have difficulties I would ask questions to guide them such as “What have you tried so far?” “What worked or didn’t work?” or “Do you see a pattern?” “Does it work for all the input/output values?” In the Appendix I revised this activity by describing and modeling two-step functions multiple times before having the students attempt to solve one on their own or with peers. I would also begin with very simple examples and increase the difficulty level of the problem as the students demonstrate increased understanding and become more confident in their abilities to solve such problems.

Activity Three: Pentominoes

Week four began with a fun, relaxing problem solving activity for the students. The activity was recorded using a digital tape recorder. *Pentominoes* is a geometry activity that is concrete in that it uses manipulative materials, square tiles, and does not include formal symbolism or definitions (Burns 2007). Square tiles and one-inch grid paper was provided to the students. I introduced the activity by asking the students if they knew the meaning of pentomino.

Student's comments were recorded directly after the class lesson and are paraphrased. John stated, "It sounds like dominoes." "That is true", I stated. "Can anyone explain to me what is a domino?" Keisha indicated that it was a game. "It is a game" I stated "but what does it look like?" Placing one square tile under the doc-camera I asked them, "How many do you need to show a domino?" They were familiar with these and knew a domino contained two squares. Next, I presented three squares and told them the term (triominoes). I then asked them how many different arrangements could I make using three tiles. I showed them the rules for combining the tiles indicating that they had to share sides. The students came up to the camera to show possible combinations. We continued this with four tiles as well. Finally, I asked the students if they knew the word pentagon. Several students indicated that they did. However, when I asked how many sides are in a pentagon Manuel stated eight. Another student indicated that was an octagon. Stacy ventured to say that there were five sides. "That is correct", I responded. "How many square tiles do you think are contained in a pentomino?" In unison the students indicated five. The goal was to develop a shared understanding of the term. I then presented the problem and discussed how to decide if two shapes are the same or different by drawing them on the graph paper, cutting them out, and move them to see if it fits on the other one. In this way I introduced another mathematical term I hoped the students would use in their writing, congruent.

The students were asked to investigate different ways to arrange five squares. They were told to consider how they found the different shapes, how

they knew they had found all the possible arrangements and how did they know that two shapes were congruent. I had introduced this word and I modeled it often during the lesson. Some students did attempt to use the mathematical term congruent in their discussions and in their journal responses that took place at the end of the activity. This activity provided a reasoning task that is different than what is required when using numbers. It taps into their logical reasoning skills in that the student must determine when all possible arrangements have been made. The activity offered visual and tactile support for those students who have auditory processing difficulties.

The students approached this task with enthusiasm, liking the idea of not working with numbers. Some students randomly placed the tiles in various arrangements while others worked systematically moving one tile at a time across the other four. Some students copied all their shapes onto the 1-inch grid paper cutting each out. Others only traced and cut those they thought might be congruent with another shape. Figures 13 and 14 show one student's work and journal entry. The numbers on the grid paper represent how many different shapes he found. Using the Process Sheet helped the students show their work but the writing was still difficult. I realized that the writing process would take time and practice; therefore, I felt that this activity worked well for my students and really didn't require revisions for the Appendix.

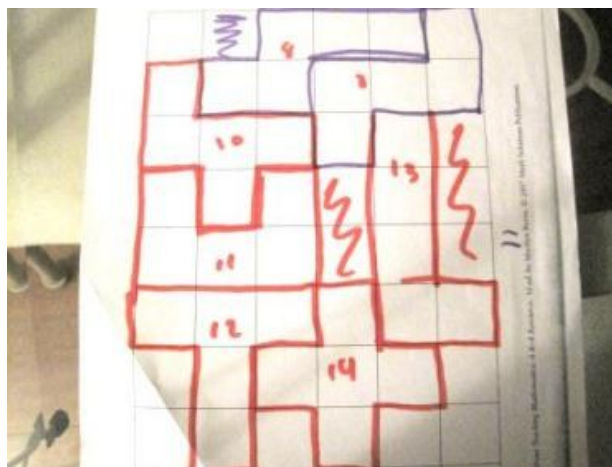


Figure 13. Pentominoes: Student work

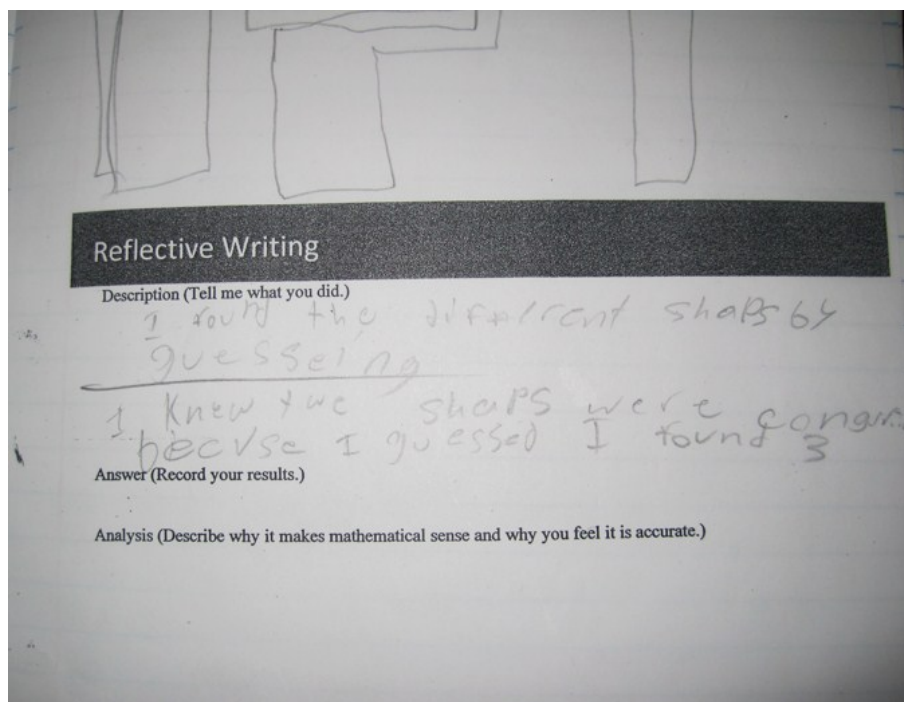


Figure 14. Pentominoes journal response

Activity Four: Sense or Nonsense?

The students had been working on percents in the general education mathematics class. I wanted to give them a chance to use what they knew about percents in problem situations. *Sense or Nonsense* provides statements that

allow the students to reason mathematically and to explain whether the statement is reasonable and explain the logic behind their choice (Burns, 2007). For example, “Cindy spends 100% of her allowance on Candy. Do you think this is sensible? Why?” Another example is “The Todd family ate out last Saturday. The bill was \$36.00. Would a 50% tip be too much to leave? Why?”

I had seven students in attendance for this activity. I divided the students into two groups, 3 boys and 4 girls. I felt the activity contained a lot of language so the larger groups would generate more discussion about the statements’ meanings. I had also found, over time, that most of the girls did not prefer to work with the boys. For the most part I honored that but since there were so few boys I would sometimes pair them up with the more willing females of the group. It was important for the students to feel comfortable enough to allow for sharing and discussions amongst themselves.

The students were very much engaged in this activity. I saw some good evidence of their understanding of percents. Michael stated, “twenty-five percent is like one fourth” and “ten percent isn’t a lot.” Another student stated, “Fifty percent for a tip is too much!” And yet another student commented, “100% of allowance on candy would be too much because that is all of it.”

I completed the activity with the following journal prompt:

The weather forecaster said there is a 60% chance of rain on Saturday and a 40 percent chance of rain on Sunday. Therefore, there is a 100 percent chance of rain on the weekend. Does this make sense? Why or why not?

All the students had difficulty with this prompt. The students did not take time to really think about the information and seeing that 40 and 60 when added together equals 100 they incorrectly answered yes. Sophie wrote in her journal “Yes it does make sence (sic) because $60 + 40 = 100\%$ and that’s how much percent is going to rain on Saturday and Sunday.” Penny wrote “Yes it is reasonable because 60% percent of rain and 40% of rain and all of that together could cause a 100% amount of rain.” Time was running short and little time was left for a rich discussion. In the future I would spend more time on the discussion piece of this assignment but I would also use a topic that was more familiar with the students and less abstract. I revised the question as seen below and included in the Appendix.

On Halloween Mike received ten pieces of candy and Kim received ten pieces of candy too! Mike ate 60 percent of his candy and Kim ate 40 percent of her candy. Therefore, they ate 100% of the candy. Does this make sense? Why or why not?

Activity Five: Cats & Birds

Cats & Birds was an activity where the students were given six clues and attempted to solve the problem of how many cats and birds Ms. Lang owned. Each clue provided information that built on the previous clues to help solve the problem. The students had to keep clues in mind and refer back to them to see if their answer met the criteria. This activity was by far the most difficult and frustrating for both the teacher and the students. When I chose it I thought it was scaffolded enough but soon realized that I should have made it simpler. I liked

the clue format because the students had done this before and I liked the algebraic expression piece because the students had experience with *Guess the Function*. *Cats & Birds* was taken from Melanese, Chung, & Forbes (2011). The vocabulary was frontloaded using a worksheet provided from the book.

Frontloading of vocabulary provides explicit instruction that focuses on specific words necessary for understanding the meaning of the problem.

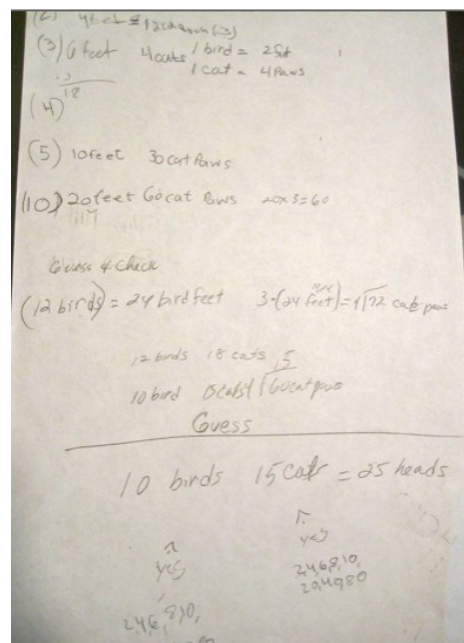
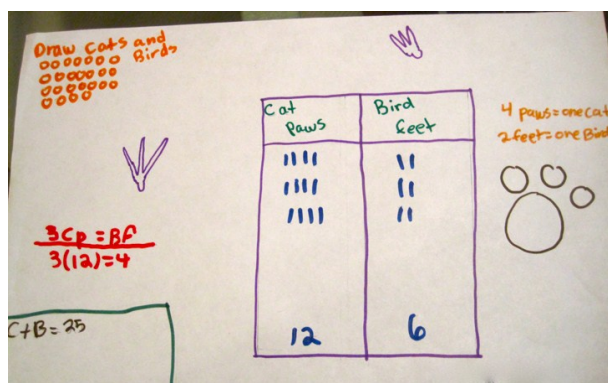
This also allows for re-teaching of any vocabulary the students may not recall.

For this activity I gave the students the handout and asked them to use pictures, words or any other means to explain their understanding of the words. I walked around the room and observed the students' attempts at representing their thinking. When the students had completed the task I pulled them back together to discuss their ideas and we came to a consensus for the definition of each term. Also provided in the book is a review of some algebraic expressions.

Again I asked the students to do the same thing with this worksheet and encouraged them to discuss it with their neighbor. I continued to roam around listening and observing the students work. The students really struggled with the language and how the sentences were structured. I pulled the group together and using a visual display on the board we teased out each sentence, discussing vocabulary words, restating the sentences in a different way, and came up with an explanation that the students copied onto their sheets. For example using the expression "There are 3 times as many apples as there are oranges" we discussed the word times again referring back to our vocabulary review. I then gave examples such as if I had one orange how many apples would I have? If I

had 2 oranges how many apples would I have? And so on until the students had a good understanding. Then I asked how could we explain this as a function if we used “o” for oranges and “a” for apples. The students finally came up with “ $a = 3o$.” I spent one class period on this giving the students approximately 35 minutes. In future implementations I would do the algebraic expression portion as a whole class activity.

Cats & Birds asks the students to use six clues to figure out how many cats and birds Ms. Lang keeps. Students were paired, with consideration of their strengths and weaknesses, in terms of reading and language ability. The pairs were each given one clue from the six but each pair had a different clue. Because of the size of the group not all clues were distributed. I randomly gave out the clues and I would not do that in future implementations. I would give a lot more thought as to which would be more appropriate for each pair of students. Based on this one clue the students had 15 minutes to write an algebraic expression and make a guess as to how many cats and birds there were. Each group was provided with poster paper and markers as well as the clue. This turned out to be too little time for this group. When completed the students shared their posters. The two examples below show the different strategies used, an organized list and a table.



Figures 15 & 16. Cats and birds posters

The students needed a great deal of help with this activity. The original classroom teacher and the instructional assistant worked with the groups as well. At this point I should have adapted the clues but for whatever reason I did not. I did, however, introduce a task organizer that I thought would be helpful for the writing process but the students did not find it useful and I abandoned it within days and switched to a problem checklist. This also proved to be ineffective because the organizer had five areas contained within five geometric shapes (of no specific significance). The first area asked for the title of the task and to describe the problem. The second suggested strategies that might work. The third asked for the student to estimate the solution. The last two areas asked for math words that may be needed in writing and representations that may help students to solve the problems. I think this organizer was asking the students to make too many choices and to integrate too much information and it

overwhelmed them. Therefore, they didn't attempt to use it. Ultimately, only two groups came up with an algebraic expression:

Ms. Lang keeps cats and birds. She has 25 heads to pet. How many cats and birds does she have? The algebraic expression generated by two of the girls was $C+B=25$.

Ms. Lang keeps cats and birds. She counted 3 times as many cat paws as bird feet. How many cats and birds does she have? Two other girls came up with $3Cp = Bf$.

The students worked on this activity over the next two classes. Two of the girls wanted to give up right away but I referred them back to the two review sheets and the $C+B=25$ that they generated the day before with their clue. They could not come up with strategies to use so we worked together to generate ideas that what might work. Working with the same clue with two boys I asked them to visualize what the clue stated. I asked them if they knew any numbers other than 25 heads and given that, could they come up with an algebraic expression to tell us what to do. Manuel hesitantly stated, "So we need a variable." I asked, "How could you write this using variables?" He was able to accomplish this. However, in 25 minutes none of the groups managed to take on more than one clue.

The following class period was a disaster. Four students were absent; therefore, I had to regroup the students but only two worked on the clues. Their partners allowed them to take over. Carmen found the solution using the first and second clues but needed a lot of support and prompting to go through each

clue to see if the answer fit the criteria. Manuel continued to try but his strategy of drawing wasn't effective.

I came to the conclusion that this activity was not within the students' current level of functioning and decided to use the following day to do a journal response and I added to the curriculum a reflective writing piece. This sheet asked the students what was the easiest and most difficult part of the activity. It also asked for one thing they learned and what they could do to help them understand better. Although this was a difficult reflection for them I added this to my features in addition to journaling the process of the problem solving activity. I think it is a beneficial activity that will help students further develop metacognitive awareness.


This problem needed a great deal of revising and I have included those revisions in the Appendix. I advise to wait until the students had stronger problem solving skills and a more varied repertoire of strategies that they were comfortable using. I would do more problems similar to the assessment assignment, Farmer Ben, which will be discussed below. I have included in my revision only four clues. I have also recommended doing the activity as a whole class so that further scaffolding could be provided through prompting, questioning, and guidance.

Activity Six: Farmer Ben (Assessment)


Farmer Ben was used as the final assessment (The Singapore Maths Teacher, 2005). Figure 17 was displayed on the board and a copy was given to each student.

Assessment

Farmer Ben has more goats than turkeys. If all the animals have 24 heads and 78 legs altogether, how many turkeys and how many goats has Farmer Ben?



Goat



Turkey

Figure 17. Final assessment

The students were told to work alone and that this problem was to assess all that they have learned over the last seven weeks. Before presenting the problem I introduced my third attempt at providing scaffolding for the writing process. The Task Organizer and the Problem Checklist did not produce the desired results. With this graphic organizer (Figure 18) I was concerned that the design might be visually overwhelming.

Write about how you solved this problem.

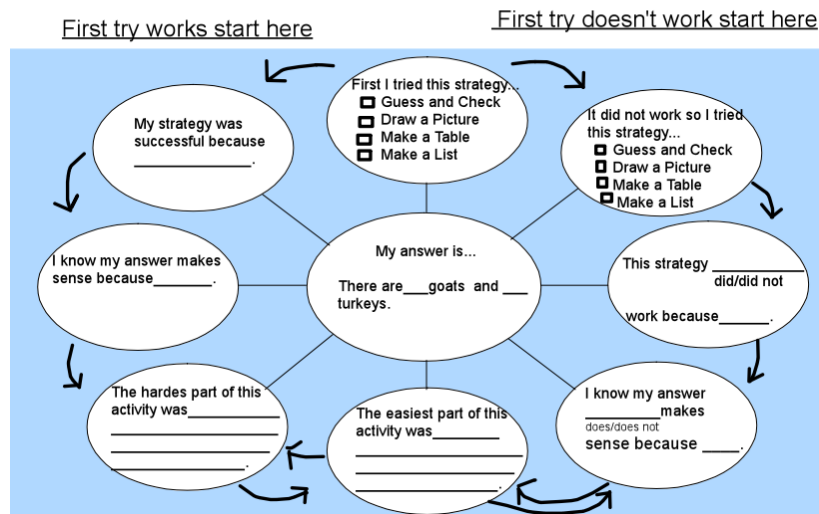


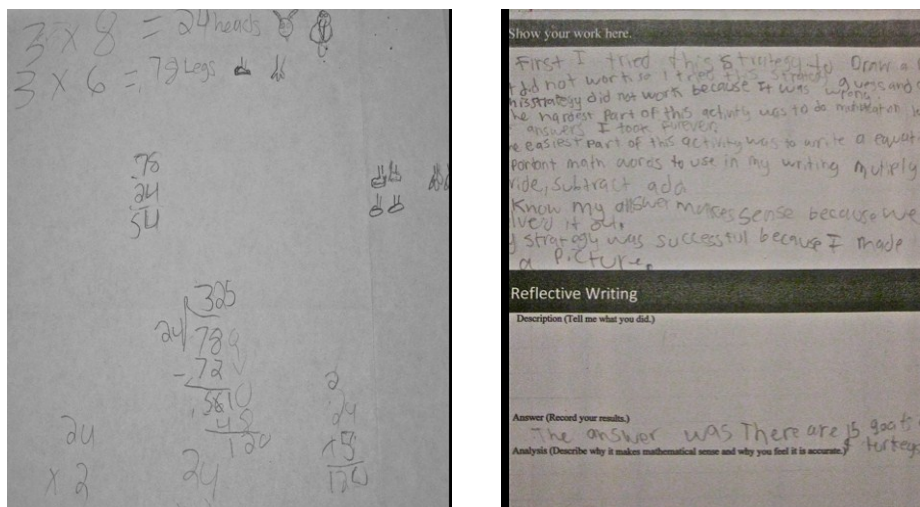
Figure 18. Graphic organizer

However, the organizer proved to be effective for the students with guidance during this initial presentation. I think, over time, the students would benefit from this support and internalize the structure of this type of response to explain the problem solving process they engaged in to solve problems. In my final revision I deleted the bubble containing math words because the students wrote this into their journals entries as part of their explanation. My purpose was to highlight words that could be incorporated into their writing but instead they inserted it as a sentence.

Originally I was not intending to provide any help but some was necessary; however, not to the extent as earlier problems required. One student, Anita, who had other emotional issues due to her family circumstances, refused to work stating she was tired because she was locked out of the house until 11:00pm. Often, we as teachers forget the everyday struggles our students

might have to contend with outside of the school setting that might affect their learning. Carmen, one of the more motivated and capable students, provided the answer of 14 goats and 10 turkeys. She did not figure out the number of legs correctly and did not show her work. When asked to explain her procedure she could not. I encouraged her to start over, which she did. Manuel thought dividing made sense until we reread the question together and I asked him to visualize the scene.

Below is Manuel's final journal entry where he wrote his description of the procedure in the wrong area of the Process Sheet. Manuel missed the last class where the students focused on using the graphic organizer to write their paragraphs. He was later pulled from class in order to transfer the information from the organizer to his journal. I think his mistake in placing the explanation of his problem solving process in the top section of the sheet occurred because the graphic organizer was new and he had never used it before (See figures 19 & 20). Although he knew the work belonged in the top section he may have become confused by the transition to using this new tool. His mistake indicated to me that it is important for the students to complete the graphic organizer and their journal writing in the same class session particularly if they are in the early stages of developing the skill of transferring information from the organizer to their journals.



Figures 19 & 20. Farmer Ben: Student work

The following day the problem was revisited and re-explained. The graphic organizer was presented and explained again. This time I was not in the classroom and the educational specialist took over. After presenting all the information the children were left to work on the problem on their own. According to the Educational Specialist they did not work together.

Upon my return the next day the students had completed the problem with some of the students having the correct solution. They, for the most part, completed the graphic organizer; however, they did not transfer this information into their journals in paragraph form. The teacher who took over for me did not understand that and although it was explained to the students just two days before they did not recall this information. They spent this period writing in their journal using the information on the graphic organizer to guide them. Anita, who refused to work at the introduction of this problem solving activity, again refused to work. The educational specialist worked with her allowing her to dictate her

response as she wrote what was said into Anita's journal. I found it interesting that she left the students to work alone the following day when I was not present but chose to work with this student on this day. It left me wondering if observing my teaching approach for the past seven weeks had left any impact on her teaching style.

One thing I learned from this activity is to introduce the graphic organizer at the beginning stages when implementing problem solving in the mathematics classroom. This organizer helped the students much more than any other attempts I made throughout the curriculum to provide support in writing. This added piece to the curriculum might propel the students' writing skills forward at a quicker pace.

Overall Revisions

The first revision I made occurred at the very beginning during the first week of implementation. The original metacognitive questionnaire I developed contained a Likert scale consisting of two choices, agree or disagree. I found this to be very limiting when I reviewed the responses. I also suspected that the students may have responded as they thought I wanted. Therefore, I revised these statements into questions that required a short answer and then re-administered the questionnaire.

During the course of the implementation of this curriculum other revisions and adaptations were required to make it more effective. In order to explicitly teach the problem solving process I created a slide to display on the board and a handout for the students to paste into their journals as a reference. The problem

solving process used in my curriculum was a recursive process. This was not clearly conveyed in the original design. I added arrows and additional information onto the sheet to represent the recursive nature of the process.

After the initial journal writing attempt I created a Problem Solving Process Sheet adapted from Burns (2007). In the students' first journal entry most did not show their work or include the answer in their writing. This sheet provided more structure to the writing process along with using the sentence frames by designating an area to show their work, describe what they did, state the solution, and tell why the answer made sense. This sheet was pasted into their journals for each activity. Along these lines, I also developed a graphic organizer to help with the writing process because throughout the implementation I felt that although the sentence frames were helping, more structure might help the student make quicker gains in their written communication skills. This proved to work well except for one part that asked the student to record important math words. The students included this in the paragraph in their responses stating "Important math words to use are...." This was not my intention. Instead it was meant to remind students to use these words within the context of their responses. The final version, found in the Appendix, does not contain this piece.

I made two final revisions to my curriculum. The first was to simplify the journal prompt for the "Sense or Nonsense" activity. The original prompt asked the students if it made sense to say that there is a 100% chance of rain for the weekend if there is a 60% chance of rain on Saturday and a 40% chance of rain on Sunday. I used a simpler concept that the students tend to be more familiar

with (candy verses the weather) and added a picture to help students visualize what was being asked. I also revised the ‘Cats and Birds” activity. There was too much language involved in the original activity and some of the structure of the sentences was too difficult for the students to comprehend. I chose four of the simpler clues to use instead of six. This can be reduced to three but this may allow for multiple answers and the teacher needs to be cognizant of that.

Overall I believe these changes provided stronger scaffolding to support students with learning disabilities when engaging in problem solving activities.

Summary

I designed this curriculum to support and develop student problem solving abilities in mathematics. It was created particularly for students with learning disabilities who often experience difficulty with higher-level mathematics where reasoning and problems solving skills are necessary (Gagnon, 2001). Often remedial mathematics instruction focuses on practice and repetition, which has proven to be ineffective for my students as evidenced by continued failing grades in their general education math classes. I immersed the students in problem solving activities that infused metacognitive teacher questioning, direct teaching of problem solving strategies, and my recursive problem solving process in conjunction with scaffolding designed to assist the students to reach a level they would not be able to attain without such supports. This first implementation of the curriculum was much more of a challenge than I anticipated. Given my experience working with students with special needs and knowing that these were challenging tasks I was placing before these students, I knew progress

would be incremental. Preliminary findings, through observations, field notes, journal and reflective writings, as well as student work, suggest some improvement, albeit small. Without fully examining the data, which will be discussed in the next chapter, I saw the following positive changes in my students by the end of experiencing this curriculum.

- Increased willingness to persevere when faced with a challenging task.
- Increased willingness to write and talk about mathematics.
- Increase in length of written responses.
- Increased use of strategies when solving problems in mathematics.

Although teachers often perceive growth in their students during the school year, when conducting educational research, data collection and evidence are necessary to make a claim. The following chapter will explore the findings and provide evidence to support them.

VII. Evaluation of Problem Solving Toward Mathematical Understanding

One of the goals of *Problem Solving Toward Mathematical Understanding* was to increase LD students' abilities to make sense of problems and persevere in solving them. Another goal was to improve their reasoning skills both abstractly and quantitatively. A final goal was for students to be better able to construct and communicate viable arguments to support their reasoning and to critique or question the reasoning of others. Meeting these goals are a challenging task for this special population of children, but it is, none-the-less, one that should be undertaken by every teacher who has special needs students in their classroom.

The activities focused on novel problem solving situations that did not have an apparent operation or procedure that could be used to obtain the solution. Understanding the problem, planning a solution, executing the plan, and monitoring for success are a part of the process. Developing these skills along with metacognitive awareness is important to the success of this program. *Problem Solving Toward Mathematical Understanding* calls for the student to be able to justify and explain why the procedure was appropriate and if their solution makes mathematical sense. Scaffolding and the use of questioning by the teacher to encourage metacognitive thinking is an integral part of this curriculum.

Data Collection and Evaluation Strategies

A variety of data collection techniques were used to evaluate *Problem Solving Toward Mathematical Understanding*. Each method brought a different

perspective and unique piece of information to the evaluation process. During the activities and discussions, whether between students or the whole class, observations, field notes, and digital audio recordings captured important data. The audio recordings provided a back up source to use when my field notes were scant or not rich in quotes from the students' comments. Data was collected through the use of the Brigance Test, which provides a standardized measure of problem solving abilities based on a grade level. This helped me to have an idea of what procedural knowledge the students had mastered. Data was collected through pre and post surveys in the areas of attitude toward mathematics. Also administered was a questionnaire to measure the students' metacognitive awareness in terms of how they approach mathematical problems and what they do when faced with a difficult task. Journals were used to evaluate improvement in the use of the problem solving process and the ability to communicate using mathematical language. Rubrics were used with the journals as the evaluation tool. Table 8 illustrates the evaluation sources as they are related to each goal. Evaluation procedures for each construct addressed in this curriculum is described below.

Table 8. Evaluation sources for goals

Data Sources	Goal 1: Makes sense of problems and persevere in solving them	Goal 2: Reason abstractly and quantitatively	Goal 3: Construct viable arguments and critique the reasoning of others
Observations	X	X	X
Pre and Post Metacognitive Questionnaire	X		
Pre and Post Math Attitude Survey	X		
Student Work	X	X	
Field Notes	X	X	X
Journals		X	
Problem Solving Rubric		X	X
Audio Recordings	X	X	X

The Problem Solving Process

I evaluated the students' abilities to use the problem solving process through observations of students at work, presentations or group discussions, and the journal writing entries. I looked for improvements in their abilities to use the steps involved in the recursive process as well as their flexibility to shift their strategy if it was not effective.

Communication

I considered communication as displayed through both written and oral abilities. The abilities to convey messages orally and in written form are very different skills, and I wanted to be able to discern improvement in both areas.

Written Expression

Written communication was evaluated through any individual writing samples. My main source of data was through the journal writings that showed their work, discussed what they did, indicated the answer, and explained why it made mathematical sense. A reflection sheet was introduced at the end of the implementation of this curriculum. Although its limited use did not contribute a lot to the data collection for this projection, I foresee it being a good source of data to monitor students understanding for teachers who might implement this approach.

Expressive Language

The ability to effectively express the mathematical process and their reasoning, as well as the ability to ask questions about the work of others was evaluated through my observations, field notes, and audio recordings of discussions. This included peer-talk, teacher and student talk, and whole class interactions. The tape recordings provided accurate information of statements made including understanding as well as misconceptions.

Perseverance

Perseverance was measured through observations of sustained attempts to find a solution to the problem. On several occasions students attempted to calculate the answer using addition, subtraction, multiplication or division. When the numbers did not make sense the students tended to draw a picture or make a list. I did not record time spent on activities, but journal responses also provide data by showing if more than one approach was used during the process. The

work represented in the journals and the descriptions of the problem solving process indicated that after a first failed attempt the student continued to try one or more strategies to find a solution. Anecdotal evidence was obtained through discussions as well.

Attitude/Motivation

I believe attitude and motivation are difficult to assess and it is my contention that they are connected to each other. I base this viewpoint on my experience of seeing poor attitude tied with poor motivation and high motivation linked to a positive attitude toward mathematics. I gathered data to evaluate improvements in this area through observations, in terms of students' willingness to persist with challenging tasks. Journal writing shed light on this as well. I also observed the students' level of engagement in class discussions. Observations were captured through my field notes and tape recordings. A pre and post survey was also administered, which was adapted from TERC Mathematics (TERC, 1997). This survey was quantitative, as the responses were assigned numerical values where higher values indicated more positive attitudes toward mathematics.

Metacognition

Pre and post surveys were used to evaluate the students' metacognitive awareness. This included their ability to know if they understood the problem, what action they would take if they did not understand the problem, what they would do if the problem appeared too difficult for them to solve, how they figured out what to do first, what they did if their first attempt did not work, how they

evaluated their solution, and how they determined if they were finished. These abilities were also evaluated through individual conversations with the students during the problem solving activities, which were documented through field notes and digital audio recordings as well as class discussions.

Data Reduction Methods

Observations, Field Notes, and Digital Audio Recordings

Students were observed throughout the implementation. After lessons I recorded anecdotal notes on students' language usage, how they approached the problems, their attitude, and how motivated they appeared. Audio recordings were made as well, which were transcribed and analyzed for language usage and the ability to communicate mathematical idea. Comparative analysis was used to analyze the data. The data was analyzed for changes in students' willingness to try a variety of strategies, a shift towards a more positive attitude, and increased motivation to persevere when faced with challenging tasks.

Pre and Post Metacognitive Questionnaire

Pre and post questionnaires were administered to the students to assess their metacognitive awareness before and after implementing the curriculum (see Appendix). The responses were analyzed using comparative and qualitative methods. The post implementation responses were broken down and coded as monitoring/self-regulation, use of a variety of strategies, and perseverance.

Pre and Post Math Attitude Survey

The students completed a survey prior to instruction and again at the completion of the seven weeks (see Appendix). The survey was adapted from

TERC, (1997). I used comparative analysis, an item-by-item analysis of the students' pre and post responses, and quantitative methods to analyze the data. The Likert scale, also taken from TERC, used a zero, one, or two-point value based on the students' responses to statements using Agreed, Disagreed, or Not sure. Higher values suggested a more positive attitude toward mathematics.

Problem Solving Rubric

I used a NCTM standard based rubric adapted Exemplars (Brewer, 2011). Journal responses and student work were evaluated using both qualitative and quantitative methods as well as a comparative analysis. The rubric evaluated the students' abilities in four domains: Problem Solving, Reasoning and Proof, Communication, and Representation. The rubric analyzed the proficiency level of students based on NCTM standards and move from less to more proficient using a four-point Likert scale. The students were given the Secret Puzzle problem to solve prior to instruction (see Appendix). At the end of seven weeks they were given the Farmer Ben problem to solve (see Appendix). Two raters, the special education teacher and myself, analyzed the students' journal responses and assigned them a rating. We then compared our ratings, discussed them, and came to a consensus on the final score. I have included the rubric in the Appendix.

Findings

Goal 1: Students will make sense of problems and persevere in solving problems (attitude and motivation).

Finding 1: Most students' attitude toward math and their willingness to persevere increased over the course of the implementation.

Math Attitude Survey

The initial Math Attitude Survey adapted from TERC was administered on the first day of the implementation of this curriculum. The questionnaire had a point system for each item answered. The higher the score, out of a possible twenty-two points, coincides with a more positive attitude toward mathematics. On the last day of implementation the students were administered the same survey in order to compare if there was an increase, decrease, or no change in attitude. The results indicated that six out of nine students' attitude toward mathematics improved. Figure 21 shows the students' improvement in attitude over a seven-week period of time.

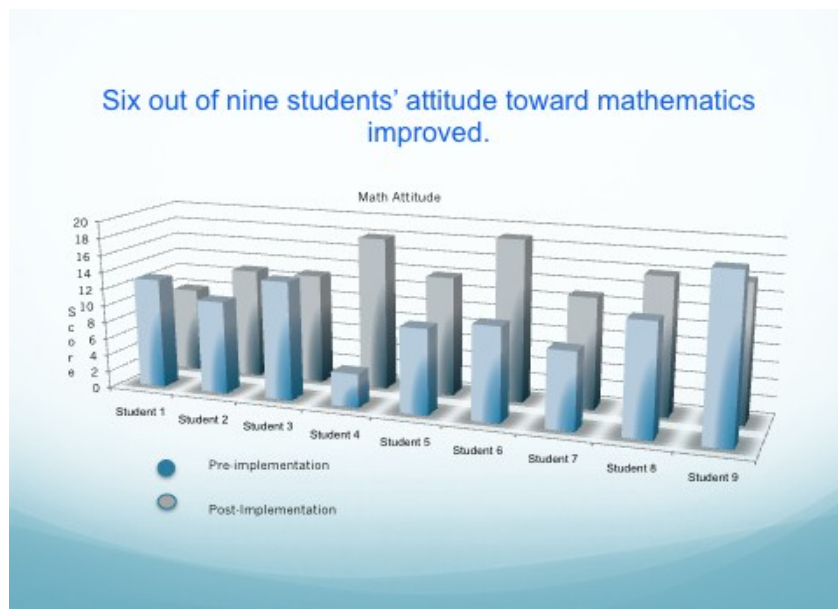


Figure 21. Results of pre- and post-implementation survey

This is an important finding because in my experience as a classroom teacher, students who have a positive attitude toward mathematics are much more motivated to work. Students who have developed intrinsic motivation enjoy learning and are more oriented to take on challenging tasks (Gottfried, Fleming, & Gottfried, 1994).

When comparing the pre-implementation and post-implementation data, student 4's attitude improved substantially. This student had failed math throughout his sixth grade year and truly did not like math. When asked if he was pretty good at math this student initially responded by disagreeing. When asked the same question again post-implementation he agreed with the statement. In the pre-implementation survey he disagreed with the comment "Doing math lets me think creatively." but changed that to be in agreement in the post-implementation response. He excelled with the curriculum in terms of motivation to work, perseverance, and participation in discussions. Five other students made gains though not as impressive as this student. Three students did not make gains. In searching for an explanation as to why this occurred I could only make conjectures. Student 1 failed all his classes for the entire year. He appears to have checked out of his education and his behavior results in multiple referrals or time out of class. This student did little work in the class and despite doing some work on occasion, for the most part, he was uncooperative and I was not able to reach him in the time I had with him. The other two students (3 and 8) were frequently absent. They often missed class and returned to participate in an activity in which they had inadequate preparation and

information. Therefore it was difficult for them to understand fully and complete the problem adequately. I am sure this resulted in frustration on their part. They often said “I missed yesterday, I don’t know what to do.”

Metacognitive Questionnaire

Students were administered a questionnaire which was designed to assess their current level of metacognitive awareness. Questions focused on their ability to know if they understand a problem, monitor their work, shift strategies if necessary, and evaluate the appropriateness of their solution. Again, as with the Math Attitude Survey, this was done both pre and post implementation.

In my analysis of the answers written by the students both before and after participating in this curriculum design I was able to code three key areas: monitoring or self-regulation, using a variety of strategies, and perseverance. Initially many students stated that if they did not understand a problem they would skip it or ask for help from the teacher. At the end of the implementation they responded to the same question with comments such as “I re-read the problem or try something similar to it,” or “I read it again.” When asked if the problem seemed too difficult what would you usually do, student moved from “I tell the teacher,” “I skip it,” or “I ask for help” to “I do similar problems,” “Try strategies to solve it,” or “Draw a picture.” When asked if something they try doesn’t work what do they usually do, initial responses changed from, “I tell teacher to help me” to “I check and make another strategy” or from “Raise my hand and ask for help” to “Try to do it again.” Although not all the children

improved in all areas, everyone did improve in at least one of the key areas mentioned above. Table 9 shows example of students' pre- and post-implementation responses to the metacognitive questionnaire.

Table 9. Pre- and post-implementation responses

Pre-implementaion responses	Post-implementation responses
I ask for help	Read it again
I tell the teacher	I skip and go back when I do similar Problems
I tell the teacher to help me	I check and make another strategy
Raise my hand and ask for help	Try to do it again
No response	I try it again or try another strategy
No response	I show different ways to figure it out

Finding 2: Students became more willing to change ineffective strategies while solving problems.

Journals and Student Work

Evidence of the use of a variety of strategies can be seen in students' journals and work samples. I asked students to show their work in their journals but I often needed to collect the scraps of paper they worked on during the process. At the beginning of the implementation of *PSTMU* the students' natural tendency was to erase their initial attempts at a solution. I instructed them to leave those failed attempts on their papers in order to refer back to it when writing about the problem solving process. The students needed guidance to realize that a procedure was not working because their number sense was not well developed. When they described what they did to solve the problem, emerging skills could be seen as they began to say they tried something and it didn't work so they tried something else. The seven weeks we had together was

not enough time for them to develop an adequate repertoire of strategies to call upon when solving challenging problems; however, many students did try using computational methods, list or tables, and drawings. The latter was the most prevalent strategy used at this early stage in their conceptual development as one can see in the student work sample below.

The image shows a student's work on a problem. At the top, there are two diagrams. The left diagram shows three animals: a cat with 4 paws, a dog with 4 paws, and a bird with 2 feet. Below them are the numbers 4, 12, and 4. The right diagram shows two birds, each with 2 feet, and a number 4 below them. In the center, the equation $3CP = BF$ is written, with $3(12) = 4$ written below it. To the right of the equations, it says "4 Paws = one cat" and "2 Feet = one Bird". Below the diagrams is a section titled "Reflective Writing". Under the heading "Description (Tell me what you did.)", the student writes: "I drew a picture to solve the problem. But first I did this $\frac{3CP = BF}{3(12) = 4}$ when I drew the picture it looks like this but it did not work." Below this text are two small diagrams of the animals. Under the heading "Answer (Record your results.)", there are no visible results.

Figure 22. Work sample: Evidence of multiple strategies

The image shows a student's journal entry. At the top is a section titled "Reflective Writing". Under the heading "Description (Tell me what you did.)", the student writes: "first I tried drawing a table and multiplying but that didn't work. Then I drew a picture and that made more sense." Below this text is the equation $C + B = 25$ heads. Under the heading "Analysis (Describe why it makes mathematical sense and why you feel it is accurate.)", the student writes: "try different ways for a problem's answer to make sense."

Figure 23. Journal writing: Evidence of multiple strategies

The Table 10 contains comments taken from students' reflective writing in their journals that supports this finding. In each excerpt the student explains the process used to arrive at a solution to problem. Each student explains what the first strategy they used and what was done if the results did not make sense. The students then explained other strategies that were attempted to arrive at a solution.

Table 10. Journal responses

"I guessed, divided, add, and use the strategy (sic) gess (sic) and check. I added $36+28$ it did not work then I added $38+32$ it did not work I finally added $46+32$ and it worked."
" first (sic) I drew a picture but that didn't work. Then I guessed and checked but that didn't make sense. Then I added and the (sic) worked."
"First tried this strategy was guess and check it did not work so I tried make a list."
" I tried (sic) using letters for it, but that didn't work then we tried multiplying also, but that didn't work eithe (sic) $C=4$, $B=2$. Didn't get the answer."
I solved the problem by using a (sic) equation. I check my answer writing a (sic) equation with varbles (sic). The strategy I used was a picture and writing a (sic) equation."

Goal 2: Students will reason abstractly and quantitatively (the problem solving process).

Finding: Students improved in their ability to solve problems.

Journals

The students' journals were rated through the use of a rubric by two raters, myself and the primary classroom teacher, to ensure that scores were accurate and had a degree of inter-rater reliability (see Appendix). When there was a disagreement we discussed the evidence and came to a consensus on the

score. The students' pre-implementation problem solving activity and their post-implementation problem solving assessment activity were rated in four separate areas. These included Problem Solving, Reasoning and Proof, Communication, and Representation. Therefore, there were 8 ratings per student with a total of 72 ratings for the group of 9 students. Out of seventy-two scores there were only eleven discrepancies between the two raters. The rubric was adapted from Exemplars, a website online that developed it based on the NTCM standards. Table 25, shown below, indicates what level each student began at and what level he or she had achieved after seven weeks of instruction. Student 4 received only four weeks of instruction and that may be why his scores did not improve. Student 1 is the child who had failed all his classes and had behaviors that interfered with his learning. Figures 24 and 25 below show scores in the areas of Problem Solving and Reasoning and Proof. The mean class level of achievement increased from 1.33 (pre) to 2.11 (post) in the area of Problem Solving and from 1.56 to 1.78 in the area of Reasoning and Proof.

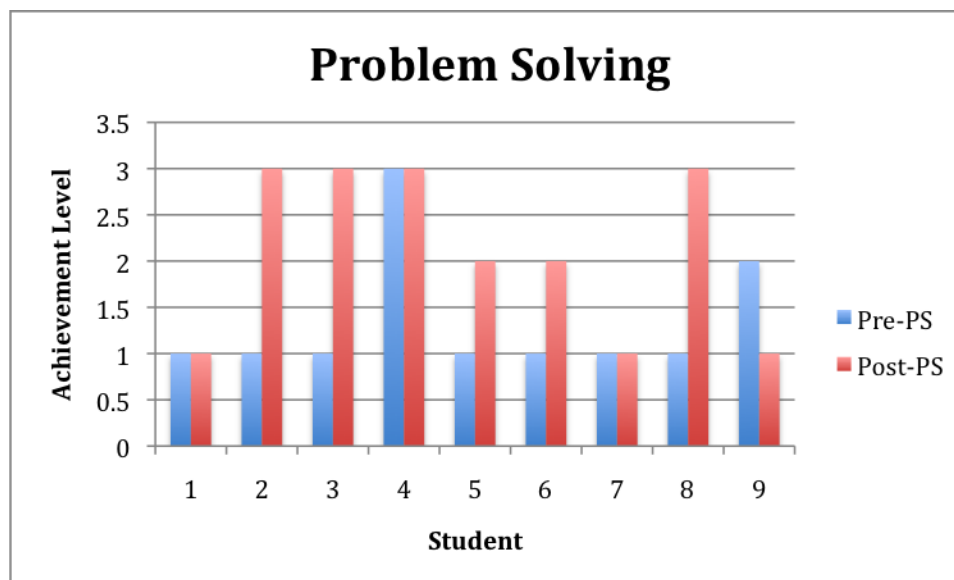


Figure 24. Achievement in problem solving abilities

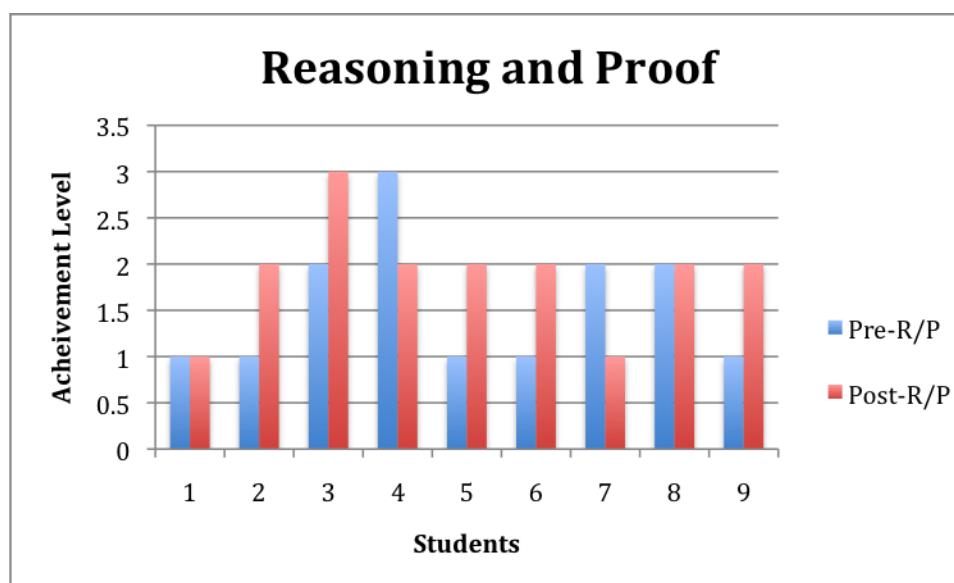


Figure 25. Achievement in the ability to provide reasoning and proof

In the area of problem solving, seven out of nine students fell in the level one range indicating that they did not use a strategy or chose a strategy that would not lead them to a solution. One student chose a partially correct strategy or one, which would enable them to solve only part of the problem. One student

chose a correct strategy and showed evidence of monitoring his progress by changing strategies when it did not produce results that made sense. This indicated work at level three. This same student received the same level in the area of reasoning and proof because he provided his reasoning and he noted patterns. Five out of the nine students scored at level one in this area because their arguments did not make mathematical sense or they did not indicate any justification for what they did. Two students showed this ability to some extent but still weak. In evaluating the post-implementation work, scores rose in the area of problem solving for five of the nine students, three remained the same and one decreased. In the area of reasoning and proof the data indicates that six of the nine students increased by one level, while one remained the same and two actually decreased. The final assessment, Farmer Ben, was a more difficult task than the Secret Number Puzzle #1 and this should be taken into consideration when interpreting these results.

Final Assessment: Student work

The students' final assessment indicated that five out of the eight students who completed this task were able to find the correct answer. This was actually surprising to me as I thought maybe one or two students would arrive at the correct answer. Figures 26 & 27 show two students' work on this problem. Both students used multiple strategies. Each used computation as well as making a drawing. The descriptions included what was tried, the correct solution, and why it made mathematical sense.

show your work here.

Reflective Writing

Description (Tell me what you did.)
I counted 4 legs for goats and 2 for Turkey until I got 78. Then I added the heads and was 24 heads.

Answer (Record your results.)
15 goats = 24 heads
9 turkeys

Analysis (Describe why it makes mathematical sense and why you feel it is accurate.)
My strategy was successful because I drew a picture. I know my answer makes sense because I added the numbers and it equaled 24 heads. I think the easiest part of this activity was adding and checking my answer until I saw that my answer was correct.

show your work here.

Reflective Writing

Description (Tell me what you did.)
First I tried guess and check. It did not work so I tried draw a picture. This strategy did not work because I added the hardest part of this activity, the division.

Answer (Record your results.)
My answer is I got 15 goats and 9 turkeys.

Analysis (Describe why it makes mathematical sense and why you feel it is accurate.)
My strategy was successful because I multiplied to get my answer.

Figures 26 & 27. Farmer Ben: Student work

I was particularly interested in the process and reasoning students have been using to problem solve and did not focus as much on the correct final response. Level three and four in the rubric requires a correct response whereas level one and two don't necessarily have this requirement.

Goal 3: Students will construct viable arguments and critique the reasoning of others (communication: oral and written)

Finding: Students improved in their ability to communicate mathematically by constructing arguments to indicate why the solution made sense; however, they showed little evidence of improving in their ability to critique the reasoning of others.

Journals

Using the rubric (see Appendix) I was able to evaluate if there was improvement in the students' ability to communicate mathematically and if they could create and use mathematical representations in their writing. Scores were based on evidence of an approach and the ability to effectively communicate this approach. They were also rated on attempts to mathematically represent their approach by organizing, recording, or modeling in some way. The levels indicated how well they have developed this skill. Figure 28 shows that six out of nine students improved in their ability to communicate mathematically based on the rubric criteria. The mean class level of achievement increased from 1.22 (pre) to 2.00 (post). For those students who initially used little or no communication to discuss their approach or used non-mathematical language to discuss ideas improvement was seen by at least one level indicating growth in their ability to communicate mathematical ideas. One student moved from level one to level three where her writing used mathematical language and conveyed a sense of purpose. Figure 29 suggests that there were also improvements in the area of mathematical representation where seven of nine students initially did not attempt to represent the data in any way. By the end of the implementation these students attempted to use drawings or lists to show how they solved the problem. The mean class level of achievement increased from 1.22 to 2.11 in the area of Representation.

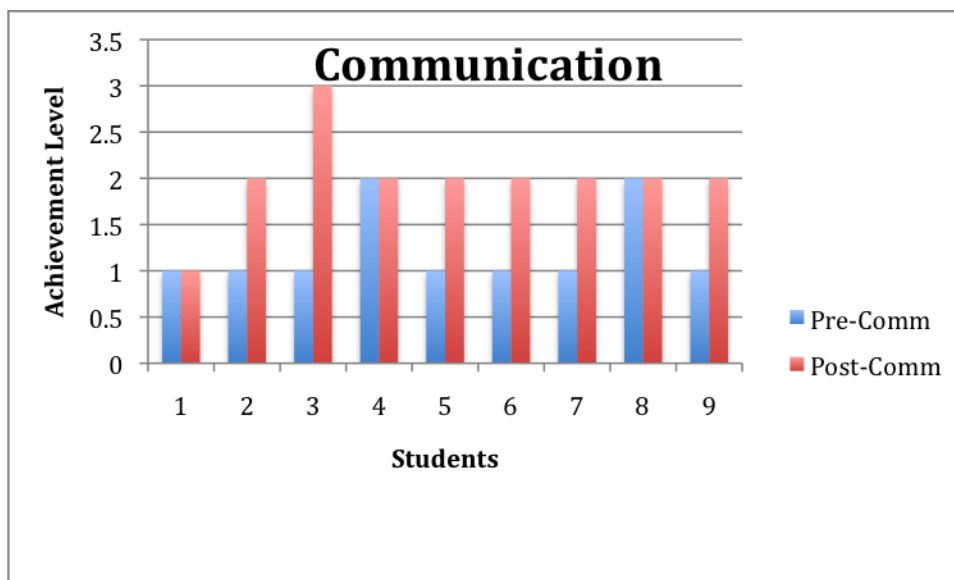


Figure 28. Achievement in mathematical communication

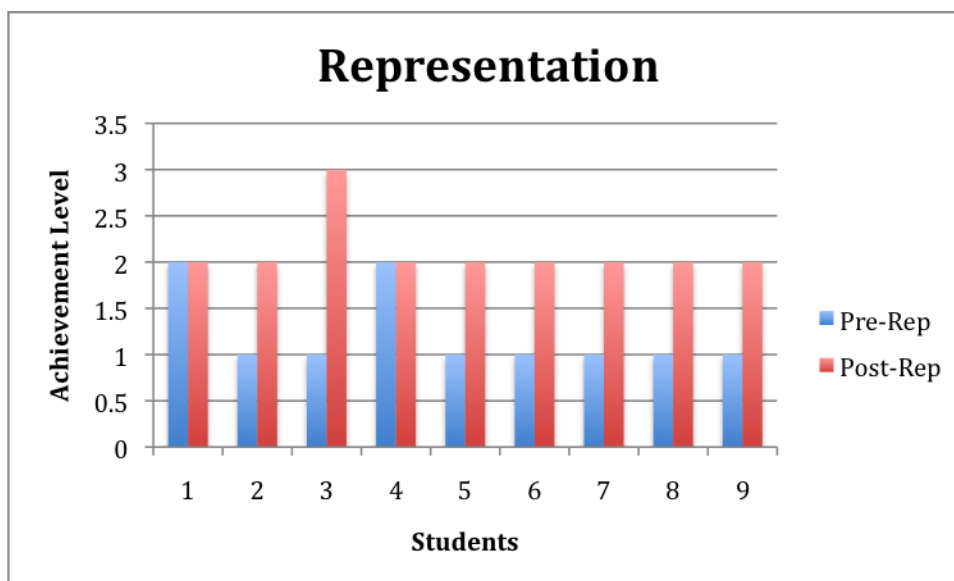


Figure 29. Achievement in ability to create and use representations

Field notes and audio digital recordings

Based on my field notes and audio digital recordings students' discourse improved in that they were more willing to share their ideas in class. Although by

the end of the implementation the students' skills in this area were still very weak, they became more willing to agree or disagree with students' answers and attempted to explain why. Students began to want to present their findings to the class even if they did not have the correct answer, instead focusing on the process they used to attempt to solve the problem. In reviewing my notes and recordings I found that a student who was very quiet during most discussions provided information about the group's process in front of the class given the support of her peers and visual cues in the form of sentence frames on the board. All of the students relied on these cues when presenting in front of the class; however they appeared much more comfortable doing so by the end of the implementation. One student who was not willing to work with others and was very resistant to work in general asked twice in the course of a lesson if they could present their problem to the group. Below are excerpts from two students' comments made during a lesson titled Guess the Function. These show not only a desire to communicate mathematically but also use of mathematical language.

Student A: After when we are done can we go up and share like we did yesterday?

Teacher: We will see if we have time.

Student A: Can we share?

Teacher: We don't have time today to share. I'm sorry.

Teacher: What did you do?

Student B: I multiplied by six and subtracted 2.

Teacher: What is the rule?

Student B: 2 times

Teacher: But we can't use words. How do you write this as a function? What are you multiplying by ?

Student B: $2n$.
Teacher: Then what did you do?
Student B: Added 3.
Teacher: So it is what?
Student B: $2n+3$.

Summary

Overall all the students benefited to varying degrees and across various areas as measured by the journals, student work, audiotapes, questionnaire, survey, and observations. Each of these assessment tools provided some support for multiple findings and each finding had at least two pieces of supporting evidence albeit somewhat weak. It must not be forgotten that seven weeks of exposure to a metacognitive and scaffolding approach to problem solving is a tiny fragment of time in these children's academic careers. Attitudes, motivation, false notions, as well as poor mathematical skills take much longer time to remediate. Given the students' learning disabilities and adding the risk factor of being an English language learner, I believe with continued and consistent use of this curriculum, the students would make even greater gains. These gains, however, may be at a slower pace than for students without learning disabilities. It is exciting to see students, who have underachieved in mathematics throughout their academic careers and have personal challenges, make academic gains. It gives me hope that higher achievement for these students in this type of setting is possible if consistently exposed to a problem solving approach to teaching mathematics.

Why some students did better than others is a multi-faceted question. There were several limitations that may have contributed to this. The sample size was small and included only nine students. Several of these students were

habitually absent. Also, during the course of the implementation, some students transferred into or out of the class. Another limitation was related to time. The curriculum was only implemented three days a week for seven weeks and was limited to 30-35 minute sessions. In addition to these limitations the students had learning disabilities and cognitive levels ranged from very low to average abilities.

Each part of this curriculum is an important piece that I do not think can be eliminated. Problems can, and should be, adjusted to meet the needs and levels of all the students within the classroom. Scaffolding is a necessary component as is the discussions and opportunities to write and reflect. This curriculum shows that providing these opportunities has a positive impact on even the most challenged student. It is a worthwhile effort for the teacher because in the end the students' learning is of higher quality and hopefully will be better retained. An investigation into this type of curriculum with LD students, for a longer period of time and on a larger scale in terms of the student population, may be an area for future research.

To illustrate the higher quality of work that can come from using this approach to teaching mathematics, I am concluding this chapter with a pre and post-implementation work for one student. This student did not include information on how she went about solving the pre-implementation problem solving activity and did not elaborate on why her answer made sense mathematically (Figure 30). The student was able to use the graphic organizer, which contained sentence frames, as a pre-writing tool to help develop her journal response (Figure 31). Her final assessment activity indicated multiple

attempts using different strategies (Figure 32) and her journal response tells the process she went through to solve the problem as well as why she feels it makes sense to her (Figures 33 & 34). Although she did not obtain the correct answer to the final assessment problem her work indicates improvement in many areas assessed during this implementation.

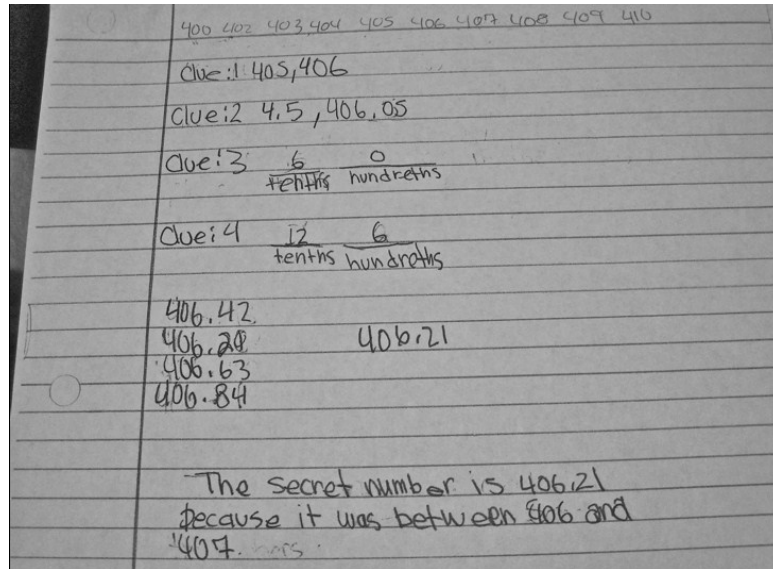


Figure 30. Pre-implementation problem solving activity

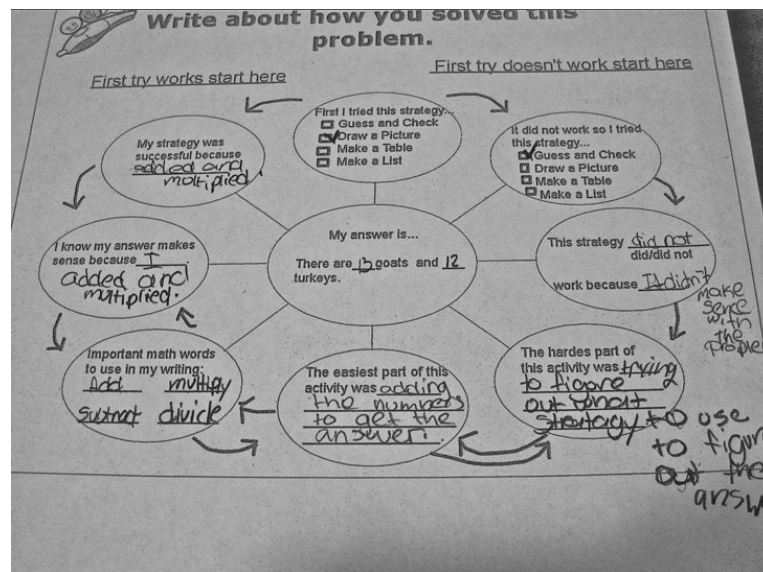


Figure 31. Final assessment graphic organizer

$$\begin{array}{r} 78 \\ + 24 \\ \hline 102 \end{array}$$

Goat = 4 legs

$$\begin{array}{r} 24 \\ 4 \\ \hline 28 \end{array}$$

Turkey = 2 legs

$$\begin{array}{r} 24 \\ 2 \\ \hline 26 \end{array}$$

$$\begin{array}{r} 78 \\ 4 \\ \hline 82 \end{array}$$

$$\begin{array}{r} 28 \\ + 26 \\ \hline 54 \end{array}$$
 heads altogether

$$\begin{array}{r} 82 \\ + 80 \\ \hline 162 \end{array}$$
 legs altogether

$$\begin{array}{r} 162 \\ + 54 \\ \hline 216 \end{array}$$
 head & legs altogether

Goat legs altogether = 82
 Turkey legs altogether = 26
 Turkey heads altogether = 28

Figure 32. Evidence of multiple attempts to solve the problem

Show your work here.

$$\begin{array}{r} 18 \\ + 24 \\ \hline 42 \end{array}$$

Goat = 4 legs
turkey = 2 legs

Reflective Writing

Description (Tell me what you did.)
first I drew a picture, but that didn't work. Then I guessed and checked but it didn't make sense. Then I added and the worked.
13 and 12 turkeys.

Analysis (Describe why it makes mathematical sense and why you feel it is accurate.)
keep trying different strategies until one mathematical strategies make sense.

Adapted from Burns, 1996, p.112

Figure 33. Final assessment journal response page one

My strategy was successful because I added and multiplied. First I tried to draw a picture. It did not so I tried guess and check. This strategy did not work because it didn't make sense and it didn't go with the problem. The hardest part of this activity was trying to figure out what strategy to use to figure out the answer. The easiest part of this activity was adding the numbers to get to the answer. Important math words to use in my writing are adding, subtracting, multiplying and dividing. I know my answer makes sense because I added and multiplied.

Figure 34. Final assessment journal response page two

VIII. Conclusion

The goals of *Problem Solving Toward Mathematical Understanding* were to improve problem solving and reasoning skills, mathematical communication, as well as attitude and motivation as measured by the students' ability to persevere when faced with challenging tasks. The curriculum was designed specifically for students with learning disabilities who receive special education support in a separate math class.

Problem Solving Toward Mathematical Understanding requires the teacher to be a facilitator of learning. By providing interesting and thought provoking problems for the students to solve, they are more apt to become engaged in the tasks before them. Working just above the students' current level the teacher provides scaffolding such as breaking down the task into smaller parts, pre-teaching vocabulary, sentence frames/stems, modeling, direct instruction of strategies and the problem solving process, as well as visuals and manipulatives when necessary, Using "think aloud" to model thought processes as you solve a problem, tapping into prior knowledge, and providing opportunities for students to talk are all forms of scaffolding which are beneficial to the diverse learners in our classrooms. The teacher asks questions to foster metacognitive awareness of the students' own thinking process, provides time for journaling or reflective writing, and allows for class discussions at every available opportunity. All these pieces are a necessary part of the curriculum, to be implemented on a consistent basis if improvements are to be achieved. Scaffolding the material presented is key to *Problem Solving Toward Mathematical Understanding*.

Without these scaffolds, the student may not access the material in a meaningful way that allows for increased conceptual understanding and continued growth in their knowledge of mathematics and their ability to use reasoning to solve problems.

Using this approach to teach students with learning disabilities how to problem solve is challenge indeed. There are many obstacles and frustrations but there are also many rewards, such as seeing a child attempt a problem that is perceived as challenging and too difficult, seeing a child develop more self-confidence, or seeing a child write about how a problem was solved. The improvement rate is slow but well worth the wait. I know if I had more time with these students or if they received this type of instruction over the course of their elementary and middle school careers they would be much better prepared to face the rigor of high school mathematics. Change must start somewhere and in some small way it did for these kids.

In conclusion, it is my hope that further research will be done on a larger population of students with learning disabilities and for an extended period of time in order to see the full potential of this problem-solving curriculum. I would also like to call for a change in how we support our students with disabilities in Special Education Programs. These programs influence the types of instructional strategies used by teachers for students with disabilities (Maccini & Gagnon, 2006). Teacher training programs need to provide both special education teacher candidates and general education math teachers with courses that address how to teach beyond procedural mathematics and how to use

problem solving to develop understanding and make sense of mathematical concepts. In a study conducted by van Garderen, middle school special education teachers from a variety of classroom settings were given a survey addressing instructional strategies used to teach LD students how to solve word problems. The one finding suggests that, regardless of the setting, teachers tended to emphasize practice exercises rather than true problems where the approach to solving the problem is not clearly evident (2008). Teacher candidates need training in how to provide scaffolding and supports to these students, whether in a substantially separate classroom on a comprehensive public school campus or within the general education setting. Only when those who teach have the knowledge and skills necessary to adapt the math curriculum for students with learning differences will these students have the opportunity to learn differently.

No longer can educators stand idly by while children's mathematical minds remain unchallenged. All children have the right to learn at a rigorous level and to access the general education curriculum. Teachers must learn and experiment with scaffolding strategies to teach problem solving in mathematics. Preparing novel problem solving activities, providing appropriate scaffolds, and allowing time for discussion may take more effort on the part of the teacher; however, the academic and motivational gains are worth the effort. Change needs to happen and it needs to happen now.

Appendix

Problem Solving Towards Mathematical Understanding

Instructional Design for Student With
Learning Disabilities

Renate Ward

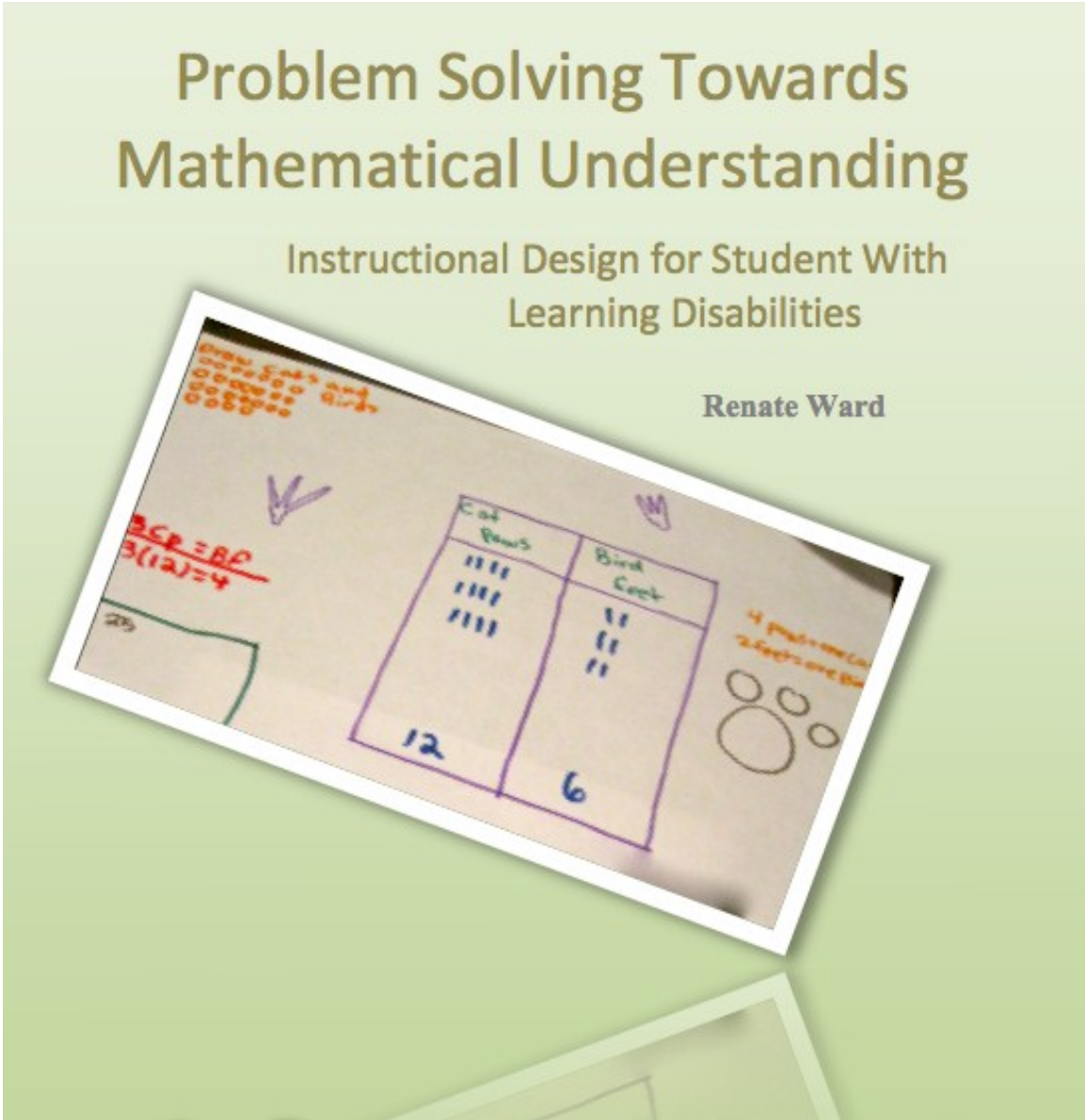


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General Materials

Sample Questions for Problem Solving

Writing Graphic Organizer

Rubric for Assessing Student Work

Dear Fellow Teachers,

As a former special education teacher I worked with deaf and hard-of-hearing students with and without additional disabilities in the area of math for 10 years at the middle school level. An additional four years were spent working with deaf students with multiple disabilities teaching predominantly math as well. I am now a school psychologist working at the middle school level and have the opportunity to work with teachers in developing interventions for struggling students in addition to my other duties.

In my current position working with both general education and special education teachers, I realized that not much has changed in terms of how we teach math to our special education students. Because they lack many of the basic skills needed in mathematics, we attempt to instill these skills through ongoing practice and repetition of computation. These students often become bored and unmotivated. In order to alleviate this dilemma and address the need to develop problem-solving skills, I created *Problem Solving Toward Mathematical Understanding: Instructional Design for Students with Learning Disabilities*. This curriculum uses problem solving to help students' improve their mathematical understanding of concepts and the problem solving process. We typically ask kids to practice computational procedures with little conceptual understanding, and as a result, they often don't retain the information. However, when students have a varied repertoire of strategies to choose from to solve problems, they are more likely to attempt a problem, monitor their work, and adjust their approach when necessary.

Problem Solving Toward Mathematical Understanding incorporates metacognition and scaffolding to improve reasoning skills, problem solving skills, self-monitoring skills, and the ability to communicate about mathematical ideas more effectively. This activity-based program engages students in investigations of novel problems, discussions, journal writing and reflections to provide valuable experiences and skills that they can use throughout their lives when facing real-life problems.

The best part of this program is that it can be used in any setting where children with learning disabilities are found. It is a supplement to your curriculum and can be used once a week or more, depending on how you chose to use it. The problems can center on the concepts you are currently teaching or they can be used as a review of previously learned material. You decide! But however you choose to use the materials, you must consider an approach that allows for consistent weekly exposure to make sure that the students do not lose the skills they have gained over time.

Have fun implementing this program and watch all your students begin to enjoy mathematics more than ever before!

Sincerely,

Renate Ward

Guidelines for Implementation

Creating a safe learning environment

Developing a safe learning environment where students feel comfortable to take risks is critical for the success of this program. Without such an environment the students will not be willing to share their ideas, procedures, mistakes and solutions. This is a critical piece as it develops the awareness that there are many ways to solve a problem, not just one right way. It is also important for students to understand that we all make mistakes in the process of problem solving, and if they monitor what they are doing and look for alternate strategies, it is possible to arrive at a solution with perseverance.

At the beginning of the implementation be sure to develop guidelines for discussion. For example I included the following: share your ideas; explaining how you reasoning; listening, with your hands down, when someone else is talking; ask questions; and offer comments about others' ideas. This can be done as a group or you can establish them prior to the first class. It is also important for the students to understand that you are looking at the process used to solve the problem as well as their thinking regardless of whether a solution was found. When the students truly believe this, I have found that they were more willing to present their work and indicate that they were unable to find the solution. For these children who have not been successful in mathematics for most of their academic careers, knowing that their reasoning and the process are the emphasis is imperative to their moving forward toward achievement of the curriculum goals.

Development of Metacognition and Communication



Metacognitive questioning

Metacognitive questioning is the term I use to refer to those questions that encourage the students to think about their thinking. Included with this curriculum are examples of questions that require the children to explain their reasoning and choices, or how they know something. Some questions ask if they agree or disagree with something and attempt to extract an explanation, thereby developing the ability to critique others and develop their own arguments using reasoning to defend them.

Completing the graphic organizer

The graphic organizer is designed to organize the students' writing (Figure 30). It includes sentence starters adapted according to the problem solving activity that the students complete. It seems overwhelming at first glance but the students worked well with it. Instruct the students to put their answer in the center. If they were successful finding the answer using one strategy they move counter clockwise around the organizer until the arrows stop going in that direction. If the students required multiple strategies, they move clockwise around the organizer again stopping where the arrows end in that direction. They then use the sentence starters and what they filled in to complete their journals (Figure 32). Remind the students not to erase any attempted strategy as they will need to include this when discussing the problem solving process in

their journals. This needs to be reiterated often as it is a natural tendency for the students to erase mistakes.

Journal writing should be completed after each activity with enough time for the children to complete the graphic organizer and transfer the information into the bottom section of the process sheet. If time is running out make sure they at least complete the graphic organizer so they can transfer the information into their journal the next day without forgetting what they did but I highly recommend completing both during the same class.

Reflective writing

This is an important piece in the curriculum, designed to develop the ability to reflect on one's work, noting what was particularly difficult or easy. Reflecting on one thing learned gives the students an opportunity to consolidate their thinking. It also provides a time to consider how they could improve their understanding. Reflective writing does not have to occur after each activity. I suggest initially using it on activities that the students do not have too much difficulty with so they can devote their cognitive energy to learning this type of critical analysis . Once they feel comfortable doing reflective writing you may want to use it only with those activities where the students particularly struggled.

Understanding by Design

This program includes an over arching design adapted from Understanding by Design by Wiggins and McTighe. This design is used for all problem-solving activities. The goals of the activities do not change; however, your content goals will follow your Common Core Standards at your grade level. For example, if you

are working on Statistics and Probability, your content goal may be that student will display numerical data in plots on a number line, including dot plots, histograms, and box plot. These “established goals” are derived from the Common Core Standards Mathematical Practices. If you wish to learn how to create units using the “backward design” espoused by Wiggins and McTighe, I recommend reading their book. It advocates for beginning your planning with finding the essential or big ideas you want students to come away with, and then moving to designing your unit from there.

On the following two pages you will find the Understanding by Design plan (UbD) for this curriculum. A template is provided as well.

Problem-Solving Toward Mathematical Understanding

Stage 1 - Desired Outcome

Established Goals:

1. Make sense of problems and persevere
In solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique
the reasoning of others.

K-12 California's Common Core Content Standards for Mathematics, Updated 10/18/10. p.1-2

Understandings:

Students will understand that...

- * There are multiple ways of approaching a math problem (strategies).
- * Oral and written communication in the math class serves two purposes: to support their learning and to help the teacher to assess their progress (Burns, 2007. p. 40).
- * That not all problems are readily solved. Many require persistence and trying different approaches before a solution can be obtained.

Essential Questions:

- How did you figure out the answer?
- Why do you think the answer is reasonable?
- Who has another way to explain (Or who has a different answer?)
- Who can explain what _____ said in your own words?
(Burns, 2007. p.37)
- Do you agree with _____? Explain your reasoning.
- Does anyone disagree? Explain your reasoning.

Students will know . . .

- Students will know that a solution isn't always apparent when faced with a problem-solving task.
- Students will know that there is not only a single approach to solving a mathematics task.

Students will be able to

- Students will be able to solve a math problem using different strategies until they find a method that allows for progress toward a solution.
- Students will be able to represent mathematics in a variety of ways.
- Students will be able to use the language of mathematics both orally and in writing to express mathematical ideas, their problem-solving process, their reasoning, and their results effectively. (Burns, 2007)

Stage 2 - Assessment Evidence

<p>Performance Tasks:</p> <ul style="list-style-type: none"> • Student work • Reflective writing • Journal writing • Farmer Ben Problem 	<p>Other Evidence:</p> <ul style="list-style-type: none"> • Observations • Classroom discussions • Group activities
<h2>Stage 3 - Learning Plan</h2>	
<p>Learning Activities:</p> <p>Journal writing Reflective writing Problem Solving Activities: Secret Number #1 Secret Number #2 Consecutive Sums Problem Guess the Function #1-3 Create your own "Guess the Function" Pentominoes Problem Sense or Nonsense? Cats & Birds</p>	

Adapted from Wiggins, G. and McTighe, J. (2005)

Understanding by Design

Stage 1 - Desired Outcome

Established Goals:

* What relevant goals (e.g. content standards, course or program objective, learning outcomes) will this design address?

Understandings:

Students will understand that...

- * What are the big ideas?
- * What specific understandings about them are desired?
- * What misunderstandings are predictable?

Essential Questions:

* What provocative questions will foster inquiry, understanding, and transfer of learning?

Students will know . . .

* What key knowledge and skills will students acquire as a result of this unit?

* What should they eventually be able to do as a result of such knowledge and skill?

Students will be able to

Stage 2 - Assessment Evidence

Performance Tasks:

* Through what authentic performance tasks will students demonstrate the desired understandings?

* By what criteria will performances of understanding be judged?

Other Evidence:

* Through what other evidence (e.g. quizzes, tests, academic prompts, observations, homework, journals) will students demonstrate achievement of the desired results?

* How will students reflect upon on self-assess their learning?

Stage 3 - Learning Plan

Learning Activities:

What learning experiences and instruction will enable students to achieve the desired results?

How will the design

W = Help the students know Where the unit is going and What is expected? Help the teacher know Where the students are coming from (prior knowledge, interests)?

H = Hook all students, and Hold their interest?

E = Equip students, help them Experience the key ideas and Explore the issue?

R = Provide opportunities to Rethink and Revise their understandings and work?

E = Allow students to Evaluate their work and its implications?

T = be Tailored (personalized) to the different needs, interests, and abilities of learners?

O = Be Organized to maximize initial and sustained engagement as well as effective learning?

Wiggins, G. and McTighe, J. (2005)

Problem Solving Toward Mathematical Understanding: Instructional Design for Students with Learning Disabilities

The Curriculum

Prior to beginning this curriculum with your students, there are several sheets that are used throughout the activities that need to be addressed. It is important to explain each sheet to the students and provide an exemplar model.

This may be yours initially, but in future years you can collect model products.

Review how to use these sheets several times early

implementation of this curriculum. These sheets

include the following and should be part of the

materials for each activity:

Graphic organizer-support for writing

Process sheet- glued into journals for each activity.

Reflection sheet



on in the

Teacher's note:

Always start with simpler problems for each activity. Based on my experience, students who struggle with math often feel overwhelmed when faced with larger numbers. Have the students start smaller, at a place where you know they will be successful. They then develop confidence, which will affect their attitude and motivation.

First session:







 Direct teaching of problem solving process.

Display and discuss the problem solving process emphasizing the recursive process. Paste a copy on the first page of students' journals.

 Direct teaching of problem solving strategies.

Display and discuss the problem solving strategies. Model the strategies with simple word problems. Write these strategies on chart paper and display where students can access them easily.

Let's Think About Thinking

Identify the Problem	Plan the Solution	Solve the Problem	Check the Solution
<p>a. Read the word problem aloud to myself twice.</p> <p>b. Do I understand the question?</p> <p>c. I understand or I don't understand.</p> <p>d. If I don't understand I talk it over with my partner or someone else who does understand.</p>	<p> a. Do I know what to do and how to do it?</p> <p>b. What should I do first?</p> <p>c. What do the key words tell me to do?</p> <p>d. This is going to be easy/hard for me.</p> <p>Return to identifying the problem as necessary. </p>	<p> a. I need to add, subtract, multiply or divide first because _____.</p> <p>b. There is one step. (There is more than one step.)</p> <p>c. I need to follow through with solving my problem.</p> <p>d. I can do this. (I can't do this.) If not, find help.</p> <p>Return to identifying the problem or planning the solution as necessary. </p>	<p> a. Did I complete everything?</p> <p>b. Does it make sense?</p> <p>c. What do I need to recheck? Do I need to return to prior steps (Identify, Plan, or Solve?)</p> <p>d. I think it's correct?</p> <p>e. It is my best try!</p> <p>Return to identifying the problem, planning the solutions, or solving the problem as necessary. </p>

Adapted from Manning and Payne, (1996). p.185
 Design adapted from Nguyen, (2008)

Problem Solving Strategies

- Look for a pattern
- Construct a table
- Make an organized list
- Represent by drawing a picture
- Guess and check
- Work backwards
- Write an equation
- Solve a simpler or similar problem



Link: <http://thesingaporemaths.com/stratf.html>

**Lets learn about some of these
Problem Solving Strategies**

Adapted from Burns, (2007)

A graphic of a spiral notebook with a blue cover and silver spiral binding, positioned vertically on the left side of the page.

Guidelines for Classroom Discussions

Share your Ideas.

Explain how you reason.

Listen, with your hand down, when someone else is talking.

Ask questions.

Offer comments about others' ideas.

Burns, (2007)

Show your work here.

Journal Writing

Description (Tell me what you did.)

Answer (Record your results)

Analysis (Tell me why it makes mathematical sense and why you feel it is accurate.)

Reflective Writing

Adapted from Burns, 1990. p.112

Name_____

Date_____

Problem Title_____

What was the most difficult part of this activity for you?

What was the easiest part of this activity for you?

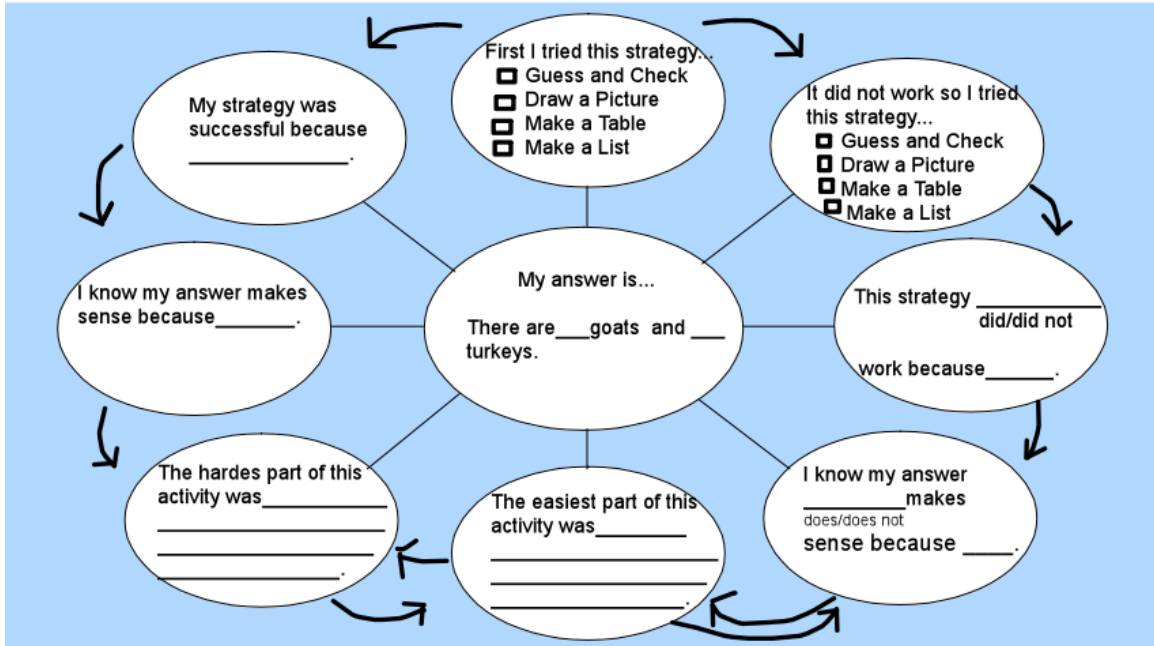
What is one thing you learned from this activity that you didn't know before?

What could you do to help you understand better?

Write about how you solved this problem.

First try works start here

First try doesn't work start here



The Problem Solving Activities

🚩 Activity One: Secret Number Puzzles (Melanese, Chung, & Forbes, 2011)

Common Core Standard: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Materials: Process worksheet
Graphic organizer
Journals
Reflective sheet

Objectives:

- Apply properties of operations as strategies to add and subtract rational numbers.
- Solve real-world and mathematical problems involving the four operations with rational numbers.

Math Goal: Students will find the “secret number” from the clues presented to them.

Procedure*:

1. Create a vocabulary bank that is displayed where the students can access it easily.
2. Tell the students they are going to solve a puzzle. Elicit a conversation about how we solve a puzzle. Write their comments on a chart or on the board.
3. Have a discussion about the term **digits** and add to vocabulary bank.
4. Display the first clue and ask students to identify a number between 408 and 450 with three digits.
5. Discuss and clarify vocabulary: Digits, clues, between, greater than, less than.
6. Explain that it is important for us to be able to justify why we chose a certain number to be the secret number. Have students write a sentence or two to explain their thinking. Have students turn to a partner and share their thinking.
7. Introduce the second clue and ask them what they think it means. Have a

discussion to clarify the meaning to the students. Ask the student to talk at their tables about what the secret number could be and why.

8. Ask the students to guess the secret number and tell how they know it is that number.
9. Provide the final clue. Give them time to solve the secret number problem.
10. Have the students give a choral response of the secret number and call on students to explain their thinking.

Extensions:

Use a number that has five places (i.e. 406.21). Discuss place value and the difference between ten and tenths; hundreds and hundredths.

Journal:

Display the sentence frames and have students show their work, the solution, and describe what they did to solve the problem on the process sheet pre-pasted into their journals. Or simply use the graphic organizer for writing.

Reflective writing

Have students complete the reflection sheet.

*For all activities it is important to always display directions and important question for the students to refer back to on the board or on chart paper

Sentence Frames

Sentence Frames

- The secret number could be ____.
- I think the secret number could be ____, because__.
- I know the secret number is ____, because_____.
- I know that ____, so the secret number could be_____.
- I know that_____, so the secret number is_____.

Melanese, Chung, & Forbes, (2011)

Secret Number

- **Clue 1:** The number has 3 digits and is between 408 and 450.
- **Clue 2:** All of the digits are even numbers greater than 2.
- **Clue 3:** the sum of the first 2 digits is equal to the last digit in the number.


Melanese, Chung, & Forbes, (2011)

Secret Number

- **Clue 1:** The number is between 400 and 410.
- **Clue 2:** The number is between 406 and 407 and has 5 places.
- _____ . _____
- Hundreds tens ones tenths hundredths
- **Clue 3:** The tenths place is bigger than the hundredths place.
- **Clue 4:** The tenths digit is twice as big as the hundredths digit.
- **Clue 5:** All of the digits are different.

- 406.21
- 406.42
- 406.63
- 406.84

Adapted from Melanese, Chung, & Forbes, (2011)



The Secret Number is...

- 406.21
- Four hundred six and twenty-one hundredths

Adapted from Melanese, Chung, & Forbes, (2011)

Activity Two: Consecutive Sums Problem (Burns, 2007)

Objectives: Reinforce the concept of consecutive numbers and sums to twenty-five.

Math Goal: Students will find all the possible ways to write the numbers one to twenty-five as the sum of consecutive numbers. They will search for patterns and describe those found.

Materials: Construction paper
Process worksheet
Graphic organizer
Journals
Reflective sheet

A row of large, blue, 3D-style numbers from 1 to 10, arranged horizontally.

Procedure:

1. Continue to build the vocabulary bank that is displayed where the students can access it easily.
2. Present and review the concept of consecutive numbers. Add to vocabulary bank. Ask for examples and non-examples.
3. Group 3-4 students to work together. Hand out one piece of paper per group to record their answers.
4. Pose a simpler problem to solve as a group.
5. Present the problem: Find all the ways to write the numbers from one to twenty-five as the sum of consecutive numbers. Some may be impossible. Is there a pattern for these?
6. Ask the students to search for other patterns and write statements describing the patterns found.
7. Display on board or chart question to consider while they are working.
8. Display the class discussion and journal writing sentence frames. Ask each group to come to the front of the class to present their findings. Have the students take turns reading the prompts and completing them in their own words.

Adaptations:

Use a smaller or larger span of numbers depending on students' abilities.

Journal:

After the presentations have the students use the sentence starters and frames or the graphic organizer to write their responses on the process sheet pre-pasted into their journals.

The Consecutive Sums Problem

What are consecutive numbers?

Give an example.

Give a non-example.

Tell me a way to write the number nine as a sum of consecutive numbers.

Burns, (2007)

Problem: Find all the ways to write the numbers from one to twenty-five as the sum of consecutive numbers. Some may be impossible. Is there a pattern for these?

Search for other patterns as well, such as how many different sums there are for different numbers.

On your recording sheet put your group label and the names of the members of your group.

Write statements that describe the patterns you found.

Burns, (2007)

Questions to Consider

Can you see a pattern to the numbers that are impossible? How could you describe that pattern in a summary statement?

What do you notice about all the number that had three possible sums?

Which numbers had only one possible sum

Burns, (2007)

Class Discussion and Journal Writing

How did your group divide up the work?

My group divided the work by _____.

Was your method a good one? Why or why not?

Our method was/was not a good one because _____.

What strategies did your group use?

Guess and check

Work backwards

Make an organized list

Look for a pattern

Others?

The strategies we used were _____. First we tried _____.

Then we tried _____.

How were you sure your group found all the possible ways to write any particular number?

We were sure we found all the possible ways for each number because _____.

Adapted from Burns, (2007)

🚩 Activity Three: Guess the Function (Melanese, Chung, & Forbes
2011)

Standard: Define, evaluate, and compare functions.

Objectives: Understand that a function is a rule that assigns to each input exactly one output.

Math Goal: Students will derive a function rule by analyzing number patterns in a table.

Materials: Guess the Function Recording Sheets
Process worksheet
Graphic Organizer
Journals
Reflective sheet

Procedure:

1. Continue to build the vocabulary bank that is displayed where the students can access it easily.
2. Introduce academic language:
 - Discuss the input/output table with students.
 - Elicit students' definition of a function and record statements on board or chart.
 - Investigate and clarify students' knowledge of vocabulary words and add to the vocabulary bank (function, rules, relationship, input and output).
3. Display on the board or a chart the definition of a function.
4. Present the task: Today we are going to look for patterns or relationship between numbers. Then we are going to guess the function.
5. Students can work independently or in pairs. Hand out the recording sheets.
6. Begin by entering data into a display of the recording sheet and ask the students to think about what is happening to the input and output numbers. Do they see a pattern or relationship?
7. Display the sentence frame and allow the students to practice the academic language by providing more examples and have the students use these sentences with a partner.

8. Display on the board or chart questions to consider while they are working.
9. Repeat this with another one-step functions then move on to two-step functions. Provide examples until you feel they are ready to try to solve a two-step function in pairs and then alone. These can be done over the course of several classes.

Provide all questions, directions, sentence frames and starters on the board/chart for students to refer to as needed.

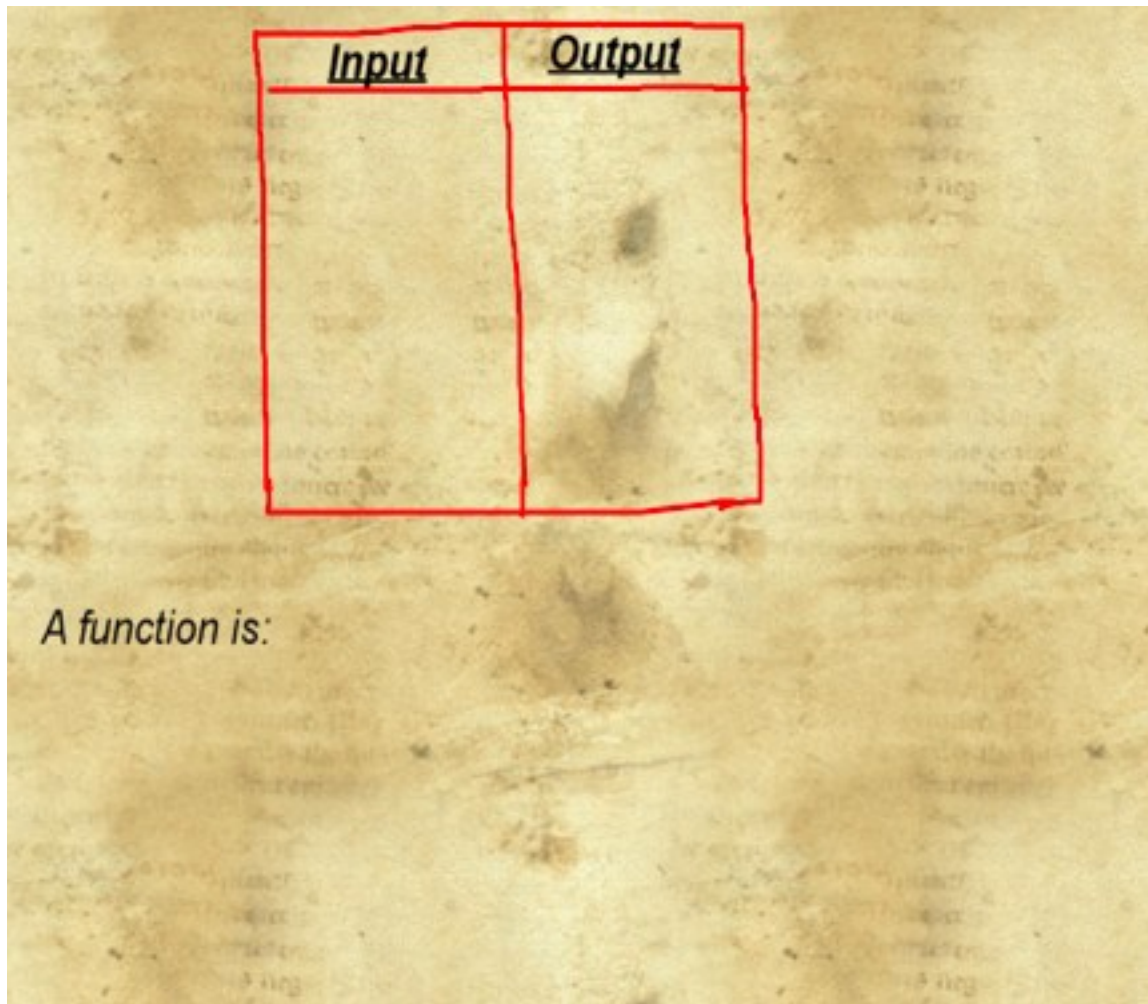
Assessment:

The final activity is an enjoyable activity for the students and allows you to assess their understandings. The students create a one or two-step function, record their data on the recording sheet and give it to a partner to guess the function. Be sure to you quickly review the students' calculations before they hand them to a partner. Mistakes in calculating input and output values make it difficult for the partner to find the function.

Have pairs come to the front of the class to share their work.

Journal:

Journals can be written after each class period and after the presentation. Have the students use the sentence starters and frames or the graphic organizer to write their responses on the process sheet pre-pasted into their journals.



Adapted from Melanese, Chung, & Forbes, (2011)

If the _____ value is _____, then
(input/output)
the _____ value is _____.
(input/output)

Guess the Function

23

22

60

Adapted from Melanese, Chung, & Forbes, (2011)

Questions:

How did you figure out the pattern?

Talk with your partner and explain what you did to get the output number (add, subtract, multiple, divide?).

See if the rule works for all of the different sets of values.

Adapted from Melanese, Chung, & Forbes, (2011)

A **function** is a **rule** that establishes a **relationship** between two quantities, called the **input** and the **output**.

Guess my Function

Input	Output
5	11
2	2
12	32
23	65
9	23
100	296
6	
n	

Adapted from Melanese, Chung, & Forbes, (2011)

Sentence Frames

You DO NOT have to use these sentence frames.
You CAN create your own sentences.

Analysis: Why does it make mathematical sense?

If the _____ value is _____, then the _____ value is _____.
input/output put a number here put a number here input/output

I know that for every input value of n , the value of the output is _____.
put your rule here

I can conclude that the function is _____ because _____.
put your function here your reason

Adapted from Melanese, Chung, & Forbes, (2011)

1. Create a one-step function (for example $2n$) or a two-step function (for example $3n-2$) like we have been working on the two days. You can not use these examples. Create your own!

2. Make an Input/Output table. Do not write your function on the table! Your partner needs to figure it out!

3. Give your Input/Output table to your partner to guess the function.

4. Show your work on Process Sheet and write about what you did, your answer and your analysis of why it makes mathematical sense with your reasoning.

You will share your work with the class.

Input	Output	Sentence Frames
		If the _____ value is _____, then the _____ value is _____.
		I know that for every input value n , the output value is _____.
		I can conclude that the function is _____ because _____.

Adapted from Melanese, Chung, & Forbes, (2011)

Activity Four: Pentominoes (Burns, 2007)

Standard: Draw, construct and describe geometrical figures and describe the relationships between them.

Objectives: Develop spatial ability and logical reasoning skills by engaging in an informal, concrete geometry experience.

Math Goal: Students will investigate different ways to arrange five squares, determine if they are congruent, and decide when they have found all the arrangements.

Materials: 1-inch grid graph paper, 2 sheets per student
 1-inch square tiles, 5 per student
 Markers
 Glue sticks
 Scissors
 Journal
 Process sheet
 Reflective sheet

Procedure:

1. Continue to build the vocabulary bank that is displayed where the students can access it easily.
2. Vocabulary:
 Ask students what is a pentomino? If students do not relate it to a domino bring this up and ask how many squares do you need for a domino?
 Place 2 square tiles under the doc-camera. Ask if they know the name for three squares (triomino) as you display the squares.
3. Pose a smaller, similar problem: If you were trying to find all the different arrangements of three squares, how many could you make? Show the students the rule for making shapes where one whole side of each square must touch at least one whole side of another square.
4. Ask a student to come up and demonstrate with the tiles the different shapes (two possible triominoes)
5. Have them try with four squares (five possible tetrominoes).
6. Ask the students how many sides are in a pentagon, then ask how many squares are in a pentomino.

7. Pose the problem: Investigate different ways to arrange five squares. Explain to the students how to find out if the shapes are congruent.
8. Pass out grid paper, tiles, markers, and scissors.
9. Display and discuss the questions to consider. The goal is to have the students think about what they are thinking and develop metacognitive awareness.
10. Discuss the results. You may use the sentence frames to guide the discussion.

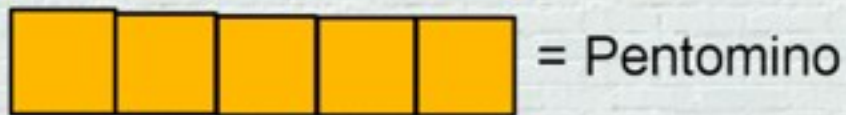
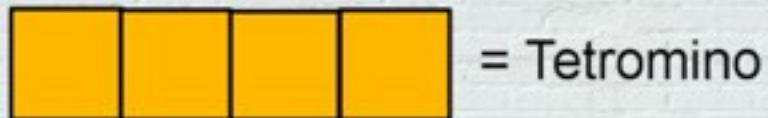
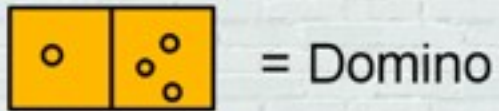
Journal writing:

Display sentence frames.

Have students draw or cut and paste their shapes into journals and write about the task using the sentence frames.

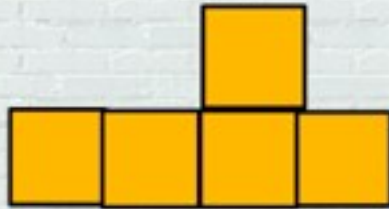
Reflective writing

What is a Pentomino?

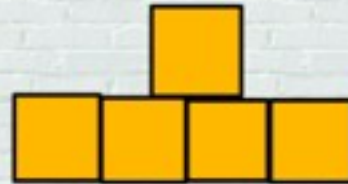


Pentominoes- Geometry like you've never seen it!

Rule for making shapes: One whole side of each square must touch at least one whole side of another square.



This is OK.



This is not OK.

Burns, (2007)

Are the shapes the same or different?

To decide, make the two shapes on paper and cut them out. Move them to see if one fits exactly on the other. You might have to flip one over or rotate one, but if they can be made to fit, they are called *congruent* and are considered to be the same.

Burns, (2007)

Suppose you were trying to find all the different arrangements of three squares. What shapes could you make?

What about with four squares?

Investigate different ways to arrange five squares. Cut each of the pentominoes you find out of the graph paper provided.

Burns, (2007)

Questions to consider:

How did you find the different shapes?

How did you know you found all of the possible arrangements?

How did you know that two shapes were congruent?

Burns, (2007)

Journal Writing

Sentence frames:

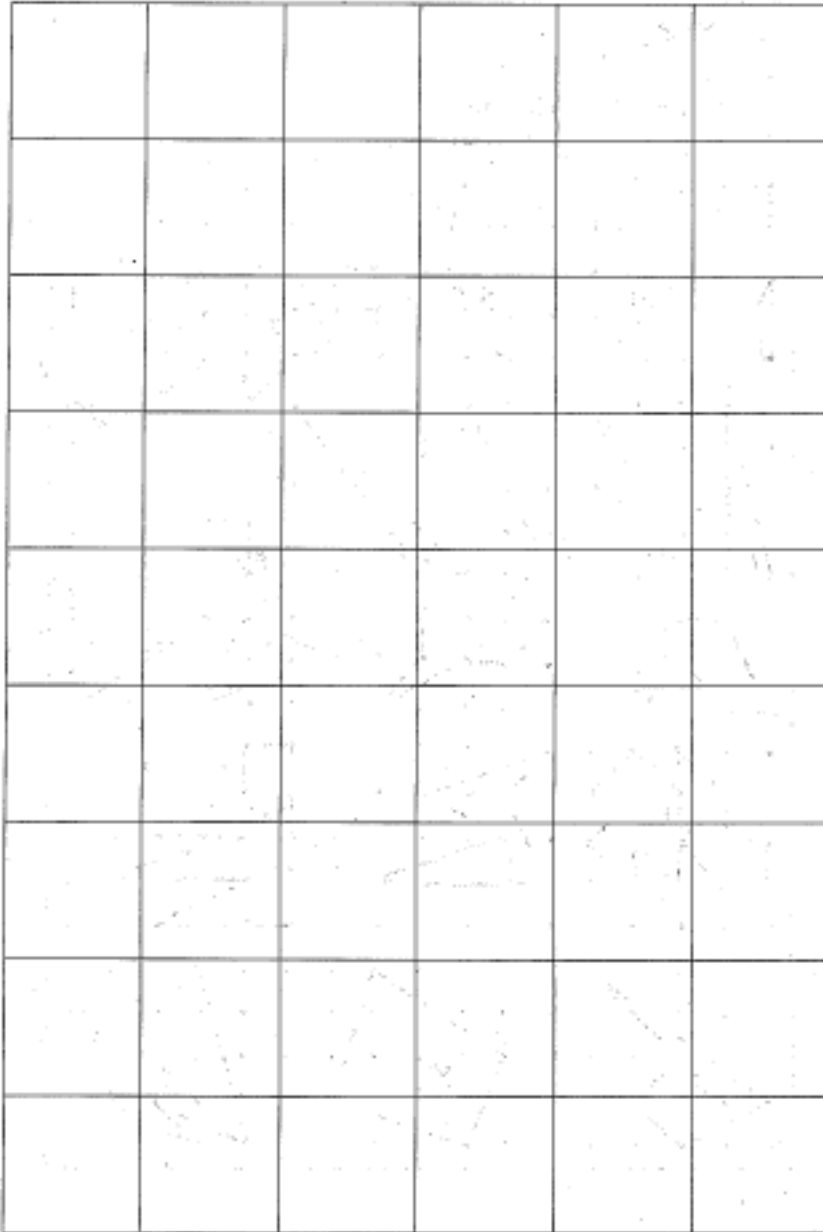
I found the different shapes by _____.

I knew I had found all be possible arrangements because _____.

I knew two shapes were congruent because _____.

Adapted from Burns, (2007)

Inch Grid Paper



From *About Teaching Mathematics: A K-8 Resource*, 3rd ed. by Marilyn Burns. © 2007 Math Solutions Publications.

Burns, (2007)

🚩 Activity Five: Sense or Nonsense? (Burns and McLaughlin, 1990)

Standard:

Objectives: Develop a better understanding of percents by using students' knowledge from their daily lives.

Math Goal: Given statements about percents, the students will decide whether they are reasonable and explain why.

Materials: Sense or Nonsense statements sheet
Reflective sheet

Procedure:

1. Continue to build the vocabulary bank that is displayed where the students can access it easily.
2. Vocabulary:
Discuss any vocabulary that you feel the students may have difficulty with from the statements.
3. Display the first statement and have a class discussion.
4. Group students.
5. Hand out statements sheet, one per group.
6. Have students discuss the ten statements and decide as a group what they think about each. Tell them to write down whether they agree or disagree and why.
7. As a group discuss the statements.

Journal writing

Pose the weather forecast question and have students write their response in their journal.

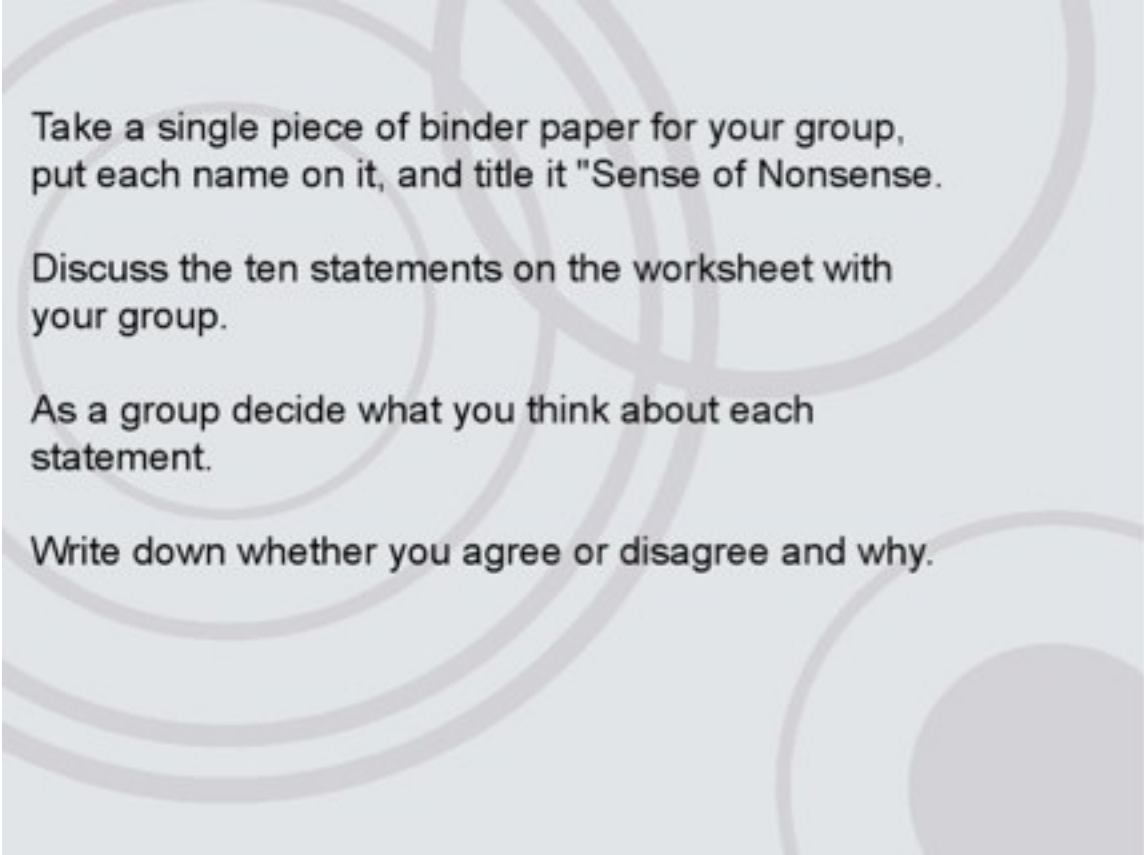
Reflective writing

Sense or Nonsense?

Decide whether the statements are reasonable.
Explain why or why not.

1. Mr. Bragg says he is right 100% of the time. Do you think Mr. Bragg is bragging? Why?

Burns and McLaughlin, (1990)

The background of the text area consists of several overlapping, light gray circles of varying sizes, creating a subtle pattern.

Take a single piece of binder paper for your group, put each name on it, and title it "Sense of Nonsense."

Discuss the ten statements on the worksheet with your group.

As a group decide what you think about each statement.

Write down whether you agree or disagree and why.

Burns and McLaughlin, (1990)

Sense or Nonsense

Decide whether these statements are reasonable.
Explain why or why not.

1. Mr. Bragg says he is right 100% of the time.
Do you think Mr. Bragg is bragging? Why?
2. The Todd family ate out last Saturday. The bill was \$36.
Would a 50% tip be too much to leave? Why?
3. Joe loaned Jeff one dollar. He said the interest would be 75% a day.
Is this a pretty good deal for Joe? Why?
4. Cindy spends 100% of her allowance on candy.
Do you think this is sensible? Why?
5. The "Never Miss" basketball team made 10% of the baskets they tried.
Do you think they should change their name?
6. Sarah missed 10 problems on the science test.
Do you think her percent is high enough for her to earn an A?
7. Rosa has a paper route. She gets to keep 25% of whatever she collects.
Do you think this is a good deal? Why?
8. The weather reporter said, "There is a 100% chance of rain for tomorrow."
Is this a reasonable prediction for this month? Why?
9. Ms. Green was complaining. "Prices have gone up at least 200% this past year," she said.
Do you think she is exaggerating? Why?

10. A store advertised "Best Sale Ever, 10% discount on all items." Is this a good sale? Why?

Burns, M. and Mclaughlin, C. (1990). p. 139. © Cuisenaire Company of America, Inc.

Journal Writing

On Halloween Mike received ten pieces of candy and Kim received ten pieces of candy too! Mike ate 60 percent of his candy and Kim ate 40 percent of her candy. Therefore, they ate 100% of the candy. Does this make sense? Why or why not?



🚩 Activity Six: Cats & Birds (Melanese, Chung, & Forbes, 2011)

Standard: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations to solve problems by reasoning about quantities.

Objectives: Students will use a variety of strategies to solve the problem. They will monitor their progress and reflect on their process

Math Goal: Students will use clues to solve a mathematical problem using a variety of strategies, including constructing simple equations.

Materials: Math Vocabulary Review sheet, 1 per student
 A Quick Review of Some Algebraic Expressions, 1 per students
 Cats and Birds: 4 Clues, 1 set per pair of students
 Markers
 Poster paper, 1 per group
 Journal
 Process sheet
 Reflective sheet
 Graphic organizer



Procedure:

1. Continue to build the vocabulary bank that is displayed where the students can access it easily.
2. Math vocabulary review: Display the vocabulary review for the class to see on the board. Ask the students to explain their understanding of the words using words, pictures, or any way they need to express their thinking. Have students share their definitions and agree to one for each word and record on the board.
3. Algebraic expressions review: As a whole group discuss and use words or drawings to represent each expression. Be sure to clarify any misconceptions.
4. Tell the students that as a group we will be solving a puzzle about cats and

birds. Explain that they will use the vocabulary and algebraic expression that were reviewed to solve the problems. Explain any words you think may be confusing for the students.

5. Display on the board the first clue and discuss its meaning. Have students generate possible answers.
6. Explain that it is important for us to be able to justify why we chose a certain number of cats and birds. Have students write a sentence or two to explain their thinking. Have students turn to a partner and share their thinking.
7. Introduce the second clue and ask them what they think it means. Have a discussion to clarify the meaning to the students. Ask the student to talk at their tables about how many cats and birds there could be and why.
8. Ask the students to guess the number of cats and the number of birds and tell how they know that there are those many.
9. Repeat with the last two clues. Display the directions and give them time to solve the problem.
10. Have the students give a choral response to the number of cats and the number of birds. Call on students to explain their thinking.
6. Discuss the results. Display and explain the graphic organizer. Have the students fill it in then use this to write a paragraph in their journals.

Journal writing:

Display graphic organizer

Reflective writing

Reproducible 4-1A *Cats and Birds: Math Vocabulary Review*

Vocabulary Word	This is my explanation:
Times	
Divisible	
Factor	
Multiple	
Altogether	
Expression	
Solution	

From *Supporting English Language Learners in Math Class, Grades 6-8* by Kathy Melanese, Luz Chang, and Cheryl Forbes.
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Melanese, Chung, and Forbes, (2011)

Key Vocabulary: altogether, clues, divisible, expression, factor, multiple, possible, solution, times

On the recording sheet titled "Math Vocabulary Review explain your understanding of these words. You may use words, pictures or any other way to represent your thinking. Talk at your tables to come up with your ideas.

Share and record

Melanese, Chung, and Forbes, (2011)

Reproducible 4-1B *Cats and Birds: A Quick Review of Some Algebraic Expressions*
 Use your own words, draw diagrams, or give examples to explain what you know.

Expression	This is my explanation:
There are 3 times as many apples as there are oranges.	
The total number of apples and oranges share common factors.	
The total number of oranges is a multiple of 4.	
The total number of apples is divisible by 3.	

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Melanese, Chung, and Forbes, (2011)

Cats and Birds



Clue Card 1

Ms. Lang keeps cats and birds.
She has 25 heads to pet.
How many cats and birds does
she have?

Adapted from Melanese, Chung, and Forbes, (2011)

Cats and Birds



Clue Card 2

Ms. Lang keeps cats and birds.
The total number of cat paws is a
multiple of 5.
How many cats and birds does she
have?

Adapted from Melanese, Chung, and Forbes, (2011)

Cats and Birds



Clue Card 3

Ms. Lang keeps cats and birds.

The total of the number of cat paws and bird feet is divisible by 2, 4, 8, 10, 20, 40, and 80.

How many cats and birds does she have?

Adapted from Melanese, Chung, and Forbes, (2011)

Cats and Birds



Clue Card 4

Ms. Lang keeps cats and birds.
The total number of bird feet is a
multiple of 5.
How many cats and birds does she
have?

Adapted from Melanese, Chung, and Forbes, (2011)

You now have all the clues to solve the problem. Work with your partner to find out how many cats and birds Ms. Lang has.

Remember you may be able to use an algebraic expression to help you solve the problem.

Do not shout out your answer. Wait for the whole class to respond.

Fill out your graphic organizer and write a paragraph in your journals to explain how you solved the problem and why you think it is correct.

✚ Activity Seven: Assessment-Farmer Ben (The Singapore Maths Teacher, 2005)

Objective: To assess students ability to problem solve without aid from the teacher or peers. Supports remain in place, however guiding questions are not supplied.

Materials: Copy of problem
Graphic organizer
Process sheet
Journal
Reflective writing sheet



Procedure:

1. Display the problem and read it to the whole class. Use pictures to depict the animals or change them to more familiar animals if you desire.
2. Discuss what it is asking for and ask the students to visualize the problem.
3. Draw students' attention to the vocabulary bank and posted strategies.
4. Hand out a copy of the problem to each student and their journals with Process sheet already glued inside.
5. Hand out the graphic organizer for students to complete prior to journal writing.
6. Allow extra time if necessary for slower performing students to complete the task.

Adaptations:

You may want to do a similar or simpler problem before the assessment. You can do this by using smaller numbers and in a different context.

Assessment

Farmer Ben has more goats than turkeys. If all the animals have 24 heads and 78 legs altogether, how many turkeys and how many goats has Farmer Ben?



Goat



Turkey

The Singapore Maths Teacher, (2005)

Additional Materials

Questions for Problem Solving

Problem Comprehension

- What is the problem about? Can you tell me about it?
- How would you explain that?
- Would you explain that in your own words?
- What do you know about this part?
- Is that information that is not important for solving the problem?

Approach and Reasoning

- What have you tried so far?
- What steps did you take so far?
- What didn't work? How did you decide that?
- How did you organize the information?
- Do you have a system to help you organize the information?
- Is there another way to solve this problem?
- Have you solved any problems like this before?
- Are there other ways to solve this problem?

Connections

- What was your estimate or prediction? Why?
- What made you think that was what you should do?
- What kinds of mathematics were used in this problem?
- How is this like the mathematics of a real-life problem?

Solution

- Are you sure your solution is correct?
- Why? How?
- Is that the only possible answer?
- How would you check the steps you have taken for your answer?
- How did you know you were done?

Communication

- Could you explain what you know right now?
- Could you write an explanation about what you did?
- Which words were most important: Why?
- Could you explain that another way?
- What pictures do you have in your mind to help you think about the task?

Adapted from Exemplars, (2011)

Standards-Based Math Rubric
(Based on revised NCTM Standards, Adapted from Exemplars, 2011)

Level	Problem Solving	Reasoning and Proof	Communication	Representation
Novice (1)	No strategy is chosen, or a strategy is chosen that will not lead to a solution Little or no evidence of engagement in the task present	Arguments are made with no mathematical basis. No correct reasoning or justification for reasoning is present.	No awareness of audience or purpose is communicated. Or Little or no communication of an approach is evident. Or Everyday, familiar language is used to communicate ideas.	No attempt is made to construct mathematical representations.
Apprentice (2)	A partially correct strategy is chosen, or a correct strategy for only solving part of the task is chosen. Evidence of drawing on some previous knowledge is present, showing some relevant engagement in the task.	Arguments are made with some mathematical basis. Some correct reasoning or justification for reasoning is present with trial and error, or unsystematic trying of several cases.	Some awareness of audience or purpose is communicated, and may take place in the form of paraphrasing of the task. Or Some communication of an approach is evident through verbal/written accounts and explanations, use of diagrams or objects, writing, and using mathematical symbols. Or Some formal math language is used, and examples are provided to communicate ideas	An attempt is made to construct mathematical representations to record and communicate problem solving.
Practitioner (3)	A correct strategy is chosen based on mathematical situation in the task. Planning or monitoring of strategy is evident. Note: The practitioner must achieve a correct answer	Arguments are constructed with adequate mathematical basis. A systematic approach and/or justification of correct reasoning is present. This may lead to... <ul style="list-style-type: none"> • clarification of the task. • exploration of mathematical phenomenon. • Noting patterns, structures and regularities. 	A sense of audience or purpose is communicated. and/or Communication of an approach is evident through a methodical, organized, coherent sequenced and labeled response. Formal math language is used throughout the solution to share and clarify ideas.	Appropriate and accurate mathematical representations are constructed and refined to solve problems or portray solutions.
Expert (4)	An efficient strategy is chosen and progress toward a solution is evaluated. Adjustments in strategy, if necessary, are made along the way, and/or alternative	Deductive arguments are used to justify decisions and may result in formal proofs. Evidence is used to justify and support decisions made and conclusions reached. This may lead to... <ul style="list-style-type: none"> • testing and accepting or rejecting a 	A sense of audience and purpose is communicated. and/or Communication at the Practitioner level is achieved, and communication of argument is supported by mathematical properties.	Abstract or symbolic mathematical representations are constructed to analyze relationships, extend thinking, and clarify or interpret phenomenon.

	<p>strategies are considered.</p> <p>Evidence of analyzing the situation in mathematical terms, and extending prior knowledge is present.</p> <p>Note: The expert must achieve a correct answer.</p>	<p>hypothesis or conjecture.</p> <ul style="list-style-type: none">• Explanation of phenomenon.• Generalizing and extending the solution to other cases.	<p>Precise math language and symbolic notation are used to consolidate math thinking and to communicate ideas.</p>	
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